# Nonrelativistic effective field theories for (exotic) double heavy hadrons

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*based on:* Phys.Rev.D 102 (2020) n°1, 014013 **arXiv:2005.00551**
*and* Phys.Rev.D 102 (2020) n°1, 014012 **arXiv:2005.00552**

**Experimental and theoretical status of and perspectives for XYZ states**, April 12th 2021.

Institut de Física
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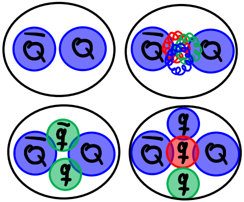
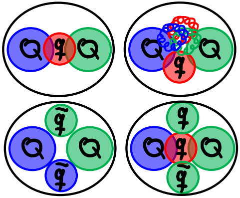
# Introduction and Motivation

- A large number of exotic quarkonium states have been discovered in the past 20 years.
- Many new structures with a heavy quark antiquark pair have been proposed.
- All double heavy hadrons have some common characteristics.





- $Q\bar{Q}+$ light quarks and gluons.
  - Heavy quarkonium.
  - Heavy hybrids $Qg\bar{Q}$.
  - Tetraquaks $Q\bar{Q}q\bar{q}$.
  - Pentaquaks $Q\bar{Q}qqq$.
  - ...
- $QQ+$ light quarks and gluons.
  - Double heavy baryons $QQq$.
  - Hybrids baryons $QQgq$.
  - Tetraquaks $QQ\bar{q}\bar{q}$.
  - Pentaquaks $QQqq\bar{q}$.
  - ...
- We propose an **unified EFT framework** to describe any **double heavy hadron**.

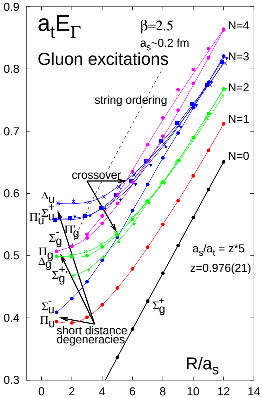
# Static Limit

- The natural starting point when studying hadrons with heavy quarks is NRQCD.
- At leading order in NRQCD the heavy quarks are static.
- In this limit the spectrum of the theory is formed by the static energies

$$E_{\kappa^p\Lambda_\eta}(\boldsymbol{r}) = \lim_{t\to\infty}\frac{i}{t}\ln\langle 0|\mathrm{Tr}\left[\mathcal{P}_{\kappa\Lambda}\mathcal{O}_{\kappa^p}(t/2,\,\boldsymbol{r},\,\boldsymbol{R})\mathcal{O}^{\dagger}_{\kappa^p}(-t/2,\,\boldsymbol{r},\,\boldsymbol{R})\right]|0\rangle$$

  - The heavy quark pair distance $r$.
  - Light-degrees of freedom quantum numbers encoded in $\mathcal{O}_{\kappa^p}$ : spin $\kappa$, parity $p$, flavor...
  - Representation $\Lambda_\eta^\sigma$ of the cylindrical symmetry group $D_{\infty h}$.
  - $\mathcal{P}_{\kappa\Lambda}$ projects the $\kappa^p$ representation of the light degrees of freedom into $\Lambda_\eta^\sigma$ representation.
- Static energies are nonperturbative and should be computed on the lattice.

# Static Limit: Hybrids




- **Quenched** lattice NRQCD.

Juge, Kuti, Morningstar Phys.Rev.Lett.90 (2003)

Recent precision computation Capitani et al Phys.Rev.D 99 (2019)

| $O(3)$ | $D_{\infty h}$ |
|---|---|
| $1^{+-}$ | $\Sigma_u^-$, $\Pi_u$ |
| $1^{--}$ | $\Sigma_g^{+\prime}$, $\Pi_g$ |
| $2^{+-}$ | $\Sigma_u^+$, $\Pi_u'$, $\Delta_u$ |
| $2^{--}$ | $\Sigma_g^-$, $\Pi_g'$, $\Delta_g$ |

$$\mathcal{P}_{10} = \mathbb{1}_3 - (\hat{\boldsymbol{r}} \cdot \boldsymbol{S}_1)^2 \ ,$$

$$\mathcal{P}_{11} = (\hat{\boldsymbol{r}} \cdot \boldsymbol{S}_1)^2 \ ,$$

$$\mathcal{P}_{20} = \mathbb{1}_5 - \frac{5}{4}(\hat{\boldsymbol{r}} \cdot \boldsymbol{S}_2)^2 + \frac{1}{4}(\hat{\boldsymbol{r}} \cdot \boldsymbol{S}_2)^4 \ ,$$

$$\mathcal{P}_{21} = \frac{4}{3}(\hat{\boldsymbol{r}} \cdot \boldsymbol{S}_2)^2 - \frac{1}{3}(\hat{\boldsymbol{r}} \cdot \boldsymbol{S}_2)^4 \ ,$$

$$\mathcal{P}_{22} = -\frac{1}{12}(\hat{\boldsymbol{r}} \cdot \boldsymbol{S}_2)^2 + \frac{1}{12}(\hat{\boldsymbol{r}} \cdot \boldsymbol{S}_2)^4 \ .$$

$$\mathcal{O}^{Q\bar{Q}}_{\kappa p}(t,\ \boldsymbol{r},\ \boldsymbol{R}) = \chi_c^\top(t,\ \boldsymbol{x}_2)\phi(t,\ \boldsymbol{x}_2\ \boldsymbol{R})\mathcal{Q}_{Q\bar{Q}\kappa p}(t,\ \boldsymbol{R})\phi(t,\ \boldsymbol{R},\ \boldsymbol{x}_1)\psi(t,\ \boldsymbol{x}_1)$$

$$\mathcal{Q}^{\alpha}_{1+-}(t,\boldsymbol{x}) = \left(\boldsymbol{e}^\dagger_\alpha \cdot \boldsymbol{B}(t,\boldsymbol{x})\right) \ ,$$

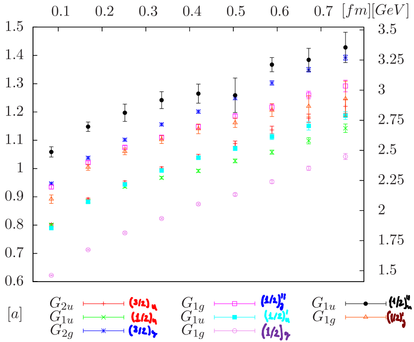
$$\mathcal{Q}^{\alpha}_{1--}(t,\boldsymbol{x}) = \left(\boldsymbol{e}^\dagger_\alpha \cdot \boldsymbol{E}(t,\boldsymbol{x})\right) \ ,$$

$$\mathcal{Q}^{\alpha}_{2+-}(t,\boldsymbol{x}) = \mathcal{C}^{2\,\alpha}_{1\,m_1\,1\,m_2}\left(\boldsymbol{e}^\dagger_{m_1} \cdot \boldsymbol{D}(t,\boldsymbol{x})\right)\left(\boldsymbol{e}^\dagger_{m_2} \cdot \boldsymbol{E}(t,\boldsymbol{x})\right) \ ,$$

$$\mathcal{Q}^{\alpha}_{2--}(t,\boldsymbol{x}) = \mathcal{C}^{2\,\alpha}_{1\,m_1\,1\,m_2}\left(\boldsymbol{e}^\dagger_{m_1} \cdot \boldsymbol{D}(t,\boldsymbol{x})\right)\left(\boldsymbol{e}^\dagger_{m_2} \cdot \boldsymbol{B}(t,\boldsymbol{x})\right) \ ,$$

$$\cdots$$

# Static Limit: Double heavy baryons




| $O(3)$ | $D_{\infty h}$ |
|---|---|
| $(1/2)^+$ | $(1/2)_g$ |
| $(3/2)^-$ | $(1/2)_u,\ (3/2)_u$ |
| $(1/2)^-$ | $(1/2)'_u$ |
| $(3/2)^+$ | $(1/2)'_g,\ (3/2)_g$ |

$$\mathcal{P}_{\frac{1}{2}\frac{1}{2}} = \mathbb{1}_2$$

$$\mathcal{P}_{\frac{3}{2}\frac{1}{2}} = \frac{9}{8}\mathbb{1}_4 - \frac{1}{2}\left(\hat{\boldsymbol{r}}\cdot\boldsymbol{S}_{3/2}\right)^2$$

$$\mathcal{P}_{\frac{3}{2}\frac{3}{2}} = -\frac{1}{8}\mathbb{1}_4 + \frac{1}{2}\left(\hat{\boldsymbol{r}}\cdot\boldsymbol{S}_{3/2}\right)^2$$

Najjar, Bali PoS LAT2009 (2009) $N_f = 2$, $a = 0.084$ fm, $L \simeq 1.3$ fm, $m_\pi \simeq 783$ MeV.

$$\mathcal{O}^{QQ}_{\kappa p}(t,\ \boldsymbol{r},\ \boldsymbol{R}) = \psi^\top(t,\ \boldsymbol{x}_2)\phi^\top(t,\ \boldsymbol{R},\ \boldsymbol{x}_2)\mathcal{Q}_{QQ\kappa p}(t,\ \boldsymbol{R})\phi(t,\ \boldsymbol{R},\ \boldsymbol{x}_1)\psi(t,\ \boldsymbol{x}_1)$$

$$\mathcal{Q}^\alpha_{(1/2)^+}(t,\boldsymbol{x}) = \underline{T}^l\left[P_+ q^l(t,\boldsymbol{x})\right]^\alpha \qquad \mathcal{Q}^\beta_{(3/2)^-}(t,\boldsymbol{x}) = \mathcal{C}^{3/2\ \beta}_{1\ m\ 1/2\ \alpha}\underline{T}^l\left[\left(\boldsymbol{e}^\dagger_m\cdot\boldsymbol{D}\right)\left(P_+ q(t,\boldsymbol{x})\right)^\alpha\right]^l$$

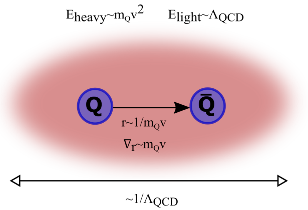
$$\mathcal{Q}^\alpha_{(1/2)^-}(t,\boldsymbol{x}) = \underline{T}^l\left[P_+\gamma^5 q^l(t,\boldsymbol{x})\right]^\alpha \qquad \mathcal{Q}^\beta_{(3/2)^+}(t,\boldsymbol{x}) = \mathcal{C}^{3/2\ \beta}_{1\ m\ 1/2\ \alpha}\underline{T}^l\left[\left(\boldsymbol{e}^\dagger_m\cdot\boldsymbol{D}\right)\left(P_+\gamma^5 q(t,\boldsymbol{x})\right)^\alpha\right]^l$$

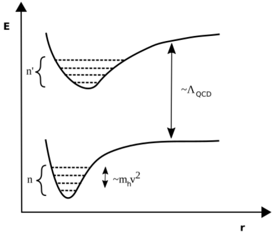
$$\ldots \qquad \ldots$$

- $\underline{T}^l$ are the $\bar{3}$ tensor invariants.

# EFT for double heavy hadrons

- We aim to build an EFT for the heavy quark-quark or heavy quark-antiquark bound states in the minima of the static energies.




- Heavy quarks are non relativistic $m_Q \gg \Lambda_{\rm QCD}$.
- The binding energy is small compared to the characteristic hadronic scale $\Lambda_{\rm QCD} \gg E_B \sim m_Q v^2$.
- No assumption on the scale of the interquark distance $r \sim 1/(m_Q v)$ respect to $\Lambda_{\rm QCD}$ as in strongly coupled pNRQCD. Brambilla, Pineda, Soto, Vairo Phys.Rev.D 63 (2001); Pineda, Vairo Phys.Rev.D 63 (2001)

# EFT for double heavy hadrons

- The fields $\Psi^{\alpha}_{\kappa^p}$ have quantum numbers matching those of $\mathcal{O}^{\alpha}_{\kappa^p}$.

$$\mathcal{L} = \sum_{\kappa^p} \Psi^{\dagger}_{\kappa^p} \left[ i\partial_t - h_{\kappa^p} \right] \Psi_{\kappa^p}$$

- $\Psi_{\kappa^p}$ has spin components corresponding to the heavy quarks and light degrees of freedom.
- The **heavy quark spin is suppressed** by $1/m_Q$ but the light degrees of freedom spin is **not**.

$$h_{\kappa^p} = \frac{\boldsymbol{p}^2}{m_Q} + \frac{\boldsymbol{P}^2}{4m_Q} + V^{(0)}_{\kappa^p}(\boldsymbol{r}) + \frac{1}{m_Q} V^{(1)}_{\kappa^p}(\boldsymbol{r}, \boldsymbol{p}) + \mathcal{O}(1/m_Q^2)$$

- The potentials are $(2\kappa+1) \times (2\kappa+1)$ matrices is the light degrees of freedom spin space.
- The static potential can be decomposed into representations of $D_{\infty h}$ with the projectors $\mathcal{P}_{\kappa\Lambda}$

$$V^{(0)}_{\kappa^p}(\boldsymbol{r}) = \sum_{\Lambda} V^{(0)}_{\kappa^p \Lambda}(r) \mathcal{P}_{\kappa\Lambda}$$

# EFT for double heavy hadrons

- Up to leading order the EFT is equivalent to the Born-Oppenheimer approximation.

$$V^{(1)}_{\kappa^p}(\boldsymbol{r}) = V^{(1)}_{\kappa^p \mathrm{SI}}(\boldsymbol{r}) + V^{(1)}_{\kappa^p \mathrm{SD}}(\boldsymbol{r})$$

- $V^{(1)}_{\kappa^p \mathrm{SI}}$ decomposes as the static potential.
- At next-to-leading order the first heavy-quark spin dependent potentials appear.

$$V^{(1)}_{\kappa^p \mathrm{SD}}(\boldsymbol{r}) = \sum_{\Lambda\Lambda'} \mathcal{P}_{\kappa\Lambda} \Big[ V^{sa}_{\kappa^p\Lambda\Lambda'}(r) \boldsymbol{S}_{QQ} \cdot (\mathcal{P}^{\mathrm{c.r.}}_{10} \cdot \boldsymbol{S}_\kappa) + V^{sb}_{\kappa^p\Lambda\Lambda'}(r) \boldsymbol{S}_{QQ} \cdot (\mathcal{P}^{\mathrm{c.r.}}_{11} \cdot \boldsymbol{S}_\kappa) + V^{l}_{\kappa^p\Lambda\Lambda'}(r) \left(\boldsymbol{L}_{QQ} \cdot \boldsymbol{S}_\kappa\right) \Big] \mathcal{P}_{\kappa\Lambda'}$$

- $2\boldsymbol{S}_{QQ} = \boldsymbol{\sigma}_{QQ} = \boldsymbol{\sigma}_{Q_1} \mathbb{1}_{2\,Q_2} + \mathbb{1}_{2\,Q_1} \boldsymbol{\sigma}_{Q_2}$, $\left(\mathcal{P}^{\mathrm{c.r.}}_{10}\right)^{ij} = \hat{\boldsymbol{r}}^i \hat{\boldsymbol{r}}^j$, $\left(\mathcal{P}^{\mathrm{c.r.}}_{11}\right)^{ij} = \delta^{ij} - \hat{\boldsymbol{r}}^i \hat{\boldsymbol{r}}^j$.
- The potentials, $V^{(0)}_{\kappa^p}$, $V^{(1)}_{\kappa^p}$ are nonperturbative and should be computed with lattice QCD.

# Matching to NRQCD

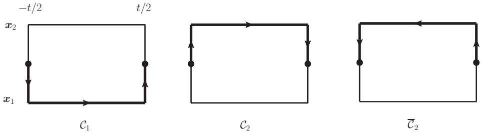
- The matching condition is

$$\mathcal{O}^{h}_{\kappa^p}(t,\, \boldsymbol{r},\, \boldsymbol{R}) = \sqrt{Z_{h\kappa^p}}\Psi_{h\kappa^p}(t,\, \boldsymbol{r},\, \boldsymbol{R})\,, \quad h = Q\bar{Q},\, QQ\,.$$

- We match the NRQCD and heavy exotic hadron EFT correlators,

$$\langle 0|\, T\{\mathcal{O}^{h}_{\kappa^p}(t/2)\mathcal{O}^{h\dagger}_{\kappa^p}(-t/2)\}|0\rangle = \sqrt{Z_{h\kappa^p}}\langle 0|\, T\{\Psi_{h\kappa^p}(t/2)\Psi^{\dagger}_{h\kappa^p}(-t/2)\}|0\rangle\sqrt{Z^{\dagger}_{h\kappa^p}}$$

- The left hand side can be expanded in terms of Wilson loops with operator insertions.




- At leading order no insertion is needed

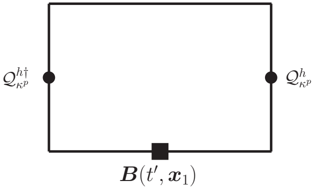
$$V^{(0)}_{\kappa^p\Lambda}(\boldsymbol{r}) = E_{\kappa^p\Lambda_\eta}(\boldsymbol{r})$$

# Matching to NRQCD

- The potentials NLO are given insertions in the Wilson loop of $\boldsymbol{D}^2(t',\, \boldsymbol{x}_1)/(2m_Q)$ or $c_F g\boldsymbol{\sigma}_1 \cdot \boldsymbol{B}(t', \boldsymbol{x}_1)/m_Q$.
- For instance:

$$V^{sa}_{\kappa^p\Lambda\Lambda'} = -\, c_F \lim_{t\to\infty} \frac{\delta_{\Lambda\Lambda'}}{t} \frac{\mathrm{Tr}\left[\mathcal{P}_{\kappa\Lambda}\right]}{\mathrm{Tr}\left[\mathcal{P}_{\kappa\Lambda}\langle 1\rangle^{h\kappa^p}_{\Box}\right]} \int_{-t/2}^{t/2} dt' \frac{\mathrm{Tr}\left[\left(\boldsymbol{S}_\kappa \cdot \mathcal{P}^{\mathrm{c.r.}}_{10}\right) \cdot \left(\mathcal{P}_{\kappa\Lambda}\langle g\boldsymbol{B}(t',\, \boldsymbol{x}_1)\rangle^{h\kappa^p}_{\Box}\mathcal{P}_{\kappa\Lambda}\right)\right]}{\mathrm{Tr}\left[\left(\boldsymbol{S}_\kappa \cdot \mathcal{P}^{\mathrm{c.r.}}_{10}\right) \cdot \left(\mathcal{P}_{\kappa\Lambda}\boldsymbol{S}_\kappa\mathcal{P}_{\kappa\Lambda}\right)\right]}$$

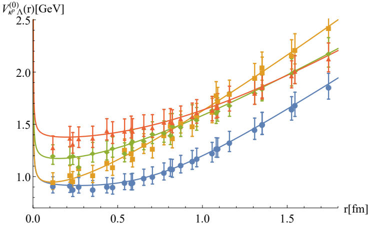
$$\langle \boldsymbol{B}(t',\boldsymbol{x}_1)\rangle^{h\kappa^p}_{\Box} =$$

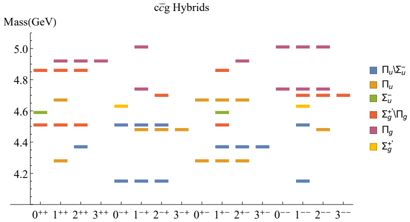



- Similar expressions (but more involved) for $V^{sb}$ and $V^{l}$ in Soto, JTC, Phys.Rev.D 102 (2020)

# Application to heavy hybrids

- Spectrum computed including nonadiabatic mixing for:
  - $\kappa = 1^{+-}$ Berwein, Brambilla, JTC, Vairo Phys.Rev.D92 (2015); Oncala, Soto Phys.Rev.D96 (2017)
  - $\kappa = 1^{--}$ Pineda, JTC, Phys.Rev.D100 (2019)

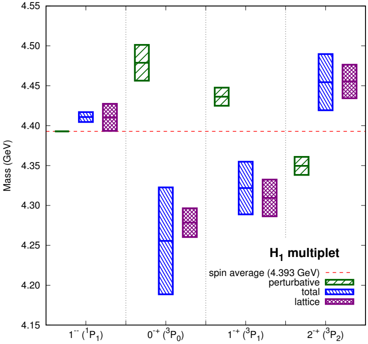



- $b\bar{b}g$ $b\bar{c}g$ also available.

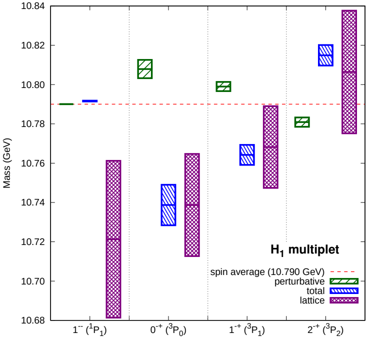
# Application to heavy hybrids

- Hyperfine contributions
  - LO Soto Nucl.Part.Phys.Proc. 294 (2018)
  - LO+NLO and predictions for $b\bar{b}g$ Brambilla, Lai, Segovia, JTC, Vairo, Phys.Rev.D99 (2019)
  - Short distance Matching Brambilla, Lai, Segovia, JTC, Phys.Rev.D101 (2020)




Fitted $c\bar{c}g$ splittings to

Cheung et al JHEP12 (2016)

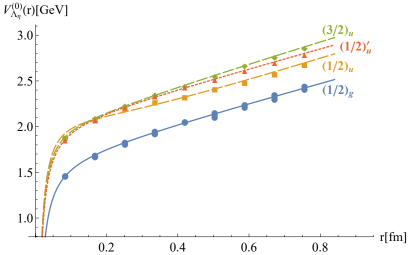



Predicted $b\bar{b}g$ compared to posterior lattice data

Ryan et al JHEP 02 (2021)

# Application to double heavy baryons

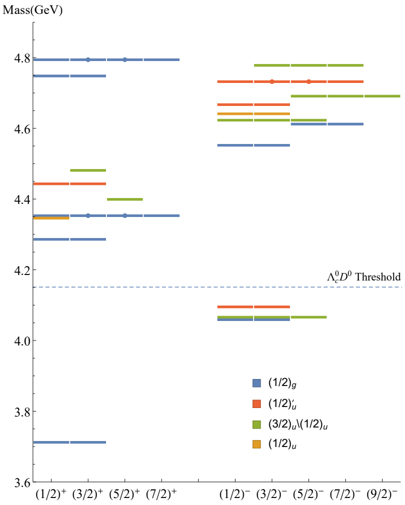
- We have applied our formalism to double heavy baryons.



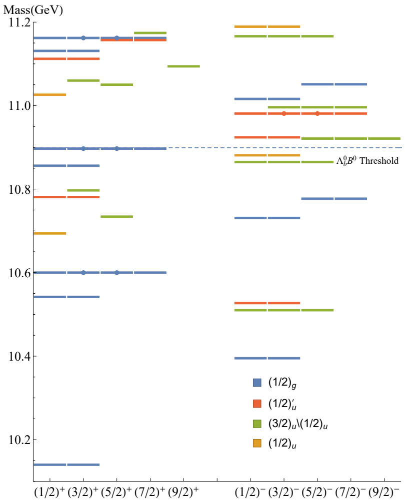

- We consider the 4 lowest laying static energies.
- $(3/2)_u$ and $(1/2)_u$ are coupled since both correspond to $\kappa = 3/2^-$.
- Lattice data: $N_f = 2$, $a = 0.084$ fm, $L \simeq 1.3$ fm, $m_\pi \simeq 783$ MeV. Najjar, Bali PoS LAT2009 (2009)

• The potentials are obtained from constrained fits to the lattice data:
  - Short distance:
    * The potentials must reproduce the perturbative quark-quark Coulombic potential plus an offset constant, i.e should match to weakly coupled pNRQCD. Brambilla, Vairo Rösch, Phys. Rev. D72 (2005)
    * Energy gaps between static energies consistent with heavy-quark-diquark symmetry. Savage, Wise, Phys. Lett. B248,(1990)
    * The offsets are shifted so the lowest one is $\overline{\Lambda}_{(1/2)^+} = 0.555(31)\ \mathrm{GeV}$ from lattice studies of D and B mesons masses. Bazavov et al. Phys. Rev. D98 (2018)
  - Long distance: fixed string-like potential $\sigma r$ with a fitted offset constant, $\sigma = 0.21$ GeV$^2$.

## $\Xi_{cc}$ spectrum




## $\Xi_{bb}$ spectrum




Soto, JTC, Phys.Rev.D 102 (2020)

# Double heavy baryons hyperfine splittings

- At $\mathcal{O}(1/m_Q)$ we have heavy-quark spin dependent operators responsible for hyperfine splittings.
- There is no lattice data for the potentials but some model independent relations can be derived.
- For $\kappa = 1/2$ there are only 3 operators:

$$V^{(1)}_{(1/2)\pm\,\mathrm{SD}}(\boldsymbol{r}) = V^{s1}_{(1/2)\pm}(r)\boldsymbol{S}_{QQ}\cdot\boldsymbol{S}_{1/2} + V^{s2}_{(1/2)\pm}(r)\boldsymbol{S}_{QQ}\cdot\left(\boldsymbol{\mathcal{T}}_2\cdot\boldsymbol{S}_{1/2}\right) + V^{l}_{(1/2)\pm}(r)\left(\boldsymbol{L}_{QQ}\cdot\boldsymbol{S}_{1/2}\right)$$

with $(\boldsymbol{\mathcal{T}}_2)^{ij} = \hat{\boldsymbol{r}}^i\hat{\boldsymbol{r}}^j - \delta^{ij}/2$

- Let us label the mass of the states as $M_{njl\ell} = M^{(0)}_{nl} + M^{(1)}_{njl\ell} + \ldots$
  - $n$ principal quantum number.
  - $l$ orbital angular momentum.
  - $\ell$ sum of orbital angular momentum and light-quark spin.
  - $j$ total angular momentum.
  - Heavy quark spin state fixed by Pauli principle, $s_{QQ} = 1$ for $l$ even and $s_{QQ} = 0$ for $l$ odd.

## Double heavy baryons hyperfine splittings

- $l = 0$ only hyperfine contributions from $V^{s1}_{(1/2)^+}$

$$M^{(1)}_{nj0\frac{1}{2}} = \frac{1}{2}\left(j(j+1) - \frac{11}{4}\right)\frac{\langle V^{s1}_{(1/2)^+}\rangle_{n0}}{m_Q}\,, \Rightarrow 2M_{n\frac{3}{2}0\frac{1}{2}} + M_{n\frac{1}{2}0\frac{1}{2}} = 3M^{(0)}_{n0}$$

- $l = 1$ only hyperfine contributions from $V^{l}_{(1/2)^+}$

$$M^{(1)}_{nj1j} = \frac{1}{2}\left(j(j+1) - \frac{11}{4}\right)\frac{\langle V^{l}_{(1/2)^+}\rangle_{n1}}{m_Q}\,, \Rightarrow 2M_{n\frac{3}{2}1\frac{3}{2}} + M_{n\frac{1}{2}1\frac{1}{2}} = 3M^{(0)}_{n1}$$

- $l = 2$ all 3 operators contribute, but since we have 6 states some model independent relations can still be found.

# Double heavy baryons hyperfine splittings: Comparison with lattice QCD

- Ground state: $1^3S$ doublet $(1/2,\,3/2)^+$. $M_{\Xi_{cc}} \equiv M_{1\frac{1}{2}0\frac{1}{2}}$, $M_{\Xi^*_{cc}} \equiv M_{1\frac{3}{2}0\frac{1}{2}}$, $\delta_{hf} = M_{\Xi^*_{cc}} - M_{\Xi_{cc}}$.

| Ref. | $\delta_{hf}$ [MeV] | spin avg. |
|---|---|---|
| Briceno et al Phys.Rev.D86 (2012) | 53(94) | 3630(50) |
| Namekawa et al Phys.Rev.D87 (2013) | 101(36) | 3672(20) |
| Brown et al Phys.Rev.D90 (2014) | 82.8(9.2) | 3665(36) |
| Alexandrou et al Phys.Rev.D90 (2014) | 84(58) | 3624(33) |
| Bali et al Phys.Rev.D92 (2015) | 85(9) | 3666(13) |
| Padmanath et al Phys.Rev.D91 (2015) | 94(12) | 3700(6) |
| Alexandrou et al Phys.Rev.D96 (2017) | 76(41) | 3657(25) |
| Our values | 136(44) | 3712(63) |

- First excitation: $1^1P$ doublet $(1/2,\,3/2)^-$. Spin average agreement with Padmanath et al Phys.Rev.D91 (2015); Bali et al Phys.Rev.D92 (2015)
- Higher excitations below $\Lambda_c - D$ threshold: $1^3(S\backslash D)$ $(\ell = 3/2)$ triplet of the $(3/2)_u - (1/2)_u$ BO potentials $(1/2,\,3/2,\,5/2)^-$ ; $1^3S$ doublet of the $(1/2)'_u$ BO potential $(1/2,\,3/2)^-$. Qualitative agreement with Padmanath et al Phys.Rev.D91 (2015).
- Bottomonium ground state doublet: our value $M^{(0)}_{10} = 10.140(77)$ MeV. Agreement with $M_{\rm s.a.} = 10.166(40)$ MeV Lewis et al Phys.Rev.D79(2009) and $M_{\rm s.a.} = 10.143(29)$ MeV Brown et al Phys.Rev.D90 (2014), compatible with $M_{\rm s.a.} = 10.099(17)$ MeV Mohanta et al Phys.Rev.D101 (2020).

## Conclusions

- We have developed an EFT framework to describe double heavy hadrons (either $QQ$ or $Q\bar{Q}$) containing light degrees of freedom (gluons or light quarks) with any quantum numbers.
  - The EFT incorporates the heavy quark mass and adiabatic expansions.
  - We have worked out the EFT up to $\mathcal{O}(1/m_Q)$.
  - The potentials, which are nonperturbative, are given by NRQCD Wilson loops.
  - No assumption on the interquark distance is made. In the short distance we can incorporate heavy-quark-diquark symmetry or perturvative computations; in the long distance string models.
- We have applied the framework to double heavy baryons
  - We have obtain the spectrum for $\Xi_{cc}$ and $\Xi_{bb}$ for the four lowest static energies including the mixing.
  - Provided model independent formulas for the hyperfine splittings of multiplets with $\kappa = 1/2$ and $l = 0, 1, 2$.
  - Results compatible with lattice QCD results.

Thank you for your attention

# Multiplets

| $\kappa^P$ | $\Lambda_\eta$ | $l$ | $\ell$ | $s_{QQ}$ | $j$ | $\eta_P$ |
|---|---|---|---|---|---|---|
| $(1/2)^{\pm}$ | $(1/2)_{g/u'}$ | 0 | $1/2$ | 1 | $(1/2,\ 3/2)$ | $\pm$ |
| | | 1 | $(1/2,\ 3/2)$ | 0 | $(1/2,\ 3/2)$ | $\mp$ |
| | | 2 | $(3/2,\ 5/2)$ | 1 | $((1/2,\ 3/2,\ 5/2),(3/2,\ 5/2,\ 7/2))$ | $\pm$ |
| | | 3 | $(5/2,\ 7/2)$ | 0 | $(5/2,\ 7/2)$ | $\mp$ |
| $(3/2)^{-}$ | $(3/2)_u\backslash(1/2)_u$ | $0\backslash 2$ | $3/2$ | 1 | $(1/2,\ 3/2,\ 5/2)$ | $-$ |
| | | $1\backslash 3$ | $5/2$ | 0 | $5/2$ | $+$ |
| | | $2\backslash 4$ | $7/2$ | 1 | $(5/2,\ 7/2,\ 9/2)$ | $-$ |
| | | $3\backslash 5$ | $9/2$ | 0 | $9/2$ | $+$ |
| | | $1\backslash 3$ | $3/2$ | 0 | $3/2$ | $+$ |
| | | $2\backslash 4$ | $5/2$ | 1 | $(3/2,\ 5/2,\ 7/2)$ | $-$ |
| | | $3\backslash 5$ | $7/2$ | 0 | $7/2$ | $+$ |
| | | $4\backslash 6$ | $9/2$ | 1 | $(7/2,\ 9/2,\ 11/2)$ | $-$ |
| | $(1/2)_u$ | 1 | $1/2$ | 0 | $1/2$ | $+$ |
| | | 2 | $1/2$ | 1 | $(1/2,\ 3/2)$ | $-$ |