Z_{cs}^{+} exotics in $B^{+} \to J/\psi\phi K^{+}$ decays at LHCb

Zehua Xu
On behalf of the LHCb collaboration

Peking University

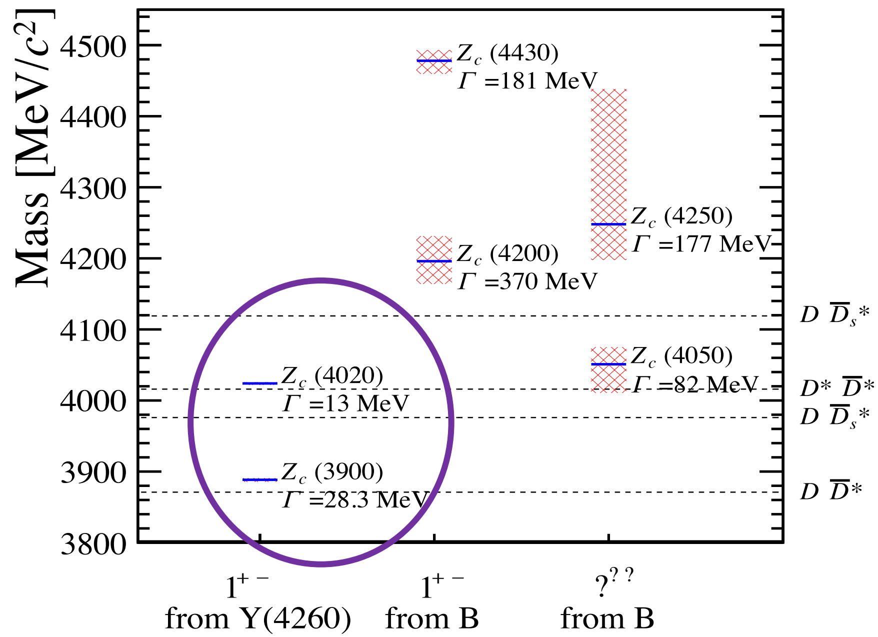
2021.04.14
@ GSI Darmstadt, Germany
Experimental and theoretical status of and perspectives for XYZ states

Z_c and predictions of Z_{cs}

- Some Z_c states were observed from B decays or $Y(4260)$
- Several papers have predicted the existence of Z_{cs} states in early time:

  [J. Korean Phys. Soc. 55 (2009) 424]
  [Phys. Rev. Lett. 110 (2013), no. 23 232001]
  [Phys. Rev. D 88 (2013), no. 9 096014]
  [Phys. Lett. B 798 (2019) 135022]

- The Z_{cs} states would be useful to distinguish different models:
  - Less exchange particles expected in the Z_{cs} molecule picture



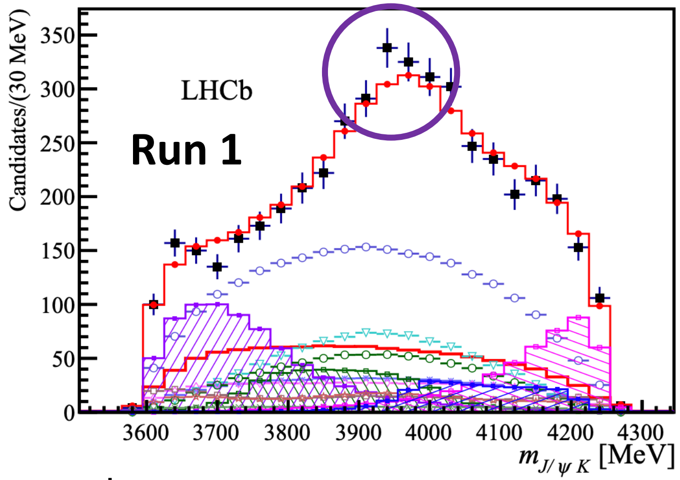
Z_c states from $Y(4260)$ are narrow;
Others from B decays are broad

> Finally, as pointed out in ref. [13], the meson-meson molecular model cannot be used to describe heavy-light tetraquarks with non-null strangeness content. The reason is that one-pion-exchange cannot take place between strange and nonstrange heavy mesons, like B and B_s. Hidden-charm and bottom mesons with strangeness are also forbidden in the

[JHEP 04 (2020) 119]

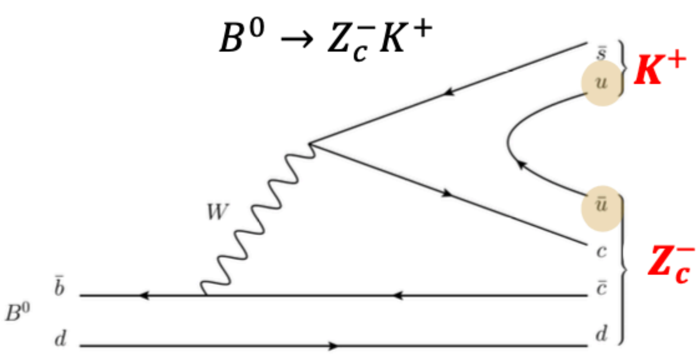
Search for Z_{cs}^{+} in $B^{+} \to J/\psi\phi K^{+}$ decay

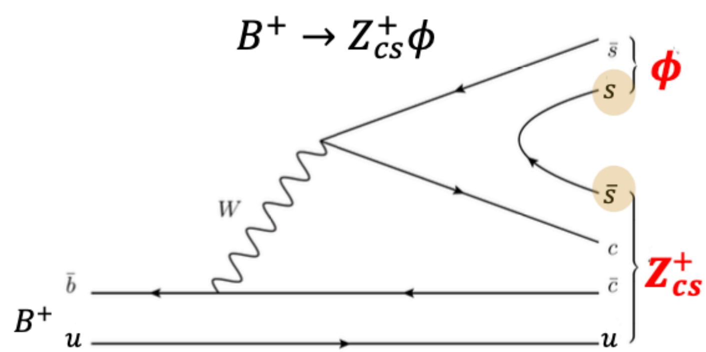
- First amplitude analysis for $B^{+} \to J/\psi\phi K^{+}$ was studied at LHCb using Run 1 sample.
- Hint of $J/\psi K^{+}$ structure in Run 1 analysis, but not significant



[PRL. 118 (2017) 022003]

- Z_{cs}^{+} in $B^{+} \to J/\psi\phi K^{+}$ decay has similar topology as Z_{c}^{+} in $B^{+} \to J/\psi K^{-}\pi^{+}$ decay, and P_{c}^{+} in $\Lambda_{b}^{0} \to J/\psi p K^{-}$





X states in $B^+ \to J/\psi\phi K^+$ decay

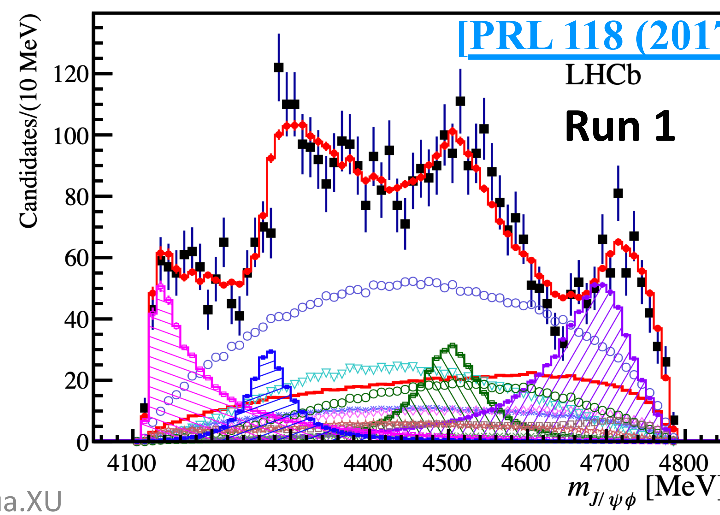
- The width of $X(4140)$ is $83 \pm 21^{+21}_{-14}$ MeV, larger than the value measured from other experiments.

[PRD 95 (2017) 012002]

Year	Experiment luminosity	$B \to J/\psi\,\phi K$ yield	$X(4140)$ peak Mass [MeV]	Width [MeV]	Sign.	Fraction %
2008	CDF 2.7 fb^{-1} [1]	58 ± 10	$4143.0 \pm 2.9 \pm 1.2$	$11.7^{+8.3}_{-5.0} \pm 3.7$	3.8σ	
2009	*Belle* [22]	325 ± 21	*4143.0 fixed*	*11.7 fixed*	1.9σ	
2011	*CDF 6.0 fb^{-1}* [29]	115 ± 12	$4143.4^{+2.9}_{-3.0} \pm 0.6$	$15.3^{+10.4}_{-6.1} \pm 2.5$	5.0σ	$14.9 \pm 3.9 \pm 2.4$
2011	LHCb 0.37 fb^{-1} [21]	346 ± 20	4143.4 fixed	15.3 fixed	1.4σ	< 7 @ 90%CL
2013	CMS 5.2 fb^{-1} [25]	2480 ± 160	$4148.0 \pm 2.4 \pm 6.3$	$28 \ ^{+15}_{-11} \pm 19$	5.0σ	10 ± 3 (stat.)
2013	D0 10.4 fb^{-1} [26]	215 ± 37	$4159.0 \pm 4.3 \pm 6.6$	$19.9 \pm 12.6^{+1.0}_{-8.0}$	3.0σ	$21 \pm 8 \pm 4$
2014	BaBar [24]	189 ± 14	4143.4 fixed	15.3 fixed	1.6σ	< 13.3 @ 90%CL
2015	D0 10.4 fb^{-1} [27]	$p\bar{p} \to J/\psi\,\phi\ldots$	$4152.5 \pm 1.7^{+6.2}_{-5.4}$	$16.3 \pm 5.6 \pm 11.4$	4.7σ (5.7σ)	
Average			4147.1 ± 2.4	15.7 ± 6.3		

- Three other $J/\psi\phi$ structures, $X(4274)$, $X(4500)$ and $X(4700)$ were observed in Run 1 analysis.

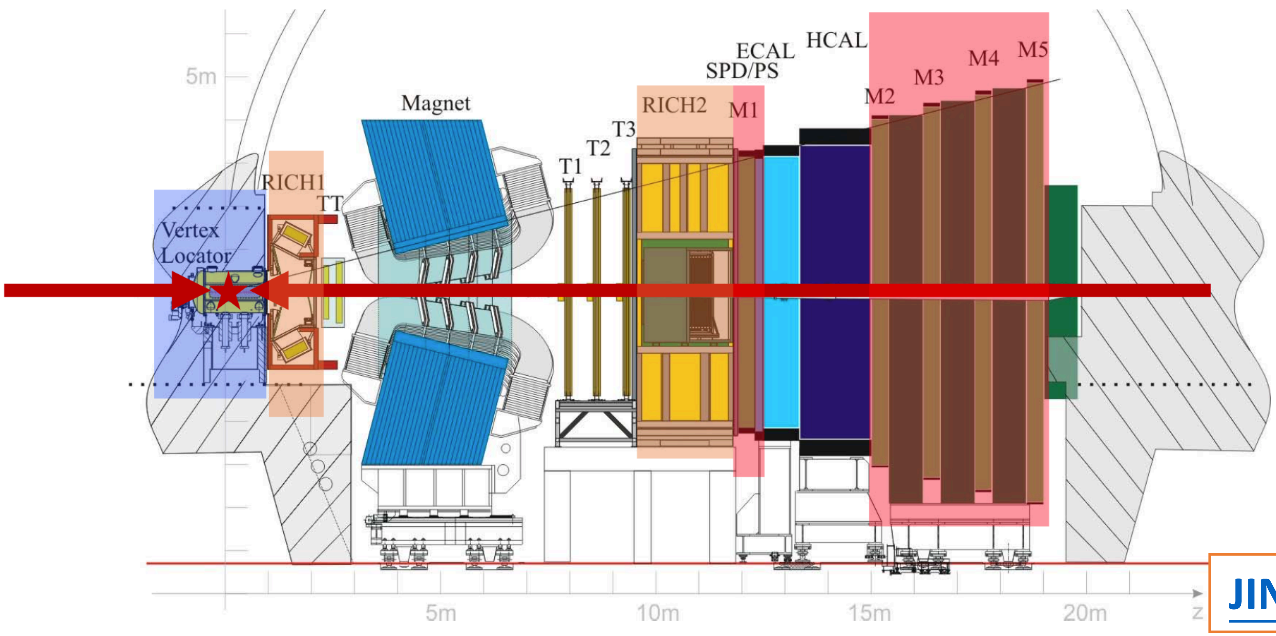
[PRL 118 (2017) 022003]

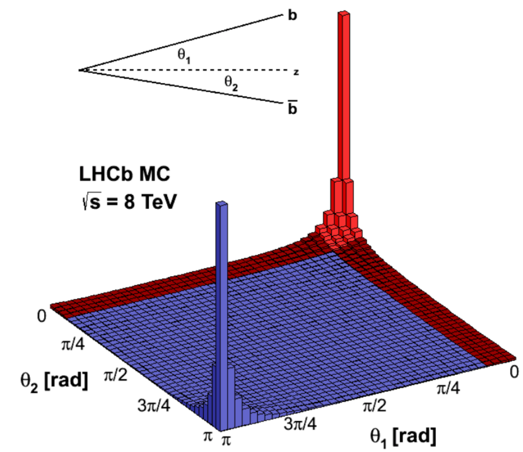


LHCb detector

LHCb is a dedicated heavy flavor physics experiment at LHC

- $\sim$20,000/s $b\bar{b}$ generated at LHCb point in Run 2
- A single-arm forward region spectrometer covering $2 < \eta < 5$



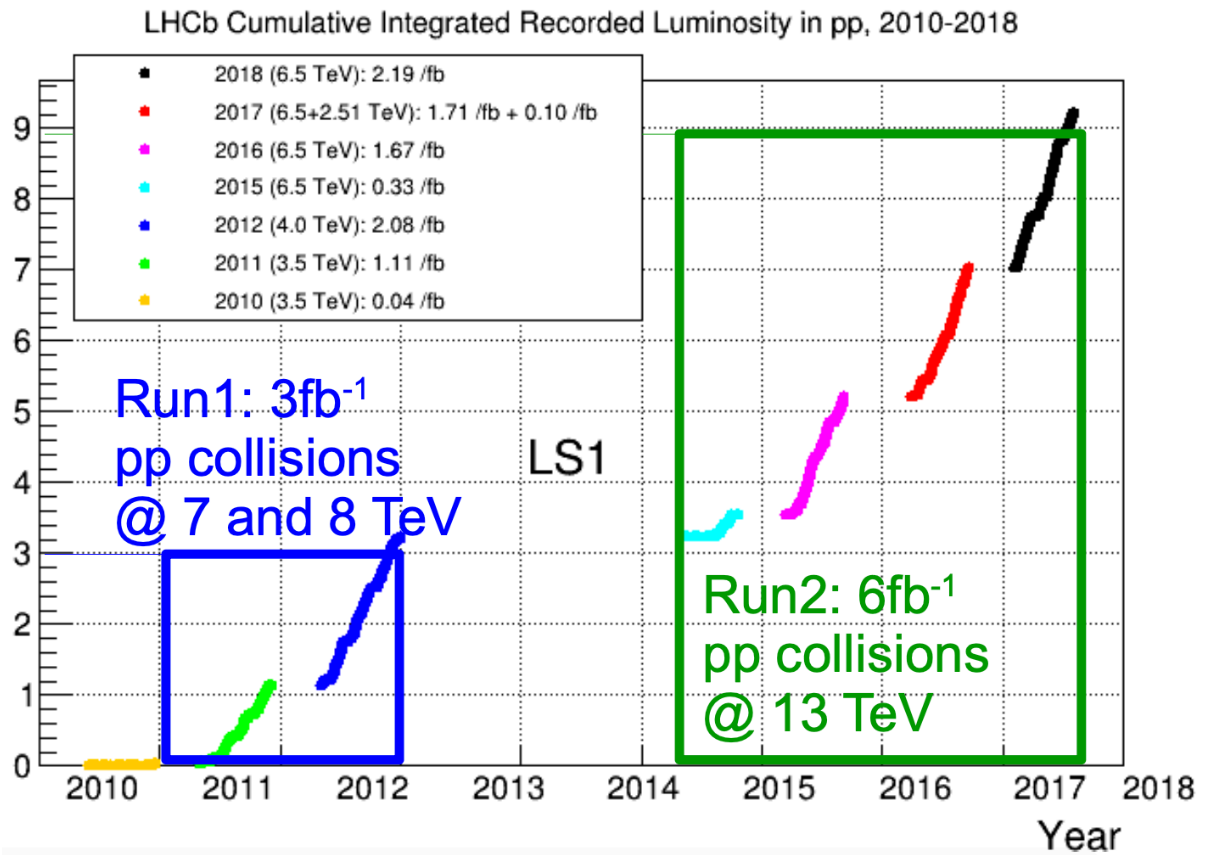


JINST 3 (2008) S08005

Vertex:	$\sigma_{IP} = 20\ \mu m$
Time:	$\sigma_\tau = 45$ fs for $B_s^0 \to J/\psi\phi$ or $D_s^+\pi^-$
Momentum:	$\Delta p/p = 0.4 \sim 0.6\%$ ($5 - 100$ GeV/c)
Mass :	$\sigma_m = 8\ \mathrm{MeV}/c^2$ for $B \to J/\psi X$ (constrained $m_{J/\psi}$)
Hadron ID:	$\varepsilon(K \to K) \sim 95\%$ mis-ID $\varepsilon(\pi \to K) \sim 5\%$
Muon ID:	$\varepsilon(\mu \to \mu) \sim 97\%$ mis-ID $\varepsilon(\pi \to \mu) \sim 1 - 3\%$
ECAL:	$\Delta E/E = 1 \oplus 10\%/\sqrt{E\ (\mathrm{GeV})}$

LHCb data taking

Large data sample used to study $B^+ \to J/\psi\phi K^+$ at LHCb

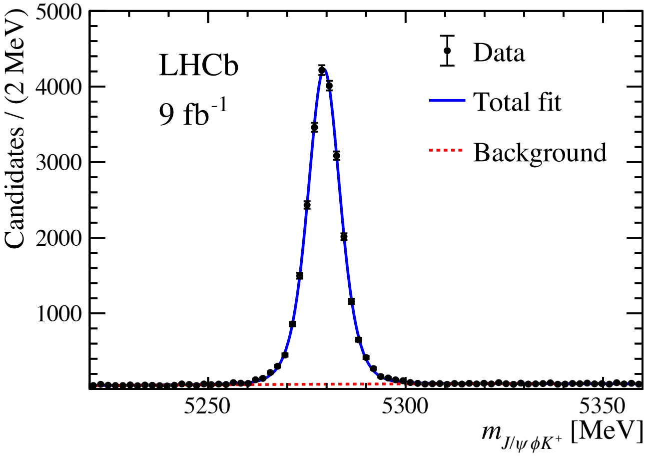


[Taken from LHCb public website]

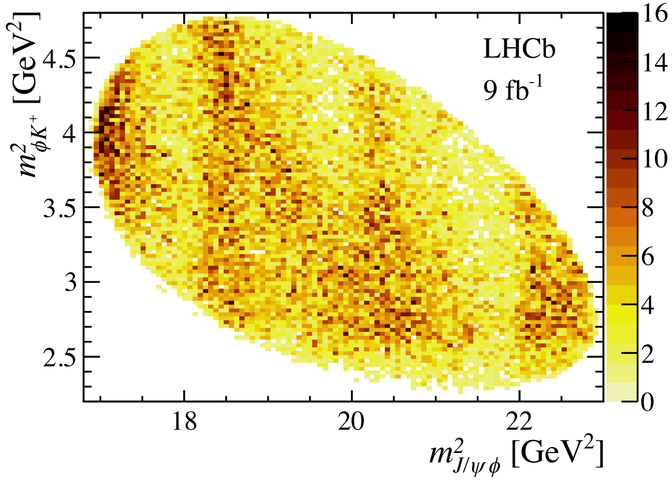
Run 1 and Run 2 sample at LHCb

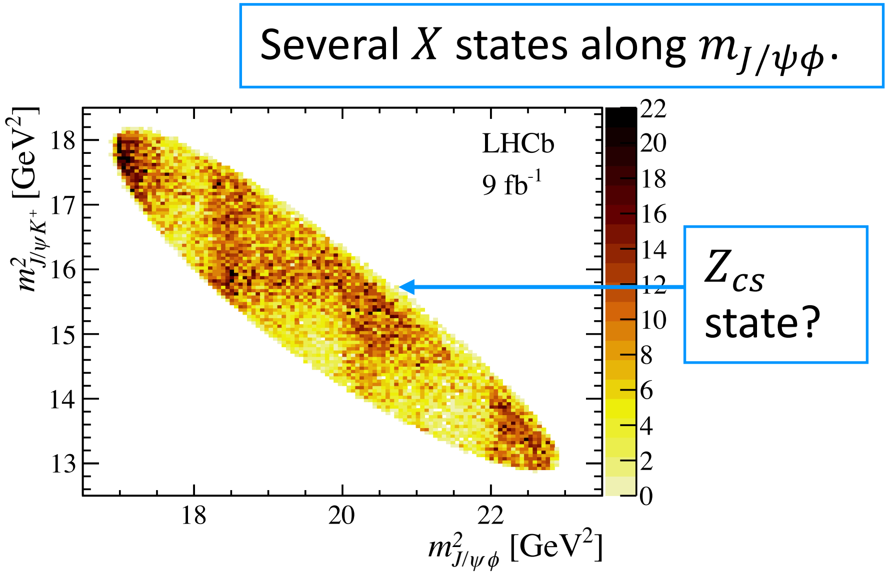
[arXiv:2103.01803]

- Selections are optimized
  - ~ 24k signal (6 times larger than the previous publication)
  - overall background fraction ~ 4% (a factor of 6 smaller)



- Clear structures in Dalitz plot





Clearly visible: 4 structures in $J/\psi\phi$ and an obvious structure in $J/\psi K$

6-D amplitude fit approach

- Signal and background components in PDF:

$$-\ln L(\vec{\omega}) = -\sum_i \ln\left[(1-\beta)\mathcal{P}_{\rm sig}(m_{\phi K\ i}, \Omega_i|\vec{\omega}) + \beta\,\mathcal{P}_{\rm bkg}(m_{\phi K\ i}, \Omega_i)\right]$$

$$= -\sum_i \ln\left[(1-\beta)\frac{\left|\mathcal{M}(m_{\phi K\ i}, \Omega_i|\vec{\omega})\right|^2 \Phi(m_{\phi K\ i})\epsilon(m_{\phi K\ i}, \Omega_i)}{I(\vec{\omega})} + \beta\frac{\mathcal{P}^u_{\rm bkg}(m_{\phi K\ i}, \Omega_i)}{I_{\rm bkg}}\right]$$

$$= -\sum_i \ln\left[\left|\mathcal{M}(m_{\phi K\ i}, \Omega_i|\vec{\omega})\right|^2 + \frac{\beta\,I(\vec{\omega})}{(1-\beta)I_{\rm bkg}}\frac{\mathcal{P}^u_{\rm bkg}(m_{\phi K\ i}, \Omega_i)}{\Phi(m_{\phi K\ i})\epsilon(m_{\phi K\ i}, \Omega_i)}\right] + N\ln I(\vec{\omega}).$$

- Each decay chain is described by 6 observables:

$$\Phi \equiv (m_{\phi K}, \theta_{K^*}, \theta_{J/\psi}, \theta_\phi, \Delta\varphi_{K^*,J/\psi}, \Delta\varphi_{K^*,\phi})$$

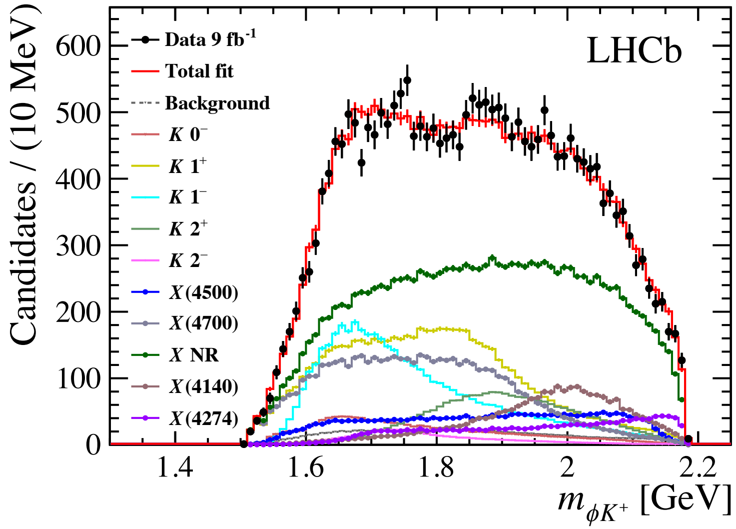
Where θ denotes the helicity angle, and $\Delta\varphi$ is the angle between two decay chains.

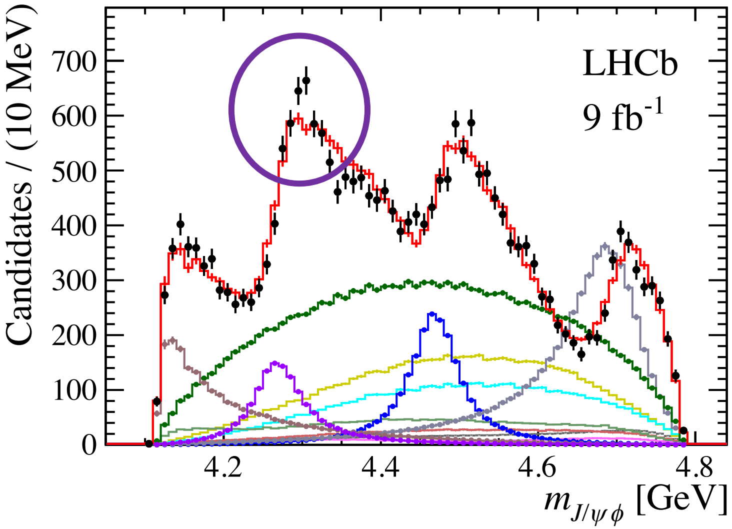
- Resonance lineshape: Breit-Wigner (default), simplified K-matrix or Flatté functions for systematic uncertainties.

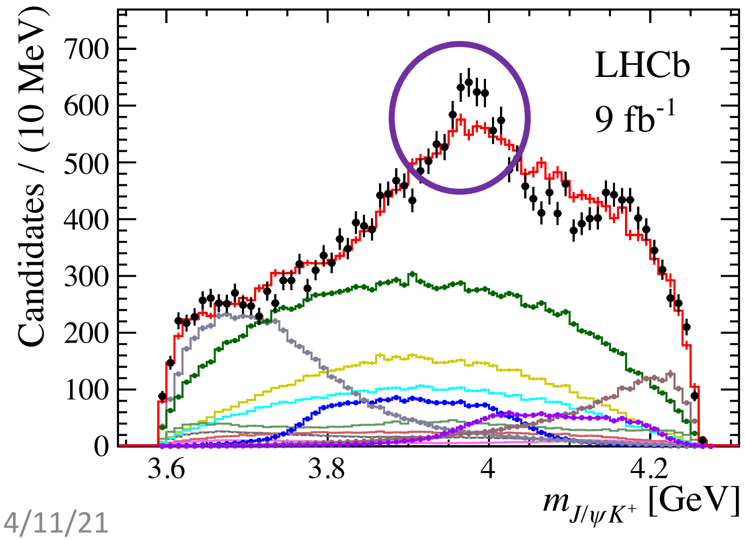
Run 1 model cannot fit well

[arXiv:2103.01803]

➢ The fit model of Run 1 analysis tested first:
[PRL 118 (2017) 022003]







- Some deficiencies in describing the $m_{J/\psi\phi}$ and $m_{J/\psi K}$ are obvious.
- The fit model needs to be improved.

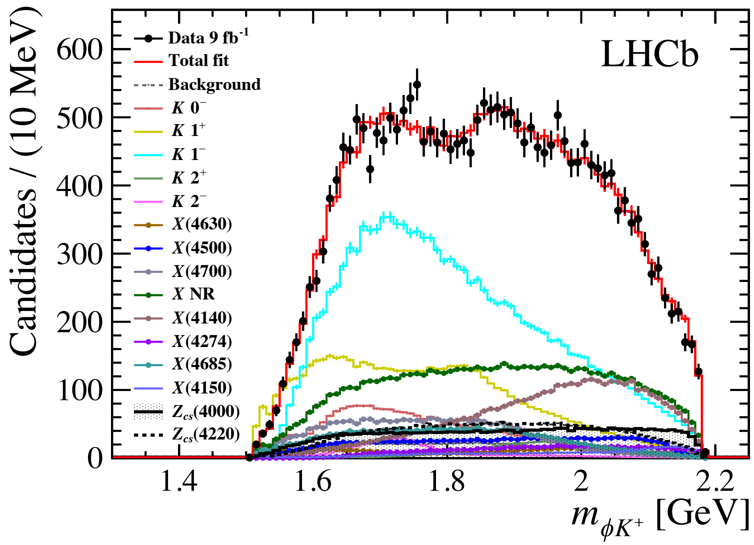
Test new exotics

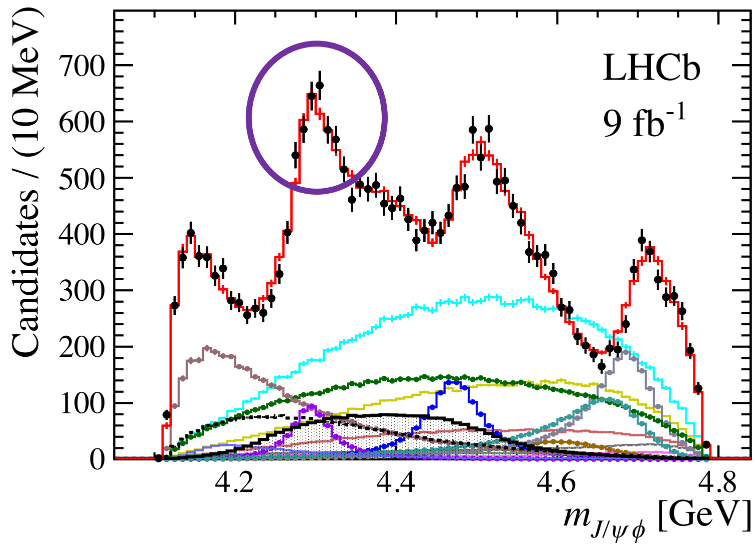
- Data cannot be described well by improving the K^* model
- New exotic states (X and Z_{cs}^+) of different J^P were tested:
  - 1^+ Z_{cs} and 1^+ X, giving the largest improvements, were first included.
  - Several states giving large fit improvements were also included in the default model: a second Z_{cs} and X states

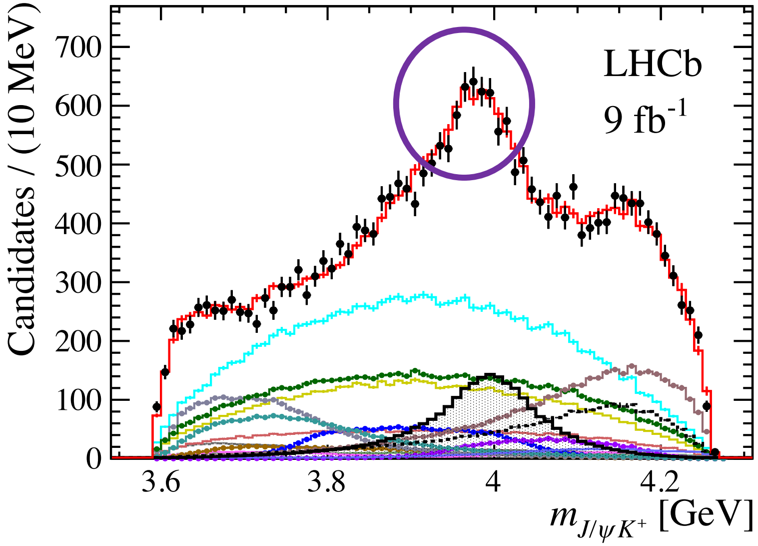
- The default model includes $9K^* + 7X + 1X(NR) + 2Z_{cs}$

Default fit

[arXiv:2103.01803]





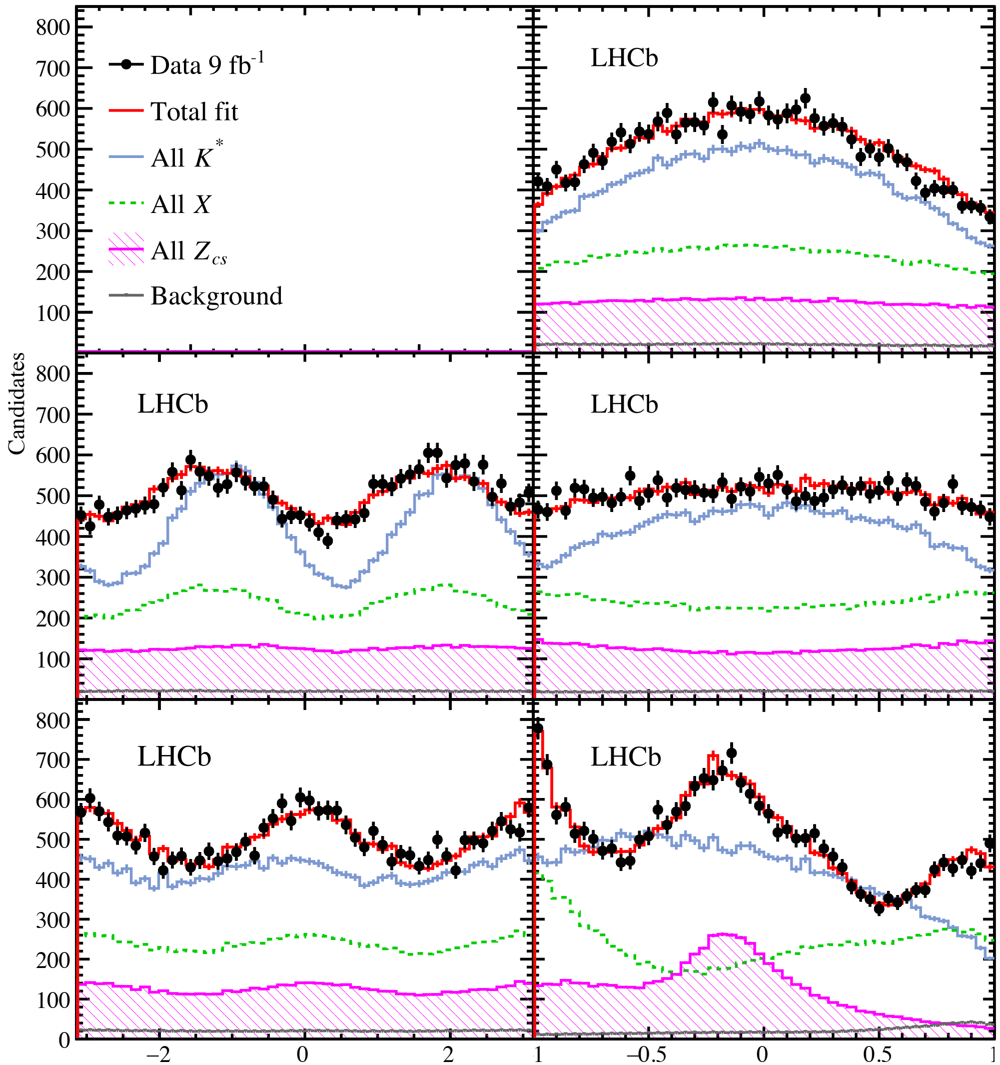


- Data is well described.

Angular distributions

[arXiv:2103.01803]

- Angular distributions in the ϕK decay chain are also well described.



Fit results

[arXiv:2103.01803]

Contribution	Significance [$\times\sigma$]	M_0 [MeV]	Γ_0 [MeV]	FF [%]
$X(2^-)$				
$X(4150)$	4.8 (8.7)	$4146 \pm 18 \pm 33$	$135 \pm 28 ^{+59}_{-30}$	$2.0 \pm 0.5 ^{+0.8}_{-1.0}$
$X(1^-)$				
$X(4630)$	5.5 (5.7)	$4626 \pm 16 ^{+18}_{-110}$	$174 \pm 27 ^{+134}_{-73}$	$2.6 \pm 0.5 ^{+2.9}_{-1.5}$
All $X(0^+)$				$20 \pm 5 ^{+14}_{-7}$
$X(4500)$	20 (20)	$4474 \pm 3 \pm 3$	$77 \pm 6 ^{+10}_{-8}$	$5.6 \pm 0.7 ^{+2.4}_{-0.6}$
$X(4700)$	17 (18)	$4694 \pm 4 ^{+16}_{-3}$	$87 \pm 8 ^{+16}_{-6}$	$8.9 \pm 1.2 ^{+4.9}_{-1.4}$
$NR_{J/\psi\phi}$	4.8 (5.7)			$28 \pm 8 ^{+19}_{-11}$
All $X(1^+)$				$26 \pm 3 ^{+8}_{-10}$
$X(4140)$	13 (16)	$4118 \pm 11 ^{+19}_{-36}$	$162 \pm 21 ^{+24}_{-49}$	$17 \pm 3 ^{+19}_{-6}$
$X(4274)$	18 (18)	$4294 \pm 4 ^{+3}_{-6}$	$53 \pm 5 \pm 5$	$2.8 \pm 0.5 ^{+0.8}_{-0.4}$
$X(4685)$	15 (15)	$4684 \pm 7 ^{+13}_{-16}$	$126 \pm 15 ^{+37}_{-41}$	$7.2 \pm 1.0 ^{+4.0}_{-2.0}$
All $Z_{cs}(1^+)$				$25 \pm 5 ^{+11}_{-12}$
$Z_{cs}(4000)$	15 (16)	$4003 \pm 6 ^{+4}_{-14}$	$131 \pm 15 \pm 26$	$9.4 \pm 2.1 \pm 3.4$
$Z_{cs}(4220)$	5.9 (8.4)	$4216 \pm 24 ^{+43}_{-30}$	$233 \pm 52 ^{+97}_{-73}$	$10 \pm 4 ^{+10}_{-7}$

Fit fraction

- Two $Z_{cs}^+ \to J/\psi K^+$ states were observed, both significance $> 5\sigma$
- New $X(4630)$ and $X(4685)$ were observed, both significance $> 5\sigma$
- Previous results using Run 1 sample were confirmed with large significance

J^P analysis

- "prefer" J^P gives best log-likelihood ($\ln\mathcal{L}$)
- The $\ln\mathcal{L}$ difference between the "prefer" one and alternative hypotheses used to estimate significance:

J^P	0^+	0^-	1^+	1^-	2^+	2^-
$X(4630)$	6.7σ	5.3σ	5.8σ	prefer	5.9σ	**3.0σ**
$X(4500)$	prefer	18σ	18σ	18σ	18σ	18σ
$X(4700)$	prefer	18σ	18σ	18σ	14σ	17σ
$X(4140)$	14σ	15σ	prefer	14σ	13σ	14σ
$X(4274)$	18σ	18σ	prefer	18σ	18σ	18σ
$X(4685)$	16σ	16σ	prefer	15σ	16σ	15σ
$Z_{cs}(4000)$	-	17σ	prefer	17σ	15σ	16σ
$Z_{cs}(4220)$	-	8.6σ	prefer	**2.4σ**	4.9σ	5.7σ

- ➢ J^P assignments to the previous 4X states are confirmed to be correct, with improved significance
- ➢ $Z_{cs}(4000)$ and $X(4685)$ J^P are determined to be $1^+ > 15\sigma$.
- ➢ J^P of the other two new X states not well determined (difference <5σ).
- ➢ $Z_{cs}(4220)$ 1^+ and 1^- cannot be distinguished.

Systematic uncertainty

Details in Backup

Following systematic sources are considered :

[arXiv:2103.01803]

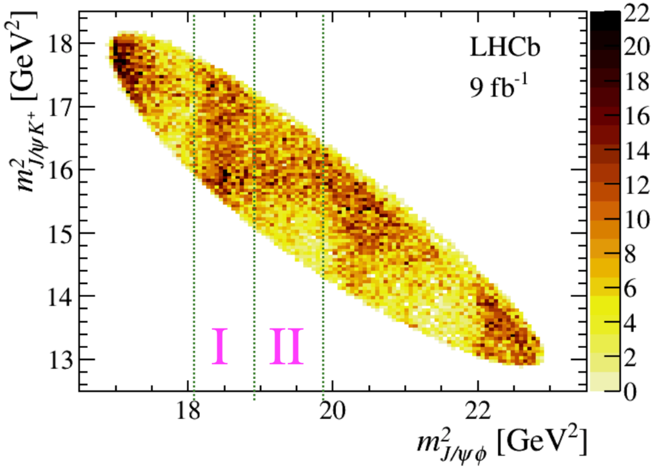
Source	$Z(4000)$ M_0	$Z(4000)$ Γ_0	$Z(4000)$ FF	$X(4685)$ M_0	$X(4685)$ Γ_0	$X(4685)$ FF
Fixed M_0&Γ_0	−0.22	−3.60	−0.83	−0.14	2.72	0.25
$\chi^2_{\rm IP}$ smearing	0.21	1.01	0.09	−0.53	1.11	0.12
Right sideband	0.01	0.58	0.11	−0.13	1.07	−0.13
Left sideband	−0.30	−1.16	−0.24	−0.09	−2.21	0.09
$\beta = 0.043$	−0.06	−0.00	0.01	0.01	−0.70	−0.09
$\beta = 0.037$	−0.02	0.26	0.02	−0.33	0.21	0.03
L0 Trigger	0.45	0.58	0.19	−0.58	1.12	0.11
PID efficiency	−1.06	−1.82	−0.69	−0.82	−4.42	−0.26
MC size	2.39	9.93	1.54	3.02	7.00	0.65
ϕ window	−4.71	−23.91	−2.75	8.60	−26.60	−1.17
Non ϕ subtraction	−2.87	−18.39	−1.79	12.40	−39.80	−1.80
Poly NR	−4.24	−16.36	−2.56	4.26	−22.07	−1.28
X NR(1^+)	1.49	−21.25	−2.53	−15.72	35.54	3.84
X NR(2^+)	2.16	3.09	1.26	1.88	−6.87	−0.03
BW d=1.5	−0.29	−5.27	−0.58	0.29	1.55	2.14
BW d=4.5	0.08	1.81	0.04	0.06	−3.53	−1.06
L	2.75	−3.19	−1.18	2.45	−24.33	−1.48
$X(4140)$ Flatté	0.52	−2.80	−0.45	−3.77	15.14	1.37
Extended model	−2.35	−6.66	−1.16	−3.61	−6.53	−0.94
Additional X	−0.68	2.07	0.30	0.74	−3.11	−0.18
1^- Z	−14.00	−21.09	−3.46	−9.41	−5.60	−1.52
K^* BW	0.08	−0.66	−0.32	−0.06	−8.09	−0.82
K-Matrix	−3.75	−20.80	−2.85	4.10	−11.95	−0.06
$Z_{cs}(4000)$ Flatté	0.18		2.83	−0.85	2.79	0.18
Background model	0.10	−0.32	−0.12	−1.04	−1.72	−0.15
Total	(−14.26 , +3.85)	(−26.26, +26.26)	(−3.43, +3.41)	(−16.05, +12.82)	(−40.85, +36.72)	(−1.96, +3.92)

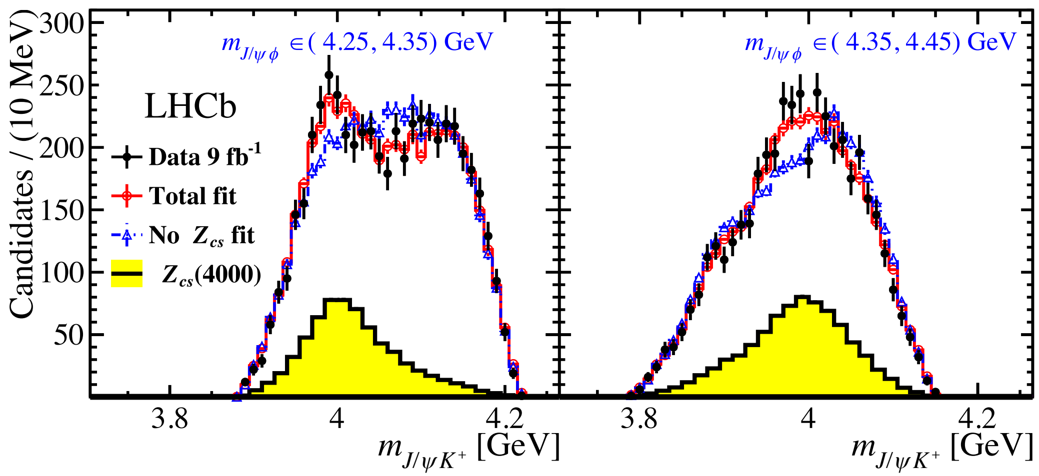
- Take the maximum deviation among the model variations, then add other sources in quadrature.
- Different J^P assignments of $Z_{cs}(4220)^+$ gives largest systematic uncertainty on $Z_{cs}(4000)^+$.

Z_{cs} results

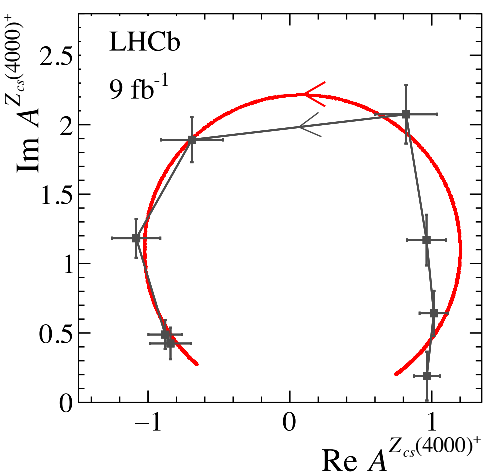
[arXiv:2103.01803]

- The fit projection onto $m(J/\psi K^+)$ in two slices of $m(J/\psi\phi)$





- Argand plot supports resonance character of $Z_{cs}(4000)^+$, obtained from lineshape-independent fitting.



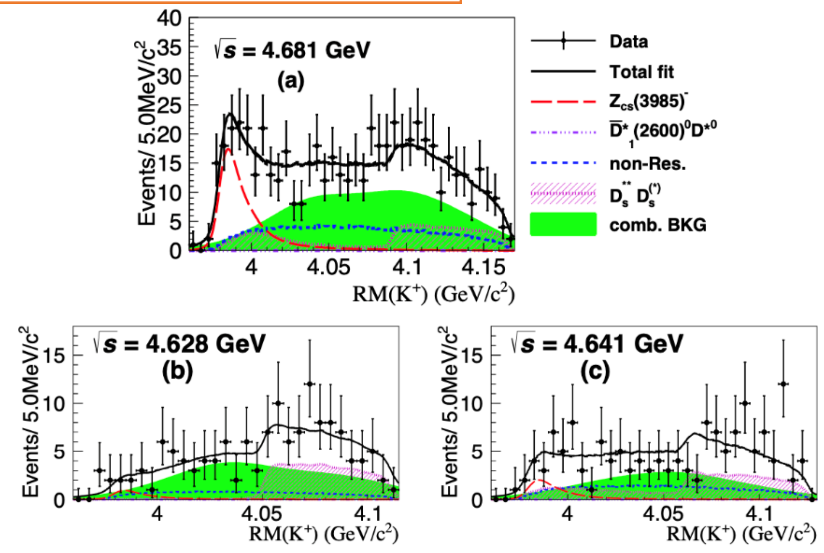
Comparison with BESIII

- BESIII experiment recently reported 5.3σ observation of a very narrow Z_{cs}^{-} in $D_s D^* + D D_s^*$ mass distributions

- Tests are applied:
  - Fix $Z_{cs}(4000)^+$ to BESIII's result, log-likelihood is much worse.
  - Adding $Z_{cs}(3985)^-$ on the default model almost doesn't improve the fit likelihood

- No evidence that $Z_{cs}(4000)^+$ state is the same as the $Z_{cs}(3985)^-$ seen by BESIII.

BESIII results

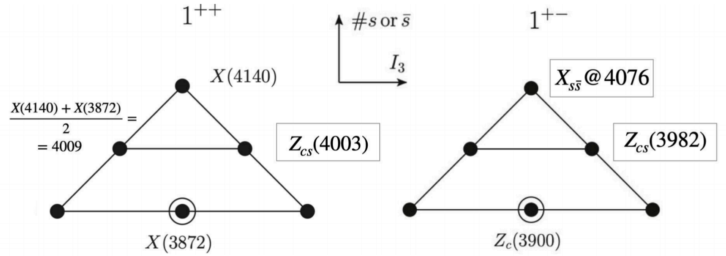


$$m_{\text{pole}}(Z_{cs}(3985)^-) = (3982.5^{+1.8}_{-2.6} \pm 2.1)\,\text{MeV}/c^2,$$
$$\Gamma_{\text{pole}}(Z_{cs}(3985)^-) = (12.8^{+5.3}_{-4.4} \pm 3.0)\,\text{MeV}.$$

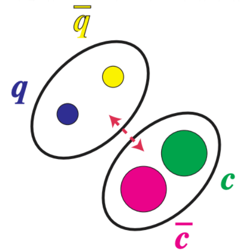
[arXiv:2011.07855]

Theory interpretations to $Z_{cs}^{+}(4000)$

- Compact tetraquark model? [arXiv:2103.08331]



- Hadronic molecule?

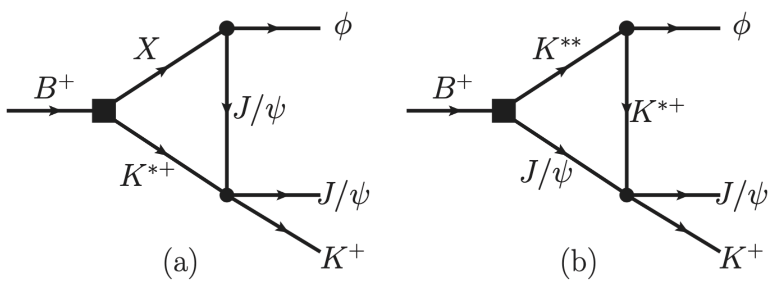


[arXiv:2011.08725]

[arXiv:2103.08586]

- $Z_{cs}(3985)$ and $Z_{cs}(4000)$ are same state in coupled-channels model? [arXiv:2103.07871]

- Threshold cusps? [arXiv:2103.05282]



- … …

X results

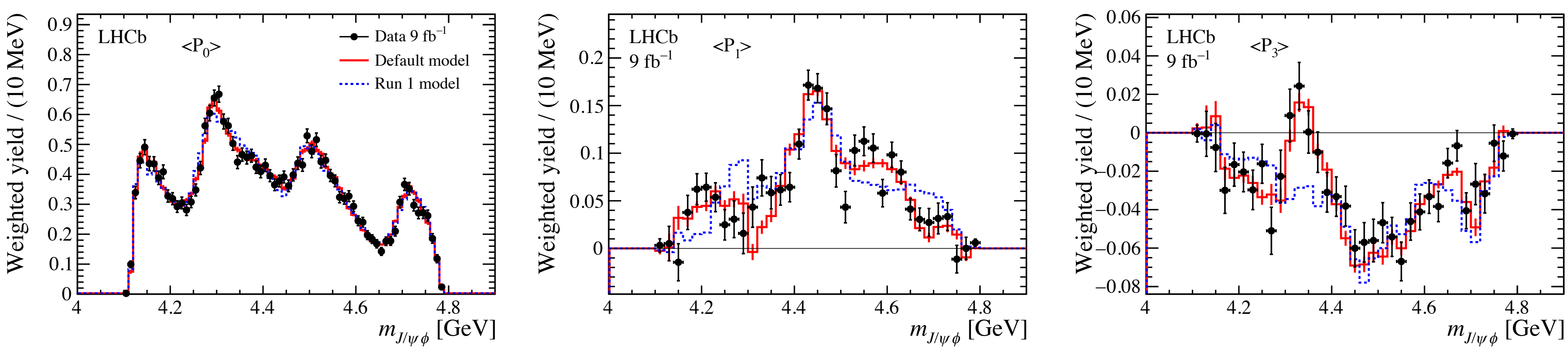
[arXiv:2103.01803]

- The measured mass of $X(4140)$ is $4118 \pm 11^{+19}_{-36}$ MeV, with width $162 \pm 21^{+24}_{-49}$ MeV, not very narrow; the mass is around the threshold of $J/\psi\phi$.
  No evidence of a narrow threshold resonance at $J/\psi\phi$ in our data

Details in Sebastian's Talk

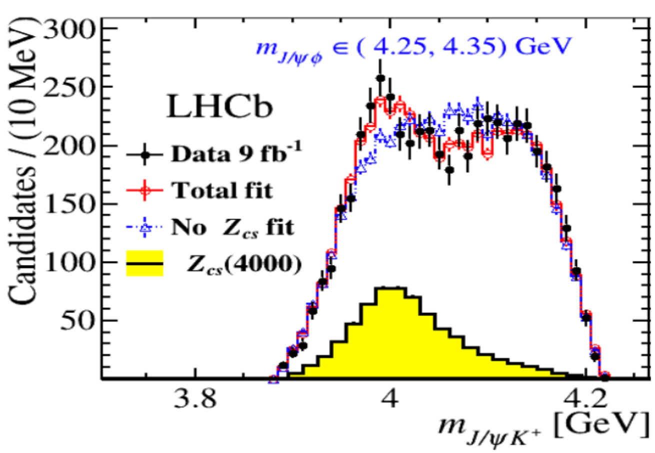
- Comparing the unnormalized Legendre moments of Run 1 model and updated model, new $X(4630)$ and $X(4685)$ are required.

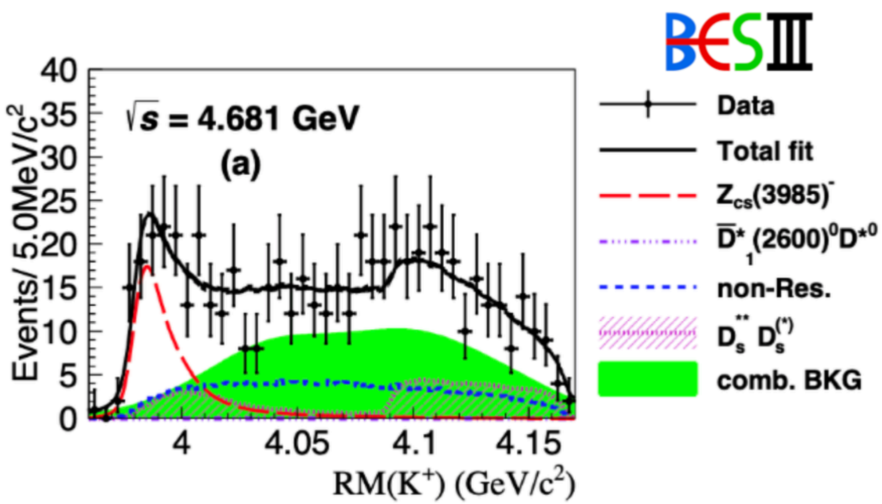
$$< P_\ell^U > = \sum_{i=1}^{N_{\text{events}}} \frac{1}{\epsilon_i} P_\ell(\cos\theta)$$



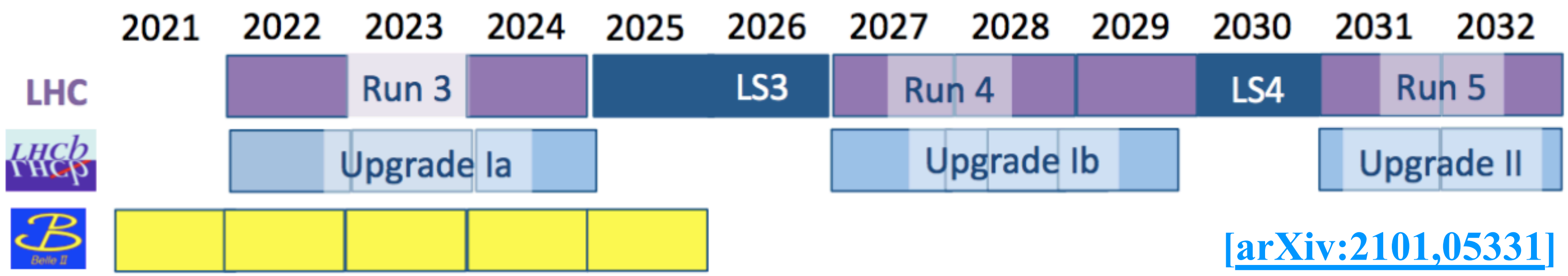
Summary

- **Four new $J/\psi K^+$ and $J/\psi\phi$ structures are observed $B^+ \to J/\psi\phi K^+$**
  1. 4 X states observed in Run 1 data sample are confirmed, and J^P determined with higher significance.
  2. $Z_{cs}(4000)^+ \to J/\psi K^+$ state is observed for first time, the significance is around 15σ, and $J^P = 1^+$ is also determined; another broad $Z_{cs}(4220)^+$ is also observed
  3. A new 1^+ $X(4685)$ is $> 15\sigma$, and new $X(4630) > 5\sigma$
- **Understanding of $Z_{cs}(4000)^+$ and $Z_{cs}(3985)^-$ may shed light on molecular and compact tetraquarks**





Prospects



[arXiv:2101,05331]

- LHCb is now boosting the data to a new level
  - Expect to 7x more data (14x hadronic events) by 2029 than current, half of these by 2024
  - The J^P of Z_{cs}^{+} (4220) could be determined with larger data sample
  - The J^{PC} of $X(4630)$ might be 1^{-+}, which is arousing interest

[arXiv:2103.03127]

- If the Z_{cs}^{+} (4000) observed at LHCb and Z_{cs}^{-} (3985) observed at BESIII are same state?
- The Z_{cs}^{0}, isospin partner of Z_{cs}^{+}, can be searched at LHCb

Thanks for listening

Backup

Many exotic states observed at LHCb

- 59 new hadron states (conventional & exotic) observed at LHC, most of them discovered at LHCb [Taken from CERNCOURIER]

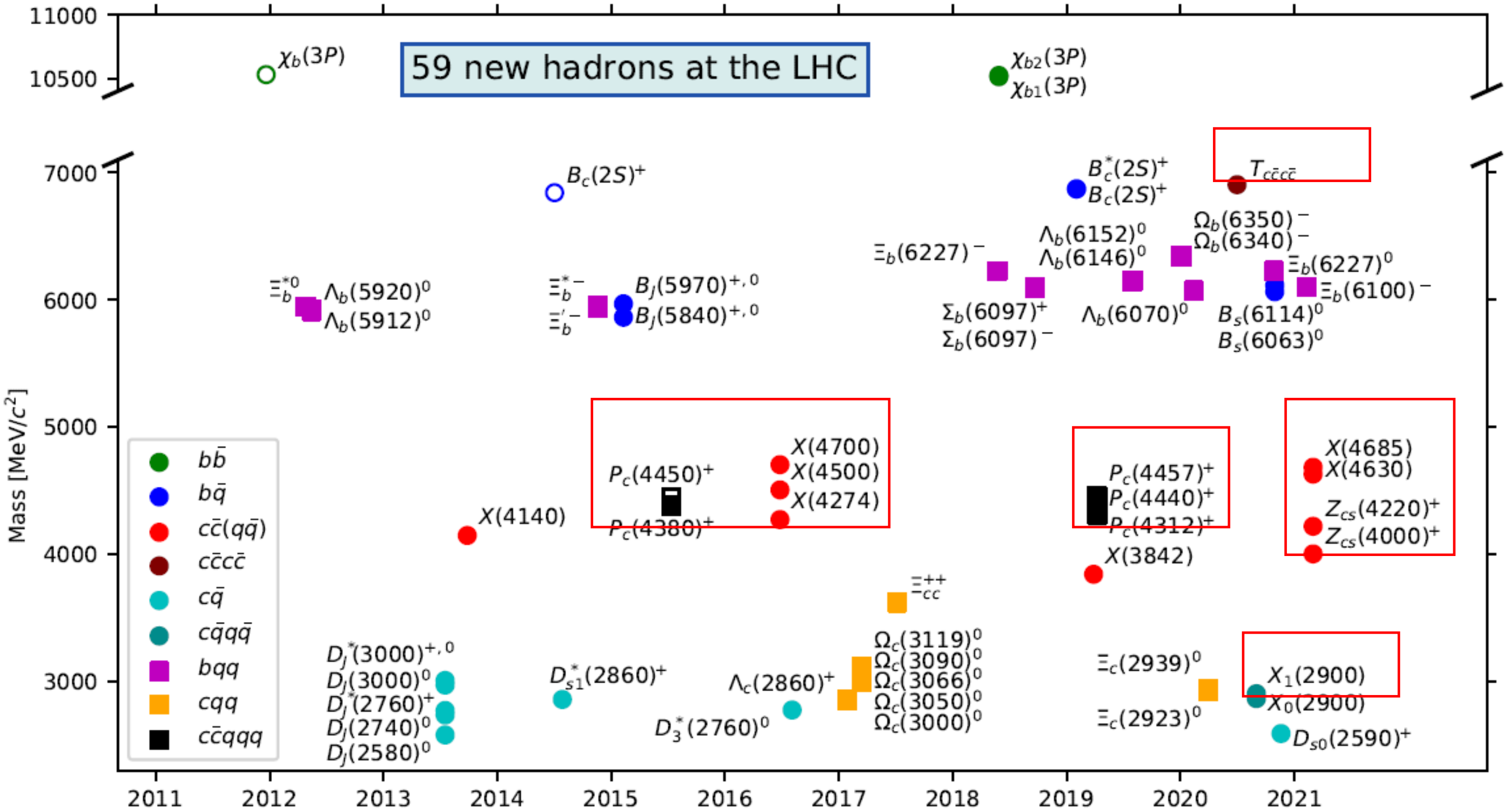
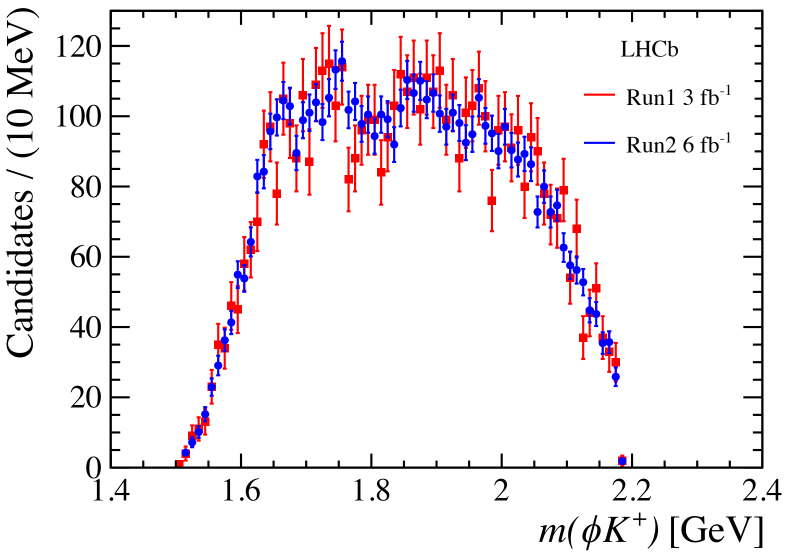
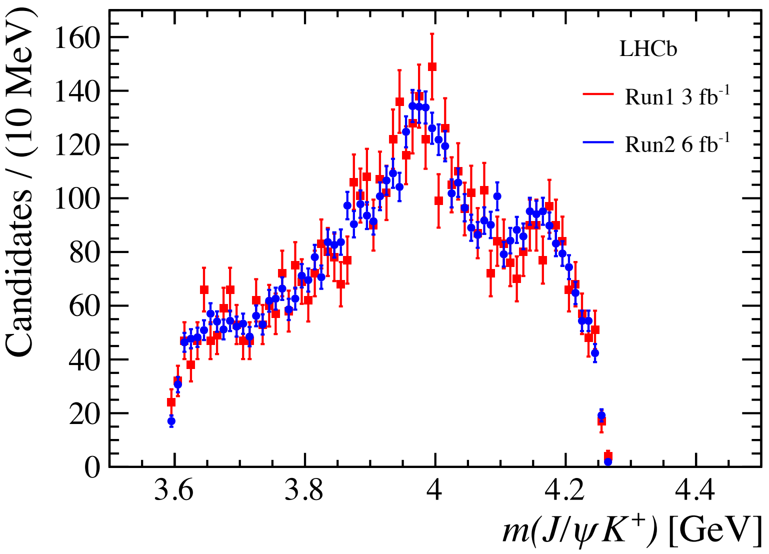


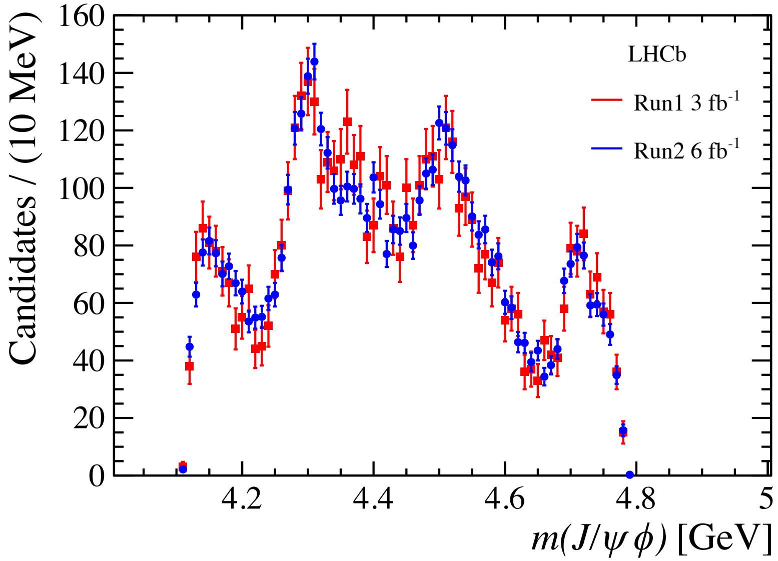
Diagram of discovery The ATLAS, CMS and LHCb collaborations have discovered 59 new hadronic states so far – the most recent being the four tetraquarks reported in this article. Credit: CERN

Run 1 and Run 2 comparison

[arXiv:2103.01803]







Legendre moments

[arXiv:2103.01803]

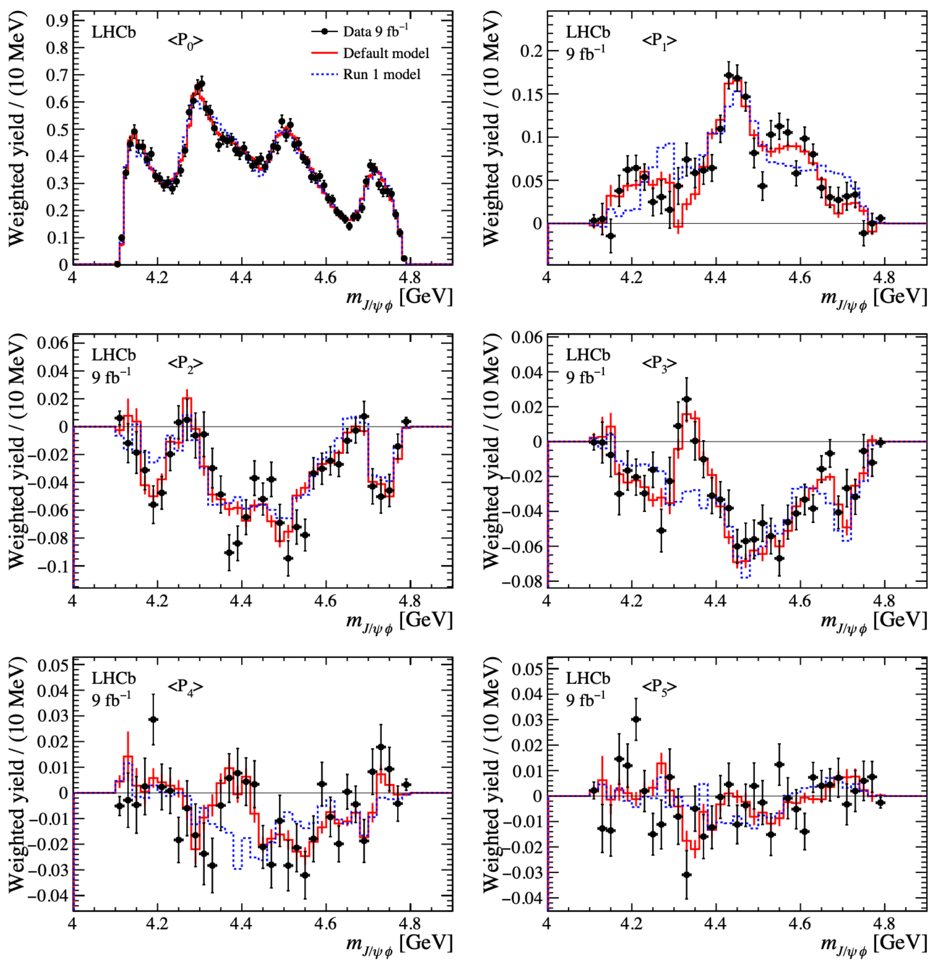
- Unnormalized Legendre moments:

$$< P_\ell^U >= \sum_{i=1}^{N_{\text{events}}} \frac{1}{\epsilon_i} P_\ell(\cos\theta)$$

Legendre polynomial of order l and efficiency for each event i.

The moments distribution is obtained by a $\frac{1}{\epsilon_i} P_l(cos\theta)$ weight.

Angular moments of $J/\psi\phi$ helicity angle

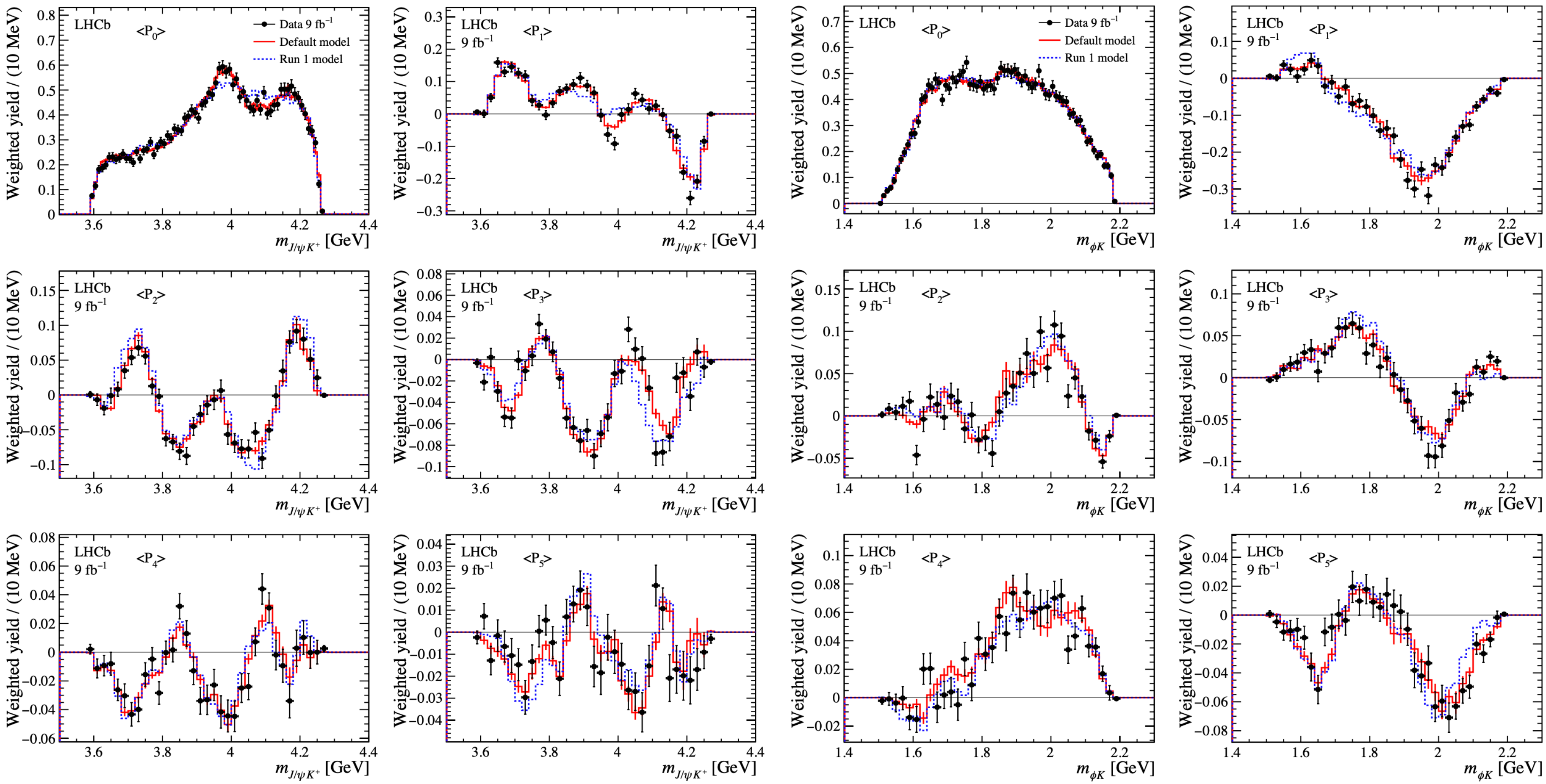


Legendre moments

[arXiv:2103.01803]

Angular moments of $J/\psi K^+$ helicity angle

Angular moments of ϕK^+ helicity angle



Systematic uncertainty 1

- To evaluate uncertainties due to the fixed masses and widths of known K^* resonances: free the masses and widths but impose Gaussian constraints to the PDG values.
- χ^2_{IP} of B^+ is not well modeled, smeared to match the data.
- To explore uncertainty in the background model, vary the B^+ sideband window.
- The uncertainty in the background fraction β: change background shape to exponential function.

- Vary the Blatt-Weisskopf barrier factor d (hadron-size parameter).
- Vary the smallest allowed orbital momentum in the resonance description function, associate the L dependent term with each LS coupling.

Orbital momentum

$$R_{K_n^*}(m_{K\phi}) = B'_{L_B^{K_n^*}}(p, p_0, d)\left(\frac{p}{M_B}\right)^{L_B^{K_n^*}} \mathrm{BW}(m_{K\phi}|M_0^{K_n^*}, \Gamma_0^{K_n^*})\, B'_{L_{K_n^*}}(q, q_0, d)\left(\frac{q}{M_0^{K_n^*}}\right)^{L_{K_n^*}}$$

Angular momentum barrier factor

Relative Breit-Wigner function

Systematic uncertainty 2

- Uncertainty due to the choice of NR component, change the constant parameterization to exponential function.
- 1^+ or 2^+ NR X contributions are optionally introduced.

- The difference between nominal model and extended model.
- Flatté function to parameterize $X(4140)$ or $Z_{cs}(4000)$ to replace BW function.

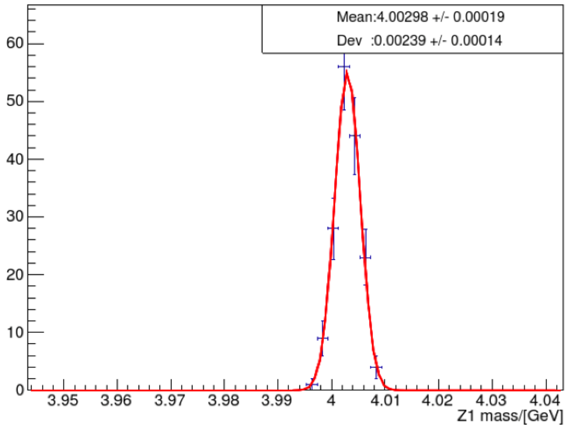
$$\mathrm{Flatte}_X(m|M_0\,,g_{J/\psi\phi}\,,g_{D_s^*D_s}) = \frac{1}{M_0^2 - m^2 - iM_0(g_{J/\psi\phi}\rho_{J/\psi\phi} + g_{D_s^*D_s}\rho_{D_s^*D_s})},$$

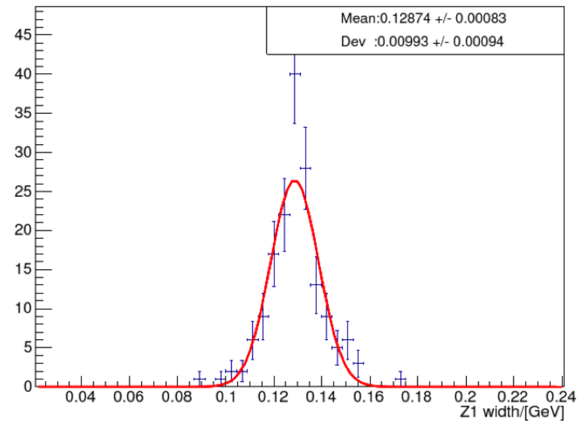
- Additional X states with different J^P in the extended model.
- Neglected no-ϕ contribution: 1)Change the ϕ mass window from $\pm$15MeV to $\pm$ 7MeV, 2) sFit to subtract no-ϕ contribution is performed as alternative to cFit

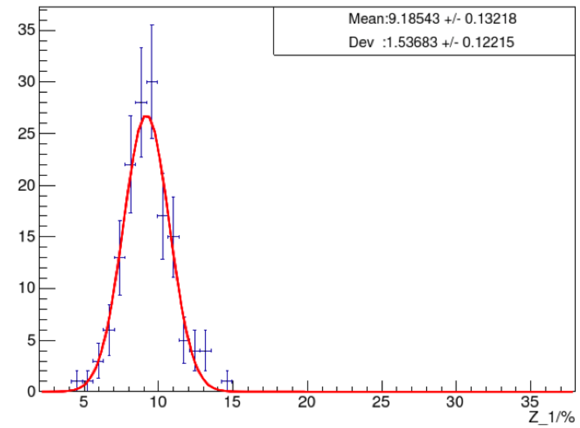
- Modification of K^* width: as the partial width to ϕK is unknown, try a fit with mass dependence of the width driven by the lowest allowed decay channel, which is $K\pi$ for the natural spin-parity and $K\omega$ for others.

Systematic uncertainty 3

- L0 trigger : change the requirement from L0 global decision to TOS on L0 Muon or Dimuon decision.
- PID correction : the nominal PID calibration is performed from PIDcorr package, recalibrate from PIDGen package. (small)
- MC size : simulation sample size is studied from bootstrap method, 200 bootstrapping samples are generated, and the deviations are taken as systematic uncertainties (see plots below)







Systematic uncertainty 4

- As an alternative to the 2D factorization of 6D background PDF, decompose the background density into multidimensional moments in the K^* decay chain variables (this uncertainty is small)
- K-Matrix model :

  1. Some K^* with the same J^P are overlapping, we use a simple K-Matrix formula to describe them as alternative

$$RKM_n(m|M_{0n},\Gamma_{0n}) = \frac{\frac{1}{M_{0n}^2 - m^2}}{1 - i(\sum_j \frac{M_{0j}\Gamma_{0j}(m)}{M_{0j}^2 - m^2} + f_{sc}\cdot\rho(m))},$$

  denominator sums over the same J^P K^* resonances, f_{sc} accounts for possible non-resonance contribution. This fit didn't change the conclusion.

  2. Alternative K-Matrix model with two coupling channels are tested, used to describe the $2^1 P_1$ and $2^3 P_1$ K^* resonances

$$\mathcal{K}_{ba}(s) = \sum_R \frac{g_b^R g_a^R}{M_R^2 - s} + \sum_{i=0}^{N_{\mathrm{b.g.}}} b_{ba}^{(i)} s^i$$

  more floating parameters are included, the nominal model is stable.