

Hadronic Molecules

Ulf-G. Meißner, Univ. Bonn & FZ Jülich



- Ulf-G. Meißner, Hadronic Molecules - EMMI workshop on XYZ states, April 12, 2021 -

Contents

- Introduction: Take home messages
- Dynamically generated states
- Theory of hadronic molecules
- Candidates for hadronic molecules
- Phenomenology of hadronic molecules
- The two-pole scenario
- Prospects

Take home messages

Introduction and Summary

The hadron spectrum is much more than a collection of quark model states \rightarrow a **new paradigm** of hadron spectroscopy emerges

Dynamically generated states are copious

 \rightarrow Hadronic molecules are a subclass of these

Hadronic molecules are characterized by particular features \rightarrow Effective Field Theories and Lattice QCD are **the** tools

The phenomenon of **two-pole** structures is tied to the molecular dynamics \rightarrow this needs to be accounted for in the RPP

Dynamically generated states

Dynmaically generated states

- Hadron-hadron (or three-hadron) interactions can dynamically generate resonances
- Molecules are a subclass of these (shallow binding, close to the real axis)
- Prime example: The light scalar mesons $f_0(500), f_0(700), f_0(980)$



Theory of hadronic molecules

What are hadronic molecules ?

- Bound states of two hadrons in an S-wave very close a 2-particle threshold or between two close-by thresholds ⇒ particular decay patterns
- weak binding entails a large spatial extension
- the classical example:

* the deuteron $m_p + m_n = 938.27 + 939.57$ MeV, $m_d = m_p + m_n - B_d \rightarrow B_d = 2.22$ MeV $B_d/m_d \simeq 1/1000$ $r_d = 2.14$ fm $[r_p = 0.85$ fm]



• other examples: $\Lambda(1405), f_0(980), X(3872), \ldots$

 \Rightarrow how to distinguish these from compact multi-quark states ?

Compositeness criterion

Weinberg (1965), Morgan (1991), Tornquist (1995), Baru et al. (2003), ...

• Wave fct. of a bound state with a compact & a two-hadron component in S-wave:

$$|\Psi
angle = egin{pmatrix} \sqrt{Z}|\psi_0
angle \ \chi(ec{k})|h_1h_2
angle \end{pmatrix}$$

compact comp. w/ probability \sqrt{Z} two-hadron comp. w/ relative w.f. $\chi(\vec{k})$

• consider the scattering amplitude and compare with the ERE:

$$a=-2rac{1-Z}{2-Z}\left(rac{1}{\gamma}
ight)+\mathcal{O}\left(rac{1}{eta}
ight) \ , \ \ r=-rac{Z}{1-Z}\left(rac{1}{\gamma}
ight)+\mathcal{O}\left(rac{1}{eta}
ight) \ \ \ \gamma=\sqrt{2\mu B}$$

 $a = \text{scattering length}, \gamma/B = \text{binding momentum/energy} (shallow b.s.)$

 μ = reduced mass of the two-particle system, $1/\beta$ = range of forces

$$\Rightarrow$$
 pure molecule ($Z=0$): maximal scattering length $a=-1/\gamma$
natural effective range $r=\mathcal{O}(1/\beta)$

$$\Rightarrow$$
 compact state ($Z = 1$): the scattering length is $a = -\mathcal{O}(1/\beta)$ effective range diverges, $r \to -\infty$

The deuteron

Weinberg (1965)

• The deuteron: shallow neutron-proton bound state ($B_d \ll m_d$):

$$B_d = 2.225\,\mathrm{MeV}
ightarrow \gamma = 45.7\,\mathrm{MeV} = 0.23\,\mathrm{fm^{-1}}$$

• range of forces set by the one-pion-exchange:

$$1/eta \sim 1/M_\pi \simeq 1.4\,{
m fm}$$

• set Z = 0 in the Weinberg formula:

$$a_{
m mol} = -(4.3 \pm 1.4)\,{
m fm}\,,~~r_{
m mol} = \mathcal{O}(1.4\,{
m fm})$$

• this is consistent with the data in the 3S_1 channel:

$$a = -5.419(7)$$
 fm , $r = 1.764(8)$ fm

One begins to suspect that Nature is doing her best to keep us from learning whether the "elementary" particles deserve that title. (Weinberg, 1965)

Extension to resonances

Baru et al. (2003), Braaten, Lu (2007), Aceti, Oset (2012), Guo, Oller (2016), ...

• Still assume closeness to a two-particle threshold:

$$T(E) = rac{g^2/2}{E-E_r+(g^2/2)(ik+\gamma)+i\Gamma_0/2}$$

with $E=k^2/(2\mu)$, Γ_0 accounts for the inelasticities of other channels

• leads to very different line shapes for compact and molecular states:



 k^2 term dominates ightarrow symmetric g^2 term dominates ightarrow asymmetric/cusp

• extension to instable particles/additional poles have also been worked out

Universality

Braaten, Hammer, Phys. Rept. **428** (2006) 259, ...

- Consider systems with the two-particle scattering length much, much bigger than the range of forces: $a \gg R_0 = 1/\beta$
- \Rightarrow physics is independent of the overall energy scale. Predictions:
 - * Two-body binding energy: $B_2 = \frac{1}{\mu a^2}$

applies for energy scales from neV (cold atoms) to GeV (charmonium, bottomonium)

Deuteron: $B_2 = 1.86$ MeV (range corrections)

* Three-body systems: Efimov effect

Efimov, Phys. Lett. B 33 (1970) 563

 \hookrightarrow another talk







Candidates for hadronic molecules

Some candidates

- Prominent examples in the light quark sector: $f_0(980), a_0(980),$ the two $\Lambda(1405), \ldots$
- Prominent examples in the $c\bar{c}$ spectrum: $X(3872), Z_c(3900), Y(4260), Y(4660), ...$
- Prominent examples of heavy-light mesons: $D_{s0}^{\star}(2317), D_{s1}(2460), D_{s1}^{\star}(2860), \dots$
- Prominent examples in the $b\bar{b}$ spectrum: $Z_b(10610), Z_b(10650)$
- and some examples of heavy baryons:

 $\Lambda_c(2595), \Lambda_c(2940), P_c(4312), P_c(4440), \ldots$



Details in: Guo, Hanhart, UGM, Wang, Zhao, Zou, Rev. Mod. Phys. 90 (2018) 015004

Phenomenology of hadronic molecules

General remarks

- Consider an hadronic molecule with w.f. Ψ , made of two hadrons h_1, h_2 , located close to the threshold $E_{
 m thr} = m(h_1) + m(h_2)$
- \Rightarrow long-distance scale $\gamma = \sqrt{2\mu B} \ll \beta \rightarrow$ scale separation!
- Two classes of decay and production processes:
 - **long-distance processes**, in which the momenta of all particles in the c.m. frame of h_1h_2 are of $\mathcal{O}(\gamma)$
 - short-distance processes, which involve particles with a momentum $\gtrsim eta$ in the c.m. frame of h_1h_2
- ⇒ only the former class of processes is entirely sensitive to the molecular component e.g. enhanced production through the triangle singularity
- ⇒ for the second class, one requires knowledge about short-distance physics and thus can often only make estimates (discuss two pitfalls often encountered)

Suitable NREFTs

- Most exotic candidates found through decays
- \rightarrow triangle diagram: anomalous triangle singularity
- \rightarrow already studied by Landau, Nambu and other in the 1950ties
- NREFT₁: all intermediate particles close to their mass shell
 - \hookrightarrow expand in powers of the average velocity and external (small) momenta
 - \hookrightarrow applied systematically to a number of charmonium transitions $\sqrt{}$ Guo, Hanhart, UGM, Zhao (2009,2010,2011), Guo, UGM (2012), ...
- **NREFT**₂: one intermediate particle further off its mass shell
 - \hookrightarrow integrate out this particle, then proceed as before
 - \hookrightarrow was originally invented as XEFT for the study of the X(3872)
 - \hookrightarrow XEFT resembles much the contact EFT of nuclear physics (π 's pert.)
 - \hookrightarrow systematic studies of processes involving the X(3872) and Z_b states Fleming et al. (2007), Braaten, Hammer, Mehen (2010), Mehen, Powell (2011), ...



The X(3872) [aka $\chi_{c1}(3872)$]

- seen at B-factories (Belle, BaBar) and colliders (D0, CDF, LHCb, ...)
- extremely close to the $D^0 \overline{D}^{*0}$ threshold:

$$B_X=0.07\pm 0.12$$
 MeV

 \rightarrow tremendously large scattering length

 \rightarrow universality

• maximal isospin violation:

$$\Gamma(X \to J/\psi \pi \pi) \simeq \Gamma(X \to J/\psi \pi \pi \pi)$$

- quantum numbers: $J^{PC} = 1^{++}$ (LHCb 2013)
- a prime candidate for a hadronic molecule:

$$|X
angle=rac{1}{\sqrt{2}}\,\left(|D^0ar{D}^{0*}
angle+|ar{D}^0D^{0*}
angle
ight)$$



Voloshin, Okun (1976)

A tribute to Misha Voloshin

• A true visionary [and Humboldt research prize fellow at Bonn]

Hydronic molecules and the charmonium atom

M. B. Voloshin and L. B. Okun'

Institute of Theoretical and Experimental Physics (February 16, 1976) Pis'ma Zh. Eksp. Teor. Fiz. 23, No. 6, 369-372 (20 March 1976)

We consider the possible existence of levels in a system consisting of a charmed particle and a charmed antiparticle; these levels result from exchange of ordinary mesons ($\omega, \rho, \epsilon, \phi$, etc.). An interpretation of the resonances in e^+e^- annihilation in the region 3.9–4.8 GeV is proposed.

PACS numbers: 12.40.Dd, 14.40.-n

A large aggregate of data indicates that ψ , ψ' , and other "psions" (χ , P_C , X) are different states of charmonium, a bound state of c and \overline{c} quarks.^[1] One cannot speak as yet, however, of a quantitative agreement between the charmonium model and the experimental data with on the level positions and widths. We wish to note here that it is necessary to superimpose on the simple quark-gluon "atomic" structure of charmonium a much more complicated and in a certain sense fundamental structure of levels of "molecular" type, which represent bound states of a charmed hadron and a charmed antihadron, for example bound and (or) resonant dimension states $D\overline{D}$ ($D^* = c\overline{n}$, $D^0 = c\overline{p}$) as well as states of the type $C\overline{C}$, where C is a charmed baryon.

X(3872) Production in e^+e^- collisions

Guo, Hanhart, UGM, Wang, Zhao, Phys. Lett. B 725 (2013) 127

• Prediction of a long-distance process: If the X(3872) is a $D\bar{D}^*$ molecule and the Y(4260) is a $D\bar{D}_1$ molecule, there will be a strong radiative transition $Y(4260) \rightarrow X(3872)\gamma$ in e^+e^- collisions

Data from BESIII



 \star Clear evidence of the X(3872)

PRL 112 (2014) 092001



★ Data hint that it proceeds through a Y state \rightarrow more data needed

Hadroproduction of the X(3872)

Guo, UGM, Wang, Yang, Eur. Phys. J. C 74 (2014) 3063

- Nice example of a process involving short-distance physics
- \hookrightarrow still, factorization is at work, best seen using EFT

Artoisenet, Braaten, Phys. Rev. D 81 (2010) 114018

 \hookrightarrow consider production at the Tevatron and at LHC

$$egin{split} \sigma[X] &= rac{1}{4m_H m_{H'}} g^2 |G|^2 igg(rac{d\sigma[HH'(k)]}{dk} igg)_{
m MC} rac{4\pi^2 \mu}{k^2} \ G(E,\Lambda) &= -rac{\mu}{\pi^2} igg[\sqrt{2\pi} \, rac{\Lambda}{4} + \sqrt{\pi} \, \gamma D \left(rac{\sqrt{2} \gamma}{\Lambda}
ight) - rac{\pi}{2} \, \gamma \, e^{2\gamma^2/\Lambda^2} igg] \end{split}$$



• typical results (using PYTHIA or HERWIG):

| $\sigma(pp/ar{p} 	o X(3872))$ | $\Lambda=0.5-1.0~{ m GeV}$ | Exp. |
|-------------------------------|----------------------------|---------------|
| Tevatron | 5 - 29 [nb] | 37 - 115 [nb] |
| LHC7 | 4 - 55 [nb] | 13 - 39 [nb] |

 \Rightarrow not very precise, but perfectly consistent with the data!

Misconcptions on hadroproduction

Albaladejo, Guo, Hanhart, UGM, Nieves, Nogga, Yang, Chin.Phys. C 41 (2017) 121001

 It is often claimed that molecules due to their large spatial extent can not be produced in high-energy collisions, say at the LHC → this is wrong!

Bignamini, Grinstein, Piccinini, Polosa, Sabelli, Phys. Rev. Lett. 103 (2009) 162001

$$egin{split} &\sigma(ar{p}p o X) \sim \left| \int d^3 \mathrm{k} \langle X | D^0 ar{D}^{*0}(\mathrm{k})
angle \langle D^0 ar{D}^{*0}(\mathrm{k}) | ar{p}p
angle
ight|^2 \ &\leq \int_{\mathcal{R}} d^3 \mathrm{k} \left| \langle D^0 ar{D}^{*0}(\mathrm{k}) | ar{p}p
angle
ight|^2 \end{split}$$

- The result depends crucially on the value of \mathcal{R} which specifies the region where the bound state wave function " $\Psi(\mathbf{k})$ is significantly different from zero"
- ullet Assumption by Bignamini et al: $\mathcal{R}\simeq 35$ MeV of the order of γ [$\simeq 0$ now]

 $\hookrightarrow \sigma(ar{p}p o X) \simeq 0.07[0.0]$ nb way smaller than experiment

- \hookrightarrow the X(3872) can not be a molecule
- \hookrightarrow so what goes wrong?

Misconceptions on hadroproduction continued

- Consider the relevant integral for the deuteron: $\bar{\Psi}_{\lambda}(\mathcal{R}) \equiv \int_{\mathcal{R}} d^3 \mathbf{k} \Psi_{\lambda}(\mathbf{k})$
- •The binding momentum is $\gamma\simeq 45$ MeV, use that for the support ${\cal R}$:



 \hookrightarrow the integral is by far not saturated for $\mathcal{R} = \gamma$, need $\mathcal{R} \simeq 2M_{\pi} \simeq 300$ MeV \rightarrow see also Eric Braaten & Alessandro Pilloni

The two-pole scenario

The first exotic hadron – the story of the two $\Lambda(1405)$ $_{\scriptscriptstyle 25}$

- Quark model: *uds* excitation with $J^P = \frac{1}{2}^-$ CLAS (2014) a few hundred MeV above the $\Lambda(1116)$ $m = 1405.1^{+1.3}_{-1.0}$ MeV, $\Gamma = 50.5 \pm 2.0$ MeV [PDG 2020]
- Prediction as early as 1959 by Dalitz and Tuan: Resonance between the coupled $\pi \Sigma$ and $\overline{K}N$ channels Dalitz, Tuan, Phys. Rev. Lett. **2** (1959) 425; J.K. Kim, PRL **14** (1965) 29
- Clearly seen in $K^-p \rightarrow \Sigma 3\pi$ reactions at 4.2 GeV at CERN Hemingway, Nucl.Phys. B **253** (1985) 742
- An enigma: Too low in mass for the quark model, but well described in unitarized chiral perturbation theory: $\phi B \to \phi B$



Kaiser, Siegel, Weise, Ramos, Oset, Oller, UGM, Lutz, ...





The two-pole scenario

• Detailed analysis found two poles in the complex energy plane

Oller, UGM, Phys. Lett. B 500 (2001) 263

CLAS, Phys. Rev. C 87, 035206 (2013)

• Group theory:

$$8\otimes 8=\underbrace{1\oplus 8_s\oplus 8_a}_{ ext{binding at LO}}\oplus 10\oplus \overline{10}\oplus 27$$

- Follow the pole movement from the SU(3) limit to the physical masses: Jido, Oller, Oset, Ramos, UGM, Nucl. Phys. A 725 (2003) 181
- Verified by various groups world-wide
- However: scattering and kaonic atom data alone do not lead to a unique solution (two poles, but spread in the complex plane)
- Further constraints from photoproduction: $\gamma p
 ightarrow K^+ \Sigma \pi$

- Ulf-G. Meißner, Hadronic Molecules - EMMI workshop on XYZ states, April 12, 2021 -



Present status of the two-pole scenario

• Two poles from scattering plus SIDDHARTA kaonic hydrogen data:



for details, see Mai, [arXiv:2010.00056 [nucl-th]]

Figures courtesy Maxim Mai

In WestGel

→ PDG 2016: http://pdg.lbl.gov/2015/reviews/rpp2015-rev-lam-1405-pole-struct.pdf

POLE STRUCTURE OF THE $\Lambda(1405)$ REGION Written November 2015 by Ulf-G. Meißner and Tetsuo Hyodo — constantly updated —

Mysteries of the charm-strange mesons

• The charm-strange mesons $D_{s0}^{\star}(2317)$ and $D_{s1}(2460)$ are prime candidates for hadronic moleluces

Barnes et al. (2003), van Beveren, Rupp (2003), Kolomeitsev, Lutz (2004), Guo et al. (2006), . . .

- A number of mysteries:
 - why are these so much lighter than predicted in the QM?
 - $\text{ why } M_{D_{s1}(2460)} M_{D_{s0}^{\star}(2317)} \\ \simeq M_{D^{\star}} M_{D} \text{ within 2 MeV?}$
 - $\text{ why } M_{D_0^\star(2300)} \simeq M_{D_{s0}^\star(2317)}$ and $M_{D_1(2430)} \simeq M_{D_{s1}(2460)}?$



 \hookrightarrow All this follows from the molecular picture combined with HQSS

Du, Albaladejo, Fernández-Soler, Guo, Hanhart, UGM, Nieves, Yao, Phys. Rev. D 98 (2018) 094018

 \hookrightarrow The resolution of the third puzzle reveals the two-pole scenario

What about the $D_0^{\star}(2300)$?

• Results for I = 1/2 coupled-channel $D\phi$ scattering (finite volume UChPT)

Albaladejo, Fernandez-Soler, Guo, Nieves, Phys. Lett. B 767 (2017) 465



• this is NOT a fit! Data from the Hadron Spectrum Collaboration

Moir, Peardon, Ryan, Thomas, Wilson, JHEP 1610 (2016) 011

• all LECs taken from an earlier study of channels with connected diagrams only

Liu, Orginos, Guo, Hanhart, UGM, PRD 87 (2013) 014508

What about the $D_0^{\star}(2300)$? continued

- reveals a two-pole scenario! [cf. $\Lambda(1405)$]
- understood from group theory

 $\overline{3} \otimes 8 = \underbrace{\overline{3} \oplus 6}_{\text{attractive}} \oplus \overline{15}$

• this was seen earlier in various calc's

Kolomeitsev, Lutz (2004), F. Guo, Shen, Chiang, Ping, Zou (2006), F. Guo, Hanhart, UGM (2009), Z. Guo, UGM, Yao (2009)

- Again: important role of chiral symmetry
- Easy lattice QCD test:

sextet pole becomes a bound state for $M_{\phi} > 575$ MeV in the SU(3) limit Du et al., Phys.Rev. D **98** (2018) 094018

Albaladejo, Fernandez-Soler, Guo, Nieves (2017)



Where is the lowest charmed scalar meson?

Du, Guo, Hanhart, Kubis, UGM, Phys. Rev. Lett. (2021) in press [2012.04599]

- Breit-Wigner description not appropriate for the S-wave but UChPT and the dispersive analysis are!
- First determination of the $D\pi$ phase shift
- The lowest charm-strange meson is located at:

 $\left(2105^{+6}_{-8}-i\,102^{+10}_{-11}
ight){
m MeV}$

• Recently confirmed by Lattice QCD! Cheung et al. [HadSpec], JHEP 02 (2021) 100



- Ulf-G. Meißner, Hadronic Molecules - EMMI workshop on XYZ states, April 12, 2021 -

Status in the Review of Particle Physics

• Two excited Λ states listed in the 2020 RPP edition:

P. A. Zyla et al. [Particle Data Group], PTEP 2020 (2020) 083C01

Citation: P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)



 $J^P = \frac{1}{2}^-$ Status: **

OMITTED FROM SUMMARY TABLE See the related review on "Pole Structure of the $\Lambda(1405)$ Region."

- a new two-star resonance at 1380 MeV
- still not in the summary table
- there are more such two-pole states!

Citation: P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)

Л(1405) 1/2⁻⁻

 $I(J^{P}) = 0(\frac{1}{2}^{-})$ Status: ****

In the 1998 Note on the $\Lambda(1405)$ in PDG 98, R.H. Dalitz discussed the S-shaped cusp behavior of the intensity at the $N-\overline{K}$ threshold observed in THOMAS 73 and HEMINGWAY 85. He commented that this behavior "is characteristic of S-wave coupling; the other below threshold hyperon, the $\Sigma(1385)$, has no such threshold distortion because its $N-\overline{K}$ coupling is P-wave. For $\Lambda(1405)$ this asymmetry is the sole direct evidence that $J^P = 1/2^-$."

A recent measurement by the CLAS collaboration, MORIYA 14, definitively established the long-assumed $J^P=1/2^-$ spin-parity assignment of the $\Lambda(1405)$. The experiment produced the $\Lambda(1405)$ spin-polarized in the photoproduction process $\gamma p \rightarrow K^+ \Lambda(1405)$ and measured the decay of the $\Lambda(1405)$ (polarized) $\rightarrow \Sigma^+$ (polarized) π^- . The observed isotropic decay of $\Lambda(1405)$ is consistent with spin J=1/2. The polarization transfer to the $\Sigma^+({\rm polarized})$ direction revealed negative parity, and thus established $J^P=1/2^-$.

See the related review(s):

Pole Structure of the $\Lambda(1405)$ Region

Hyodo, UGM

- this is a fascinating phenomenon intimately tied to molecular structures
- for a review, see UGM, Symmetry 12 (2020) 981

Prospects: Survey of hadronic molecules I

• Simple model with contact interactions \rightarrow spectrum of heavy-antiheavy molecules



X(3872) and related states

Dong, Guo, Zou, Progr. Phys. 41 (2021) 65

- \bullet X(3872) as a $\bar{\boldsymbol{D}}\boldsymbol{D}^{\star}$ bound state
- Negative C-parity partner seen at COMPASS
 Phys. Lett. B 783 (2018) 334
- *DD* bound state predicted
 from lattice QCD
 Prelovsek et al., 2011.02542
- ullet Evidence for a $D_s^\star ar{D}_s^\star$ virtual state



33

Prospects: Survey of hadronic molecules II

Isoscalar vectors and related states



- \bullet $Y(4260)/\psi(4230)$ as a $ar{D}D_1$ bound state
- Vector charmonia around 4.4 GeV unclear
- Evidence for a $1^{--} \Lambda_c \overline{\Lambda}_c$ bound state in BESIII data BESIII, PRL **120** (2018) 132001



- Sommerfeld factor
- Near-threshold pole
- Different from Y(4630/4660)

• Many 1⁻⁻ states above 4.8 GeV

SPARES

Summary and outlook

- Hadronic molecules are a particular manifestation of non-conventional states
 → they appear in nuclear and hadronic physics (also 3-body states)
- Closeness to two-particle thresholds allows to formulate suitable NREFTs
 - \hookrightarrow systematic access to production, decay and other processes
- Must differentiate between long-distance and short-distance processes
 - \hookrightarrow can lead to misconceptions about the dynamcis of such states

More than 60 years after Weinberg's groundbreaking work on the compositeness of the deuteron, we are now in the position to identity and understand many more of such loosely bound states through an interplay of experiment, theory and lattice simulations. This leads to a paradigm shift: The QCD spectrum is much more than a collection of quark model states!

Mysteries in the quarkonium spectrum



- many of these close to two-particle thresholds \hookrightarrow hadronic molecules
- some are charged \hookrightarrow these must be exotic (at least four quarks)

More mysteries: Charm-strange mesons

- observed 2003 by BaBar & CLEO, isospin-violating strong decays
- ullet mass much lower than in quark models, just below the KD/KD^* threshold



Salient features of QCD

QCD Lagrangian

•
$$\mathcal{L}_{\text{QCD}} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu,a} + \sum_f \bar{q}_f (i \not \!\!\!D - \mathcal{M}) q_f + \dots$$

$$D_{\mu} = \partial_{\mu} - ig A^a_{\mu} \lambda^a / 2$$

$$G^a_{\mu\nu} = \partial_{\mu} A^a_{\nu} - \partial_{\nu} A^a_{\mu} - g [A^b_{\mu}, A^c_{\nu}]$$

$$f = (u, d, s, c, b, t)$$
• running of $\alpha_s = \frac{g^2}{4\pi} \Rightarrow \Lambda_{\text{QCD}} = 210 \pm 14 \text{ MeV}$ $(N_f = 5, \overline{MS}, \mu = 2 \text{ GeV})$

• light (u,d,s) and heavy (c,b,t) quark flavors [two different worlds]:

 $egin{aligned} m_{ ext{light}} \ll \Lambda_{ ext{QCD}} & m_{ ext{heavy}} \gg \Lambda_{ ext{QCD}} & \ m_u &= 2.2^{+0.6}_{-0.4} \, ext{MeV} & m_c &= 1.28 \pm 0.03 \, ext{GeV} & \ m_d &= 4.7^{+0.5}_{-0.4} \, ext{MeV} & m_b &= 4.18^{+0.04}_{-0.03} \, ext{GeV} & \ m_s &= 96^{+8}_{-4} \, ext{MeV} & m_t &= 173.1 \pm 0.6 \, ext{GeV} & \end{aligned}$



Limits of QCD

Iight quarks:

$$\mathcal{L}_{ ext{QCD}} = ar{q}_L \, i D \hspace{-.5mm}/ q_L + ar{q}_R \, i D \hspace{-.5mm}/ q_R + \mathcal{O}(m_f/\Lambda_{ ext{QCD}})$$

q = (u, d, s)

- L and R quarks decouple \Rightarrow chiral symmetry

- spontaneous chiral symmetry breaking \Rightarrow pseudo-Goldstone bosons

- pertinent EFT \Rightarrow chiral perturbation theory (CHPT)

• heavy quarks:

$${\cal L}_{
m QCD} = ar Q \, i v \cdot D \, Q + {\cal O}(\Lambda_{
m QCD}/m_f) \, .$$

Q = (c, b)

- independent of quark spin and flavor

 \Rightarrow SU(2) spin and SU(2) flavor symmetries (HQSS and HQFS)

- pertinent EFT \Rightarrow heavy quark effective field theory (HQEFT)

• heavy-light systems:

- heavy quarks act as matter fields coupled to light pions
- combine CHPT and HQEFT