Bottomonium resonances in the Born-Oppenheimer approximation using static potentials from lattice QCD

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"Experimental and theoretical status of and perspectives for XYZ states"

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Study heavy-heavy-light-light tetraquarks with lattice QCD using the Born Oppenheimer approximation

- heavy quarks are regarded as static color charges
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$\rightarrow$ Similar efforts for I=1 corresponding to the Z_b-states ($Z_b(10610)$, $Z_b(10650)$ by [S. Prelovsek, H. Bahtiyar, J. Petkovic, Phys. Lett. B 805 (2020) 135467, arXiv:1912.02656 [hep-lat]]

Coupled channel Schroedinger equation

Consider two channels for now:

- Quarkonium channel $\bar{Q}Q$ with orbital angular momentum $L = 0$
- Heavy-light meson-meson channel, $\bar{M}M$ with $M = \bar{Q}q$

Assumptions and symmetries

- Heavy quark spins are conserved quantities
- Only considering the lightest decay channel, two parity negative mesons which corresponds to $S_q^{PC} = 1^{--}$

One can derive a 2×2 Schroedinger-equation

$$\left(-\frac{1}{2}\begin{pmatrix} 1/\mu_Q & 0 \\ 0 & 1/\mu_M \end{pmatrix}\partial_r^2 + \frac{1}{2r^2}\begin{pmatrix} 0 & 0 \\ 0 & 2/\mu_M \end{pmatrix} + \begin{pmatrix} V_{\bar{Q}Q}(r) & V_{\mathrm{mix}}(r) \\ V_{\mathrm{mix}}(r) & V_{\bar{M}M,\parallel}(r) \end{pmatrix} + 2m_M - E\right)\begin{pmatrix} u(r) \\ \chi_{\bar{M}M}(r) \end{pmatrix} = -\begin{pmatrix} V_{\mathrm{mix}}(r) \\ V_{\bar{M}M,\parallel}(r) \end{pmatrix} krj_1(kr) \quad (1)$$

to be solved numerically with boundary conditions

$$u(r) = 0 \quad \text{and} \quad \chi_{\bar{M}M} = i t_{\bar{M}M}\, kr\, h_L^{(1)}(kr) \quad \text{for} \quad r \to \infty. \qquad (2)$$

$V_{\bar{Q}Q}(r)$, V_{mix}, $V_{\bar{M}M,\parallel}$ and $V_{\bar{M}M,\perp}$ can be related to lattice results for static potentials from QCD.

Static potentials from lattice QCD

Lattice computation of string breaking with optimized operators:

[G. S. Bali, H. Neff, T. Duessel, T. Lippert, and K. Schilling (SESAM), Phys. Rev. D 71, 114513 (2005), arXiv:hep-lat/0505012 [hep-lat]], [J. Bulava, B. Hörz, F. Knechtli, V. Koch, G. Moir, C. Morningstar, and M. Peardon, Phys. Lett. B 793,493 (2019), arXiv:1902.04006 [hep-lat]]

Treat heavy quarks as static quarks with frozen positions at $\mathbf{0}$ and $\mathbf{r}$.

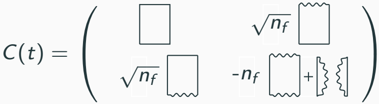
$$C(t) = \begin{pmatrix} \langle \mathcal{O}_{Q\bar{Q}} | \mathcal{O}_{Q\bar{Q}} \rangle & \langle \mathcal{O}_{Q\bar{Q}} | \mathcal{O}_{M\bar{M}} \rangle \\ \langle \mathcal{O}_{M\bar{M}} | \mathcal{O}_{Q\bar{Q}} \rangle & \langle \mathcal{O}_{M\bar{M}} | \mathcal{O}_{M\bar{M}} \rangle \end{pmatrix} \tag{3}$$

$$\mathcal{O}_{Q\bar{Q}} = (\Gamma_Q)_{AB} \qquad \left(\bar{Q}_A(\mathbf{0})\ U(\mathbf{0};\mathbf{r})\ Q_B(\mathbf{r})\right) \tag{4}$$

$$\mathcal{O}_{M\bar{M}} = (\Gamma_Q)_{AB}(\Gamma_q)_{CD} \qquad \left(\bar{Q}_A(\mathbf{0})\ u_D(\mathbf{0})\ \bar{u}_C(\mathbf{r})\ Q_B(\mathbf{r}) + (u \to d)\right) \tag{5}$$

$$\langle \mathcal{O}_{Q\bar{Q}} | \mathcal{O}_{Q\bar{Q}} \rangle_U \propto \left\langle \mathrm{tr}\left(V_t^\dagger(\mathbf{r},\mathbf{0}) U_\mathbf{r}(t,0) V_0(\mathbf{r},\mathbf{0}) U_\mathbf{0}^\dagger(t,0) \right) \right\rangle_U \tag{6}$$

$$\langle \mathcal{O}_{Q\bar{Q}} | \mathcal{O}_{M\bar{M}} \rangle_U \propto \left\langle \mathrm{tr}\left(\Gamma_Q M^{-1}_{(\mathbf{0},t);(\mathbf{r},t)} U_\mathbf{r}(t,0) V_0(\mathbf{r},\mathbf{0}) U_\mathbf{0}^\dagger(t,0) \right) \right\rangle_U \tag{7}$$



— gauge transporter

〰 light quark propagators

n_f number of degenerate flavours

Relating V(r) to static potentials from lattice QCD

From C(t) the potentials can be extracted in the limit of large Euclidean time separations:

$$[C(t)]_{ij} \propto \sum_k a_k(r) \mathrm{e}^{-V_k(r)t} \qquad \text{for} \quad t \to \infty \tag{8}$$

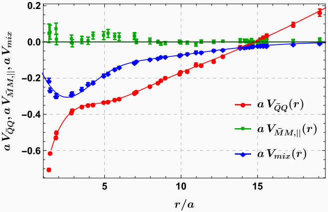
One can derive a relation between these $V_k(r)$ and $V_{\bar{Q}Q}(r)$, $V_{mix}(r)$ and $V_{\bar{M}M}(r)$.

$$V_{\bar{Q}Q}(r) = \cos^2(\theta(r)) V_0^{\Sigma_g^+}(r) + \sin^2(\theta(r)) V_1^{\Sigma_g^+}(r)$$

$$V_{\bar{M}M,\parallel}(r) = \sin^2(\theta(r)) V_0^{\Sigma_g^+}(r) + \cos^2(\theta(r)) V_1^{\Sigma_g^+}(r)$$

$$V_{\mathrm{mix}}(r) = \cos(\theta(r)) \sin(\theta(r)) \left(V_0^{\Sigma_g^+}(r) + V_1^{\Sigma_g^+}(r) \right)$$

$$V_{\bar{M}M,\perp}(r) = V^{\Pi_g^+}(r) = 0$$



where $V_0^{\Sigma_g^+}(r)$ denotes the ground state potential and $V_1^{\Sigma_g^+}(r)$ its first excitation. We use existing results from

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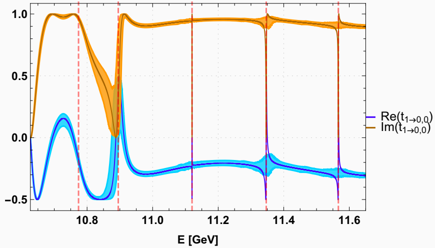
Scattering amplitude and scattering phase

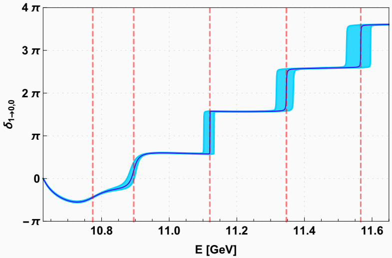
Solved SE using two independent methods:

- Discretization of spacetime rewriting the SE as a system of linear equations $M(E)\mathbf{x} = \mathbf{b}$, solved by Matrix inversion
- 4th order Runge-Kutta algorithm

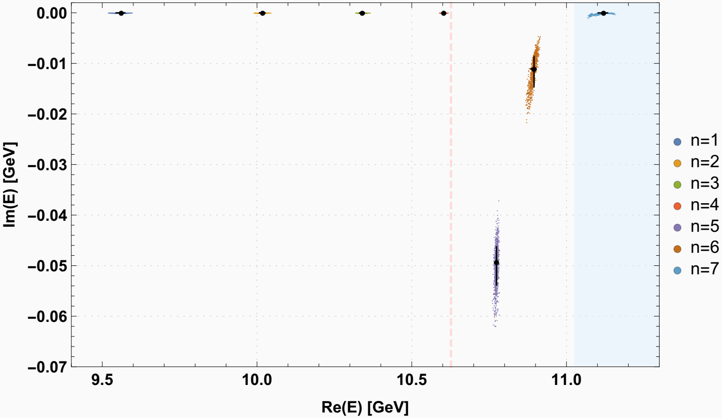
Propagating the errors of the lattice data by resampling and computing the 16th and 84th percentile.

$$\text{scattering phase:} \qquad e^{2i\delta_{\bar{M}M}} = 1 + 2it_{\bar{M}M} \tag{9}$$





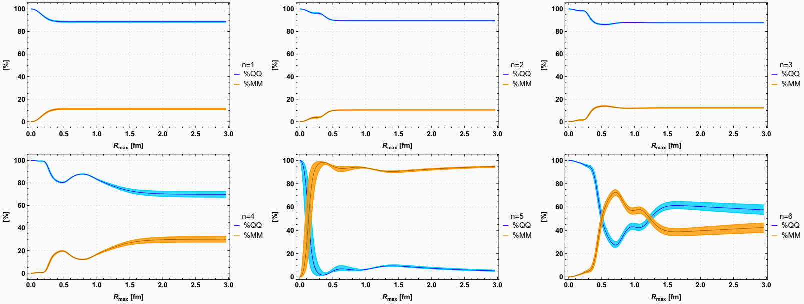
Pole positions in the complex plane



- Analytic continuation of our scattering problem to the complex plane
- Poles found using a Newton-Raphson shooting algorithm.
- Pole positions are related to masses and decay width via

$$m = \mathrm{Re}(E) \qquad \text{and} \qquad \Gamma = -2\,\mathrm{Im}(E)$$

Quarkonium and meson-meson content



Investigation whether bound states and resonances are conventional $\bar{Q}Q$ quarkonia or there is a sizable $\bar{Q}Q\bar{q}q$ component.

$$\%\bar{Q}Q = \frac{Q}{Q+M} \quad , \quad \%\bar{M}M = \frac{M}{Q+M} \tag{10}$$

with

$$Q = \int_0^{R_{\mathrm{max}}} dr \left|u(r)\right|^2 \quad , \quad M = \int_0^{R_{\mathrm{max}}} dr \left|\chi_{\bar{M}M}(r)\right|^2. \tag{11}$$

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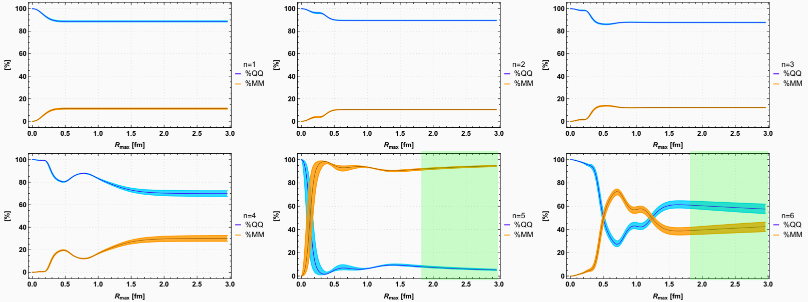
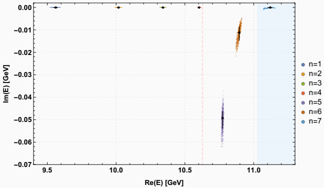


bound states:

- independent of $R_{\max}$

resonances:

- at the real part of the poleposition $\mathrm{Re}(E)$
- M linearly rising for large $R_{\max}$
- use $1.8\mathrm{fm} \leq R_{\max} \leq 3.0\mathrm{fm}$ as estimate



$$Q = \int_0^{R_{\max}} dr \left| u(r) \right|^2 \quad , \quad M = \int_0^{R_{\max}} dr \left| \chi_{\bar{M}M}(r) \right|^2 .$$

Extension to the three coupled channel case

We extend the Schroedinger-equation by an additional $\bar{B}_s^{(*)} B_s^{(*)}$-channel using the same string breaking potentials. We expect this to be reasonable as the light quark mass used in the lattice data is between the physical u/d quark mass and the physical s quark mass.

$$\left(-\frac{1}{2}\begin{pmatrix} 1/\mu_Q & 0 & 0 \\ 0 & 1/\mu_M & 0 \\ 0 & 0 & 1/\mu_{M_s} \end{pmatrix}\partial_r^2 + \frac{1}{2r^2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 2/\mu_M & 0 \\ 0 & 0 & 2/\mu_{M_s} \end{pmatrix} + \right.$$
$$\left. + \begin{pmatrix} V_{\bar{Q}Q}(r) & V_{\text{mix}}(r) & V_{\text{mix}}(r)/\sqrt{2} \\ V_{\text{mix}}(r) & 0 & 0 \\ V_{\text{mix}}(r)/\sqrt{2} & 0 & 0 \end{pmatrix} + \begin{pmatrix} E_{\text{threshold}} & 0 & 0 \\ 0 & 2m_M & 0 \\ 0 & 0 & 2m_{M_s} \end{pmatrix} - E\right) \times$$
$$\times \begin{pmatrix} u(r) \\ \chi_{\bar{M}M}(r) \\ \chi_{\bar{M}_s M_s}(r) \end{pmatrix} = -\begin{pmatrix} V_{\text{mix}}(r) \\ 0 \\ 0 \end{pmatrix}\left(\alpha k r j_1(kr) + \alpha_s k_s r j_1(k_s r)/\sqrt{2}\right). \tag{12}$$

- $V_{\text{mix}}(r)$ needs an additional factor of $1/\sqrt{2}$
- all meson-meson interactions are expected to vanish
- $E_{\text{threshold}}$ is the meson-meson threshold of the lattice data
- there are two types of incoming waves (right hand side of the equation)

Scattering matrix in the three coupled channel case

$$\left(-\frac{1}{2}\begin{pmatrix} 1/\mu_Q & 0 & 0 \\ 0 & 1/\mu_M & 0 \\ 0 & 0 & 1/\mu_{M_s}\end{pmatrix}\partial_r^2 + \frac{1}{2r^2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 2/\mu_M & 0 \\ 0 & 0 & 2/\mu_{M_s}\end{pmatrix} + \right.$$
$$+\begin{pmatrix} V_{\bar{Q}Q}(r) & V_{\rm mix}(r) & V_{\rm mix}(r)/\sqrt{2} \\ V_{\rm mix}(r) & 0 & 0 \\ V_{\rm mix}(r)/\sqrt{2} & 0 & 0 \end{pmatrix} + \left.\begin{pmatrix} E_{\rm threshold} & 0 & 0 \\ 0 & 2m_M & 0 \\ 0 & 0 & 2m_{M_s}\end{pmatrix} - E\right)\times$$
$$\times\begin{pmatrix} u(r) \\ \chi_{\bar{M}M}(r) \\ \chi_{\bar{M}_s M_s}(r)\end{pmatrix} = -\begin{pmatrix} V_{\rm mix}(r) \\ 0 \\ 0 \end{pmatrix}\left(\alpha k r j_1(kr) + \alpha_s k_s r j_1(k_s r)/\sqrt{2}\right). \tag{13}$$

Incident $\bar{B}^{(*)}B^{(*)}$ wave (i.e. $(\alpha, \alpha_s) = (1, 0)$):

$$\chi_{\bar{M}M}(r) = i t_{\bar{M}M;\bar{M}M} k r h_1^{(1)}(kr) \; ,$$
$$\chi_{\bar{M}_s M_s}(r) = i t_{\bar{M}M;\bar{M}_s M_s} k_s r h_1^{(1)}(k_s r)$$
$$\text{for } r \to \infty. \tag{14}$$

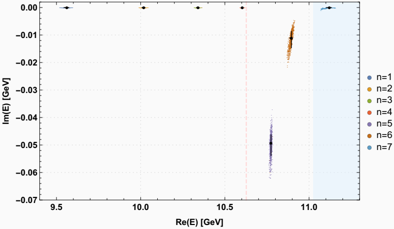
Incident $\bar{B}_s^{(*)}B_s^{(*)}$ wave (i.e. $(\alpha, \alpha_s) = (0, 1)$):

$$\chi_{\bar{M}M}(r) = i t_{\bar{M}_s M_s;\bar{M}M} k r h_1^{(1)}(kr) \; ,$$
$$\chi_{\bar{M}_s M_s}(r) = i t_{\bar{M}_s M_s;\bar{M}_s M_s} k_s r h_1^{(1)}(k_s r)$$
$$\text{for } r \to \infty. \tag{15}$$

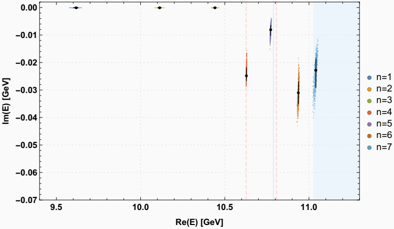
This defines the 2×2 matrices S and T,

$$\mathrm{S} = 1 + 2i\mathrm{T} \quad , \quad \mathrm{T} = \begin{pmatrix} t_{\bar{M}M;\bar{M}M} & t_{\bar{M}_s M_s;\bar{M}M} \\ t_{\bar{M}M;\bar{M}_s M_s} & t_{\bar{M}_s M_s;\bar{M}_s M_s}\end{pmatrix}. \tag{16}$$

pole positions in the three coupled channel case



two channels



three channels

Results

	from poles of $t_{\bar{M}M}$, two channels				from poles of T, three channels					from experiment		
n	m [GeV]	Γ [MeV]	$\%\bar{Q}Q$ [%]	$\%\bar{M}M$ [%]	m [GeV]	Γ [MeV]	$\%\bar{Q}Q$ [%]	$\%\bar{M}M$ [%]	$\%\bar{M}_s M_s$ [%]	name	m [GeV]	Γ [MeV]
1										$\eta_b(1S)$	9.399^{+2}_{-2}	10^{+5}_{-4}
1	9.562^{+11}_{-17}	0	89^{+1}_{-0}	11^{+0}_{-1}	9.618^{+10}_{-15}	0	84^{+1}_{-1}	12^{+0}_{-0}	5^{+0}_{-0}	$\Upsilon(1S)$	9.460^{+0}_{-0}	≈ 0
2	10.018^{+8}_{-10}	0	90^{+0}_{-0}	10^{+0}_{-0}	10.114^{+7}_{-11}	0	84^{+0}_{-0}	12^{+0}_{-0}	4^{+0}_{-0}	$\Upsilon(2S)$	10.023^{+0}_{-0}	≈ 0
3	10.340^{+7}_{-9}	0	88^{+0}_{-0}	12^{+0}_{-0}	10.442^{+7}_{-9}	0	79^{+0}_{-0}	17^{+0}_{-0}	4^{+0}_{-0}	$\Upsilon(3S)$	10.355^{+0}_{-0}	≈ 0
4	10.603^{+5}_{-6}	0	70^{+3}_{-2}	30^{+2}_{-3}	10.629^{+1}_{-1}	$49.3^{+5.4}_{-3.9}$	$67^{+0}_{-5}{}^{+1}_{-1}$	29^{+5+1}_{-0-1}	4^{+0+0}_{-0-0}	$\Upsilon(4S)$	10.579^{+1}_{-1}	21^{+3}_{-3}
5	10.774^{+4}_{-4}	$98.5^{+9.2}_{-5.9}$	6^{+1+2}_{-0-1}	94^{+0+1}_{-1-2}	10.773^{+1}_{-2}	$15.9^{+2.9}_{-4.4}$	$24^{+3}_{-3}{}^{+1}_{-1}$	60^{+4+1}_{-4-2}	16^{+1+1}_{-2-1}	$\Upsilon(10753)$	10.753^{+7}_{-7}	36^{+22}_{-14}
6	10.895^{+7}_{-10}	$22.2^{+7.1}_{-4.9}$	59^{+4+2}_{-4-2}	41^{+4+2}_{-4-2}	10.938^{+2}_{-2}	$61.8^{+7.6}_{-8.0}$	35^{+11+4}_{-7-3}	40^{+3+3}_{-6-3}	25^{+5+0}_{-6-0}	$\Upsilon(10860)$	10.885^{+3}_{-2}	37^{+4}_{-4}

Still large systematic errors $\mathcal{O}(50\mathrm{MeV})$ mainly due to

- neglecting heavy-spins and the $B - B^*$ mass splitting
- lack of more suitable lattice data

Results - n = 1,2,3

	from poles of $t_{\bar{M}M}$, two channels				from poles of T, three channels					from experiment		
n	m [GeV]	Γ [MeV]	$\%\bar{Q}Q$ [%]	$\%\bar{M}M$ [%]	m [GeV]	Γ [MeV]	$\%\bar{Q}Q$ [%]	$\%\bar{M}M$ [%]	$\%\bar{M}_sM_s$ [%]	name	m [GeV]	Γ [MeV]
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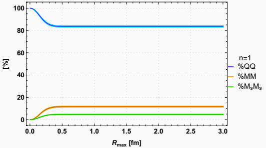

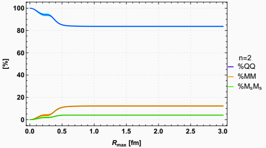

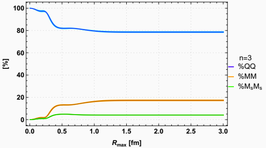

- mostly $\bar{Q}Q$ - states with $\%\bar{Q}Q$ of around 80%

Results - n = 4

	from poles of $t_{\bar{M}M}$, two channels				from poles of T, three channels					from experiment		
n	m [GeV]	Γ [MeV]	$\%\bar{Q}Q$ [%]	$\%\bar{M}M$ [%]	m [GeV]	Γ [MeV]	$\%\bar{Q}Q$ [%]	$\%\bar{M}M$ [%]	$\%\bar{M}_sM_s$ [%]	name	m [GeV]	Γ [MeV]
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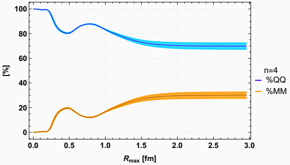
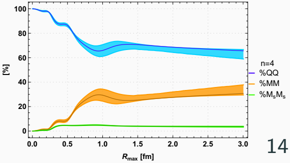
5	10.774^{+4}_{-4}	$98.5^{+9.2}_{-5.9}$	6^{+1+2}_{-0-1}	94^{+0+1}_{-1-2}	10.773^{+1}_{-2}	$15.9^{+2.9}_{-4.4}$	$24^{+3\ +1}_{-3\ -1}$	60^{+4+1}_{-4-2}	16^{+1+1}_{-2-1}	$\Upsilon(10753)$	10.753^{+7}_{-7}	36^{+22}_{-14}
6	10.895^{+7}_{-10}	$22.2^{+7.1}_{-4.9}$	59^{+4+2}_{-4-2}	41^{+4+2}_{-4-2}	10.938^{+2}_{-2}	$61.8^{+7.6}_{-8.0}$	$35^{+11+4}_{-7\ -3}$	40^{+3+3}_{-6-3}	25^{+5+0}_{-6-0}	$\Upsilon(10860)$	10.885^{+3}_{-2}	37^{+4}_{-4}



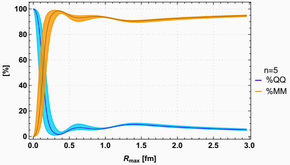
- bound state in the two-channel case, resonance for three channels
- $\%\bar{Q}Q$ remains similar at $\approx 70\%$
- shows a sizeable meson-meson component of $\approx 30\%$

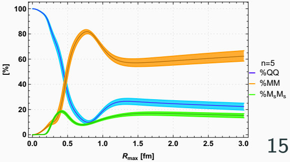


Results - n = 5

	from poles of $t_{\bar{M}M}$, two channels				from poles of T, three channels					from experiment		
n	m [GeV]	Γ [MeV]	$\%\bar{Q}Q$ [%]	$\%\bar{M}M$ [%]	m [GeV]	Γ [MeV]	$\%\bar{Q}Q$ [%]	$\%\bar{M}M$ [%]	$\%\bar{M}_sM_s$ [%]	name	m [GeV]	Γ [MeV]
1										$\eta_b(1S)$	9.399^{+2}_{-2}	10^{+5}_{-4}
1	9.562^{+11}_{-17}	0	89^{+1}_{-0}	11^{+0}_{-1}	9.618^{+10}_{-15}	0	84^{+1}_{-1}	12^{+0}_{-0}	5^{+0}_{-0}	$\Upsilon(1S)$	9.460^{+0}_{-0}	≈ 0
2	10.018^{+8}_{-10}	0	90^{+0}_{-0}	10^{+0}_{-0}	10.114^{+7}_{-11}	0	84^{+0}_{-0}	12^{+0}_{-0}	4^{+0}_{-0}	$\Upsilon(2S)$	10.023^{+0}_{-0}	≈ 0
3	10.340^{+7}_{-9}	0	88^{+0}_{-0}	12^{+0}_{-0}	10.442^{+7}_{-9}	0	79^{+0}_{-0}	17^{+0}_{-0}	4^{+0}_{-0}	$\Upsilon(3S)$	10.355^{+0}_{-0}	≈ 0
4	10.603^{+5}_{-6}	0	70^{+3}_{-2}	30^{+2}_{-3}	10.629^{+1}_{-1}	$49.3^{+5.4}_{-3.9}$	$67^{+0\ +1}_{-5\ -1}$	29^{+5+1}_{-0-1}	4^{+0+0}_{-0-0}	$\Upsilon(4S)$	10.579^{+1}_{-1}	21^{+3}_{-3}
5	10.774^{+4}_{-4}	$98.5^{+9.2}_{-5.9}$	6^{+1+2}_{-0-1}	94^{+0+1}_{-1-2}	10.773^{+1}_{-2}	$15.9^{+2.9}_{-4.4}$	$24^{+3\ +1}_{-3\ -1}$	60^{+4+1}_{-4-2}	16^{+1+1}_{-2-1}	$\Upsilon(10753)$	10.753^{+7}_{-7}	36^{+22}_{-14}
6	10.895^{+7}_{-10}	$22.2^{+7.1}_{-4.9}$	59^{+4+2}_{-4-2}	41^{+4+2}_{-4-2}	10.938^{+2}_{-2}	$61.8^{+7.6}_{-8.0}$	$35^{+11+4}_{-7\ -3}$	40^{+3+3}_{-6-3}	25^{+5+0}_{-6-0}	$\Upsilon(10860)$	10.885^{+3}_{-2}	37^{+4}_{-4}

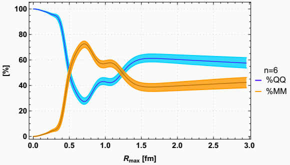
- more realistic decay width in the three-channel-case
- quarkonium component increases
- Additional $\%M_sM_s$ of around 16%
- does not exist in a pure quarkonium spectrum.
  It is dynamically generated by coupling to a meson-meson coupling.



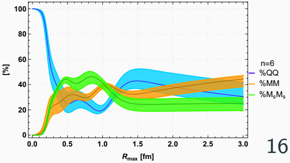


Results - n = 6

	from poles of $t_{\bar{M}M}$, two channels				from poles of T, three channels					from experiment		
n	m [GeV]	Γ [MeV]	$\%\bar{Q}Q$ [%]	$\%\bar{M}M$ [%]	m [GeV]	Γ [MeV]	$\%\bar{Q}Q$ [%]	$\%\bar{M}M$ [%]	$\%\bar{M}_sM_s$ [%]	name	m [GeV]	Γ [MeV]
1										$\eta_b(1S)$	9.399^{+2}_{-2}	10^{+5}_{-4}
1	9.562^{+11}_{-17}	0	89^{+1}_{-0}	11^{+0}_{-1}	9.618^{+10}_{-15}	0	84^{+1}_{-1}	12^{+0}_{-0}	5^{+0}_{-0}	$\Upsilon(1S)$	9.460^{+0}_{-0}	≈ 0
2	10.018^{+8}_{-10}	0	90^{+0}_{-0}	10^{+0}_{-0}	10.114^{+7}_{-11}	0	84^{+0}_{-0}	12^{+0}_{-0}	4^{+0}_{-0}	$\Upsilon(2S)$	10.023^{+0}_{-0}	≈ 0
3	10.340^{+7}_{-9}	0	88^{+0}_{-0}	12^{+0}_{-0}	10.442^{+7}_{-9}	0	79^{+0}_{-0}	17^{+0}_{-0}	4^{+0}_{-0}	$\Upsilon(3S)$	10.355^{+0}_{-0}	≈ 0
4	10.603^{+5}_{-6}	0	70^{+3}_{-2}	30^{+2}_{-3}	10.629^{+1}_{-1}	$49.3^{+5.4}_{-3.9}$	$67^{+0\ +1}_{-5\ -1}$	29^{+5+1}_{-0-1}	4^{+0+0}_{-0-0}	$\Upsilon(4S)$	10.579^{+1}_{-1}	21^{+3}_{-3}
5	10.774^{+4}_{-4}	$98.5^{+9.2}_{-5.9}$	6^{+1+2}_{-0-1}	94^{+0+1}_{-1-2}	10.773^{+1}_{-2}	$15.9^{+2.9}_{-4.4}$	$24^{+3\ +1}_{-3\ -1}$	60^{+4+1}_{-4-2}	16^{+1+1}_{-2-1}	$\Upsilon(10753)$	10.753^{+7}_{-7}	36^{+22}_{-14}
6	10.895^{+7}_{-10}	$22.2^{+7.1}_{-4.9}$	59^{+4+2}_{-4-2}	41^{+4+2}_{-4-2}	10.938^{+2}_{-2}	$61.8^{+7.6}_{-8.0}$	$35^{+11+4}_{-7\ -3}$	40^{+3+3}_{-6-3}	25^{+5+0}_{-6-0}	$\Upsilon(10860)$	10.885^{+3}_{-2}	37^{+4}_{-4}



- increased meson-meson contribution with the inclusion of the $\bar{B}_s^{(*)}B_s^{(*)}$-channel
  $\rightarrow$ not surprising since above $\bar{B}_s^{(*)}B_s^{(*)}$-threshold



Conclusion and Outlook

We

- explored the nature of the $I = 0$ bottomonium S wave bound states and resonances
- $\Upsilon(1S)$, $\Upsilon(2S)$, $\Upsilon(3S)$ have rather small meson-meson-components
- find that $\Upsilon(4S)$ is quarkonium dominated with a sizable meson-meson component
- find a S wave state close to the energy of $\Upsilon(10860)$ which is mostly a meson-meson-state
- find a pole near the energy of $\Upsilon(10753)_{Belle}$ which is dynamically generated by coupling to a meson-meson-channel and has a large meson-meson content

Outlook:

- Extend the study to P wave, D wave and F wave
  $\rightarrow$ Investigate possible D-wave nature of $\Upsilon(10860)$ and $\Upsilon(11020)$
- perform a dedicated lattice QCD computation of the static potentials
- include heavy spin effects and $B - B^*$ mass splitting