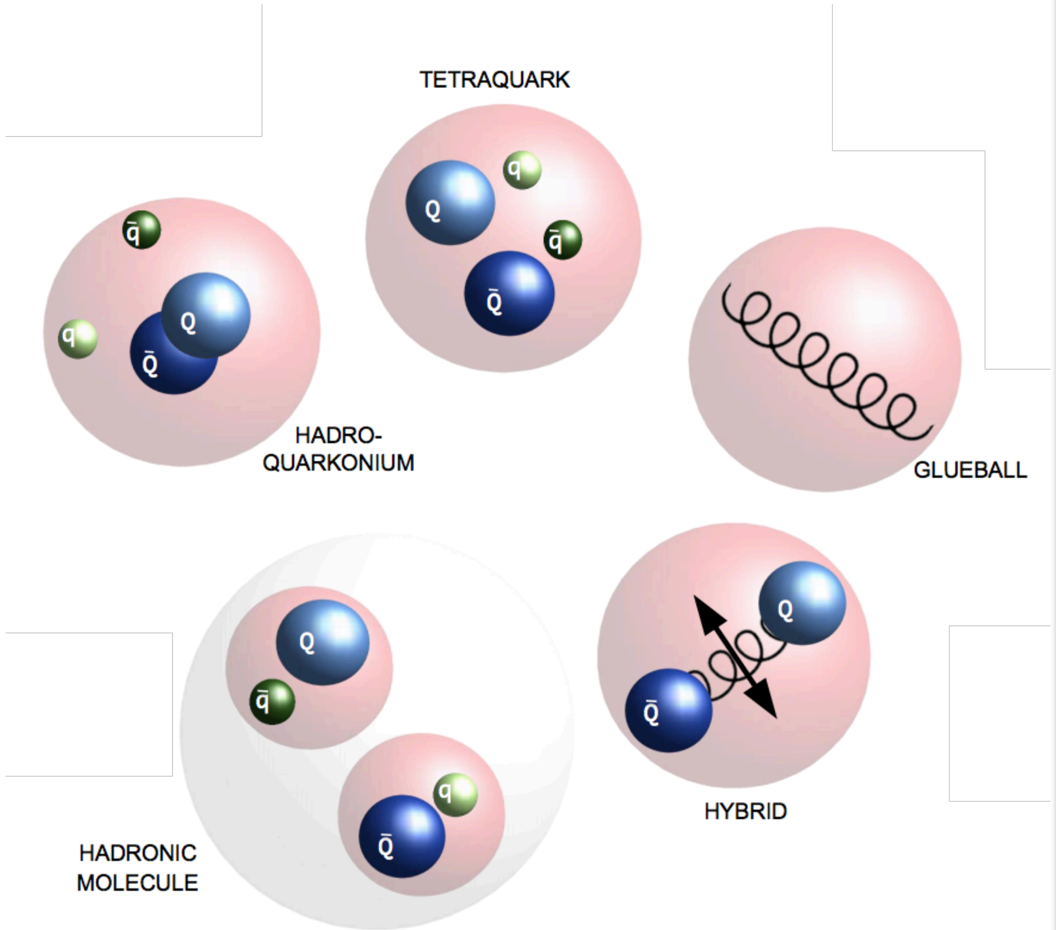

Inclusion of $\pi\pi$ interaction in heavy meson decays

Igor Danilkin

in coll. with Daniel Molnar, Oleksandra Deineka and Marc Vanderhaeghen

Phys. Lett. B 797 134851, (2019)
Phys. Rev. D 102 1, 016019, (2020)
arXiv: 2012.11636 [hep-ph], (2020)

April 14, 2021

SFB 1044
THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

JG|U

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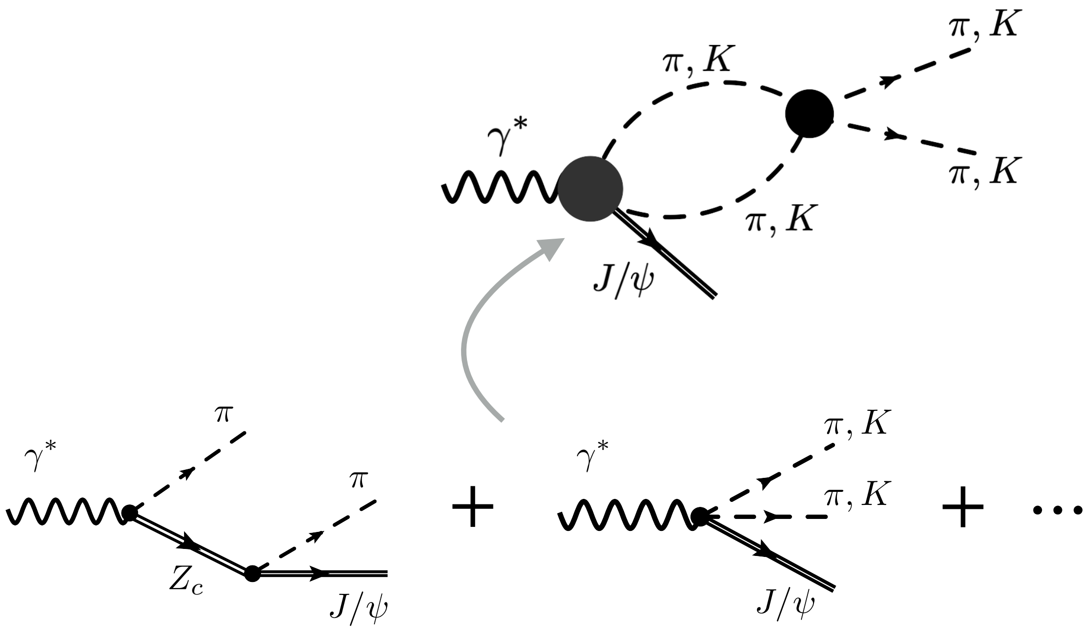


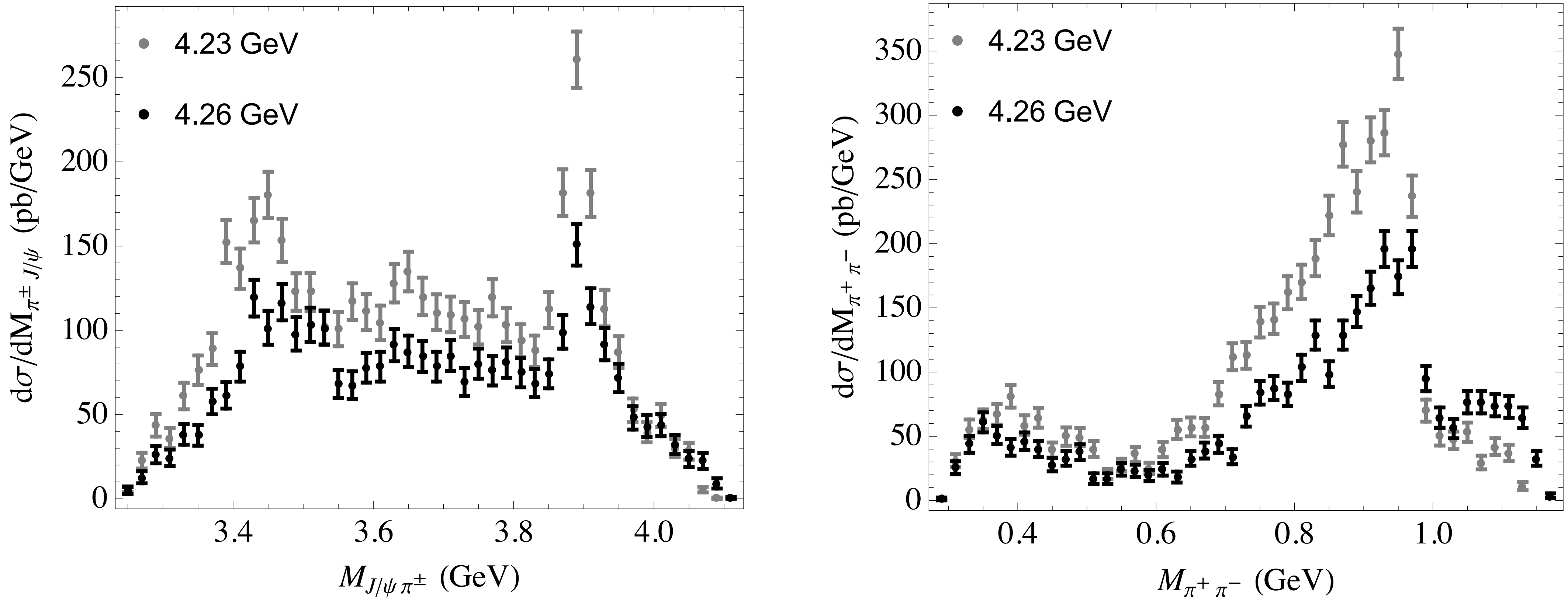
Introduction and Motivation

- New era of high precision data from BESIII, Belle II and LHCb Collaborations

- In our analysis, the recent **BESIII data** on $e^+e^- \to \pi\pi(KK)J/\psi$ and $e^+e^- \to \pi\pi\psi(2S)$ play the central role

  ➢ In these channels the discoveries of new resonances have been reported
  In 2013, Zc(3900) was discovered by BESIII and Belle Collaborations and later on by CLEO-c
  In 2015, the neutral partner of Zc(3900) was observed by BESIII Collaboration
  In 2018, $e^+e^- \to KKJ/\psi$ cross sections have been measured by BESIII Collaboration
  In 2017, indications of new Zc(4030) have been observed in $e^+e^- \to \pi\pi\psi(2S)$

$e^+e^- \to \pi\pi J/\psi$



Zc(3900)

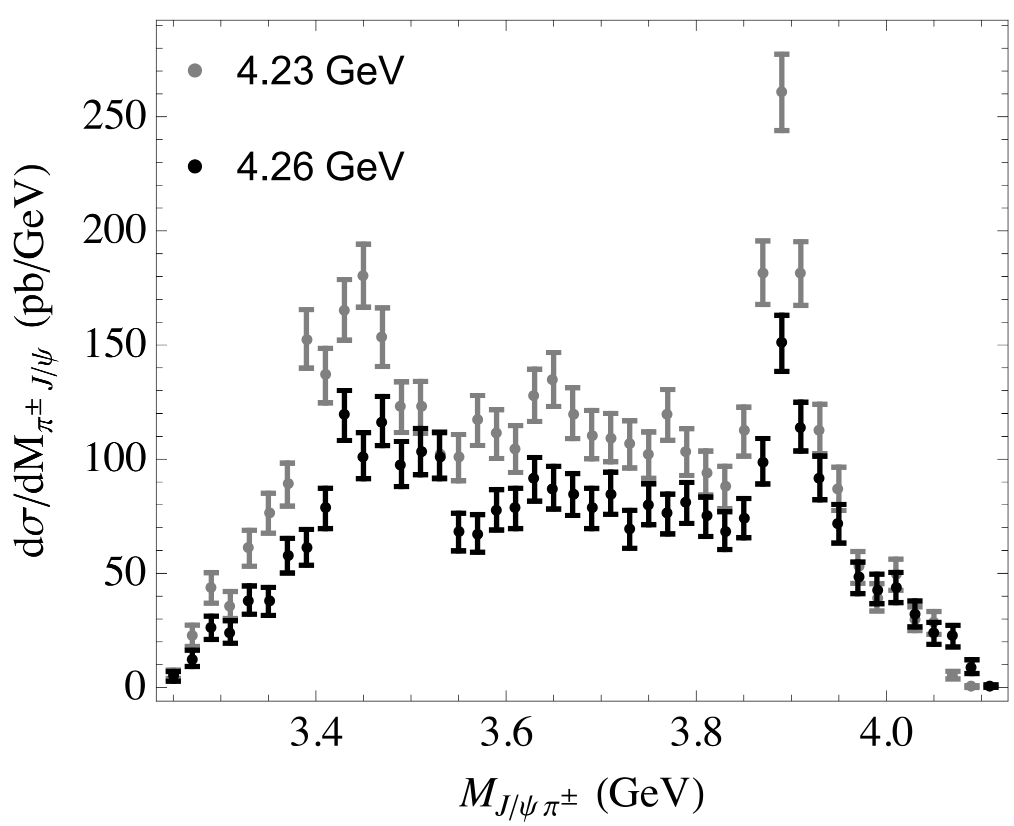
- Zc as a "smoking gun" for 4-quark meson: decays to J/ψ (must contain ccbar pair) and electrically charged (must contain udbar pair)

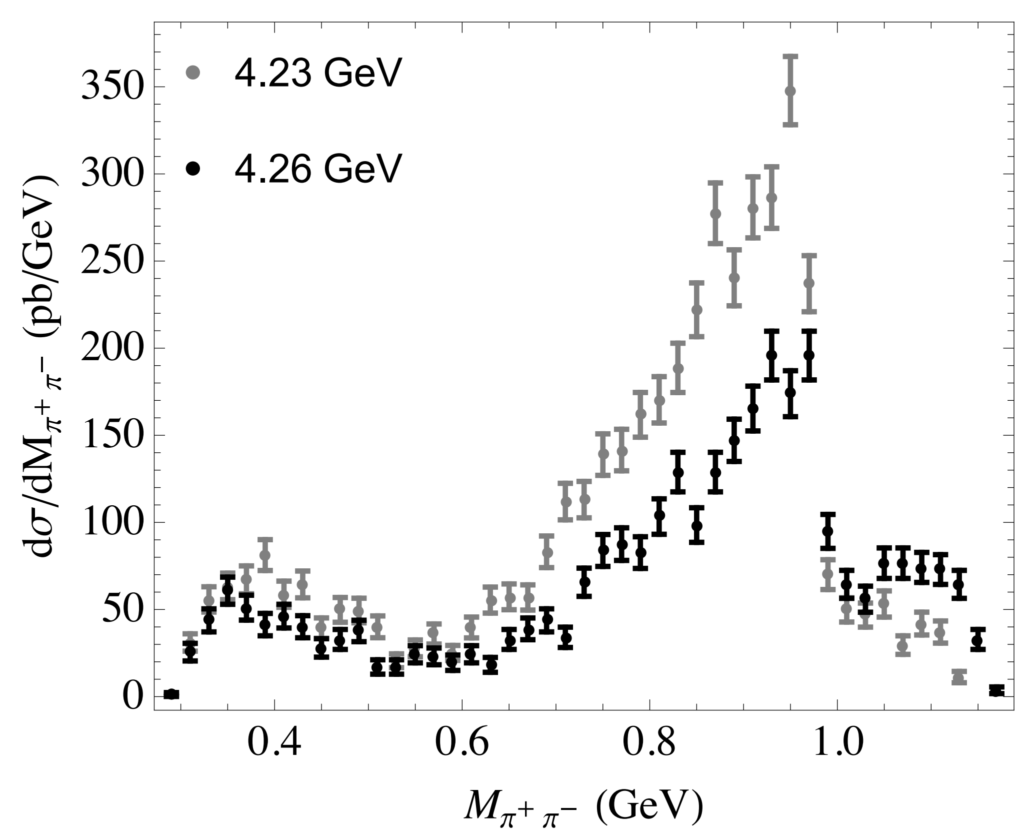
➢Tetraquark
➢DD* molecular state
➢Virtual state
➢Hadrocharmonium

pole in the unphysical Riemann sheet

Recent Reviews:
Esposito et al. (2017)
Olsen et al. (2018)
Brambilla et al. (2019)
Liu et al. (2019)
Guo et al. (2020)

➢Kinematical effect ($D_0(2300)D^*D$ or $D_1(2420)D^*D$ loops)
seems to be excluded: in both cases left-hand cut branch point stays far away from the physical region and the data at 4.23 GeV is more enhanced compared to 4.26 GeV





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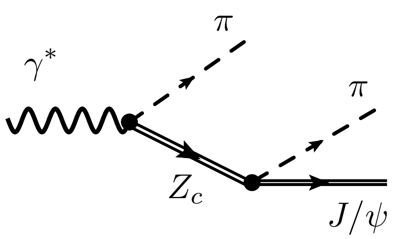
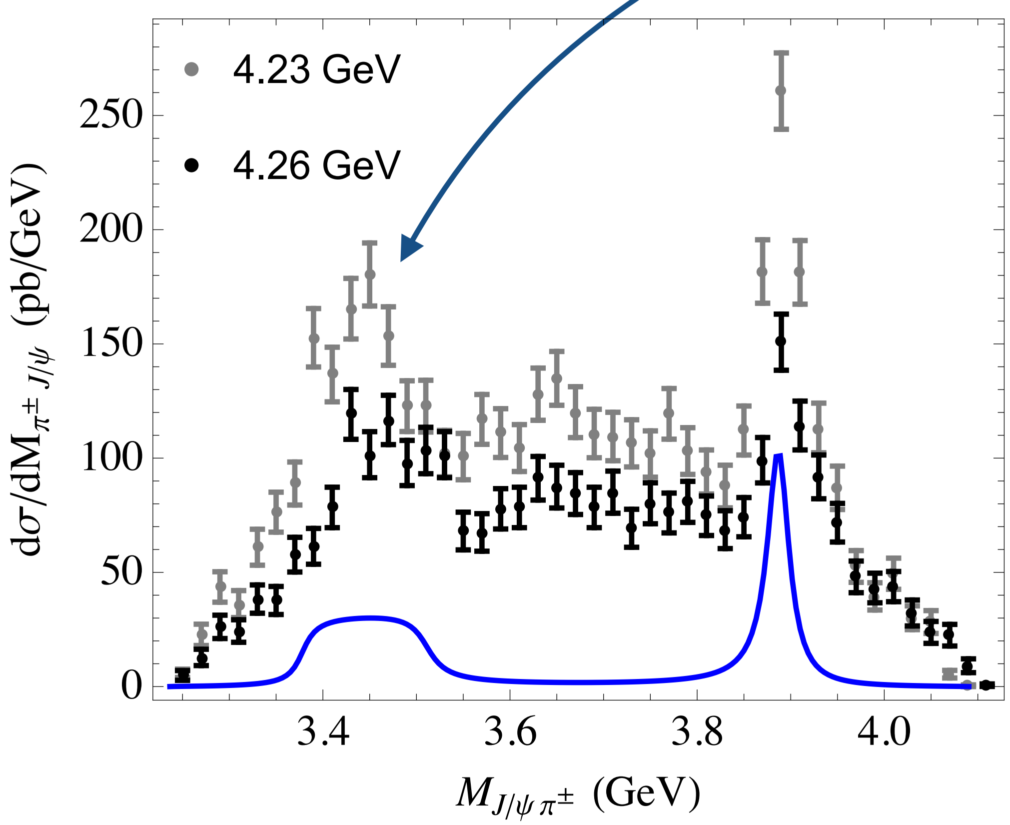
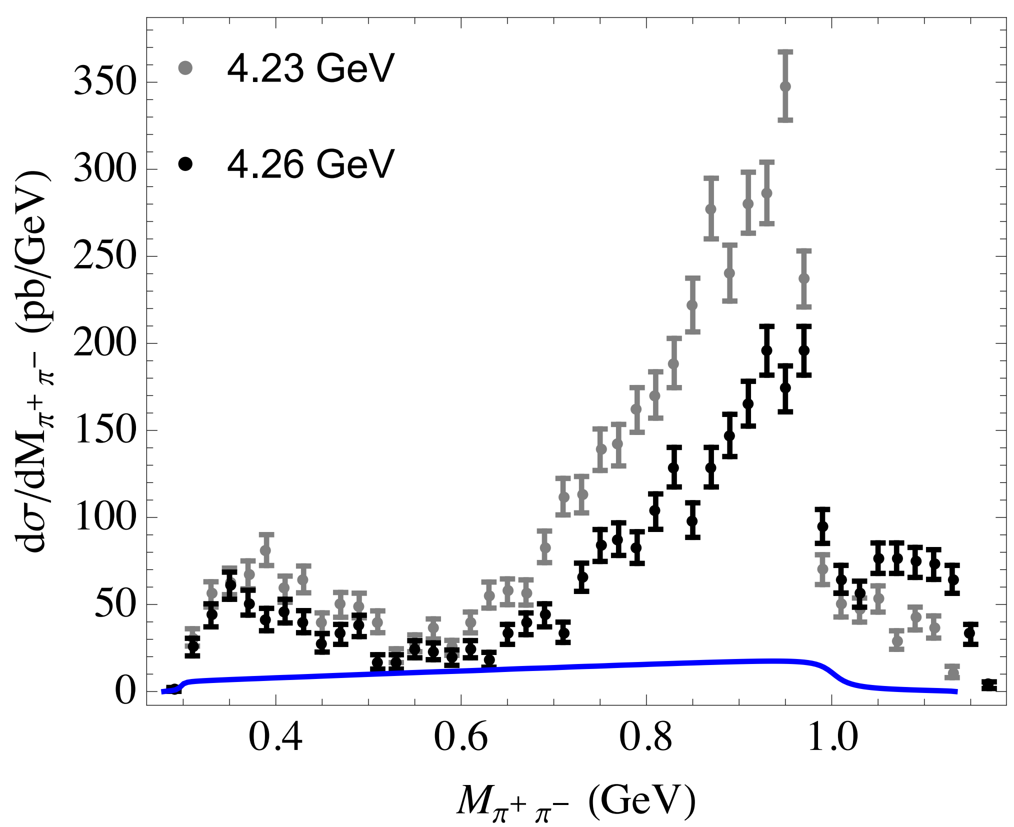
  pole in the unphysical Riemann sheet

- PDG data: mass **3888.4±2.5 MeV** and width **28.3±2.5 MeV**
  In data we see the peak and its **kinematic reflection**



+ crossed





Zc(3900)

- Zc as a "smoking gun" for 4-quark meson: decays to J/ψ (must contain ccbar pair) and electrically charged (must contain udbar pair)

  ➢Tetraquark
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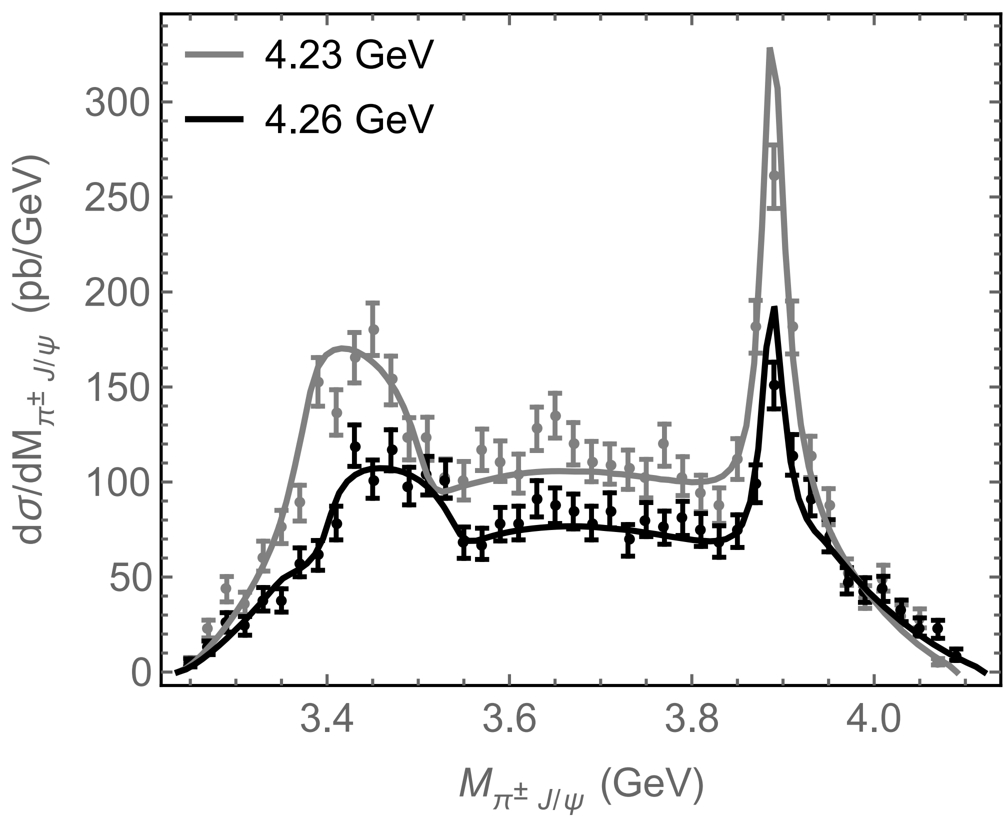
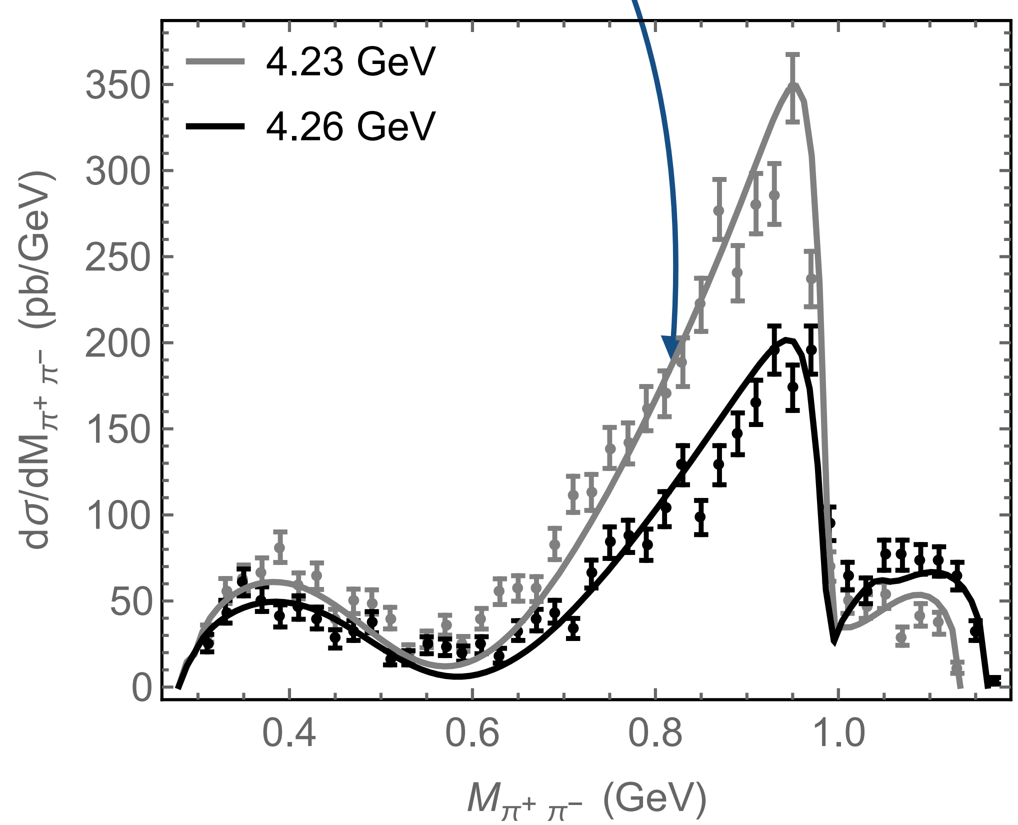
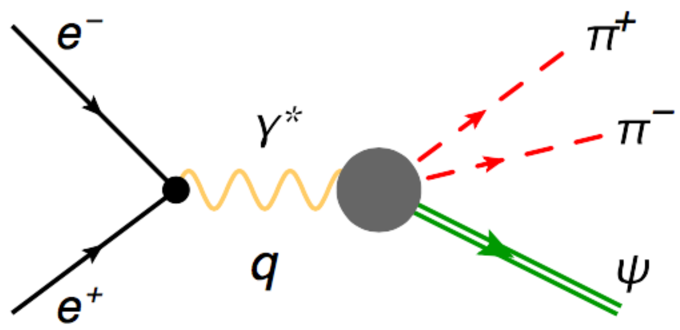
  } pole in the unphysical Riemann sheet

- One needs to include **ππ(KK) FSI (isospin=0)** in a model independent way
  ➢it serves as a background in J/ψπ channel and **affects Zc resonance parameters determination**





General scheme + assumptions

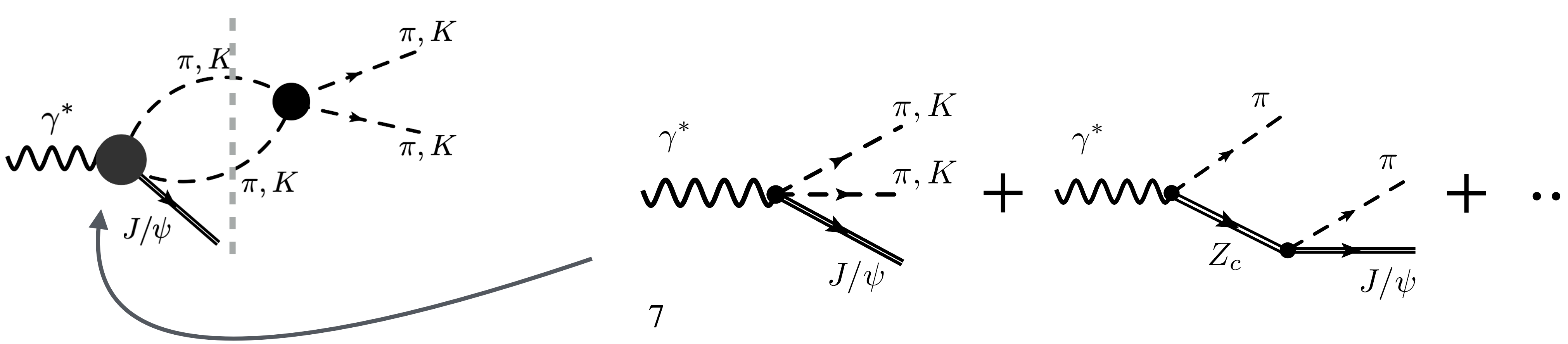


- We do not aim at a description of the full $e^+e^- \to \gamma^* \to \pi\pi(KK)\psi$ cross-section and instead apply our formalism for each value e^+e^- c.m. energy independently

- Perform a **simultaneous** description of $\pi\pi$(KK) and $\pi\psi$ invariant mass distributions

- Take **Zc(3900) as explicit d.o.f.** (i.e. minimum assumptions about its the nature)
  ➢a strange partner of Zc [recently observed Zcs(3985)] cannot be seen as peak in the KJ/ψ invariant mass distribution at 4.23 GeV and 4.26 GeV c.m. energies

- Consider **ππ(KK) FSI** in **S and D waves**
  ➢Direct interaction of two pions
  ➢Rescattering through Zc as a left-hand cut (so-called crossed-channel 3-body effect)
  ➢Other left-hand cuts absorbed in the subtraction constants



What has been done so far?

- Most of the approaches focus on the description of $\pi J/\psi$ invariant mass projection and use "effective" parameterizations in the $\pi\pi$ distribution

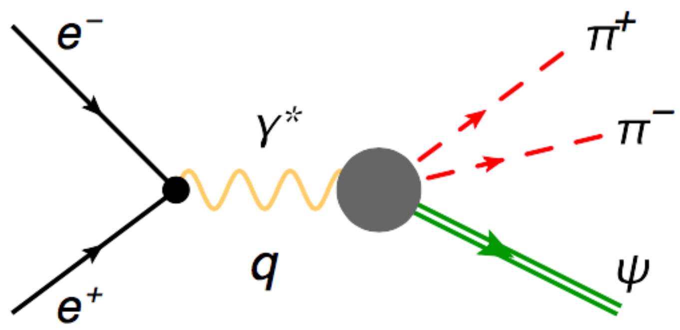
  - ➢ simplistic parameterization $a/(t-3.6)^b+c+d\,t$
  - ➢ polynomial multiplied $\pi\pi$ amplitude
  - ➢ Breit-Wigner functions ($f_0(500)$, $f_0(980)$, $f_2(1270)$,..)
  - ➢ sometimes $f_0(980)$ described with a Flatte formula

BESIII (2013)
Wang et al. (2013)
Albaladejo et al. (2015)
Pilloni et al. (2016)
BESIII (2017)
…

- Dispersion theory for $\pi\pi$(KK) rescattering has been first applied in Chen et al. (2019)

  - ➢ focused only on the $\pi\pi$ invariant mass distribution
  - ➢ similar in spirit but different in details dispersive implementation
    (Omnes function for the S-wave, kinematical singularities for the D-wave)
  - ➢ imply particular dynamics on the contact interaction to get insights into the structure of the Y(4260)
  - ➢ different outcome

Kinematics



$$s = (p_{\pi^+} + p_{\pi^-})^2 \equiv M^2_{\pi^+\pi^-}$$
$$t = (p_\psi + p_{\pi^-})^2 \equiv M^2_{\pi^-\psi}$$
$$u = (p_\psi + p_{\pi^+})^2 \equiv M^2_{\pi^+\psi}$$

- The double differential cross section is fully determined by the helicity amplitudes of the process $\gamma^*(\lambda_1) \rightarrow \pi\pi\psi(\lambda_2)$ with 5 independent contributions: ++, +-, +0, 0+, 00

$$\frac{d^2\sigma}{ds\,dt} = \frac{e^2}{2^5(2\pi)^3\,q^6}\,\frac{1}{3}\left[\sum_{\lambda_1\lambda_2} |\mathcal{H}_{\lambda_1\lambda_2}|^2\right]$$

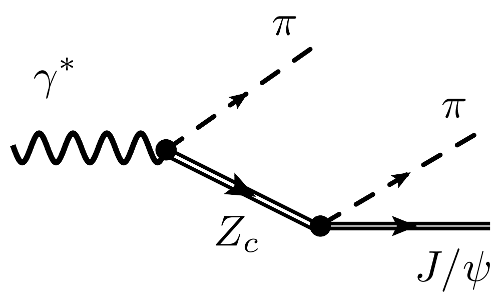
- The hadron tensor can be decomposed into a suitable set of Lorentz structures

$$\mathcal{H}_{\lambda_1\lambda_2} \equiv \mathcal{H}^{\mu\nu}\epsilon_\mu(p_{\gamma^*},\lambda_1)\,\epsilon^*_\nu(p_\psi,\lambda_2) = \left(\sum_{i=1}^{5} F_i L_i^{\mu\nu}\right)\epsilon_\mu(p_{\gamma^*},\lambda_1)\,\epsilon^*_\nu(p_\psi,\lambda_2)$$

pole contribution of Zc
(immune to the choice of the field representation)

identify all the **kinematic constraints/singularities** of p.w. helicity amplitudes
(needed for dispersive **$\pi\pi$ FSI**)

Zc exchange: pole contribution



+ crossed

- The helicity amplitude can be expressed in a general form as follows

$$\mathcal{H}^{Z_c}_{\lambda_1\lambda_2} = (V_{Z_c\psi\pi})^{\beta\nu}\, S_{\nu\mu}(Q_z)\, (V_{\gamma^*\pi Z_c})^{\mu\alpha}\epsilon_\alpha(p_{\gamma^*},\lambda_1)\,\epsilon^*_\beta(p_\psi,\lambda_2)$$
$$= \left(\sum_{i=1}^{5} F_i^{Z_c} L_i^{\mu\nu}\right)\epsilon_\mu(p_{\gamma^*},\lambda_1)\,\epsilon^*_\nu(p_\psi,\lambda_2)$$

Roca et al. (2004)
Lichard Juran (2006)

Invariant amplitudes

$$F_1^{Z_c} = -\frac{\mathcal{F}_{\gamma^*\pi Z}\, C_{Z\psi\pi}}{8}\left(\frac{4\overset{m_Z^2}{\cancel{t}} + q^2 + m_\psi^2}{t - m_Z^2} + \frac{4\overset{m_Z^2}{\cancel{u}} + q^2 + m_\psi^2}{u - m_Z^2}\right)$$
$$F_2^{Z_c} = -\frac{\mathcal{F}_{\gamma^*\pi Z}\, C_{Z\psi\pi}}{8}\left(\frac{1}{t - m_Z^2} + \frac{1}{u - m_Z^2}\right)$$

...

Tarrach (1975)
Drechsel et al. (1998)
Colangelo et al. (2015)
Danilkin et al. (2020)

- For the considered cases everything can be written in terms of H_{++} only

$$\sum_{\lambda_1\lambda_2} |\mathcal{H}_{\lambda_1\lambda_2}|^2 \approx 3\,|\mathcal{H}_{++}|^2$$

similar observation
was also made in
Chen et al. (2016, 2017, 2019)

Kinematic constraints of p.w. amplitudes

- p.w. helicity amplitudes suffers from kinematical constraints/ singularities

$$\mathcal{H}_{\lambda_1\lambda_2} \equiv \mathcal{H}^{\mu\nu}\epsilon_\mu(p_{\gamma^*},\lambda_1)\,\epsilon^*_\nu(p_\psi,\lambda_2) = \sum_{J=0}^{\infty}(2J+1)\,d^{(J)}_{\Lambda,0}(\theta_s)\,h^{(J)}_{\lambda_1\lambda_2}(s)$$
$$= \left(\sum_{i=1}^{5} F_i L_i^{\mu\nu}\right)\epsilon_\mu(p_{\gamma^*},\lambda_1)\,\epsilon^*_\nu(p_\psi,\lambda_2)$$

which can be fully identified using an expansion in a suitable set of Lorentz structures and invariant amplitudes

Danilkin et al. (2020)

$$h^{(0)}_{++}(s) \pm h^{(0)}_{00}(s) \sim \mathcal{O}(s-(q\pm m_\psi)^2)$$
$$h^{(2)}_{++}(s) \pm \ldots$$

$$\sum_{\lambda_1\lambda_2}|\mathcal{H}_{\lambda_1\lambda_2}|^2 \approx 3\,|\mathcal{H}_{++}|^2$$

$$h^{(0)}_{++}(s) \sim \text{no constraints}$$
$$h^{(2)}_{++}(s) \sim \mathcal{O}(s-4m_\pi^2)(s-(q-m_\psi)^2)$$

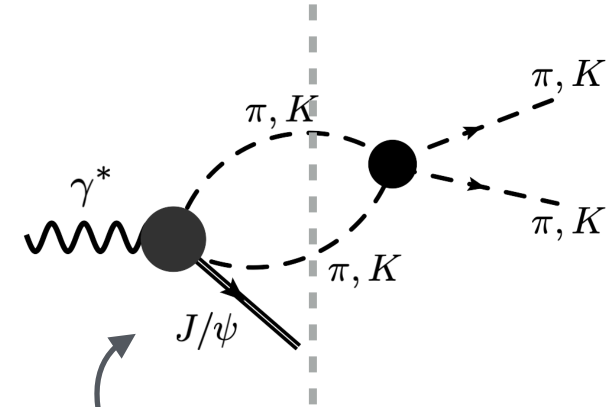
also a minimum constraint that needed to compensate the singularities of $cos\theta_s$ in the physical region

$$d^{(2)}_{0,0}(\theta_s) \sim \cos^2\theta_s, \;\; \cos\theta_s = \frac{t-u}{\kappa(s)}$$

Final state interaction

- Unitarity relation

$$\mathrm{Disc}\, h^{(J)}_{++}(s) = t^{(J)*}(s)\,\rho(s)\,h^{(J)}_{++}(s)$$



- Split contributions from the left and right-hand cuts

$$h^{(J)}_{++}(s) = h^{(J),L}_{++}(s) + h^{(J),R}_{++}(s)$$

$$\mathcal{H}_{++}(s,t) = \mathcal{H}^{L}_{++}(s,t) + \sum_{J=0}^{2}(2J+1)\,d^{(2)}_{0,0}(\theta_s)\,h^{(J),R}_{++}(s)$$

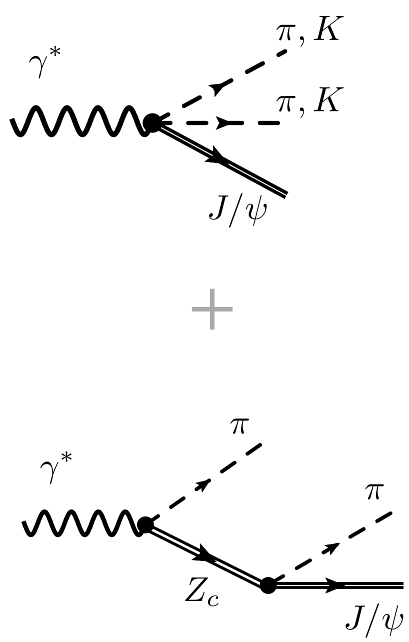
- Direct rescattering

$$\begin{bmatrix} h^{(0),R}_{++}(s) \\ k^{(0),R}_{++}(s) \end{bmatrix} = \Omega^{(0)}(s)\left\{ \begin{bmatrix} a' + .. \\ c' + .. \end{bmatrix} \right\}$$

hard to distinguish these contributions when combined

- Rescattering through Zc (crossed-channel effects)

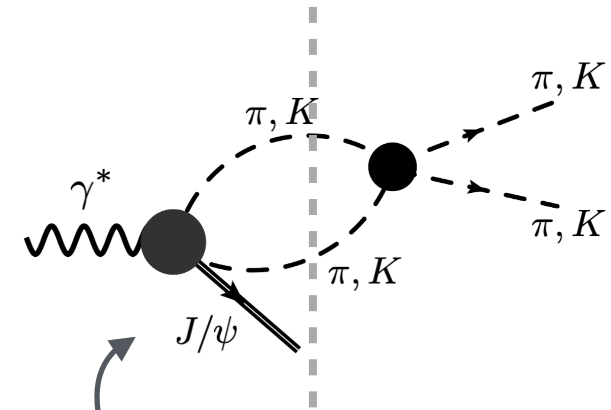
$$\begin{bmatrix} h^{(0),R}_{++}(s) \\ k^{(0),R}_{++}(s) \end{bmatrix} = \Omega^{(0)}(s)\left\{ \begin{bmatrix} a'' \\ c'' \end{bmatrix} - \frac{s}{\pi}\int_{4m_\pi^2}^{\infty} \frac{ds'}{s'}\,\frac{\mathrm{Disc}\,(\Omega^{(0)}(s'))^{-1}}{s'-s} \begin{bmatrix} h^{(0),L}_{++}(s') \\ k^{(0),L}_{++}(s') \end{bmatrix} \right\}$$



Final state interaction

- Unitarity relation

$$\mathrm{Disc}\, h_{++}^{(J)}(s) = t^{(J)*}(s)\,\rho(s)\,h_{++}^{(J)}(s)$$



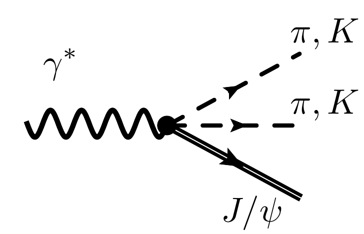
- Split contributions from the left and right-hand cuts

$$h_{++}^{(J)}(s) = h_{++}^{(J),L}(s) + h_{++}^{(J),R}(s)$$

$$\mathcal{H}_{++}(s,t) = \mathcal{H}_{++}^{L}(s,t) + \sum_{J=0}^{2}(2J+1)\,d_{0,0}^{(2)}(\theta_s)\,h_{++}^{(J),R}(s)$$

- Direct rescattering

$$\begin{bmatrix} h_{++}^{(0),R}(s) \\ k_{++}^{(0),R}(s) \end{bmatrix} = \Omega^{(0)}(s)\left\{ \begin{bmatrix} a' + b's \\ c' + d's \end{bmatrix} \right\}$$



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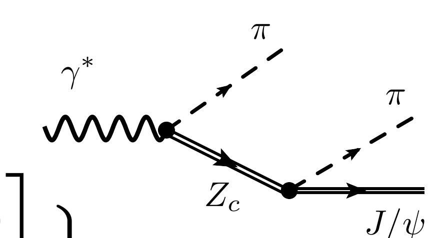
one needs to look at **over-subtracted DR** and **sum rule values**

- Rescattering through Zc (crossed-channel effects)



$$\begin{bmatrix} h_{++}^{(0),R}(s) \\ k_{++}^{(0),R}(s) \end{bmatrix} = \Omega^{(0)}(s)\left\{ \begin{bmatrix} a'' + b''\,s \\ c'' + d''\,s \end{bmatrix} - \frac{s^2}{\pi}\int_{4m_\pi^2}^{\infty} \frac{ds'}{s'^2}\,\frac{\mathrm{Disc}\,(\Omega^{(0)}(s'))^{-1}}{s'-s} \begin{bmatrix} h_{++}^{(0),L}(s') \\ k_{++}^{(0),L}(s') \end{bmatrix} \right\}$$

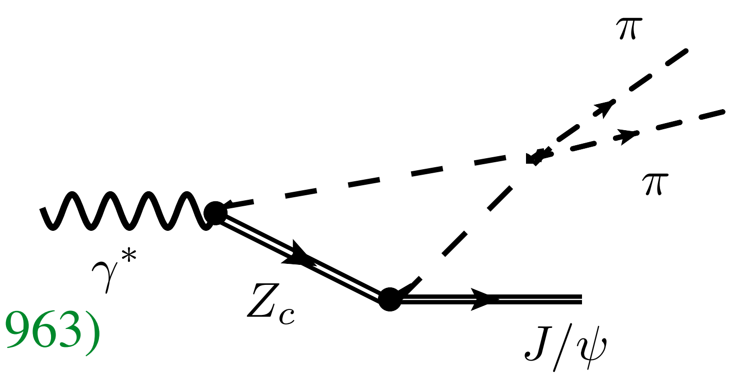
+

...

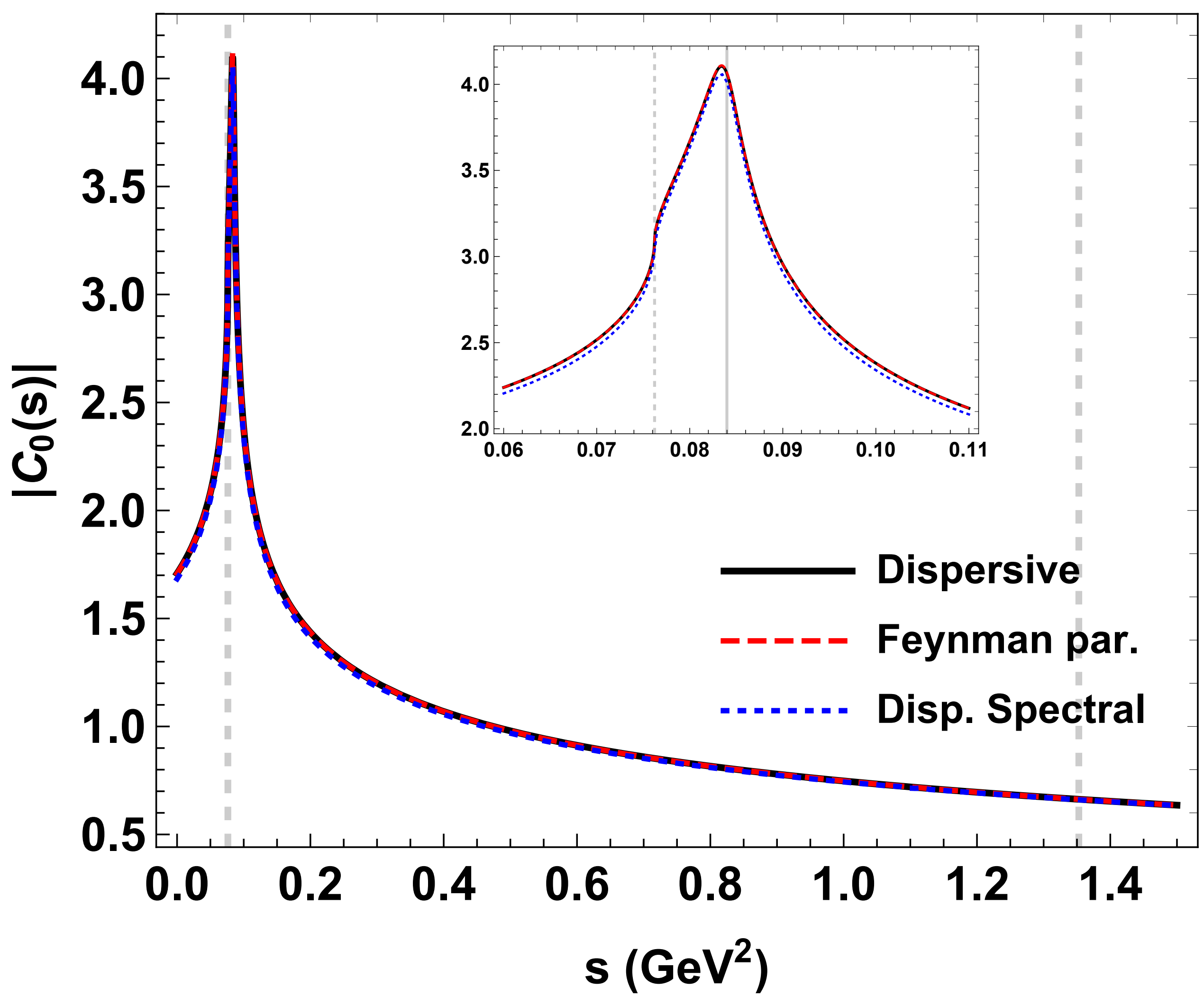
- Other l.h.c. effects → absorb in over-subtracted dispersion relation

Overlap of the cuts

- Triangle singularity condition associated with Zcππ loop
  - ➢ Zc(3900) can be produced on shell
  - ➢ l.h.c branch point located just above two pion threshold (pinching condition)
  - ➢ proper analytical continuation $q^2 \to q^2 + i0$



Bronzan, Kacser (1963)
Moussallam (2013)
Szczepaniak (2015)



Naive way (can be cross-checked using scalar triangle loop)

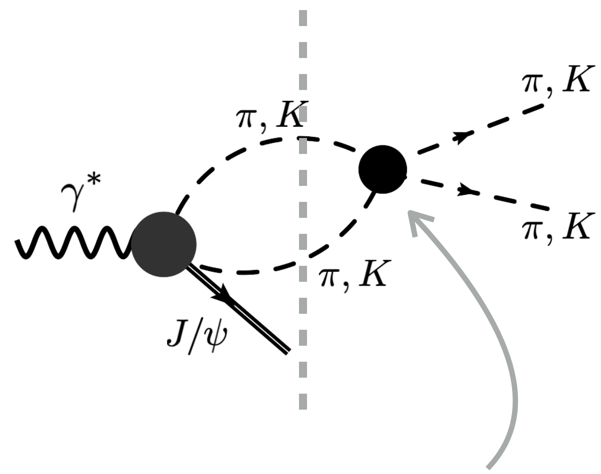
$$\frac{1}{t - m_Z^2} \to \frac{1}{t - m_Z^2 + i m_Z \Gamma_Z}$$

Spectral representation (does not bring much change)

$$\Gamma_Z \to \Gamma_Z(t)$$

$$\frac{1}{t - m_Z^2} \to \int_{t_{th}}^{\infty} \frac{dt'}{\pi} \frac{\text{Disc BW}(t')}{t' - t}$$

Omnes function for {ππ, KK}



- Even though the ππ→ππ (and to lesser extent ππ→KK) amplitudes are known very well from the Roy (Roy-Steiner) analyses, in practical dispersive applications the Final State Interactions are implemented with the help of the so-called Omnes function, which does not have left-hand cut

$$\mathrm{Disc}\,\Omega_{ab}(s) = \sum_c t^*_{ac}(s)\,\rho_c(s)\,\Omega_{cb}(s)\,, \quad s > s_{th}$$

- We employ the data-driven N/D formalism, where the Omnes functions come out naturally, as the inverse of the D-functions

p.w. dispersion relation

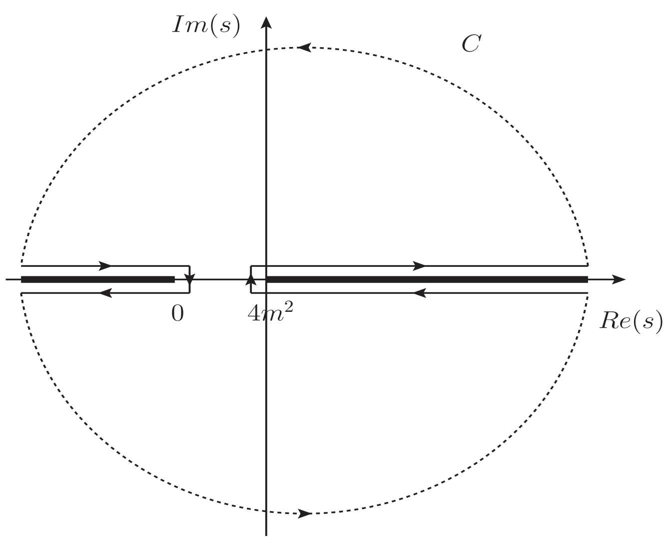
- **Unitarity relation** for the p.w. amplitude
  ➢ guarantees that the p.w. amplitudes behave asymptotically no worse **than a constant**

$$\mathrm{Disc}\, t_{ab}(s) = \sum_c t_{ac}(s)\, \rho_c(s)\, t^*_{cb}(s)$$

$$-\frac{1}{2\rho_1} \le \mathrm{Re}\, t_{11}(s) \le \frac{1}{2\rho_1}, \qquad 0 < \mathrm{Im}\, t_{11}(s) \le \frac{1}{\rho_1}$$

$$\ldots$$

- Based on maximal analyticity principle on can write **p.w. dispersion relation**



$$t_{ab}(s) = \int_{-\infty}^{s_L} \frac{ds'}{\pi} \frac{\mathrm{Disc}\ t_{ab}(s')}{s'-s} + \int_{s_{th}}^{\infty} \frac{ds'}{\pi} \frac{\mathrm{Disc}\ t_{ab}(s')}{s'-s}$$

which we subtract once in accordance with **unitarity bound**

$$t_{ab}(s) = U_{ab}(s) + \frac{s}{\pi} \int_{s_{th}}^{\infty} \frac{ds'}{s'} \frac{\mathrm{Disc}\ t_{ab}(s')}{s'-s}$$

included subtraction constant and left-hand cuts, asymptotically bounded **unknown function**

N/D method

- Once-subtracted p.w. dispersion relation

$$t_{ab}(s) = U_{ab}(s) + \frac{s}{\pi}\int_{s_{th}}^{\infty}\frac{ds'}{s'}\frac{\text{Disc}\; t_{ab}(s')}{s'-s}$$

can be solved using N/D method with input from $U_{ab}(s)$ **above threshold**

Chew, Mandelstam (1960)
Luming (1964)
Johnson, Warnock (1981)

$$t_{ab}(s) = \sum_c D^{-1}_{ac}(s)\, N_{cb}(s)$$

$$N_{ab}(s) = U_{ab}(s) + \frac{s}{\pi}\sum_c\int_{s_{th}}^{\infty}\frac{ds'}{s'}\frac{N_{ac}(s')\,\rho_c(s')\,(U_{cb}(s') - U_{cb}(s))}{s'-s}$$

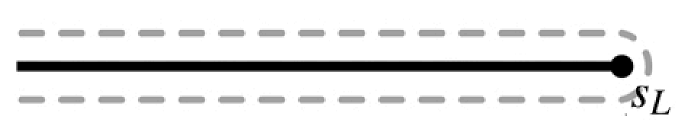
$$D_{ab}(s) = \delta_{ab} - \frac{s}{\pi}\int_{s_{th}}^{\infty}\frac{ds'}{s'}\frac{N_{ab}(s')\,\rho_b(s')}{s'-s} = \Omega^{-1}_{ab}(s)$$

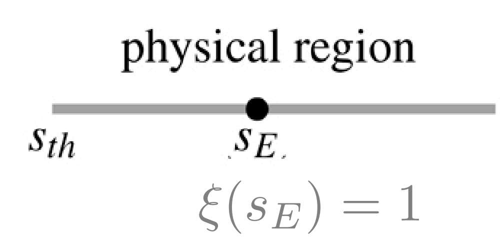
Gasparyan, Lutz (2010)

- In general scattering problem, little is known about left-hand cuts, except their analytical structure in the complex plane. We approximate them as an expansion in a **conformal mapping variable** $\xi(s)$

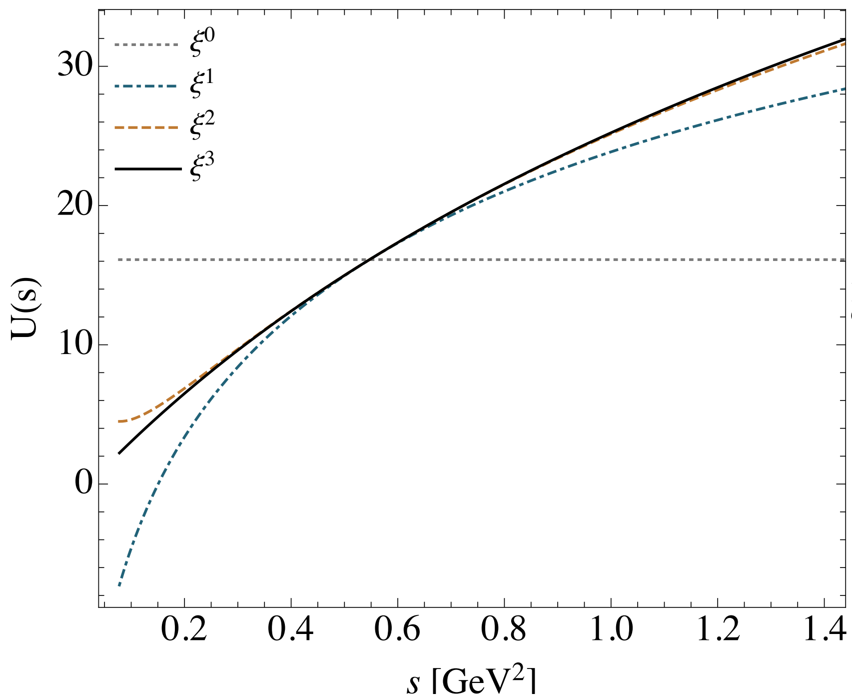
$$U_{ab}(s) = \sum_{n=0}^{\infty} C_{ab,n}\,(\xi_{ab}(s))^n$$

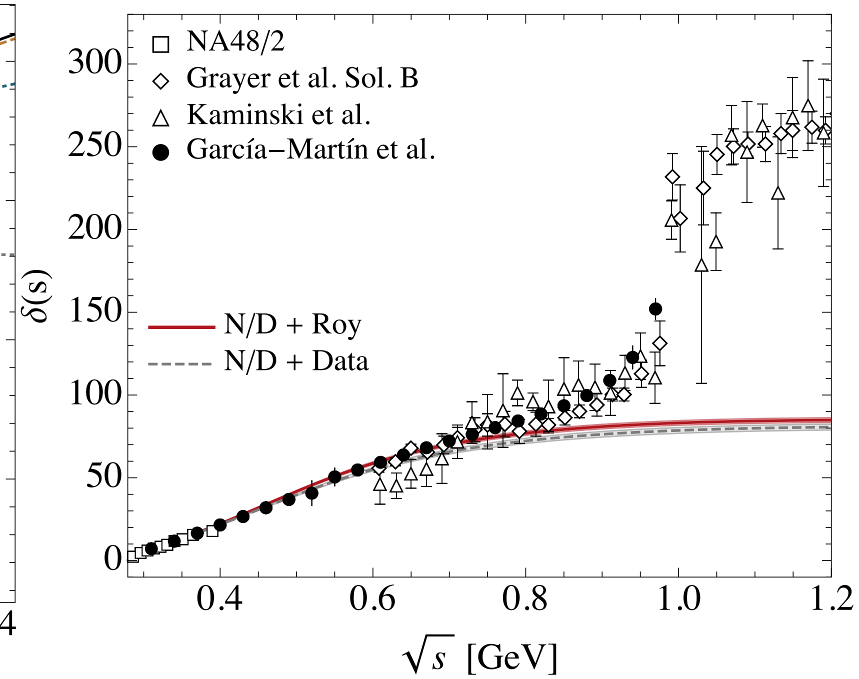
unknown coefficients fitted to data

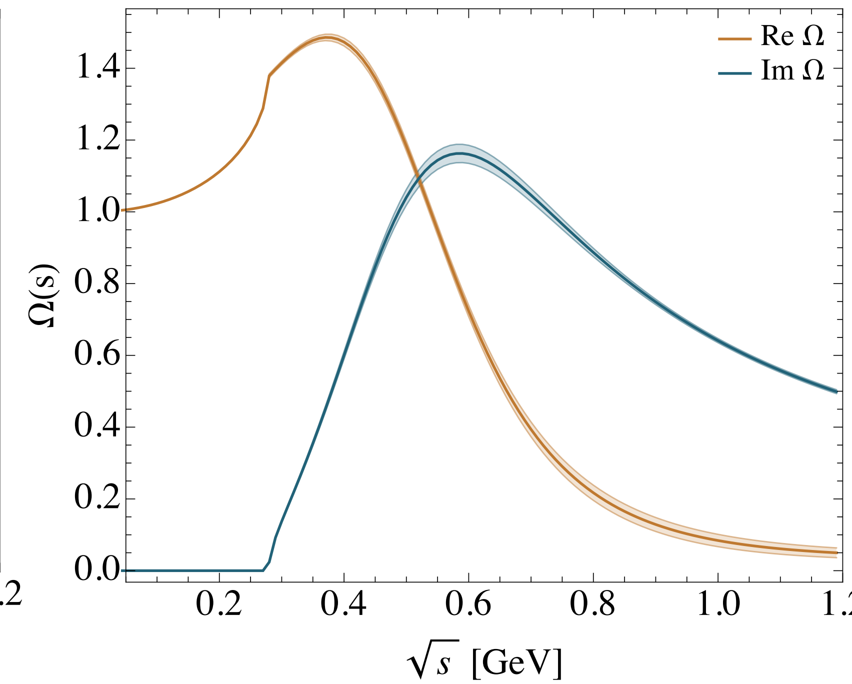




single-channel {ππ}







	Our results		Roy-like analyses	
	pole position, MeV	coupling, GeV	pole position, MeV	coupling, GeV
$\sigma/f_0(500)$	$458(7)^{+4}_{-10} - i\,245(6)^{+7}_{-10}$	$\pi\pi : 3.15(5)^{+0.11}_{-0.20}$	$449^{+22}_{-16} - i\,275(15)$	$\pi\pi : 3.45^{+0.25}_{-0.29}$

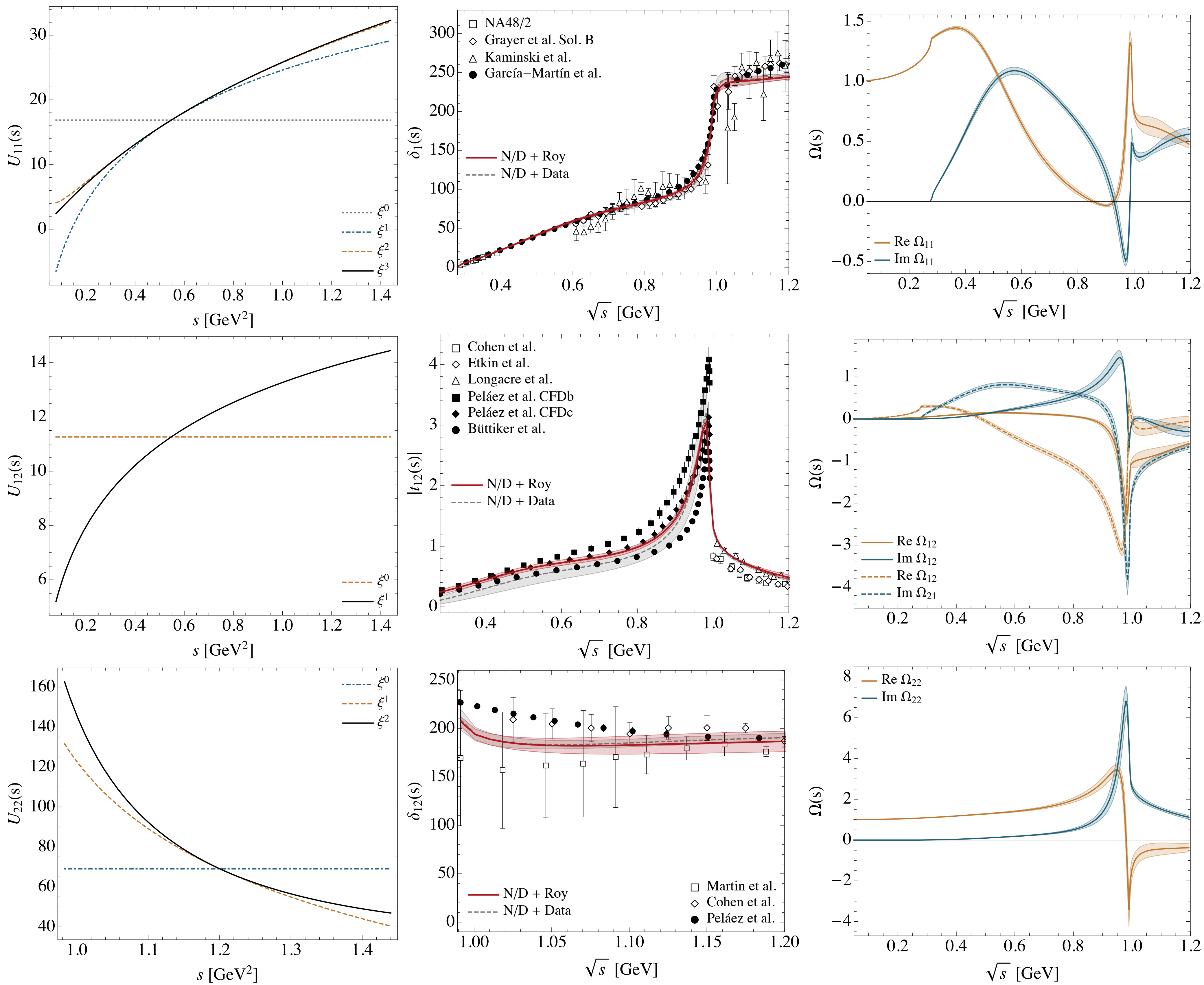
Caprini et al. (2006)
Garcia-Martin et al. (2011)

$$\Omega(s) = D^{-1}(s) = \exp\left(\frac{s}{\pi}\int_{s_{th}}^{\infty}\frac{ds'}{s'}\frac{\delta(s')}{s'-s}\right)$$

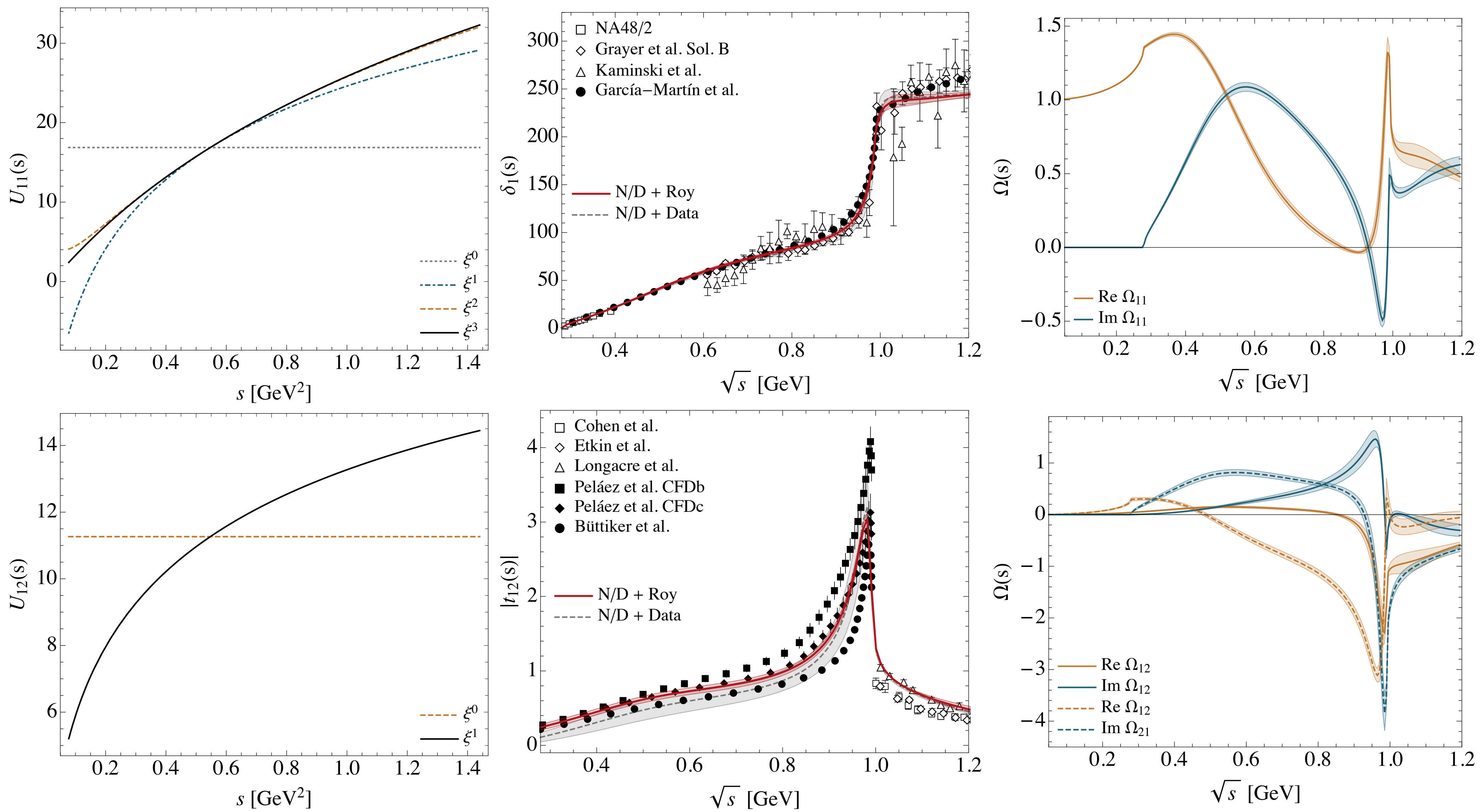
- Similar results for single-channel ππ phase-shift and Omnes function can obtained by using mIAM with the ChPT input for the left-hand cuts and subtraction constants

Gomez Nicola et al. (2008)
Hanhart et al. (2008)
Nebreda et al. (2010)
Pelaez et al. (2010)

coupled-channel {ππ, KK}



coupled-channel {ππ, KK}



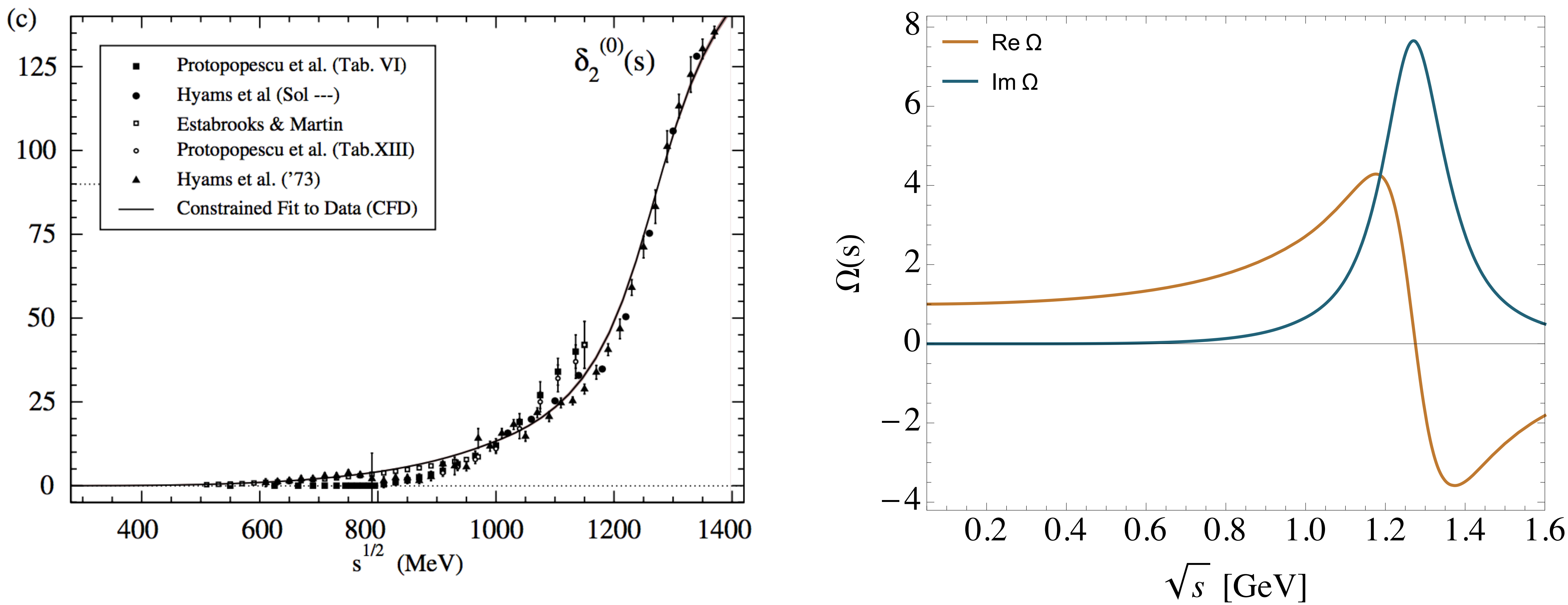
	Our results		Roy-like analyses	
	pole position, MeV	couplings, GeV	pole position, MeV	couplings, GeV
$\sigma/f_0(500)$	$458(10)^{+7}_{-15} - i\,256(9)^{+5}_{-8}$	$\pi\pi : 3.33(8)^{+0.12}_{-0.20}$ $K\bar{K} : 2.11(17)^{+0.27}_{-0.11}$	$449^{+22}_{-16} - i\,275(15)$	$\pi\pi : 3.45^{+0.25}_{-0.29}$ $K\bar{K} : -$
$f_0(980)$	$993(2)^{+2}_{-1} - i\,21(3)^{+2}_{-4}$	$\pi\pi : 1.93(15)^{+0.07}_{-0.12}$ $K\bar{K} : 5.31(24)^{+0.04}_{-0.24}$	$996^{+7}_{-14} - i\,25^{+11}_{-6}$	$\pi\pi : 2.3(2)$ $K\bar{K} : -$

d-wave ππ

- In the N/D formalism $f_2(1270)$ could be implemented as a CDD pole
  - ➢ predominantly decays to ππ and therefor can be easily implemented using

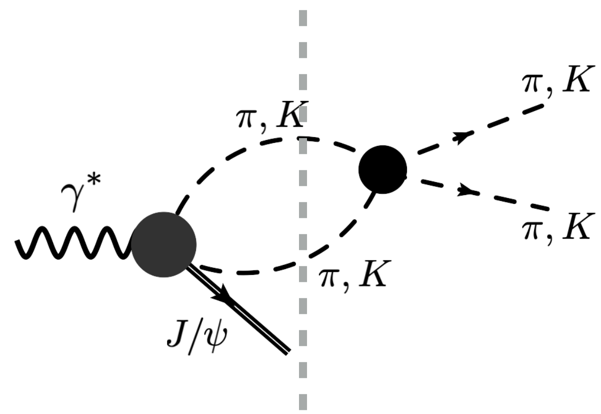
$$\Omega^{(2)}(s) = \exp\left(\frac{s}{\pi}\int_{s_{th}}^{\infty}\frac{ds'}{s'}\frac{\delta^{(2)}_{I=0}(s')}{s'-s}\right)$$

  - ➢ input: Madrid-Krakow fit [Garcia-Martin et al. (2011)] which is smoothly continued to180°



Results for $e^+e^-\to\pi\pi J/\psi$

- Are these the main ingredients?



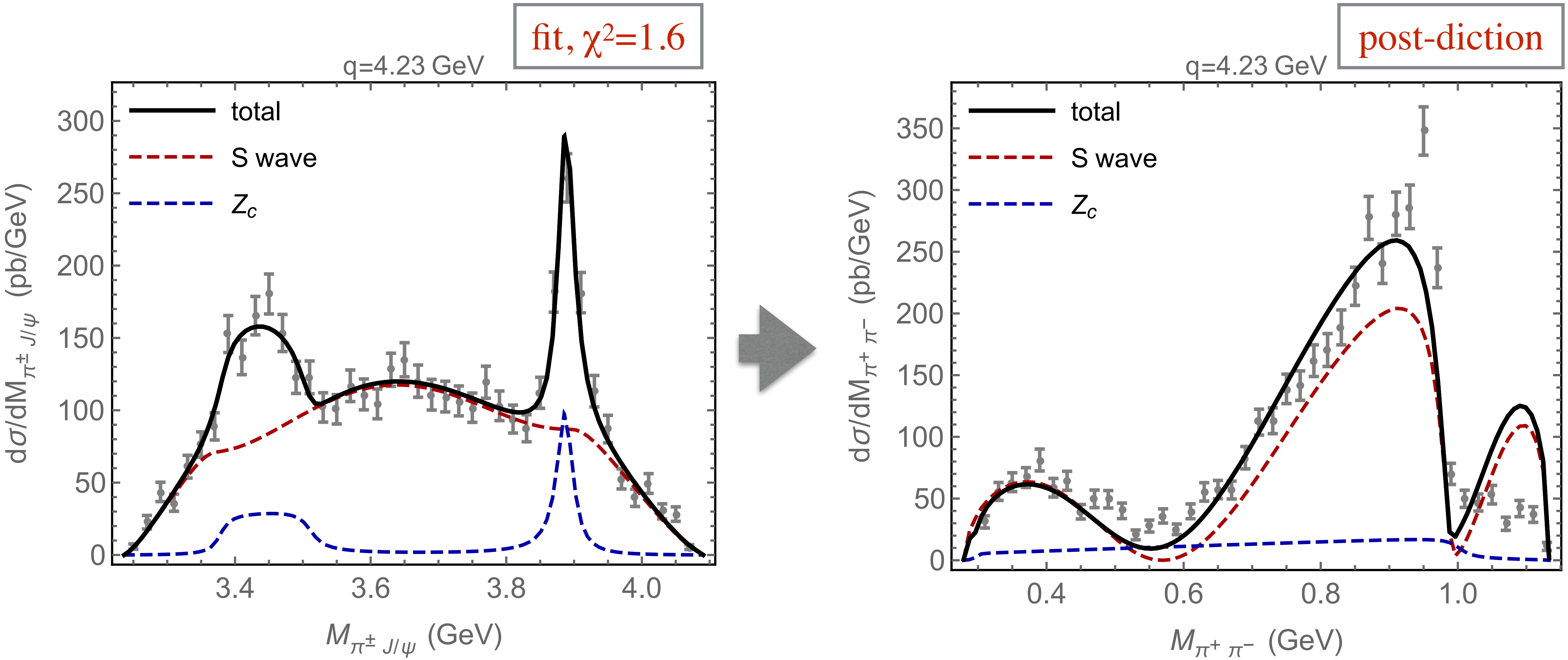
➢Take **Zc(3900) as explicit d.o.f.** (i.e. minimum assumptions about its the nature)
➢at 4.23 GeV and 4.26 GeV c.m. energies can not see **Zcs** as a peak
➢Consider **rescattering between pions** (kaons) in the final state only in **S and D waves**

$$
\begin{aligned}
\mathcal{H}_{++}(s,t) = \frac{1}{\sqrt{3}}\Bigg[\mathcal{H}^{Z_c}_{0,++}(s,t) &+ \Omega^{(0)}_{11}\left\{a + b\,s - \frac{s^2}{\pi}\int_{4m_\pi^2}^{\infty}\frac{ds'}{s'^2}\frac{\mathrm{Disc}\,(\Omega^{(0)}(s'))^{-1}_{11}}{s'-s}h^{(0),Z_c}_{0,++}(s')\right\}\\
&+ \Omega^{(0)}_{12}\left\{c + d\,s - \frac{s^2}{\pi}\int_{4m_\pi^2}^{\infty}\frac{ds'}{s'^2}\frac{\mathrm{Disc}\,(\Omega^{(0)}(s'))^{-1}_{21}}{s'-s}h^{(0),Z_c}_{0,++}(s')\right\}\\
&+ 5\,P_2(z)\,\gamma(s)\,\Omega^{(2)}\left\{e - \frac{s}{\pi}\int_{4m_\pi^2}^{\infty}\frac{ds'}{s'}\frac{\mathrm{Disc}\,(\Omega^{(2)}(s'))^{-1}}{s'-s}\frac{h^{(2),Z_c}_{0,++}(s')}{\gamma(s')}\right\}\Bigg]
\end{aligned}
$$

$$\gamma(s) \equiv (s-4m_\pi^2)(s-(q-m_\psi)^2)$$

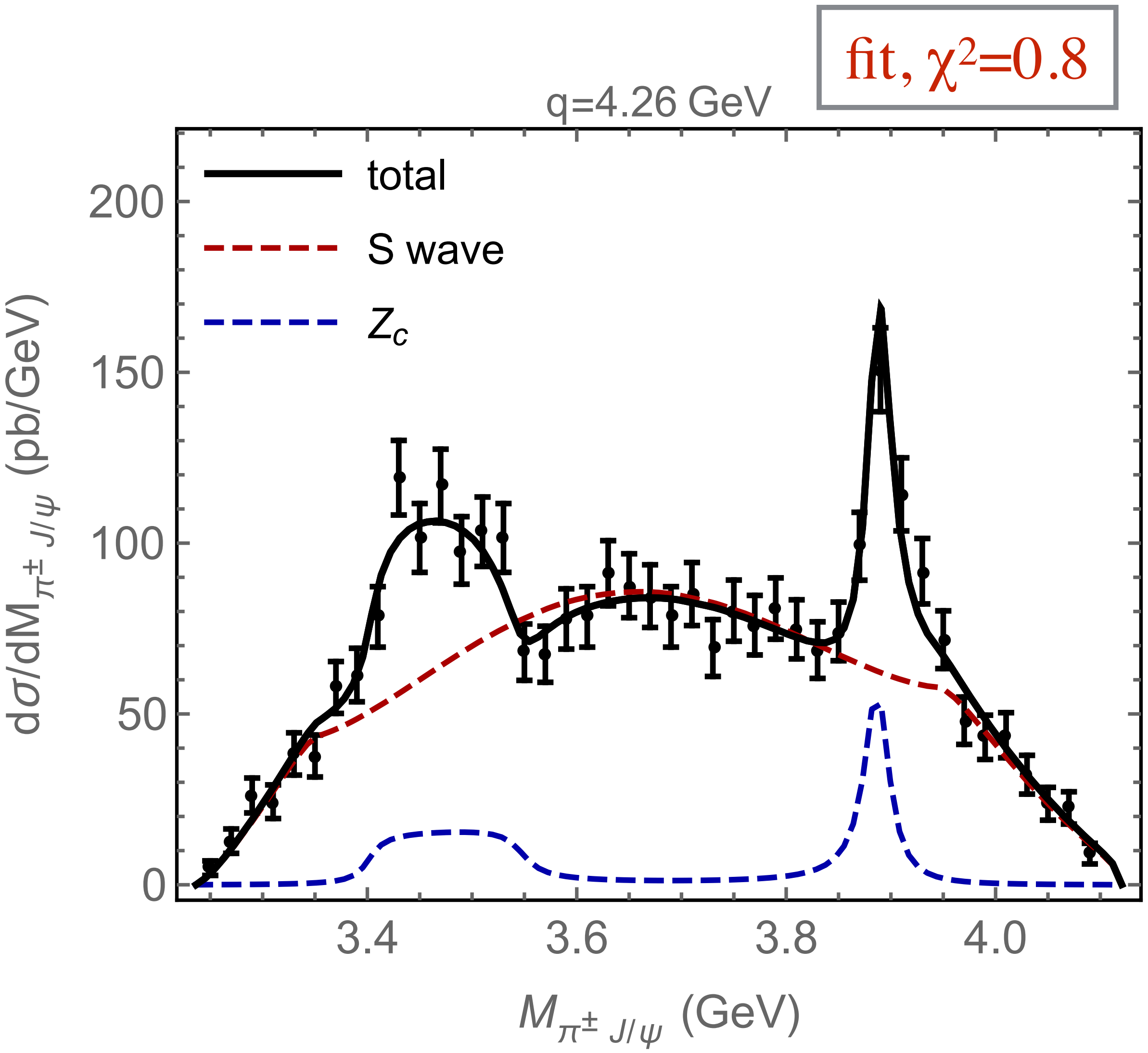
Results for $e^+e^- \to \pi\pi J/\psi$

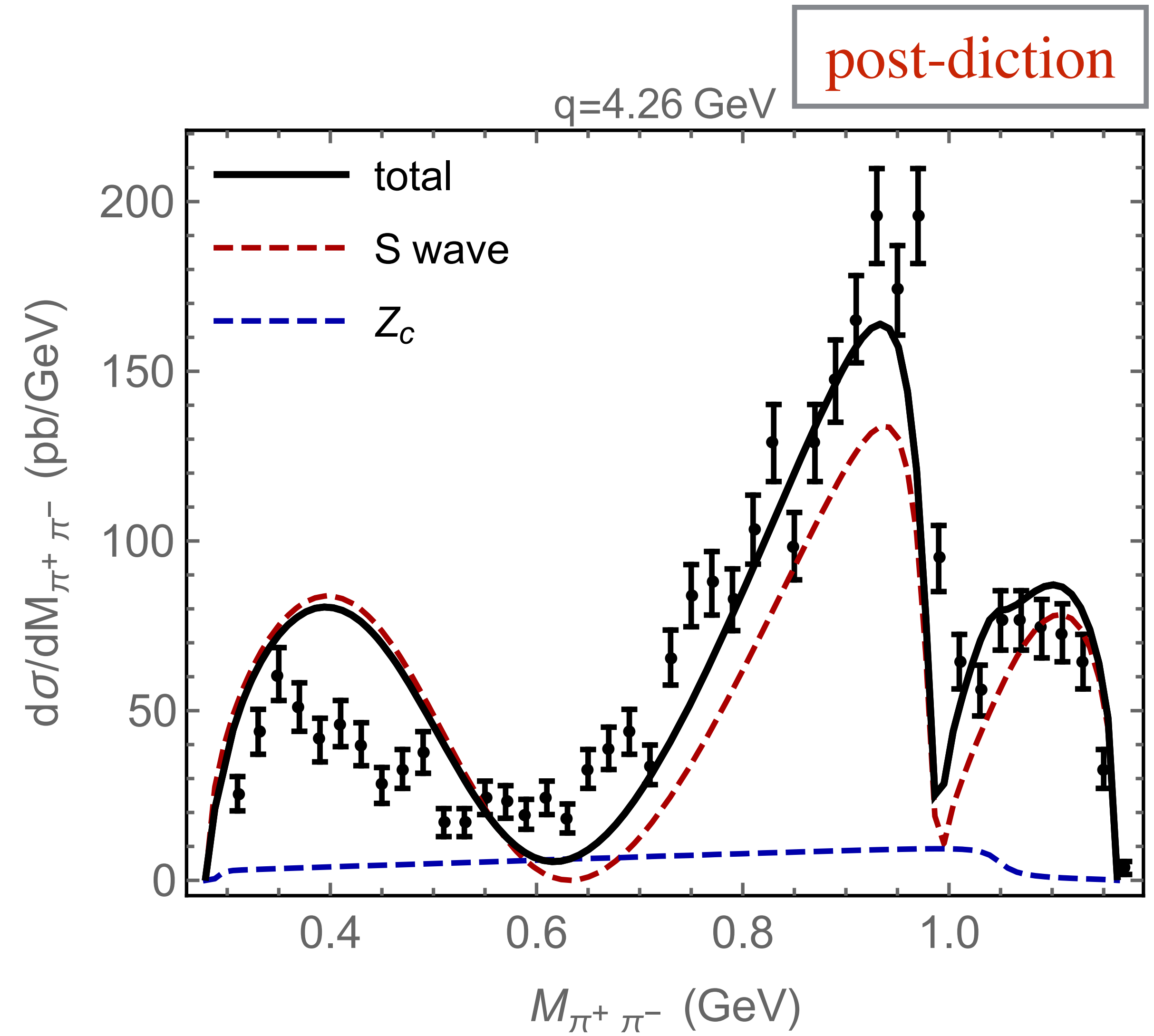
➢ Economic fits: **S-wave** a, b, (c), d real, **D-wave** no subtractions (4 parameters)



Results for $e^+e^- \to \pi\pi J/\psi$

➢ Economic fits: **S-wave** a, b, (c), d real, **D-wave** no subtractions (4 parameters)



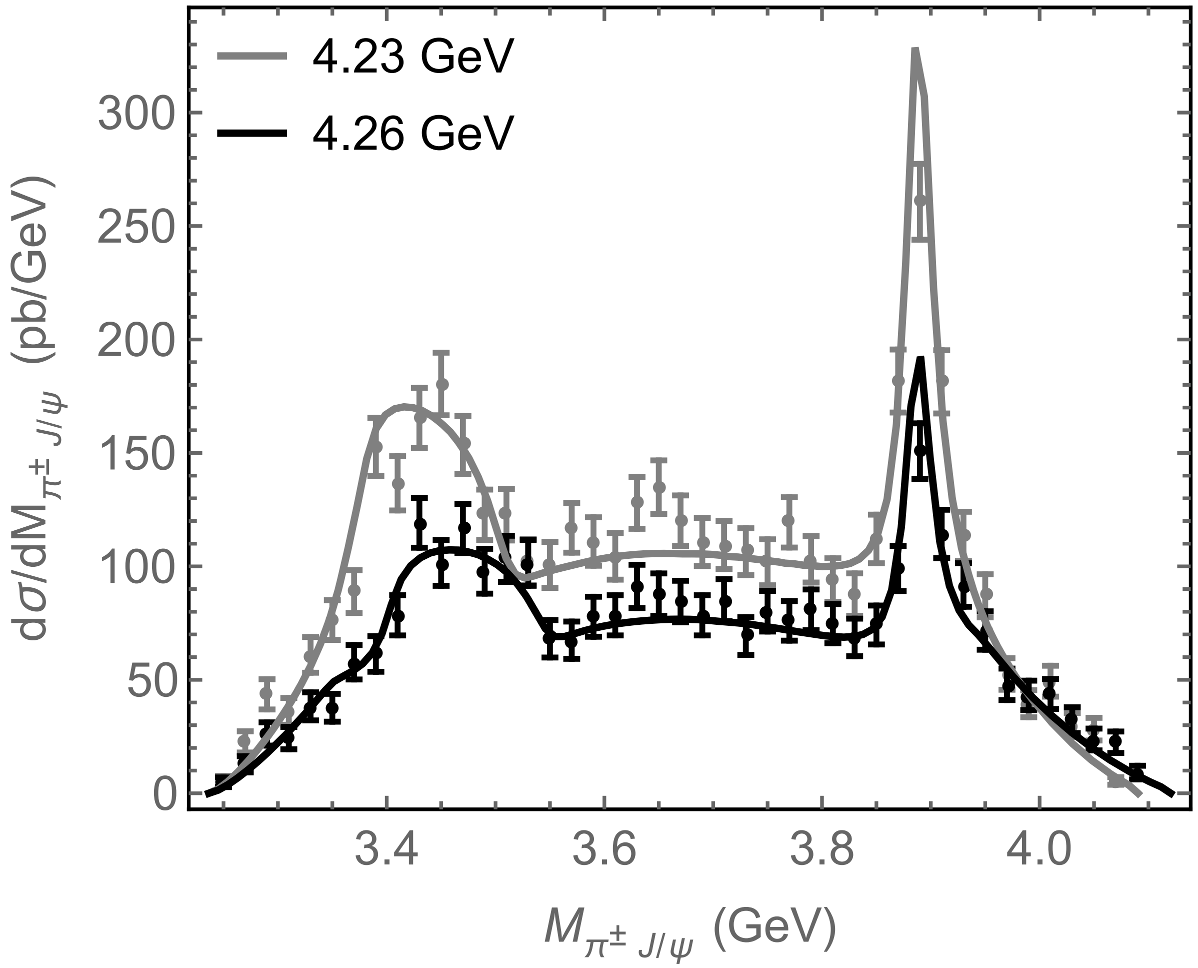


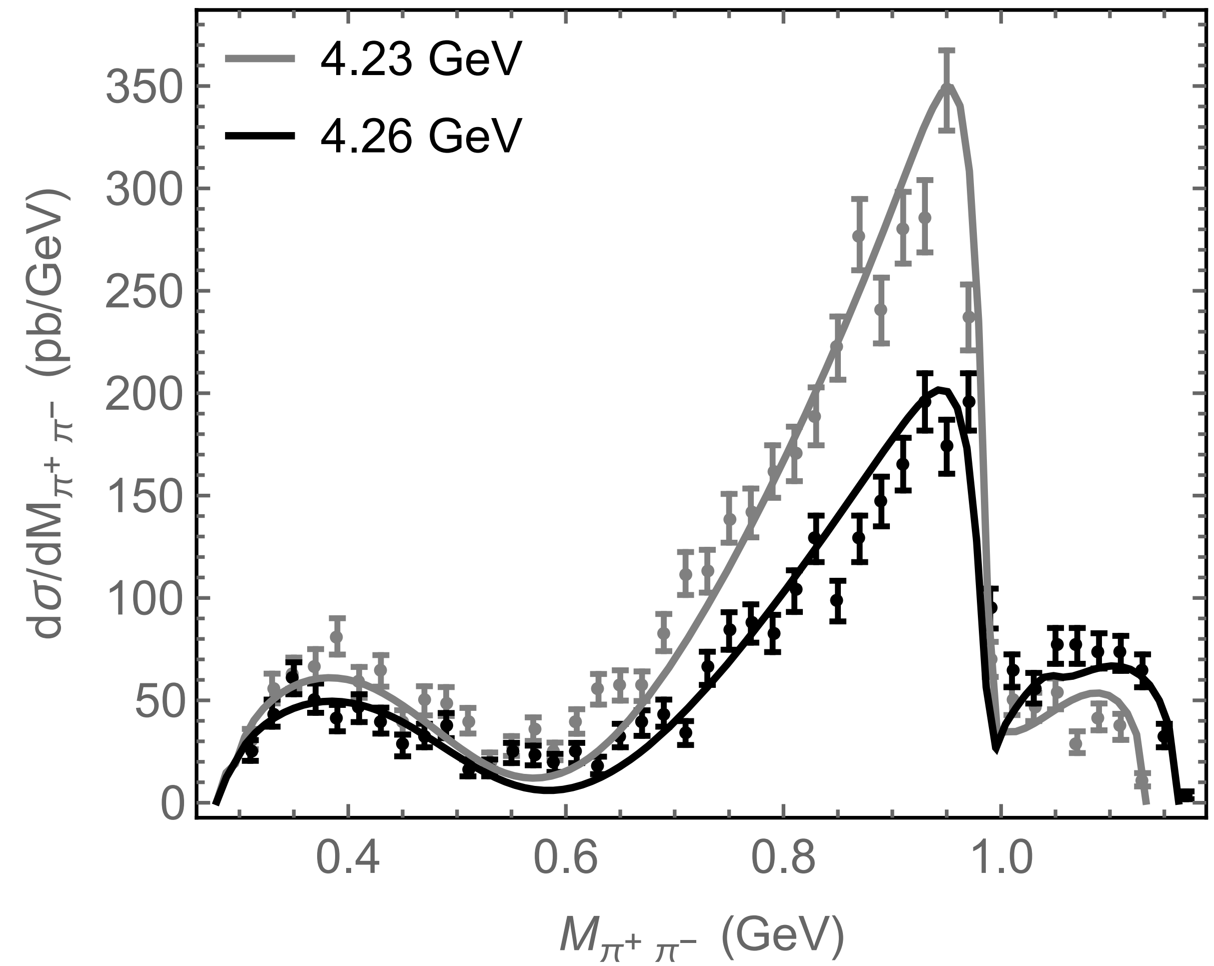
⇒ Framework has the correct ingredients in the **simultaneous** description of the $\pi\pi$ and $\pi\psi$ mass distributions

Results for $e^+e^- \to \pi\pi J/\psi$

➢ Economic fits: **S-wave** a, b, (c), d real, **D-wave** no subtractions (4 parameters)

combined fit & $\sigma(e^+e^- \to KKJ/\psi)$





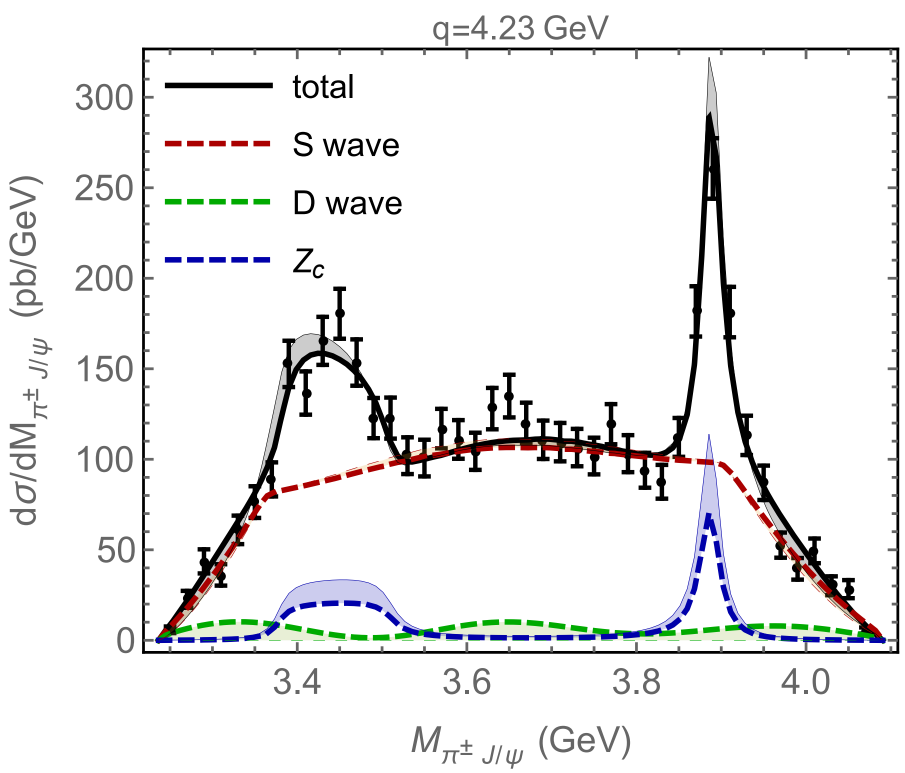
➢ the obtained fit parameters do not vary much between q=4.23 and 4.26 GeV

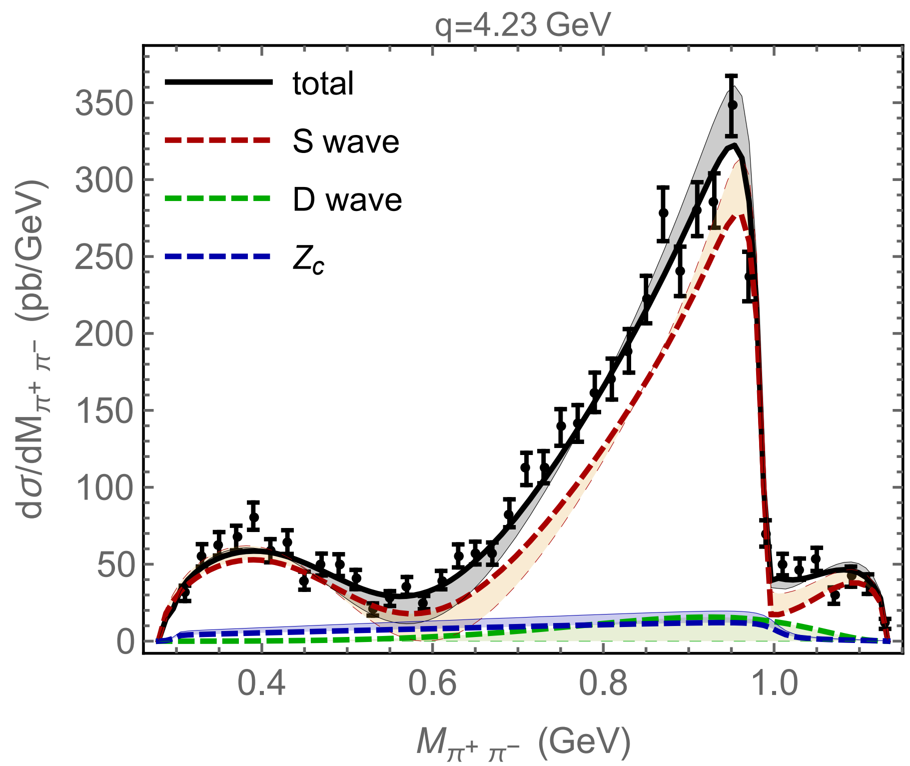
➢ serve as starting values of our best fit

e^+e^- cm energy	4.23 GeV	4.26 GeV
$\sigma(J/\psi K^+K^-)^{\rm Exp}$ [pb]	5.3(1.0)	3.1(6)
$\sigma(J/\psi K^+K^-)^{\rm Th}$ [pb]	4.4(5)	2.9(4)
$\chi^2_{\rm tot}$	3.4	2.5

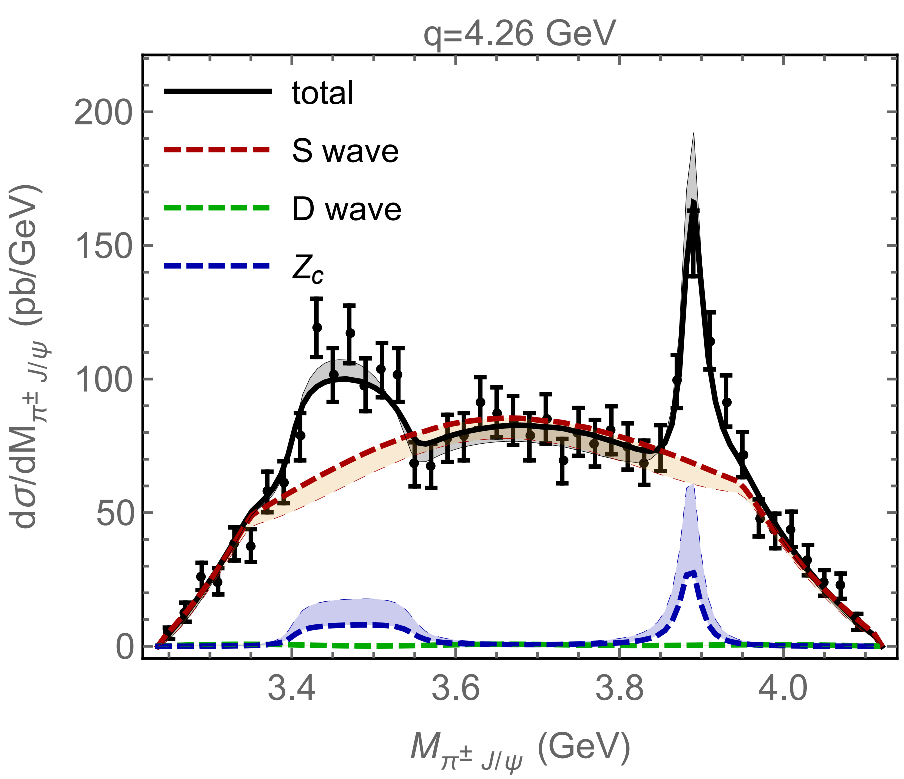
Results for $e^+e^- \to \pi\pi J/\psi$

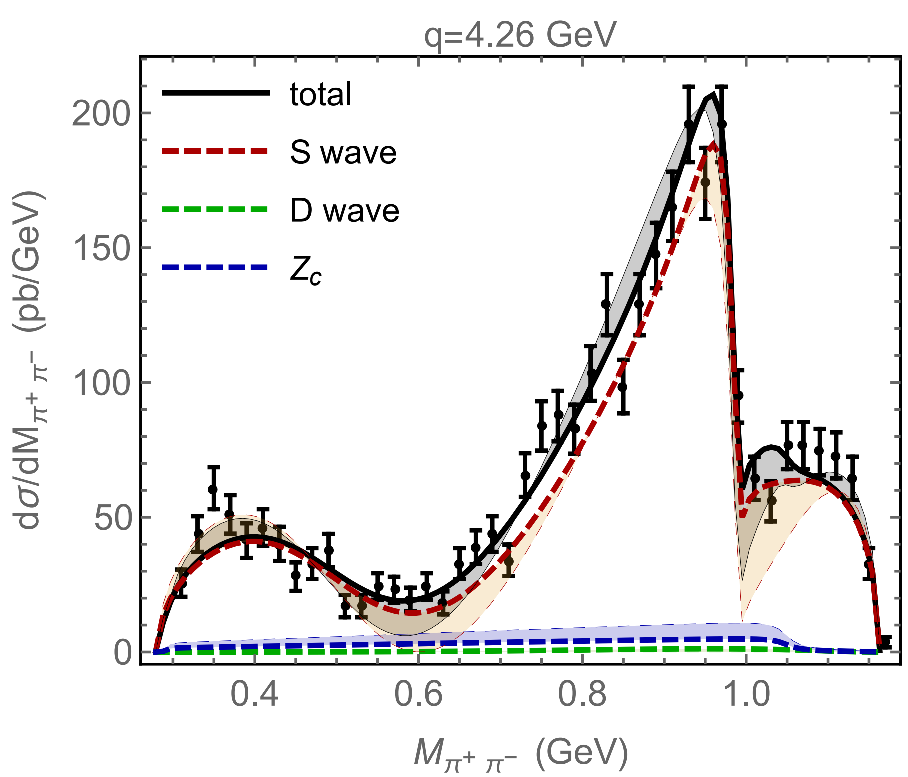
➢ Best fits: **S-wave** a, b, c, d real/complex, **D-wave** e real (7 parameters)





e^+e^- cm energy	4.23 GeV
$\sigma(J/\psi K^+K^-)^{\text{Exp}}$ [pb]	5.3(1.0)
$\sigma(J/\psi K^+K^-)^{\text{Th}}$ [pb]	5.2(2)
χ^2_{tot}	1.7





e^+e^- cm energy	4.26 GeV
$\sigma(J/\psi K^+K^-)^{\text{Exp}}$ [pb]	3.1(6)
$\sigma(J/\psi K^+K^-)^{\text{Th}}$ [pb]	3.0(3)
χ^2_{tot}	1.3

➢ D-wave contribution is small

different from
Chen et al. (2019)

Sum rule values / rescattering mechanism

➢ We use over subtracted DR

$$\mathcal{H}_{++}(s,t) = \frac{1}{\sqrt{3}}\Bigg[\mathcal{H}^{Z_c}_{0,++}(s,t) + \Omega^{(0)}_{11}\left\{a + b\,s - \frac{s^2}{\pi}\int_{4m_\pi^2}^{\infty}\frac{ds'}{s'^2}\frac{\mathrm{Disc}\,(\Omega^{(0)}(s'))^{-1}_{11}}{s'-s}h^{(0),Z_c}_{0,++}(s')\right\}$$

$$+\,\Omega^{(0)}_{12}\left\{c + d\,s - \frac{s^2}{\pi}\int_{4m_\pi^2}^{\infty}\frac{ds'}{s'^2}\frac{\mathrm{Disc}\,(\Omega^{(0)}(s'))^{-1}_{21}}{s'-s}h^{(0),Z_c}_{0,++}(s')\right\}$$

$$+\,5\,P_2(z)\,\gamma(s)\,\Omega^{(2)}\left\{e - \frac{s}{\pi}\int_{4m_\pi^2}^{\infty}\frac{ds'}{s'}\frac{\mathrm{Disc}\,(\Omega^{(2)}(s'))^{-1}}{s'-s}\frac{h^{(2),Z_c}_{0,++}(s')}{\gamma(s')}\right\}\Bigg]$$

$$\gamma(s) \equiv (s-4m_\pi^2)(s-(q-m_\psi)^2)$$

➢ One can compare fitted values with sum rule values

$$b^{SR} = -\int_{4m_\pi^2}^{\infty}\frac{ds'}{\pi}\frac{\mathrm{Disc}\,(\Omega^{(0)}(s'))^{-1}_{11}}{s'^2}\,h^{(0),Z_c}_{0,++}(s')$$

$$q = 4.23\ \mathrm{GeV}$$
$$b^{SR}\times 10^{-3} = -0.6\,e^{0.9i}$$
$$b^{Fit}\times 10^{-3} = -11.2\,e^{-0.2i}$$

➢ Rescattering of ππ **mainly goes through the contact interaction** (direct rescattering) and **not through Zc(3900)**

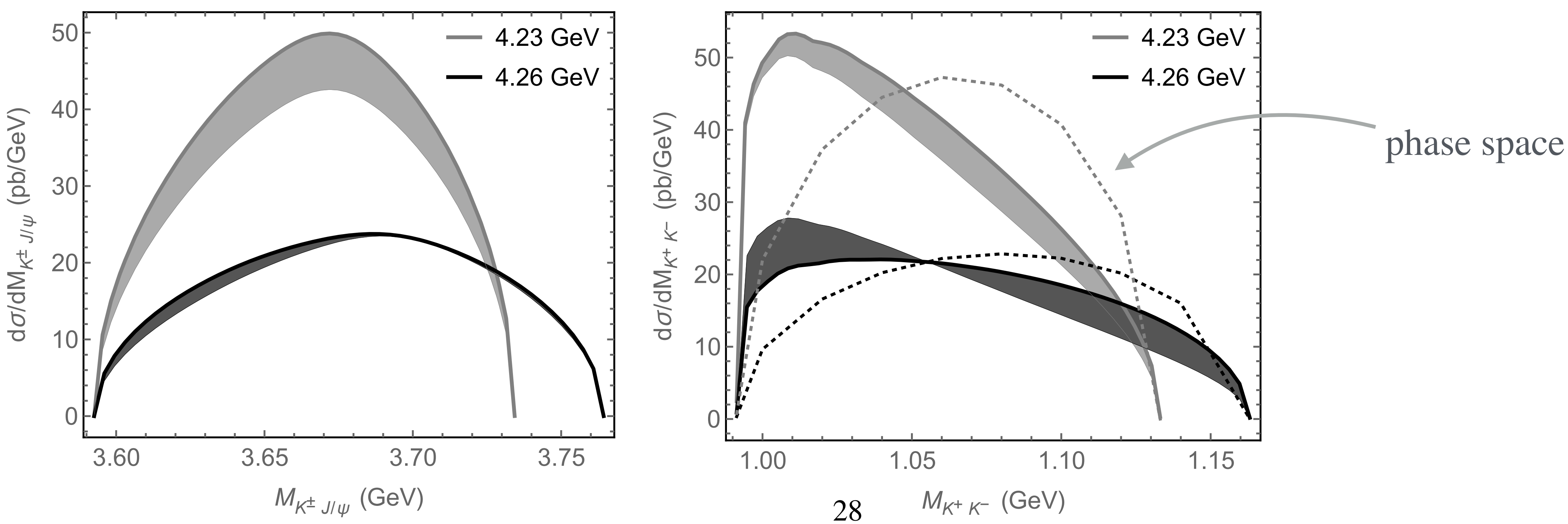
different from
Chen et al. (2019)

Prediction for $e^+e^- \to KKJ/\psi$

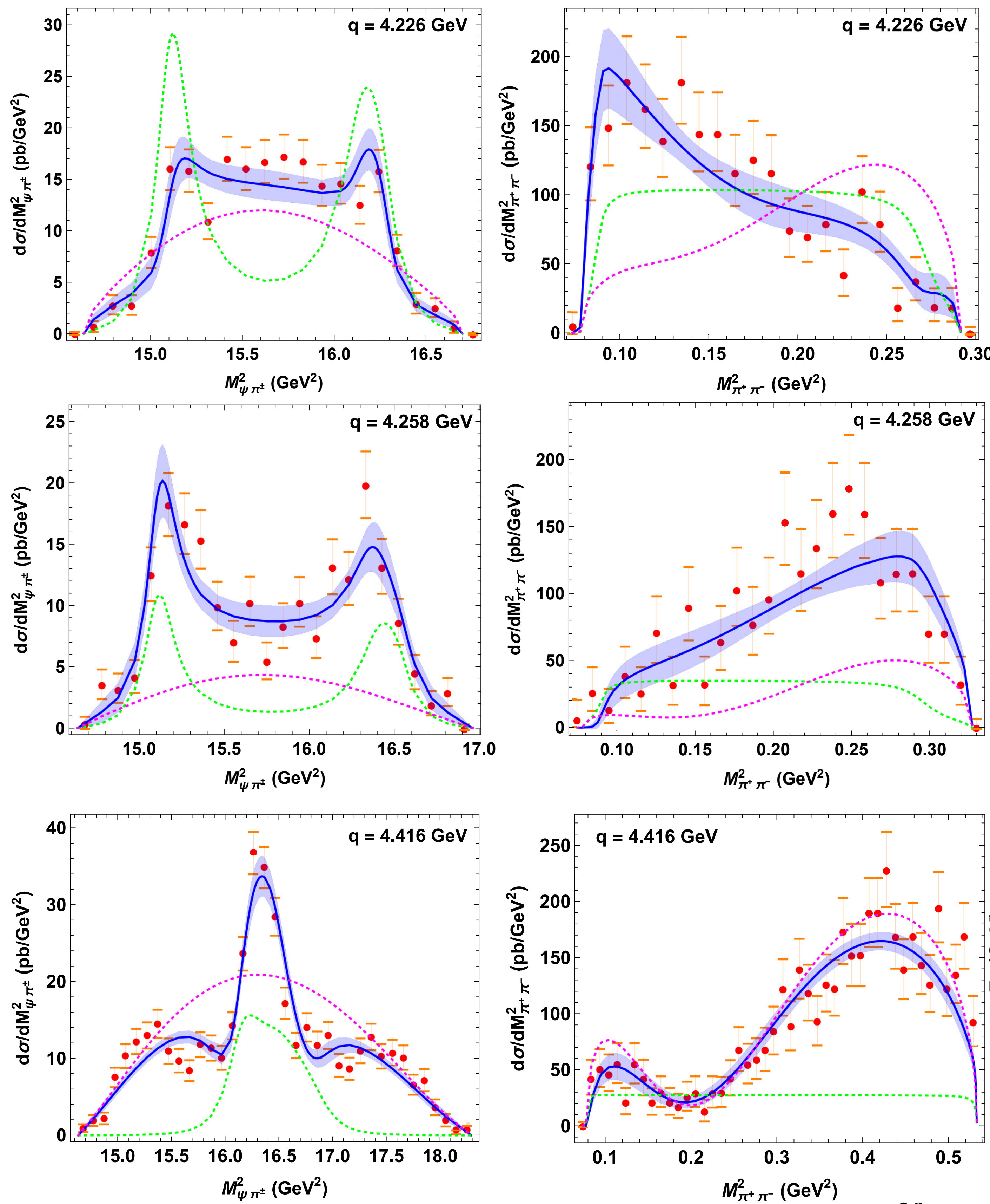
- KJ/ψ mass distribution is a pure phase space in the absence of Zcs
- **KK mass distribution** comes naturally from the coupled-channel analysis

$$\mathcal{K}_{++}(s,t) = \frac{\Omega^{(0)}_{21}}{2}\left\{a + b\,s - \frac{s^2}{\pi}\int_{4m_\pi^2}^{\infty}\frac{ds'}{s'^2}\frac{\mathrm{Disc}\,(\Omega^{(0)}(s'))^{-1}_{11}}{s'-s}\,h^{(0),Z_c}_{0,++}(s')\right\}$$
$$+\frac{\Omega^{(0)}_{22}}{2}\left\{c + d\,s - \frac{s^2}{\pi}\int_{4m_\pi^2}^{\infty}\frac{ds'}{s'^2}\frac{\mathrm{Disc}\,(\Omega^{(0)}(s'))^{-1}_{21}}{s'-s}\,h^{(0),Z_c}_{0,++}(s')\right\}$$

➢ We observe a rapid rise just above threshold due to **f_0(980) resonance**
➢ Expect to see it in future experimental results (BESIII already collected data)



Results for $e^+e^- \to \pi\pi\psi(2S)$



➢ single-channel ππ rescattering in S-wave

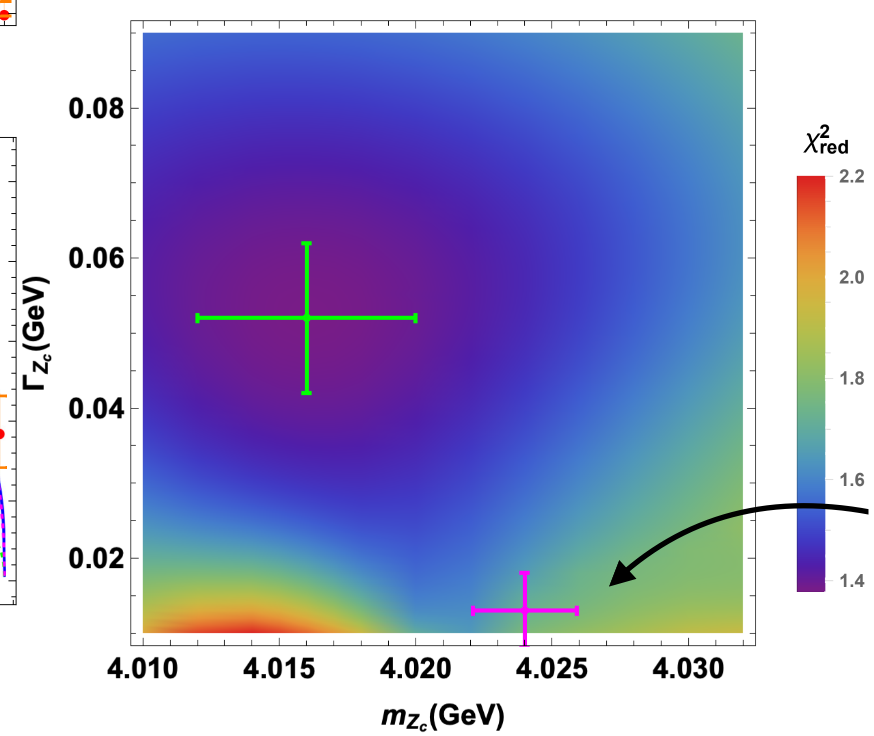
➢ Overlap of left and right-hand cuts leads to an additional anomalous piece in the dispersive integral

➢ One can describe 4.226 GeV and 4.258 GeV under an assumption of dominant Zc(3900)

➢ **Heavier Zc** is required at 4.416 GeV (mass 4.016(10), width 52(10))

➢For all e^+e^- c.m. energies an overlap of Zc(3900) and higher Zc is possible

→ more detailed analyses (combined with $e^+e^- \to \pi\pi h_c$ data) is being performed



Zc(4020)
$e^+e^- \to \pi\pi h_c$
BESIII (2014)

Summary and outlook

Summary

- We provided a quantitative and simultaneous description of the ππ and πJ/ψ mass distributions of the recent BESIII data on $e^+e^- \to \pi\pi J/\psi$ together with the total cross sections $\sigma(e^+e^- \to KKJ/\psi)$ at 4.23 GeV and 4.26 GeV e^+e^- c.m. energies

- Crucial elements in our analysis:
  ➢ explicit inclusion of Zc(3900) as a pole contribution in *t*- and *u*-channels
  (Mass and Width fixed to PDG)
  ➢ dispersive treatment of the ππ (KK) final state interaction
  (employed N/D method for Omnes function)

- For $e^+e^- \to KKJ/\psi$ we provided the first theoretical prediction for the two kaon invariant mass distribution, which is significantly different from the pure phase space

Outlook

- Proposed method is currently being used to analyse BESIII data (such as $e^+e^- \to \pi\pi h_c$, …) and can be applied to future Belle II data
- Will allow to extract Zc parameters more precisely