Heavy four-quark states from lattice QCD

main topic: Z_b

$\bar{b}\, b\, \bar{d}\, u$ $\qquad$ $\bar{b}\, \bar{b}\, d\, u$

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XYZ workshop @ GSI (online)
14th April 2021

in collaboration with: H. Bahtiyar, J. Petkovic and M. Sadl

Two exotic systems in exp and Lattice QCD

	open-bottom	closed-bottom
	$\bar{b}\,\bar{b}\,d\,u$ $J^P=1^+, I=0$	$\bar{b}\,b\,\bar{d}\,u$ $J^P=1^+, I=1$
nearby threshold	BB^*	$B\bar{B}^*,\ B^*\bar{B}^*$
exp	no state found prospect: not easy	two resonances Z_b discovered (Belle 2011)
lattice QCD	strong bound state predicted “straightforward” : done	resonances with many strong decay ch. very challenging $\bar{b}\,b\,\bar{d}\,u \to \bar{b}b + \bar{d}u$

$\bar{b}\, b\, \bar{d}\, u$ Z_b in experiment

discovered by Belle in 2011
[PRL 108 (2012) 122001]

$Z_b^+(10610)$, $Z_b^+(10650)$ **I=1, $J^{PC}=1^{+-}$**

observed decays
$\Upsilon(1S)\,\pi$, $\Upsilon(2S)\,\pi$, $\Upsilon(3S)\,\pi$
$h_b(1S)\,\pi$, $h_b(2S)\,\pi$

$$\bar{b}\, b\, \bar{d}\, u \rightarrow \bar{b}b + \bar{d}u$$

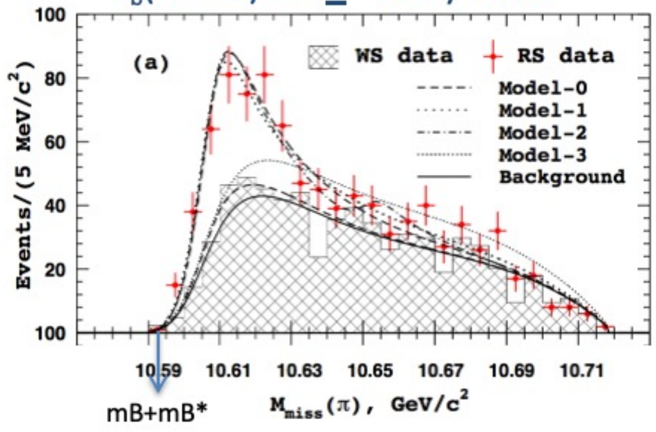
dominant Br: B $\underline{B}$* , B* $\underline{B}$*

$$\bar{b}\, b\, \bar{d}\, u \rightarrow \bar{b}u + \bar{d}b$$

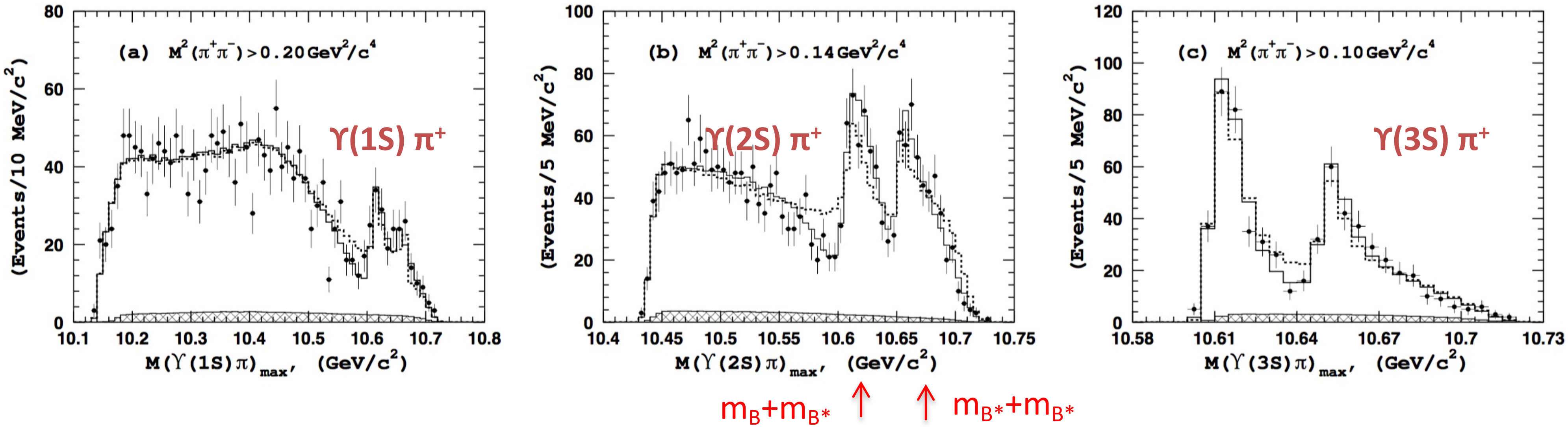
allowed (unobserved) $\eta_b\ \rho$

[Belle, 1512.07419, PRL 2016]

$Z_b(10610)$ in B$\underline{B}$* decay mode Br≈85 %



Belle PRD 91 (2015) 072003



Lattice QCD with non-static b quarks and Luscher's method

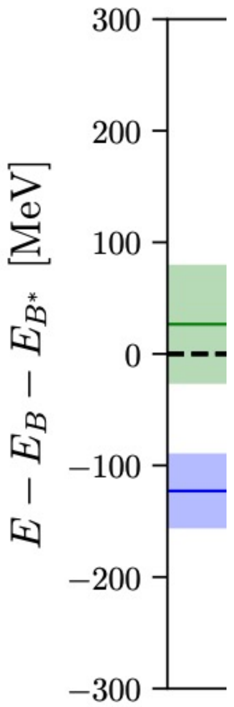
$\bar{b}\,\bar{b}\,d\,u$ $\quad$ $\bar{b}\,b\,\bar{d}\,u$

$\bar{b}\,\bar{b}\,d\,u$ with Lattice QCD, non-static b quarks and Luscher's method : **done**

because it a strongly bound state !

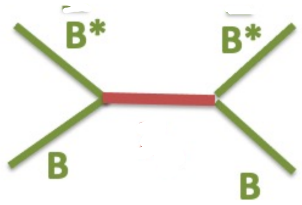
NRQCD Luscher interpolation pole in T(E)

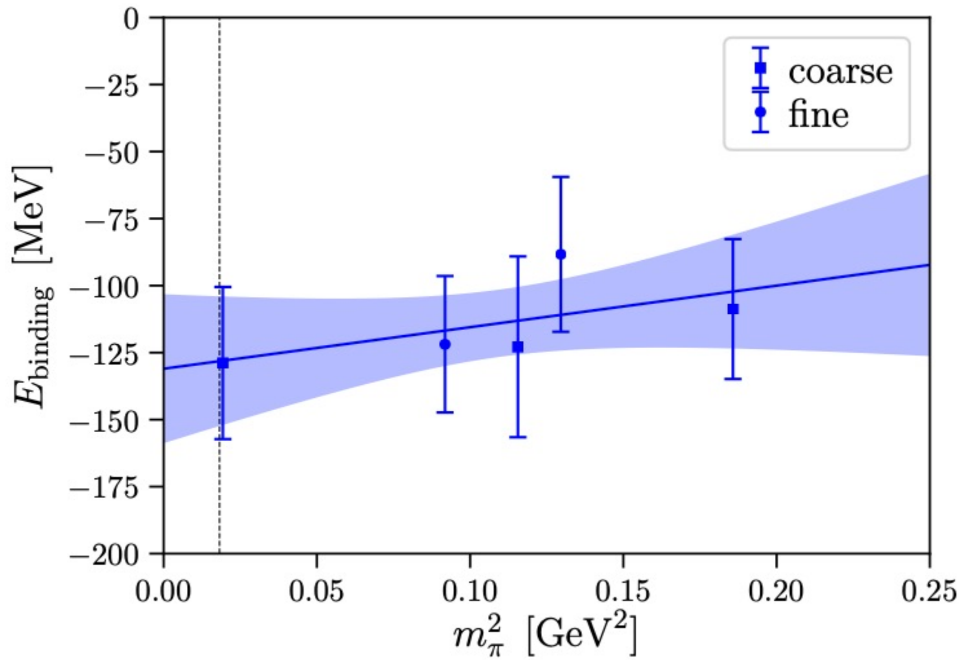
$$E_n \;\rightarrow\; T(E_n) \;\rightarrow\; T(E) \;\rightarrow\; m$$



one-channel

BB* scattering





above: 1904.04197 Leskovec, Meinel, Pflaumer, Wagner

also works by: Lewis & Francis & Maltman & Hudspith; Orginos & Brown; Padmanath & Junnakar & Mathur;

$\bar{b}b\bar{d}u$ with Lattice QCD, non-static b quarks and Luscher's method : **to challenging** !

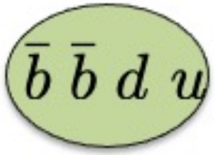
because Zb are resonances above many (>5) thresholds

problem: $\bar{b}b\bar{d}u \rightarrow \bar{b}b + \bar{d}u$

Rigorous treatment to challenging:

- at least 7 two-particle channels coupled $\longrightarrow$ $\Upsilon(1S)\,\pi$, $\Upsilon(2S)\,\pi$, $\Upsilon(3S)\,\pi$; $h_b(1S)\,\pi$, $h_b(2S)\,\pi$; $B\,\underline{B}^*$, $B^*\,\underline{B}^*$
- very dense $B\underline{B}^*$ and $B^*\underline{B}^*$ energy levels

Lattice QCD with static b quarks and Born-Oppenheimer approach

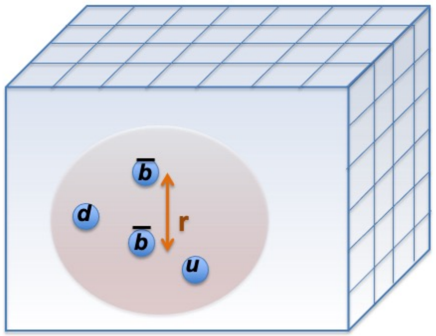


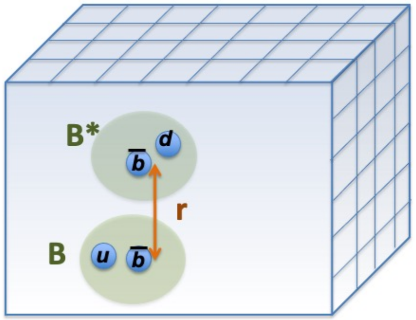
with Lattice QCD, static b quarks and Born-Oppenheimer : **done**

Lattice QCD

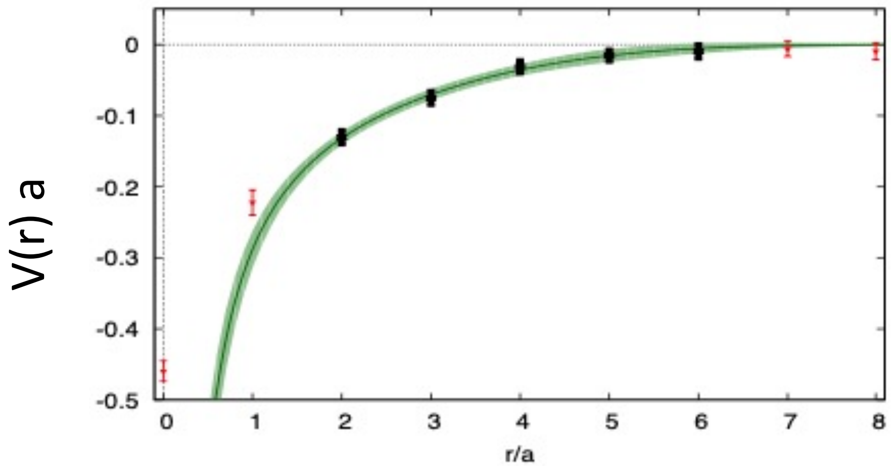
non-relativisic Schr. eq.

$$E_n(r) \;\rightarrow\; V(r) \;\rightarrow\; m$$



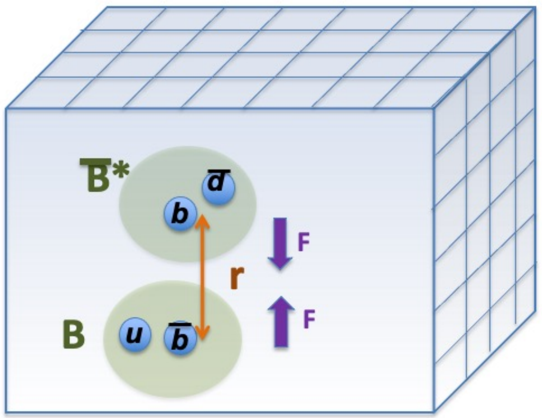


$m - m_B - m_{B*} = -\ 38\ (18)$ MeV



a number of works by Bicudo, Wagner, Peters, Cichy (above 1209.6274)

$\bar{b}\,b\,\bar{d}\,u$ with Lattice QCD, static b quarks and Born-Oppenheimer



Take-home message ... with more details to follow

- attractive potential between B and $\underline{B}$* at small distance r (for Sh=1)
- this attraction leads to one bound state not far below $m_B+m_{B^*}$

[S.P., Bahtiyar, Petkovic arXiv:1912.02656v4;
Wagner, Peres, Bicudo, Cichy 1602.07621]

Z_b with static b and $\underline{b}$

general idea: talks by
Nora Brambilla
Jaume Tarrus

Idea and the only previous lat study

Bicudo, Cichy, Peters, Wagner [proceedings : Lat16: 1602.07621]

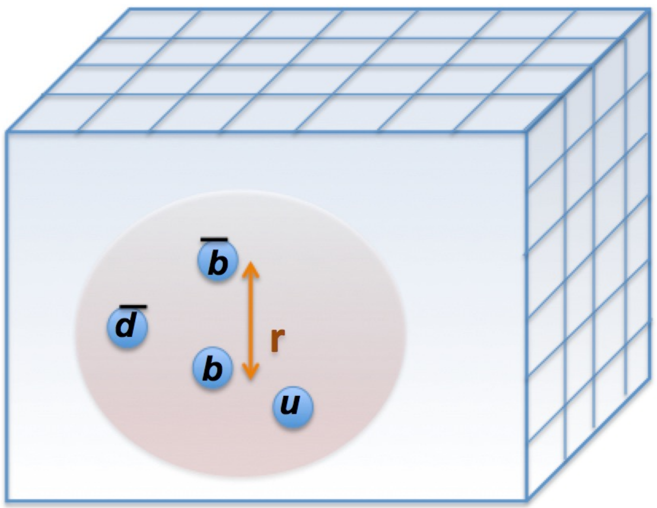
Born-Oppenheimer approach

h = heavy: b, $\underline{b}$ l=light:u,d,gluons

Step 1: fix static b and $\underline{b}$ at distance r: determine $E_n(r)$ for light d.o.f.: lattice QCD

Step 2: consider motion of heavy d.o.f. in the potential determined in step 1 with non-relativistic Schrodinger equation

[Braaten et al PRD 1402.0438 , Brambilla et al PRD 1707.09647, Bali et al. hep-lat/0505012 PRD, Bicudo & Wagner 1209.6274 + many others ..]



Quantum numbers relevant for Zb

h=heavy=b,$\bar{b}$ $\quad \vec{S}_h = \vec{S}_b + \vec{S}_{\bar{b}}$

l=light=u,d,gluons

exp+pheno

continuum

$J^{PC}=1^{+-}$

$Z_b(10610)$ as $B\bar{B}^*$ molecule

$$\bar{b}\gamma_5 q \ \bar{q}\gamma_z b + \bar{b}\gamma_z q \ \bar{q}\gamma_5 b \propto (S_h=1)(J_l=0) + (S_h=0)(J_l=1)$$

B $\quad \bar{B}^*$ $\quad$ B* $\quad \bar{B}$

Bondar, Garmash, Milstein, Mizuk, Voloshin PRD84 054010, Voloshin PRD84 (2011) 031502

Wang, Baru, Filin, Hanhart, Nefediev, Wynen, 1805.07453, PRD 2018

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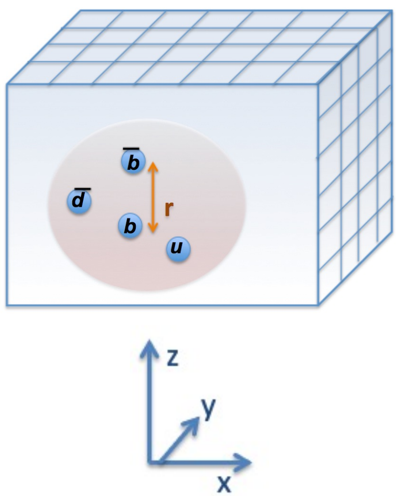
lattice with static b

$m_b = \infty$



static b $\rightarrow$ b quark can not flip spin via gluon exchange

S_h is conserved

quantum numbers of light degrees of freedom in static limit

$$S_h = 1 \ \& \ J_l = 0 \ (J_l^z = 0, CP = -1, \epsilon = -1 \ : \Sigma_u^-)$$

$$S_h = 0 \ \& \ J_l = 1 \ (J_l^z = 0, CP = +1, \epsilon = +1 \ : \Sigma_g^+)$$

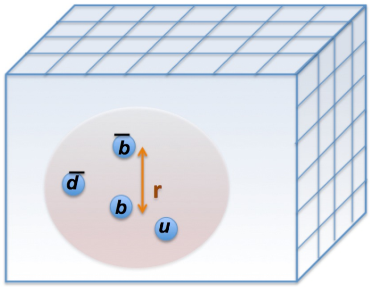
seperate channels, considered seperately

J^x, J^y not goot q.n.

reflection over yz plane

$S_h = 1 \ \& \ J_l = 0 \ (J_l^z = 0, CP = -1, \epsilon = -1 \ : \Sigma_u^-)$

Z_b channel with S_h=1 & J_l=0

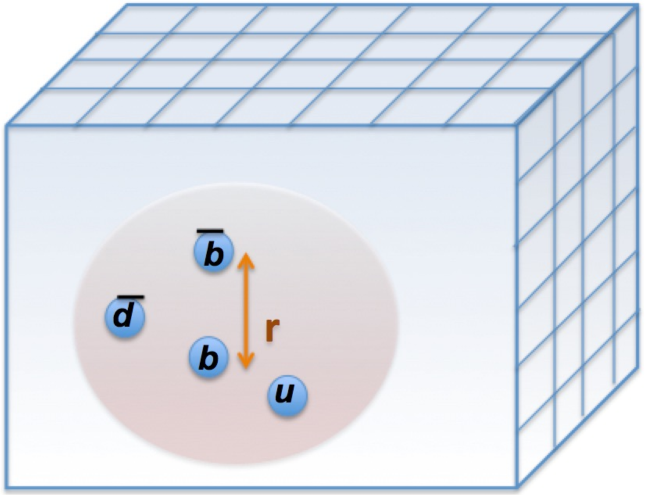


$$\bar{b}\, b\, \bar{d}\, u$$

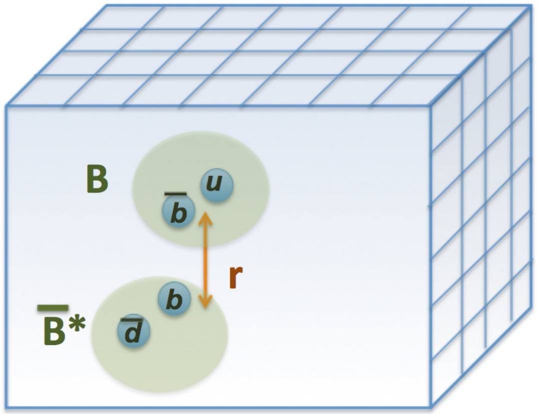
$$\vec{S}_h = \vec{S}_b + \vec{S}_{\bar{b}}$$

[S.P., H. Bahtiyar, J. Petkovic: arXiv:1912.02656v4]

Fock components incorporated



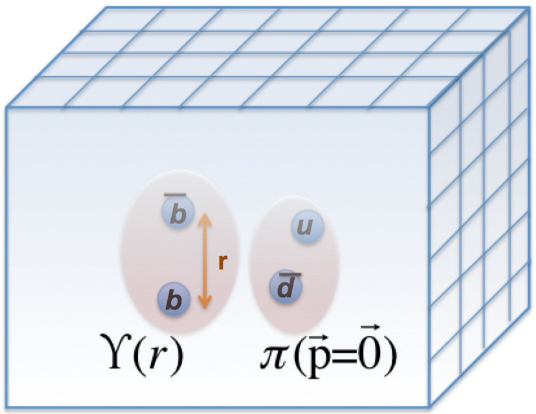
main challenge: B$\underline{B}$* is not the gound state !

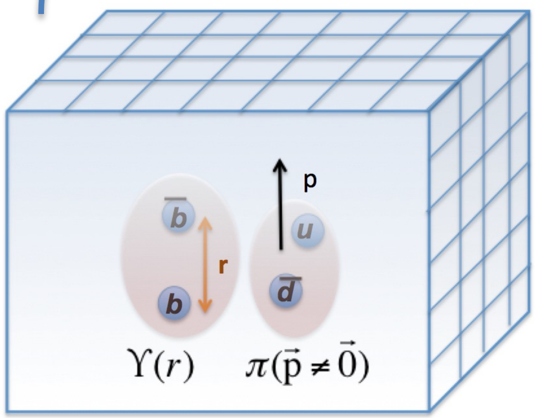


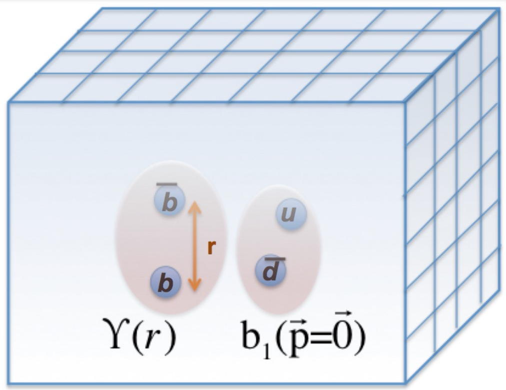
- main aim: extract static potential V(r) between B and $\underline{B}$*
- momentum of light degrees of freedom not conserved

  in presence of static quarks

not incorporated before







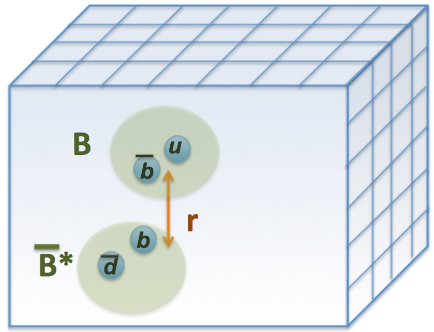
this state does not have J_l=0 but it has

$$J_l^z = 0, CP = +1, \epsilon = +1 \;\; : \Sigma_g^+$$

Determine E_n with operators O that annihilate/create the system

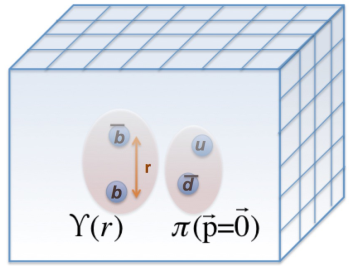
in this way S_h=1 and $(Sz)_h$=0 and $(Jz)_l$=0 and are indeed separately good quantum num. (inspired by Wagner et al.)

$$O^{B\bar{B}*} = \sum_{a,b} \sum_{A,B,C,D} \Gamma_{BA}\tilde{\Gamma}_{CD}\; \bar{b}^a_C(0) q^a_A(0)\; \bar{q}^b_B(r) b^b_D(r)\;, \quad \Gamma = P_-\gamma_5 \;\tilde{\Gamma} = \gamma_z P_+,$$



[..] indicate color singlets

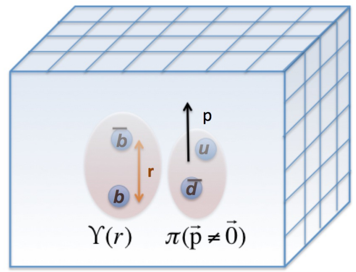
$$O^{\Upsilon\pi(0)} = \Upsilon_z\; \pi_{p=000} = [\bar{b}(0)\gamma_z P_+ b(r)]\; [\bar{q}\gamma_5 q]_{p=000}$$



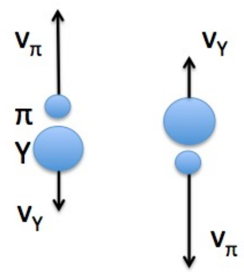
$$O^{\Upsilon\pi(1)} = \Upsilon_z\; (\pi_{p=001} + \pi_{p=00-1}) = [\bar{b}(0)\gamma_z P_+ b(r)] \left([\bar{q}\gamma_5 q]_{p=001} + [\bar{q}\gamma_5 q]_{p=00-1} \right)$$

p=n 2π/L

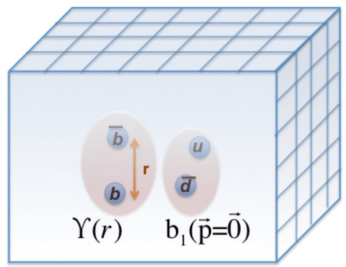
$$O^{\Upsilon\pi(2)} = \Upsilon_z\; (\pi_{p=002} + \pi_{p=00-2}) = [\bar{b}(0)\gamma_z P_+ b(r)] \left([\bar{q}\gamma_5 q]_{p=002} + [\bar{q}\gamma_5 q]_{p=00-2} \right)$$



- momentum in z direction ensures that $(Jz)_{light}$=0
- the sum ensures that CP is good q.n.



$$O^{\Upsilon\; b1(0)} = \Upsilon_z\; (b1_z)_{p=000} = [\bar{b}(0)\gamma_z P_+ b(r)]\; [\bar{q}\gamma_x\gamma_y q]_{p=000}$$



For each r : correlation matrix (6x6) renders E_n and Z_i^n

$$C_{ij}(t) = \left\langle 0 \right| \mathcal{O}_i(t)\ \mathcal{O}_j^+(0) \left| 0 \right\rangle = \sum_n Z_i^n\ Z_j^{n*}\ e^{-E_n t}$$

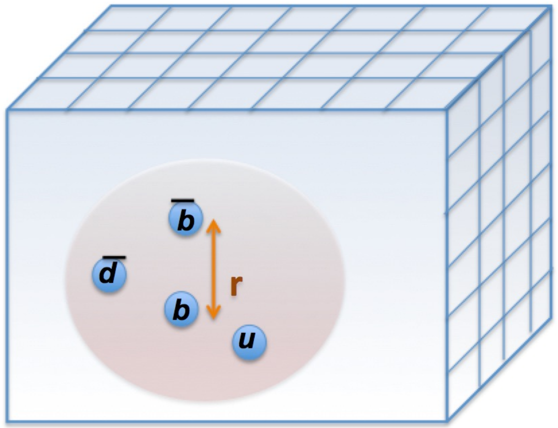
$$Z_i^n \equiv \left\langle 0 \right| \mathcal{O}_i \left| n \right\rangle$$

overlaps

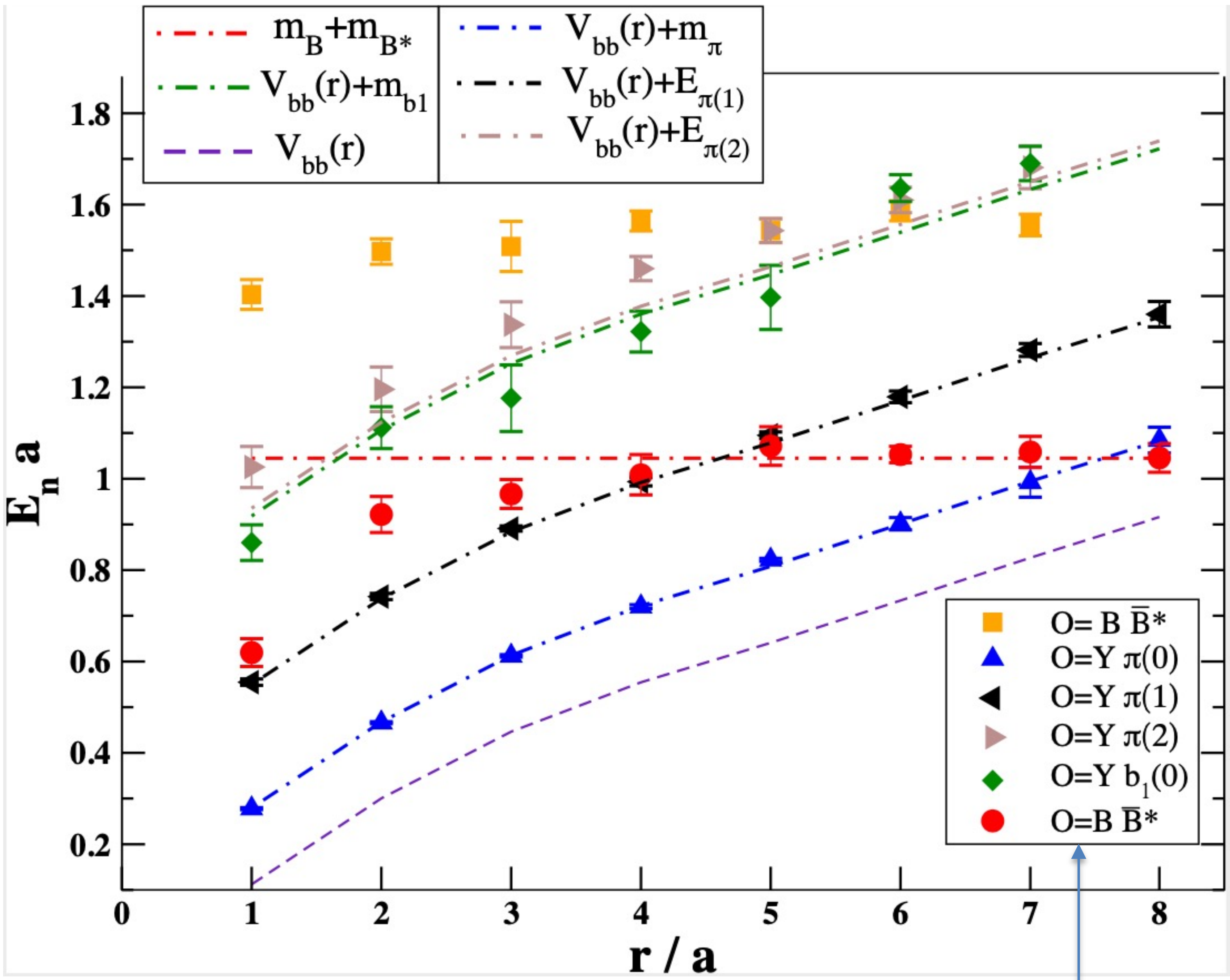
ensemble used: N_f=2, m_π≈266 MeV, a≈0.124 fm, L≈2 fm

[larger L would require Υπ(p=003) ...]

full distillation method to compute Wick contractions



Eigen-energies $E_n(r)$: channel $S_h=1, J_l=0$ (CP=-1, ε=-1)

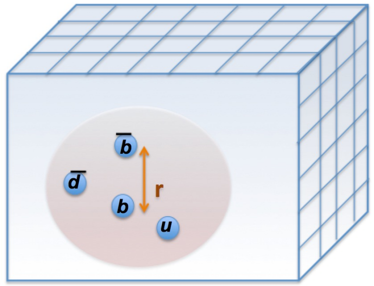


m_B+m_{B*}

dominant operator

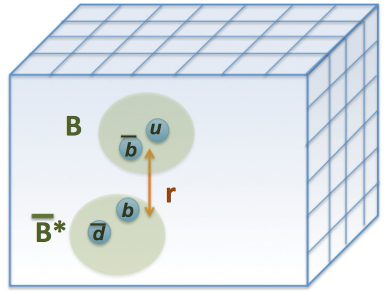
in each |n>

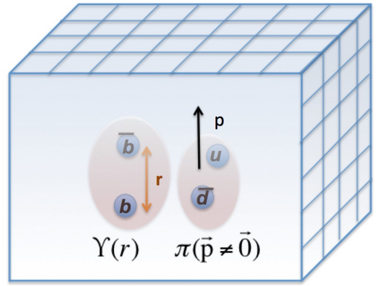
according to $\langle O_i | n \rangle$

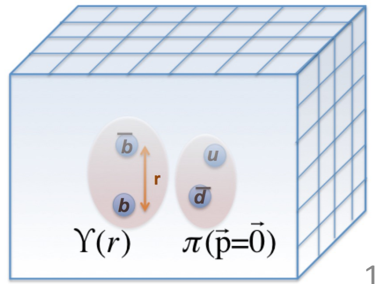


dot-dashed-lines:

$E_n^{\text{non-int}}$

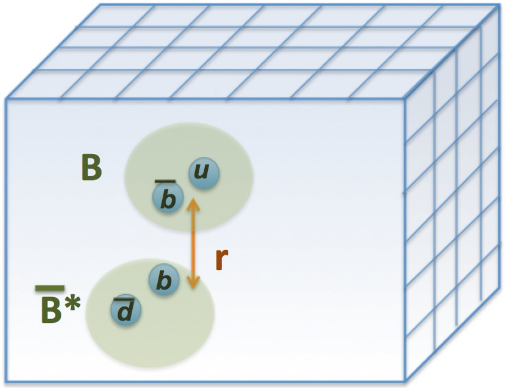


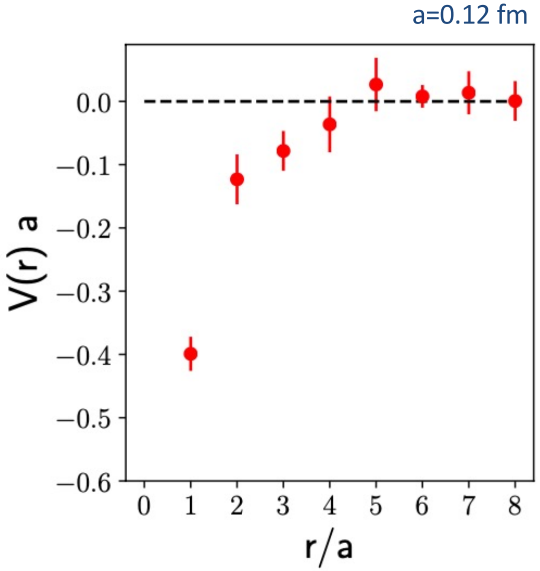




V(r) for interaction between B and $\underline{B}$*

Assuming that B $\underline{B}$* eigenstate is decoupled from $\Upsilon\pi$ and Υb_1 channels (overlaps support that)





Open problems

- V(r/a<1) = ?
- V(r/a>>1) = ?
- analytic fit form for V(r) = ?

request to the community to consider this

Employed form V(r) for various choices F and fits r/a=[1,4] and [2,4] $$V(r) = -A\, e^{-(r/d)^F} \qquad \mu = \tfrac{1}{2} m_B^{\exp}$$

- all parametrizations of V give one bound state
- the binding energy depends on the parametrization

$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2} + \frac{\hbar^2 L(L+1)}{2\mu r^2} + V(r)\right]u(r) = Eu(r)$$

we find one bound state with binding enery

[S.P., H. Bahtiyar, J. Petkovic: 1912.02656v4]

$$M - m_B - m_{B^*} = -48\ ^{+41}_{-108}\ \mathrm{MeV}$$

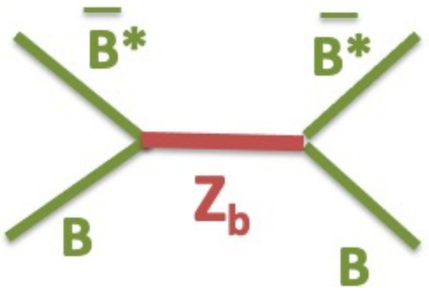
- in agreement with only previous lattice study

Bicudo, Cichy, Peters, Wagner [proceedings : Lat16: 1602.07621]

$$M - m_B - m_{B^*} = (-58 \pm 71)\,\mathrm{MeV}$$

Peak above B$\underline{\text{B}}$* for shallow bound state Z_b

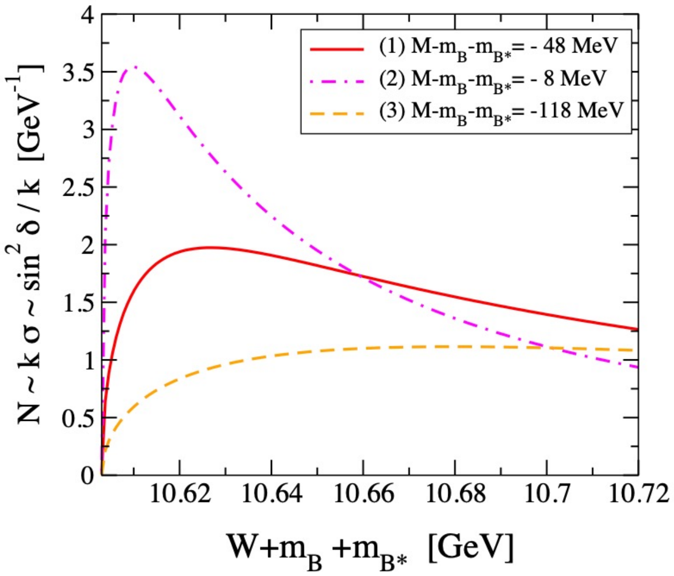
Schrodinger equation for B$\underline{\text{B}}$* motion -> scattering phase shift $\boldsymbol{\delta}$ -> cross section $\boldsymbol{\sigma}$



$$M - m_B - m_{B^*} = -48\,^{+41}_{-108}\ \text{MeV}$$

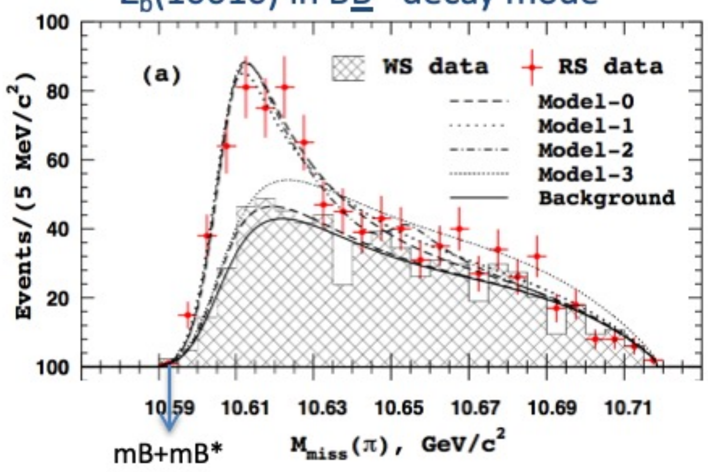
$$N_{B\bar{B}^*} \propto k\ \sigma$$

for three parametrizations consistent with lattice V(r)



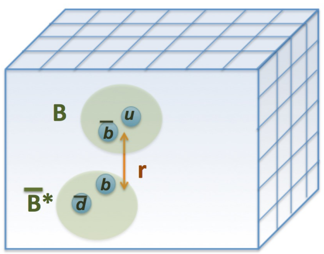
[Belle, 1512.07419, PRL 2016]

Z_b(10610) in B$\underline{\text{B}}$* decay mode



Conclusion from our lattice study [in agreement with Wagner & Bicudo & Peters]

- attraction between B and $\underline{\text{B}}^*$ renders bound state Zb
- for certain parametrizations bound state is close below threshold and renders peak in B$\underline{\text{B}}$* cross-section above threshold



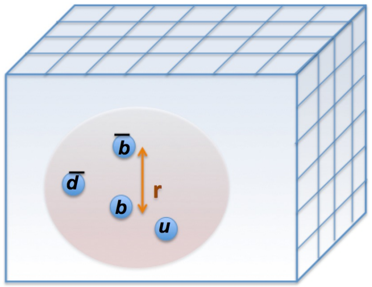
Re-analysis of exp data

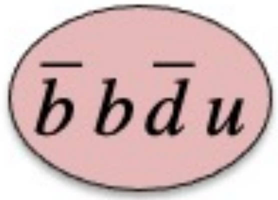
[Wang, Baru, Filin, Hanhart, Nefediev, Wynen, 1805.07453, PRD 2018]:

- Zb is virtual bound state few MeV below B$\underline{\text{B}}$*
  [when coupling to ($\underline{\text{b}}$b)($\underline{\text{d}}$u) omitted]
- renders peak above threshold

$S_h = 0 \ \& \ J_l = 1 \ (J_l^z = 0, CP = +1, \epsilon = +1 \ : \Sigma_g^+)$

Z_b channel with S_h=0 & J_l=1





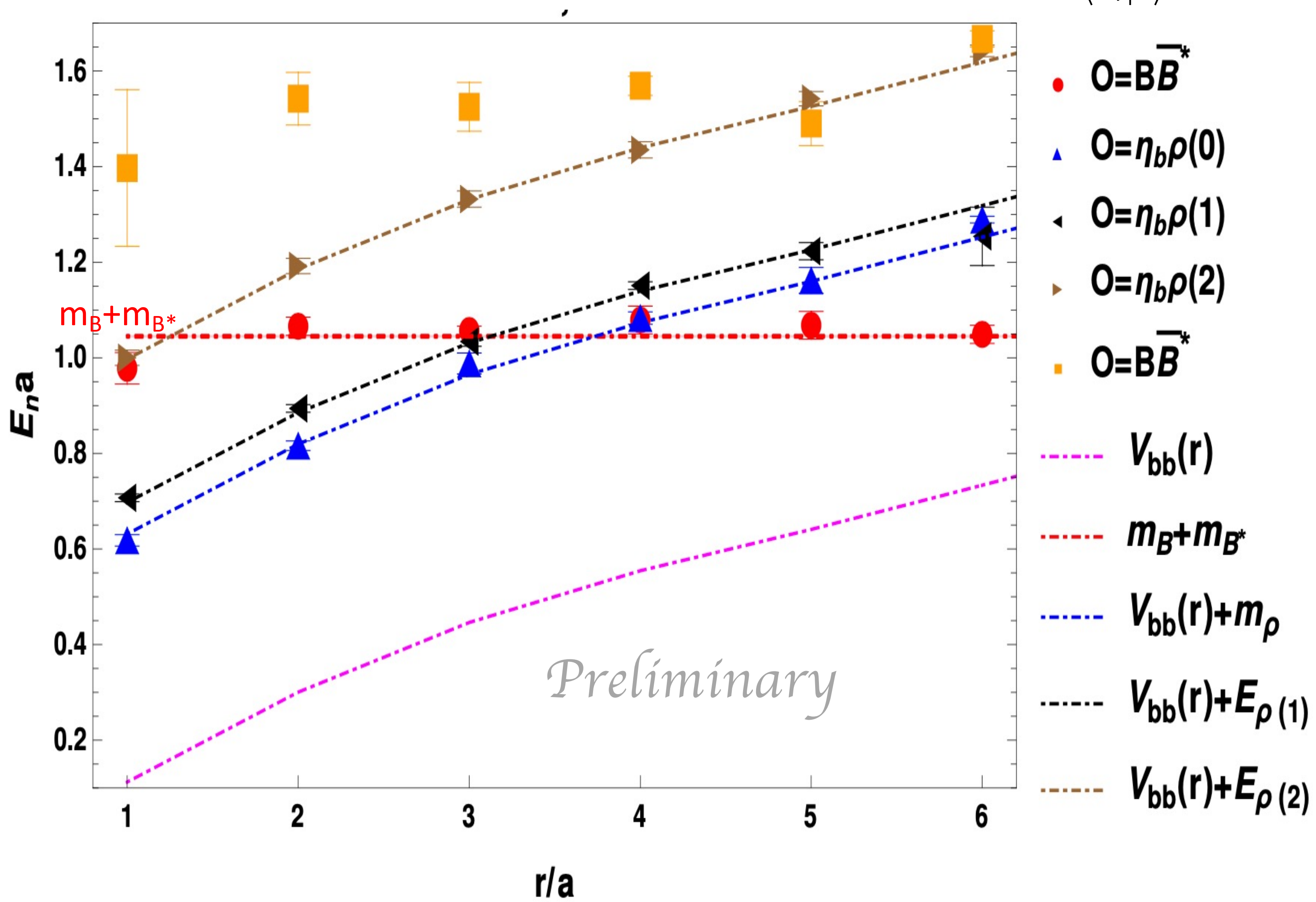
$$\vec{S}_h = \vec{S}_b + \vec{S}_{\bar{b}}$$

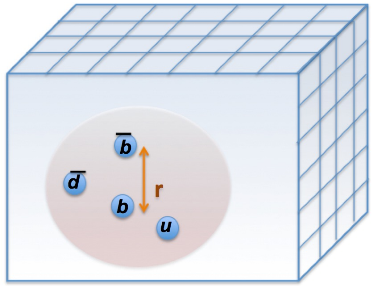
Mitja Sadl and S.P., in progress, preliminary result

Eigen-energies $E_n(r)$: channel S_h=0, J_l=1 (CP=1, ε=1)

dominant

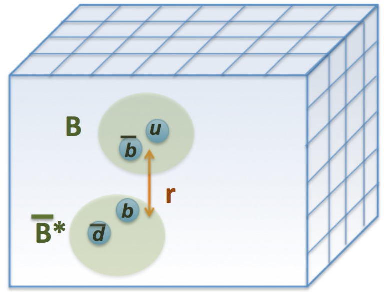
$\langle O_i | n \rangle$

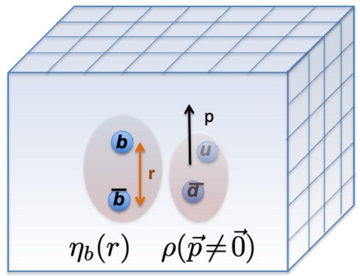


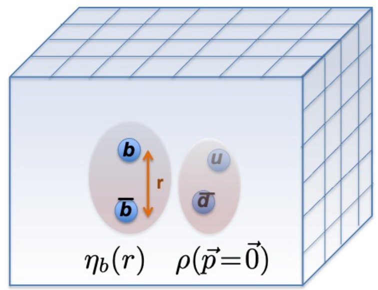


dot-dashed-lines:

$E_n^{\text{non-int}}$







No sizable attraction between B and <u>B</u>*

is found in this channel.

Conclusions

$\bar{b}\,\bar{b}\,d\,u$ exp: no state, very difficult lattice QCD: done

$\bar{b}\,b\,\bar{d}\,u$ exp: Zb lattice QCD: difficult

$\bar{b}\,b\,\bar{d}\,u$ channel with lattice QCD

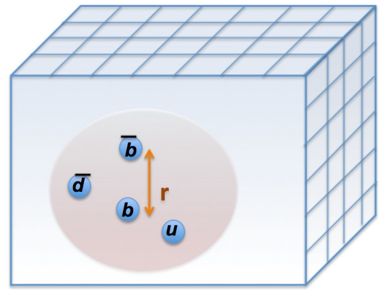
- study with non-static b-quarks and rigorous Luscher's approach to challenging

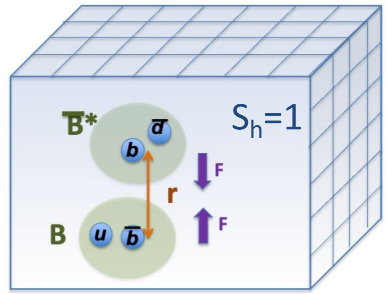
- study with static b, $\underline{b}$:

  attraction between B and $\underline{B}$* at small distances in channel $S_h=1$

  this attraction is most likely responsible for the existence of Zb exotic hadron

  no sizable attraction between B and $\underline{B}$* in channel $S_h=0$





Request to the community: determine BO potentials at very small r

determine analytic "fit" form for BO potentials

Much more work within lattice QCD is required to overcome all the simplifications ...

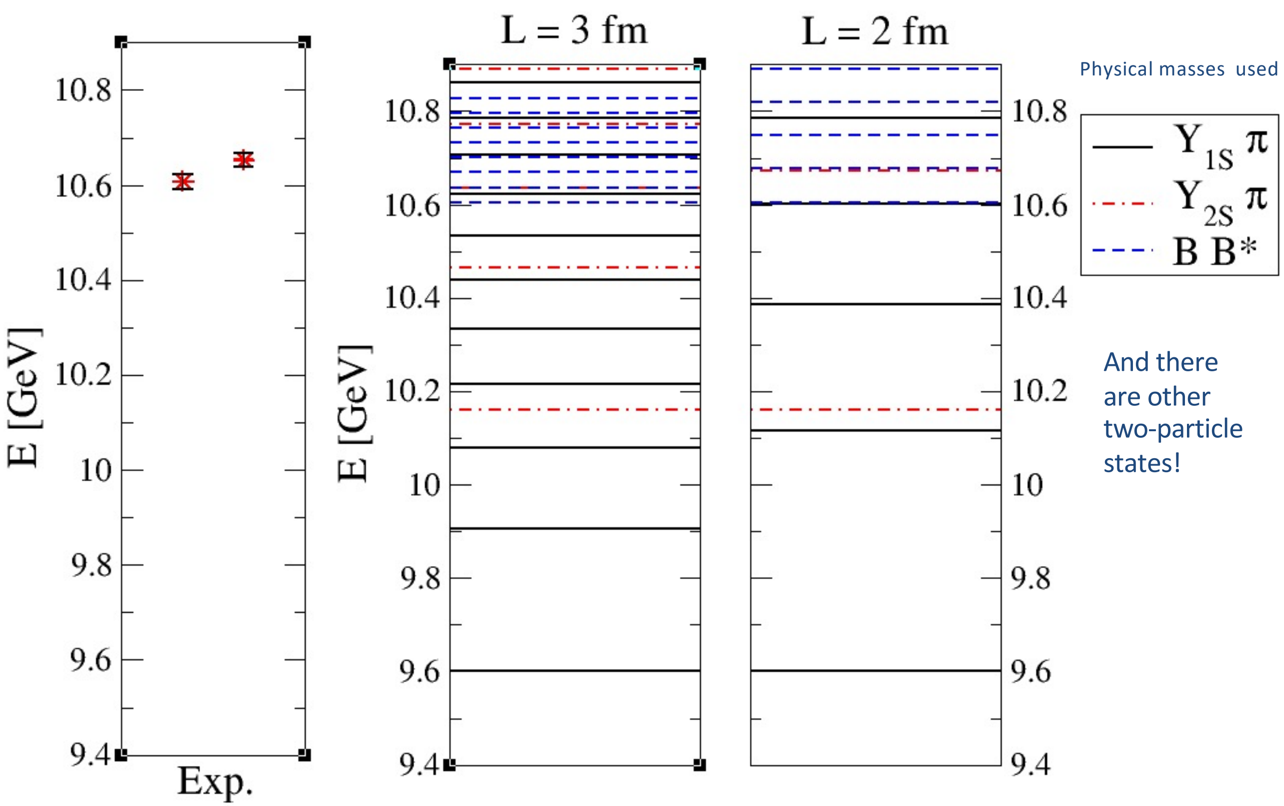
It would be great to see experimental confirmation of Belle's Zb at another exp (LHCb)

Backup

$\bar{b} b \bar{d} u$ with Lattice QCD, non-static b quarks and Luscher's method : **to challenging** !

because Zb are resonances above many (>5) thresholds

Eigen-energies in non-interacting limit



$$E^{n.i.}(L) = \sqrt{m_1^2 + \vec{p}^{\,2}} + \sqrt{m_2^2 + (-\vec{p})^2}$$

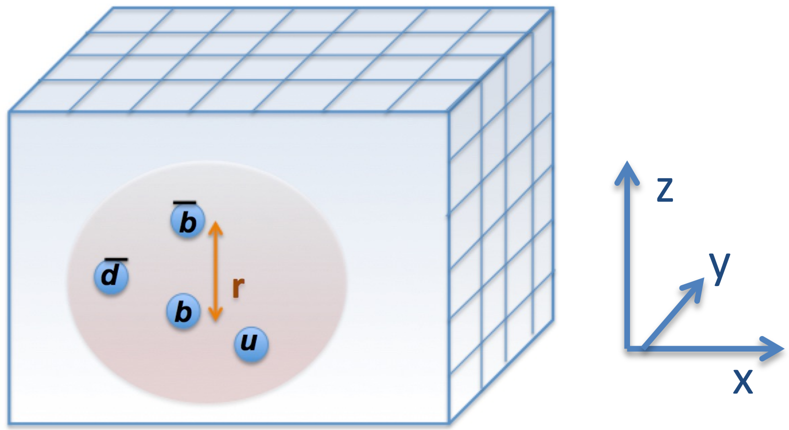
$$\vec{\mathrm{p}} = \frac{2\pi}{L}\vec{n}$$

Rigorous treatment to challenging:

- at least 7 two-particle channels coupled →
- very dense B$\underline{\mathrm{B}}$* and B*$\underline{\mathrm{B}}$* energy levels

Υ(1S) π , Υ(2S) π, Υ(3S) π
h_b(1S) π, h_b(2S) π
B $\underline{\mathrm{B}}$* , B* $\underline{\mathrm{B}}$*

Symmetries and quantum numbers



$I=1$ $I_3=0$ (consider neutral Z_b)

$S_h=1$ $(Sz)_h=0$

$(Jz)_l=0$ [J_x and J_y not conserved]

C•P= -1 (P= inversion over midpoint between b and $\underline{b}$)

R_l= reflection over yz plane = $P_l * R_l(y,\pi)$: $\varepsilon=-1$

momentum of light degrees of freedom: not conserved

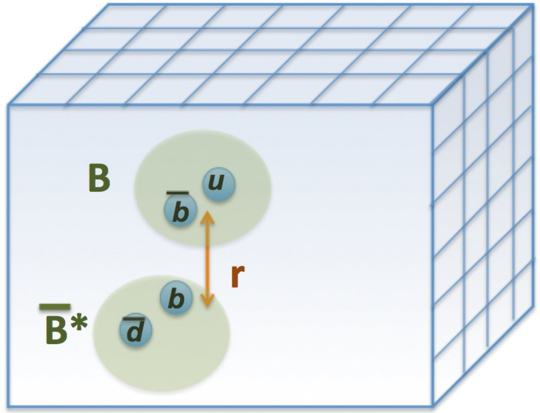
$$\vec{J} = \vec{S}_h + \vec{J}_l$$

h = heavy: b, $\underline{b}$

l = light:u,d,gluons

$$(J_z^l)_{CP}^{\epsilon} = \Sigma_u^-$$

Operators O_{BB*} with given quantum numbers



in this way $(Jz)_{light}$, S_{heavy} and $(Sz)_{heavy}$

are indeed separately good quantum num.

inspired by Wagner et al.

$$O^{B\bar{B}*} = \sum_{a,b} \sum_{A,B,C,D} \Gamma_{BA}\tilde{\Gamma}_{CD}\ \bar{b}^a_C(0) q^a_A(0)\ \bar{q}^b_B(r) b^b_D(r)\ , \quad \Gamma = P_-\gamma_5\ \tilde{\Gamma} = \gamma_z P_+,$$

$$= \sum_{a,b} \sum_{A,B,C,D} \bar{q}^b_B(r)\Gamma_{BA} q^a_A(0)\ \bar{b}^a_C(0)\tilde{\Gamma}_{CD} b^b_D(r)$$

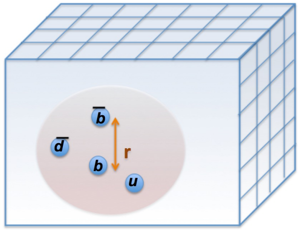
$$\propto [\bar{b}(0)P_-\gamma_5 q(0)]\ [\bar{q}(r)\gamma_z P_+ b(r)] + [\bar{b}(0)P_-\gamma_z q(0)]\ [\bar{q}(r)\gamma_5 P_+ b(r)]$$

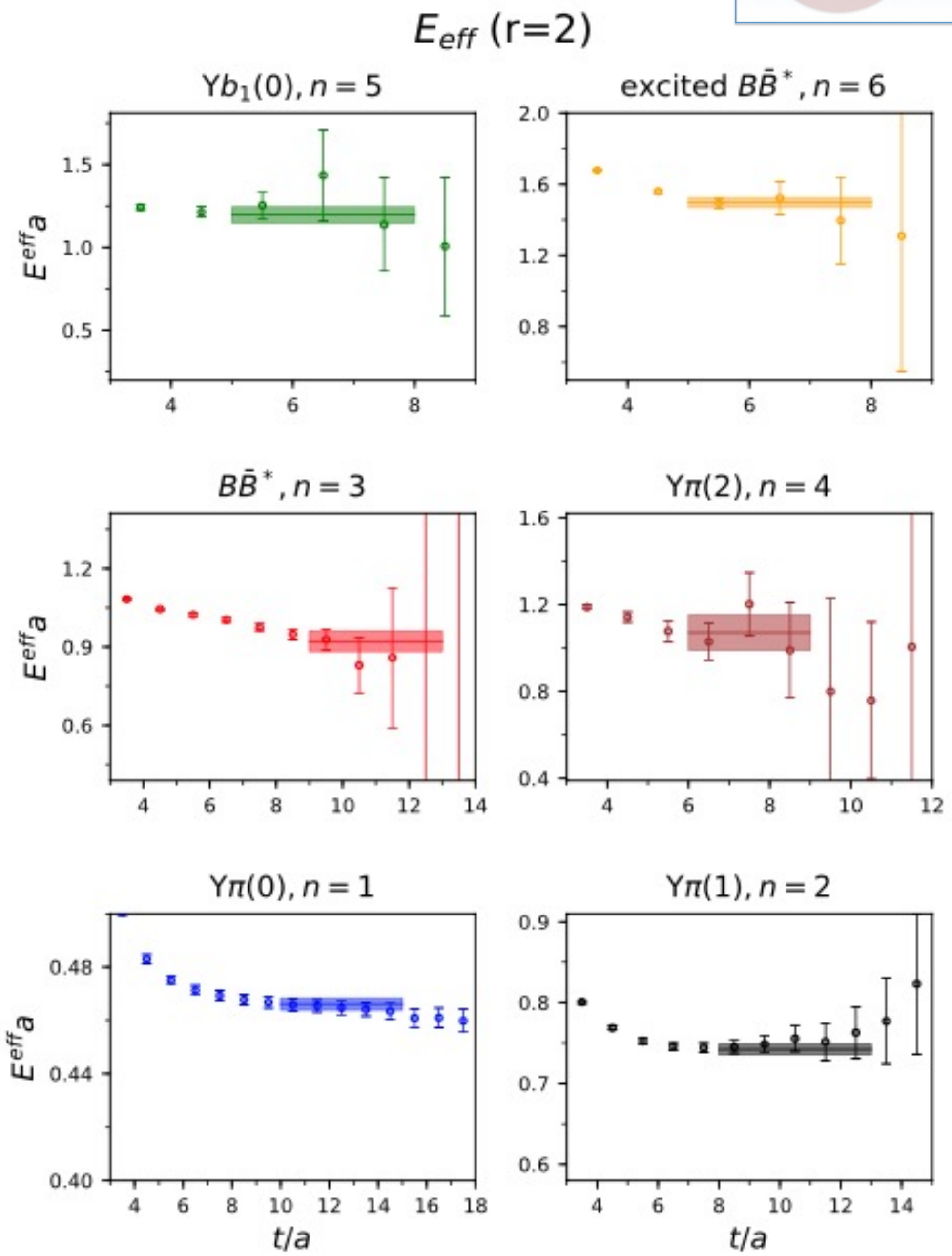
$$- [\bar{b}(0)P_-\gamma_x q(0)]\ [\bar{q}(r)\gamma_y P_+ b(r)] + [\bar{b}(0)P_-\gamma_y q(0)]\ [\bar{q}(r)\gamma_x P_+ b(r)]$$

$$O^{B\bar{B}*} = \sum_{a,b} \sum_{A,B,C,D} \Gamma_{BA}\tilde{\Gamma}_{CD}\ \bar{b}^a_C(0)\nabla^2 q^a_A(0)\ \bar{q}^b_B(r)\nabla^2 b^b_D(r)\ , \quad \Gamma = P_-\gamma_5\ \tilde{\Gamma} = \gamma_z P_+,$$

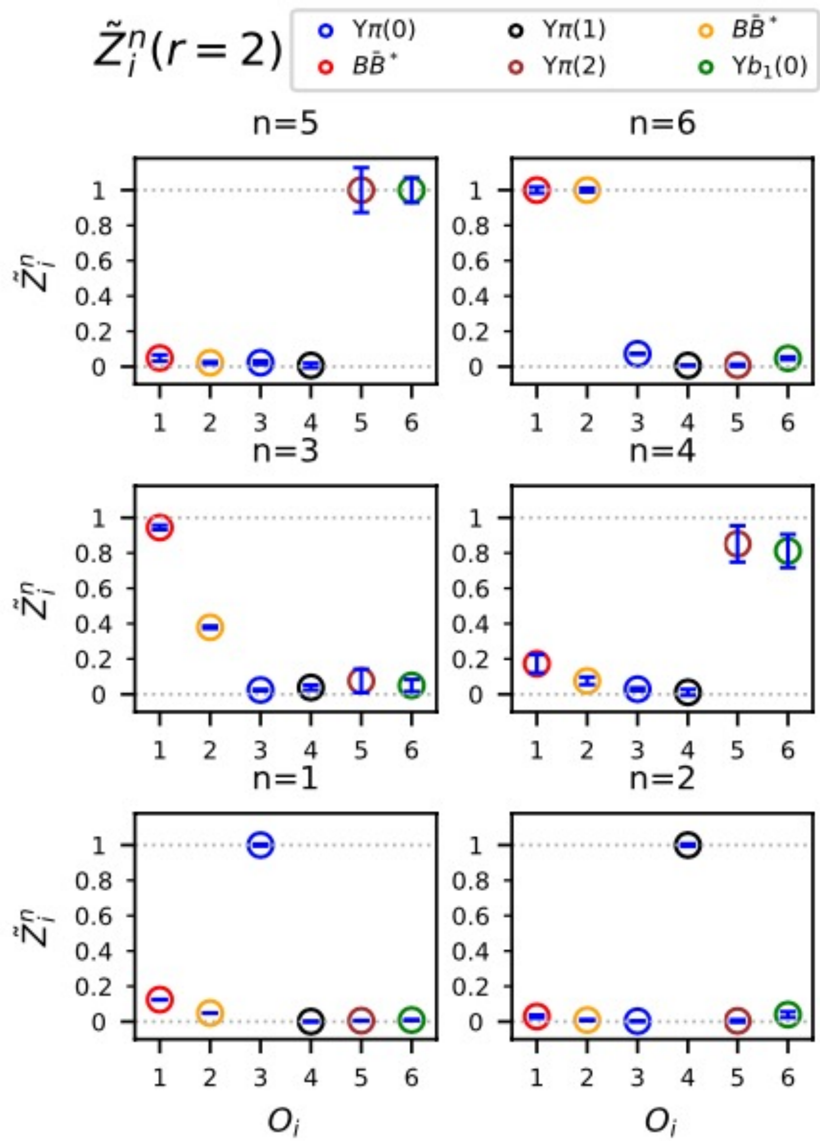
I=1 I_3=0 : $\quad \bar{q}q \to \bar{u}u - \bar{d}d$

Example: r=2





$$\tilde{Z}_i^n \equiv \langle O_i|n\rangle / \max_m \langle O_i|m\rangle$$



ark states fr

Position of pole in S-matrix for bound state and virtual bund state

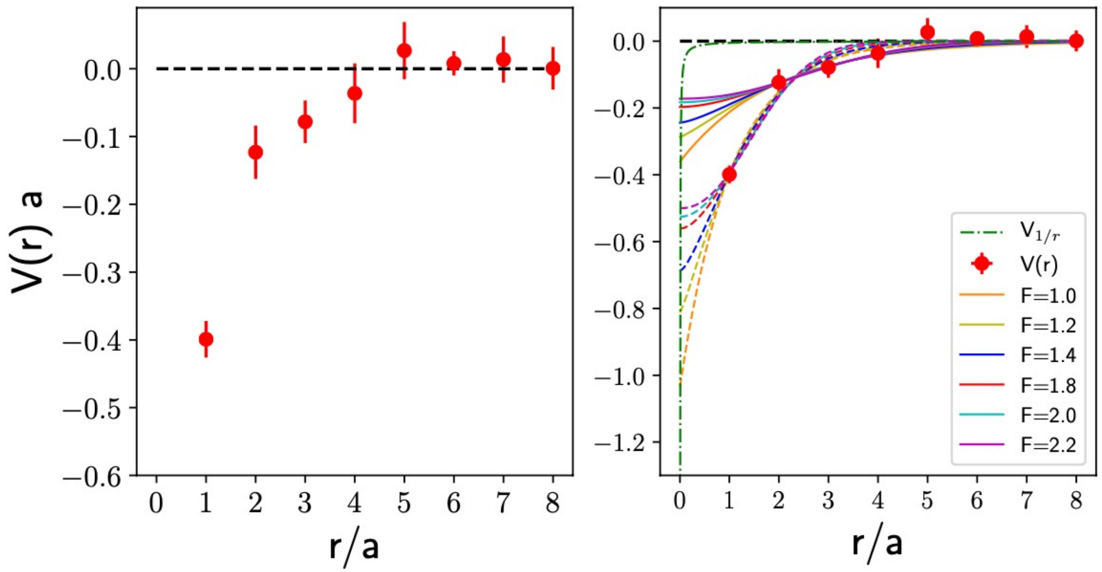
$$W_B = \sqrt{m_B^2 + p^2} + \sqrt{m_{B^*}^2 + p^2}$$

p= + i |p| for bound state

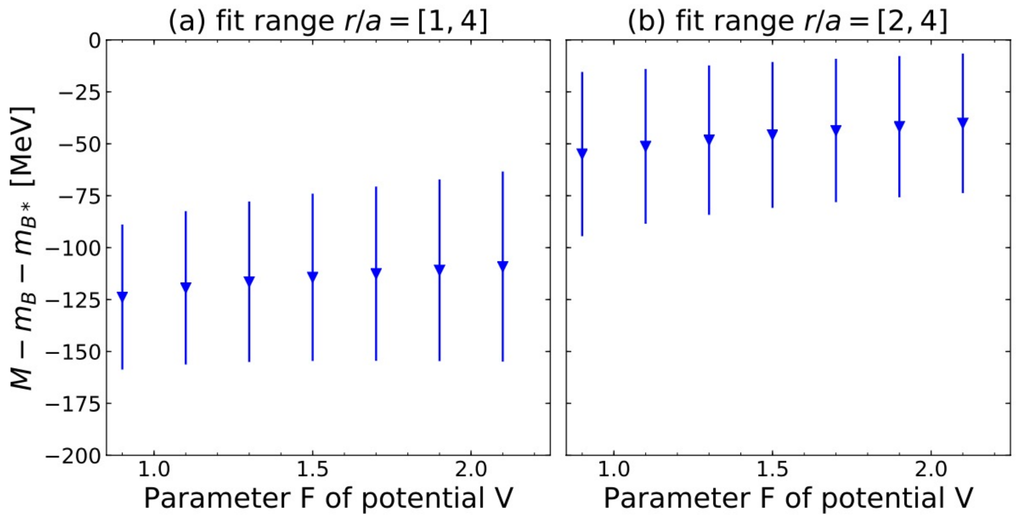
p= - i |p| for virtual bound state

a=0.12 fm

V(r) for interaction between B and $\underline{B}$*



binding enery



Open problems

- $V(r/a<1)$ = ???

reqest to the community: determine V(r) for small r in this and similar channels

perturbative estimate at very small r
supp. S4 1912.02656 & backup-slides
green line : negligible effect

$$V_{1/r}(r) = -C\ \alpha_s^3/r$$

- $V(r/a>>1)$ = ???
- analytic fit form for V(r) = ???

reqest to the community:

determine analytic form for V(r)

We employ form below for various choices F

$$V(r) = -A\ e^{-(r/d)^F}$$

$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2} + \frac{\hbar^2 L(L+1)}{2\mu r^2} + V(r)\right]u(r) = Eu(r)$$

we find one bound state with binding enery

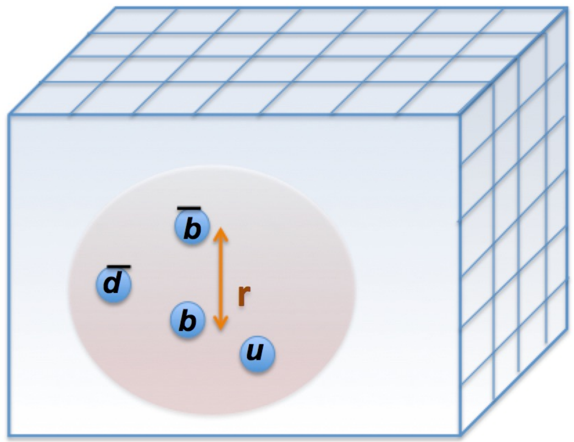
$$M - m_B - m_{B^*} = -48\ ^{+41}_{-108}\ \mathrm{MeV}$$

Analytic study of interest .. yet to be done

- Consider resulting lattice potentials in channels Sh=1 and 0
- Think about analytic expectation for potentials at small r (maybe also large r)
- Take into account that $B\underline{B}^*$ and $B^*\underline{B}^*$ are linear combinations of channels Sh=1 and 0
- consequences concerning Zb ?
- ... not easy/straightforward ... also because of sizable errors on lattice eigen-energies $E_{B\underline{B}^*}(r)$

... these are partly due to the fact that $B\underline{B}^*$ is not the ground state

Z_b with static b and $\underline{b}$



static b → b quark can not flip spin via gluon exchange

$m_b = \infty$

$\vec{S}_h = \vec{S}_b + \vec{S}_{\bar{b}}$ conserved : S_h=1 and S_h=0 are seperate channels

$\Upsilon_b \nleftrightarrow \eta_b$, h_b

$\Upsilon_b \pi \nleftrightarrow \eta_b \rho$, $h_b \pi$

$m_b = \text{finite}$

$\vec{S}_h = \vec{S}_b + \vec{S}_{\bar{b}}$ not conserved : S_h=1 and S_h=0 couple

$\Upsilon_b \leftrightarrow \eta_b$, h_b

$\Upsilon_b \pi \leftrightarrow \eta_b \rho$, $h_b \pi$