Long ranged part of the static potential

M. Naeem Anwar

Forschungszentrum Jülich

Experimental and theoretical status of and perspectives for XYZ states
GSI Darmstadt, Germany, April 12-15, 2021

April 15, 2021

Overview

$\hookrightarrow$ The static potential (long & intermediate ranged) between two B mesons have following main contributions:

1. One pion exchange, having range $\mathcal{O}\left(\frac{1}{m_\pi}\right)$
2. 2 pion exchange, driven by S-wave πB interaction - range $\mathcal{O}\left(\frac{1}{2m_\pi}\right)$
3. 2 pion exchange, driven by P-wave πB interaction - range $\mathcal{O}\left(\frac{1}{2m_\pi}\right)$

Tools: Chiral perturbation theory + Dispersion theory

Overview

$\hookrightarrow$ Subsystem interactions are very crucial here

$\hookrightarrow$ Treatment of the two pion exchange potential at $m_\pi = 340$ MeV is highly non-trivial;

1. at this pion mass the $f_0(500)$ pole is very close to the $2m_\pi$ threshold- $\pi\pi$ rescattering is very essential Hanhart, Pelaez, Rios, PRL 100(2008)152001

2. around $m_\pi = 340$ MeV, the πB S-wave interaction also develops a bound state Liu et al. PRD 87, 014508(2013)

Goal: Proper inclusion of the subsystem interactions and their interplay

One Pion Exchaneg Potential

- The LO contribution is one pion exchange - OPE
  Mark B. Wise, PRD 45(1992)2188; G. Burdman, J. F. Donoghue, PLB 280(1992)287

$$\mathcal{L}_{\rm LO} = i\mathrm{Tr}[\bar{H}_a v_\mu D^\mu_{ba} H_b] + g_\pi \mathrm{Tr}[\bar{H}_a H_b \gamma_\nu \gamma_5] u^\nu_{ba} \tag{1}$$

heavy field $H = \frac{1+\slashed{v}}{2}\left[\slashed{V} + iP\gamma_5\right]$, $U = \exp\left(\frac{\sqrt{2}i\phi}{F}\right)$

- OPE potential in the momentum space is

$$V_\pi(q) = (I_1 \cdot I_2)\,\frac{g_\pi^2}{F_\pi^2}\,\frac{(q\cdot\epsilon_2)(q\cdot\epsilon_4^*)}{q^2 - m_\pi^2} \qquad I_1 \cdot I_2 = \begin{cases} -\frac{3}{4} & \text{for } I = 0 \\ \frac{1}{4} & \text{for } I = 1 \end{cases}$$

- The S-wave position space potential is

$$V_\pi(r) = (I_1 \cdot I_2)\,\frac{g_\pi^2}{3F_\pi^2}\left(m_\pi^2\,\frac{e^{-m_\pi r}}{4\pi r} - \delta(r)\right) \tag{2}$$

OPE Comparison

- $g_\pi = 0.5$, almost m_π independent
  UKQCD Collaboration, JHEP10(1998)010; Detmold et al. PRD 85,114508(2012)
- $F_\pi = 114$ MeV at $m_\pi = 340$ MeV Bicudo et al. PRD 96, 054510 (2017)

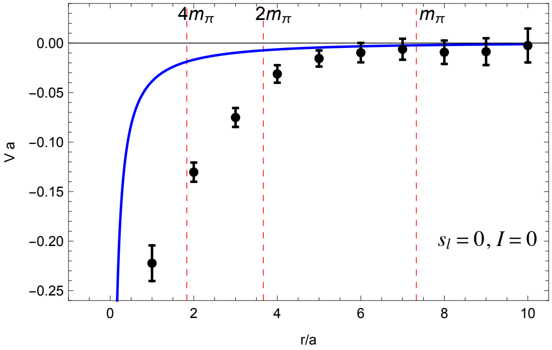


Figure: OPE potential in lattice units ($a = 0.079$ fm), data Bicudo et al. PRD96,054510(2017)

Intermediate Range Potential

- Possible two pion exchange diagrams - without $\pi\pi$ correlation

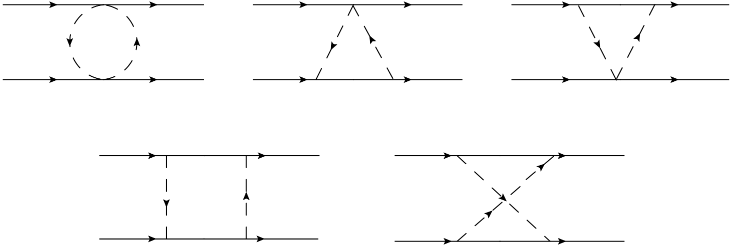
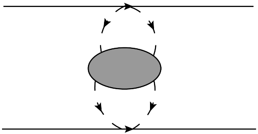


Figure: Two Pion exchange between two B mesons.

- Contributions from the correlated pions are discussed in the following

General prescription to deal with 2PE

- 2PE potential can be calculated by appropriately multiplying together the relevant $B\pi$ scattering diagrams, as NN Donoghue, PLB 643(2006)165
- For the $\pi\pi$ subsystem, unitarity requires the inclusion of the $\pi\pi$ rescattering Hanhart, Pelaez, Rios, PRL 100(2008)152001



- $\pi\pi$ amplitudes are described by a polynomial times Omnès function - Omnès solution Omnès, Nuovo Cim. 8 (1958) 316; Hanhart, PLB 715(2012)170
- Ladder-type two pion exchanges, which are already present in the iteration of the OPE, need to subtract

2 pion exchange, driven by S-wave πB interaction

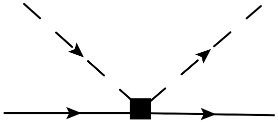


Figure: πB scattering

πB Scattering Amplitude

- The $B\pi$ scattering amplitude using $\mathcal{L}_{\rm NLO}$ from Liu PRD 87, 014508(2013)

$$V(s,t,u) = \frac{1}{F_\pi^2}\left[\frac{C_{\rm LO}}{4}(s-u) - 4C_0 h_0 + 2C_1 h_1 - 2C_{24} H_{24}(s,t,u) + 2C_{35} H_{35}(s,t,u)\right]$$

LECs $h_{0,1,24,35} \sim m_Q = M_H$. For arbitrary heavy mass $h_i^Q = h_i^c \ \frac{M_H}{M_D}$

- At threshold, the S-wave amplitude is

$$V_{\rm thr}^{H\pi,I} = \frac{m_\pi^2 M_H}{F_\pi^2}\left[\frac{C_{\rm LO}^I}{m_\pi} - \frac{2}{M_D}\left(2h_0^c + h_1^c + 2h_{24}^c - h_{35}^c\right)\right] \equiv M_H \tilde{V}^I \qquad (3)$$

- The S-wave $H\pi$ scattering length Guo, Hanhart, Meißner, EPJ A 40,(2009)171

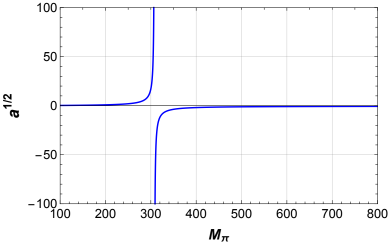
$$a_0^I = -\frac{M_H T_{\rm thr,NR}^I}{4\pi(M_H + m_\pi)}, \qquad \text{with} \qquad T_{\rm thr}^I = \frac{V_{\rm thr}^{H\pi,I}}{\left[1 - V_{\rm thr}^{H\pi,I} G_{\rm thr}^\Lambda\right]} \qquad (4)$$

Scattering Length Approximation

- Using non-rel. unitarized amplitude

$$a_0^I = \frac{\tilde{V}^I}{4\pi}\left\{1 - \frac{\tilde{V}^I}{16\pi^2}\left[-2\Lambda + M_\pi \log \frac{M_\pi^2}{(\sqrt{M_\pi^2+\Lambda^2}+\Lambda)^2}\right]\right\}^{-1} \quad (5)$$

Hard cutoff $\Lambda \sim 700$ MeV is matched (at $B\pi$ threshold) to the sub. constant of the dim. reg. scalar loop function Guo et al. NPA 773(2006)78



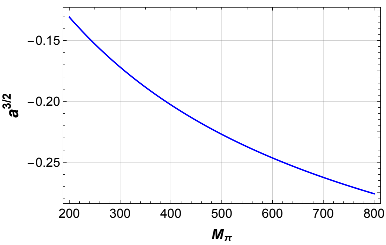


Figure: $B\pi$ scattering length in the HQL: $a^{1/2}$ (Left), $a^{3/2}$ (Right). Bound state in 1/2 channel around $m_\pi = 340$ MeV. Also for the $D\pi$ case Liu PRD 87, 014508(2013)

2PE via Dispersion Relation

$\hookrightarrow$ The scalar-isoscalar potential can be written as Donoghue, PLB 643(2006)165

$$V_\sigma(q^2) = \frac{2}{\pi} \int_{2m_\pi}^{\infty} d\mu \; \mu \frac{\mathrm{Im}\mathcal{M}(s,\mu^2)}{\mu^2 + q^2} \tag{6}$$

$\hookrightarrow$ $\mathrm{Im}\mathcal{M}(s,\mu^2)$ is the discontinuity in the $B\bar{B} \to \pi\pi$ transition amplitude, properly matched to $\pi B \to \pi B$ scattering length, following Hanhart, PLB 715(2012)170

$$\mathrm{Im}\mathcal{M}^{I=0} = -\frac{3}{2} i\pi \left(a^{(+)}\right)^2 \sqrt{1 - \frac{4m_\pi^2}{s}} \left|\Omega^0(s)\right|^2 \Theta(s - 4m_\pi^2) \tag{7}$$

$a^{(+)} = \frac{1}{3}(a^{1/2} + 2a^{3/2}) = -0.54^{+0.19}_{-0.59}$ fm by fitting h_is to the lattice data

$\hookrightarrow$ The 2PE potential in the scalar-isoscalar channel is

$$V_\sigma(r) = -3 \left(a^{(+)}\right)^2 \int_{2m_\pi}^{\infty} d\mu \sqrt{\mu^2 - 4m_\pi^2} \; \frac{e^{-\mu r}}{4\pi r} \left|\Omega^0(\mu^2)\right|^2 \tag{8}$$

Comparison with lattice data

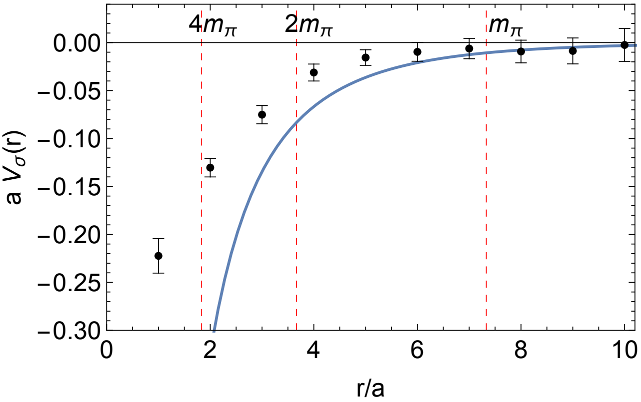
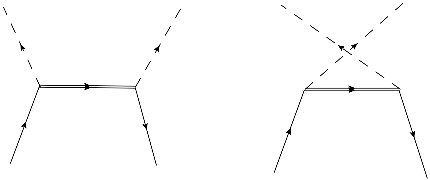


Figure: Scalar-isoscalar 2PE potential driven by S-wave πB interaction. Data is from Bicudo et al. PRD 96,054510(2017)

2 pion exchange, driven by P-wave πB interaction

Inclusion of the πB P-wave Interaction

- $\pi B \to B^* \to \pi B$ process is equivalent to $B\bar{B} \to \pi\pi$ in the crossed channel



- The B^* exchange amplitude

$$\hat{A}(s) = -\frac{\sqrt{3}g_\pi^2}{\sqrt{2}F_\pi^2}(\vec{q_1}\cdot\vec{q_2})m_{B^*}\left(\frac{1}{t-m_{B^*}^2}+\frac{1}{u-m_{B^*}^2}\right) \tag{9}$$

projected on $\pi\pi$ S-wave is

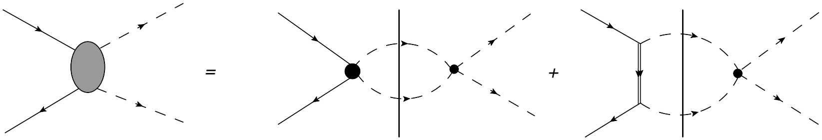
$$\hat{A}_0(s) = \frac{\sqrt{3}\pi g_\pi^2}{4\sqrt{2}F_\pi^2}\sqrt{s-4M_\pi^2} \tag{10}$$

- $\hat{A}_0(s)$ with $\pi\pi$ rescattering and πB scattering is included-framework?

The Khuri-Treiman Formalism

Khuri and Treiman PR119(1960)1115; Niecknig, Kubis, Schneider EPJC72(2012)2014

Amplitude $\Gamma(s) = A(s) + \hat{A}(s)$, where $A(\hat{A})$ has only right (left) hand cut



- $\Gamma(s)$ can be reconstructed dispersively, for $\pi\pi$ S-wave

$$\Gamma_0(s) = \hat{A}_0(s) + \Omega_0(s)\left[P_0 + \frac{s}{\pi}\int_{4M_\pi^2}^{\infty} dz \frac{\hat{A}_0(z)\sin\delta_0(z)}{z(z-s-i\epsilon)|\Omega_0(z)|}\right], \quad P_0 = 2\sqrt{6}\pi a^{(+)}$$

- Ladder-type 2PE, identified as $(\hat{A}_{0,t-\mathrm{channel}})^2$, must be subtracted from the $|\Gamma_0|^2$

$$\mathrm{Im}\mathcal{M}(s,t) = -\left[\Gamma_0(s)\sigma\Gamma_0^*(s) - |\hat{A}_0|^2\sigma\right]. \tag{11}$$

- The modified potentials will be (using Donoghue, PLB 643 (2006) 165)

$$V_\sigma(r) = -\frac{1}{32\pi^3}\int_{2m_\pi}^{\infty} d\mu\, \frac{e^{-\mu r}}{r}\sqrt{\mu^2 - 4m_\pi^2}\left[\left|\Gamma_0(\mu^2)\right|^2 - \left|\hat{A}_0\right|^2\right]. \tag{12}$$

Results comparison with data

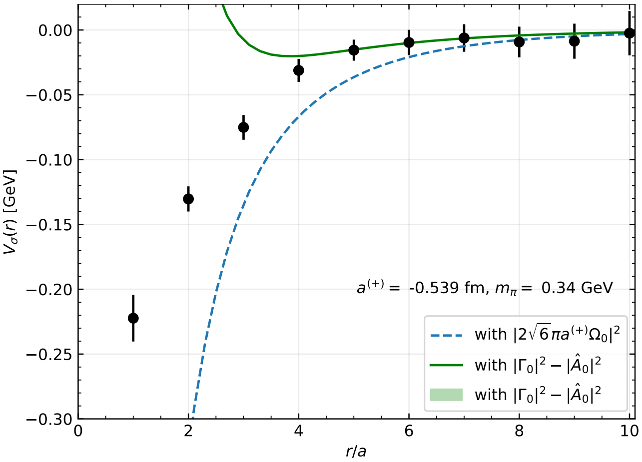


Figure: Scalar-isoscalar 2PE potential with LHCs and subtracted ladder-type 2PE. Uncertainty is from fitting h_is to πD lattice data, $a^{(+)} = -0.54^{+0.19}_{-0.59}$ fm

Conclusions

Summary:

- The long range part of the lattice potential is well-described by the OPE and scalar-isoscalar 2PE potentials
- The uncertainty band arising from $a^{(+)}$ is large for the intermediate range
- Finite volume corrections (not discussed) are analyzed and found small

Outlook:

- Due to the $B\pi$ bound state, $a^{(+)}$ gets large at $m_\pi = 340$ MeV. Better to use energy dependent $B\pi$ scattering amplitude instead of s.l. approx.
- Other intermediate range contributions are needed to be considered, e.g. the vector-isovector channel, namely the ρ exchange

Thanks for your attention!

Finite volume corrections

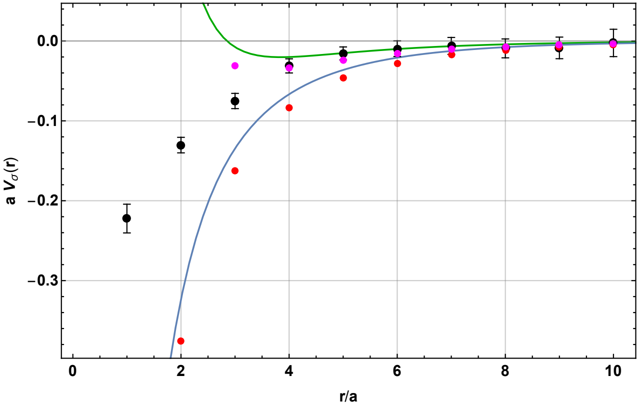


Figure: Full sigma exchange potential with finite volume corrections.

Final Results