Triangle singularities and new ideas about the X(3872)

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Experimental and theoretical status of and perspectives for XYZ states

In memory of Mikhail Voloshin

12-15 April 2021

Triangle singularities (TSs):

M. Bayar, F. Aceti, FKG, E. Oset, PRD 94 (2016) 074039;

FKG, X.-H. Liu, S. Sakai, PPNP 112 (2020) 103757

Measuring the X(3872) binding energy using TS:

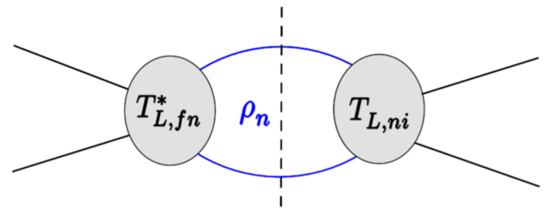
FKG, PRL122 (2019) 202002; S. Sakai, H.-J. Jing, FKG, PRD 102 (2020) 114041

X atom:

Z.-H. Zhang, FKG, arXiv:2012.08281

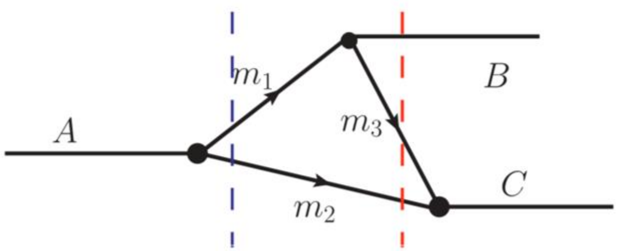
S-matrix singularities

- Dynamics: poles, corresponding to hadron resonances, genuine physical states such as the ordinary and exotic mesons, baryons; location fixed by the dynamics
- Kinematics: (normally) branch points of the S-matrix, due to on-shell intermediate particles, results of unitarity; location fixed by the involved kinematic variables: masses, energies



  - ☞ normal two-body threshold cusp
  - ☞ triangle singularity
  - ☞ . . .

  traps/tools in hadron spectroscopy



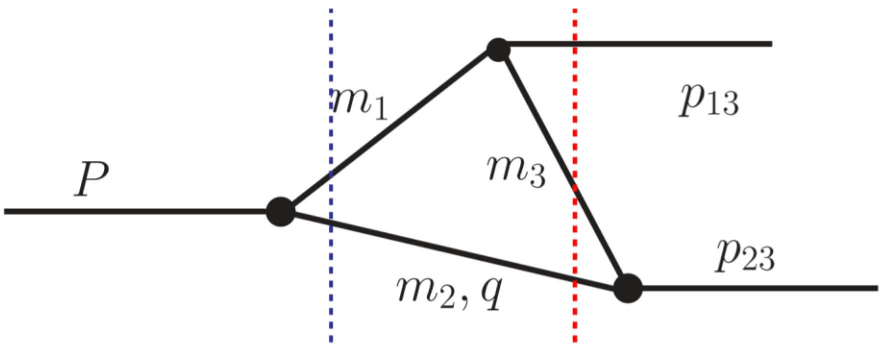
Known since long: Landau singularity

L.D.Landau (1959); J.D.Bjorken (1959); J.Mathews (1959); N.Nakanishi (1959)

Derivation of the TS equation

Bayar, Aceti, FKG, Oset, PRD94(2016)074039

TS: Leading Landau singularity for a triangle diagram



Consider the scalar three-point loop integral

$$I = i\int \frac{d^4q}{(2\pi)^4} \frac{1}{[(P-q)^2 - m_1^2 + i\epsilon]\,(q^2 - m_2^2 + i\epsilon)\,[(p_{23}-q)^2 - m_3^2 + i\epsilon]}$$

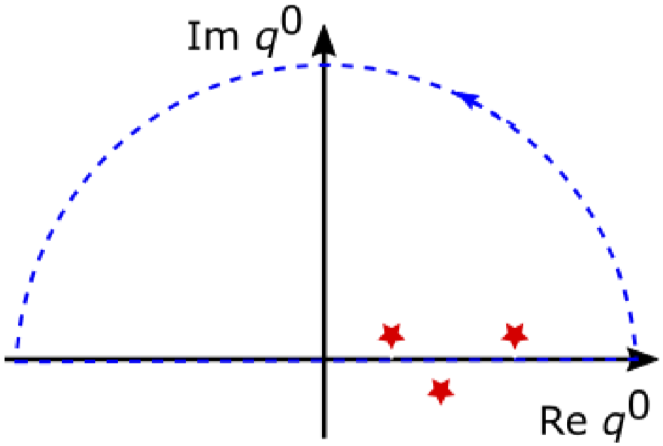
Rewriting a propagator into two poles:

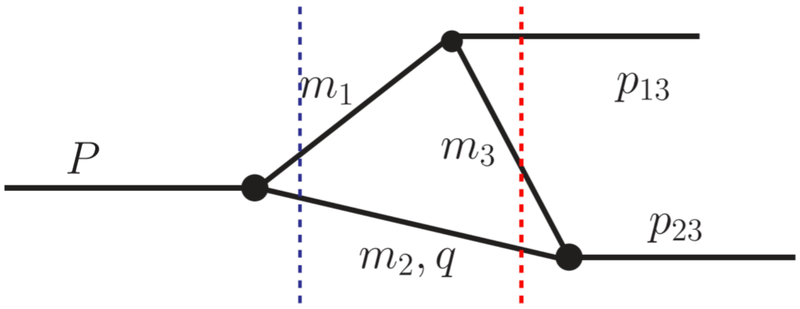
$$\frac{1}{q^2 - m_2^2 + i\epsilon} = \frac{1}{(q^0 - \omega_2 + i\epsilon)\,(q^0 + \omega_2 - i\epsilon)} \quad \text{with} \quad \omega_2 = \sqrt{m_2^2 + \vec{q}^{\,2}}$$

focus on the positive-energy poles

$$I \simeq \frac{i}{8m_1m_2m_3}\int \frac{dq^0 d^3\vec{q}}{(2\pi)^4} \frac{1}{(P^0 - q^0 - \omega_1 + i\epsilon)\,(q^0 - \omega_2 + i\epsilon)\,(p_{23}^0 - q^0 - \omega_3 + i\epsilon)}$$

Derivation of the TS equation





Contour integral over q^0 $\Rightarrow$

cut-1 cut-2

$$I \propto \int \frac{d^3\vec{q}}{(2\pi)^3} \frac{1}{[P^0 - \omega_1(q) - \omega_2(q) + i\,\epsilon][p_{23}^0 - \omega_2(q) - \omega_3(\vec{p}_{23} - \vec{q}\,) + i\,\epsilon]}$$

$$\propto \int_0^\infty dq \, \frac{q^2}{P^0 - \omega_1(q) - \omega_2(q) + i\,\epsilon} f(q)$$

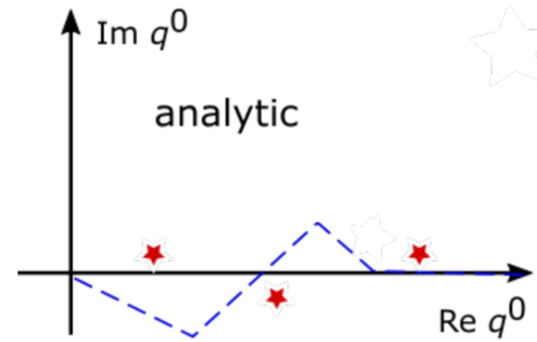
The second cut:

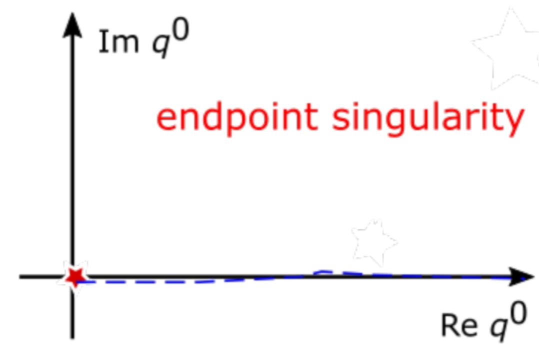
$$f(q) = \int_{-1}^{1} dz \, \frac{1}{p_{23}^0 - \omega_2(q) - \sqrt{m_3^2 + q^2 + p_{23}^2 - 2p_{23}qz} + i\,\epsilon}$$

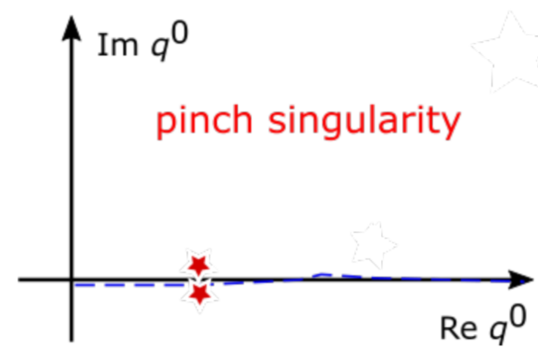
A little math

Relation between singularities of integrand and integral

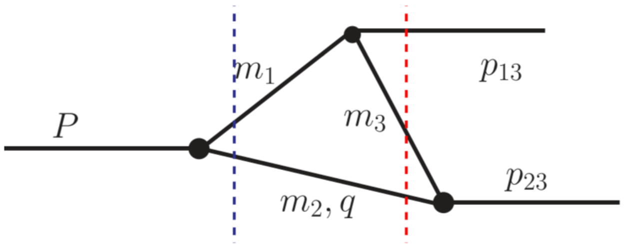
- singularity of integrand does not necessarily give a singularity of integral:
  integral contour may be deformed to avoid the singularity
- Two cases that a singularity cannot be avoided:
  - ☞ endpoint singularity
  - ☞ pinch singularity







Derivation of the TS equation



$$I \propto \int_0^\infty dq \, \frac{q^2}{P^0 - \omega_1(q) - \omega_2(q) + i\,\epsilon} f(q)$$

$$f(q) = \int_{-1}^{1} dz \, \frac{1}{A(q,z)} \equiv \int_{-1}^{1} dz \, \frac{1}{p_{23}^0 - \omega_2(q) - \sqrt{m_3^2 + q^2 + p_{23}^2 - 2p_{23}qz} + i\,\epsilon}$$

Singularities of the **integrand of I** in the rest frame of initial particle ($P^0 = M$):

- 1st cut: $M - \omega_1(l) - \omega_2(l) + i\,\epsilon = 0 \Rightarrow$
  $$q_{\text{on}\pm} \equiv \pm \left(\frac{1}{2M} \sqrt{\lambda(M^2, m_1^2, m_2^2)} + i\,\epsilon \right)$$
- 2nd cut: $A(q, \pm 1) = 0 \Rightarrow$ endpoint singularities of $f(q)$

$$z = +1: \quad q_{a+} = \gamma\,(\beta\,E_2^* + p_2^*) + i\,\epsilon\,, \qquad q_{a-} = \gamma\,(\beta\,E_2^* - p_2^*) - i\,\epsilon\,,$$

$$z = -1: \quad q_{b+} = \gamma\,(-\beta\,E_2^* + p_2^*) + i\,\epsilon\,, \quad q_{b-} = -\gamma\,(\beta\,E_2^* + p_2^*) - i\,\epsilon$$

$$\beta = |\vec{p}_{23}|/E_{23}, \qquad \gamma = 1/\sqrt{1-\beta^2} = E_{23}/m_{23}$$

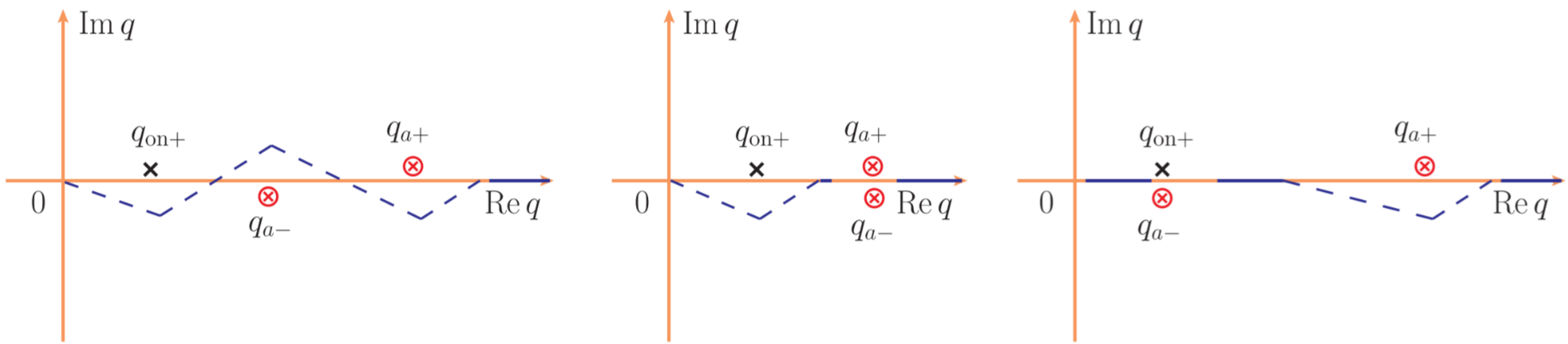
$E_2^*(p_2^*)$: energy (momentum) of particle-2 in the cmf of the (2,3) system

Derivation of the TS equation

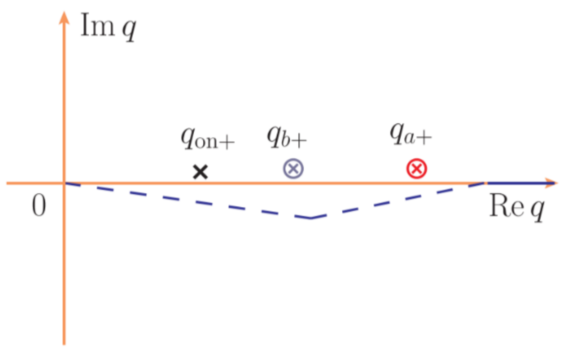
All singularities of the integrand of I:

$$q_{\text{on}+}, \qquad q_{a+} = \gamma\,(\beta\,E_2^* + p_2^*) + i\,\epsilon, \qquad q_{a-} = \gamma\,(\beta\,E_2^* - p_2^*) - i\,\epsilon,$$

$$q_{\text{on}-} < 0, \quad q_{b-} = -q_{a+} < 0 \text{ (for } \epsilon = 0), \qquad q_{b+} = -q_{a-},$$



(a) (b) (c)



2-body threshold singularity at

$m_{23} = m_2 + m_3$

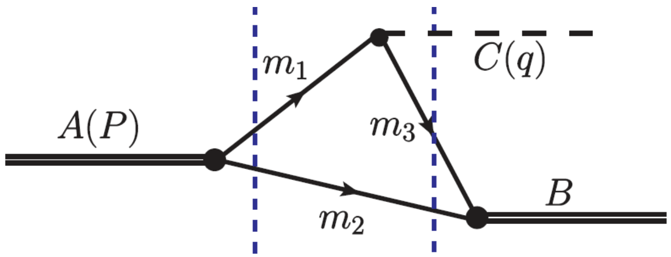
triangle singularity at

$\boxed{p_{2,\text{left}} = p_{2,\text{right}}}$

here $p_{2,\text{left}} = q_{\text{on}+}$, $p_{2,\text{right}} = q_{a-}$

Triangle singularity (TS)

Logarithmic singularity



$$\frac{1}{2m_A}\sqrt{\lambda(m_A^2, m_1^2, m_2^2)} \equiv \boxed{p_{2,\text{left}} = p_{2,\text{right}}} \equiv \gamma\,(\beta\, E_2^* - p_2^*)$$

on-shell momentum of m_2 at the left and right cuts in the A rest frame

$\beta = |\vec{p}_{23}|/E_{23},\, \gamma = 1/\sqrt{1-\beta^2}$

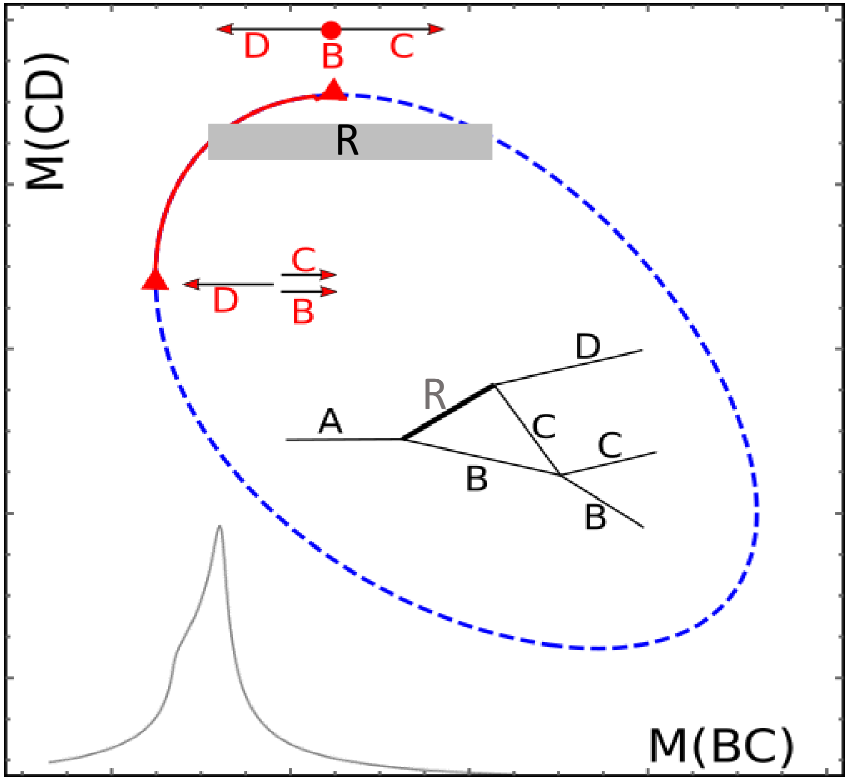
Bayar, Aceti, FKG, Oset, PRD94(2016)074039

- $p_2 > 0,\, p_3 = \gamma\,(\beta\, E_3^* + p_2^*) > 0 \Rightarrow$ m_2 and m_3 move in the same direction
- velocities in the A rest frame: $v_3 > \beta > v_2$

$$v_2 = \beta\,\frac{E_2^* - p_2^*/\beta}{E_2^* - \beta\, p_2^*} < \beta\,, \qquad v_3 = \beta\,\frac{E_3^* + p_2^*/\beta}{E_3^* + \beta\, p_2^*} > \beta$$

- Conditions (Coleman–Norton theorem): Coleman, Norton (1965); Bronzan (1964)
  - ☞ all three intermediate particles can go on shell simultaneously
  - ☞ $\vec{p}_2 \parallel \vec{p}_3$, particle-3 can catch up with particle-2 (as a classical process)
- needs very special kinematics $\Rightarrow$ process dependent! (contrary to pole position)

Triangle singularity (TS)

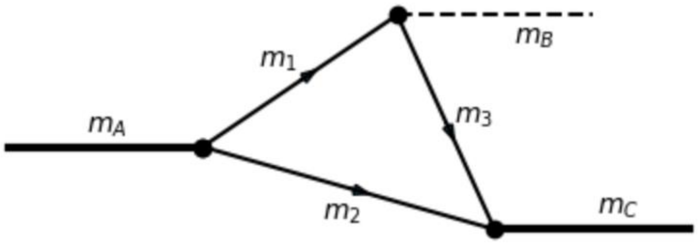


Features of TS

- logarithmic branch point, can produce a peak,
  mimicking a resonance
- normally close to a two-body threshold
- very sensitive to kinematic variables

- Given masses of intermediate particles and the external m_B, TS can produce peaks in both m_A and m_C distributions.
- For fixed $m_{2,3}$, m_A and m_B, TS in the physical region only happens when:

$$m_1^2 \in \left[\frac{m_A^2 m_3 + m_B^2 m_2}{m_2 + m_3} - m_2 m_3, (m_A - m_2)^2\right]$$



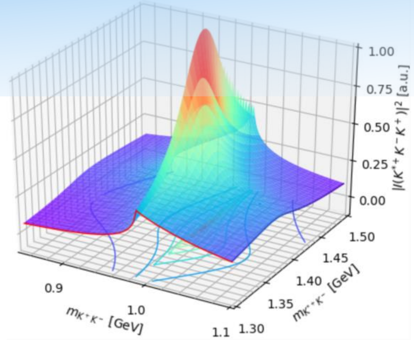
- TS in the physical region only happens when

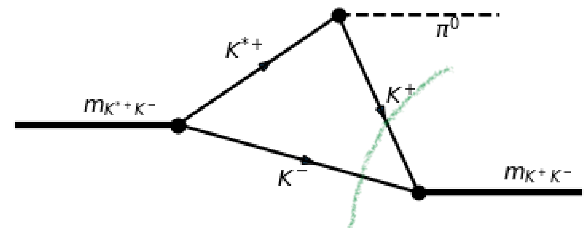
$$m_A^2 \in \left[(m_1 + m_2)^2, (m_1 + m_2)^2 + \frac{m_2}{m_3}\left[(m_1 - m_3)^2 - m_B^2\right]\right]$$

$$m_C^2 \in \left[(m_2 + m_3)^2, (m_2 + m_3)^2 + \frac{m_2}{m_1}\left[(m_1 - m_3)^2 - m_B^2\right]\right]$$

Features of TS

- Phase motion of triangle diagram in the presence of a TS

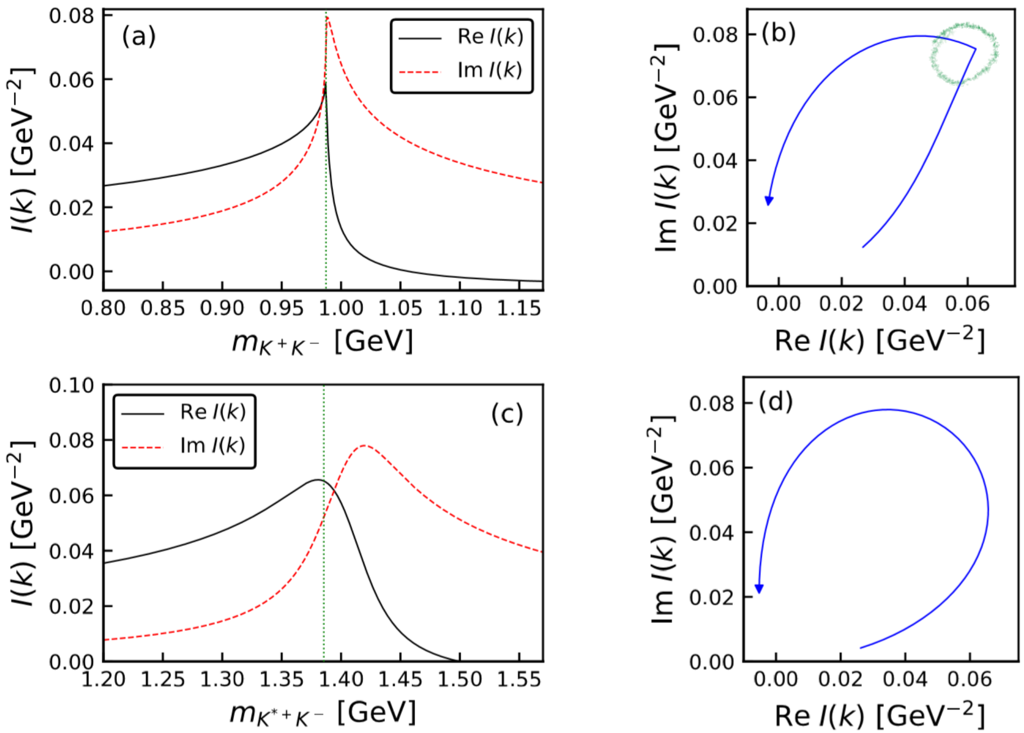




$K\overline{K}$ threshold

$m_{K^*\overline{K}} = 1.42$ GeV

$m_{K\overline{K}} = 0.99$ GeV



The argand plot is counterclockwise, resembling that of a resonance

Reactions with TS

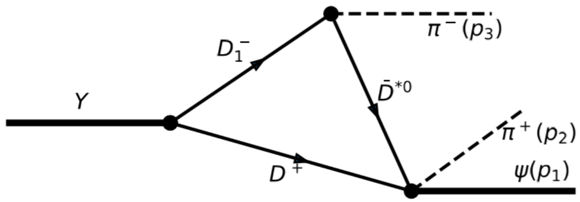
Structures	Processes	Loops	I/F	Refs.
$\rho(1480)$ [78,79]	$\pi^- p \to \phi\pi^0 n$	$K^*\bar{K}K$	I	[80,81]
$\eta(1405/1475)$ [82–86]	$\eta(1405/1475) \to \pi f_0$	$K^*\bar{K}K$	I	[87–91][a,b]
$f_1(1420)$ [92]	$f_1(1285) \to \pi a_0/\pi f_0$	$K^*\bar{K}K$	I	[89,93–95][b]
$a_1(1420)$ [96,97]	$a_1(1260) \to f_0\pi \to 3\pi$	$K^*\bar{K}K$	I	[97–99]
1.4 GeV [100]	$J/\psi \to \phi\pi^0\eta/\phi\pi^0\pi^0$	$K^*\bar{K}K$	I	[101][b]
1.42 GeV	$B^- \to D^{*0}\pi^- f_0(a_0)$, $\tau \to \nu_\tau\pi^- f_0(a_0)$	$K^*\bar{K}K$	I	[102,103]
	$D_s^+ \to \pi^+\pi^0 f_0(a_0)$, $\bar{B}_s^0 \to J/\psi\pi^0 f_0(a_0)$	$K^*\bar{K}K$	I	[104,105]
$f_2(1810)$ [10]	$f_2(1640) \to \pi\pi\rho$	$K^*\bar{K}^*K$	I	[106]
1.65 GeV	$\tau \to \nu_\tau\pi^- f_1(1285)$	$K^*\bar{K}^*K$	I	[107]
1515 MeV	$J/\psi \to K^+K^- f_0\ (a_0)$	$\phi\bar{K}K$	I	[108]
2.85 GeV, 3.0 GeV	$B^- \to K^-\pi^- D_{s0}^*/K^-\pi^- D_{s1}$	$K^{*0}D^{(*)0}K^+$	I	[109,110]
5.78 GeV	$B_c^+ \to \pi^0\pi^+ B_s^0$	$\bar{K}^{*0}B^+\bar{K}$	F	[111]
[4.01, 4.02] GeV	$[\bar{D}^{*0}D^{*0}] \to \gamma X$	$D^{*0}\bar{D}^{*0}D^0$	I	[112]
4015 MeV	$e^+e^- \to \gamma X$	$D^{*0}\bar{D}^{*0}D^0$	I	[113,114]
4015 MeV	$B \to KX\pi$, $pp/p\bar{p} \to X\pi$+anything	$D^{*0}\bar{D}^{*0}D^0$	I	[115,116]
$\Upsilon(11020)$ [117,118]	$e^+e^- \to Z_b\pi$	$B_1(5721)\bar{B}B^*$	I	[119,120]
3.73 GeV	$X \to \pi^0\pi^+\pi^-$	$D^{*0}\bar{D}^0D^0$	F	[121]
[4.22, 4.24] GeV	$e^+e^- \to \gamma J/\psi\phi/\pi^0 J/\psi\eta$	$D_{s0(s1)}^*\bar{D}_s^{(*)}D_s^{(*)}$	F	[122]
[4.08, 4.09] GeV	$e^+e^- \to \pi^0 J/\psi\eta$	$D_{s0(s1)}^*\bar{D}_s^{(*)}D_s^{(*)}$	F	[122]
$Z_c(3900)$ [31,32]	$e^+e^- \to J/\psi\pi^+\pi^-$	$D_1\bar{D}D^*$	F	[119,123–127][c]
		$D_0^*(2400)\bar{D}^*D$	F	[128,129]
$Z_c(4020, 4030)$ [33,130]	$e^+e^- \to \pi^+\pi^- h_c(\psi')$	$D_{1(2)}\bar{D}^{(*)}D^{(*)}$	F	[125]
$X(4700)$ [131,132]	$B^+ \to K^+J/\psi\phi$	$K_1(1650)\psi'\phi$	F	[133]
$Z_c(4430)$ [30,134]	$\bar{B}^0 \to K^-\pi^+J/\psi$	$\bar{K}^{*0}\psi(4260)\pi^+$	F	[135]
$Z_c(4200)$ [136,137]	$\bar{B}^0 \to K^-\pi^+\psi(2S)$	$\bar{K}_2^*\psi(3770)\pi^+$	F	[135]
	$\Lambda_b^0 \to p\pi^- J/\psi$	$N^*\psi(3770)\pi^-$	F	[135]
$X(4050)^\pm$ [138]	$\bar{B}^0 \to K^-\pi^+\chi_{c1}$	$\bar{K}^{*0}X\pi^+$	F	[139]
$X(4250)^\pm$ [138]	$\bar{B}^0 \to K^-\pi^+\chi_{c1}$	$\bar{K}_2^*\psi(3770)\pi^+$	F	[139]
$Z_b(10610)$ [34]	$e^+e^- \to \Upsilon(1S)\pi^+\pi^-$	$B_J^*\bar{B}^*B$	F	[128]

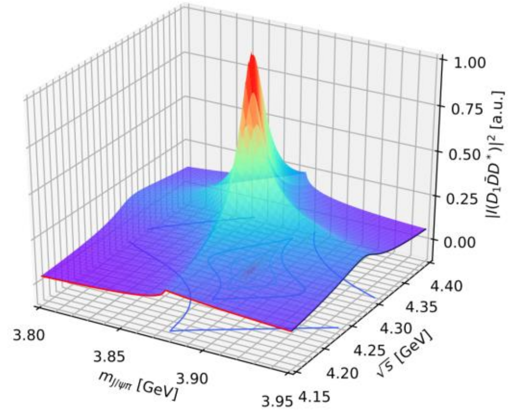
Structures	Processes	Loops	I/F	Refs.
2.1 GeV [141]	$\gamma p^+ \to N^*(2030) \to K^+\Lambda(1405)$	$K^*\Sigma\pi$	I	[142]
2.1 GeV	$\pi^- p^+ \to K^0\Lambda(1405)$, $pp \to pK^+\Lambda(1405)$	$K^*\Sigma\pi$	I	[143]
1.88 GeV	$\Lambda_c^+ \to \pi^+\pi^0\pi\Sigma$	$\bar{K}^*N\bar{K}$	I	[144,145][a]
$N(1700)$ [10]	$N(1700) \to \pi\Delta$	$\rho N\pi$	I	[146]
$N(1875)$ [10]	$N(1875) \to \pi N(1535)$	$\Sigma^*K\Lambda$	I	[147]
$\Delta(1700)$ [148–150]	$\gamma p \to \Delta(1700) \to \pi N(1535) \to p\pi^0\eta$	$\Delta\eta p$	I	[151]
2.2 GeV [152]	$\Lambda_c^+ \to \pi^0\phi p$	$\Sigma^*K^*\Lambda$	F	[153]
1.66 GeV [154,155]	$\Lambda_c^+ \to \pi^+K^-p$	$a_0\Lambda\eta$, $\Sigma^*\eta\Lambda$	F	[156]
$P_c(4450)$ [35]	$\Lambda_b^0 \to K^-J/\psi p$	$\Lambda(1890)\chi_{c1}p$	F	[157–160][b]
		$N(1900)\chi_{c1}p$	F	[159]
Peaks relevant for P_c	$\Lambda_b^0 \to K^-J/\psi p$	$\bar{D}_{sJ}\Lambda_c^{(*)}\bar{D}^{(*)}$	F	[36,158]

Reviewed in FKG, X.-H. Liu, S. Sakai, PPNP 112 (2020) 103757

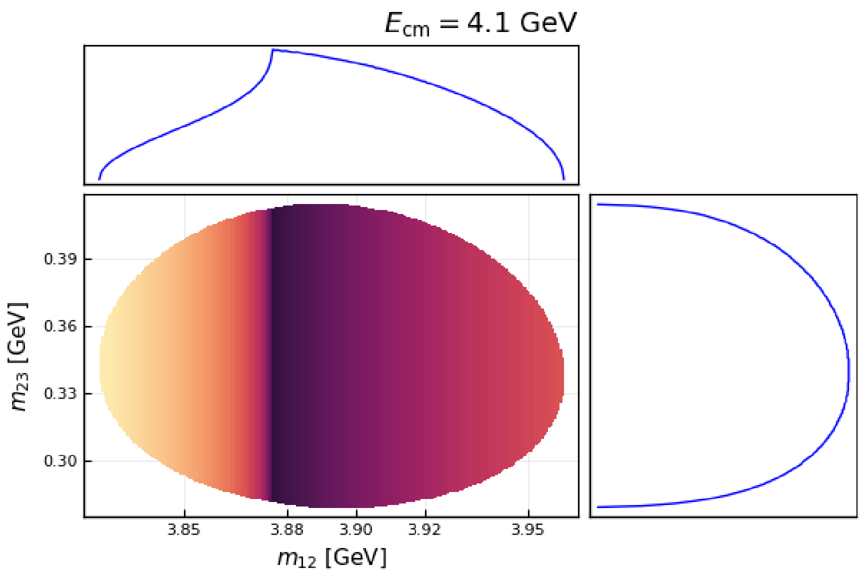
Example: $Z_c(3900)$

Example: $D_1\bar{D}D^* + c.c.$ triangles relevant for an analysis of $Z_c(3900)$: $e^+e^- \to J/\psi\pi\pi$

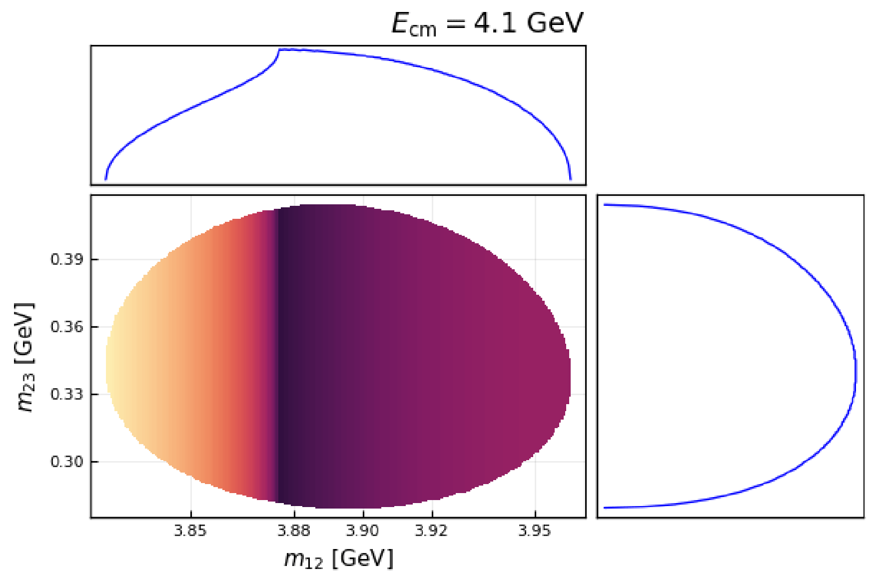




Very sensitive to energy!



Scalar 3-point $D_1\bar{D}D^*$ loop



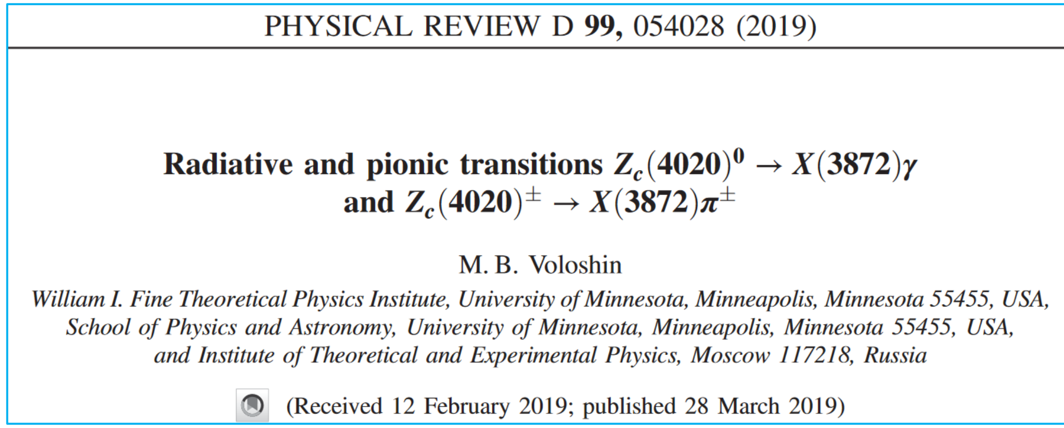
Sum of $D_1\bar{D}D^*, D_1\bar{D}^*D^*, D_2\bar{D}^*D^*$loops

TSs for $Z_c(3900)$: Wang et al. (2013); Albaladejo et al. (2015); JPAC(2016); Gong et al. (2018)

Despite the TSs, the $Z_c(3900)$ is still needed, see also the talk by I. Danilkin

TS as a tool: precise measurement of X(3872)

FKG, PRL122 (2019) 202002


PHYSICAL REVIEW D **99,** 054028 (2019)

Radiative and pionic transitions $Z_c(4020)^0 \to X(3872)\gamma$ and $Z_c(4020)^\pm \to X(3872)\pi^\pm$

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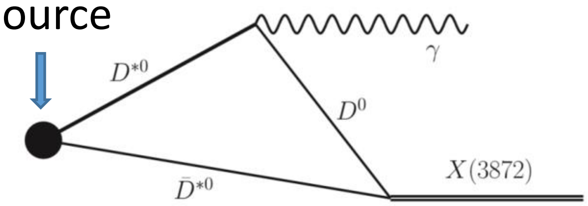
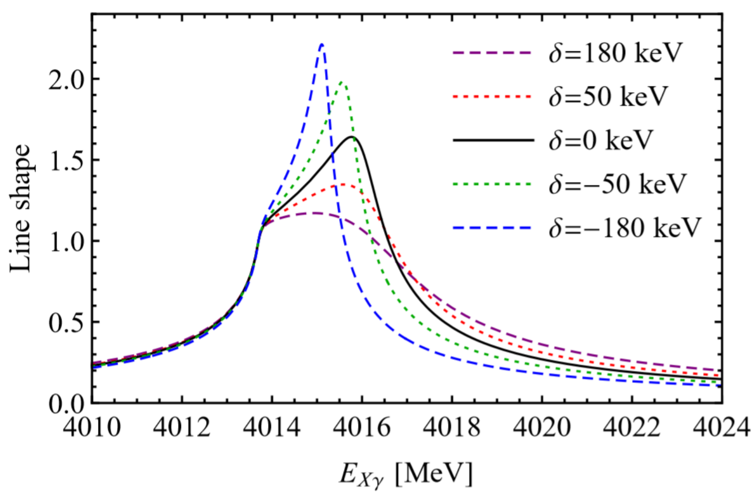
(Received 12 February 2019; published 28 March 2019)

Short distance $D^{*0}\overline{D}^{*0}$ source



$$E_{X\gamma}^{\text{TS}} \simeq 2M_{D^{*0}} + \frac{1}{2M_{D^0}}\left(M_{D^{*0}} - M_{D^0} - 2\sqrt{-M_{D^0}\delta} + \delta\right)^2$$

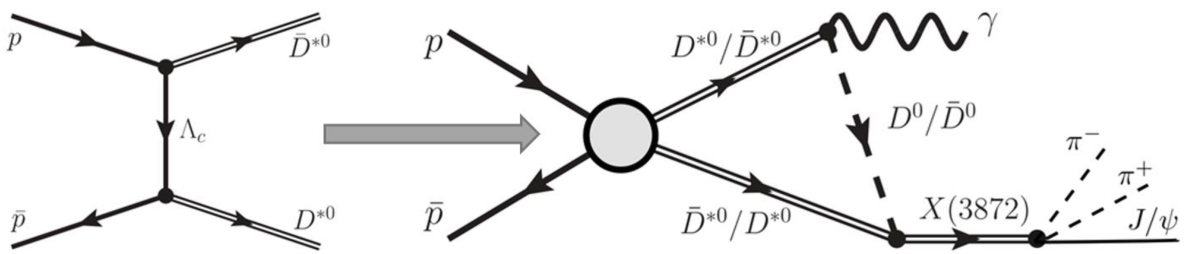


- Cusp fixed at the $D^{*0}\bar{D}^{*0}$ threshold
- Peak fixed at the TS energy:

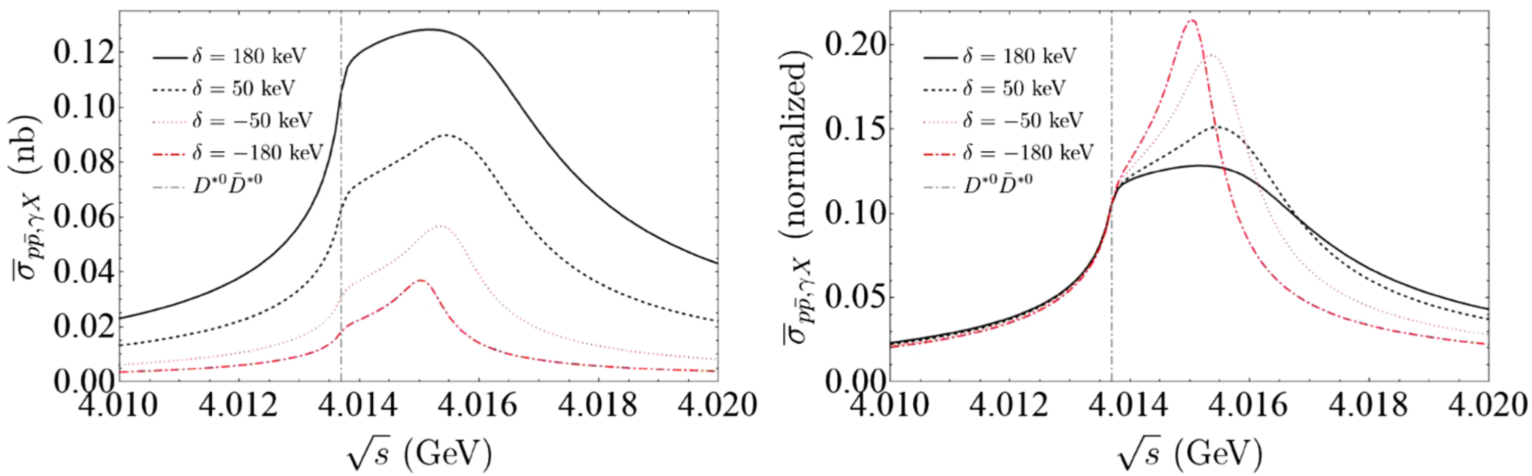
δ (keV)	$E_{X\gamma}^{\text{TS}}$ (MeV)
-180	$4015.2 - i0.1$
-50	$4015.7 - i0.2$
0	$4016.0 - i0.4$

TS as a tool: precise measurement of X(3872)

Cross section estimate: $p\bar{p} \to \gamma X(3872)$ S. Sakai, H.-J. Jing, FKG, PRD 102 (2020) 114041



- $\sigma(p\bar{p} \to \gamma X) \times \mathcal{B}(X \to J/\psi\pi^+\pi^-) = \mathcal{O}(10\ \mathrm{pb})$



- $\mathcal{O}(2 \times 10^3)$ events taking into account $\mathcal{B}(J/\psi \to \ell^+\ell^-) \simeq 12\%$ for an integrated luminosity of 2 fb^{-1} at PANDA

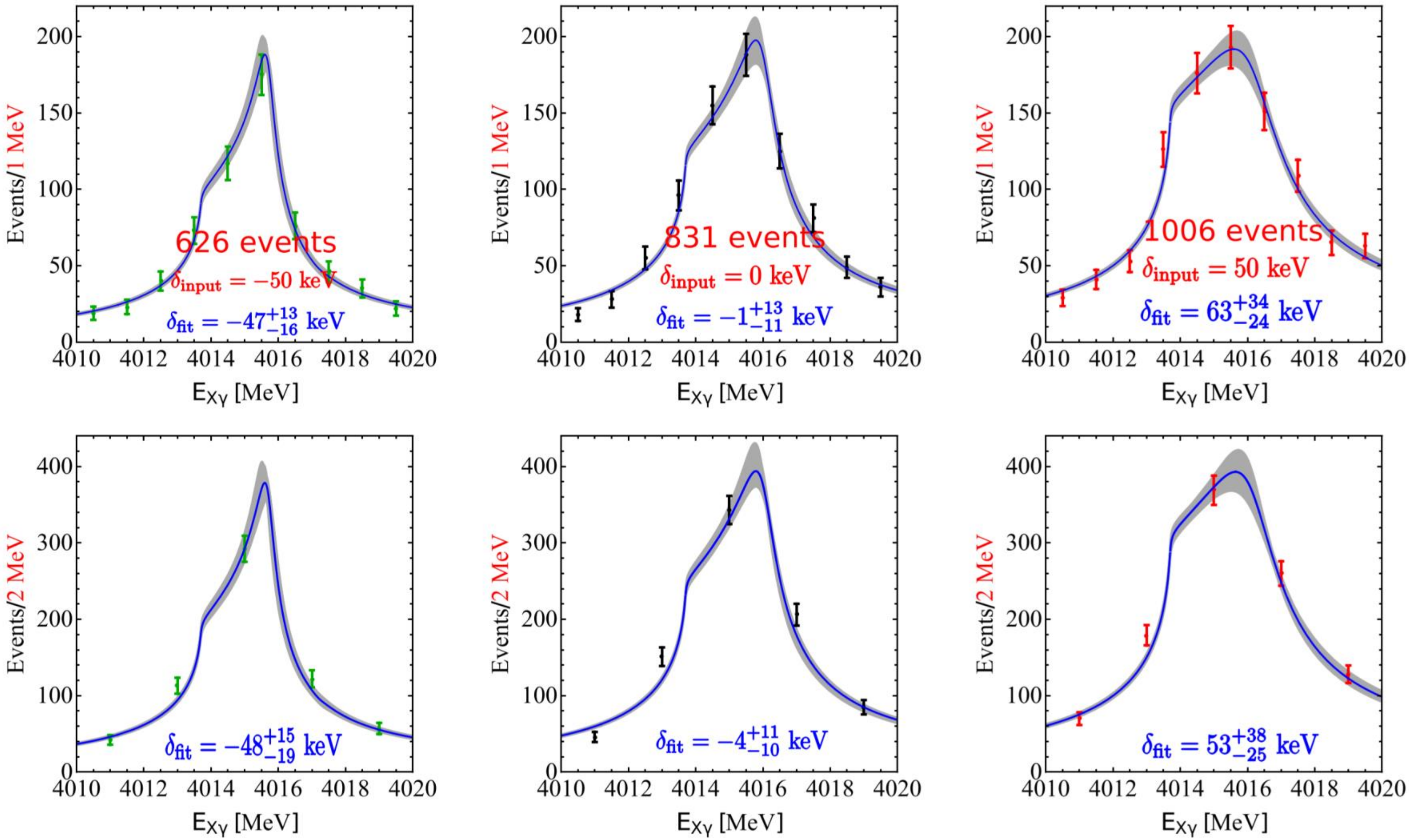
PANDA, EPJA55(2019)42

TS as a tool: precise measurement of X(3872)

- Directly probe δ, uncertainty can be smaller than that of the $D^{(*0)}$ masses

Monte Carlo simulation of the sensitivity:

FKG, PRL122(2019)202002



Effects of energy resolution studied in P. G. Ortega, E. Ruiz Arriola, arXiv:2007.11608

Other works on TS effects for X(3872)

Talk by E. Braaten

$B \to K\pi X(3872)$

E. Braaten, L.-P. He, K. Ingles, Phys.Rev.D 100 (2019) 074028

S. Sakai, E. Oset, FKG, Phys.Rev.D 101 (2020) 054030

S. Nakamura, Phys.Rev.D 102 (2020) 074004

Production of $X(3872)$ at hadron collider

E. Braaten, L.-P. He, K. Ingles, Phys.Rev.D 100 (2019) 094006

$e^+e^- \to \gamma X(3872)$

E. Braaten, L.-P. He, K. Ingles, Phys.Rev.D 101 (2020) 014021

$e^+e^- \to \gamma D^0\bar{D}^{*0}$

E. Braaten, L.-P. He, K. Ingles, Phys.Rev.D 101 (2020) 096020

X atom

- $X(3872)$: strong coupling to $D^0\bar{D}^{*0}$

  Unavoidably extended, large radius, $r_X \simeq \frac{1}{\sqrt{2\mu_0\delta}} \gtrsim 10\text{ fm}$

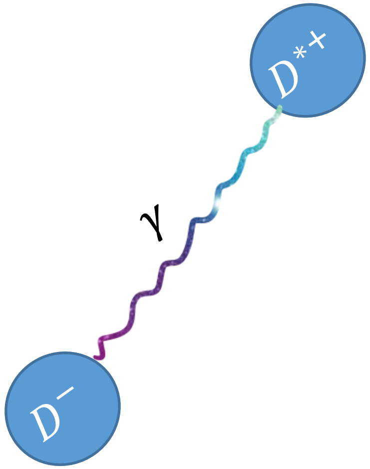
- The same order as the Bohr radius of Coulomb bound state of D^-D^{*+}, D^+D^{*-} : hadronic atoms $r_B = \frac{1}{\alpha\mu_c} = 27.86\text{ fm}$

  $$\mu_0 = \frac{m_{D^0}m_{D^{*0}}}{\Sigma_0} \qquad \mu_c = \frac{m_D m_{D^*}}{\Sigma_c}$$ thresholds: $\Sigma_{0,c}$

- Coulomb binding energies: $E_n = \frac{\alpha^2\mu_c}{2n^2} = \frac{25.81\text{ keV}}{n^2}$

- For production: the more extended, the more difficult to be produced. ($\sigma \propto \delta^{1/2}$)

- X atom: The ground state $D^-D^{*+} - D^+D^{*-}$ atom with $C = +$; correction due to strong interaction



X atom

- Nonrelativistic effective field theory (NREFT) for coupled channels:
  - $1^{++}\ D^0\bar{D}^{*0}$
  - $1^{++}\ D^+D^{*-}$, the Green function contains both Coulomb bound states and continuum
- Around the threshold, LO in NREFT: constant contact terms for strong interaction

$$
\begin{aligned}
\mathcal{L} = & -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \sum_{\phi=D^\pm,D^0,\bar{D}^0} \phi^\dagger \left(iD_t - m_\phi + \frac{\nabla^2}{2m_\phi} \right) \phi \\
& + \sum_{\phi=D^{*\pm},D^{*0},\bar{D}^{*0}} \phi^\dagger \left(iD_t - m_\phi + i\frac{\Gamma_\phi}{2} + \frac{\nabla^2}{2m_\phi} \right) \phi \\
& - \frac{C_0}{2}(D^+D^{*-} - D^-D^{*+})^\dagger (D^+D^{*-} - D^-D^{*+}) \\
& - \frac{C_0}{2}\left[(D^+D^{*-} - D^-D^{*+})^\dagger (D^0\bar{D}^{*0} - \bar{D}^0D^{*0}) + \text{h.c.} \right] \\
& - \frac{C_0}{2}(D^0\bar{D}^{*0} - \bar{D}^0D^{*0})^\dagger (D^0\bar{D}^{*0} - \bar{D}^0D^{*0}) + \cdots ,
\end{aligned}
$$

- Approximation: Isospin-1 strong interaction neglected
  - No isovector state was found
  - Isospin breaking in the couplings is small: Hanhart et al., PRD85(2012)011501

$$
\frac{g_{X\rho}}{g_{X\omega}} = 0.26^{+0.08}_{-0.05}
$$

X atom

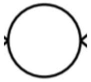
- The T-matrix for positive C parity channels: $T(E) = V[1 - G(E)V]^{-1}$

$$V = C_0 \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \quad G(E) = \begin{pmatrix} J_0(E) & 0 \\ 0 & J_c(E) + J_{|\Psi\rangle}(E) \end{pmatrix}$$

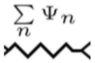
$$J_0(E) = \frac{\mu_0}{2\pi}\left(-\frac{2\Lambda}{\pi} + \sqrt{-2\mu_0(E + \Delta + i\Gamma_0/2)}\right),$$

$\Delta = \Sigma_c - \Sigma_0$

$$J_c(E) = \frac{\mu_c}{2\pi}\left(-\frac{2\Lambda}{\pi} + \sqrt{-2\mu_c(E + i\Gamma_c/2)}\right),$$



$$J_{|\Psi\rangle}(E) = \sum_{n=1}^{\infty} \frac{\alpha^3 \mu_c^3}{\pi n^3} \frac{1}{E + E_n + i\Gamma_c/2},$$



- The T-matrix has infinity of poles: $X(3872)$, hadronic atoms

$$T(E) = \frac{1}{C_0^{-1} - \left[J_0(E) + J_c(E) + J_{|\Psi\rangle}(E)\right]} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

Renormalization: $C_{0R}^{-1} = C_0^{-1} + \Lambda(\mu_0 + \mu_c)/\pi^2$

X atom

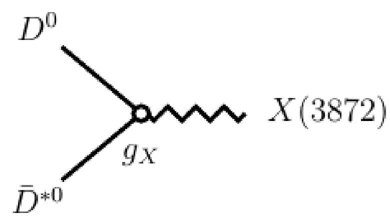
- X atom binding energy and decay width (due to decays of D^{*-} and into $D^0\bar{D}^{*0}$)

$$\mathrm{Re}\, E_{A1} = E_1 - \frac{\alpha^3\mu_c^2}{\sqrt{2\mu_c\Delta}} \simeq 22.92\,\mathrm{keV}$$

$$\boxed{M_{A1} = (3879.89 \pm 0.07)\ \mathrm{MeV}}$$

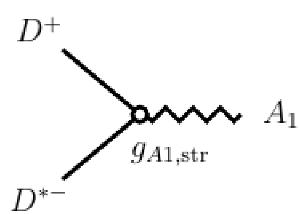
$$\underbrace{\Gamma_c}_{(83.4 \pm 1.8)\ \mathrm{KeV}} + 2\,\mathrm{Im}\, E_{A1} = \Gamma_c + \frac{2\alpha^3\mu_c^2}{\sqrt{2\mu_c\Delta}} = (89.2 \pm 1.8)\,\mathrm{keV}$$

- Effective coupling for $D^0\bar{D}^{*0} \to X(3872)$



$$g_X^2 = \lim_{E \to -\Delta-\delta-i\frac{\Gamma_0}{2}} \left(E + \Delta + \delta + i\frac{\Gamma_0}{2}\right) T_{11}(E) = \frac{2\pi}{\mu_0^2}\sqrt{2\mu_0\delta}\left[1 + \mathcal{O}\left(\frac{\delta^{1/2}}{\Delta^{1/2}}\right)\right]^{-1}$$

- Effective coupling for $D^+D^{*-} \to A_1$



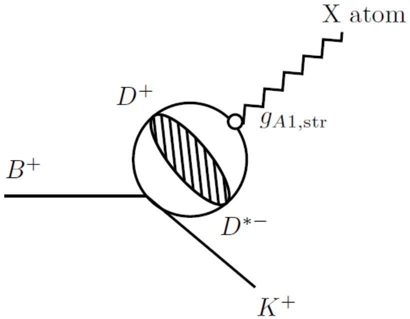
$$g_{A1,\mathrm{str}}^2 = \lim_{E \to -E_{A1}-i\frac{\Gamma_c}{2}} \left(E + E_{A1} + i\frac{\Gamma_c}{2}\right) T_{22}(E) = -i\frac{\pi\alpha^3}{\Delta}\left[1 + \mathcal{O}\left(\frac{\alpha^2\mu_c}{\Delta}\right)\right]^{-1}$$

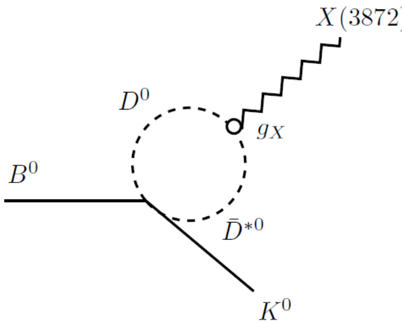
X atom

- Productions of the $X(3872)$ and the X atom
- Scale separation: factorization

Braaten, Kusunoki, PRD72(2005)014012

$$\mathcal{A}_{B^+\to AK^+} = \mathcal{A}^{\text{s.d.}}_{B^+\to(DD^*)_+K^+}\, g_{A1,\text{str}}$$
$$\mathcal{A}_{B^0\to XK^0} = \mathcal{A}^{\text{s.d.}}_{B^0\to(DD^*)^0_+K^0}\, g_X\,,$$





- Isospin symmetry: the short-distance parts are the same, productions of the X(3872) and the X atom are related

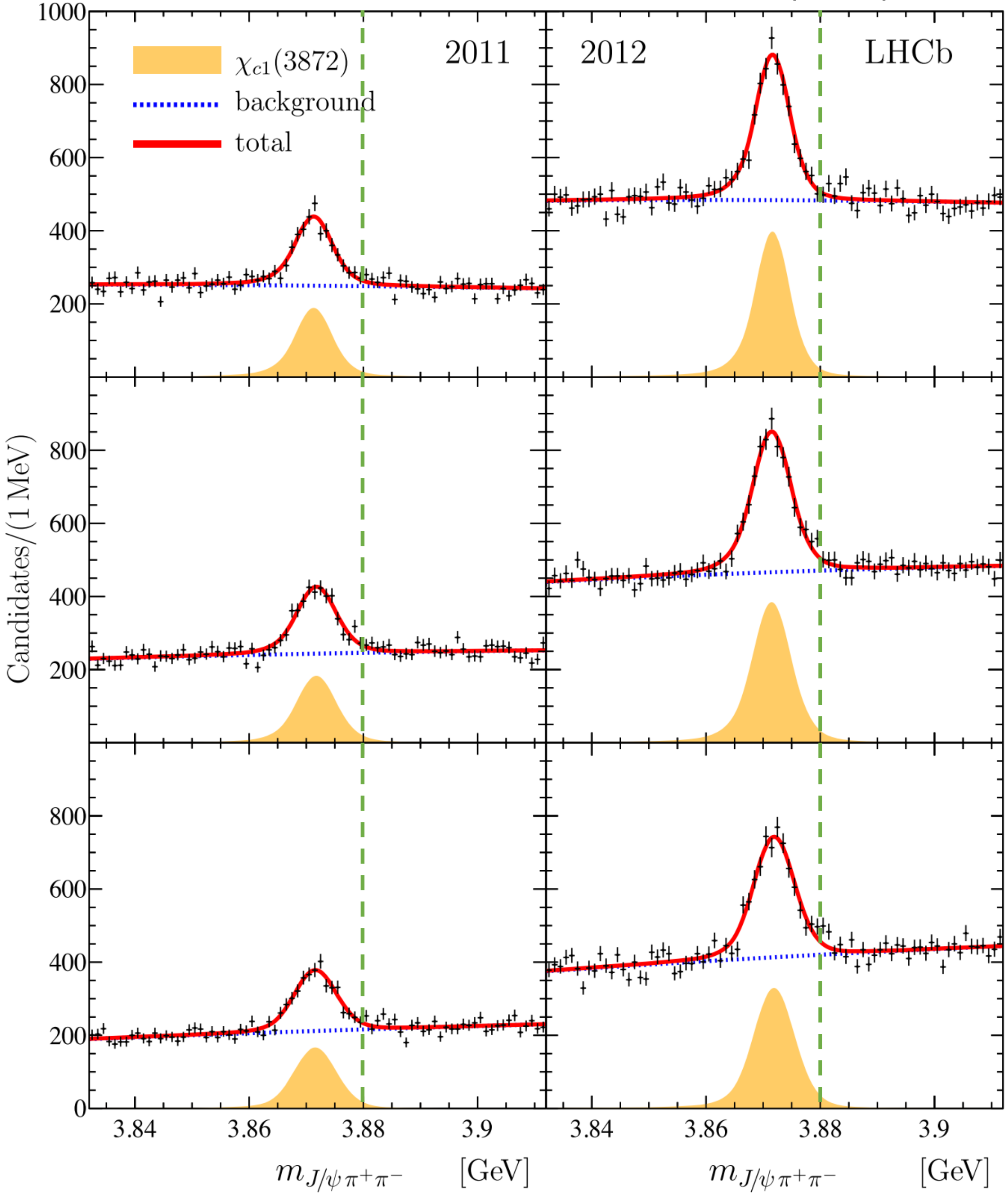
$$R_\Gamma \equiv \frac{\Gamma_{B^+\to AK^+}}{\Gamma_{B^0\to XK^0}} = \frac{|g_{A1,\text{str}}|^2}{|g_X|^2} \qquad R_\sigma \equiv \frac{d\sigma_{pp\to A+y}}{d\sigma_{pp\to X+y}} = \frac{|g_{A1,\text{str}}|^2}{|g_X|^2}$$

- Production rate for the X atom: $\boxed{R_\Gamma \simeq R_\sigma \gtrsim 1\times 10^{-3}}$
- Null signal leads to a lower bound on the $X(3872)$ binding energy

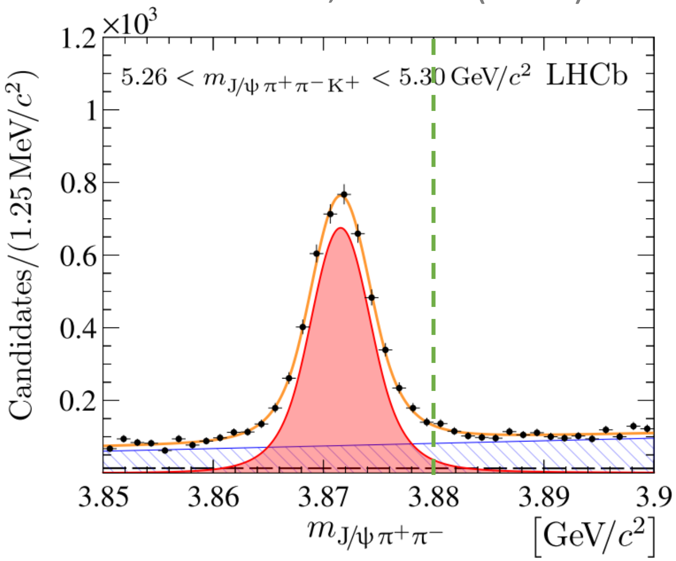
$$\boxed{\delta \simeq \frac{0.25\ \text{eV}}{R_\Gamma^2} \simeq \frac{0.25\ \text{eV}}{R_\sigma^2}}$$

X atom

LHCb, PRD102(2020)092005

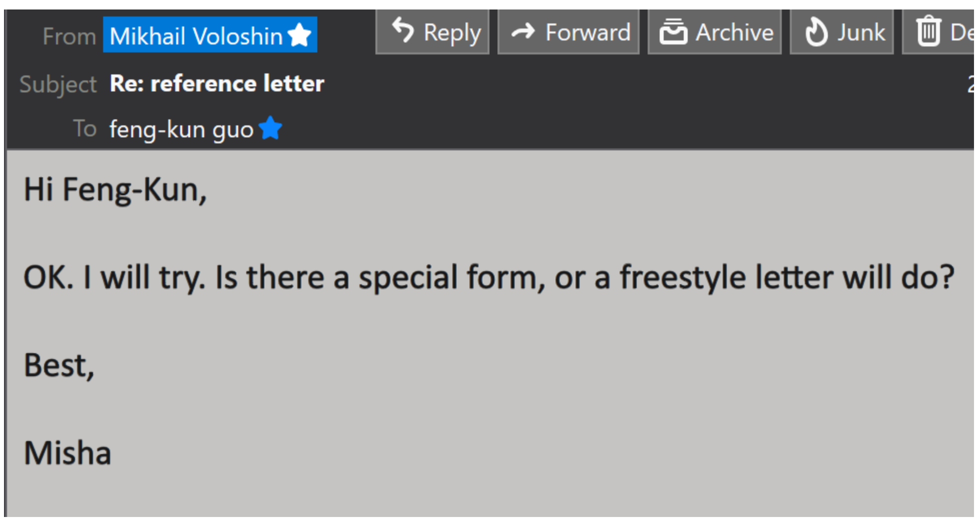


LHCb, JHEP08(2020)123



X atom can be searched for at LHCb and PANDA

- **In memory of Misha Voloshin**



From Mikhail Voloshin
Reply Forward Archive Junk

Subject **Re: reference letter**

To feng-kun guo

Hi Feng-Kun,

OK. I will try. Is there a special form, or a freestyle letter will do?

Best,

Misha

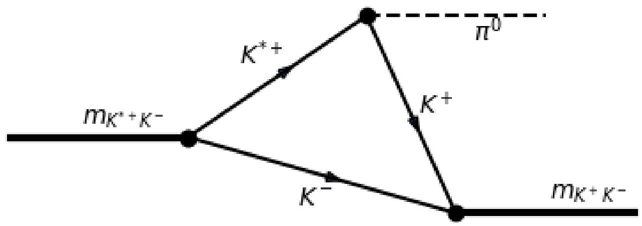
Conclusion

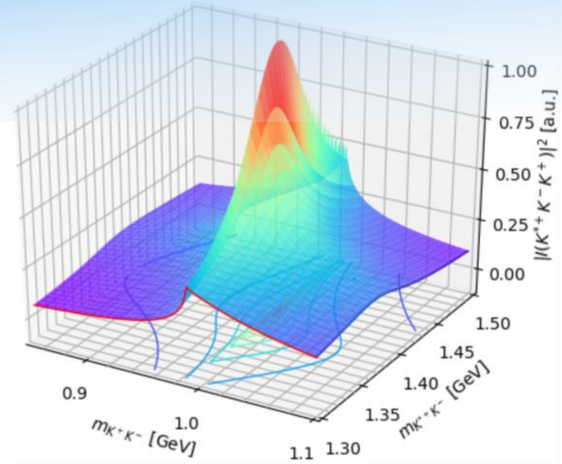
- Triangle singularity effects are sensitive to energies:

  TS-induced peak for the initial-state (final-state) energy distribution may be checked by varying the invariant mass of final (initial) states
- The X(3872) has been discovered for 17 years, debates continue
- New ideas regarding X(3872):
  - Measuring the binding energy using TS, uncertainty not limited by that of $D^{(*)0}$ masses, best at PANDA (high production rate and high energy resolution)
  - X atom can be used to set a lower limit on the X(3872) binding energy and to settle the debates regarding the production

Thank you for your attention!

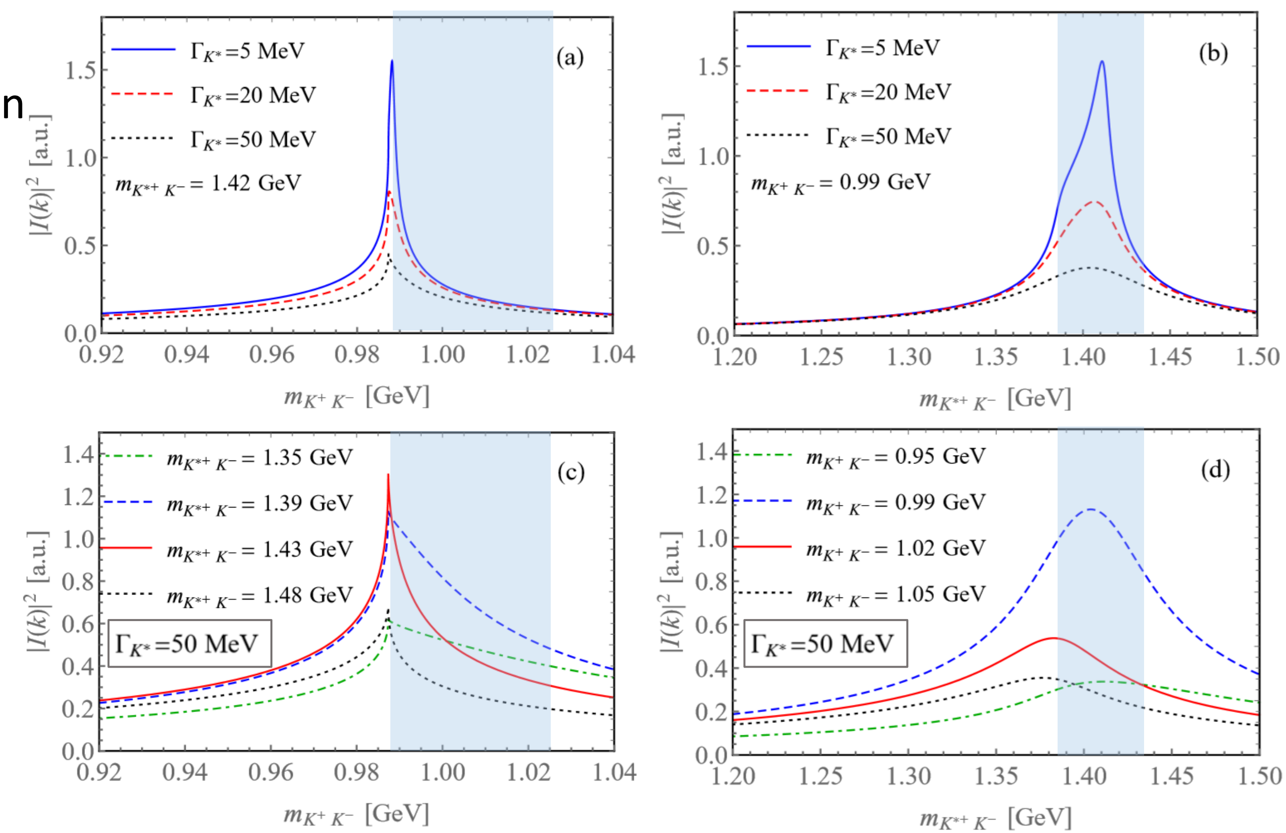
Features of TS





While a resonance would persist independent of energy.

TS in the physical region only when $m_{K\bar{K}} \in [987,1026]$ MeV

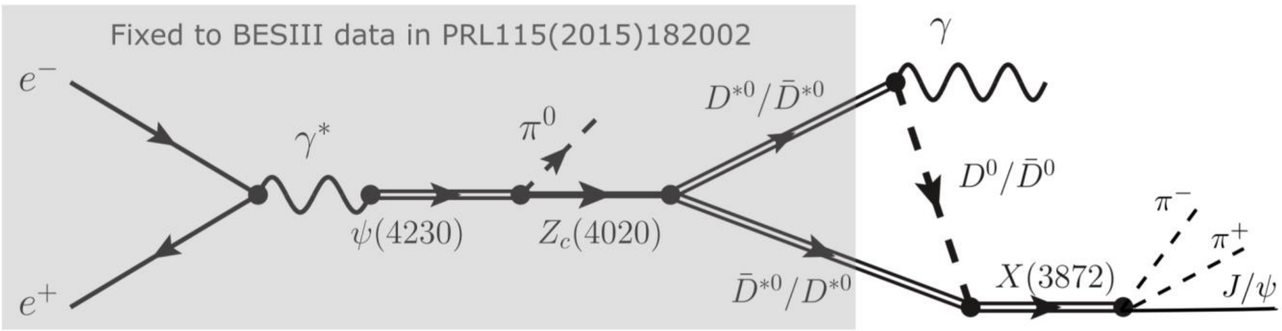


$m_{K^*\bar{K}} \in [1385,1435]$ MeV

TS as a tool: precise measurement of X(3872)

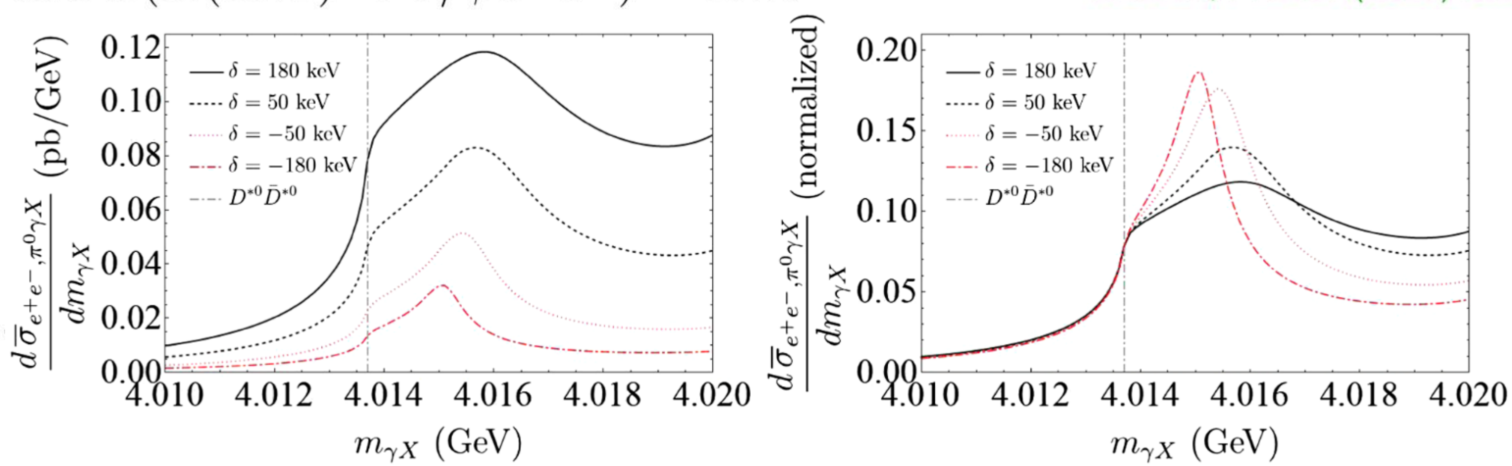
Cross section estimate (not precise prediction): S. Sakai, H.-J. Jing, FKG, arXiv:2008.10829

- Consider $e^+e^- \to \psi(4230) \to \pi^0 Z_c(4020)^0 \to \pi^0\gamma X(3872)$,



- Consider $X(3872) \to J/\psi\pi^+\pi^-$ with a $X(3872)$ width of 100 keV

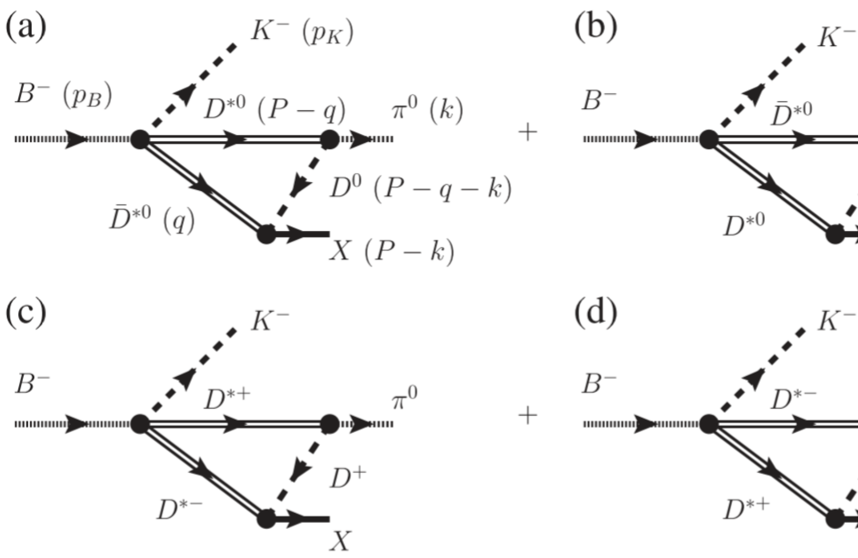
  take $\mathcal{B}(X(3872) \to J/\psi\pi^+\pi^-) = 4.1\%$ BABAR, PRL124(2020)152001



The cross section can be higher at higher energies, e.g. above 4.4 GeV, but seems still difficult for STCF…

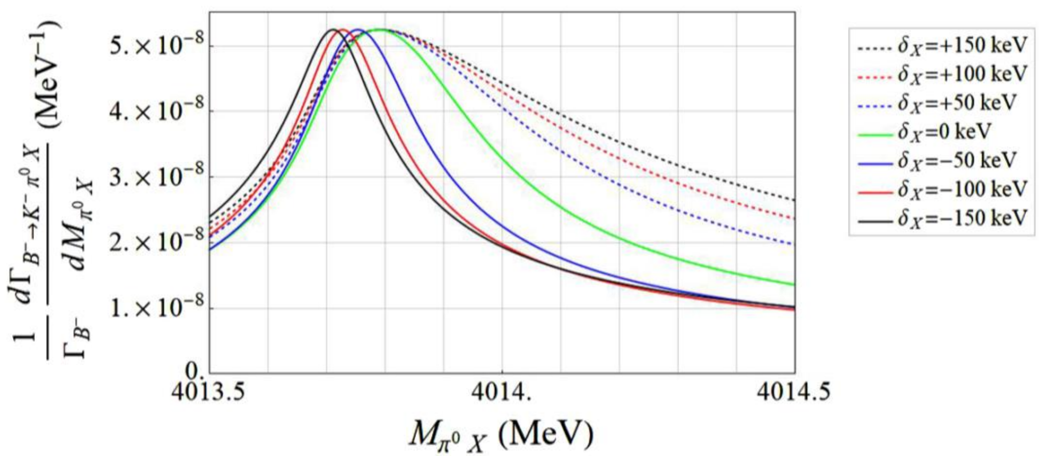
$B \to K\pi X(3872)$

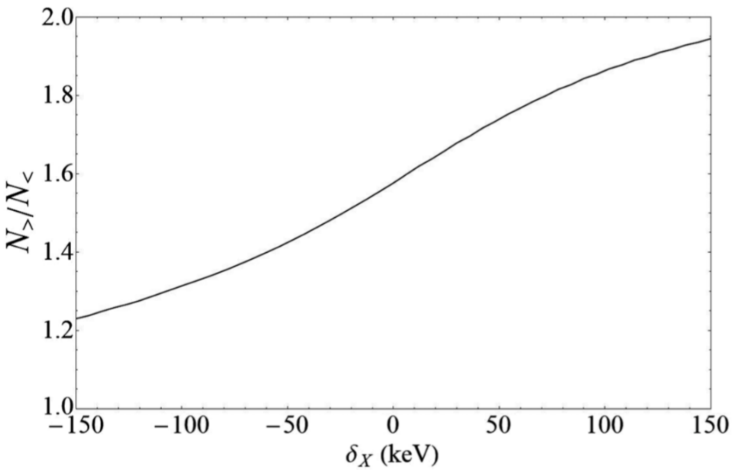
S. Sakai, E. Oset, FKG, Phys.Rev.D101(2020)054030



$$\frac{N_>}{N_<} \equiv \frac{\int_{M^{\max}}^{M^{\max}+\delta} dM_{\pi^0 X} \left(\overline{d\Gamma_{B^-\to K^-\pi^0 X}/dM_{\pi^0 X}}\right)}{\int_{M^{\max}-\delta}^{M^{\max}} dM_{\pi^0 X} \left(\overline{d\Gamma_{B^-\to K^-\pi^0 X}/dM_{\pi^0 X}}\right)}$$

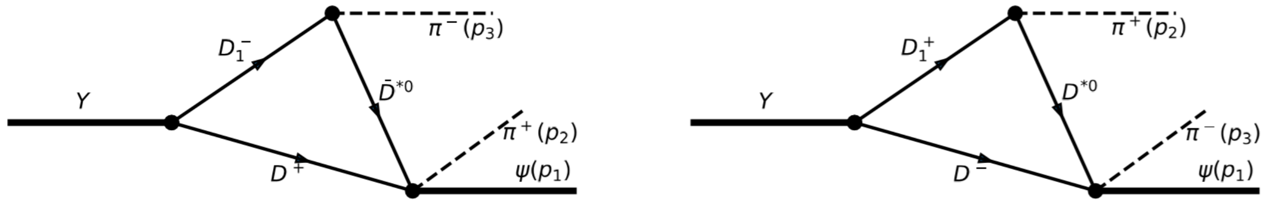
Asymmetric line shape





$Z_c(3900)$

Example: $D_1\bar{D}D^* + c.c.$ triangles relevant for an analysis of $Z_c(3900)$: $e^+e^- \to J/\psi\pi\pi$



(a) (b)

- Importance of TS pointed out, but a $Z_c(3900)$ is still needed for $e^+e^- \to J/\psi\pi\pi$

  Q.Wang, Hanhart, Q.Zhao, PRL111(2013)132002; PLB725(2013)106;

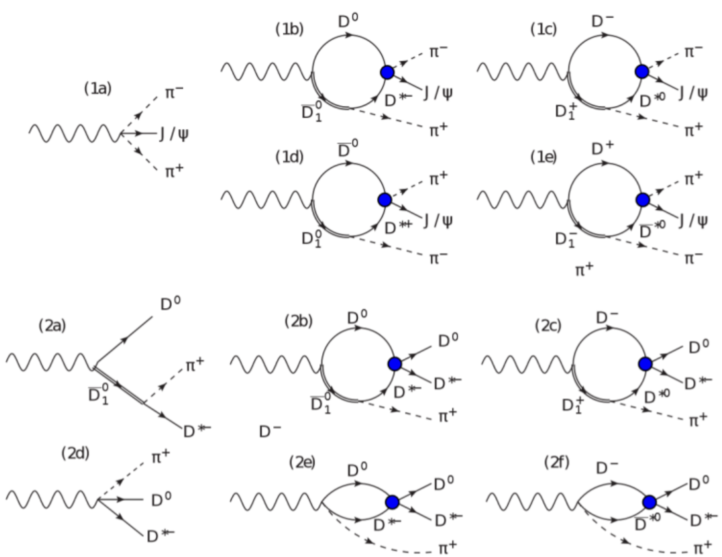
  Q.-R. Gong, J.-L. Pang, Y.-F. Wang, H.-Q. Zheng, EPJC78(2018)276

- Analyses of $e^+e^- \to J/\psi\pi\pi, D\bar{D}^*\pi$ data in Albaladejo, FKG, Hidalgo-Duque, Nieves, PLB755(2016)337

  ✓ triangle diagrams

  ✓ T-matrix for $J/\psi\pi$- $D\bar{D}^*$ coupled channels

  ✓ T-matrix may or may not have a pole, data tell

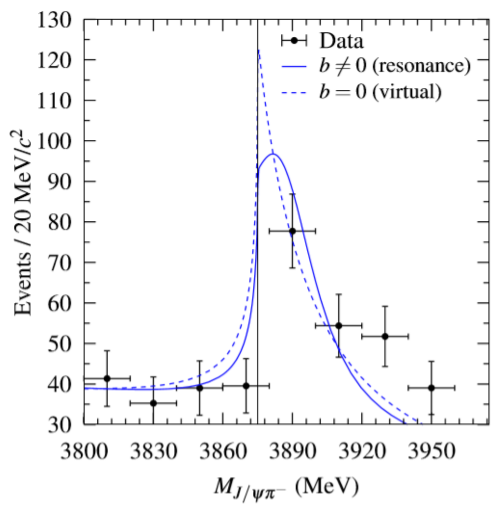


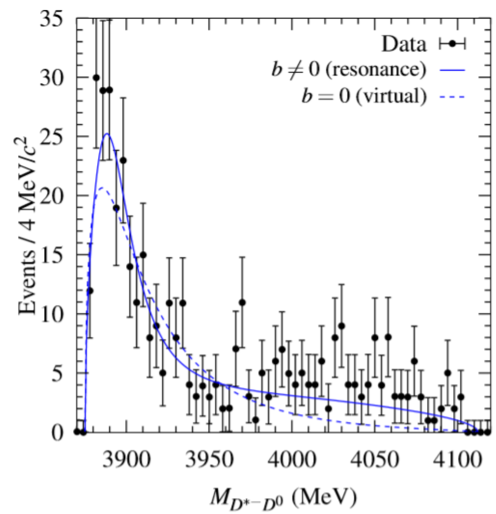
used D-wave $D_1(2420)D^*\pi$ coupling

$Z_c(3900)$

The T-matrix from the best has a pole

Albaladejo, FKG, Hidalgo-Duque, Nieves, PLB755(2016)337





M_{Z_c} (MeV)	$\Gamma_{Z_c}/2$ (MeV)	Ref.	Final state
3899 ± 6	23 ± 11	[1] (BESIII)	$J/\psi\ \pi$
3895 ± 8	32 ± 18	[2] (Belle)	$J/\psi\ \pi$
3886 ± 5	19 ± 5	[3] (CLEO-c)	$J/\psi\ \pi$
3884 ± 5	12 ± 6	[4] (BESIII)	$\bar{D}^* D$
3882 ± 3	13 ± 5	[5] (BESIII)	$\bar{D}^* D$
$3894 \pm 6 \pm 1$	$30 \pm 12 \pm 6$	$\Lambda_2 = 1.0$ GeV	$J/\psi\ \pi$, $\bar{D}^* D$
$3886 \pm 4 \pm 1$	$22 \pm 6 \pm 4$	$\Lambda_2 = 0.5$ GeV	$J/\psi\ \pi$, $\bar{D}^* D$
$3831 \pm 26^{+7}_{-28}$	virtual state	$\Lambda_2 = 1.0$ GeV	$J/\psi\ \pi$, $\bar{D}^* D$
$3844 \pm 19^{+12}_{-21}$	virtual state	$\Lambda_2 = 0.5$ GeV	$J/\psi\ \pi$, $\bar{D}^* D$

resonance pole $\chi^2/\text{dof} = 1.09$

or virtual state $\chi^2/\text{dof} = 1.36$

The same parameters produce FV energy levels consistent with lattice results by Prelovsek et al.

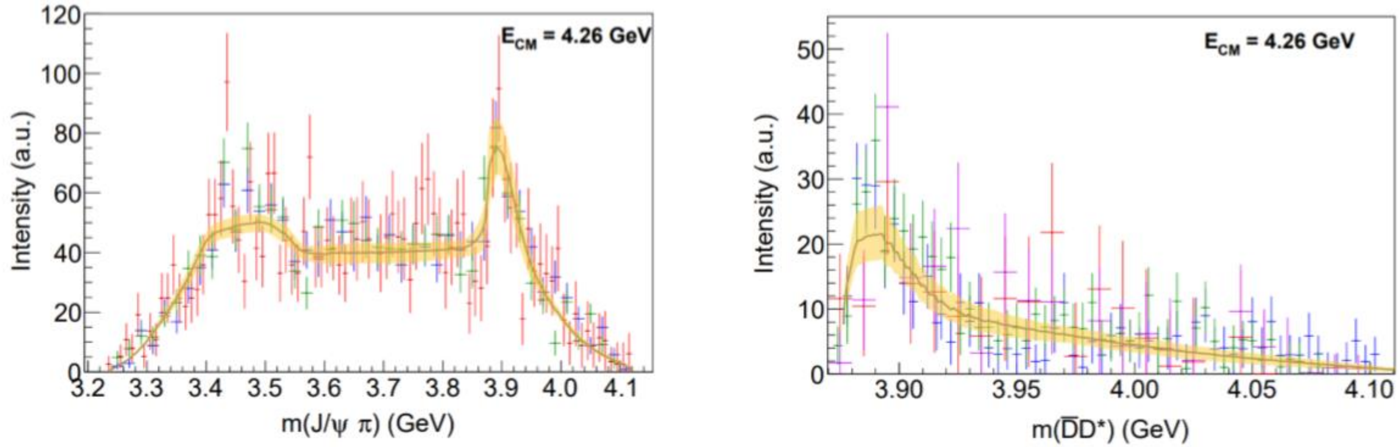
Albaladejo, Fernandez-Soler, Nieves, EPJC76(2016)573

$Z_c(3900)$

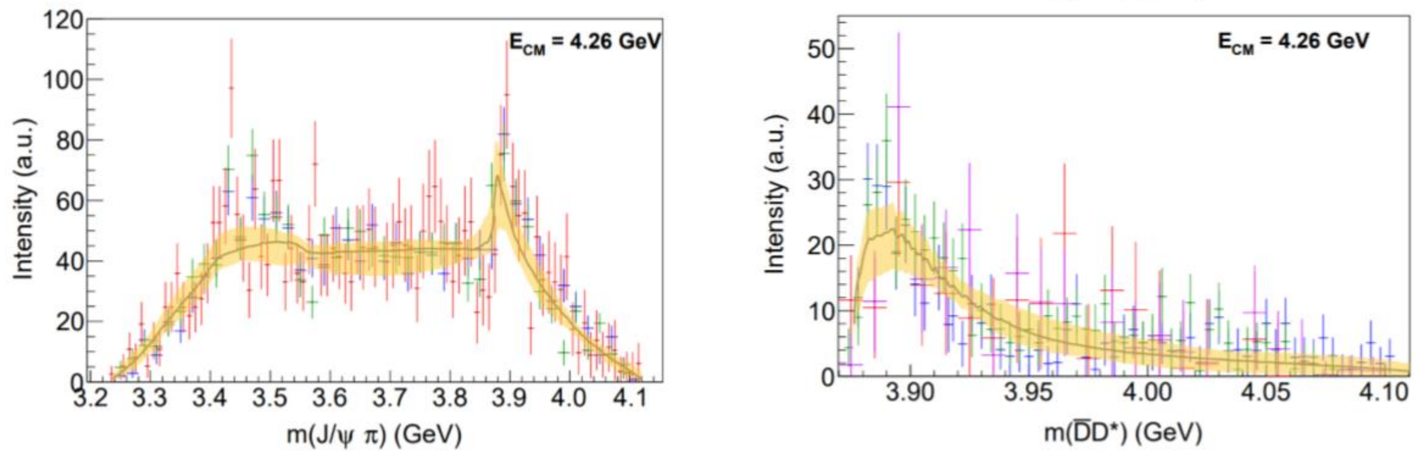
- However, the JPAC analysis has a different conclusion

Pilloni et al. (JPAC), PLB772(2017)200

Pole+TS:



Only TS:

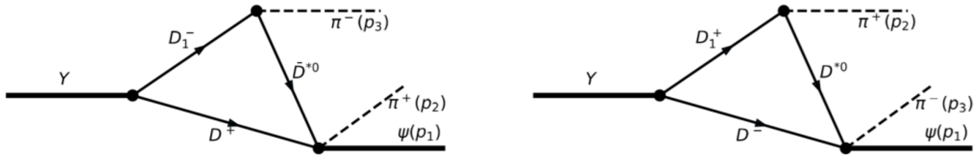


used S-wave $D_1(2420)D^*\pi$ coupling

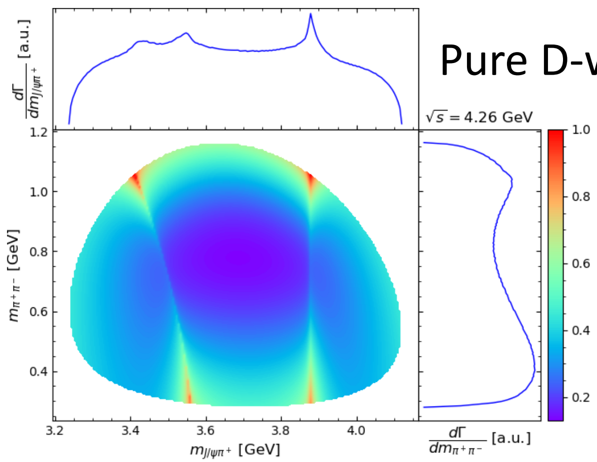
$Z_c(3900)$

- Partial waves for the $D_1(2420)D^*\pi$ coupling make a difference

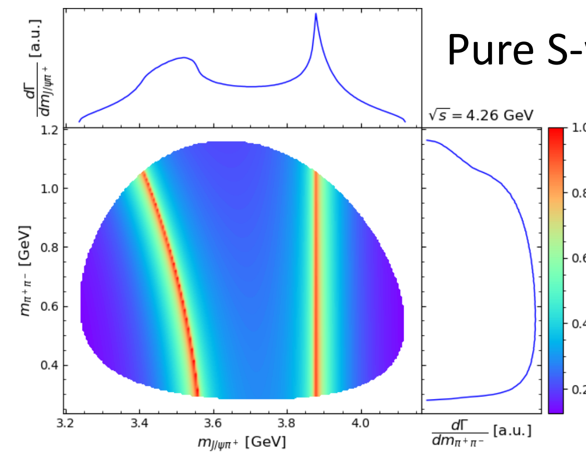
FKG, arXiv:2001.05884



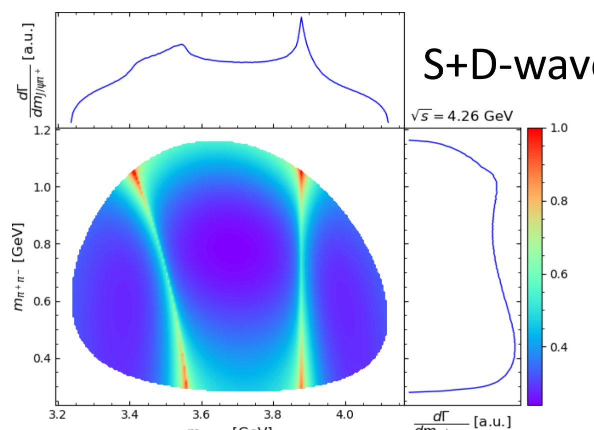
(a) (b)



Pure D-wave



Pure S-wave



S+D-wave

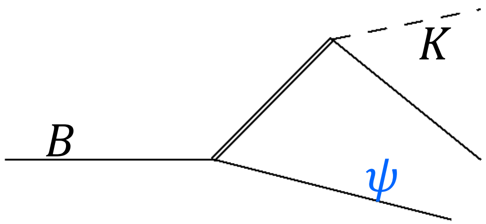
Here the D-wave $D_1(2420)D^*\pi$ coupling is fixed from the width of $D_2(2460)$ [spin partner of $D_1(2420)$]:

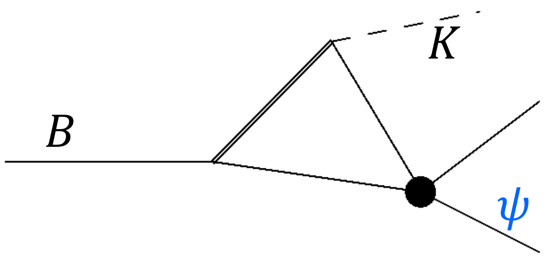
$$\Gamma_D(D_1) = 15.2 \text{ MeV}$$
$$\Gamma_S(D_1) = 16.5 \text{ MeV}$$

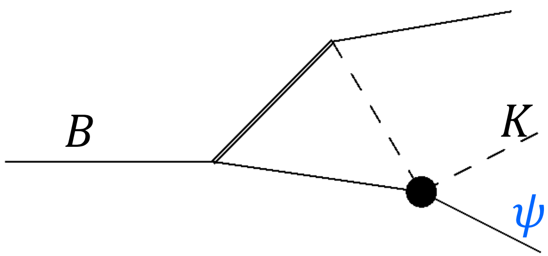
A reanalysis is ongoing (Yun-Hua Chen, FKG)

Complications in resonance searching due to TS

- Difficulties for multi-hadron final states
  - ✓ Many resonances from the cross channel:
    branching fractions often unknown, interference between overlapping resonances, …







  - ✓ Complicated 3-body FSI:
    intermediate states can be different from external ones; threshold cusps; triangle singularities

Kinematical singularities (threshold cusp, TS) and resonances are NOT exclusive

Schmid theorem

Schmid, PR154(1967)1363; Debastiani, Sakai, Oset, EPJC79(2019)69; FKG, Liu, Sakai, arXiv:1912.07030

- Channels with both tree-level and triangle diagrams: interference may give rich structure