

# Experimental and Theoretical Status of and Perspectives for XYZ States

In memory of Mikhail Voloshin († March 20, 2020)

GSI Darmstadt, Germany, April 12-15, 2021



## *The properties and production of fully-heavy Tetraquarks in a strongly interacting medium*

Jiaxing Zhao(赵佳星)

Tsinghua University

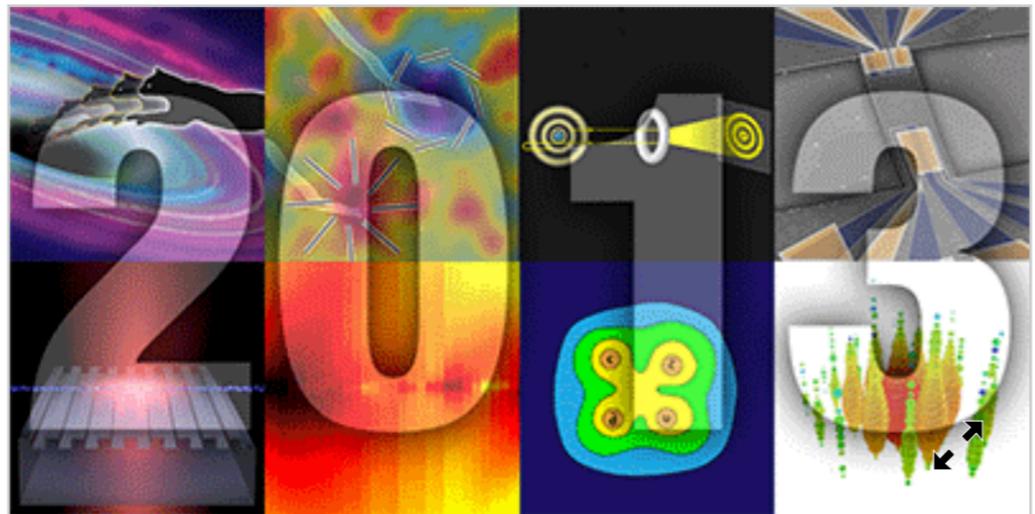
*In collaboration with : Dr. Shuzhe Shi and Prof. Pengfei Zhuang*

*Email: jiaxingzhao@mail.tsinghua.edu.cn*

# Outline

- *Brief review about heavy flavor exotic hadrons and heavy ion collisions*
- *Study the static properties of fully-heavy Tetraquarks in vacuum and finite temperature.*
- *Production of fully-heavy Tetraquarks in relativistic heavy ion collisions*
- *Summary and outlook*

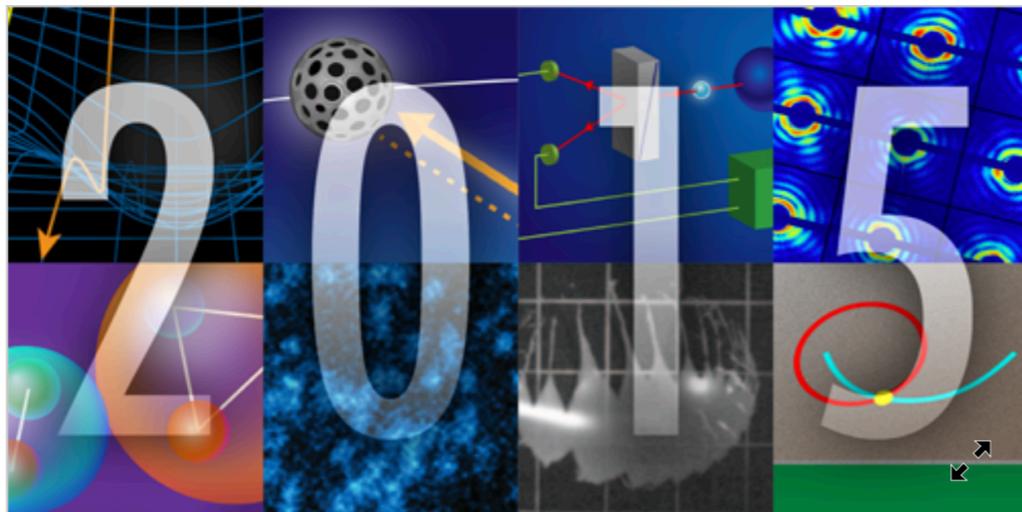
# Heavy Flavor Exotic Hadrons



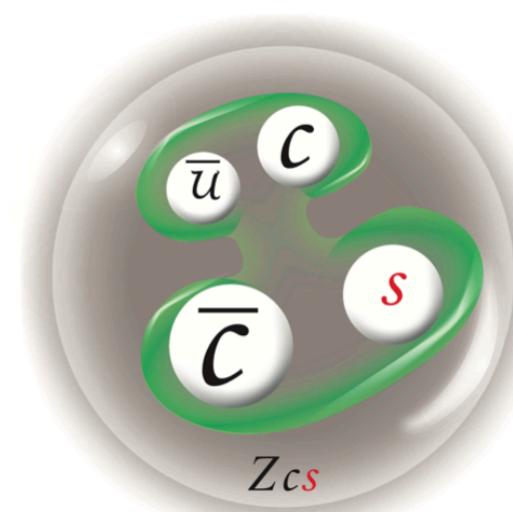
*Research highlights of 2013:  
Discovery of Zc(3900) at BESIII and Belle*



*2020: Discovery of Full-heavy  
Tetraquarks X(6900) at LHCb*



*Research highlights of 2015:  
Discovery of Pentaquark at LHCb*



*2021: Discovery of Zcs(3985) at BESIII  
and Zcs(4000), Zcs(4220) at LHCb*

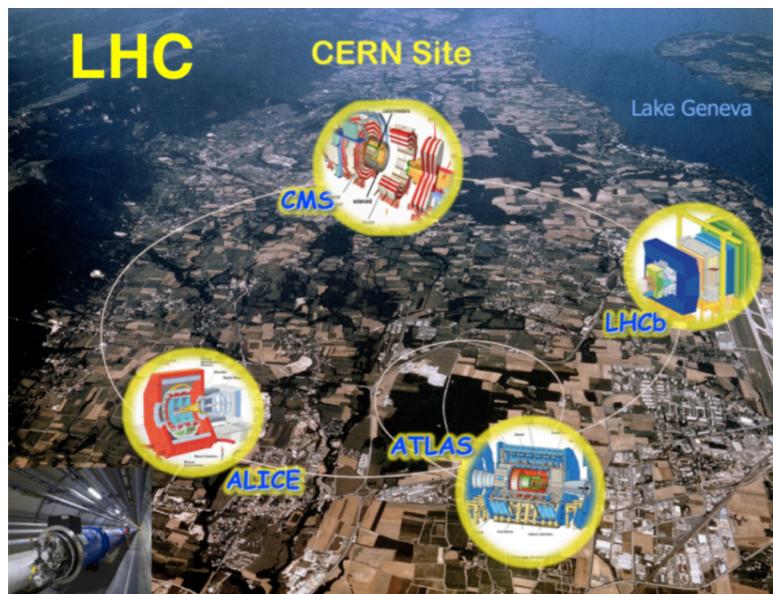
*A good platform to study the nature of strong interaction.  
Deepen our understanding of low-energy dynamic behavior of QCD theory.*

## *Heavy Flavor Exotic Hadrons in Exp.*

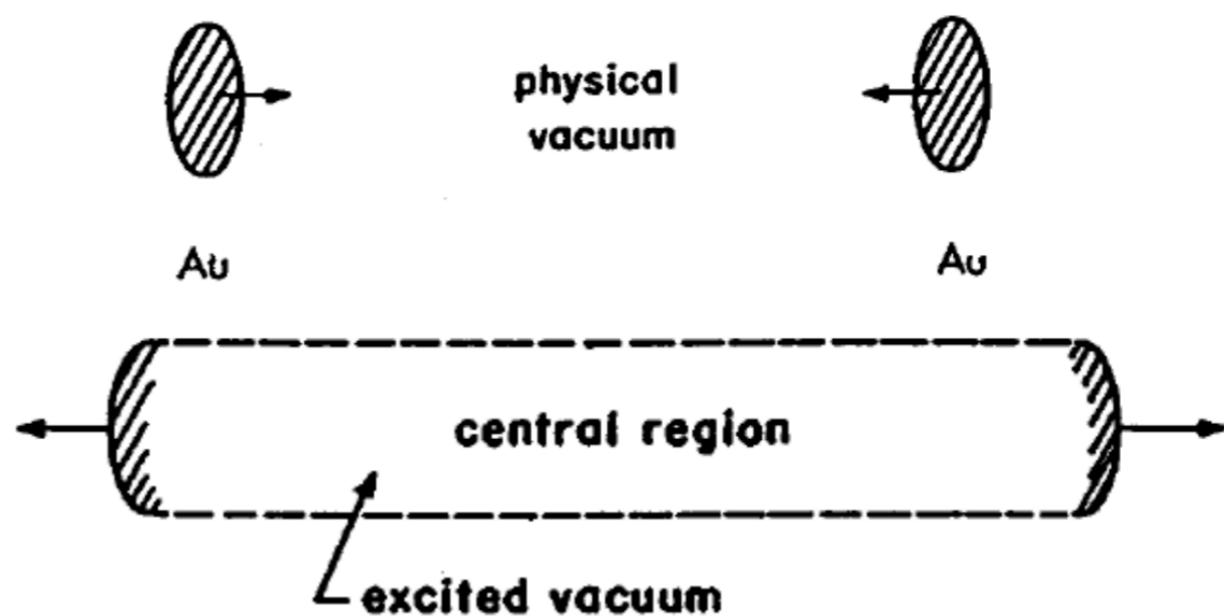
- *e+e- collisions: BESIII, Babar, Belle and CLEO*  
*Very clean experimental environment and various production mechanisms*
- *p-antiproton collisions: CDF, D0; pp collisions: LHCb, CMS, ATLAS...*  
*High statistic, decay of b hadrons (B and B<sub>s</sub> mesons as well as the Λ<sub>b</sub> baryon)*

# Heavy Flavor Exotic Hadrons in Exp.

- $e+e-$  collisions: *BESIII, Babar, Belle and CLEO*  
*Very clean experimental environment and various production mechanisms*
- $p$ -antiproton collisions: *CDF, D0; pp collisions: LHCb, CMS, ATLAS...*  
*High statistic, decay of b hadrons (B and  $B_s$  mesons as well as the  $\Lambda_b$  baryon)*
- *Relativistic Heavy-ion Collisions at RHIC and LHC*



$PbPb$ ,  $\sqrt{s_{NN}} \sim 2.76\text{-}5.02\text{TeV}$

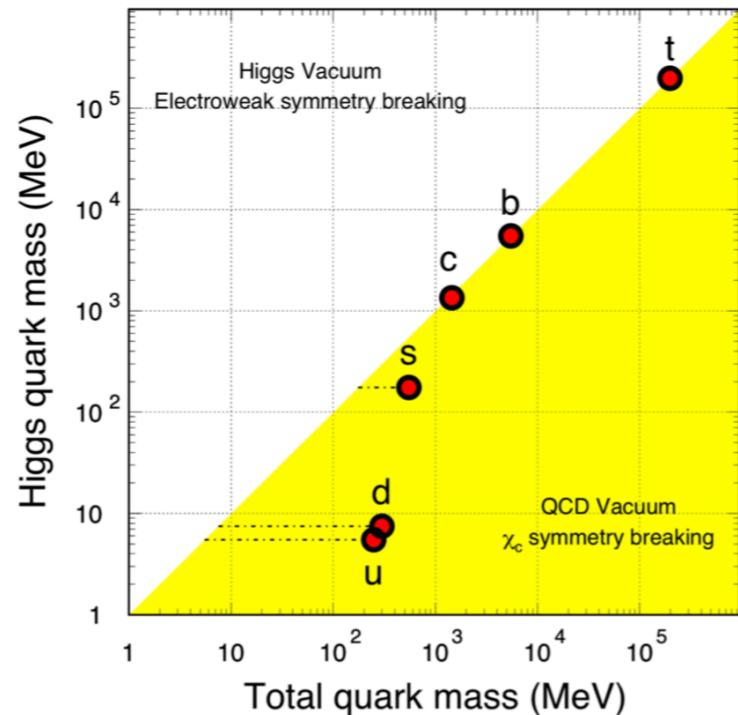


*A new state of matter: Quark-Gluon Plasma(QGP) !*

*QGP: color deconfinement, chiral restoration, strong coupling("prefect liquid"),...*

# Heavy Flavor: a sensitive probe of QGP

- Heavy flavor can be used to probe and study the QGP !

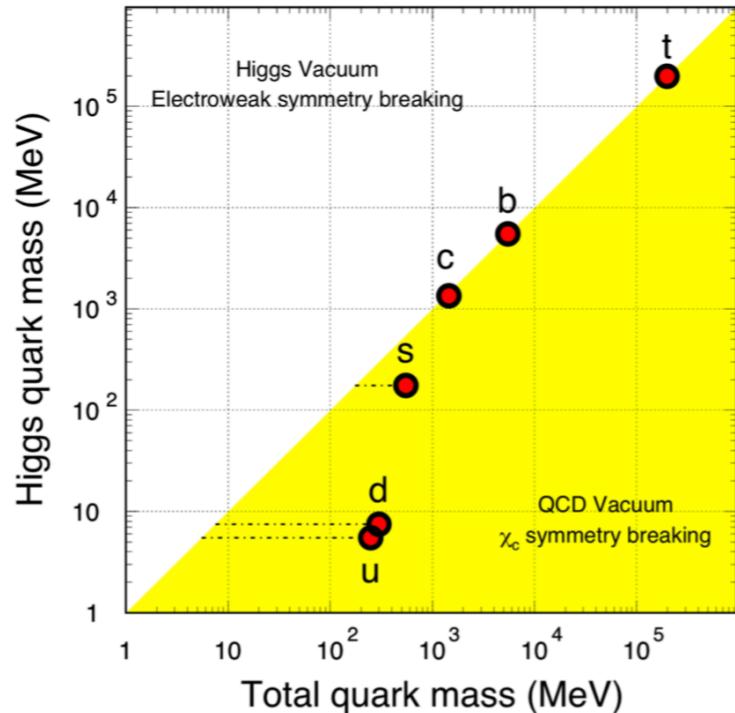


- $M_c, M_b \gg \Lambda_{QCD}$ , produced by initial hard scattering and can be described by pQCD.
- Mass not change in QGP medium, number conserved. strong interaction with the hot medium.
- Heavy flavor hadrons produced on the boundary of QCD phase transition. Clear decay mode and easy to distinguish

[JX. Zhao K. Zhou, ShL. Chen, PF. Zhuang, Prog. Part. Nucl. Phys. 114 \(2020\) 103801.](#)

# Heavy Flavor: a sensitive probe of QGP

- Heavy flavor can be used to probe and study the QGP !



- $M_c, M_b \gg \Lambda_{QCD}$ , produced by initial hard scattering and can be described by pQCD.
- Mass not change in QGP medium, number conserved. strong interaction with the hot medium.
- Heavy flavor hadrons produced on the boundary of QCD phase transition. Clear decay mode and easy to distinguish

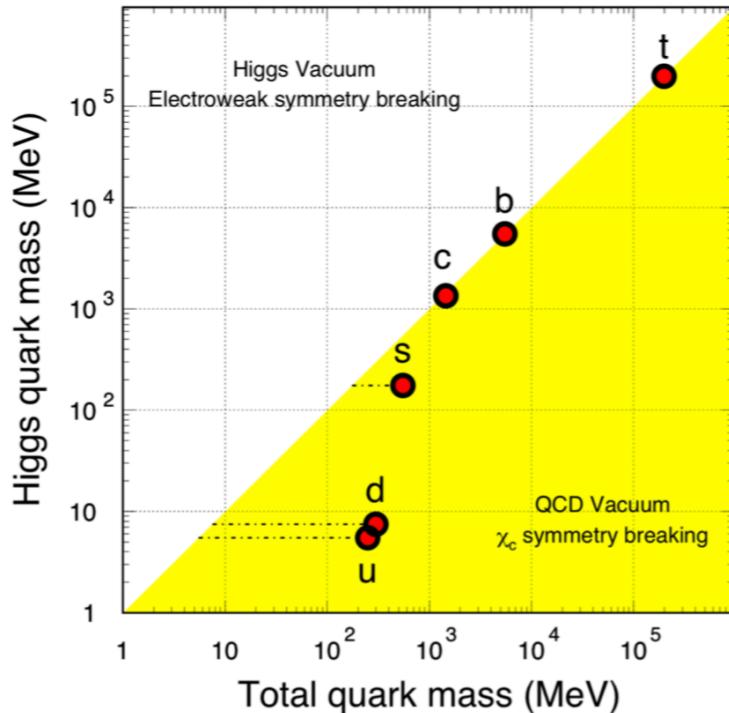
[JX. Zhao K. Zhou, ShL. Chen, PF. Zhuang, Prog. Part. Nucl. Phys. 114 \(2020\) 103801.](#)

- The appearance of QGP play a crucial role in heavy flavor hadrons production !

- [A. Andronic, P. Braun-Munzinger, K. Redlich, J. Stachel, Phys. Lett. B 571 \(2003\) 36.](#)  
[JX. Zhao, H. He, PF. Zhuang, Phys. Lett. B. 771 \(2017\) 349–353. Few Body Syst. 58 \(2017\) 2, 100.](#)  
[ExHIC Collaboration, Sungtae Cho et al, Phys. Rev. Lett. 106 \(2011\) 212001.](#)  
[ExHIC Collaboration, Sungtae Cho et al, Prog. Part. Nucl. Phys. 95 \(2017\)279-322.](#)  
[A. Torres, K. Khemchandani, F. Navarra, and M. Nielsen, Phys. Rev. D 90 \(2014\) 11, 114023.](#)  
[B. Wu, X. Du, M. Sibila, R. Rapp, arXiv: 2006.09945.](#)  
[H. Zhang, J. Liao, E. Wang, Q. Wang, H. Xing, Phys.Rev.Lett. 126 \(2021\) 1, 012301.](#)  
...

# Heavy Flavor: a sensitive probe of QGP

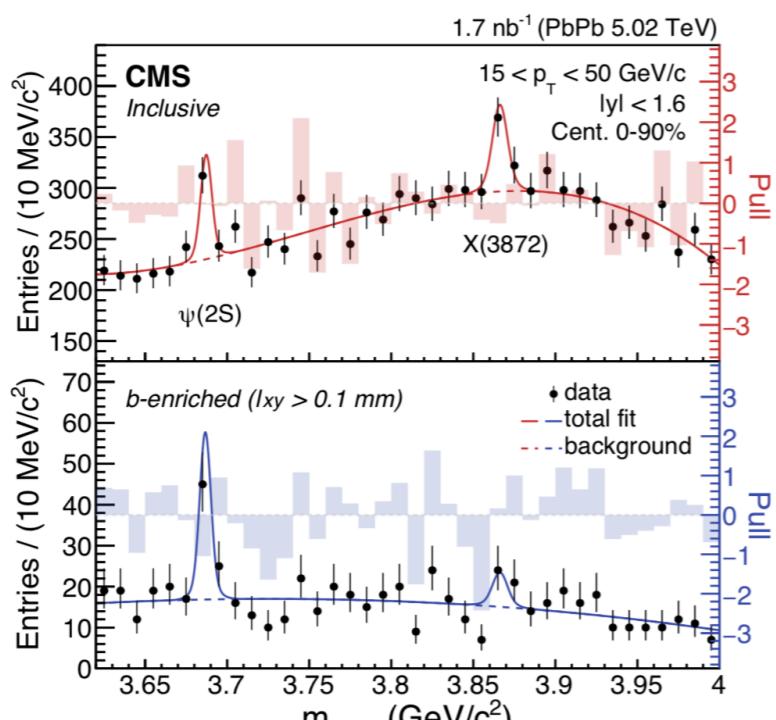
- Heavy flavor can be used to probe and study the QGP !



- $M_c, M_b \gg \Lambda_{QCD}$ , produced by initial hard scattering and can be described by pQCD.
- Mass not change in QGP medium, number conserved. strong interaction with the hot medium.
- Heavy flavor hadrons produced on the boundary of QCD phase transition. Clear decay mode and easy to distinguish

JX. Zhao K. Zhou, ShL. Chen, PF. Zhuang, Prog. Part. Nucl. Phys. 114 (2020) 103801.

- The appearance of QGP play a crucial role in heavy flavor hadrons production !



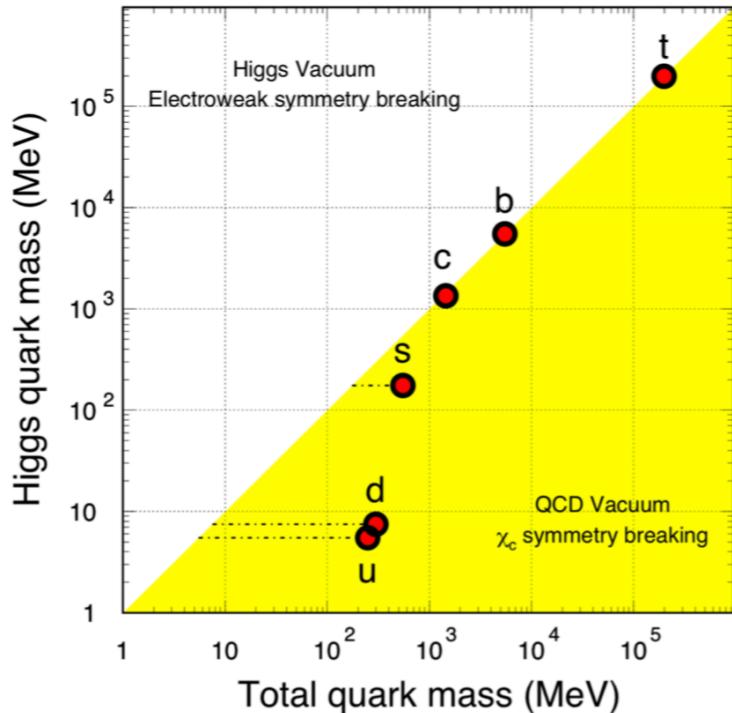
CMS Collaboration, arXiv: 2102.13048.

- A. Andronic, P. Braun-Munzinger, K. Redlich, J. Stachel, Phys. Lett. B 571 (2003) 36.  
JX. Zhao, H. He, PF. Zhuang, Phys. Lett. B. 771 (2017) 349–353. Few Body Syst. 58 (2017) 2, 100.  
ExHIC Collaboration, Sungtae Cho et al, Phys. Rev. Lett. 106 (2011) 212001.  
ExHIC Collaboration, Sungtae Cho et al, Prog. Part. Nucl. Phys. 95 (2017) 279-322.  
A. Torres, K. Khemchandani, F. Navarra, and M. Nielsen, Phys. Rev. D 90 (2014) 11, 114023.  
B. Wu, X. Du, M. Sibila, R. Rapp, arXiv: 2006.09945.  
H. Zhang, J. Liao, E. Wang, Q. Wang, H. Xing, Phys. Rev. Lett. 126 (2021) 1, 012301.  
...

First evidence of **X(3872)** production in **heavy ion collisions!**

# Heavy Flavor: a sensitive probe of QGP

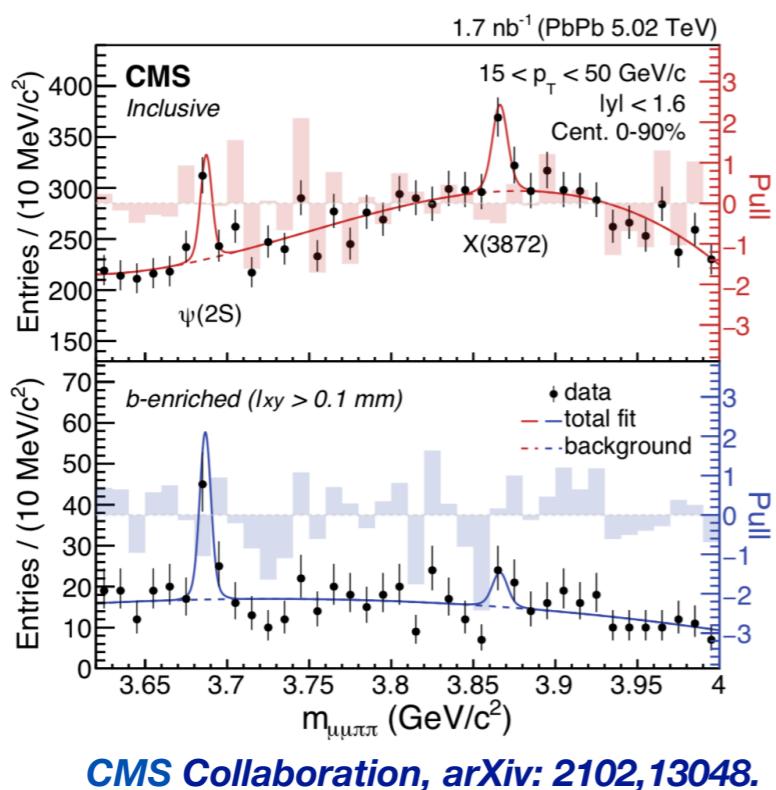
- Heavy flavor can be used to probe and study the QGP !



- $M_c, M_b \gg \Lambda_{QCD}$ , produced by initial hard scattering and can be described by pQCD.
- Mass not change in QGP medium, number conserved. strong interaction with the hot medium.
- Heavy flavor hadrons produced on the boundary of QCD phase transition. Clear decay mode and easy to distinguish

JX. Zhao K. Zhou, ShL. Chen, PF. Zhuang, Prog. Part. Nucl. Phys. 114 (2020) 103801.

- The appearance of QGP play a crucial role in heavy flavor hadrons production !



- A. Andronic, P. Braun-Munzinger, K. Redlich, J. Stachel, Phys. Lett. B 571 (2003) 36.  
JX. Zhao, H. He, PF. Zhuang, Phys. Lett. B. 771 (2017) 349–353. Few Body Syst. 58 (2017) 2, 100.  
ExHIC Collaboration, Sungtae Cho et al, Phys. Rev. Lett. 106 (2011) 212001.  
ExHIC Collaboration, Sungtae Cho et al, Prog. Part. Nucl. Phys. 95 (2017) 279-322.  
A. Torres, K. Khemchandani, F. Navarra, and M. Nielsen, Phys. Rev. D 90 (2014) 11, 114023.  
B. Wu, X. Du, M. Sibila, R. Rapp, arXiv: 2006.09945.  
H. Zhang, J. Liao, E. Wang, Q. Wang, H. Xing, Phys. Rev. Lett. 126 (2021) 1, 012301.  
...

First evidence of **X(3872)** production in **heavy ion collisions!**

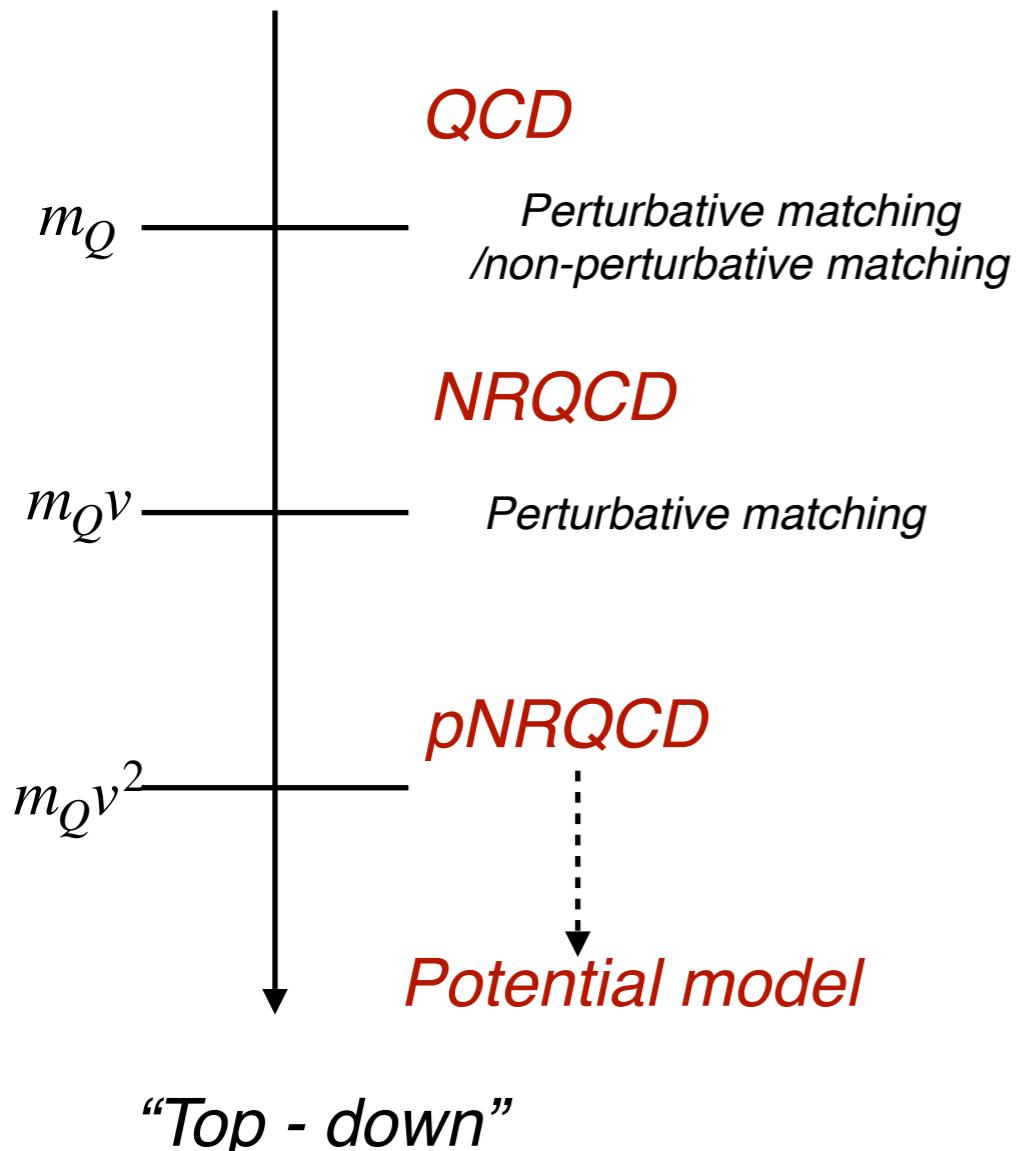
**Fully-heavy Tetraquark properties and production in QGP !**

# *Heavy Flavor Effective Theory*

$$m_c \sim 1.5\text{GeV}, m_b \sim 4.7\text{GeV}$$

*Separation of scales:*

$$m_Q \gg m_Q^\nu \gg m_Q^{\nu^2}$$

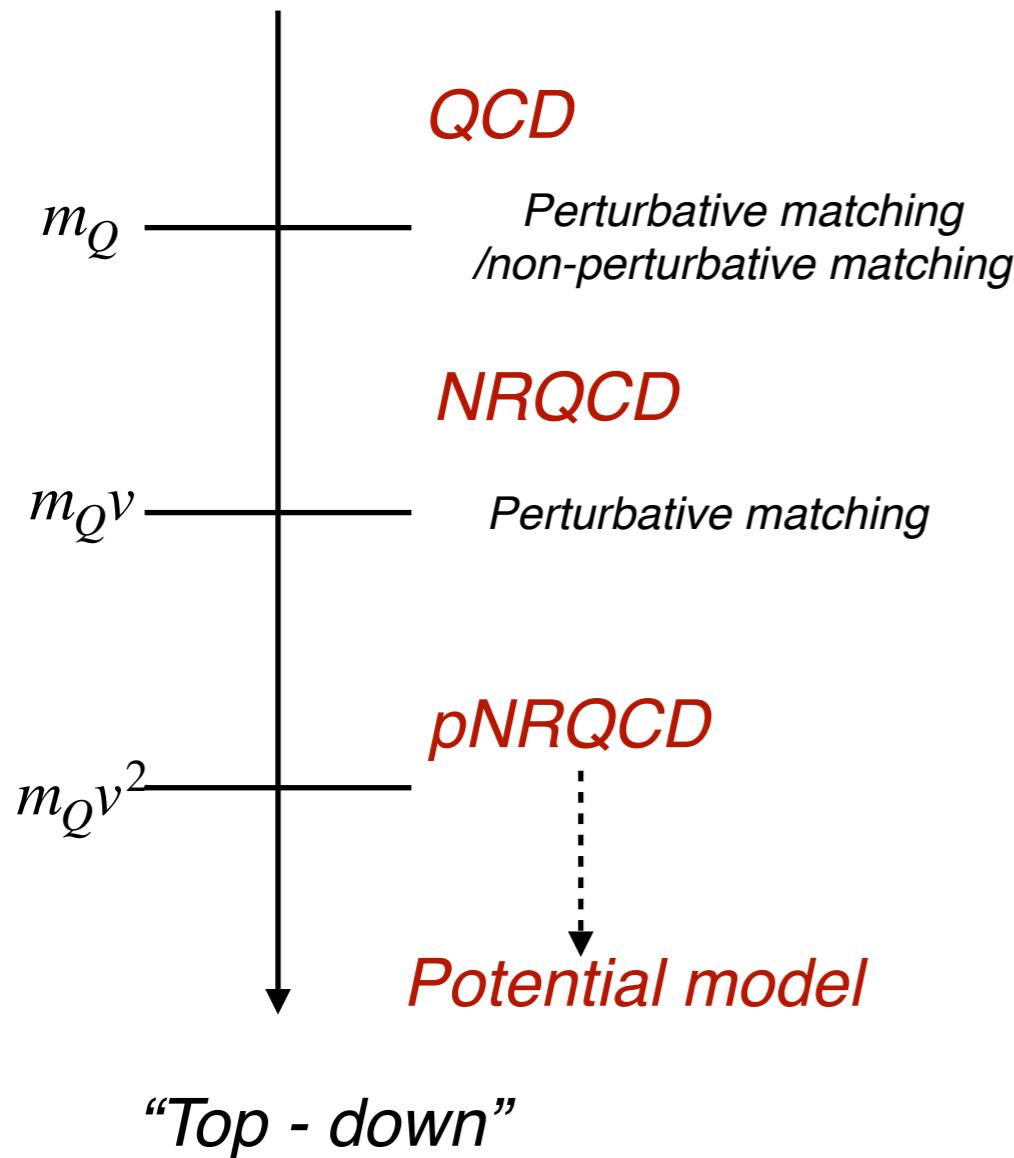


# Heavy Flavor Effective Theory

$$m_c \sim 1.5 \text{GeV}, m_b \sim 4.7 \text{GeV}$$

*Separation of scales:*

$$m_Q \gg m_Q v \gg m_Q v^2$$



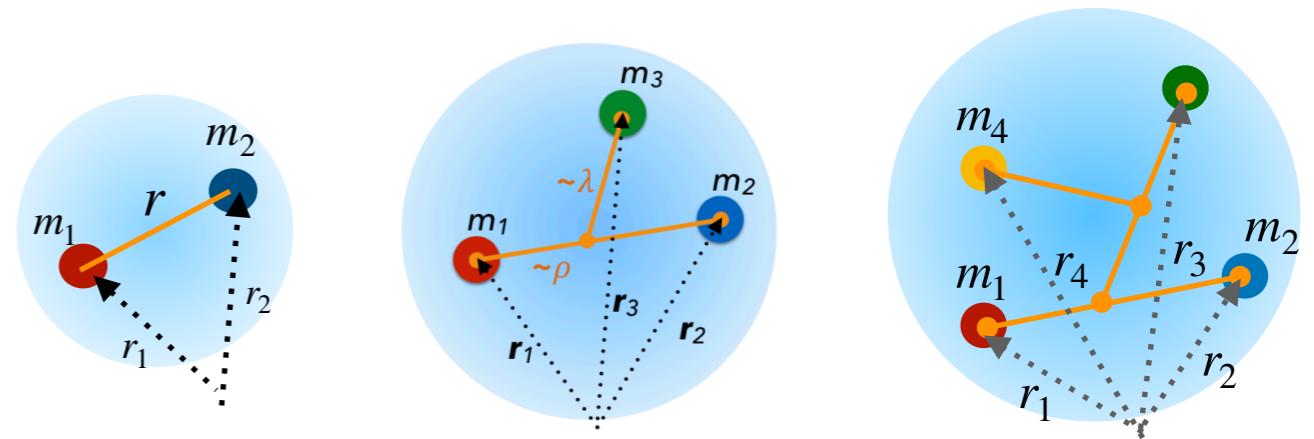
*N-Body Schroedinger Equation Framework*

$$\left( \sum_{i=1}^N \frac{\hat{p}_i^2}{2m_i} + \sum_{i < j} V_{ij} \right) \Psi(\mathbf{r}_1, \dots, \mathbf{r}_N) = E \Psi(\mathbf{r}_1, \dots, \mathbf{r}_N)$$

*Jacobi coordinates :*

$$\mathbf{R} = \frac{1}{M} \sum_{i=1}^N m_i \mathbf{r}_i,$$

$$\mathbf{x}_j = \sqrt{\frac{M_j m_{j+1}}{M_{j+1} \mu}} \left( \mathbf{r}_{j+1} - \frac{1}{M_j} \sum_{i=1}^j m_i \mathbf{r}_i \right)$$



*Then, factorize the N-body motion into a center-of-mass motion and a relative motion*

$$\Psi(\mathbf{r}_1, \dots, \mathbf{r}_N) = \Theta(\mathbf{R}) \Phi(\mathbf{x}_1, \dots, \mathbf{x}_{N-1}),$$

## *N-Body Schroedinger Equation Framework*

*Further, N-1 relative coordinates can be transformed to a single hyperradial coordinate and 3N-4 hyperangular coordinates.*

$$(\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_{N-1}) \rightarrow (\rho, \alpha_{N-1}, \dots, \alpha_2, \theta_1, \phi_1, \dots, \theta_{N-1}, \phi_{N-1})$$

$$\rho = \sqrt{\mathbf{x}_1^2 + \dots + \mathbf{x}_{N-1}^2} \quad \sin \alpha_i = x_i / \rho_i \quad \hat{x}_i = (\theta_i, \phi_i)$$

*N. Barnea, et al. Phys. Rev. C 61.054001(2000)  
FBS Colloquium. Few-Body System 25, 199-238(1998)*

## N-Body Schroedinger Equation Framework

Further,  $N-1$  relative coordinates can be transformed to a **single** hyperradial coordinate and  **$3N-4$**  hyperangular coordinates.

$$(\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_{N-1}) \rightarrow (\rho, \alpha_{N-1}, \dots, \alpha_2, \theta_1, \phi_1, \dots, \theta_{N-1}, \phi_{N-1})$$

$$\rho = \sqrt{\mathbf{x}_1^2 + \dots + \mathbf{x}_{N-1}^2} \quad \sin \alpha_i = x_i / \rho_i \quad \hat{x}_i = (\theta_i, \phi_i)$$

The relative motion is controlled by :

*N. Barnea, et al. Phys. Rev. C 61.054001(2000)*

*FBS Colloquium. Few-Body System 25, 199-238(1998)*

$$\left[ \frac{1}{2\mu} \left( -\frac{1}{\rho^{3N-4}} \frac{d}{d\rho} \rho^{3N-4} \frac{d}{d\rho} + \frac{\hat{K}_{N-1}^2}{\rho^2} \right) + V(\rho, \Omega) \right] \Phi(\rho, \Omega) = E_r \Phi(\rho, \Omega),$$

$$\hat{K}_{N-1}^2 = -\frac{\partial^2}{\partial \alpha_{N-1}^2} + \frac{(3N-9) - (3N-5) \cos(2\alpha_{N-1})}{\sin(2\alpha_{N-1})} \frac{\partial}{\partial \alpha_{N-1}} + \frac{1}{\cos^2 \alpha_{N-1}} \hat{K}_{N-2}^2 + \frac{1}{\sin^2 \alpha_{N-1}} \hat{l}_{N-1}^2,$$

$$\hat{K}_{N-1}^2 \mathcal{Y}_\kappa(\Omega) = K(K+3N-5) \mathcal{Y}_\kappa(\Omega). \quad \text{hyper-angular momentum operator}$$

## N-Body Schroedinger Equation Framework

Further,  $N-1$  relative coordinates can be transformed to a **single** hyperradial coordinate and  **$3N-4$**  hyperangular coordinates.

$$(\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_{N-1}) \rightarrow (\rho, \alpha_{N-1}, \dots, \alpha_2, \theta_1, \phi_1, \dots, \theta_{N-1}, \phi_{N-1})$$

$$\rho = \sqrt{\mathbf{x}_1^2 + \dots + \mathbf{x}_{N-1}^2} \quad \sin \alpha_i = x_i / \rho_i \quad \hat{x}_i = (\theta_i, \phi_i)$$

The relative motion is controlled by :

N. Barnea, et al. Phys. Rev. C 61.054001(2000)

FBS Colloquium. Few-Body System 25, 199-238(1998)

$$\left[ \frac{1}{2\mu} \left( -\frac{1}{\rho^{3N-4}} \frac{d}{d\rho} \rho^{3N-4} \frac{d}{d\rho} + \frac{\hat{K}_{N-1}^2}{\rho^2} \right) + V(\rho, \Omega) \right] \Phi(\rho, \Omega) = E_r \Phi(\rho, \Omega),$$

$$\hat{K}_{N-1}^2 = -\frac{\partial^2}{\partial \alpha_{N-1}^2} + \frac{(3N-9) - (3N-5) \cos(2\alpha_{N-1})}{\sin(2\alpha_{N-1})} \frac{\partial}{\partial \alpha_{N-1}} + \frac{1}{\cos^2 \alpha_{N-1}} \hat{K}_{N-2}^2 + \frac{1}{\sin^2 \alpha_{N-1}} \hat{l}_{N-1}^2,$$

$$\hat{K}_{N-1}^2 \mathcal{Y}_\kappa(\Omega) = K(K+3N-5) \mathcal{Y}_\kappa(\Omega). \quad \text{hyper-angular momentum operator}$$

$$\Phi(\rho, \Omega) = \sum_\kappa R_\kappa(\rho) \mathcal{Y}_\kappa(\Omega) \quad \text{hyper-spherical harmonic function expansion}$$

→  $\left[ \frac{1}{2\mu} \left( \frac{1}{\rho^{3N-4}} \frac{d}{d\rho} \rho^{3N-4} \frac{d}{d\rho} - \frac{K(K+3N-5)}{\rho^2} \right) + E_r \right] R_\kappa = \sum_{\kappa'} V_{\kappa\kappa'} R_{\kappa'}$

Now, we apply this tool to deal with fully-heavy Tetraquark states !

## Symmetric Analysis

Pauli exclusion principle requires the wave-function to be **anti-symmetric** when exchanging two identical fermions

$$\Psi = \psi \cdot \cancel{\phi_f} \cdot \chi_s \cdot \phi_c$$

Color  $(3_c \otimes 3_c) \otimes (\bar{3}_c \otimes \bar{3}_c) = \bar{3}_c \otimes 3_c \oplus 6_c \otimes \bar{6}_c \oplus \bar{3}_c \otimes \bar{6}_c \oplus 6_c \otimes 3_c$

$$|\phi_1\rangle = |(QQ)_{\bar{3}_c}(\bar{Q}\bar{Q})_{3_c}\rangle, \quad |\phi_2\rangle = |(QQ)_{6_c}(\bar{Q}\bar{Q})_{\bar{6}_c}\rangle$$

Spin  $2 \otimes 2 \otimes 2 \otimes 2 = 1 \otimes 1 \oplus 1 \otimes 3 \oplus 3 \otimes 1 \oplus 3 \otimes 3$

$$s=0 : \quad |\chi_1\rangle = |(QQ)_0(\bar{Q}\bar{Q})_0\rangle_0, \quad |\chi_2\rangle = |(QQ)_1(\bar{Q}\bar{Q})_1\rangle_0$$

$$s=1 : \quad |\chi_3\rangle = |(QQ)_0(\bar{Q}\bar{Q})_1\rangle_1, \quad |\chi_4\rangle = |(QQ)_1(\bar{Q}\bar{Q})_0\rangle_1, \quad |\chi_5\rangle = |(QQ)_1(\bar{Q}\bar{Q})_1\rangle_1$$

$$s=2 : \quad |\chi_6\rangle = |(QQ)_1(\bar{Q}\bar{Q})_1\rangle_2$$

So, for  $cc\bar{c}\bar{c}$  and  $bb\bar{b}\bar{b}$

$$J^{PC} = 0^{++} : \quad |\phi_1\chi_2\rangle \quad \& \quad |\phi_2\chi_1\rangle$$

$$J^{PC} = 1^{+-} : \quad |\phi_1\chi_5\rangle$$

$$J^{PC} = 2^{++} : \quad |\phi_1\chi_6\rangle$$

## Solving the Coupled Equations

$J^{PC} = 0^{++}$  :

$$\Phi(\rho, \Omega) = \sum_{\kappa} R_{\kappa}^{(1)}(\rho) \mathcal{Y}_{\kappa}(\Omega) |\phi_1 \chi_2\rangle + R_{\kappa}^{(2)}(\rho) \mathcal{Y}_{\kappa}(\Omega) |\phi_2 \chi_1\rangle$$

$$-\frac{1}{2\mu} \left( \frac{d^2}{d\rho^2} + \frac{8}{\rho} \frac{d}{d\rho} - \frac{K(K+7)}{\rho^2} \right) R_{\kappa}^{(1)} + \sum_{\kappa'} V_1^{\kappa\kappa'} R_{\kappa'}^{(1)} + \sum_{\kappa'} V_m^{\kappa\kappa'} R_{\kappa'}^{(2)} = \textcolor{red}{E_r} R_{\kappa}^{(1)}$$

$$-\frac{1}{2\mu} \left( \frac{d^2}{d\rho^2} + \frac{8}{\rho} \frac{d}{d\rho} - \frac{K(K+7)}{\rho^2} \right) R_{\kappa}^{(2)} + \sum_{\kappa'} V_2^{\kappa\kappa'} R_{\kappa'}^{(2)} + \sum_{\kappa'} V_m^{\kappa\kappa'} R_{\kappa'}^{(1)} = \textcolor{red}{E_r} R_{\kappa}^{(2)}$$

$J^{PC} = 1^{+-}, 2^{++}$  :

$$\Phi(\rho, \Omega) = \sum_{\kappa} R_{\kappa}(\rho) \mathcal{Y}_{\kappa}(\Omega) |\phi_1 \chi_i\rangle$$

$$-\frac{1}{2\mu} \left( \frac{d^2}{d\rho^2} + \frac{8}{\rho} \frac{d}{d\rho} - \frac{K(K+7)}{\rho^2} \right) R_{\kappa} + \sum_{\kappa'} V^{\kappa\kappa'} R_{\kappa'} = \textcolor{red}{E_r} R_{\kappa}$$

$$V^{\kappa\kappa'} = \int V(\rho, \Omega) \mathcal{Y}_{\kappa}^*(\Omega) \mathcal{Y}_{\kappa'}(\Omega) d\Omega \quad \textit{potential matrix element in angular momentum space}$$

- Numerically solve the coupled differential equations with inverse power method
- Focus on the states with  $L=M=0$ , choose all hyperspherical harmonic functions with hyperangular quantum number  $K \leq 3$ .

# Heavy Quark Potential

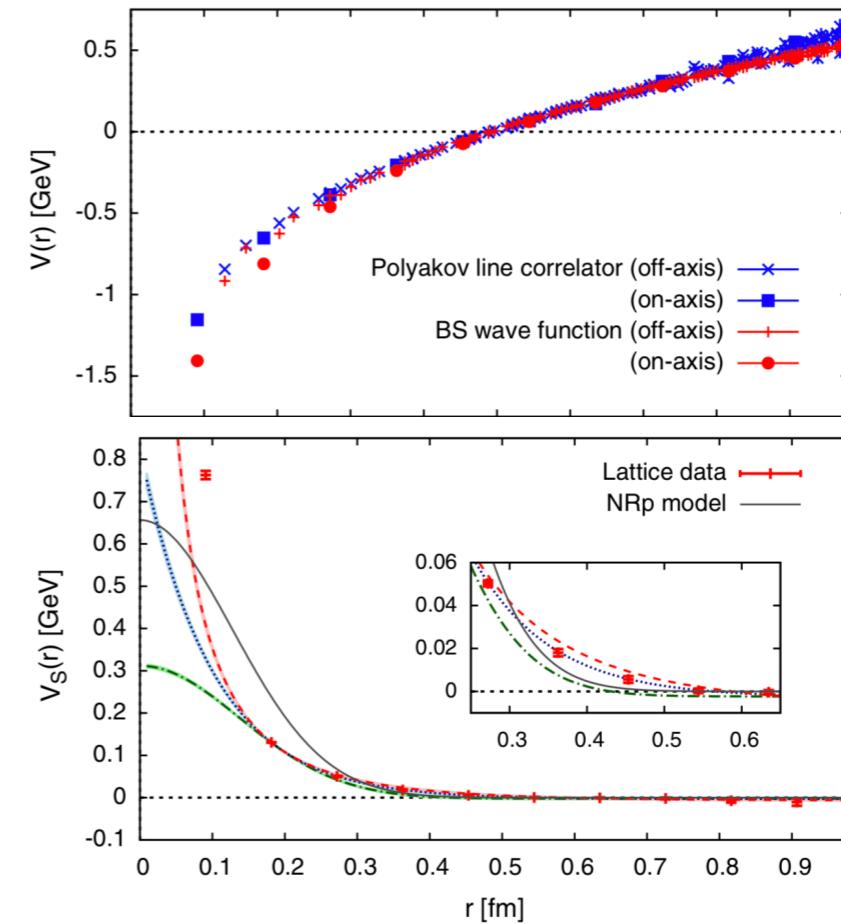
## One-Gluon Exchange (OGE)

$$V_{ij}(|\mathbf{r}_{ij}|) = -\frac{1}{4} \lambda_i^a \cdot \lambda_j^a \left( V_{ij}^c(|\mathbf{r}_{ij}|) + V_{ij}^{ss}(|\mathbf{r}_{ij}|) \mathbf{s}_i \cdot \mathbf{s}_j \right)$$

$\lambda_i^a$  ( $a = 1, \dots, 8$ )  $SU(3)$  Gell-Mann matrices

## Cornell potential

$$V_{ij}^c(|\mathbf{r}_{ij}|) = -\frac{\alpha}{|\mathbf{r}_{ij}|} + \sigma |\mathbf{r}_{ij}|$$



Supported by lattice QCD simulations:

T. Kawanai, S. Sasaki, Phys. Rev. D 85 (2012) 091503.

# Heavy Quark Potential

## One-Gluon Exchange (OGE)

$$V_{ij}(|\mathbf{r}_{ij}|) = -\frac{1}{4} \lambda_i^a \cdot \lambda_j^a \left( V_{ij}^c(|\mathbf{r}_{ij}|) + V_{ij}^{ss}(|\mathbf{r}_{ij}|) \mathbf{s}_i \cdot \mathbf{s}_j \right)$$

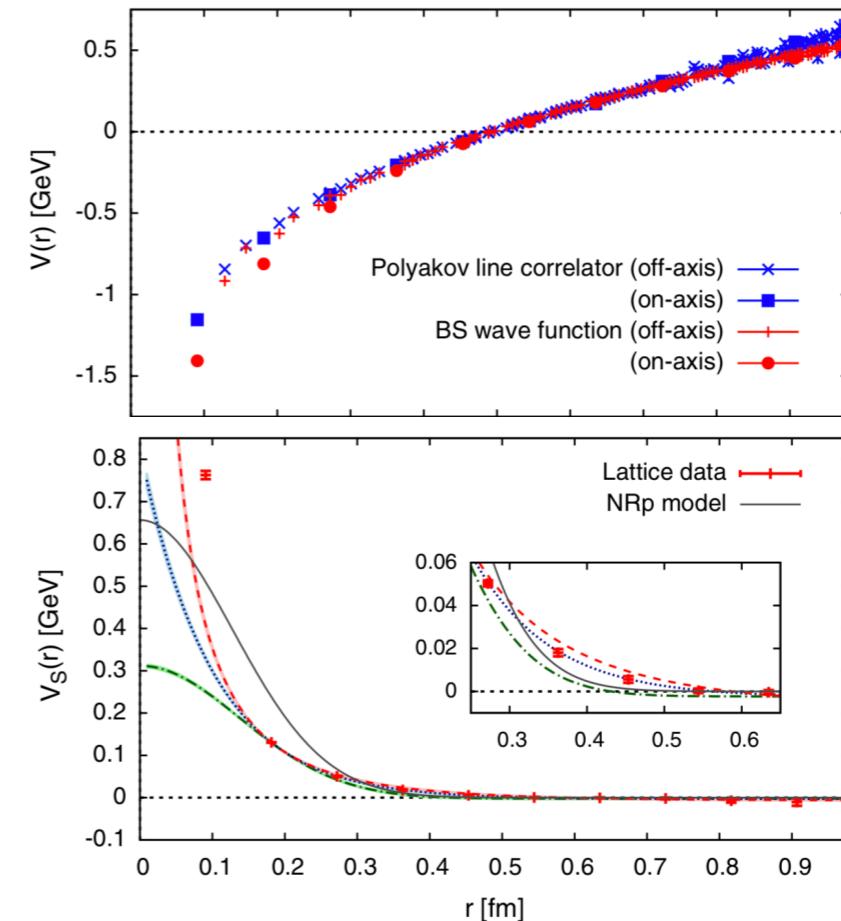
$\lambda_i^a$  ( $a = 1, \dots, 8$ )  $SU(3)$  Gell-Mann matrices

## Cornell potential

$$V_{ij}^c(|\mathbf{r}_{ij}|) = -\frac{\alpha}{|\mathbf{r}_{ij}|} + \sigma |\mathbf{r}_{ij}|$$

## Parameters

Fixed by quarkonium mass in vacuum.



Supported by lattice QCD simulations:

T. Kawanai, S. Sasaki, Phys. Rev. D 85 (2012) 091503.

$m_b$	$m_c$	$\alpha$	$\sigma$	$\gamma$	$\beta_b$	$\beta_c$
4.7 GeV	1.29 GeV	0.308	$0.15 \text{ GeV}^2$	1.982 GeV	0.239 GeV	1.545 GeV

# Heavy Quark Potential

## One-Gluon Exchange (OGE)

$$V_{ij}(|\mathbf{r}_{ij}|) = -\frac{1}{4} \lambda_i^a \cdot \lambda_j^a \left( V_{ij}^c(|\mathbf{r}_{ij}|) + V_{ij}^{ss}(|\mathbf{r}_{ij}|) \mathbf{s}_i \cdot \mathbf{s}_j \right)$$

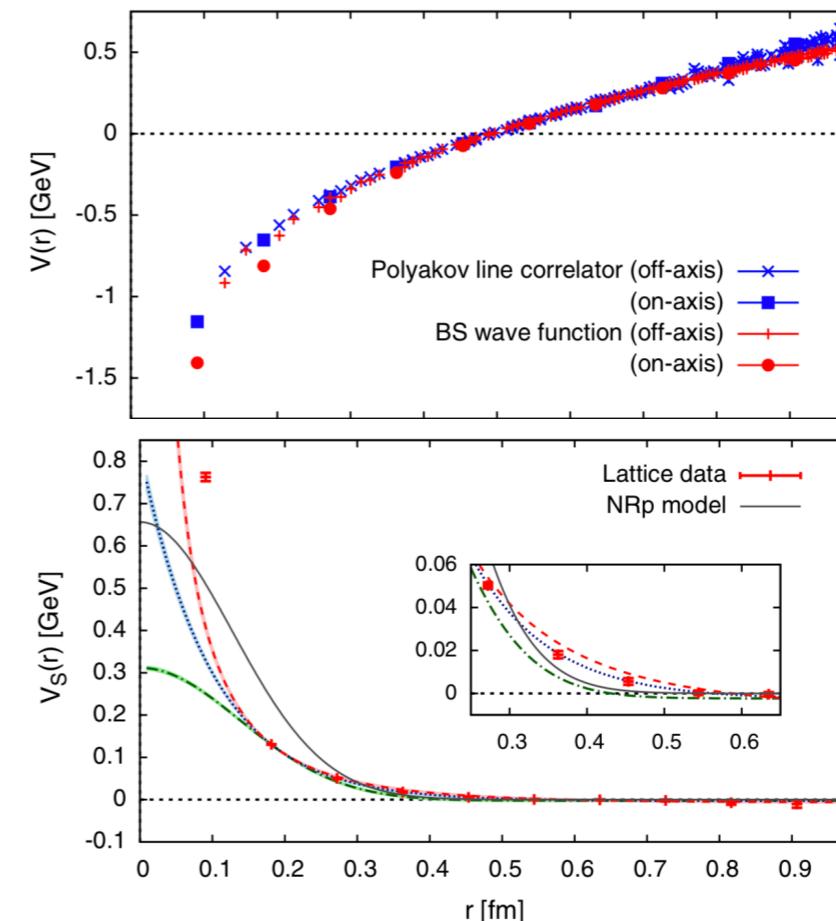
$\lambda_i^a (a = 1, \dots, 8)$  *SU(3) Gell-Mann matrices*

## Cornell potential

$$V_{ij}^c(|\mathbf{r}_{ij}|) = -\frac{\alpha}{|\mathbf{r}_{ij}|} + \sigma |\mathbf{r}_{ij}|$$

## Parameters

Fixed by quarkonium mass in vacuum.



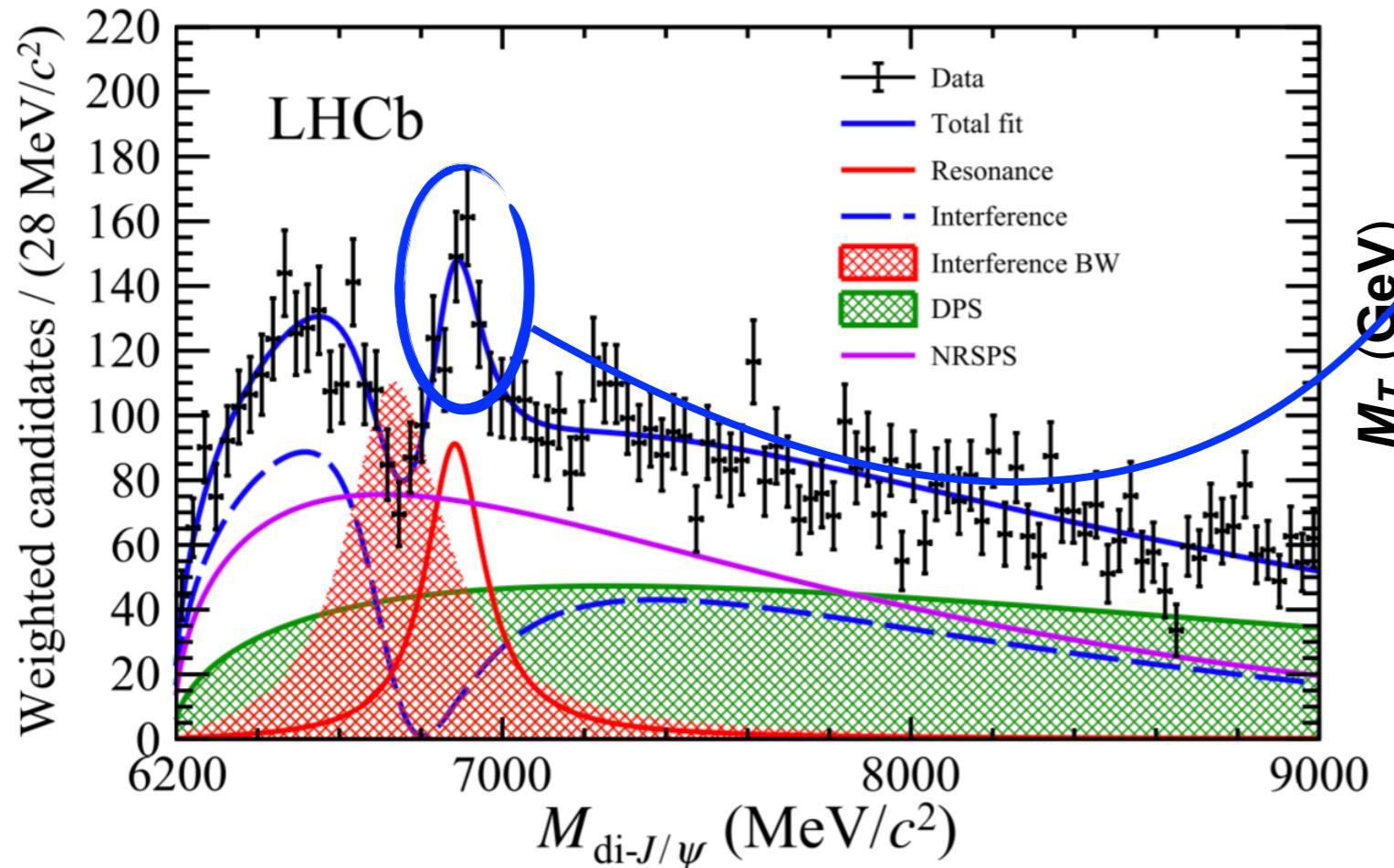
Supported by lattice QCD simulations:

T. Kawanai, S. Sasaki, Phys. Rev. D 85 (2012) 091503.

	$m_b$	$m_c$	$\alpha$	$\sigma$	$\gamma$	$\beta_b$	$\beta_c$
	4.7 GeV	1.29 GeV	0.308	0.15 GeV <sup>2</sup>	1.982 GeV	0.239 GeV	1.545 GeV
<hr/>							
State	$\eta_c$	$J/\psi$	$h_c(1P)$	$\chi_c(1P)$	$\eta_c(2S)$	$\psi(2S)$	$\chi_c(2P)$
$M_E$ (GeV)	2.981	3.097	3.525	3.556	3.639	3.696	3.927
$M_T$ (GeV)	2.968	3.102	3.480	3.500	3.654	3.720	4.000
<hr/>							
State	$\eta_b$	$\Upsilon(1S)$	$h_b(1P)$	$\chi_b(1P)$	$\eta_b(2S)$	$\Upsilon(2S)$	$\chi_b(2P)$
$M_E$ (GeV)	9.398	9.460	9.898	9.912	9.999	10.023	10.269
$M_T$ (GeV)	9.397	9.459	9.845	9.860	9.957	9.977	10.221

Agree very well with  
the experiment data !

# Results

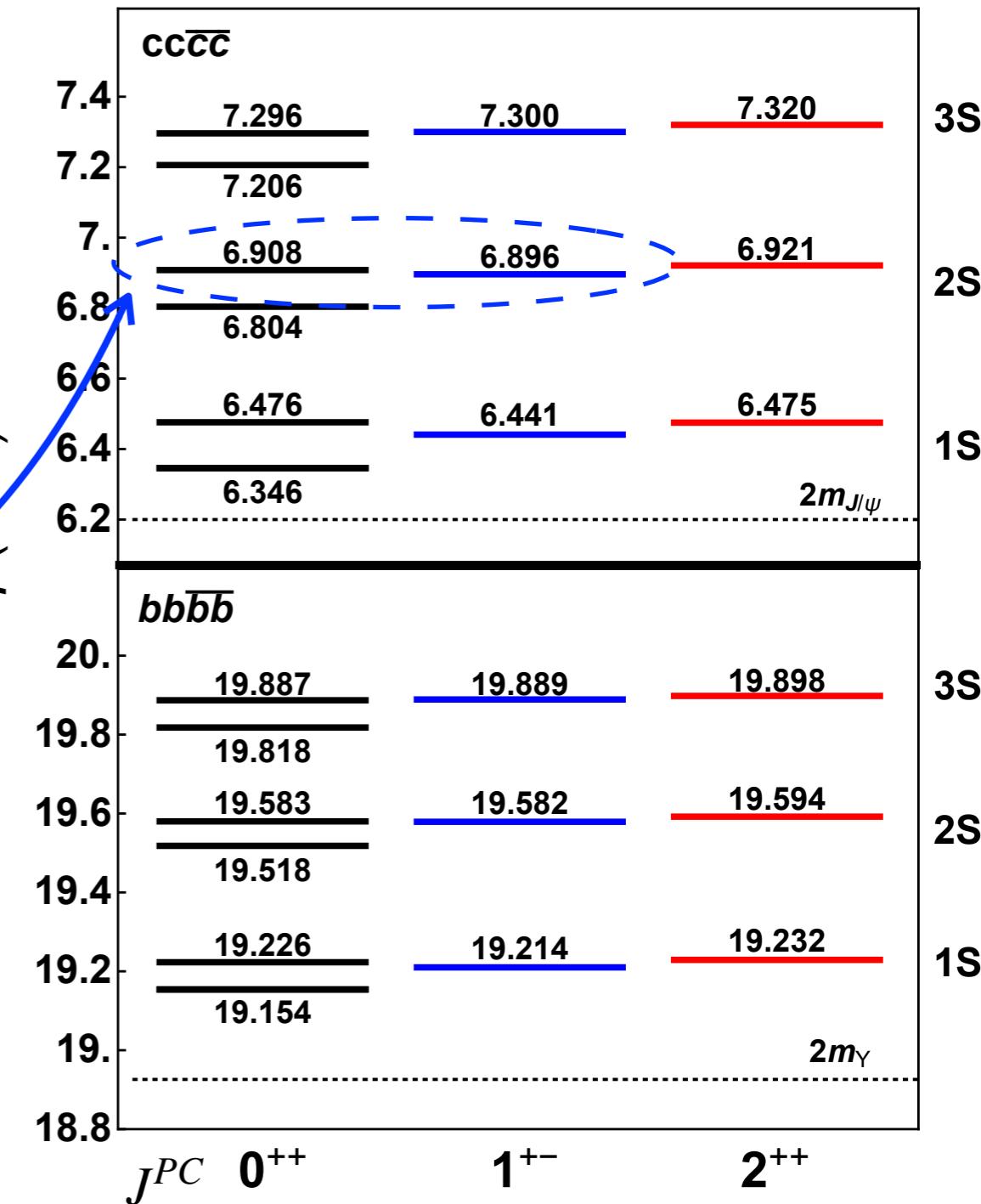


$$m[X(6900)] = 6905 \pm 11 \pm 7 \text{ MeV}/c^2$$

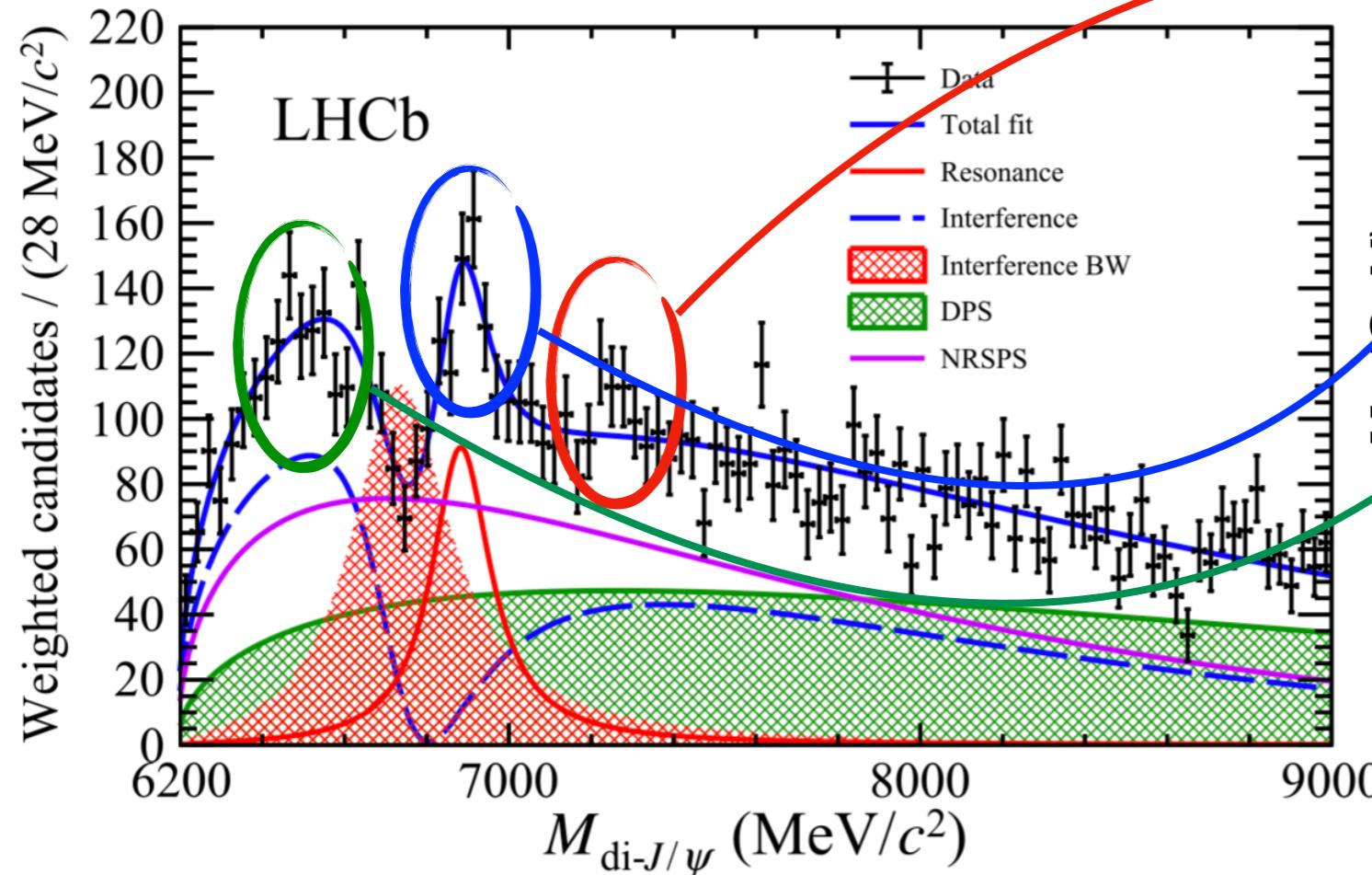
$$\Gamma[X(6900)] = 80 \pm 19 \pm 33 \text{ MeV}$$

*LHCb Collaboration, Science Bulletin, 2020, 65(23) 1983-1993*

- The  $X(6900)$  is probably the 1st excited state of  $cc\bar{c}\bar{c}$  with  $J^{PC} = 0^{++}$  or  $1^{+-}$



# Results

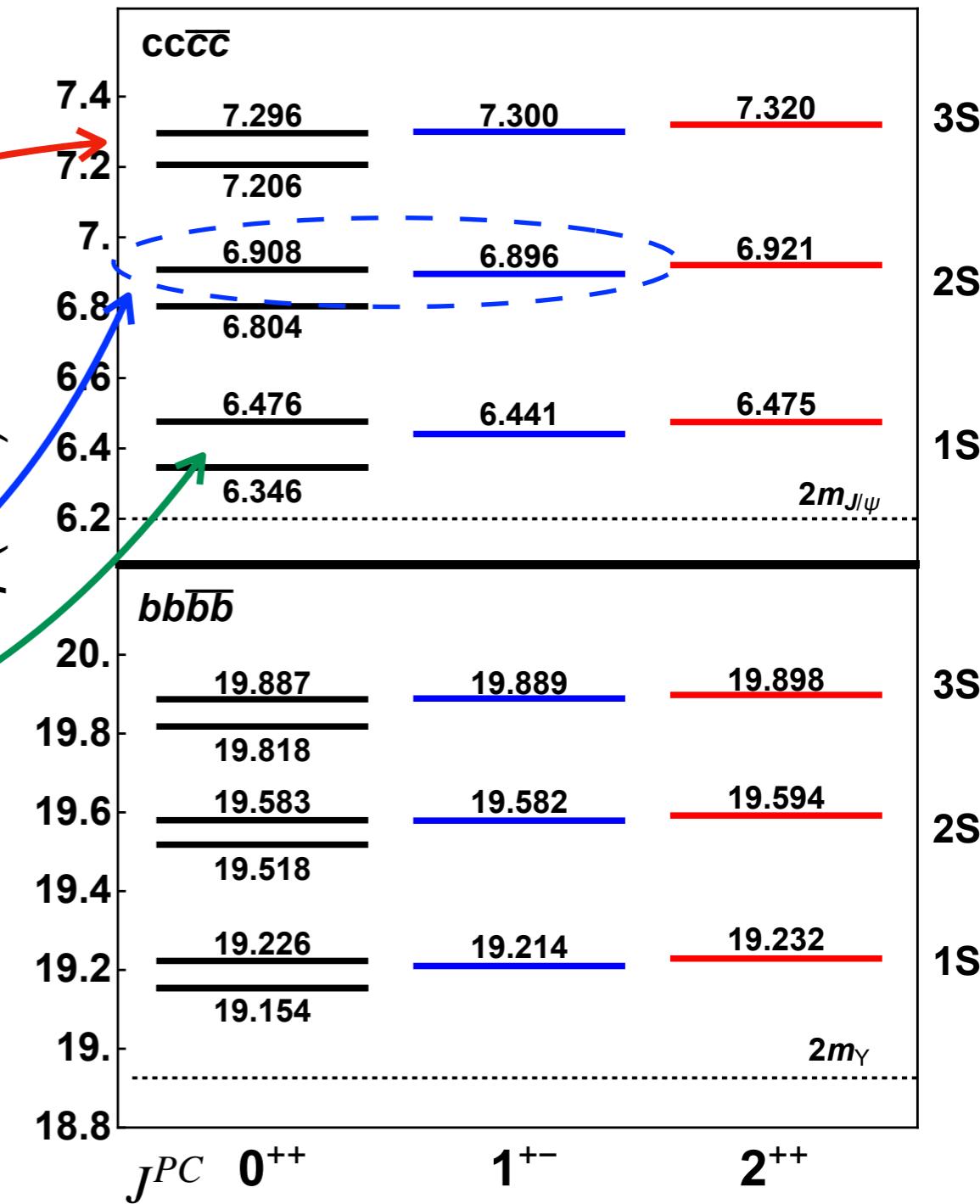


$$m[X(6900)] = 6905 \pm 11 \pm 7 \text{ MeV}/c^2$$

$$\Gamma[X(6900)] = 80 \pm 19 \pm 33 \text{ MeV}$$

*LHCb Collaboration, Science Bulletin, 2020, 65(23) 1983-1993*

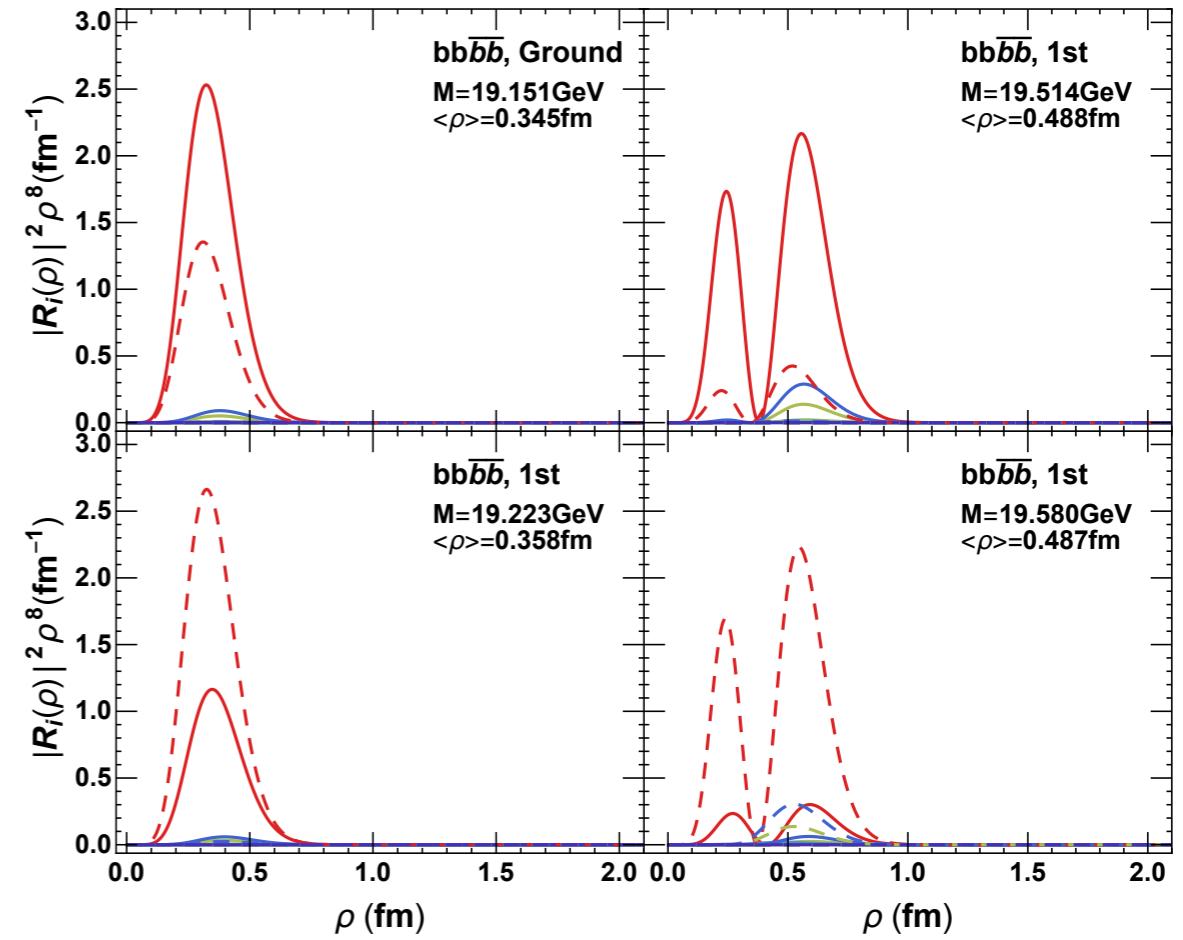
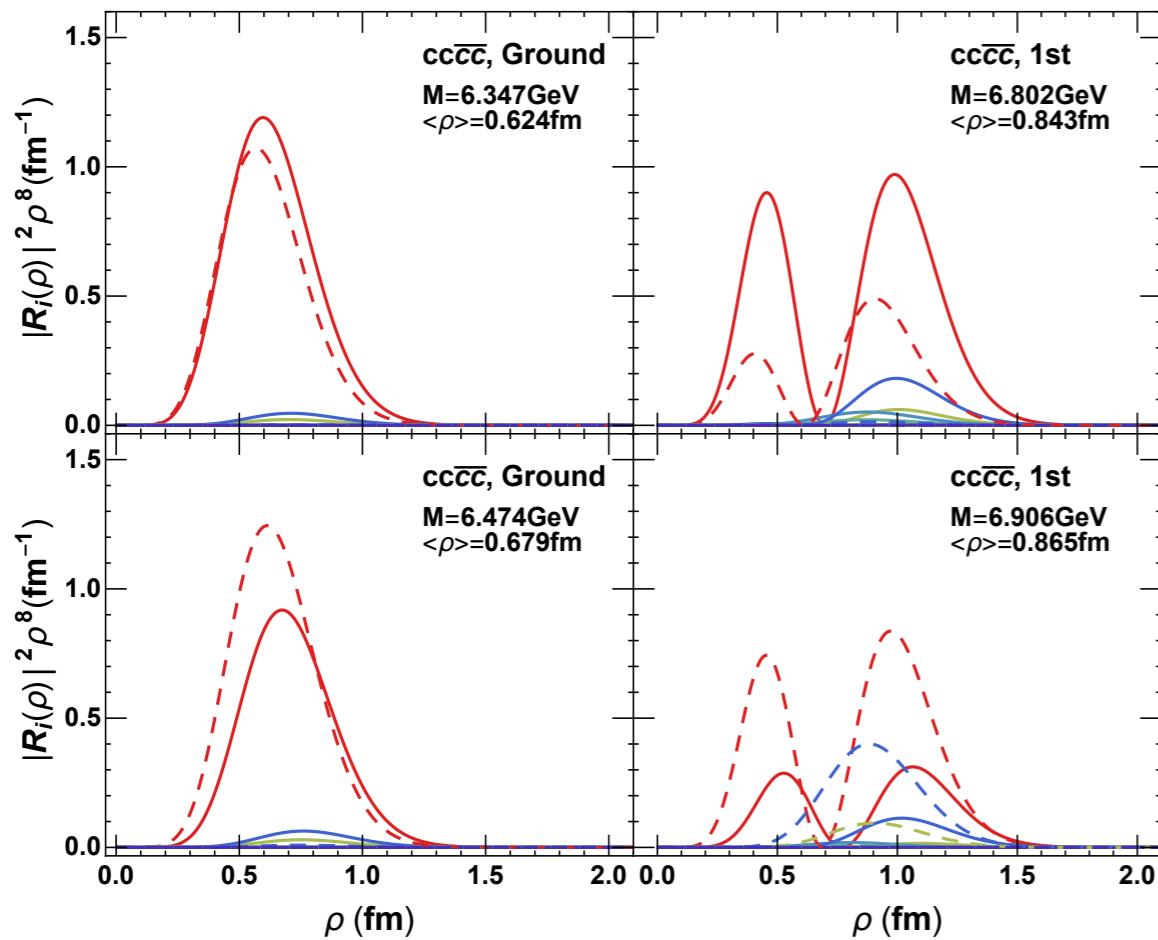
- The  $X(6900)$  is probably the 1st excited state of  $cc\bar{c}\bar{c}$  with  $J^{PC} = 0^{++}$  or  $1^{+-}$
- The potential states (~6.4 GeV and ~7.2 GeV) might be the ground and 2nd excited states.



# Results

JX Zhao, ShZh. shi, and Pf. Zhuang, Phys.Rev.D 102 (2020) 11, 114001

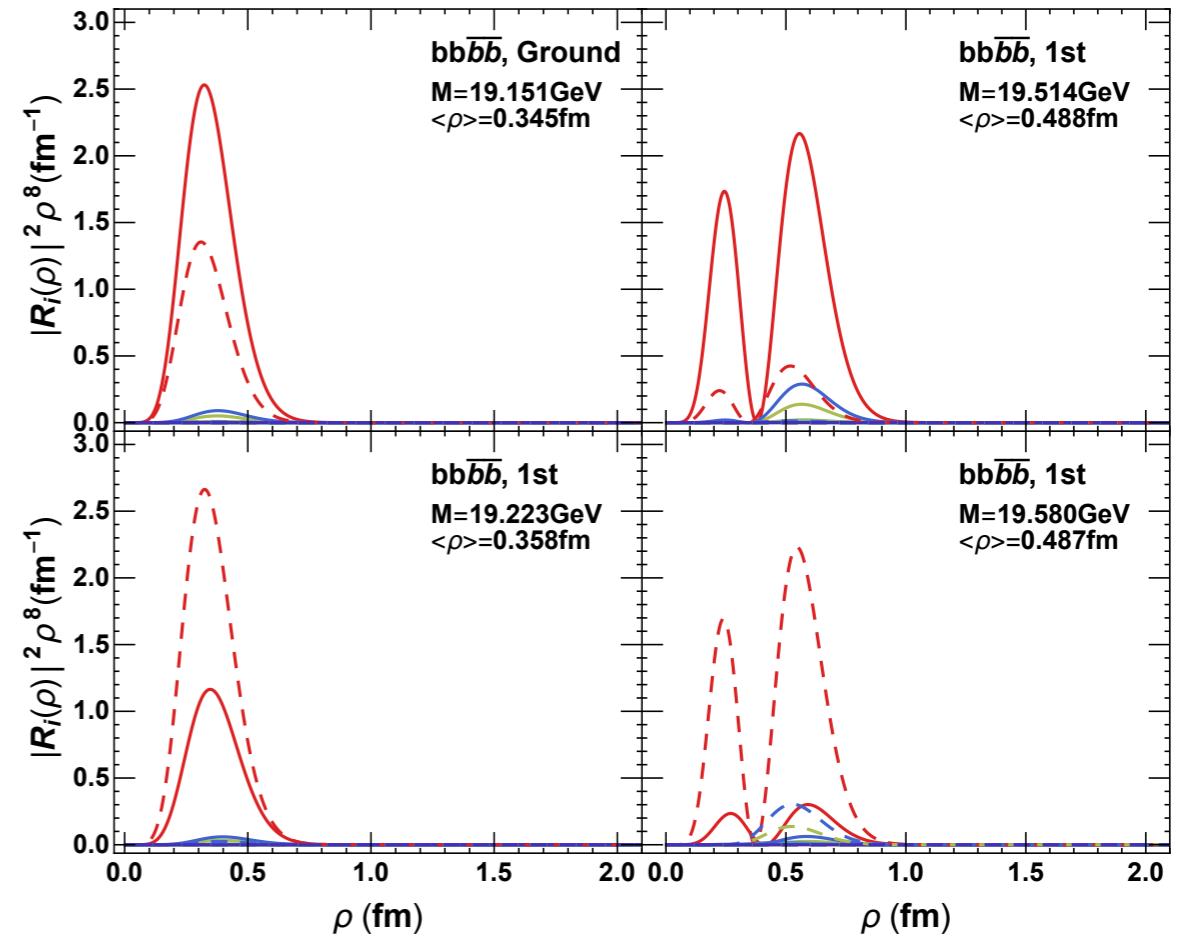
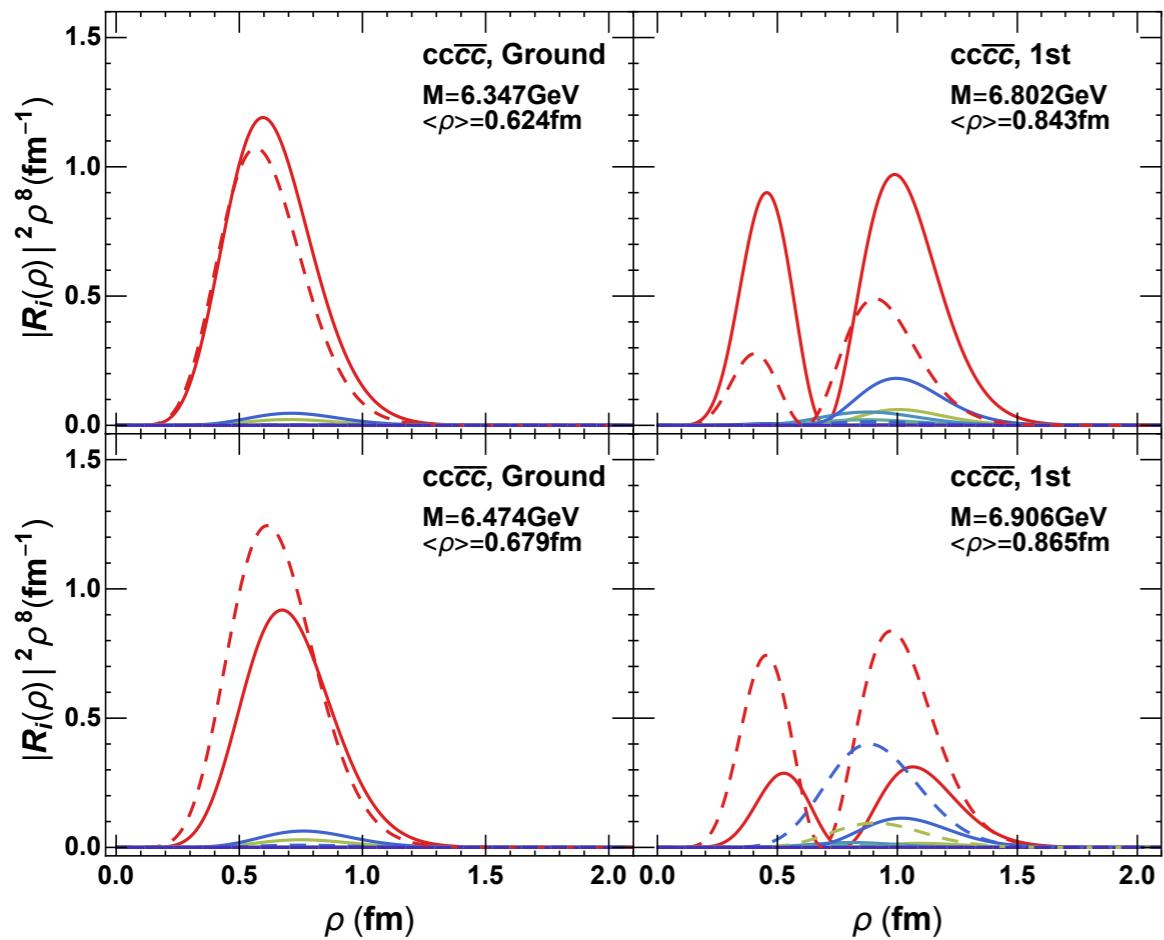
$J^{PC}$	0 <sup>++</sup>						1 <sup>-+</sup>			2 <sup>++</sup>			
State	1S		2S		3S		1S	2S	3S	1S	2S	3S	
$cc\bar{c}\bar{c}$	$M_T$ (GeV)	6.346	6.476	6.804	6.908	7.206	7.296	6.441	6.896	7.300	6.475	6.921	7.320
	$r_{\text{rms}}$ (fm)	0.323	0.351	0.445	0.457	0.550	0.530	0.331	0.446	0.547	0.339	0.452	0.552
$bb\bar{b}\bar{b}$	$M_T$ (GeV)	19.154	19.226	19.518	19.583	19.818	19.887	19.214	19.582	19.889	19.232	19.594	19.898
	$r_{\text{rms}}$ (fm)	0.180	0.186	0.259	0.259	0.328	0.325	0.181	0.257	0.324	0.183	0.259	0.326



# Results

JX Zhao, ShZh. shi, and Pf. Zhuang, Phys.Rev.D 102 (2020) 11, 114001

$J^{PC}$	0 <sup>++</sup>						1 <sup>-+</sup>			2 <sup>++</sup>			
State	1S		2S		3S		1S	2S	3S	1S	2S	3S	
$cc\bar{c}\bar{c}$	$M_T$ (GeV)	6.346	6.476	6.804	6.908	7.206	7.296	6.441	6.896	7.300	6.475	6.921	7.320
	$r_{\text{rms}}$ (fm)	0.323	0.351	0.445	0.457	0.550	0.530	0.331	0.446	0.547	0.339	0.452	0.552
$bb\bar{b}\bar{b}$	$M_T$ (GeV)	19.154	19.226	19.518	19.583	19.818	19.887	19.214	19.582	19.889	19.232	19.594	19.898
	$r_{\text{rms}}$ (fm)	0.180	0.186	0.259	0.259	0.328	0.325	0.181	0.257	0.324	0.183	0.259	0.326



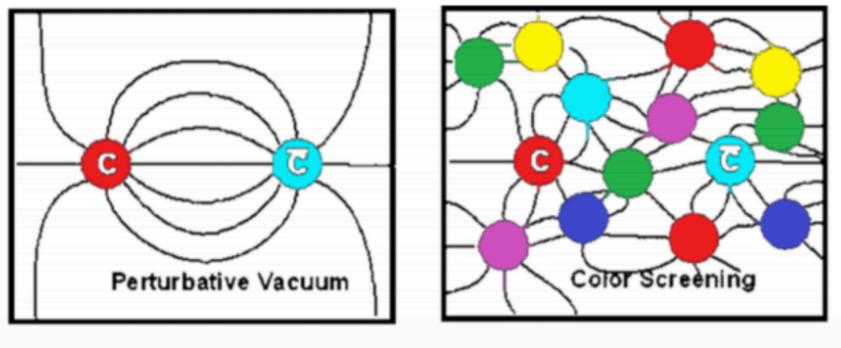
*Yield and decay width:*

F. Feng, Y. Huang, Y. Jia, WL. Sang, X. Xiong, J-Y. Zhang, arXiv: 2009.08450.

C. Becchi, A. Giachino, L. Maiani, E. Santopinto, arXiv: 2006.14388.

# Heavy Quark Potential at Finite-temperature

*Color Screening in hot dense medium:*



*Hard Thermal Loop (HTL):*

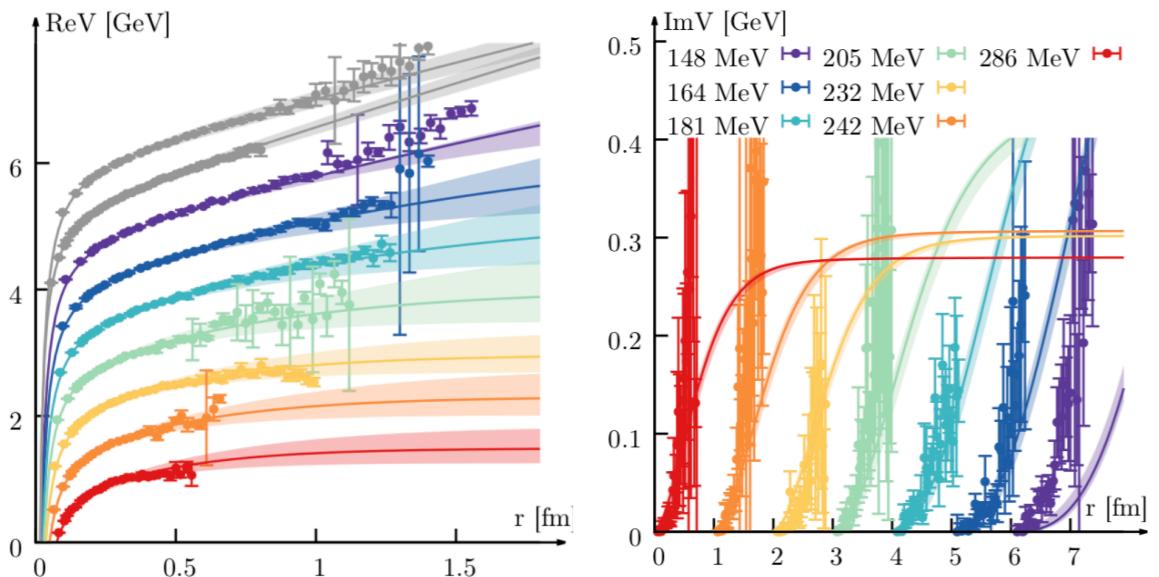
$$-\frac{\alpha}{r} \rightarrow -\frac{\alpha}{r} e^{-m_D r} \quad m_D^2 = \frac{1}{3} g^2 T^2 \left( N_c + \frac{N_f}{2} \right)$$

**N. Brambilla, J. Ghiglieri, A. Vairo, and P. Petreczky, Phys. Rev. D 78, 014017 (2008).**

**N. Brambilla, M. A. Escobedo, J. Ghiglieri, J. Soto, and A. Vairo, JHEP 09, 038 (2010).**

**M. Laine, O. Philipsen, P. Romatschke, and M. Tassler, JHEP 03, 054 (2007).**

*Lattice QCD results:*

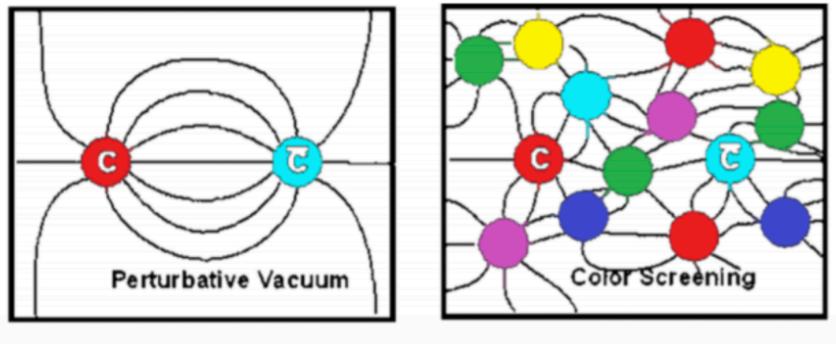


**Burnier Y, Kaczmarek O, Rothkopf A. JHEP, 2015, 12:101.**

**D. Lafferty, A. Rothkopf, Phys.Rev.D 101 (2020) 5, 056010 .**

# Heavy Quark Potential at Finite-temperature

*Color Screening in hot dense medium:*



*Hard Thermal Loop (HTL):*

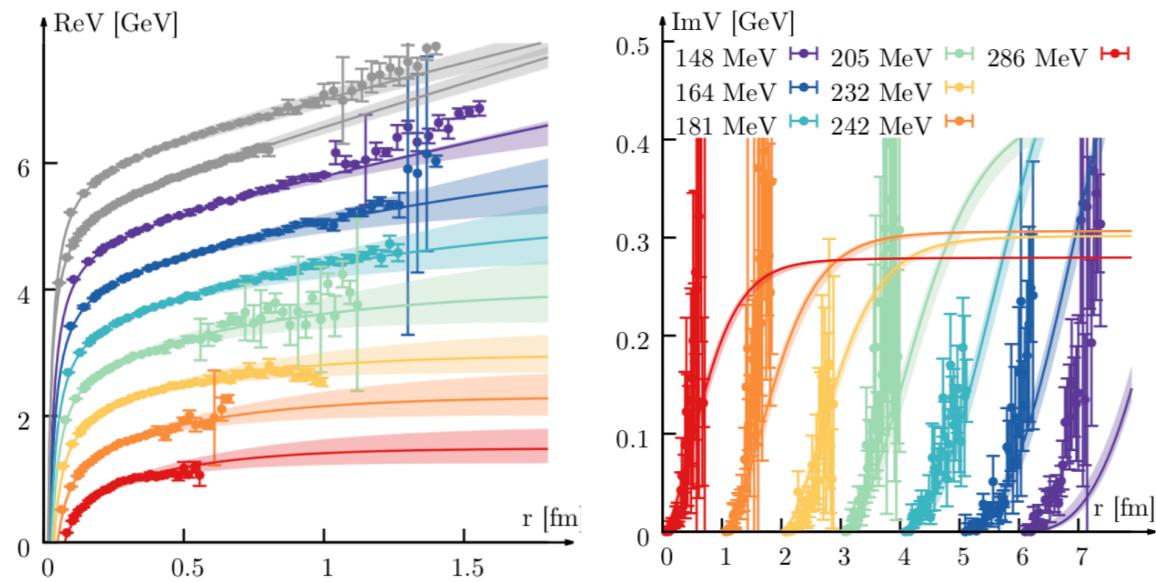
$$-\frac{\alpha}{r} \rightarrow -\frac{\alpha}{r} e^{-m_D r} \quad m_D^2 = \frac{1}{3} g^2 T^2 \left( N_c + \frac{N_f}{2} \right)$$

N. Brambilla, J. Ghiglieri, A. Vairo, and P. Petreczky, Phys. Rev. D 78, 014017 (2008).

N. Brambilla, M. A. Escobedo, J. Ghiglieri, J. Soto, and A. Vairo, JHEP 09, 038 (2010).

M. Laine, O. Philipsen, P. Romatschke, and M. Tassler, JHEP 03, 054 (2007).

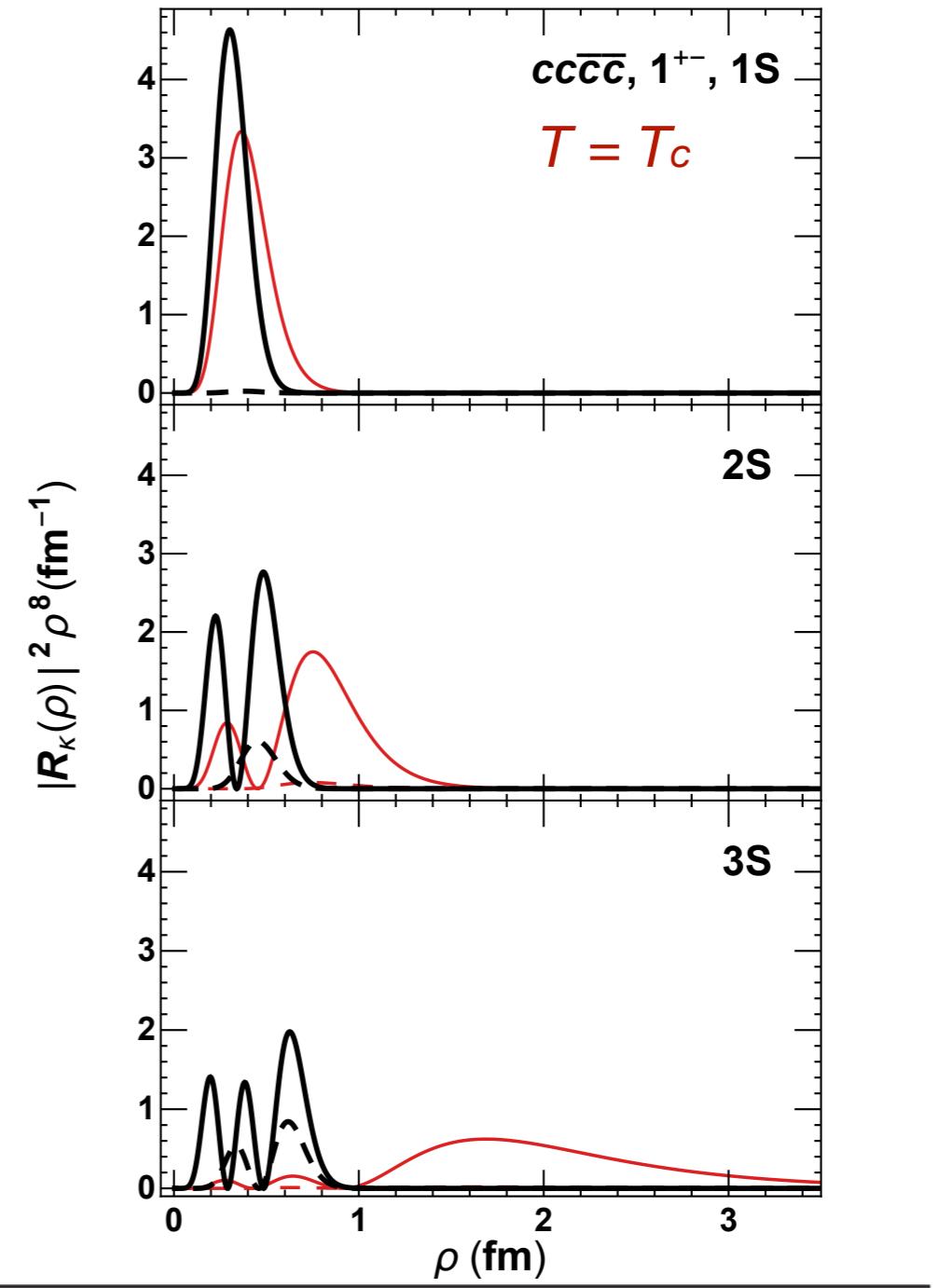
*Lattice QCD results:*



Burnier Y, Kaczmarek O, Rothkopf A. JHEP, 2015, 12:101.

D. Lafferty, A. Rothkopf, Phys.Rev.D 101 (2020) 5, 056010 .

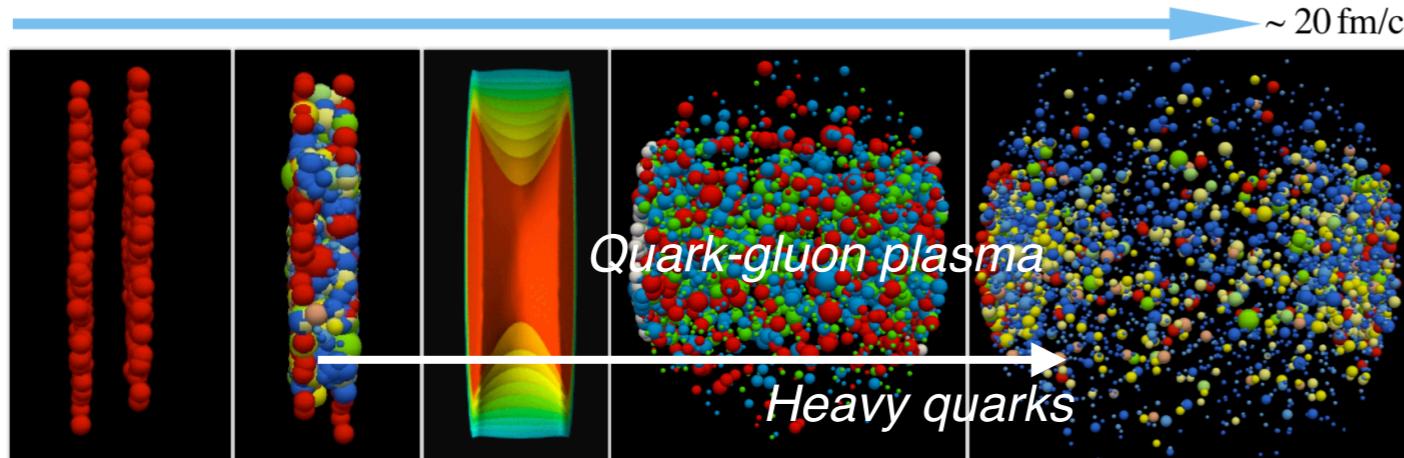
*Wave function at QCD phase transition temperature  $T_c$*



*Dissociation temperature*

	$cc\bar{c}\bar{c}$			$bb\bar{b}\bar{b}$		
	1S	2S	3S	1S	2S	3S
$T_d/T_c$	1.08	1.02	1.0	2.40	1.85	1.30

## Dynamical production (color recombination on the phase boundary)

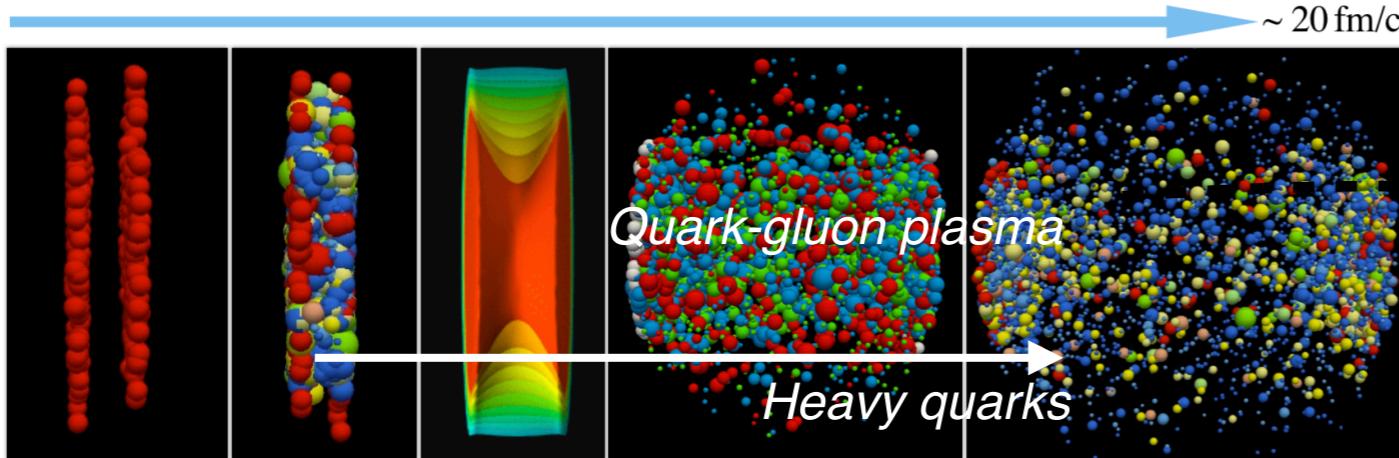


*Heavy flavor initial production: pQCD*

*Evolve in the QGP medium: Langevin equation*

*Dynamic production: Coalescence model*

# Dynamical production (color recombination on the phase boundary)



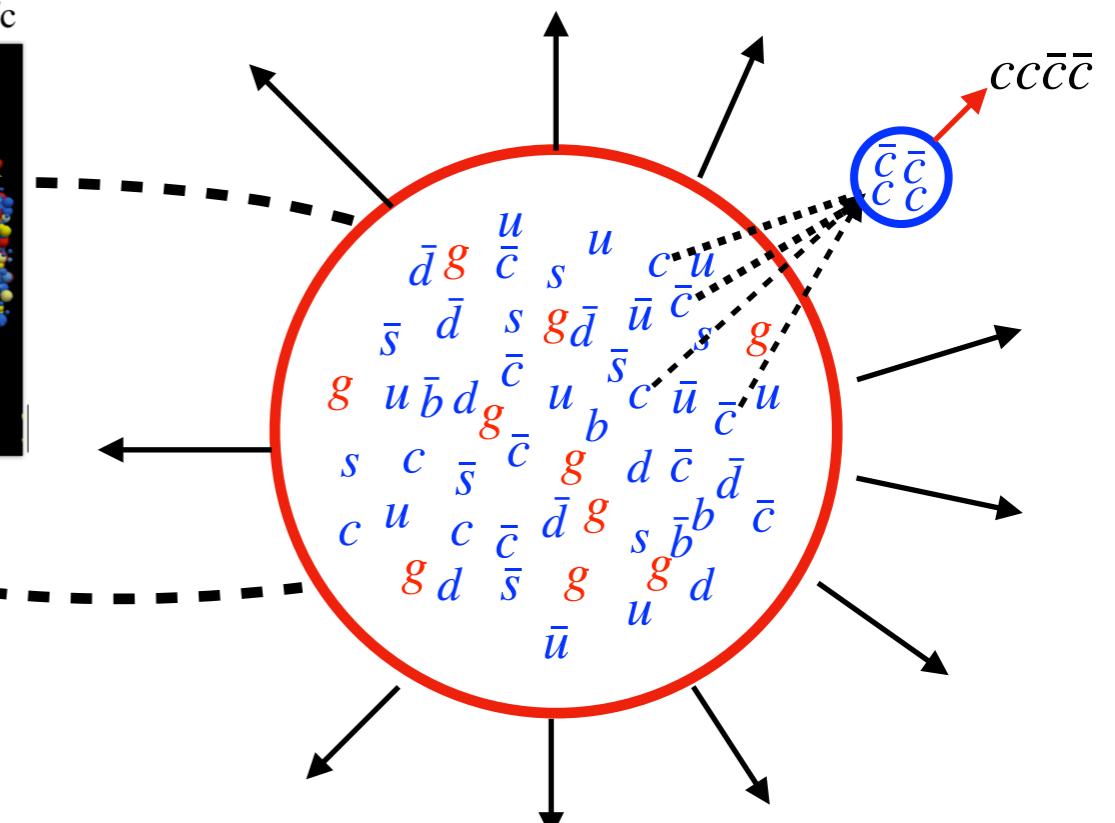
*Heavy flavor initial production: pQCD*

*Evolve in the QGP medium: Langevin equation*

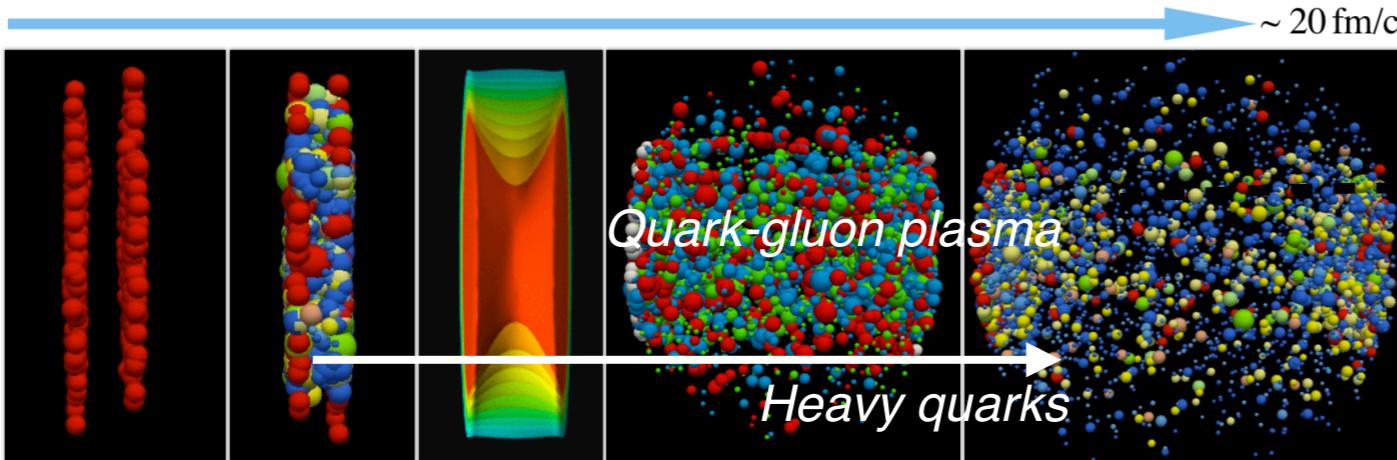
*Dynamic production: Coalescence model*

$$\frac{dN}{d^2\mathbf{P}_T d\eta} = C \int_{\Sigma} \frac{P_\mu d\sigma_\mu}{(2\pi)^3} \int \frac{d^9\mathbf{x} d^9\mathbf{y}}{(2\pi)^9} F(r_1, r_2, r_3, r_4, p_1, p_2, p_3, p_4) W(\mathbf{x}, \mathbf{p})$$

V. Greco, C. M. Ko and R. Rapp, Phys. Lett. B 595, 202 (2004).  
 D. Molnar and S. A. Voloshin, Phys. Rev. Lett. 91, 092301 (2003).  
 R. J. Fries, B. Muller, C. Nonaka and S. A. Bass. Phys. Rev. C68, 044902(2004).



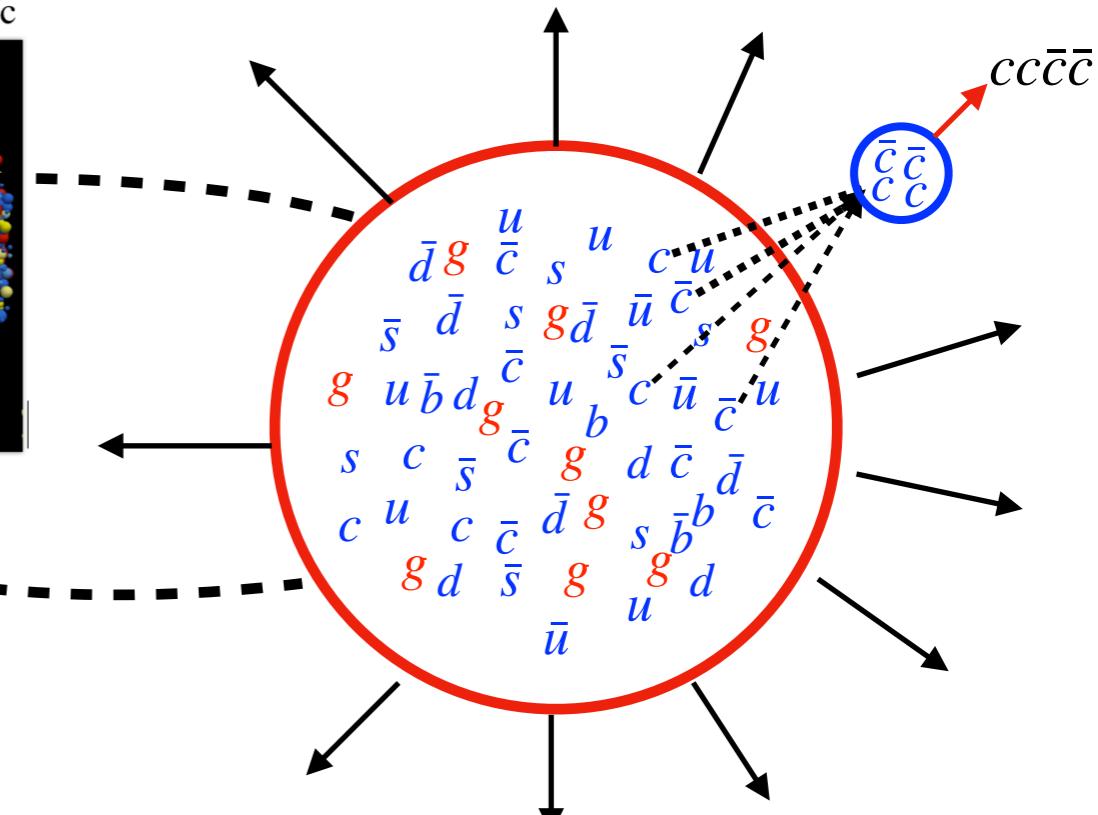
## *Dynamical production (color recombination on the phase boundary)*



Heavy flavor initial production: pQCD

## *Evolve in the QGP medium: Langevin equation*

## *Dynamic production: Coalescence model*



$$\frac{dN}{d^2\mathbf{P}_T d\eta} = C \int_{\Sigma} \frac{P_\mu d\sigma_\mu}{(2\pi)^3} \int \frac{d^9\mathbf{x} d^9\mathbf{y}}{(2\pi)^9} F(r_1, r_2, r_3, r_4, p_1, p_2, p_3, p_4) W(\mathbf{x}, \mathbf{p})$$

*V. Greco, C. M. Ko and R. Rapp, Phys. Lett. B 595, 202 (2004).  
 D. Molnar and S. A. Voloshin, Phys. Rev. Lett. 91, 092301 (2003).  
 R. J. Fries, B. Muller, C. Nonaka and S. A. Bass, Phys. Rev. C68, 044902(2004).*

- The hadronization hypersurface is determined by the hydrodynamics.
  - The Wigner function can self-consistently be determined by the wavefunction.

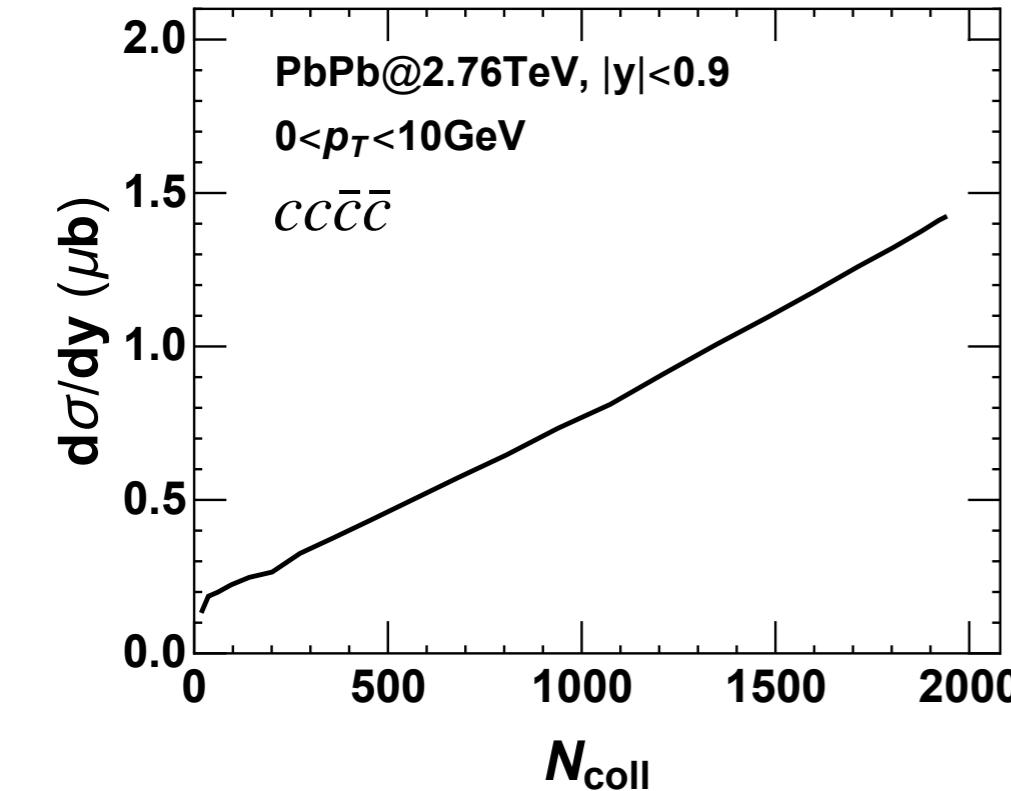
$$W(\mathbf{x}, \mathbf{p}, T) = \int d^9\mathbf{y} e^{-i\mathbf{p}\cdot\mathbf{y}} \Phi\left(\mathbf{x} + \frac{\mathbf{y}}{2}, T\right) \Phi\left(\mathbf{x} - \frac{\mathbf{y}}{2}, T\right).$$

- ## ■ *Heavy quark distribution functions*

$$F(r_1, r_2, r_3, r_4, p_1, p_2, p_3, p_4) = \frac{1}{4} f_Q(r_1, p_1) f_Q(r_2, p_2) f_{\bar{Q}}(r_3, p_3) f_{\bar{Q}}(r_4, p_4)$$

## Results

*Fully-heavy Tetraquark state production in heavy-ion collisions!*



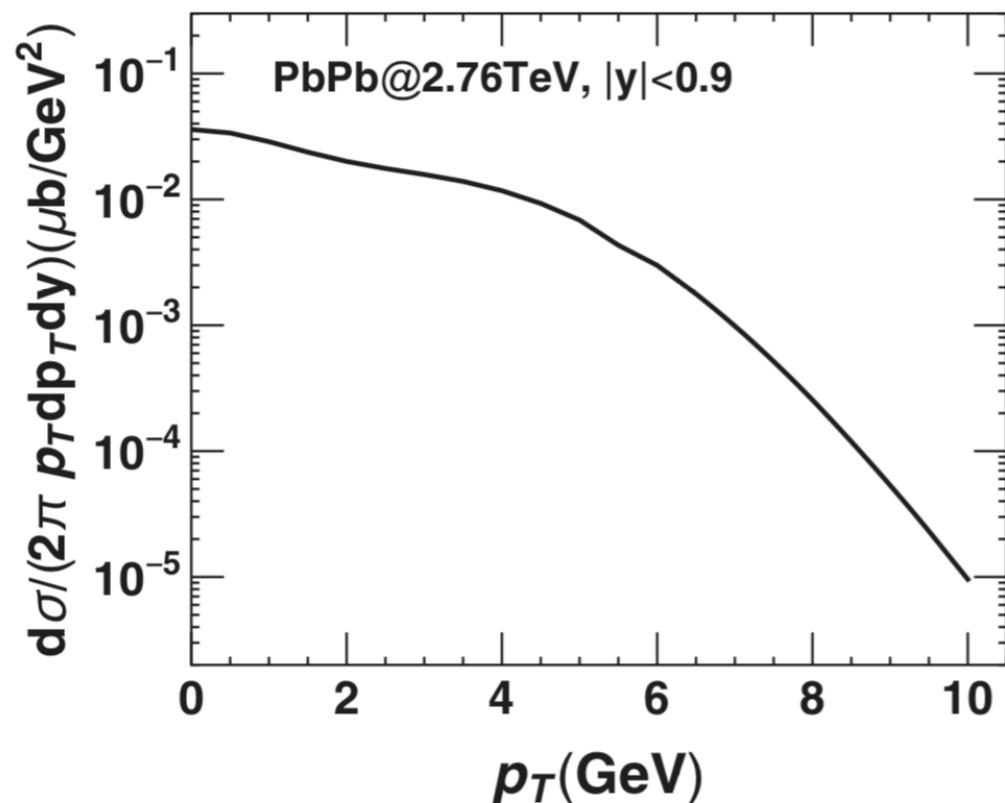
*1 Pb+Pb collisions  $\approx$  2000 p+p collisions.*

$$\left. \frac{d\sigma}{N_{coll} dy} \right|_{AA} \approx 770 pb \text{ in AA at } 5.02 \text{ TeV}$$

$$\left. \frac{d\sigma}{dy} \right|_{pp} = 78 pb \quad \text{in pp at } 7 \text{ TeV}$$

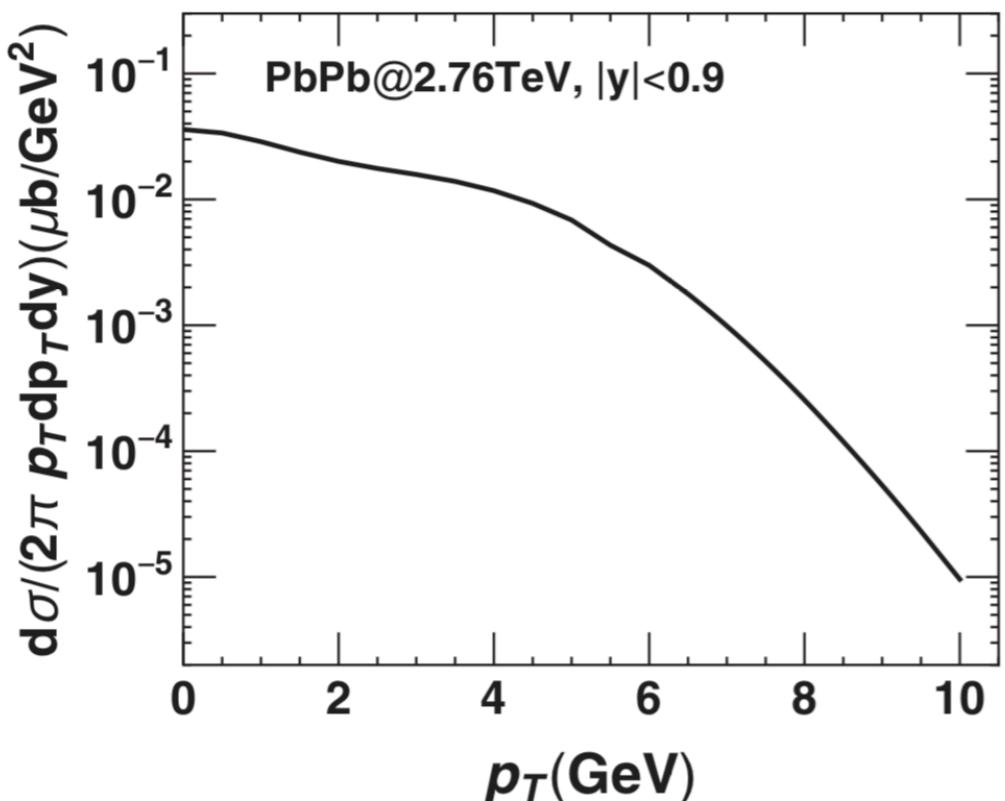
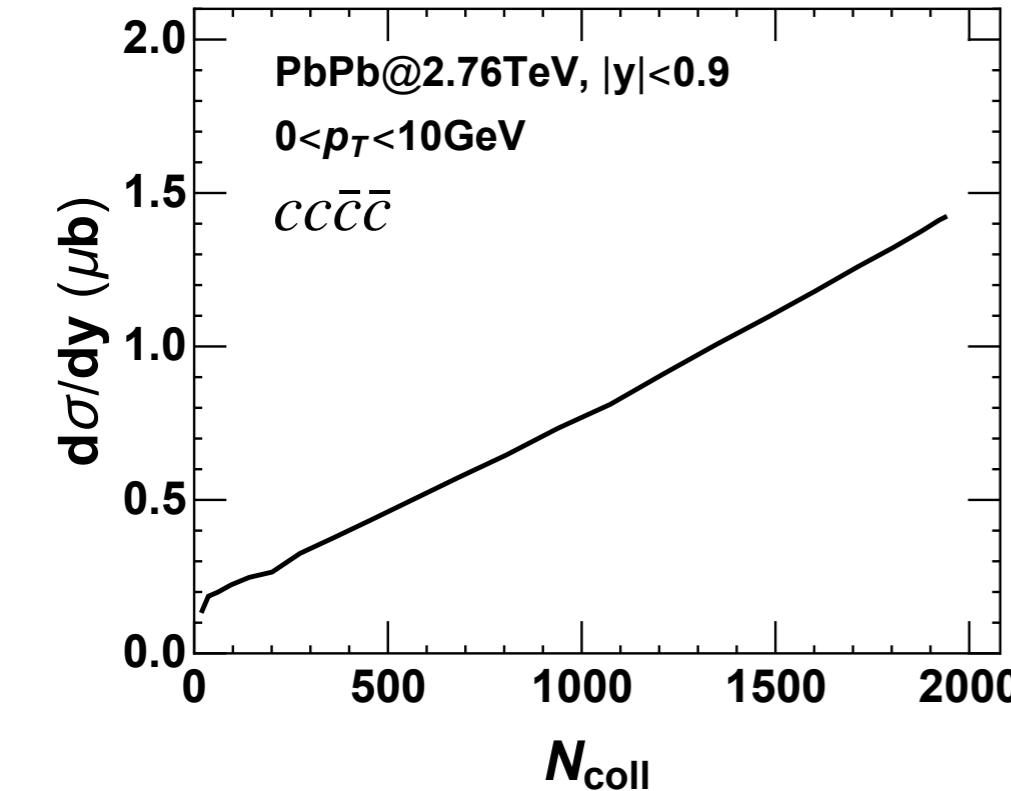
Marek Karliner et al, Phys.Rev.D 95 (2017) 3, 034011.  
Ruilin Zhu, arXiv: 2010.09082.

*yield significantly enhanced !*



## Results

Fully-heavy Tetraquark state production in heavy-ion collisions!



1  $Pb+Pb$  collisions  $\approx 2000$   $p+p$  collisions.

$$\left. \frac{d\sigma}{N_{coll} dy} \right|_{AA} \approx 770 pb \text{ in AA at } 5.02\text{TeV}$$

$$\left. \frac{d\sigma}{dy} \right|_{pp} = 78 pb \quad \text{in pp at } 7\text{TeV}$$

Marek Karliner et al, Phys.Rev.D 95 (2017) 3, 034011.  
Ruilin Zhu, arXiv: 2010.09082.

yield significantly enhanced !

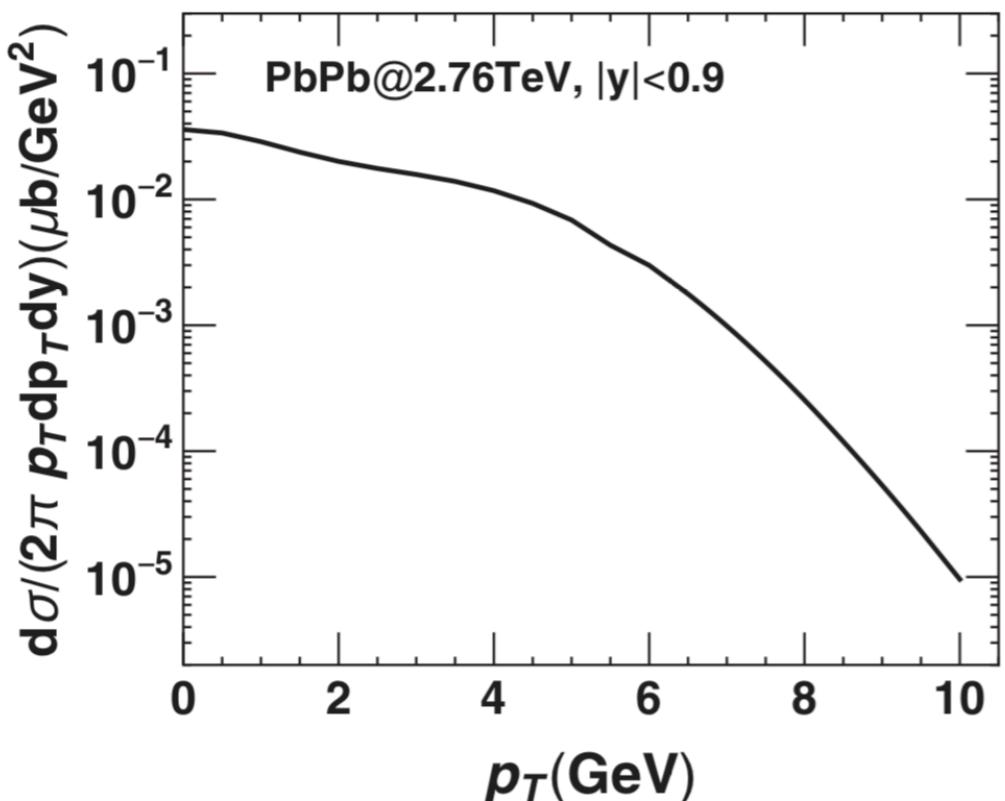
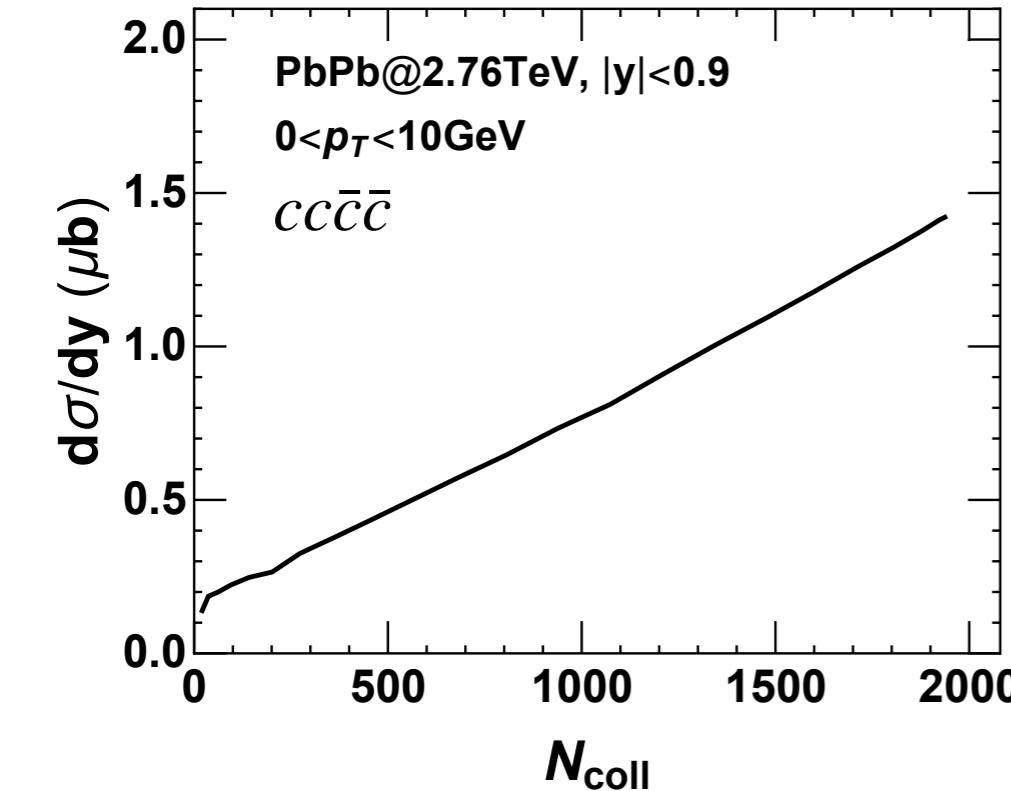
The four-lepton decay channel

$$X(cc\bar{c}\bar{c}) \rightarrow l_1^+ l_2^- l_3^+ l_4^-$$

can be well separated from the bulk back ground  
and makes it possible to find such exotic states in  
heavy-ion collision even in low  $p_T$  region!

# Results

*Fully-heavy Tetraquark state production in heavy-ion collisions!*



*1 Pb+Pb collisions  $\approx$  2000 p+p collisions.*

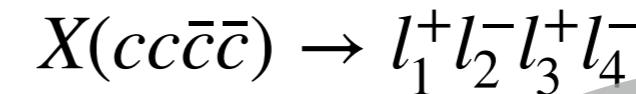
$$\left. \frac{d\sigma}{N_{coll} dy} \right|_{AA} \approx 770 \text{ pb} \text{ in AA at } 5.02 \text{ TeV}$$

$$\left. \frac{d\sigma}{dy} \right|_{pp} = 78 \text{ pb} \text{ in pp at } 7 \text{ TeV}$$

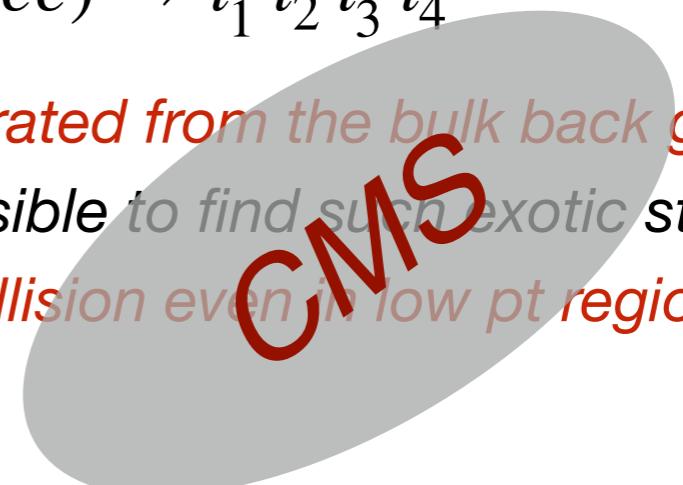
Marek Karliner et al, Phys.Rev.D 95 (2017) 3, 034011.  
Ruilin Zhu, arXiv: 2010.09082.

*yield significantly enhanced !*

*The four-lepton decay channel*



can be well separated from the bulk back ground  
and makes it possible to find such exotic states in  
heavy-ion collision even in low  $p_T$  region!



## Summary

- We studied fully-heavy Tetraquark in vacuum and finite-temperature via 4-body Schroedinger equation.



- Due to the color recombination of uncorrelated charm quarks, the yield significantly enhanced comparing with p+p collisions !

This supply a way to searching for more fully-heavy Tetraquark states in the experiment.

## Outlook

- The angle excited( $L=1, 2\dots$ ) states of fully-heavy Tetraquark states.
- Searching for new observables of fully-heavy Tetraquark states in heavy-ion collisions.

*Thanks for your attention!*

# Backup

*hyper-spherical harmonic function expansion  
( $L = M = 0$ )*

$$\mathcal{Y}_1 = \sqrt{\frac{105}{32}} \frac{1}{\pi^2},$$

$$\mathcal{Y}_2 = \sqrt{\frac{385}{6}} \frac{3}{16\pi^2} (3 \cos(2\alpha_3) - 1),$$

$$\mathcal{Y}_3 = \sqrt{\frac{385}{2}} \frac{3}{8\pi^2} \cos(2\alpha_2) \cos^2(\alpha_3),$$

$$\begin{aligned} \mathcal{Y}_4 = & -\sqrt{\frac{385}{2}} \frac{3}{4\pi^2} \cos \alpha_2 \sin \alpha_2 \cos^2 \alpha_3 \\ & \times [\cos \theta_1 \cos \theta_2 + \cos(\phi_1 - \phi_2) \sin \theta_1 \sin \theta_2], \end{aligned}$$

$$\begin{aligned} \mathcal{Y}_5 = & -\sqrt{\frac{385}{2}} \frac{3}{4\pi^2} \cos \alpha_2 \cos \alpha_3 \sin \alpha_3 \\ & \times [\cos \theta_1 \cos \theta_3 + \cos(\phi_1 - \phi_3) \sin \theta_1 \sin \theta_3], \end{aligned}$$

$$\begin{aligned} \mathcal{Y}_6 = & -\sqrt{\frac{385}{2}} \frac{3}{4\pi^2} \sin \alpha_2 \cos \alpha_3 \sin \alpha_3 \\ & \times [\cos \theta_2 \cos \theta_3 + \cos(\phi_2 - \phi_3) \sin \theta_2 \sin \theta_3], \end{aligned}$$

$$\begin{aligned} \mathcal{Y}_7 = & i\sqrt{5005} \frac{3}{8\pi^2} \sin \alpha_2 \cos \alpha_2 \sin \alpha_3 \cos^2 \alpha_3 \\ & \times [\cos \theta_3 \sin \theta_1 \sin \theta_2 \sin(\phi_1 - \phi_2) \\ & - \sin \theta_3 \cos \theta_2 \sin \theta_1 \sin(\phi_1 - \phi_3) \\ & + \sin \theta_3 \cos \theta_1 \sin \theta_2 \sin(\phi_2 - \phi_3)]. \end{aligned}$$

$$\begin{aligned} V_1 = & \langle \phi_1 \chi_2 | \sum_{i < j} V_{ij} | \phi_1 \chi_2 \rangle \\ = & \frac{2}{3} (V_{12}^c + V_{34}^c) + \frac{1}{3} (V_{13}^c + V_{14}^c + V_{23}^c + V_{24}^c) \\ & + \frac{1}{6} (V_{12}^{ss} + V_{34}^{ss}) - \frac{1}{6} (V_{13}^{ss} + V_{14}^{ss} + V_{23}^{ss} + V_{24}^{ss}), \end{aligned}$$

$$\begin{aligned} V_2 = & \langle \phi_2 \chi_1 | \sum_{i < j} V_{ij} | \phi_2 \chi_1 \rangle \\ = & -\frac{1}{3} (V_{12}^c + V_{34}^c) + \frac{5}{6} (V_{13}^c + V_{14}^c + V_{23}^c + V_{24}^c) \\ & + \frac{1}{4} (V_{12}^{ss} + V_{34}^{ss}), \end{aligned}$$

$$\begin{aligned} V_m = & \langle \phi_1 \chi_2 | \sum_{i < j} V_{ij} | \phi_2 \chi_1 \rangle \\ = & \langle \phi_2 \chi_1 | \sum_{i < j} V_{ij} | \phi_1 \chi_2 \rangle \\ = & -\frac{\sqrt{6}}{8} (V_{13}^{ss} + V_{14}^{ss} + V_{23}^{ss} + V_{24}^{ss}), \end{aligned}$$

$$\begin{aligned} V = & \langle \phi_1 \chi_5 | \sum_{i < j} V_{ij} | \phi_1 \chi_5 \rangle \\ = & \frac{2}{3} (V_{12}^c + V_{34}^c) + \frac{1}{3} (V_{13}^c + V_{14}^c + V_{23}^c + V_{24}^c) \\ & + \frac{1}{6} (V_{12}^{ss} + V_{34}^{ss}) - \frac{1}{12} (V_{13}^{ss} + V_{14}^{ss} + V_{23}^{ss} + V_{24}^{ss}), \end{aligned}$$

*Many excellent review papers :*

**E. Klempt, A. Zaitsev, Phys. Rept. 454 (2007) 1–202.**

**E. Klempt, JM. Richard, Rev. Mod. Phys. 82 (2010) 1095–1153.**

**YR Liu, HX Che, W. Chen, X. Liu, SL Zhu. Phys. Rept. 639 (2016) 1–121.**

**A. Ali, JS. Lange, S. Stone. Prog. Part. Nucl. Phys. 97 (2017) 123–198.**

**FK. Guo, C. Hanhart, U. Meißner, Q. Wang, Q. Zhao. Rev. Mod. Phys. 90 (2018) 015004.**

**YR. Liu, HX. Chen, W. Chen, X. Liu, SL. Zhu. Prog. Part. Nucl. Phys. 107 (2019) 237–320.**

**N. Brambilla, S. Eidelman, C. Hanhart, A. Nefediev, CP. Shen, CE. Thomas, A. Vairo, ChZh. Yuan. Phys. Rept. 873 (2020) 1-154.**

....