Heavy four-quarks and six-quarks states from lattice QCD

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Tata Institute of Fundamental Research, India

Collaborators : P. Junnarkar and M. Padmanath

Phys. Rev. D99 034507 (2019), Phys. Rev. Lett. 123, 162003 (2019)

Experimental and theoretical status of and perspectives for XYZ, GSI, Darmstadt

Exotic Multiquark States

Four and five quark states	Six quark states
• Multiple states, with two heavy quarks (mainly charm), having four quark configurations, have been discovered. • Charmed pentaquark states have been discovered.	• Beside deuteron, only one more state with six quark configurations (***d*(2380)***) have been observed in one experiment. • A few hypernuclei with very shallow binding have been observed.

Lattice study on heavy multiquark states

LQCD for heavy quark physics

Requirement: $ma << 1$

- **Charm :** ma = 1.275 GeV,
  - ma = 0.5 → a ~ 0.075 fm
  - ma = 0.3 → a ~ 0.046 fm
- **Bottom :** ma = 4.66 GeV
  - ma = 0.5 → a = 0.021 fm
  - ma = 0.3 → a = 0.013 fm

Computational cost $\propto \frac{1}{a^{4-6}}$!

- **Heavy light hadrons:**

$$\frac{1}{L} \ll m_\pi \ll m_H \ll \frac{1}{a}$$

- **Being heavy, lattice correlation functions for heavy quarks decay rapidly.**
- **Relativistic charm quark calculations are now possible.**
- **However, relativistic bottom-quark is still very costly.**
- **Most of the lattice calculations with bottom quarks utilize NRQCD.**

Lattice study of heavy exotics

- Nonrelativistic *b* quark with relativistic other quarks (calculations with relativistic *b* quarks are starting. …S. Ryan's talk)

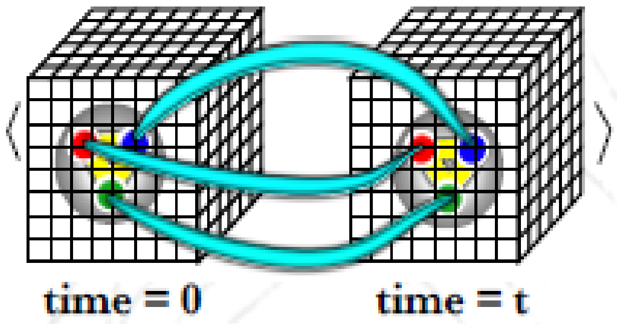
- Potential:
  - Static quark potential (Born-Oppenheimer)…

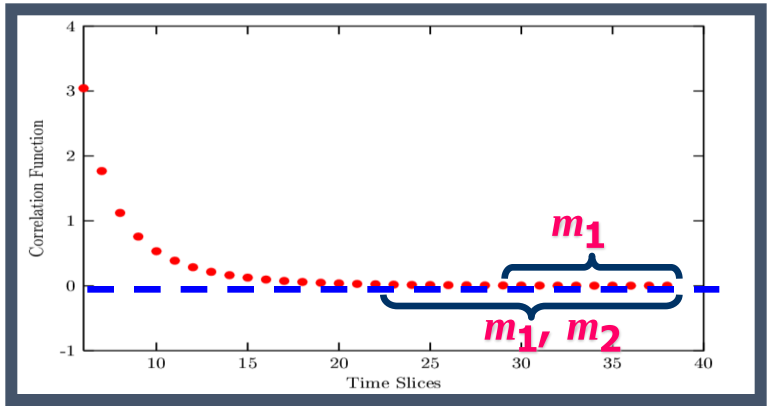
    P. Bicudo et al, Phys. Rev. D95, 034502(2017)
  - HALQCD lattice potential ..HAL QCD ('16,'18)

Hadron Spectra: What can Lattice QCD do?

- Energy excitations can be obtained from Euclidean two-point correlation functions

$$\left\langle C^{2pt}_{ab}(t,\vec{P})\right\rangle = \quad = \sum_n Z_{b,n} Z^{\dagger}_{a,n} e^{-E_n}$$





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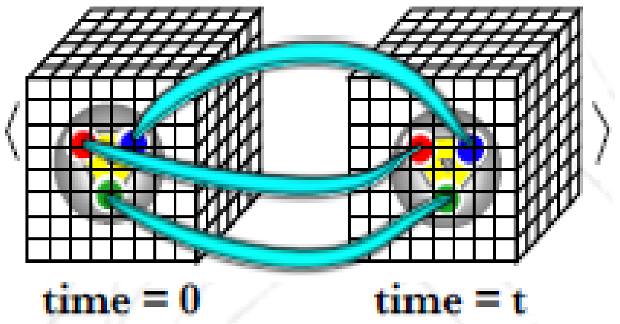
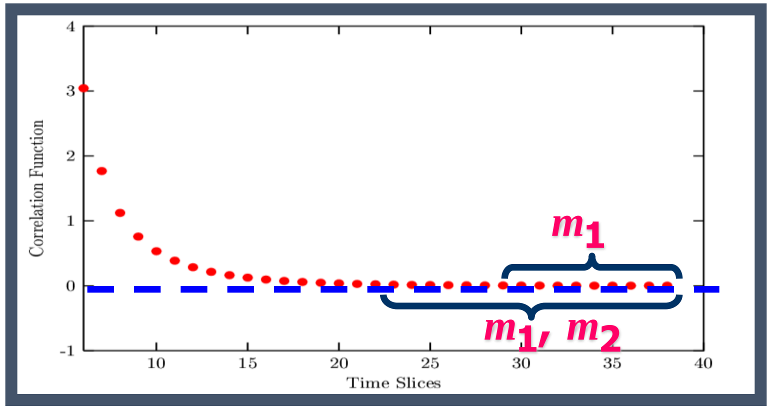


$$= \sum_n Z_{b,n} Z_{a,n}^{\dagger} e^{-E_n}$$



- Suitable quark smearing (including distillation method) can improve overlap to the desired energy levels

$$q(\boldsymbol{x}, t) = \sum_{y} S(\boldsymbol{x}, \boldsymbol{y}) q_b(\boldsymbol{y}, t)$$

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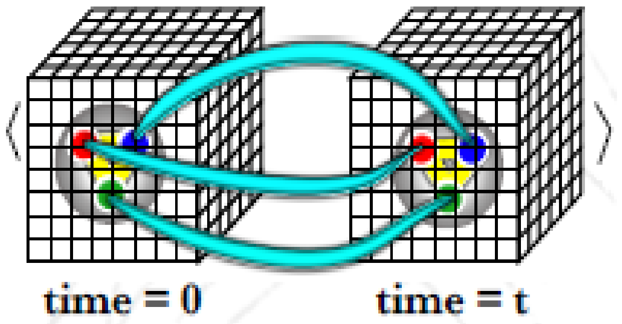

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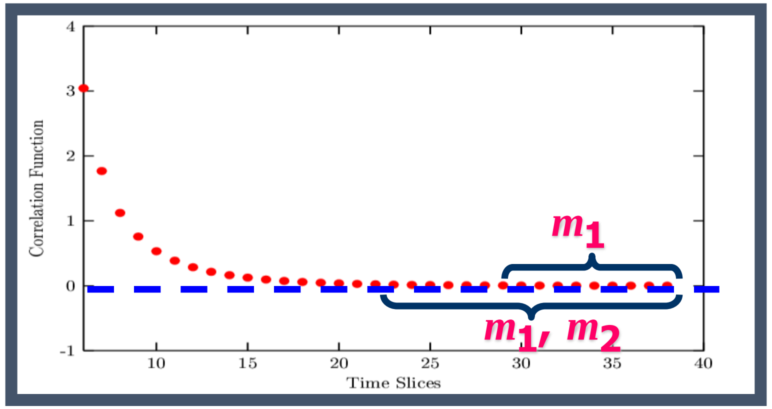


- Suitable quark smearing (including distillation method) can improve overlap to the desired energy levels

$$q(\boldsymbol{x},t) = \sum_y S(\boldsymbol{x},\boldsymbol{y})\, q_b(\boldsymbol{y},t)$$

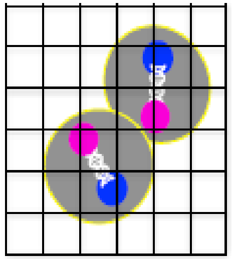
- Correlation matrices of a large basis of hadronic interpolating fields can provide multiple energy levels.

$$C(t) = \begin{bmatrix} \langle 0|\phi_1(t)\phi_1^{+}(0)|0\rangle & \langle 0|\phi_1(t)\phi_2^{+}(0)|0\rangle & \dots & \dots \\ \langle 0|\phi_2(t)\phi_1^{+}(0)|0\rangle & \langle 0|\phi_2(t)\phi_2^{+}(0)|0\rangle & \dots & \dots \\ \dots\dots & \dots\dots\dots & & \dots\dots \\ \dots\dots & \dots\dots & & \dots\dots \end{bmatrix}$$

- Variational analysis with a good basis of operators can be utilized to obtain energy excitations

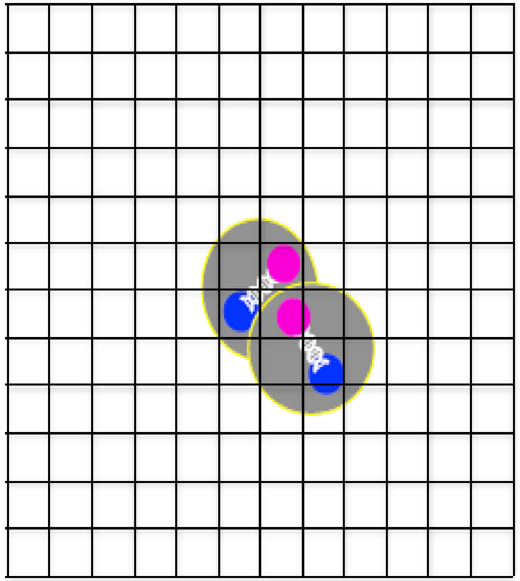
- Finite volume analysis using Luscher formula can help to find out pole structure along with scattering information from the extracted Euclidean energy levels

Energy spectra within finite volume



Infinite-volume
bound state

$$[E(L) - E(\infty)] \propto \frac{e^{-\gamma L}}{\gamma L}$$



Huang, Yang, Phys. Rev. 105 (1957)

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Beane et al, Phys. Lett. B 585 (2004)

...

Recent review:

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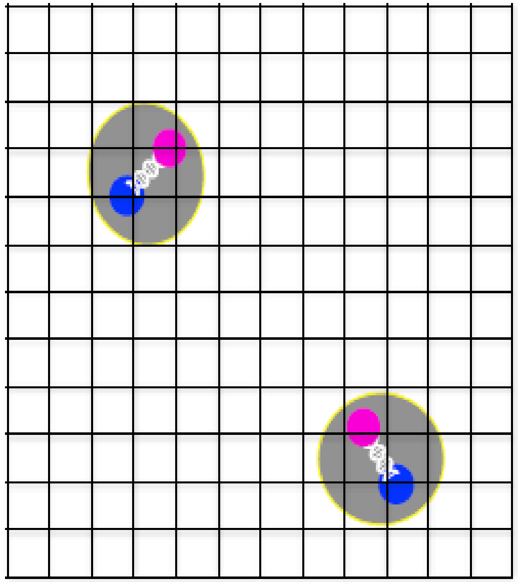
Infinite-volume
scattering state

$$[E(L) - E(\infty)] \propto \frac{a}{ML^3}$$

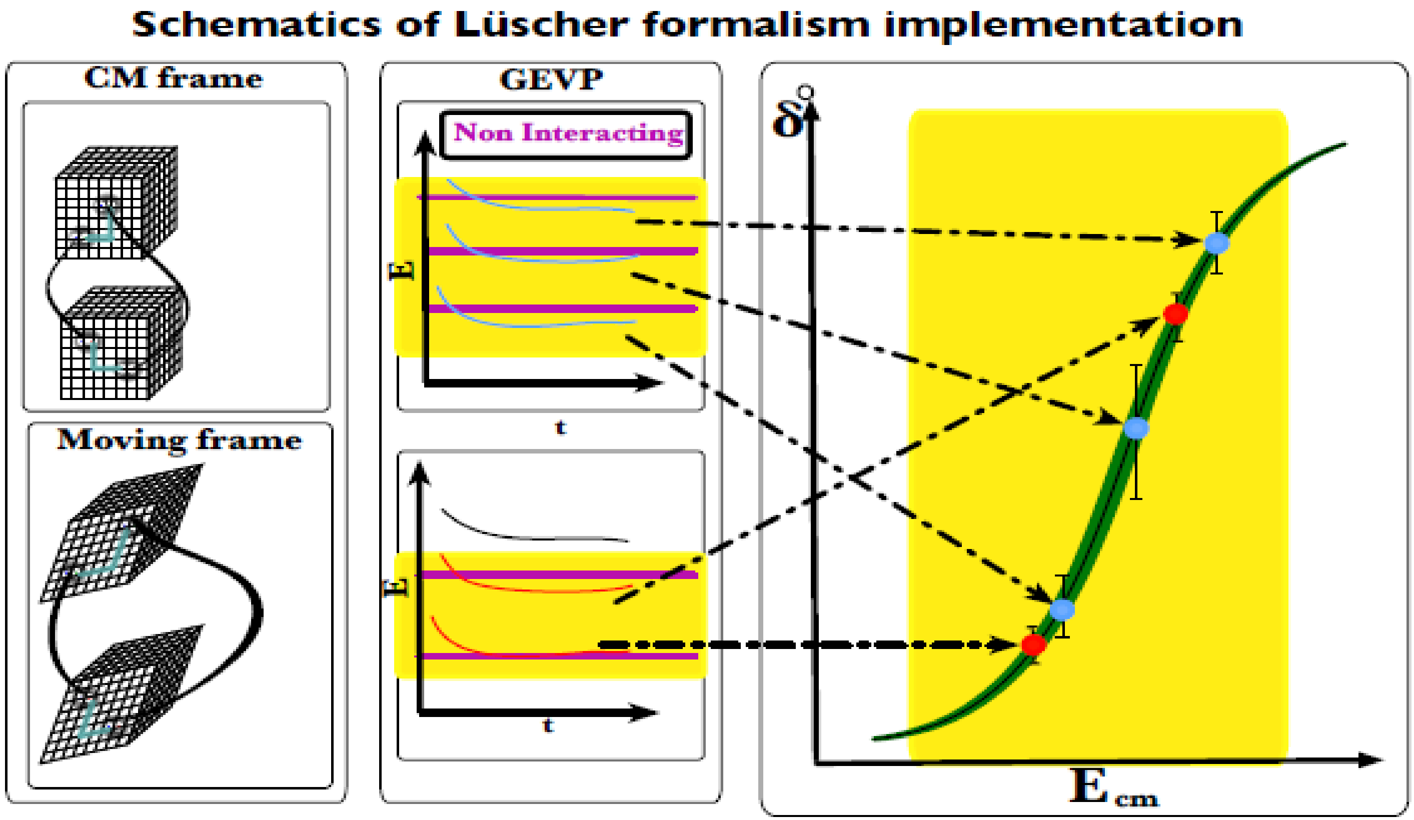


@M. Wagner

Finite volume Euclidean time spectra ⟺ Infinite volume real time scattering amplitudes

$$\det\left(1 + i\,\boxed{t_\ell(s)}\,(1 + i\,\boxed{\mathcal{M}^{\vec{P}}})\right) = 0$$

Luscher



Hadron Spectra: What can Lattice QCD do?

Relatively Easy	Moderately difficult	Difficult

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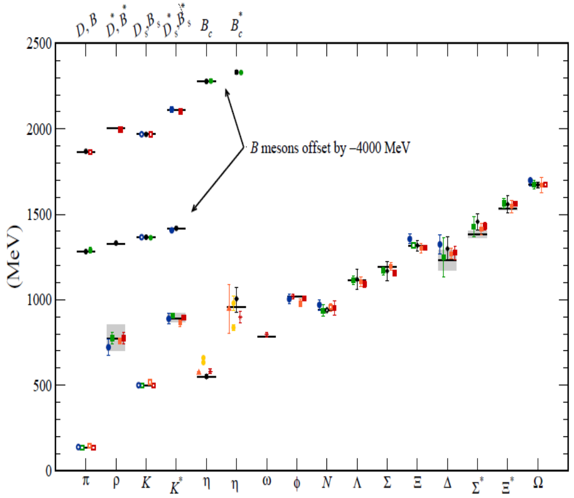 Ground state masses of stable single hadrons (pion, nucleon, ground state charmonia etc.) can reliably be obtained (provided reliable control over statistical and systematics errors are achieved)		

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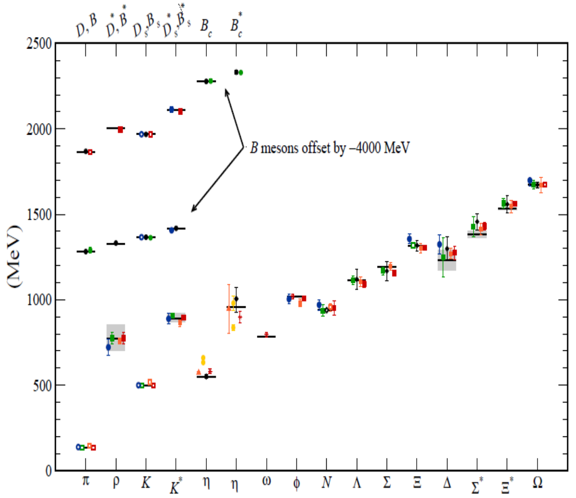

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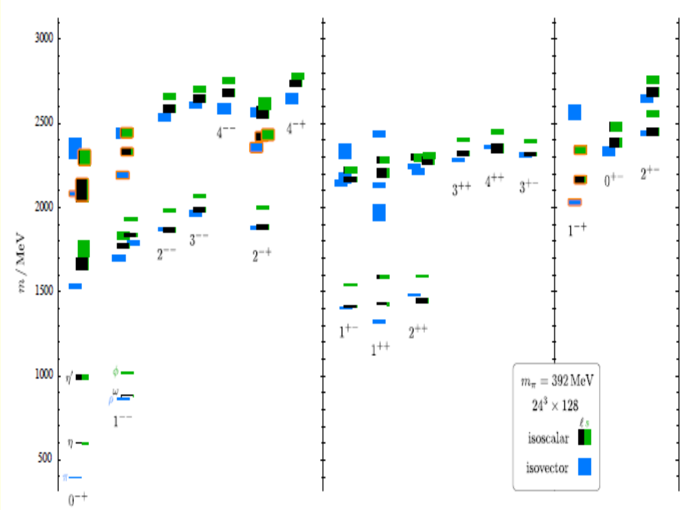

Excited state masses.
(excited states of mesons, heavy baryons etc. away from multi-hadron thresholds)

Hadron Spectra: What can Lattice QCD do?

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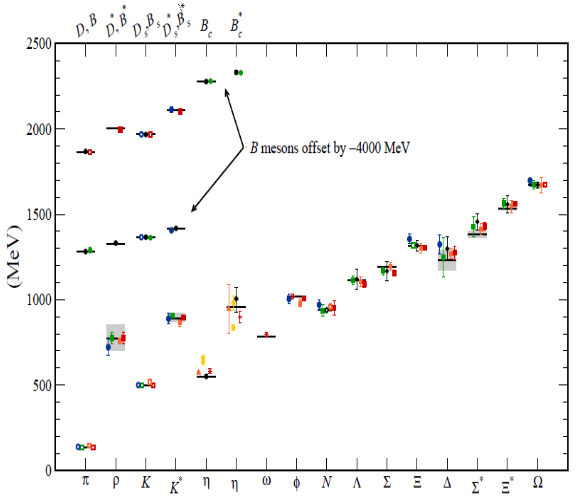
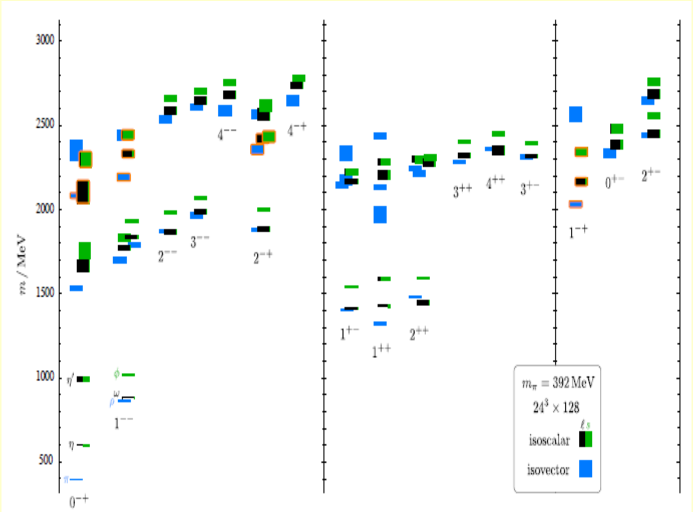
Relatively Easy



Ground state masses of stable single hadrons (pion, nucleon, ground state charmonia etc.) can reliably be obtained (provided reliable control over statistical and systematics errors are achieved)

Moderately difficult



Excited state masses.
(excited states of mesons, heavy baryons etc.)

Difficult

Hadrons

- **particularly close to thresholds (like deuteron, X(3872), Z(4430), pentaquarks, H-dibaryon etc)**
- **hadrons decaying/mixing to multi-hadrons (1^{-+} light exotics, light baryon resonances, glueballs etc)**
- **multi-hadrons (deuteron, dineutron, helium etc multi-baryons).**

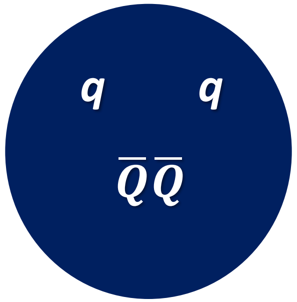
Heavy Four-quark states

- Fourquark states have been observed experimentally with heavy quark contents. ….LHC, Belle, BES

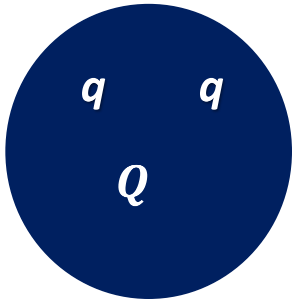
- Is their possibility to find more of those?

- What can lattice studies do?

Heavy four-quark states



$[\bar{Q}\bar{Q}]_3 \to Q$



Diquark properties:

- $[\bar{Q}\bar{Q}]_3^{m_Q\to\infty} \Longrightarrow$ compact
- $\{qq'\}_{\bar{3}}$ **attractive**
- $\{qq'\} \equiv (qC\gamma_5 q')$ **lightest**
- $m(\{ud\}) < m(\{us\})$

A possible structure:

$(qC\gamma_5 q')(\bar{Q}C\gamma_i\bar{Q}')$

How about ? $\{qq'\} \quad \{\bar{Q}\bar{Q}'\}$

How to build heavy tetraquarks? A way

- Two heavy quarks with two light quarks
- $C_l = \bar{3}$, good light diquark
  - ➢ $F = \bar{3}, J_l = 0 \Rightarrow J_h = 1, C_h = 3,\ J^P = 1^+$

$$(\boldsymbol{qC\gamma_5 q'})(\boldsymbol{\bar{Q}C\gamma_i\bar{Q}'})$$

$$\{\bar{\mathbf{3}}, \boldsymbol{J} = \mathbf{0}\} \quad \{\mathbf{3}, \boldsymbol{J} = \mathbf{1}\}$$

➢ Spin dependent interaction $\propto 1/m_h$. For threshold $J^P = 1^+$ states, like $B^*B, B^*D, B_s^*B_s, D^*D$, this interaction will be supressed.

➢ With $C_h = 3$, colour Coulomb attraction, this is not present for two-meson thresholds.

Possible states? : $\boldsymbol{\bar{b}\bar{b}ud, \bar{b}\bar{b}us, \bar{b}\bar{b}uc, \bar{b}\bar{b}sc,}$
$\boldsymbol{\bar{b}\bar{c}ud, \bar{b}\bar{c}us\ \ etc.}$

$$\boldsymbol{J = 1, l_1 l_2 \bar{Q}\bar{Q}} \qquad\qquad \boldsymbol{J = 0, ll\bar{Q}\bar{Q}}$$

Interpolating Fields

$$J = 1, l_1 l_2 \bar{Q}\bar{Q}$$

$$(l_1, l_2) \to (\bar{\mathbf{3}}_c, 0, F_A), \qquad (\bar{Q}, \bar{Q}) \to (\mathbf{3}_c, 1, F_s)$$

$$(l_1, l_2; l_1 \neq l_2) \subset (u, d, s, c). \qquad Q \neq l_1 \neq l_2 \subset (\bar{c}, \bar{b})$$

- **Tetraquark type :**

$$\mathcal{T}^1(x) = (l_1)^a_\alpha(x)\ (C\gamma_5)_{\alpha\beta}\ (l_2)^b_\beta(x)\ \bar{Q}^a_\kappa(x)(C\gamma_i)_{\kappa\rho}\ \bar{Q}^b_\rho(x)$$

- **Two mesons type:**

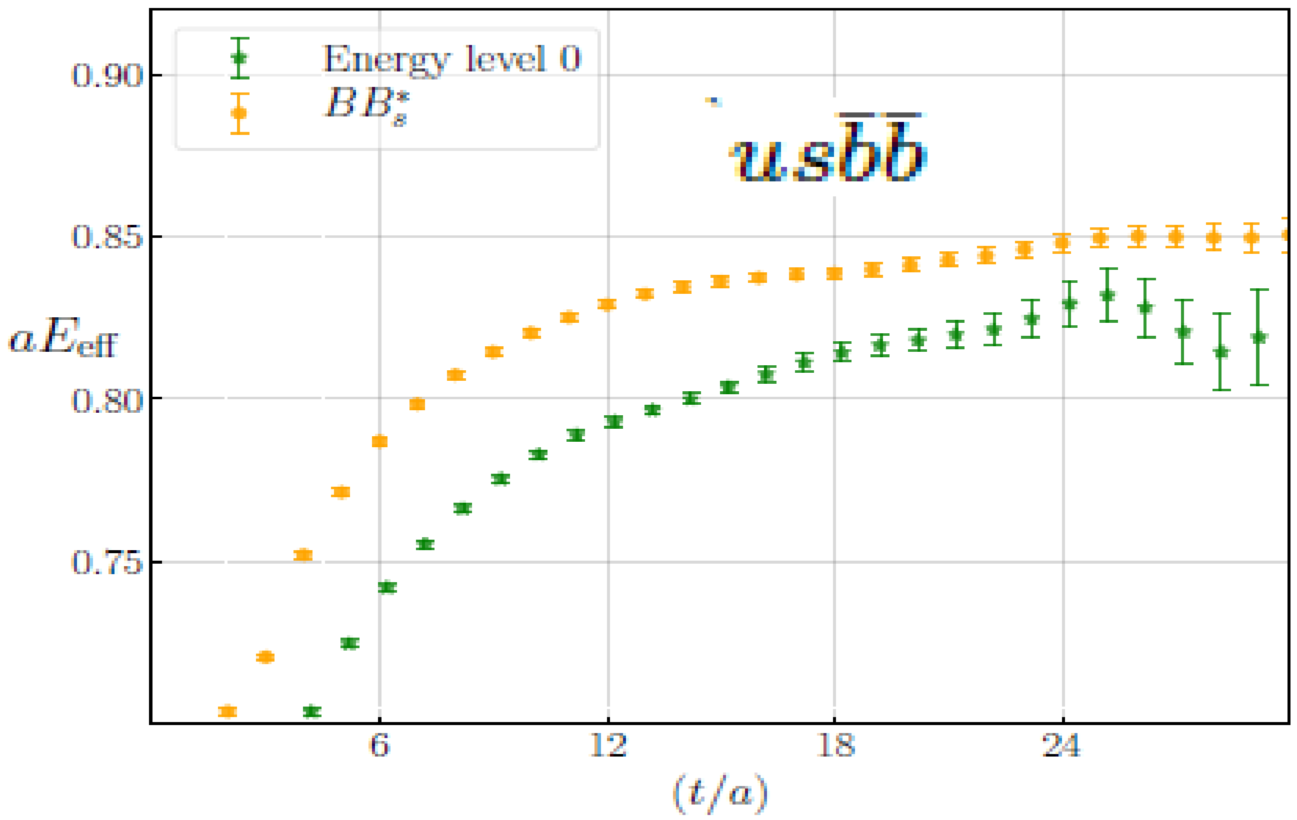
$$\mathcal{M}^1(x) = M_1(x)M_2^*(x) - M_2(x)M_1^*(x)$$
$$M_{1,2}(x) = (l_{1,2})^a_\alpha(x)\ (\gamma_5)_{\alpha\beta}\ \bar{Q}^a_\beta(x)$$
$$M^*_{1,2}(x) = (l_{1,2})^a_\alpha(x)\ (\gamma_i)_{\alpha\beta}\ \bar{Q}^a_\beta(x).$$

$$C_{ij}(t) = \sum_x \langle 0|\mathcal{O}_i(x,t)\mathcal{O}_j^\dagger(0,0)|0\rangle$$

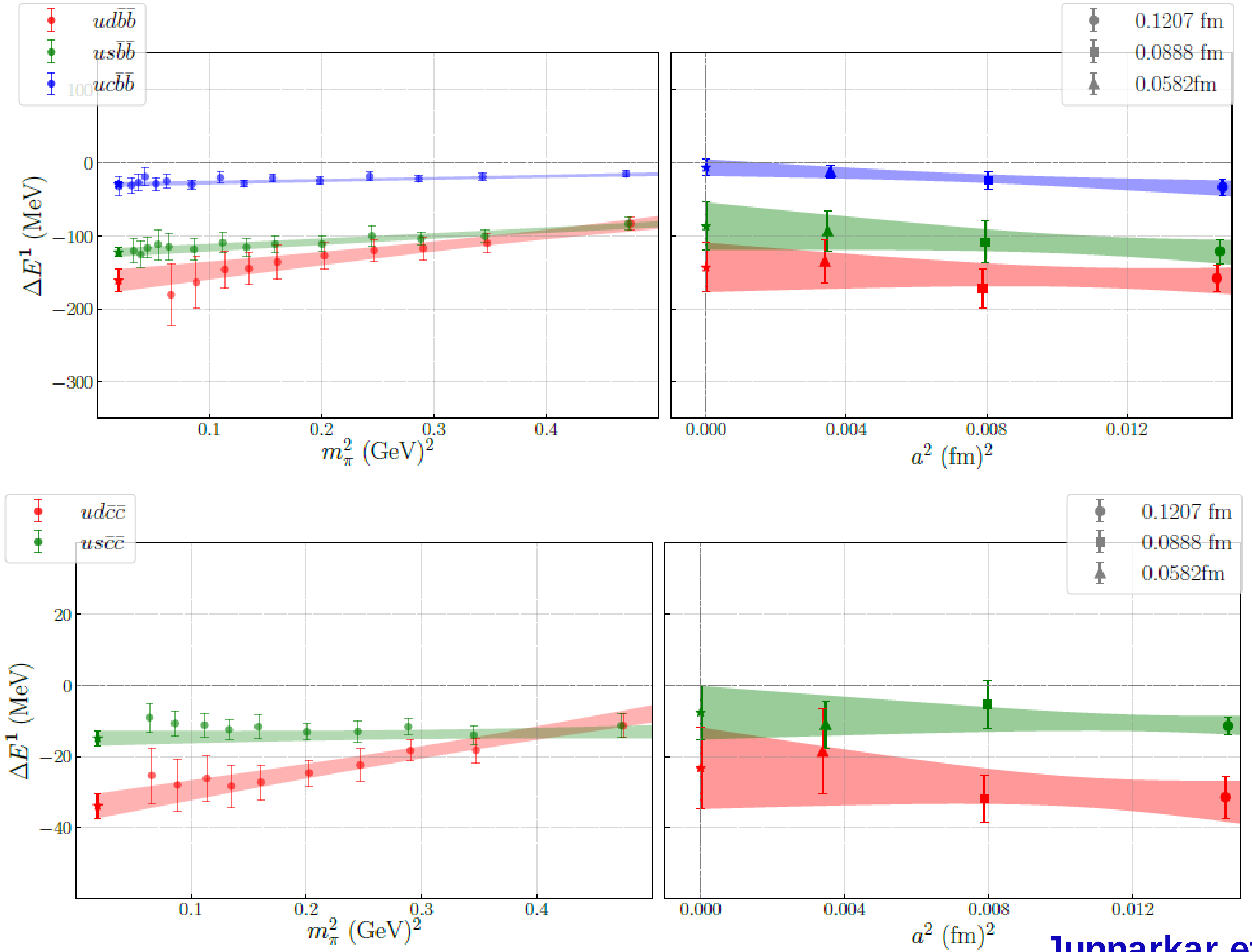
$$\mathcal{O}_i(x,t) \in \{T(x,t), M(x,t)\}$$

Phys. Rev. D99 034507 (2019),

$(l_1 l_2 \bar{Q}\bar{Q})$	$[(M_1 M_2^*)(M_2 M_1^*)]$	I	m_π (MeV)
$ud\bar{b}\bar{b}$	$(BB^{0*})(B^0B^*)$	0	(257 - 688)
$us\bar{b}\bar{b}$	$(BB_s^*)(B_sB^*)$	$\frac{1}{2}$	(186 - 688)
$uc\bar{b}\bar{b}$	$(BB_c^*)(B_cB^*)$	$\frac{1}{2}$	(153 - 688)
$ud\bar{c}\bar{c}$	$(DD^{0*})(D^0D^*)$	0	(257 - 688)
$us\bar{c}\bar{c}$	$(DD_s^*)(D_sD^*)$	$\frac{1}{2}$	(257 - 688)

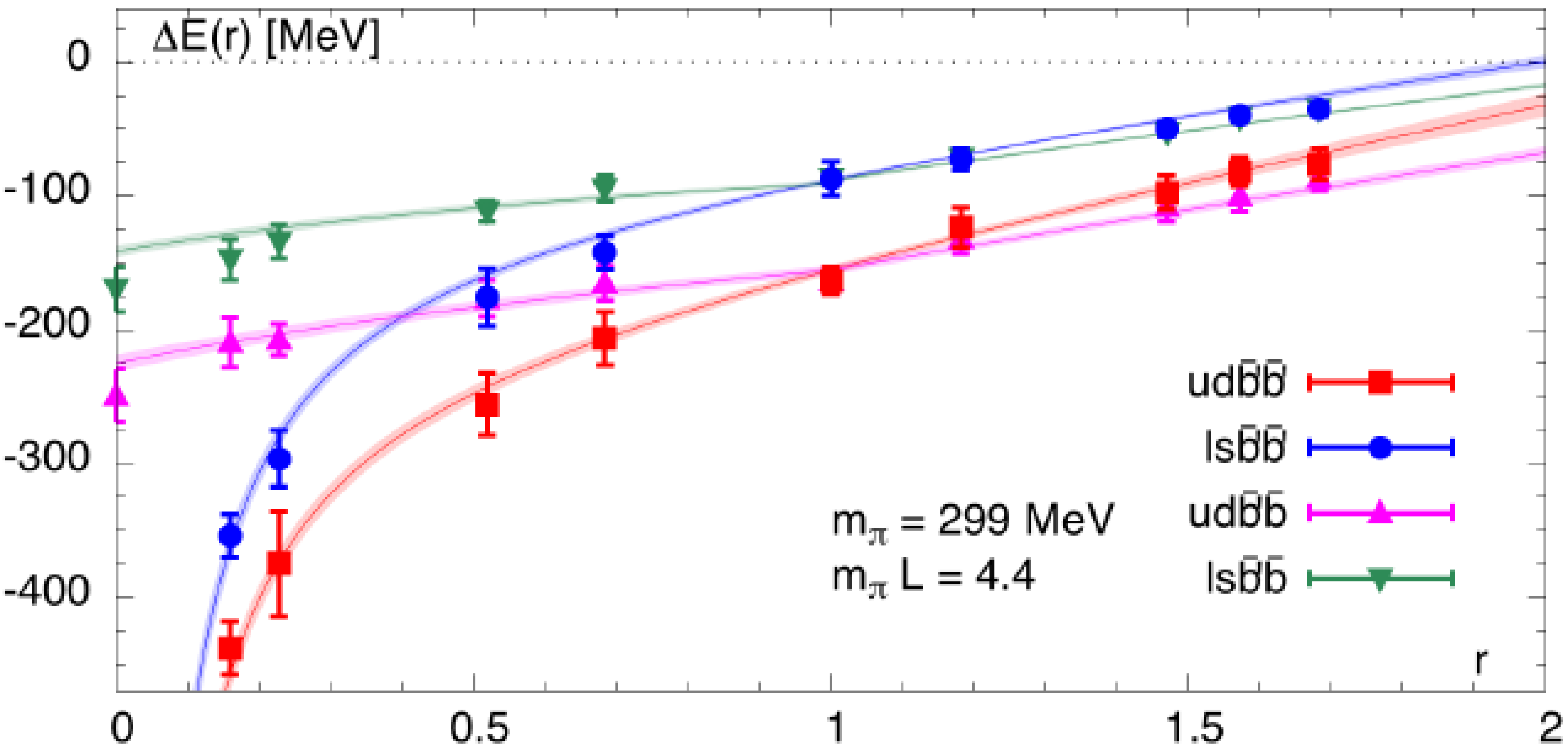


Energy level 0
BB_s^*
$us\bar{b}\bar{b}$
0.90
0.85
0.80
0.75
$aE_{\rm eff}$
6
12
18
24
(t/a)



Junnarkar et al,
Phys. Rev. D99 034507 (2019)

Francis et al. ('18)



- **Heavier the heavy quark masses, deeper the binding**
- **Lighter the light quark masses, deeper the binding**

$J = 0, ll\bar{Q}\bar{Q}$

$$(l,l) \to (6_c, 0, F_S), \qquad (\bar{Q}\bar{Q}) \to (\overline{6}_c, 0, F_s).$$

$$l \in (u, s, c) \qquad h \in (c, b).$$

Tetraquark-type :

$$\mathcal{T}^0(x) = l^a_\alpha(x)(C\gamma_5)_{\alpha\beta} l^b_\beta(x)\ \bar{Q}^b_\kappa(x)(C\gamma_5)_{\kappa\rho}\bar{Q}^a_\rho(x).$$

Two mesons-type :

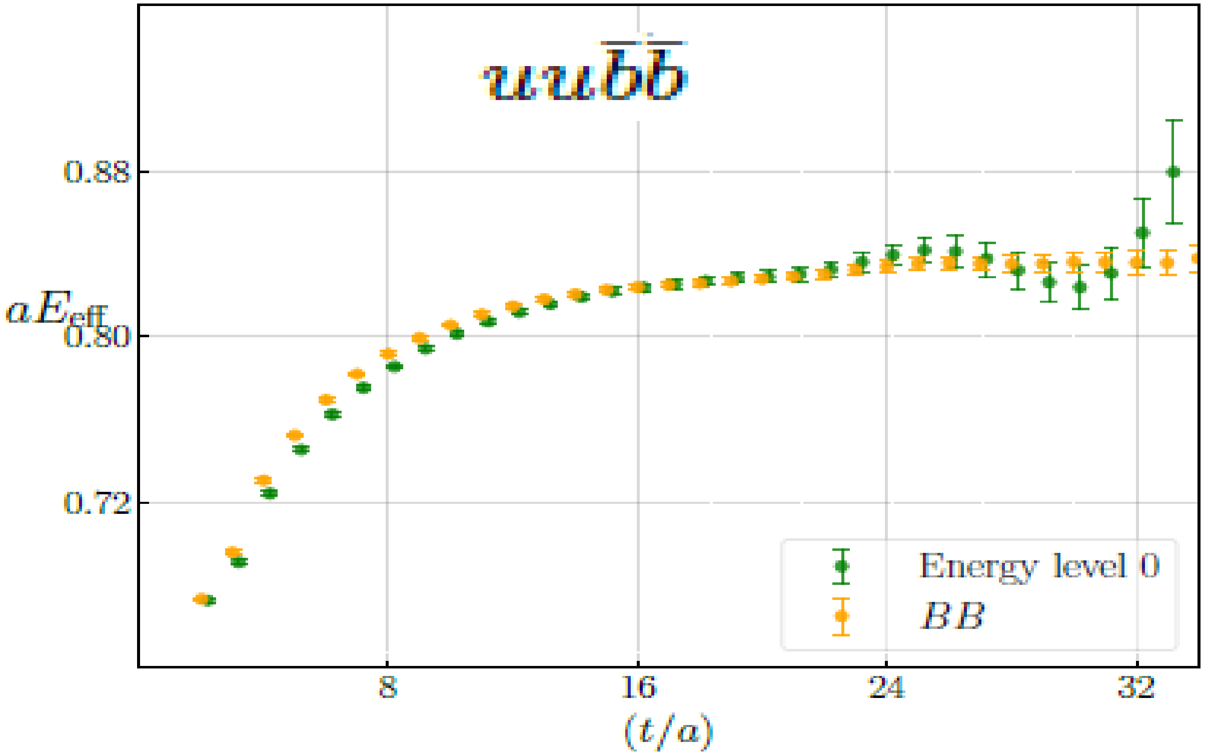
$$\mathcal{M}^0(x) = \bar{Q}^a_\alpha(x)(\gamma_5)_{\alpha\beta} l^a_\beta(x)\ \bar{Q}^b_\kappa(x)(\gamma_5)_{\kappa\rho} l^b_\rho(x).$$

$$C_{ij}(t) = \sum_x \langle 0|\mathcal{O}_i(x,t)\mathcal{O}^\dagger_j(0,0)|0\rangle$$

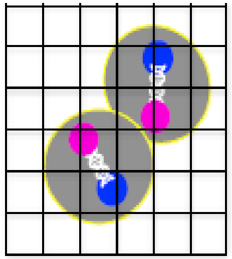
$$\mathcal{O}_i(x,t) \in \{T(x,t), M(x,t)\}$$

$(l_1 l_2 \bar{Q}\bar{Q})$	$(M_1 M_2)$	I	m_π (MeV)
$uu\bar{b}\bar{b}$	(BB)	1	(337 - 688)
$uu\bar{c}\bar{c}$	(DD)	1	(297 - 688)
$ss\bar{b}\bar{b}$	$(B_s B_s)$	0	-
$cc\bar{b}\bar{b}$	$(B_c B_c)$	0	-
$ss\bar{c}\bar{c}$	$(D_s D_s)$	0	-

Phys. Rev. D99 034507 (2019)

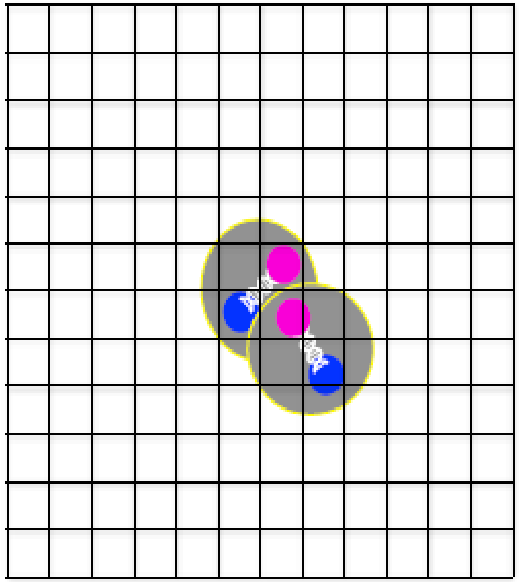


$uu\bar{b}\bar{b}$
0.88
0.80
0.72
$aE_{\rm eff}$
8
16
24
32
(t/a)
Energy level 0
BB



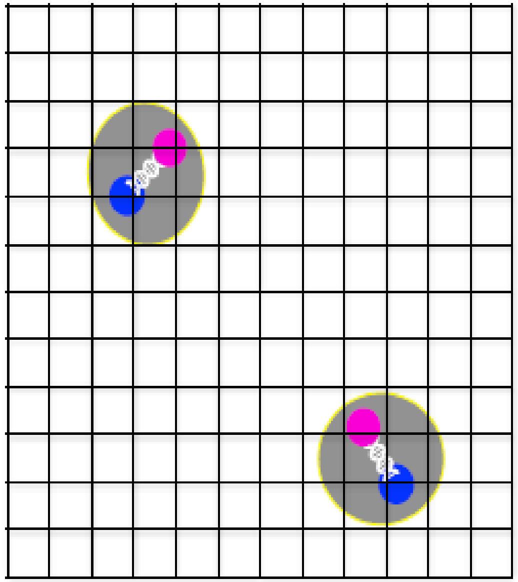
Infinite-volume
bound state

$$[E(L) - E(\infty)] \propto \frac{e^{-\gamma L}}{\gamma L}$$



Infinite-volume
scattering state

$$[E(L) - E(\infty)] \propto \frac{a}{ML^3}$$



Huang, Yang, Phys. Rev. 105 (1957)

Lüscher, Commun. Math. Phys. 105 (1986)

Beane et al, Phys. Lett. B 585 (2004)

…

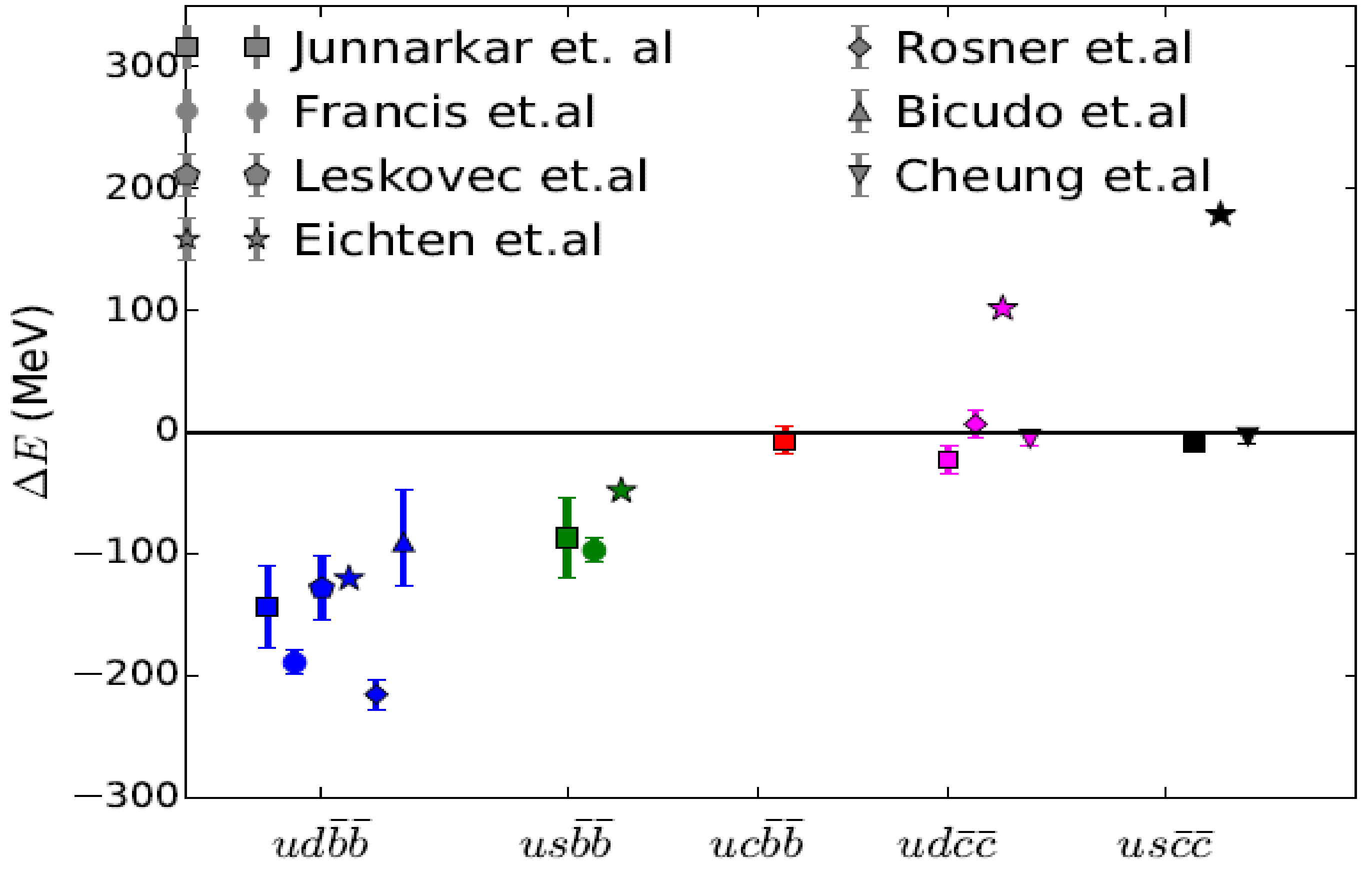
Recent review:

Briceño, Dudek, Young, Rev. Mod. Phys. 90 (2018)

Finite volume effects for heavy hadrons:

$$\Delta_{FV} = E_{FV} - E_{\infty} \propto \mathcal{O}(e^{-k_{\infty}L})/L,$$
$$\text{with} \quad k_{\infty} = \sqrt{(m_1 + m_2)B_{\infty}}\,,$$

Beane et al Phys. Lett. B585 (2004)
Davoudi etal : Phys. Rev. D84 (2011) 114502

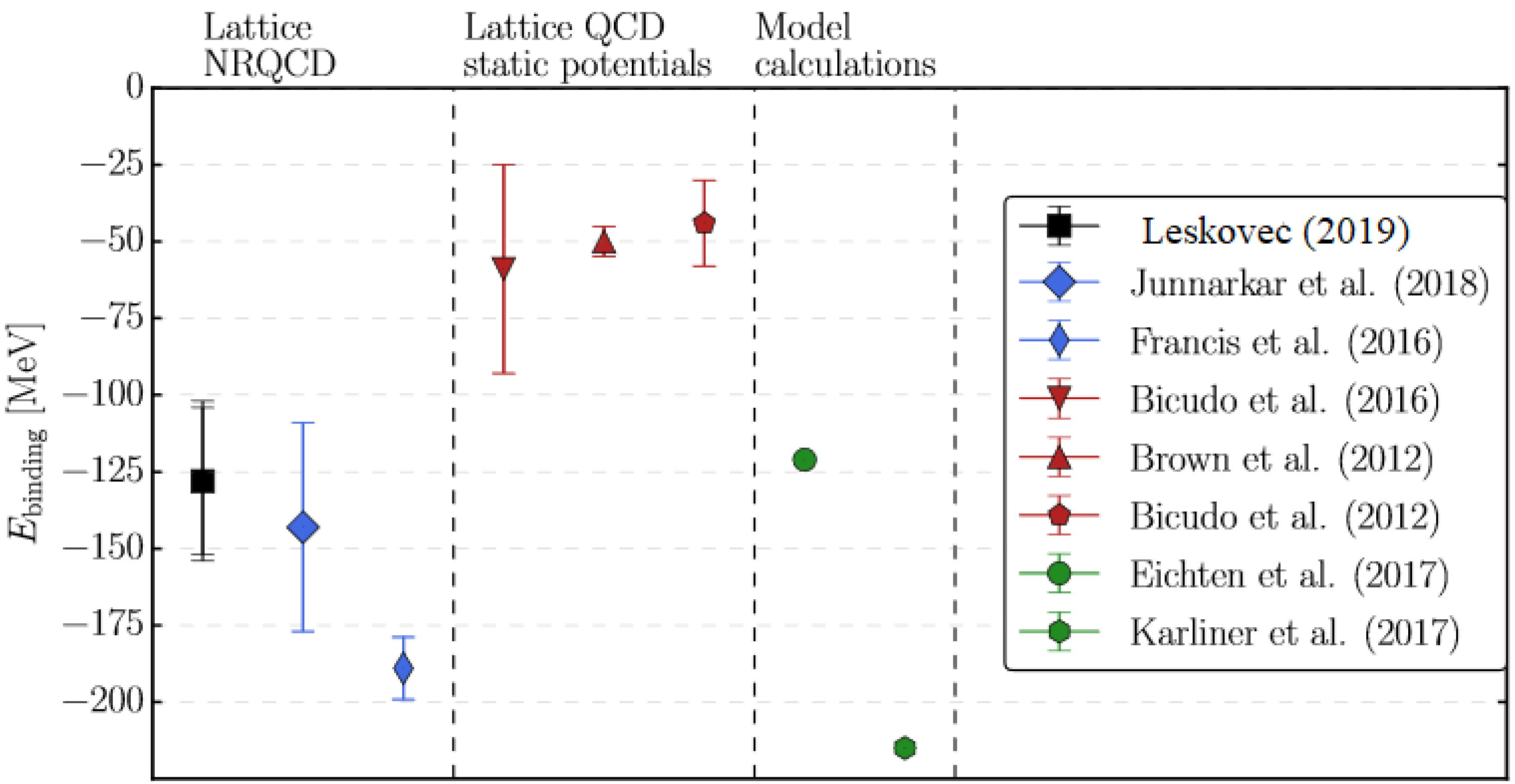


Junnarkar et al, Phys. Rev. D99 034507 (2019)

State	ΔE^1 [MeV]	State	ΔE^1 [MeV]
$ud\bar{b}\bar{b}$	-143(34)	$us\bar{b}\bar{b}$	-87(32)
$uc\bar{b}\bar{b}$	-6(11)	$sc\bar{b}\bar{b}$	-8(3)
$ud\bar{c}\bar{c}$	-23(11)	$us\bar{c}\bar{c}$	-8(8)

State	ΔE^0 [MeV]	State	ΔE^0 [MeV]
$uu\bar{b}\bar{b}$	-5(18)	$uu\bar{c}\bar{c}$	26(11)
$ss\bar{b}\bar{b}$	3(9)	$ss\bar{c}\bar{c}$	14(4)
$cc\bar{b}\bar{b}$	16(1)		

$ud\bar{b}\bar{b}\ (1^+)$



Leskovec et al, Phys. Rev. D 100, 014503 (2019)

$ud\bar{b}\bar{c}\ (1^+)$

$-61\ \mathrm{MeV} < \Delta E_{ud\bar{b}\bar{c}} \sim -15\ \mathrm{MeV}$

$\bar{D}B^*$threshold

A. Francis et al:

Phys. Rev. D99 (2019) no.5, 054505

No state below thresholds: Hudspith et al, Physical Review D102, 114506 (2020)

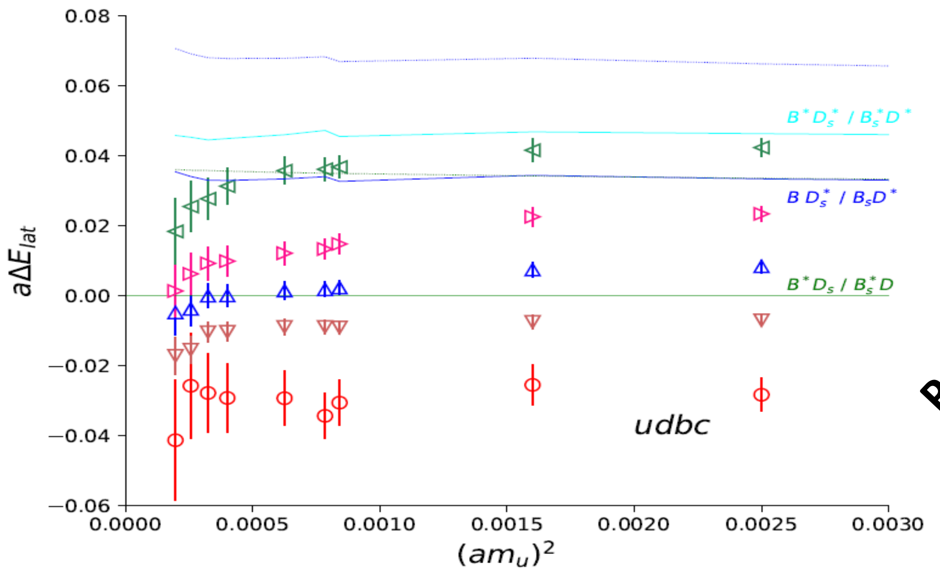
$ud\bar{b}\bar{c}\ (1^{+})$

New:

- ❑ **We use more number of operators including** diquark-antidiquark type operators with the light diquark having a spin 1 keeping in mind that the charm quark is much lighter compared to the bottom quark.

❑ Preliminary Findings:

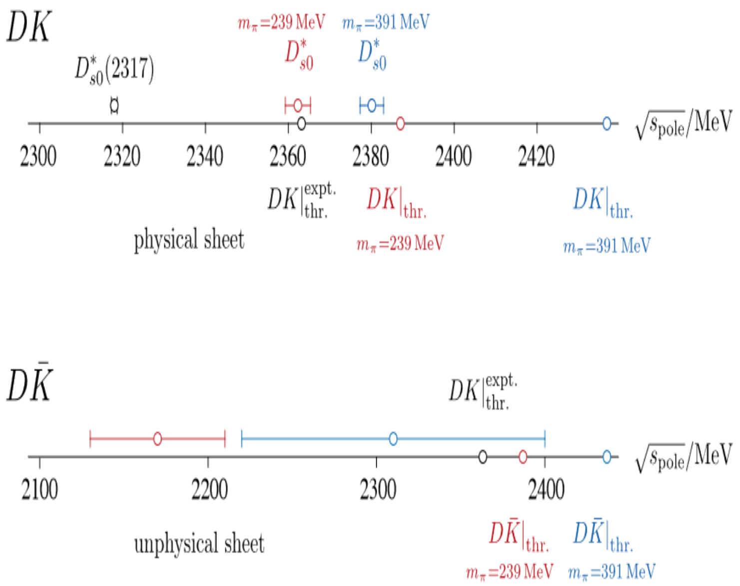
- **Multiple energy levels around elastic threshold.**
- **At least one energy level below two-meson thresholds.**



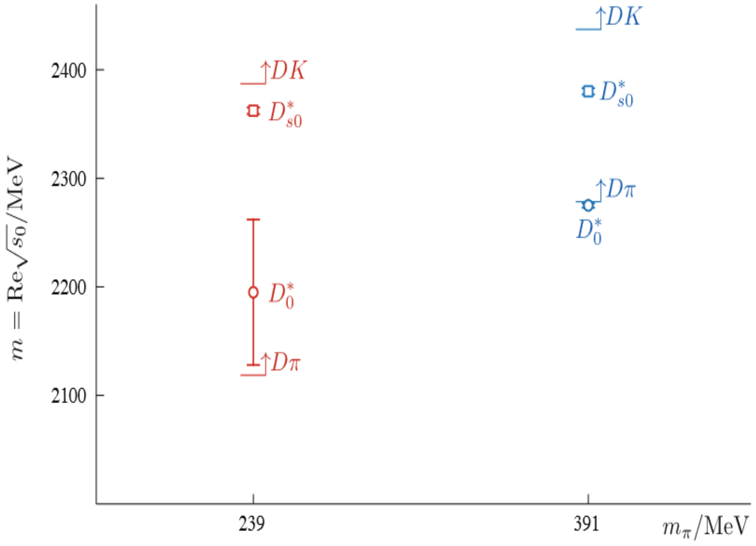
Preliminary

More work in progress on to cross-check this finding with more statistics and another volume ---stay tuned

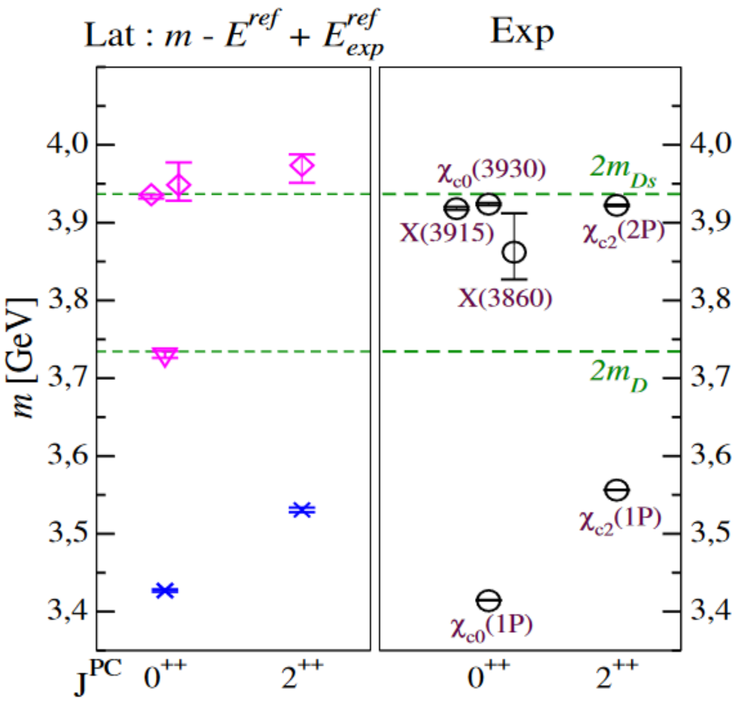
Recent calculations on charmed resonances



Hadspec@JHEP 02 (2021) 100



Hadspec@arXiv:2102.04973



Prelovsek et al: arXiv:2011.02542

Dibaryons

- Deuteron is the only dibaryon that we know as a bound state.
- A resonance peak is observed ($d^*(2380)$) in polarized proton-neutron scattering

M. Bashkanov *et al.*
(CELSIUS/WASA Collaboration)
Phys. Rev. Lett. 102, 052301 (2009)
Phys. Rev. Lett. 124, 132001 (2020)
P. Adlarson et al
Phys. Rev. Lett. 112, 202301 (2014)

- Experimentally a few hypernuclei with shallow bindings have also been observed

J-PARC E07 Collaboration: Phys. Rev. Lett. 126, 062501
J. K. Ahnet al., Phys. Rev. C88, 014003 (2013).
S. Acharya et al., Phys. Rev. Lett.123, 112002 (2019).
ALICE Collaboration, Nature, 588, 232 (2020).

- Multi-baryon states, including hypernuclei have been investigated by lattice collaborations:
NPLQCD and particularly, HALQCD
- How about dibaryons with heavy quarks?
    - Predicted many by potential models

d^*(2380)

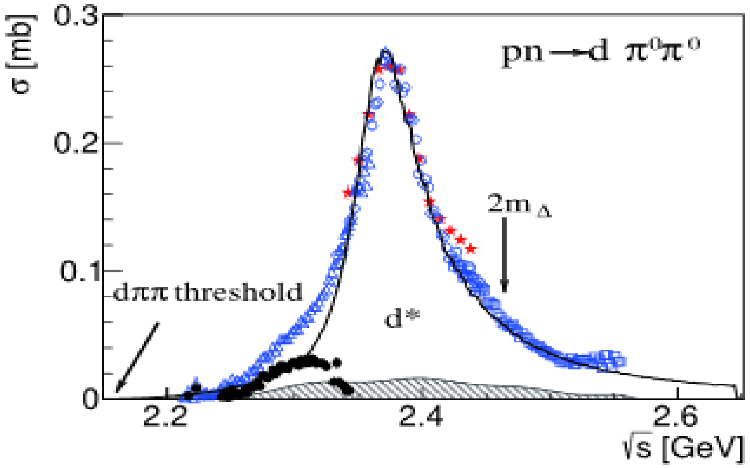
Resonance peak in polarized proton-neutron scattering

$$pn \rightarrow d\pi^0\pi^0$$
$$pn \rightarrow d\pi^+\pi^-$$

A resonance structure
at M ≈ 2380 MeV with a width of Γ ≈ 70 MeV
$I(J^P) = 0(3^+)$

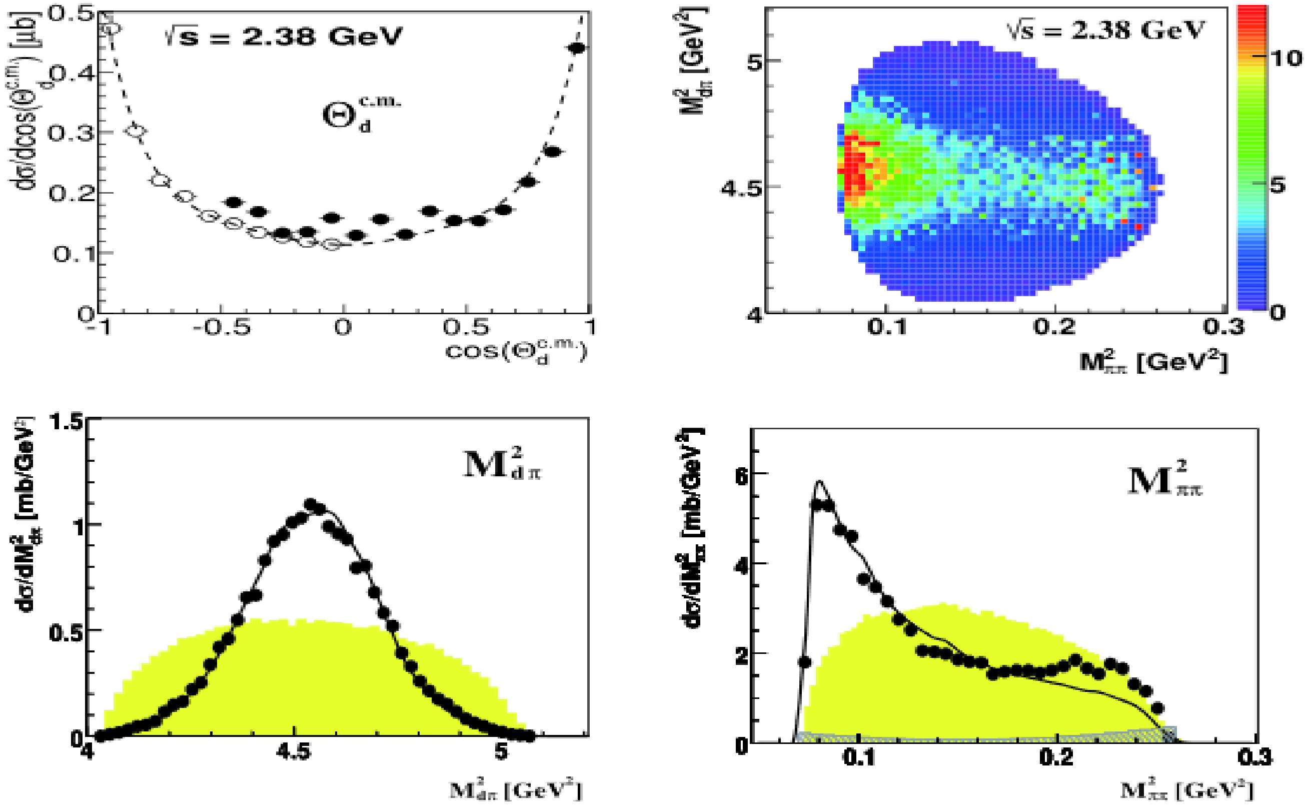
M. Bashkanov *et al.*
(CELSIUS/WASA Collaboration)
Phys. Rev. Lett. 102, 052301 (2009)
Phys. Rev. Lett. 124, 132001 (2020)
P. Adlarson et alPhys. Rev. Lett. 112, 202301 (2014)

A six quark state made of 3 *u* and 3 *d* quarks (*uuddud*)



M. Bashkanov *et al.*
(CELSIUS/WASA Collaboration)
Phys. Rev. Lett. 102, 052301 (2009)
Phys. Rev. Lett. 124, 132001 (2020)

J. Clement et al,
Chinese Physics C Vol. 45, No. 2 (2021) 02200

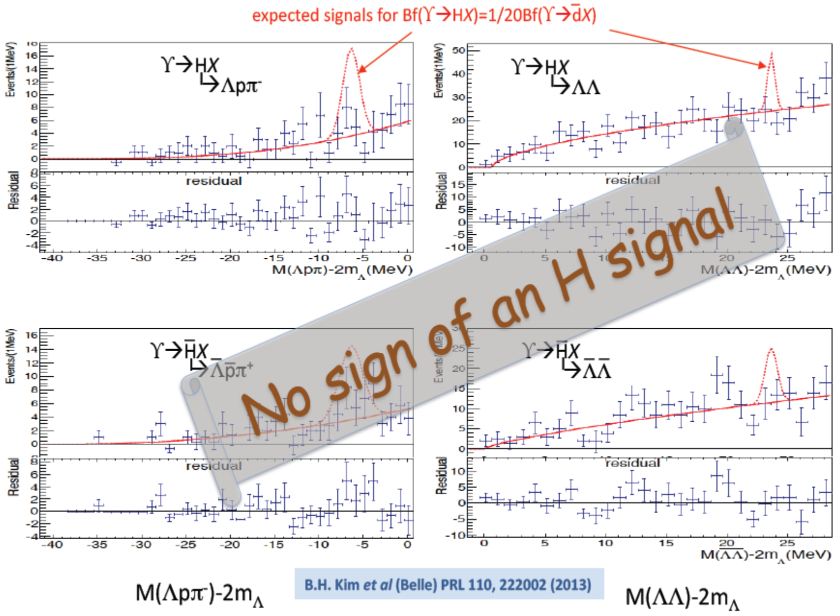


H Dibaryon

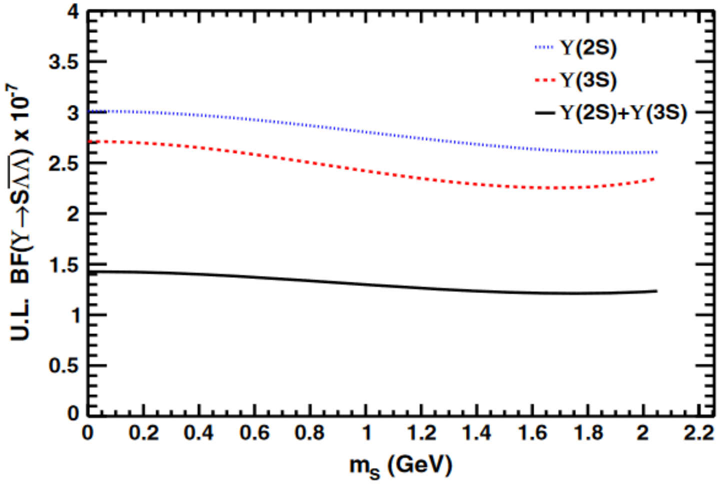
Bound state of two Λ $\Lambda\Lambda\ (udssud)$ Proposed by Jaffe (1976)

- Has to be below the two proton threshold. Then it will be bound
- If it exists it is extremely stable and could be a candidate for SM dark matter?
  May not be as oxygen may not exist with that!

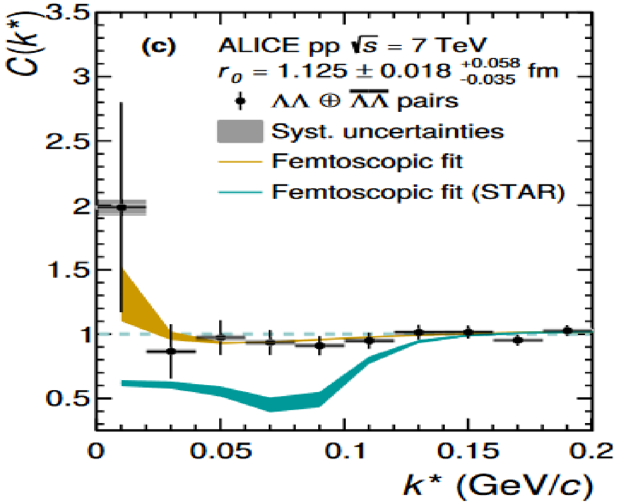
No H dibaryon



Belle: PRL 110,222002(2013)

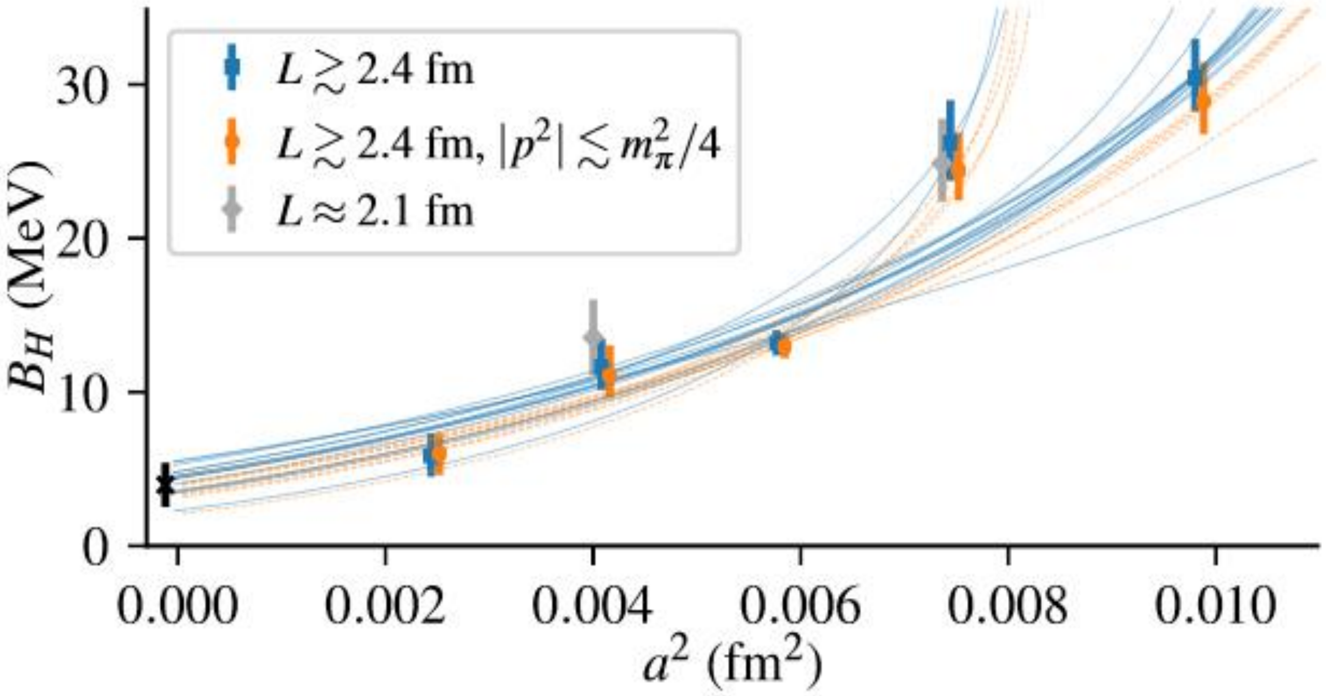


BABAR Collaboration:
Phys. Rev. Lett. 122, 072002 (2019)
No signal is observed
in Υ **decays** (90% confidence limit)



Alice: Phys. Rev. C 99, 024001 (2019)

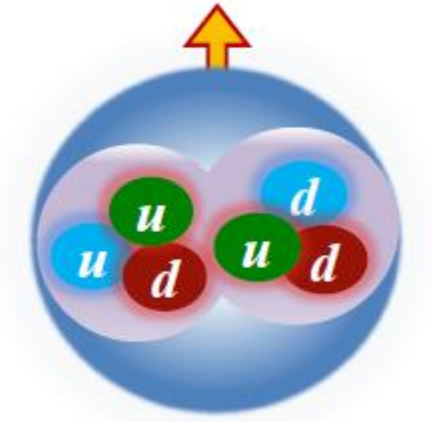
H-dibaryon at SU(3) symmetric point



$$B_H = 3.97 \pm 1.16 \pm 0.86 \text{ MeV}$$

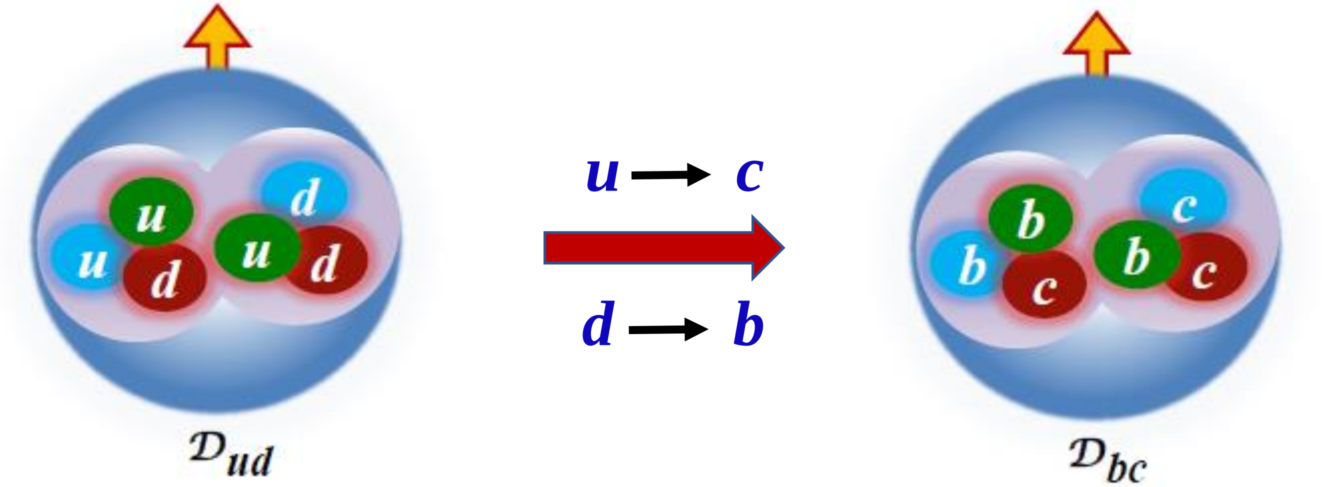
Green et al : arXiv: 2103.01054

Deuteron-like heavy dibaryons

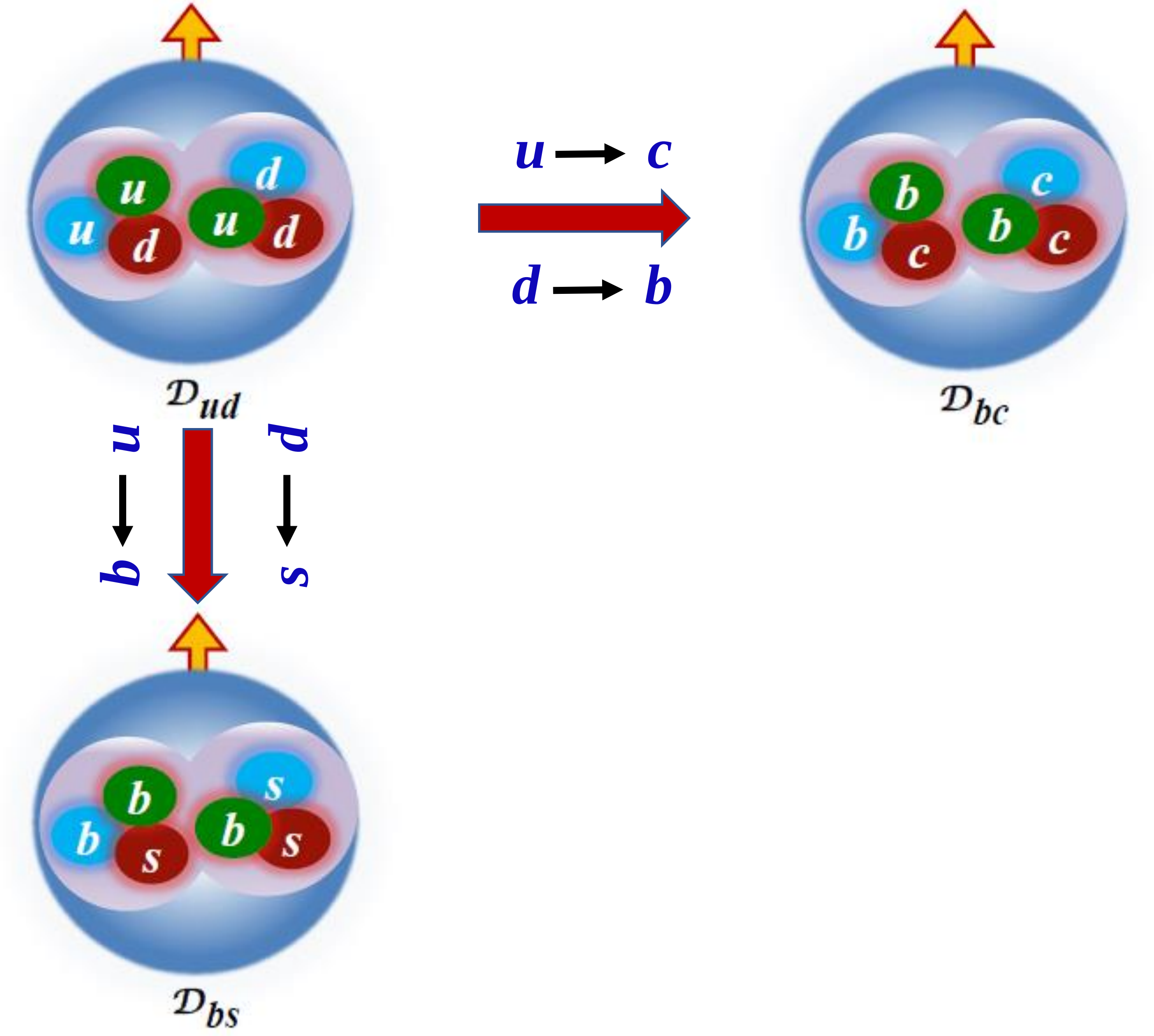


$\mathcal{D}_{ud}$

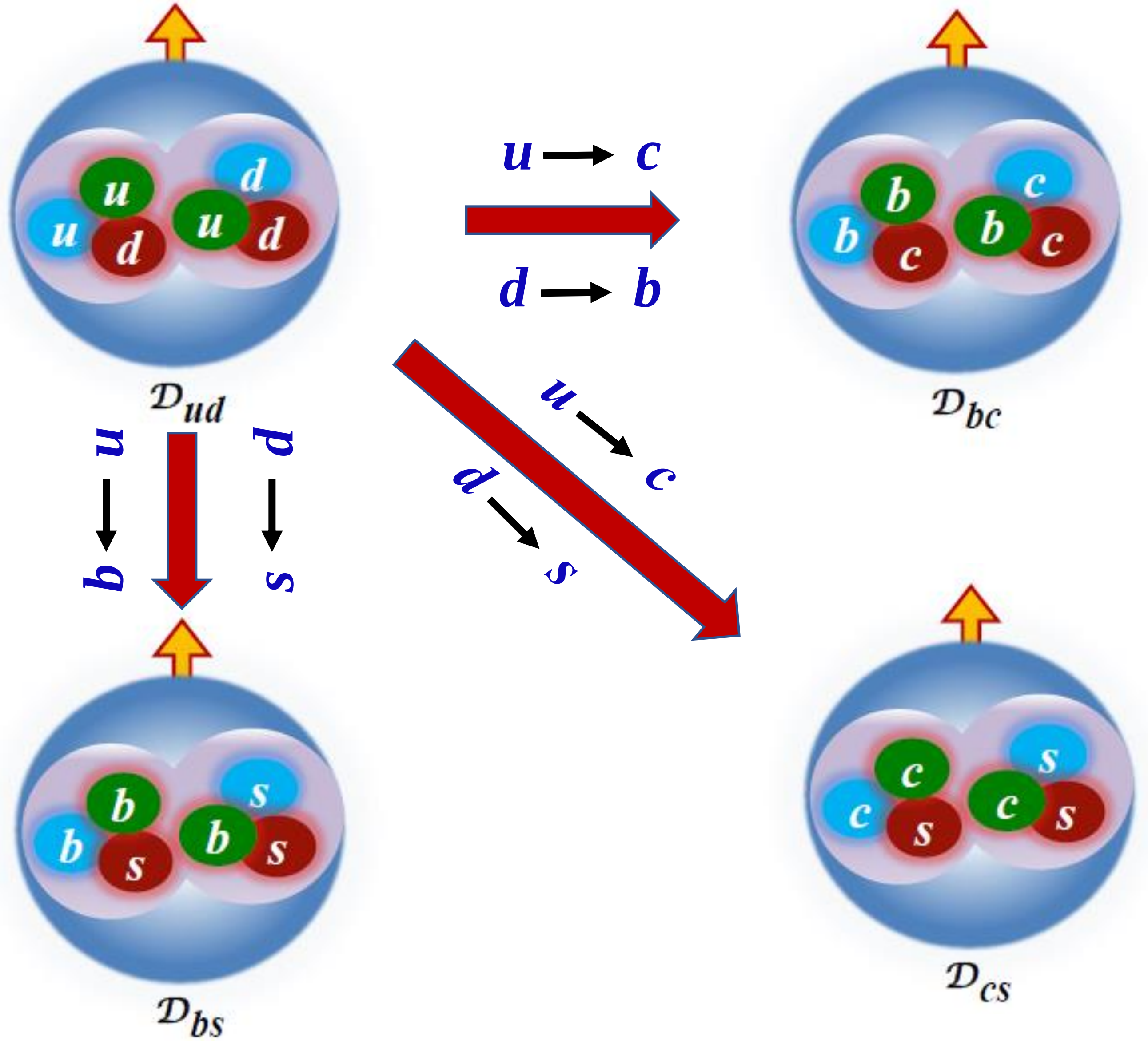
Deuteron-like heavy dibaryons



Deuteron-like heavy dibaryons

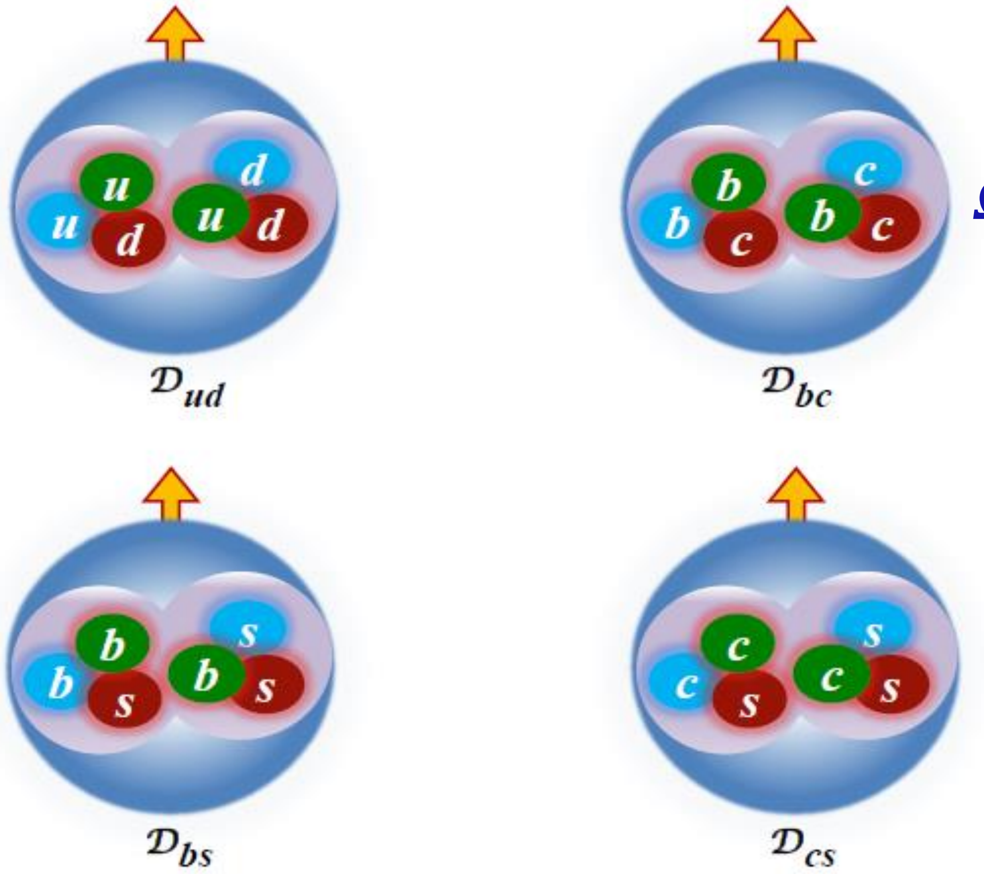


Deuteron-like heavy dibaryons



Deuteron-like heavy dibaryons

np $\Omega_{ccb}\Omega_{cbb}$



$\Omega_b\Omega_{bb}$ $\Omega_c\Omega_{cc}$

Junnarkar and NM : Phys. Rev. Lett. 123, 162003(2019)

Strong-decay thresholds

Two thresholds :
Spin-1/2 baryons as well as spin-3/2 baryons

Charm

$$M^{\frac{1}{2}}_{\Sigma_c} + M^{\frac{1}{2}}_{\Xi_{cc}} < M^{\frac{3}{2}}_{\Delta_u} + M^{\frac{3}{2}}_{\Omega_{ccc}}$$

$$M^{\frac{1}{2}}_{\Omega_c} + M^{\frac{1}{2}}_{\Omega_{cc}} < M^{\frac{3}{2}}_{\Omega_s} + M^{\frac{3}{2}}_{\Omega_{ccc}}$$

Bottom

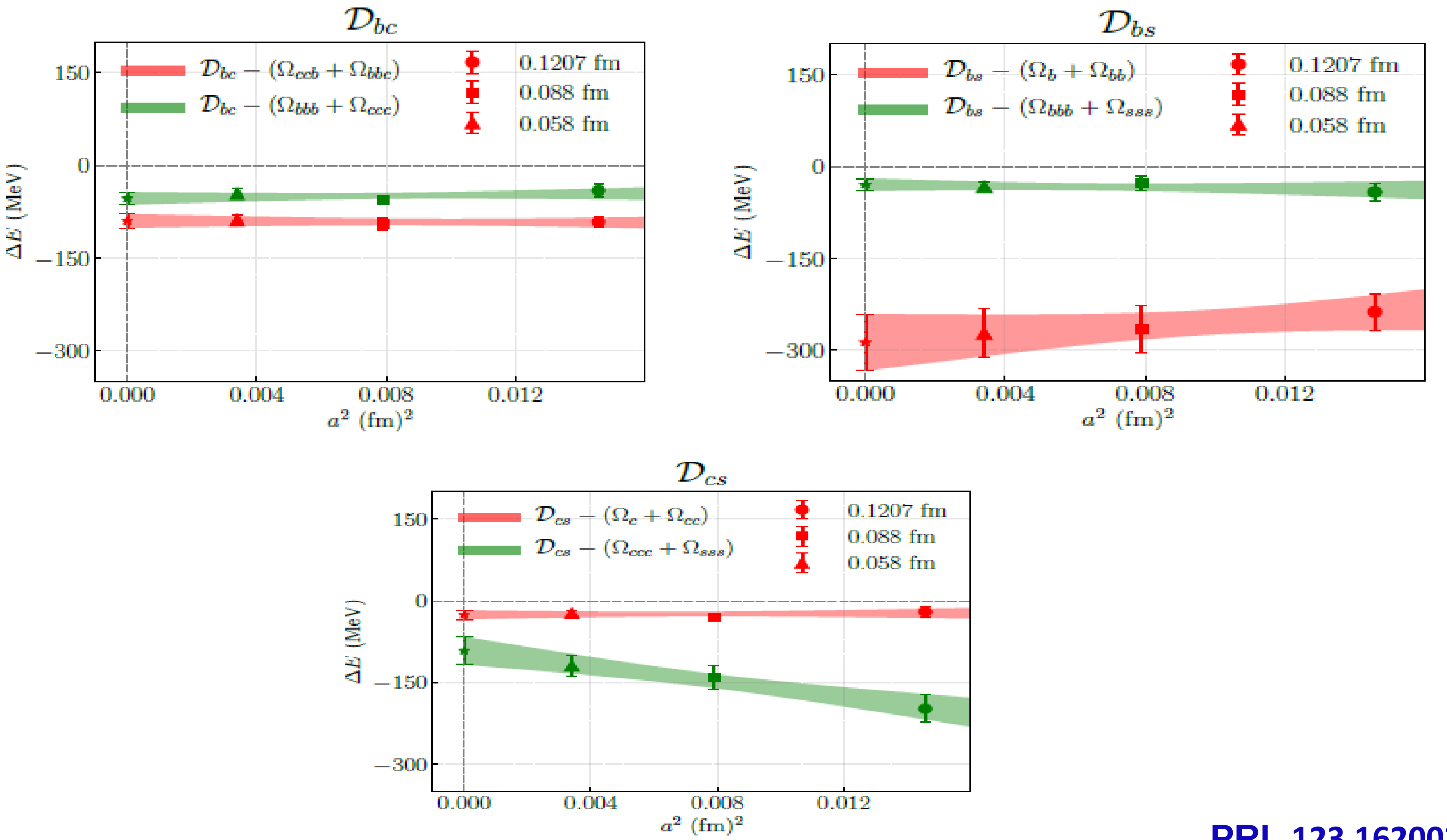
$$M^{\frac{1}{2}}_{\Sigma_b} + M^{\frac{1}{2}}_{\Xi_{bb}} > M^{\frac{3}{2}}_{\Delta_u} + M^{\frac{3}{2}}_{\Omega_{bbb}}$$

$$M^{\frac{1}{2}}_{\Omega_b} + M^{\frac{1}{2}}_{\Omega_{bb}} > M^{\frac{3}{2}}_{\Omega_s} + M^{\frac{3}{2}}_{\Omega_{bbb}}$$

$$M^{\frac{1}{2}}_{\Omega_{ccb}} + M^{\frac{1}{2}}_{\Omega_{cbb}} > M^{\frac{3}{2}}_{\Omega_{ccc}} + M^{\frac{3}{2}}_{\Omega_{bbb}}$$

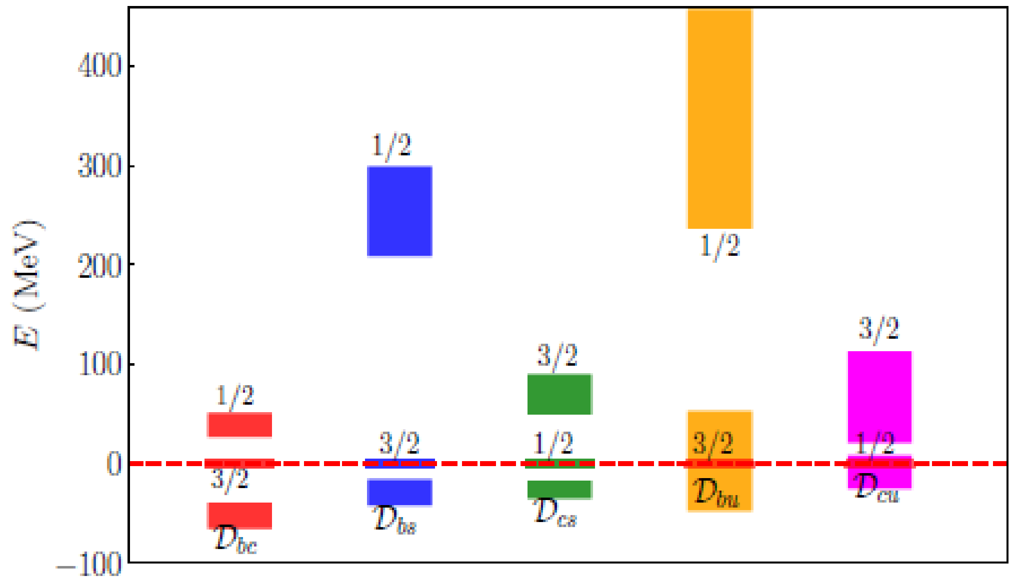
PDG
Phys. Rev. Lett. 121 (2018) no.20, 202002, N. Mathur et al
Phys. Rev. D99, 031501 (2019), N. Mathur et al
Phys. Rev. D90 (2014) no.9, 094507, Z. Brown et al

Deuteron-like heavy dibaryons



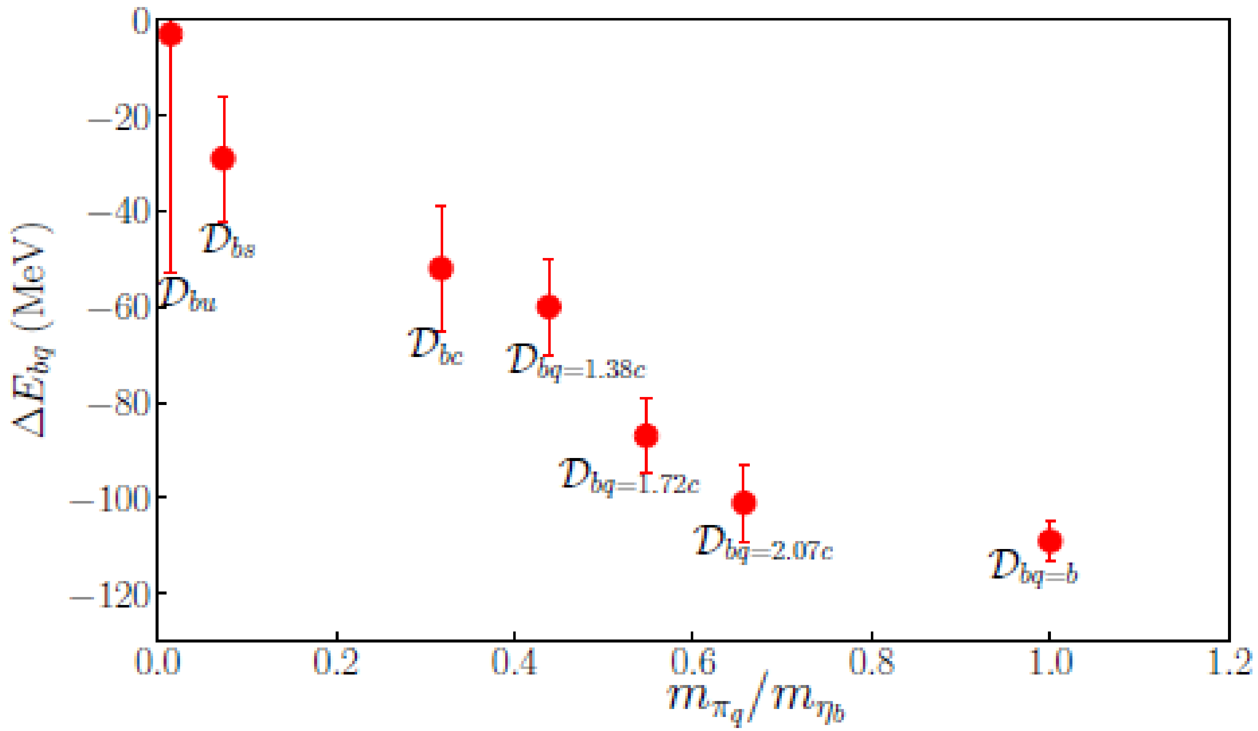
PRL 123,162003 (2019)

Results



Dibaryon	Energy difference from spin-1/2 threshold [MeV]	Energy difference from spin-3/2 threshold [MeV]	Mass [MeV]
$\mathcal{D}_{bc}$	−91(12)	−52(13)	19105(21)
$\mathcal{D}_{bs}$	−287(45)	−29(13)	16004(17)
$\mathcal{D}_{cs}$	−26(9)	−90(20)	6381(20)
$\mathcal{D}_{bu}$	−350(110)	3(50)	
$\mathcal{D}_{cu}$	−8(17)	−75(46)	
$\mathcal{D}_{bq(m_q=1.38m_c)}$		−60(10)	
$\mathcal{D}_{bq(m_q=1.72m_c)}$		−87(8)	
$\mathcal{D}_{bq(m_q=2.07m_c)}$		−101(8)	
$\mathcal{D}_{bq(m_q=m_b)}$		−109(5)	
$\mathcal{D}_{bq(m_q=2m_b)}$		−85(10)	
$\mathcal{D}_{bq(m_q=5m_b)}$		−15(8)	

PRL 123,162003 (2019)



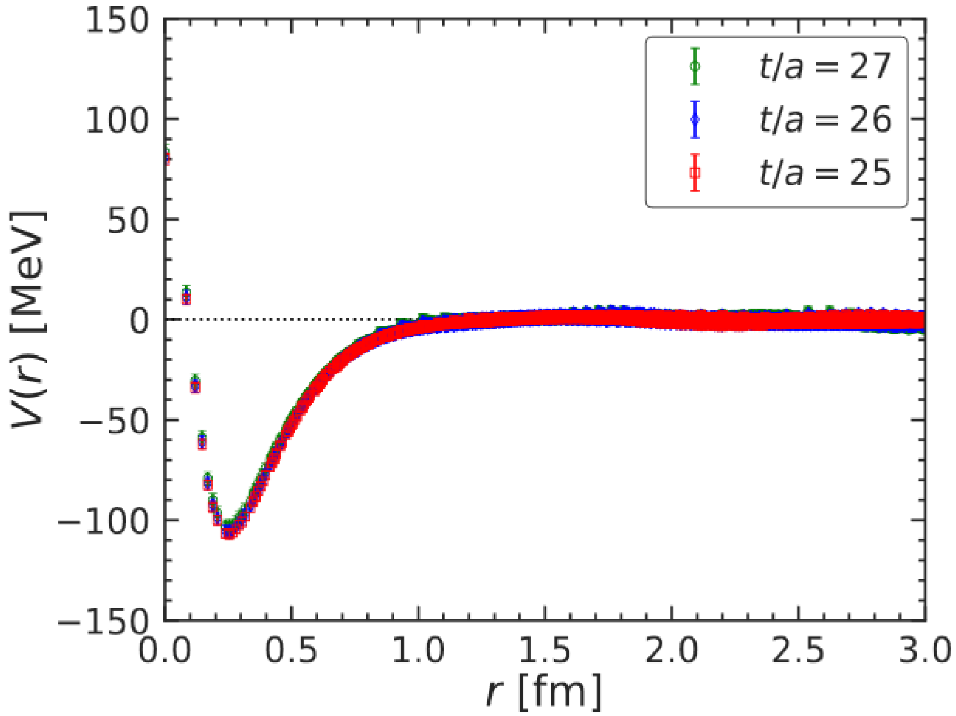
- **Heavier the heavy quark mass, deeper the binding**
- **Heavier the light quark mass up to a heavy mass, deeper the binding**

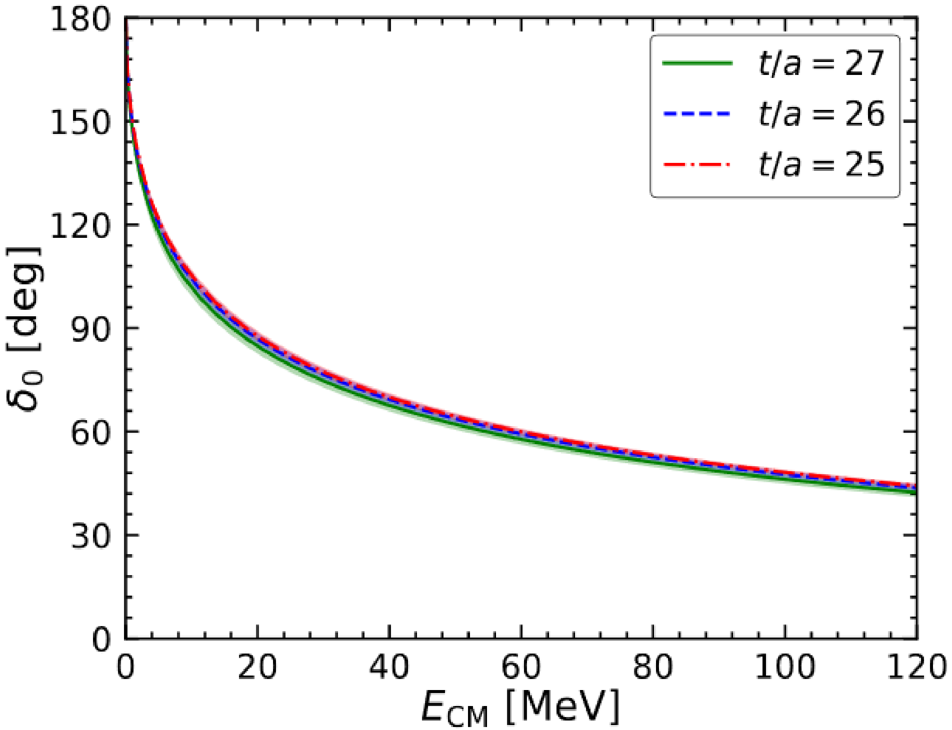
PRL 123,162003 (2019)

Three flavor heavy dibaryons (like H-dibaryon)

New: Preliminary results from similar calculations on three flavor heavy dibaryons (like H-dibaryon) indicates that extra energy levels exist below the respective thresholds **only when quark masses are heavy.**

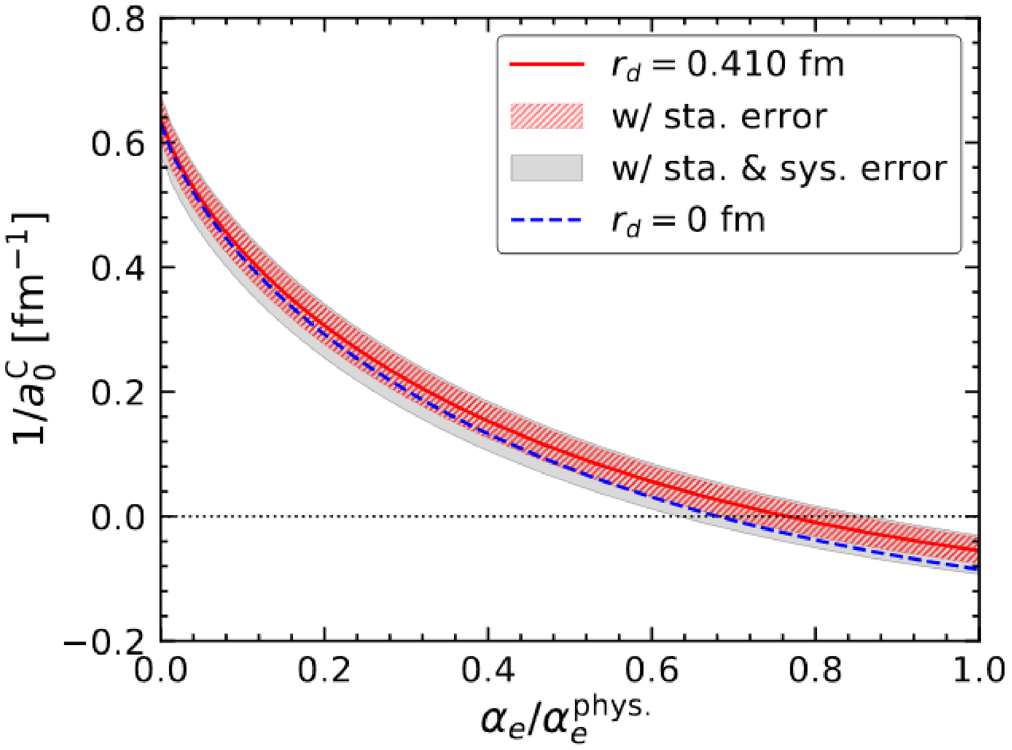
$\Omega_{ccc}\,\Omega_{ccc}(^1S_0)$

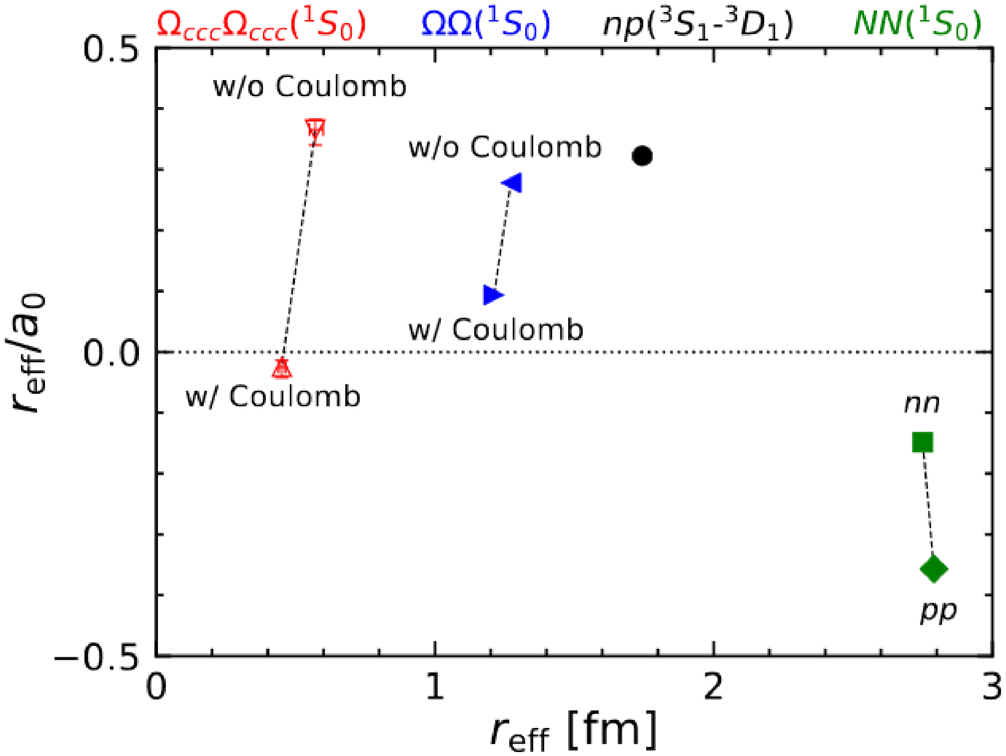




arXiv: HALQCD@2102.00181

$\Omega_{ccc}\Omega_{ccc}(^1S_0)$





arXiv: HALQCD@2102.00181

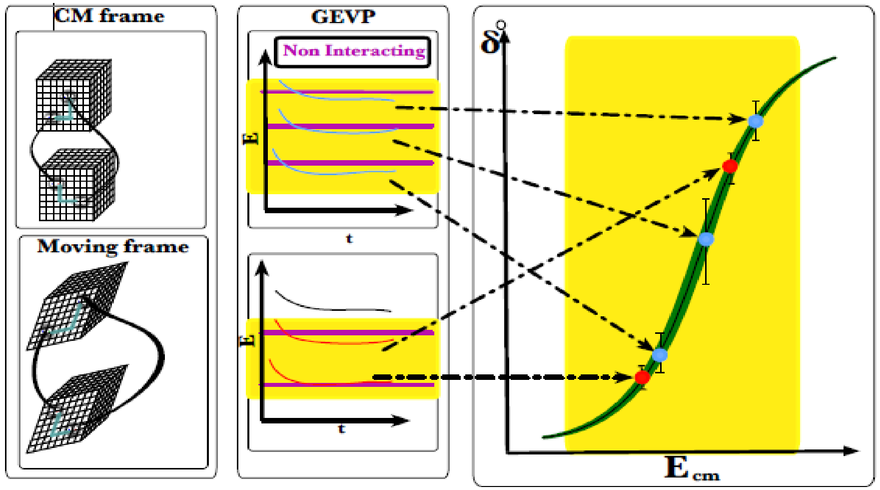
Recent potential model calculations

- No binding for *bbbccc* and similar configurations …J-M. Richard et al, PRL 124,212001 (2020)
- Many triply heavy dibaryons are found as a bound states …Pan et al, PRD 102,054025 (2020)
- Bound $\Omega_{ccc}\Omega_{ccc}$ and $\Omega_{bbb}\Omega_{bbb}$ dibaryons …Huang et al 2011.00513

What to do next? When can we be sure?

- Do a volume study to see the dependence of energy levels on volumes. (quick fix, lattice groups are working on this).

- Employ Luscher method with multi operators and multi-volume: phase (lengthy and computationally intensive procedure).



- Do a calculation with relativistic bottom quarks just to make sure no problem with NRQCD (working on this).

- Calculate form factors and/or distribution functions to find out the structure of these states (lengthy procedure, may need large statistics).

Conclusions and Outlooks

- There is a resurgence of interest in the study of bound states with heavy quark(s).
- Exotic states with heavy quarks have been observed having four and five quark configurations.
- Lattice QCD calculations are playing a crucial role in studying these states.
- Multiple lattice QCD studies suggest the existence of a deeply bound doubly-bottom four-quark state. More works are needed to firmly confirm this. Lattice groups are also studying on other heavy four-quark states.
- LQCD study also predicts the existence of two-flavoured spin-1 bound heavy dibaryons.
- Is it possible to form nuclei with heavy quarks? Lattice QCD can study this but will be computationally expensive.
- Will look forward to new discoveries.

Deuteron-like heavy dibaryons

$\mathcal{D}_{Qq}$	Interpolating fields
$\mathcal{D}_{bc}$	$\frac{1}{\sqrt{2}}\left(\Omega_{ccb}\Omega_{bbc} - \Omega_{bbc}\Omega_{ccb}\right)$
$\mathcal{D}_{bs}$	$\frac{1}{\sqrt{2}}\left(\Omega_{b}\Omega_{bb} - \Omega_{bb}\Omega_{b}\right)$
$\mathcal{D}_{cs}$	$\frac{1}{\sqrt{2}}\left(\Omega_{c}\Omega_{cc} - \Omega_{cc}\Omega_{c}\right)$
$\mathcal{D}_{bu}$	$\frac{1}{\sqrt{2}}\left(\Sigma_{b}\Xi_{bb} - \Xi_{bb}\Sigma_{b}\right)$
$\mathcal{D}_{cu}$	$\frac{1}{\sqrt{2}}\left(\Sigma_{c}\Xi_{cc} - \Xi_{cc}\Sigma_{c}\right)$

$$\mathcal{D}_{qQ} = \frac{1}{\sqrt{2}}\left(\Omega_{qqQ}(C\gamma^j)\Omega_{QQq} - \Omega_{QQq}(C\gamma^j)\Omega_{qqQ}\right)$$