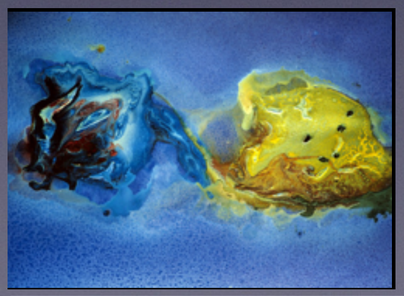
TECHNISCHE UNIVERSITÄT MÜNCHEN

EFTs (and lattice) for Quarkonium and X Y Z

in memory of Misha Voloshin



Nora Brambilla

Physik Department TUM
T30f

- Exotic states i.e. states different for qqbar or qqq have been predicted before and after the inception of QCD: in the last decades they (X Y Z) have been observed in the sector with two heavy quarks QQbar, at or above the quarkonium strong decay threshold

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- X Y Z offer the unique possibility to investigate the dynamical properties of strongly correlated systems in QCD

- This is very interesting for two reasons: we have now the tools to describe quarkonium in Quantum Field Theorys —Nonrelativistic Effective Quantum Field Theories plus lattice ; —we can extend these tools to X Y Z

Plan of the talk

- Quarkonium: multiscale system -> hierarchy of scales/hierarchy of NREFTs

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- in relation to X(6900) :quarkonium-quarkonium van der Waals interaction: bound states?

Material for discussion/references

Heavy quarkonium: progress, puzzles, and opportunities
N. Brambilla (Munich, Tech. U.) *et al.*. Oct 2010. 181 pp.
Published in **Eur.Phys.J. C71 (2011) 1534**
e-Print: **arXiv:1010.5827 [hep-ph]**-

QCD and Strongly Coupled Gauge Theories: Challenges and Perspectives
N. Brambilla (Munich, Tech. U.) *et al.*. Apr 2014. 241 pp.
Published in **Eur.Phys.J. C74 (2014) no.10, 2981**
e-Print: **arXiv:1404.3723**

chapter on exotics

N. Brambilla, S. Eidelman, C. Hanhart, A. Nefediev, C. P. Shen, C. E. Thomas, A. Vairo and C. Z. Yuan

The XYZ states: experimental and theoretical status and perspectives

Phys.Rept. 873 (2020) 1-154 • e-Print: 1907.07583 [hep-ex]

Effective field theories for heavy quarkonium
Nora Brambilla, Antonio Pineda, Joan Soto, Antonio Vairo
Rev.Mod.Phys. 77 (2005) 1423
e-Print: **hep-ph/0410047**

Born-Oppenheimer approximation in an effective field theory language
Nora Brambilla , Gastão Krein, Jaume Tarrús Castellà, Antonio Vairo
Phys.Rev. D97 (2018) no.1, 016016
e-Print: **arXiv:1707.09647**

Quarkonium Hybrids with Nonrelativistic Effective Field Theories
Matthias Berwein , Nora Brambilla, Jaume Tarrús Castellà, Antonio Vairo
Phys.Rev. D92 (2015) no.11, 114019
e-Print: **arXiv:1510.04299**

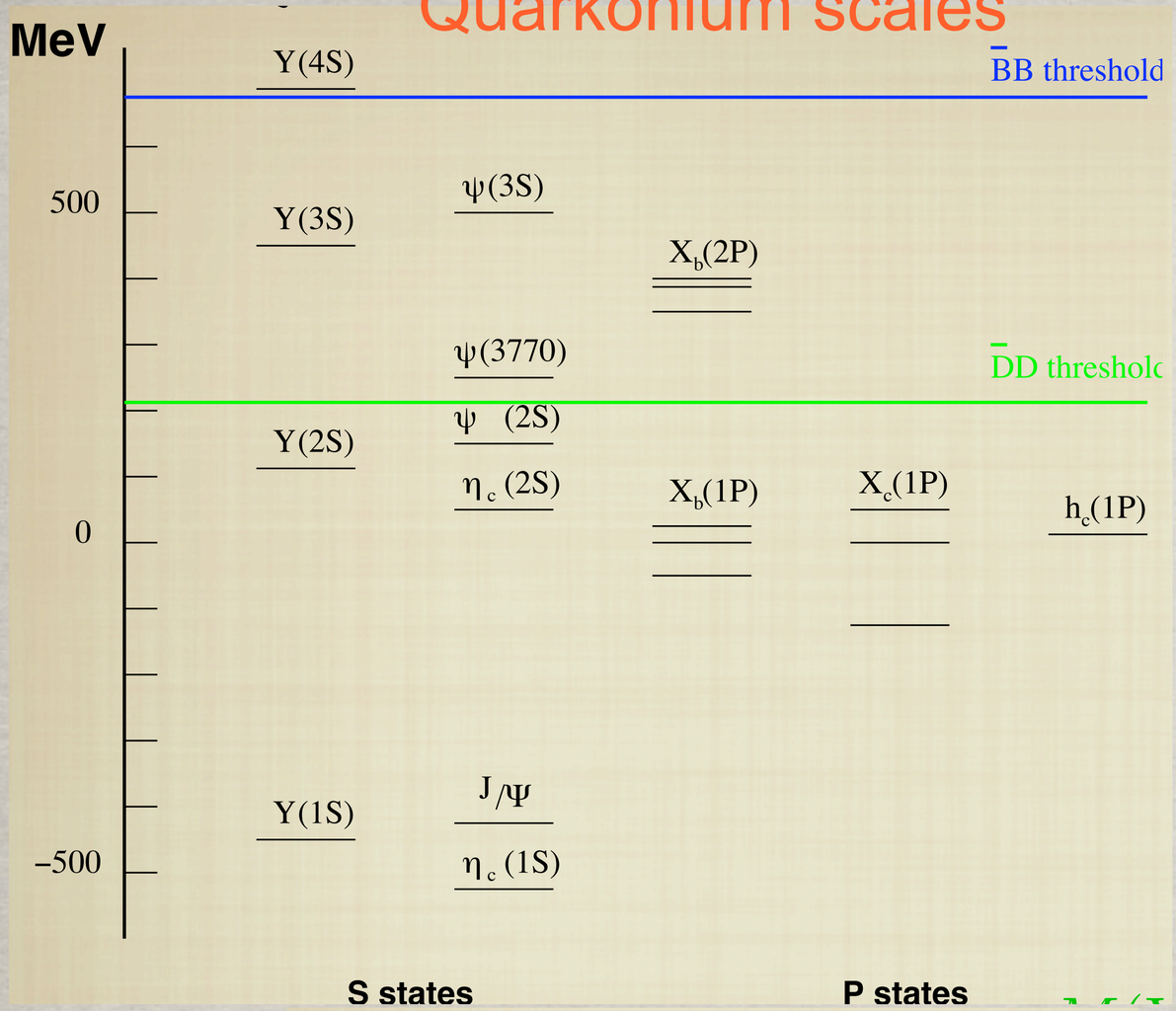
QCD spin effects in the heavy hybrid potentials and spectra Nora Brambilla, Wai Kin Lai, J. Segovia, J. Tarrus *Phys.Rev.D* 101 (2020) 5, 054040 • e-Print: 1908.11699

Spin structure of heavy-quark hybrids
N. Brambilla, Wai KIn Lai, J. Segovia, J. Tarrus
A. Vairo
Phys.Rev.D 99 (2019) 1, 014017, e-Print: 1805.07713 [hep-ph]

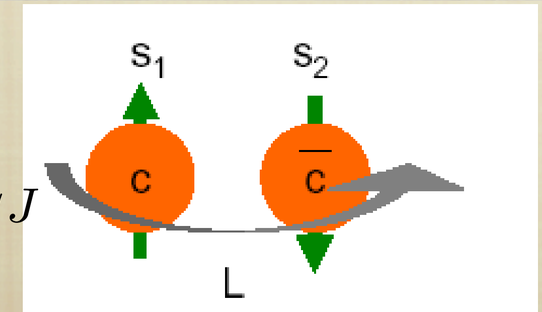
Long range properties of 1S bottomonium states
N. Brambilla, G. Krein, J.. Tarrus, A. Vairo
Phys.Rev.D 93 (2016) 5, 054002 • e-Print: 1510.05895

Quarkonium: Physical Scales and physical significance

Quarkonium scales



$^{2S+1}L_J$

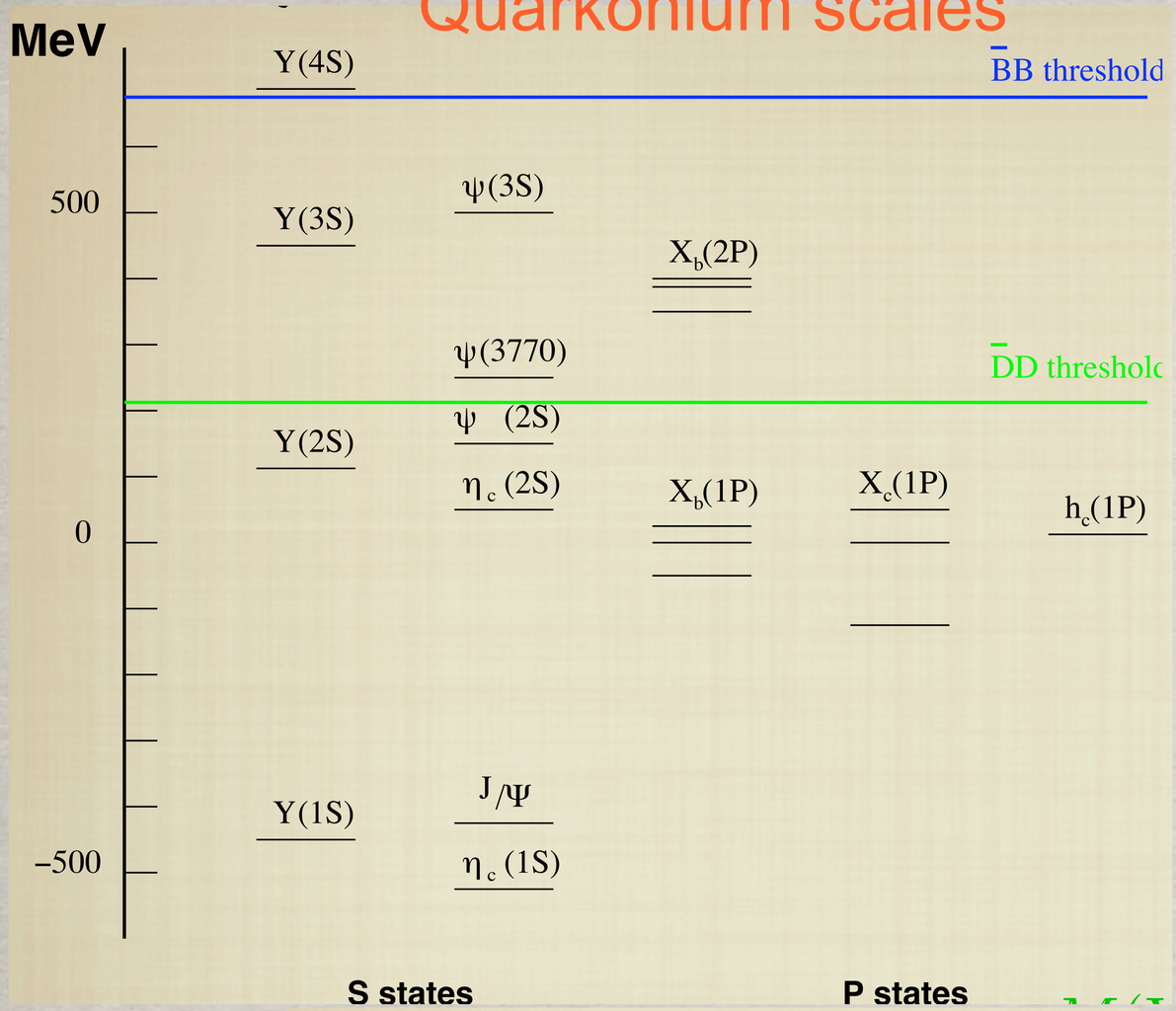


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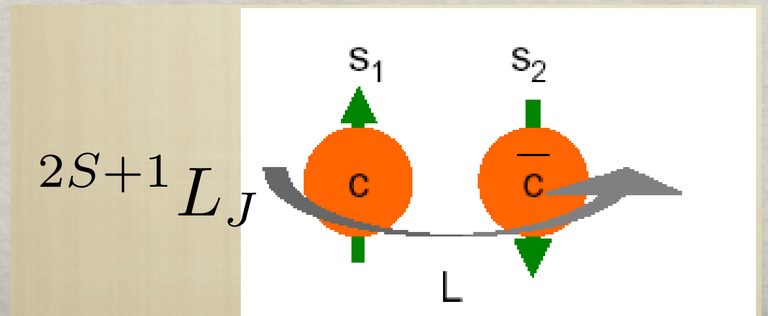
$$m_Q \gg \Lambda_{\rm QCD}$$

$$m_b \simeq 5\,{\rm GeV}; m_c \simeq 1.5\,{\rm GeV}$$

Quarkonium scales



Normalized with respect to $\chi_b(1P)$ and $\chi_c(1P)$



The system is nonrelativistic(NR)

$$\Delta E \sim mv^2, \Delta_{fs} E \sim mv^4$$

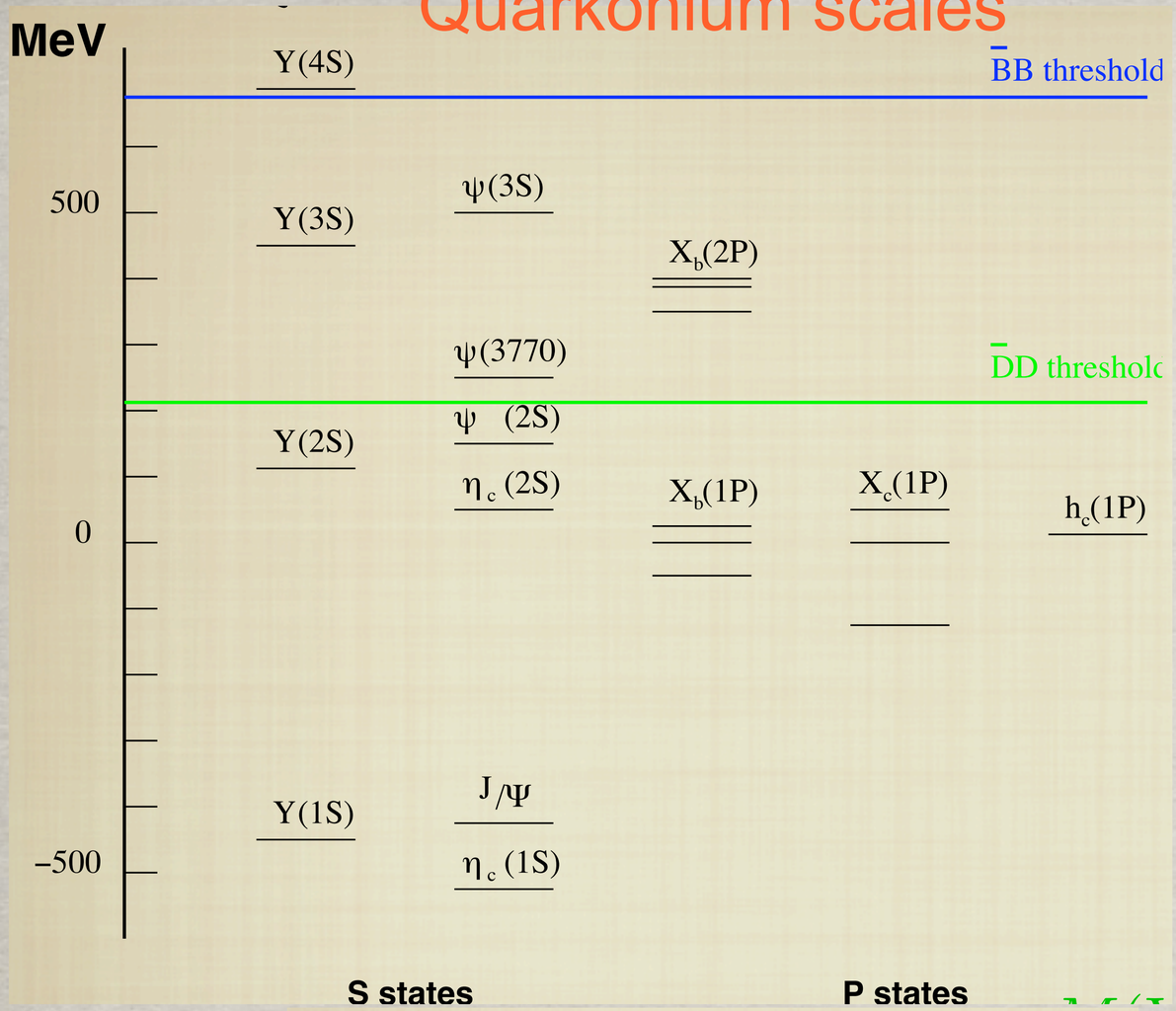
$$v_b^2 \sim 0.1, v_c^2 \sim 0.3$$

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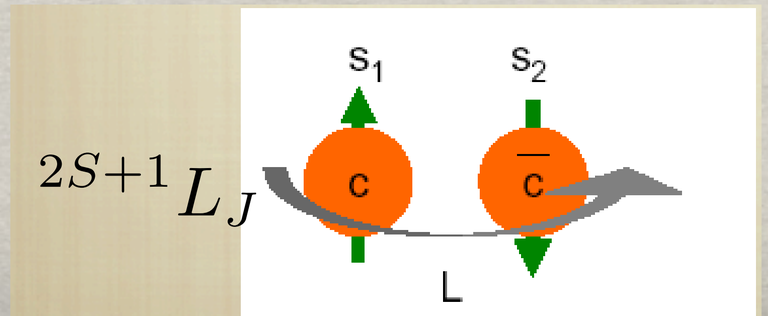
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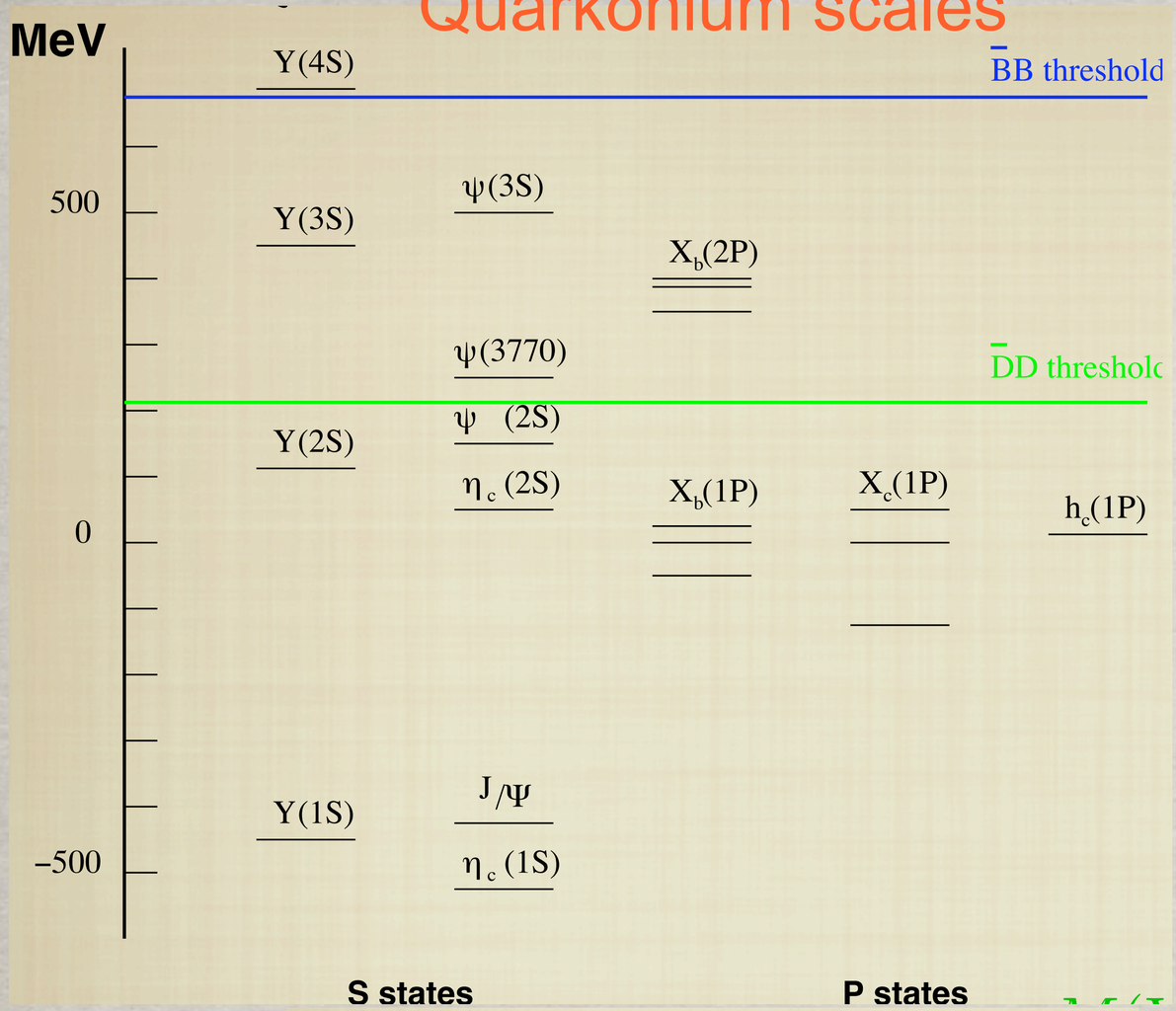
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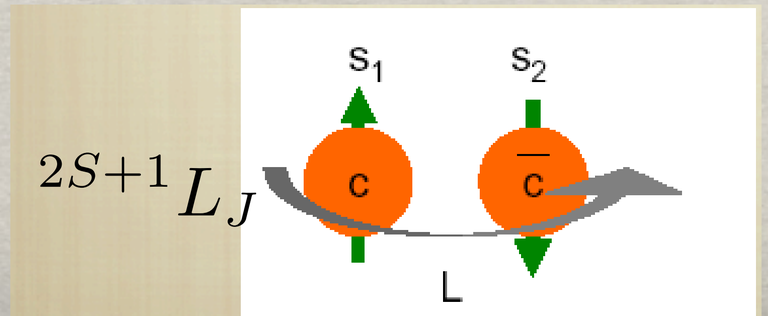
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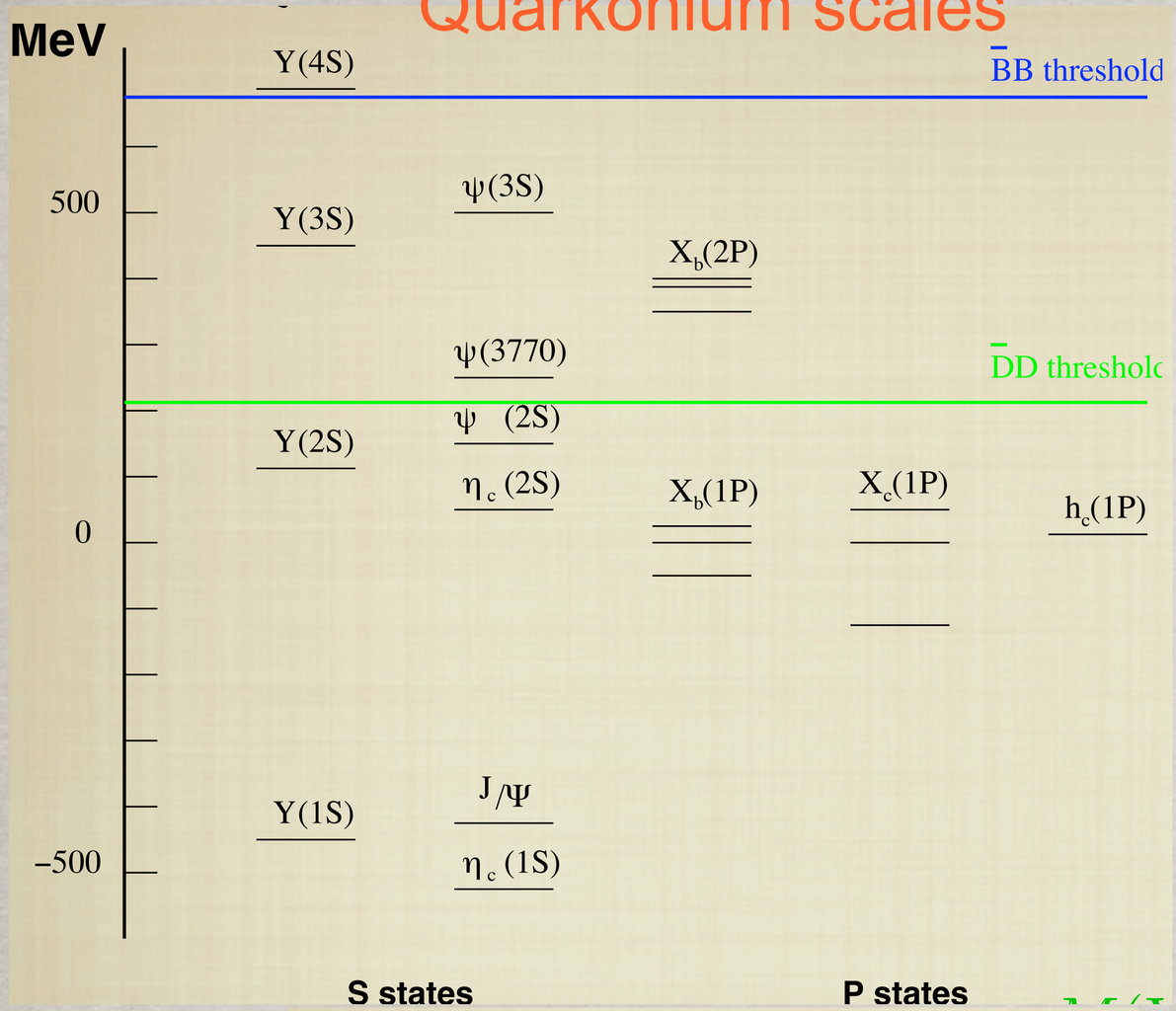
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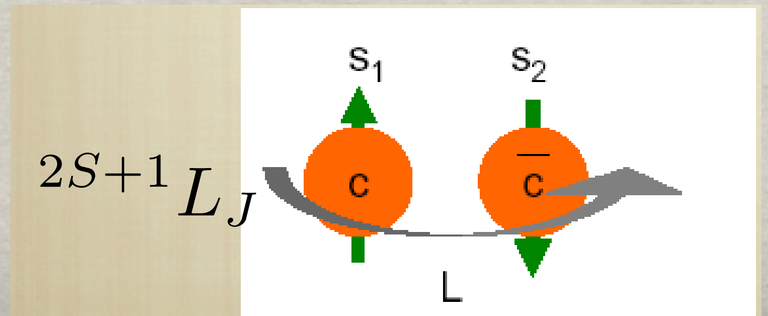
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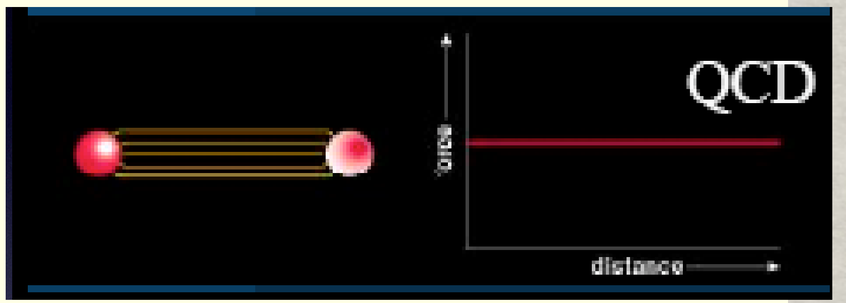
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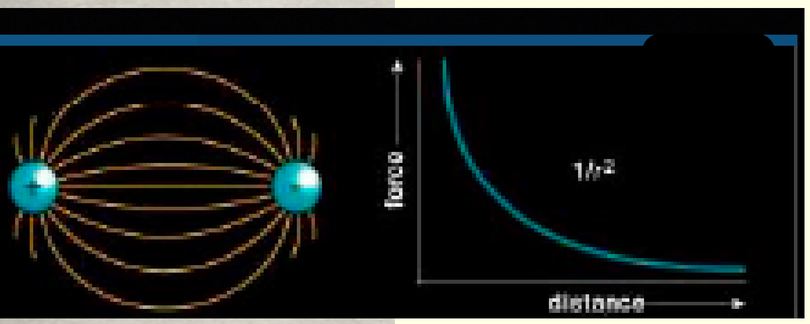
Quarkonium as a confinement and deconfinement probe

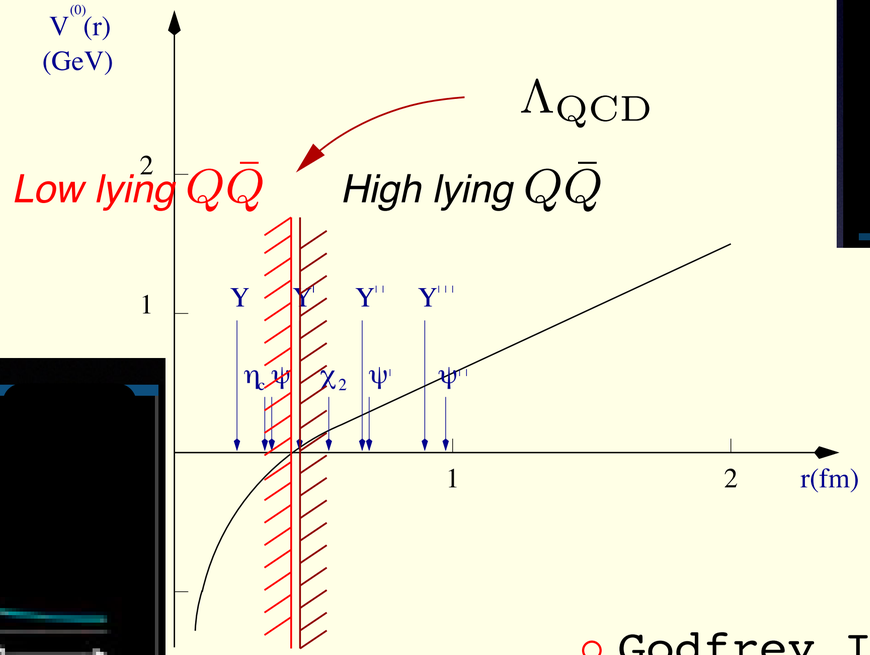
The rich structure of separated energy scales makes QQbar an ideal probe

At zero temperature

- The different quarkonium radii provide different measures of the transition from a Coulombic to a confined bound state.





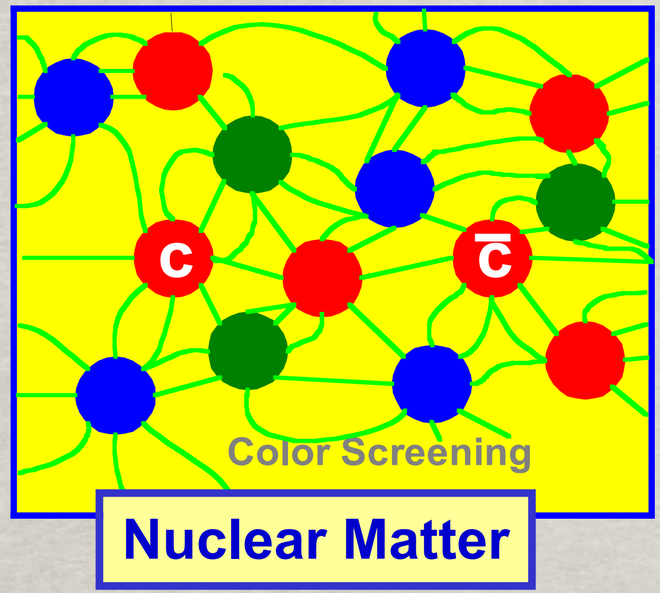


- Godfrey Isgur PRD 32(85)189

quarkonia probe the perturbative (high energy) and non perturbative region (low energy) as well as the transition region in dependence of their radius r: they help us to understand the structure of the nonperturbative corrections in a strongly coupled theory

Quarkonium as a confinement and deconfinement probe

At finite temperature T they are sensitive to the formation of a quark gluon plasma via color screening



Debye charge screening $m_D \sim gT$

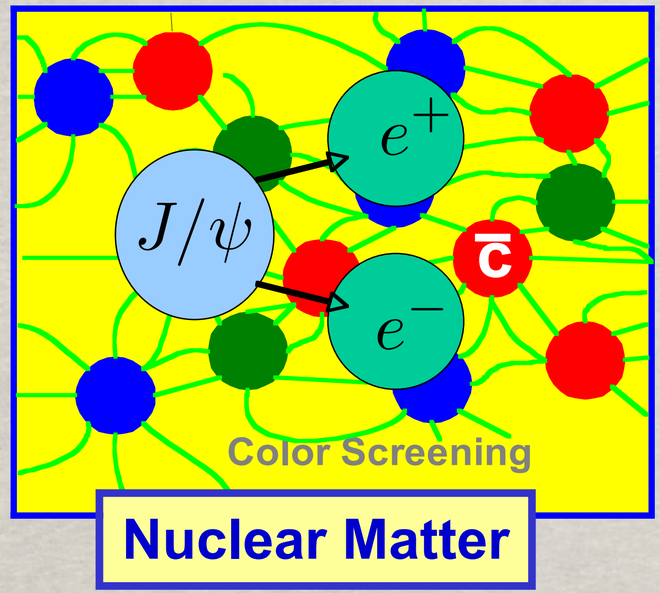
$$V(r) \sim -\alpha_s \frac{e^{-m_D r}}{r}$$

Matsui Satz 1986

A simulated collision of lead ions, courtesy the A
experiment at CERN

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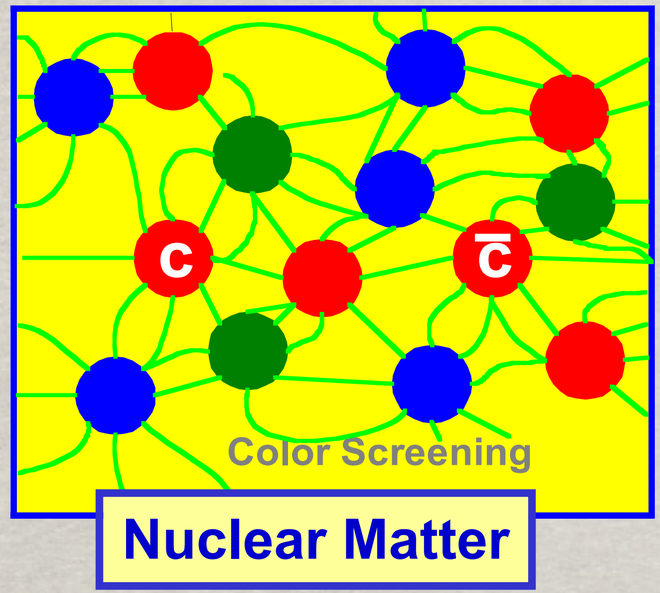
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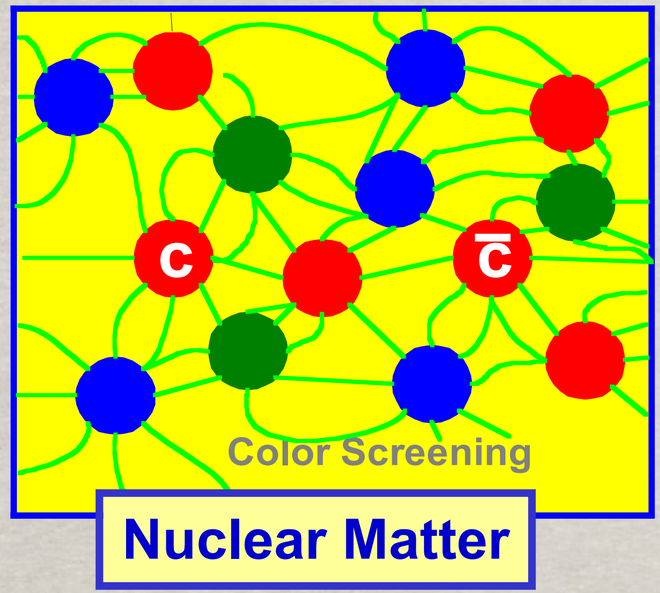
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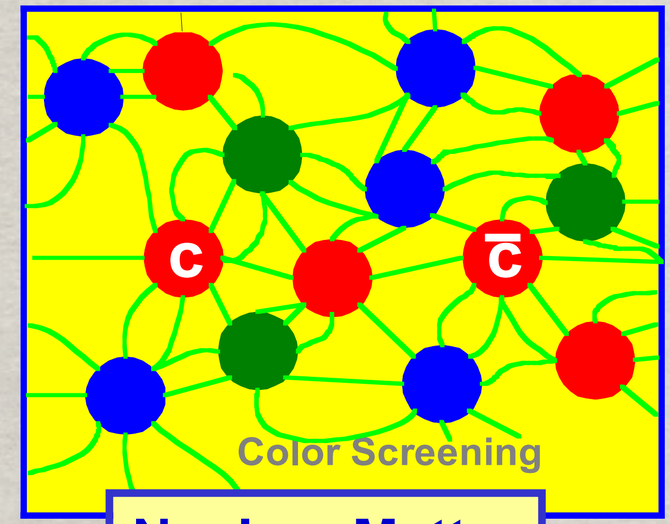
Matsui Satz 1986

$$r \sim \frac{1}{m_D} \longrightarrow$$ Bound state dissolves

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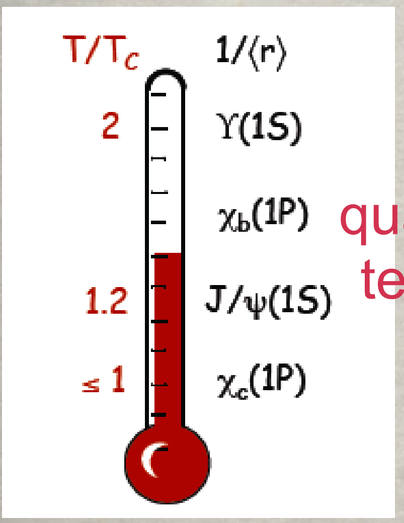


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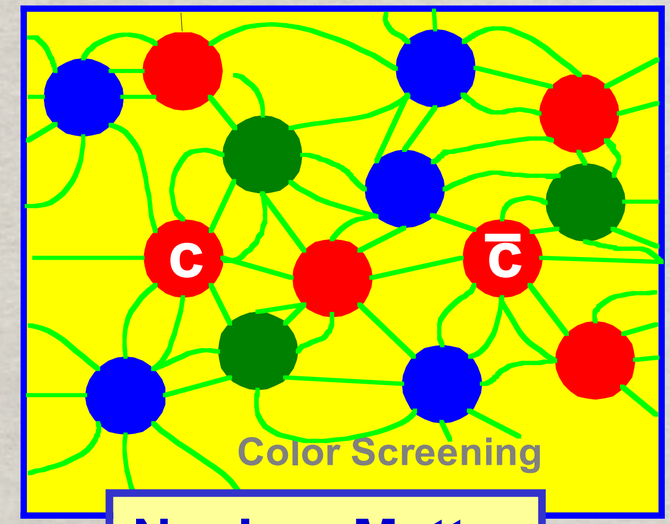


quarkonia dissociate at different temperature in dependence of their radius: they are a Quark Gluon Plasma thermometer

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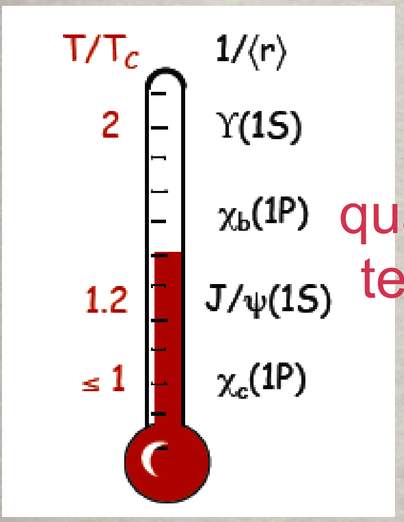
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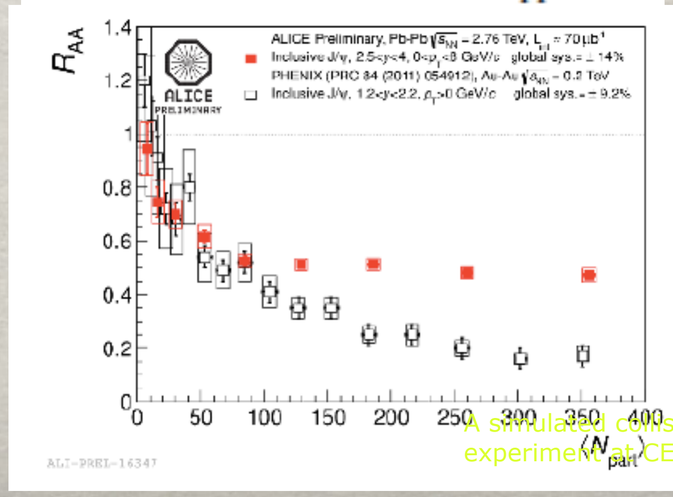
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nuclear modificatio

$$R_{AA} = \frac{Yield^{q\bar{q}}_{AA}}{\langle N_{coll} \rangle \times Yield^{q\bar{q}}_{pp}}$$



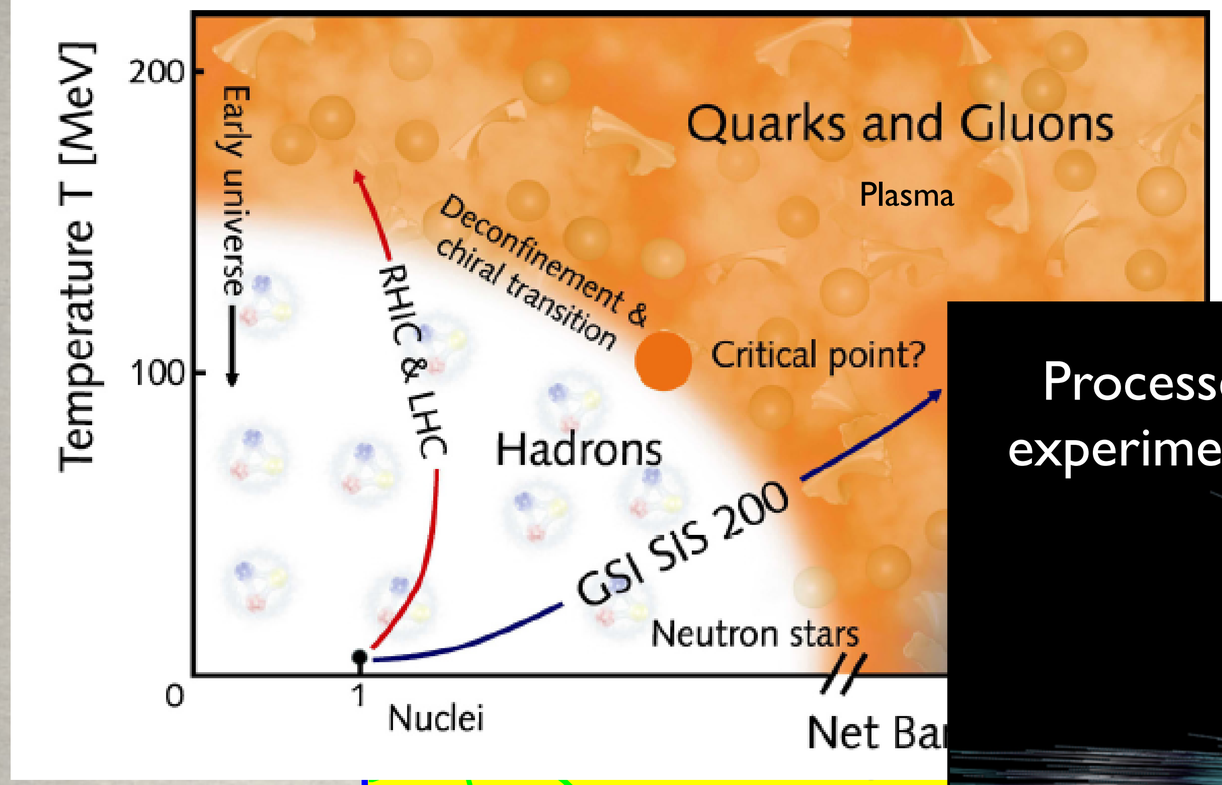
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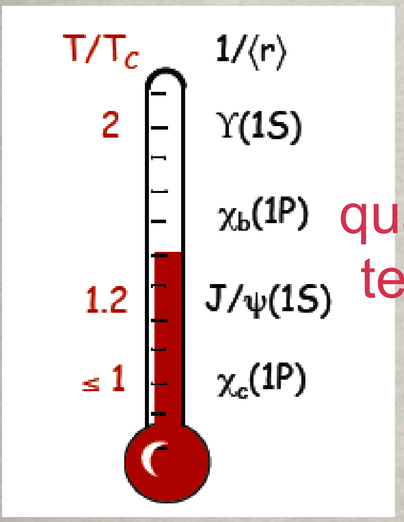
A simulated collision of lead ions, courtesy the ALICE experiment at CERN

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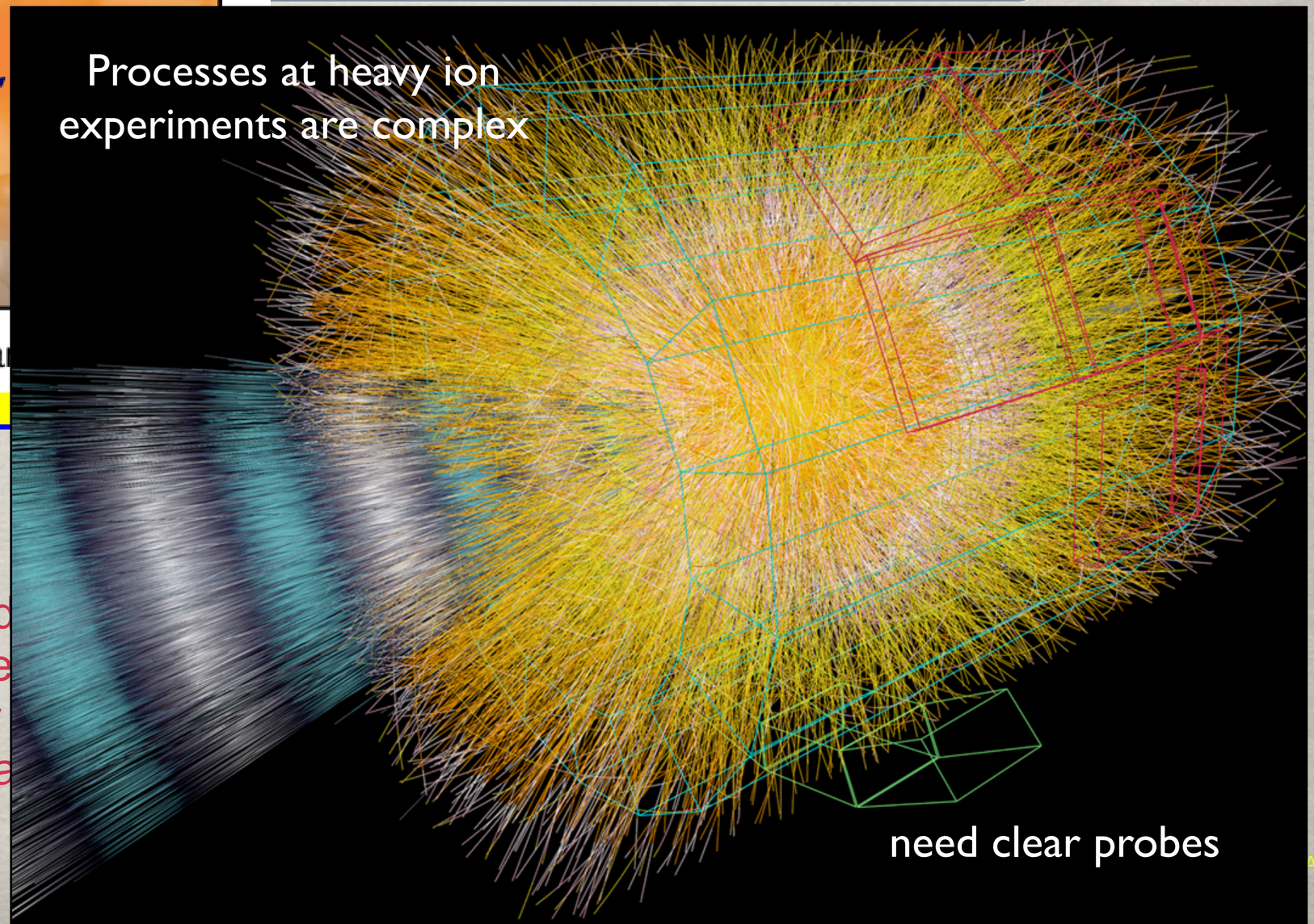
sitive to the formation of a quark gluon



Nuclear Matter



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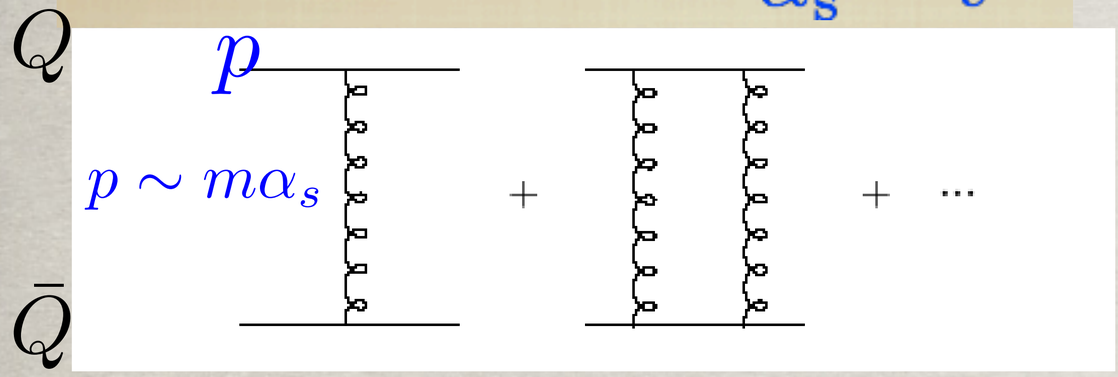
QCD THEORY OF QUARKONIUM: A VERY CHALLENGING PROBLEM

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Close to the bound state $\alpha_s \sim v$

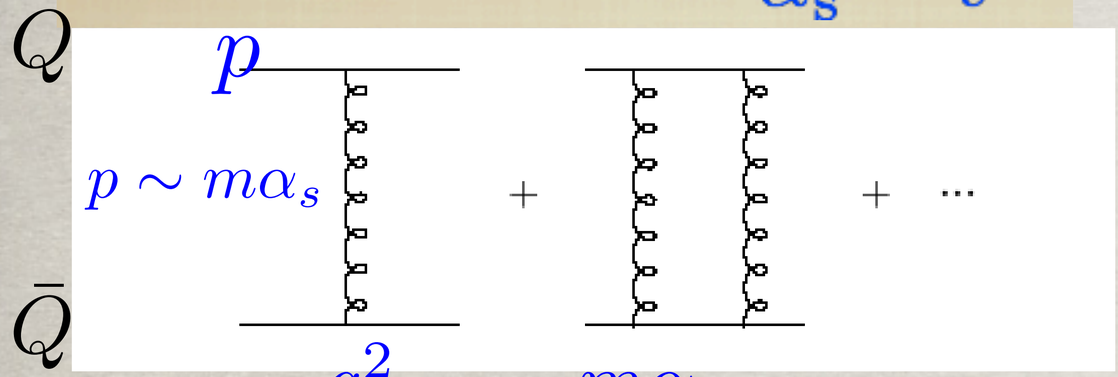
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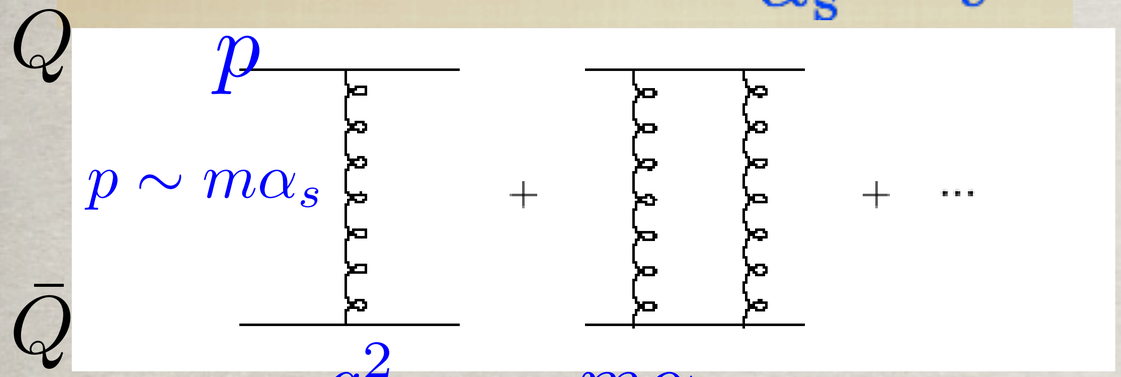
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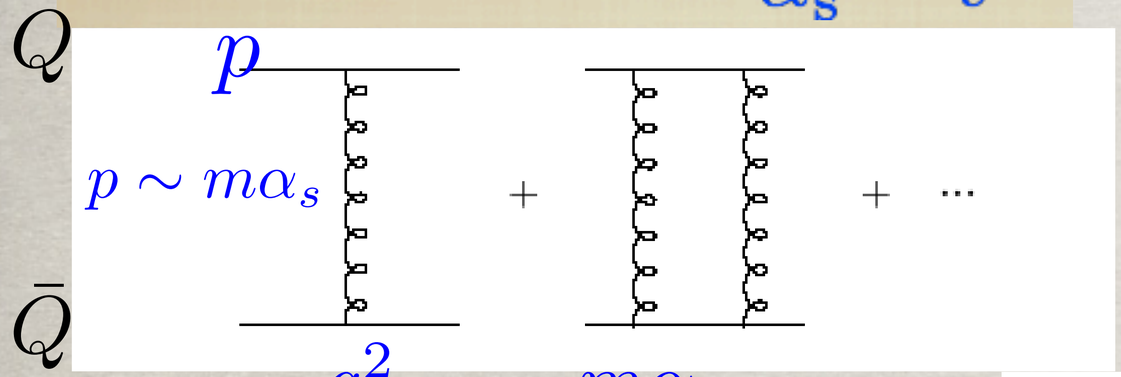


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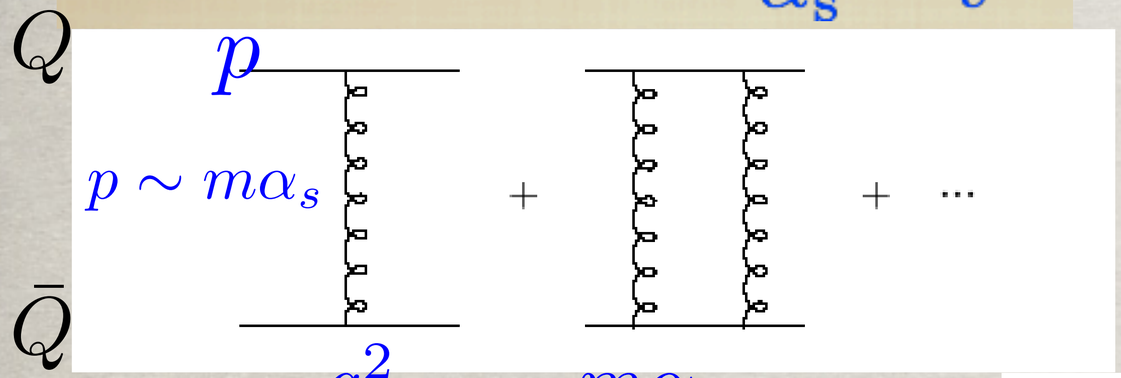
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- From $\left(\frac{p^2}{m}+V\right)\phi = E\phi \rightarrow p \sim mv$ and $E = \frac{p^2}{m}+V \sim mv^2$.

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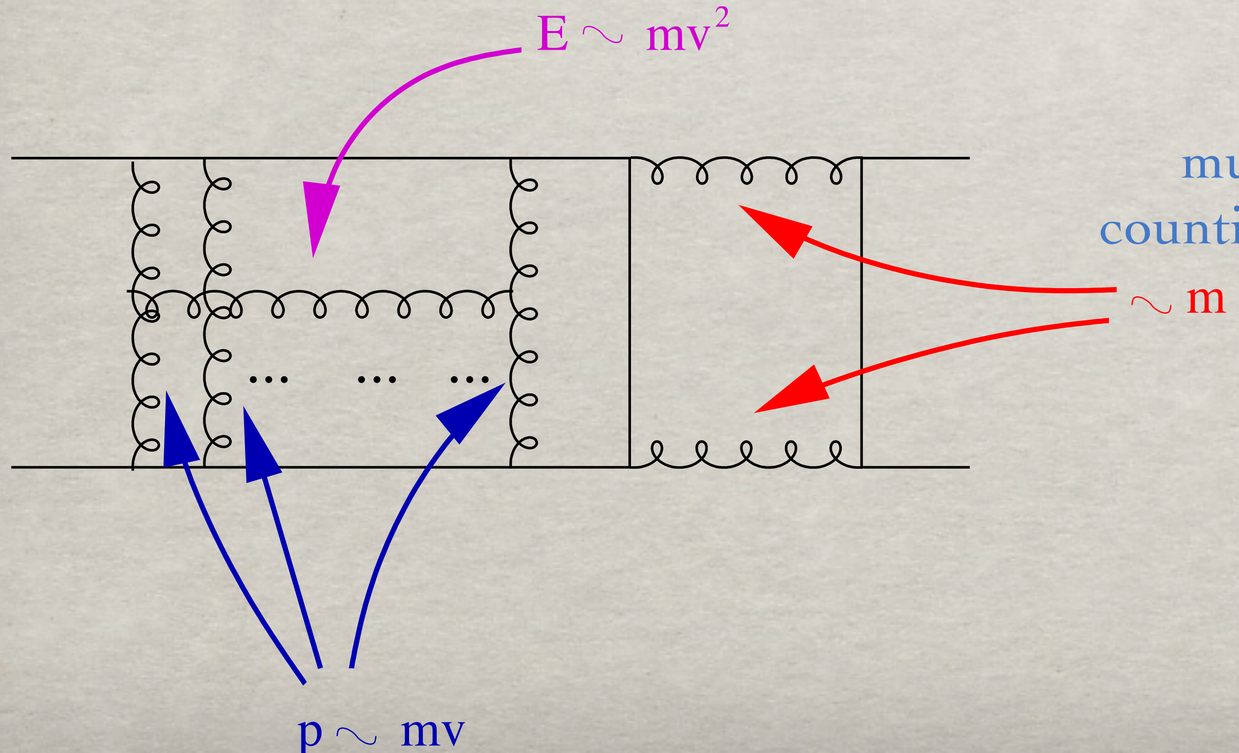
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- From $\left(\frac{p^2}{m}+V\right)\phi = E\phi \rightarrow p \sim mv$ and $E = \frac{p^2}{m}+V \sim mv^2$.



multiscale diagrams have a complicate power counting and contribute to all orders in the coupling

Difficult also for the lattice!

$$L^{-1} \ll \lambda \ll \Lambda \ll a^{-1}$$

NREFTS for quarkonium below threshold construction and applications

a hierarchy of EFTS can be formulated in correspondence to the hierarchy of scales



An effective field theory makes the expansion in λ/Λ explicit at the Lagrangian level.

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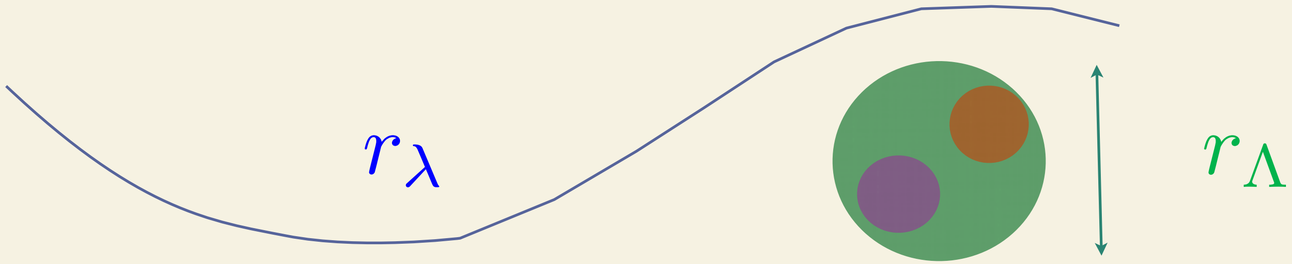
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RANGE OF VALIDITY OF THE EFT: ENERGY $< \mu$

$\Rightarrow \mathcal{L}_{\rm EFT}$ is made of all operators O_n that may be built from the effective degrees of freedom and are consistent with the symmetries of $\mathcal{L}$.

Effective Field Theories

$$\mathcal{L}_{\mathrm{EFT}} = \sum_n c_n(\Lambda, \mu) \frac{O_n(\mu, \lambda)}{\Lambda^n}$$

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low energy operator

Wilson coefficient

large scale

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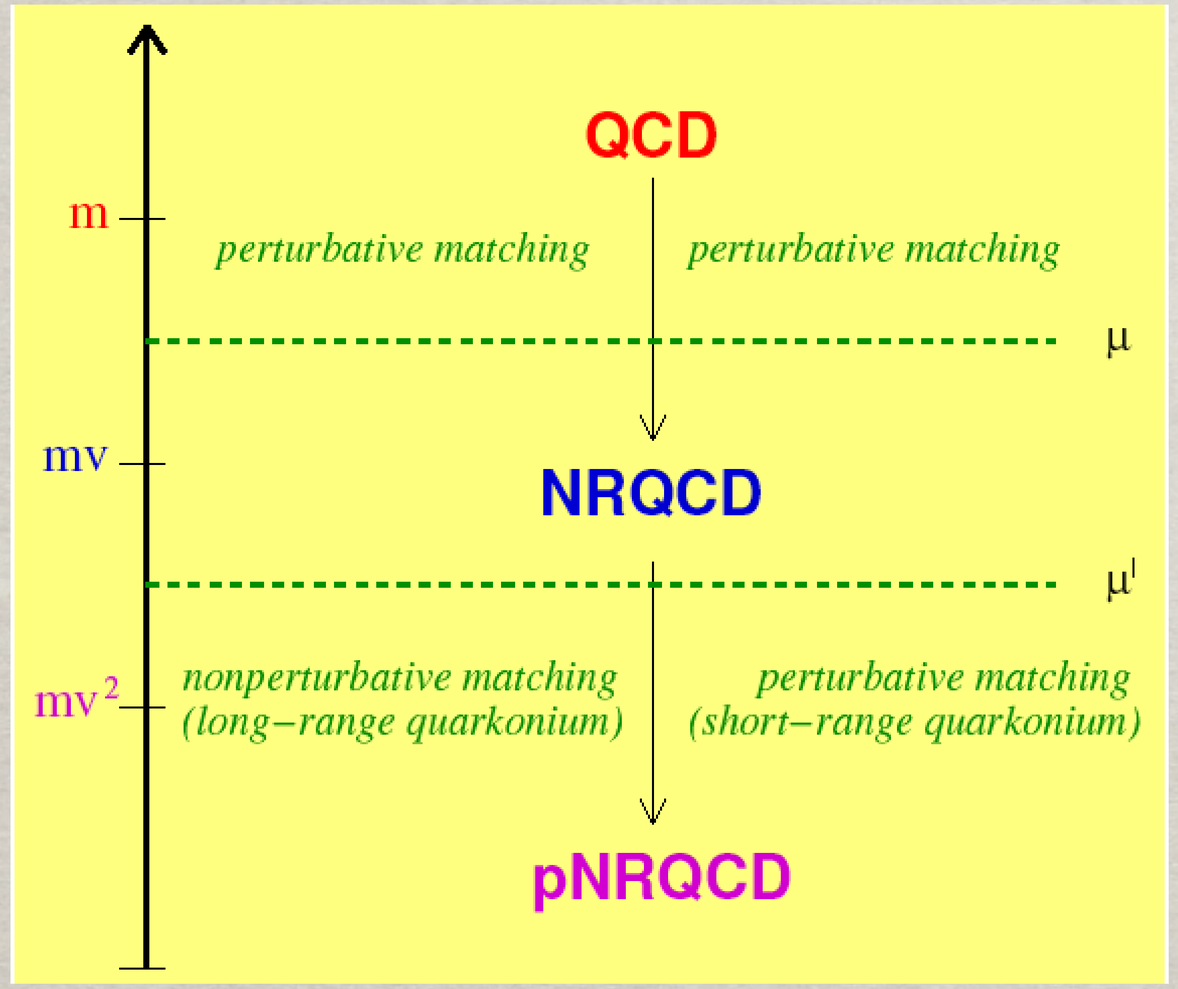
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- If $\Lambda \gg \Lambda_{\rm QCD}$ then $c_n(\Lambda/\mu)$ may be calculated in perturbation theory.

- Symmetries of the system become manifest; MORE PREDICTIVE POWER !!!
- Large $\log(\Lambda/\lambda)$ can be resummed via RG. (Renormalization group)

Quarkonium with Non relativistic Effective Field Theories

Color degrees of freedom
3X3bar=1+8
singlet and octet QQbar



Hard

Soft
(relative
momentum)

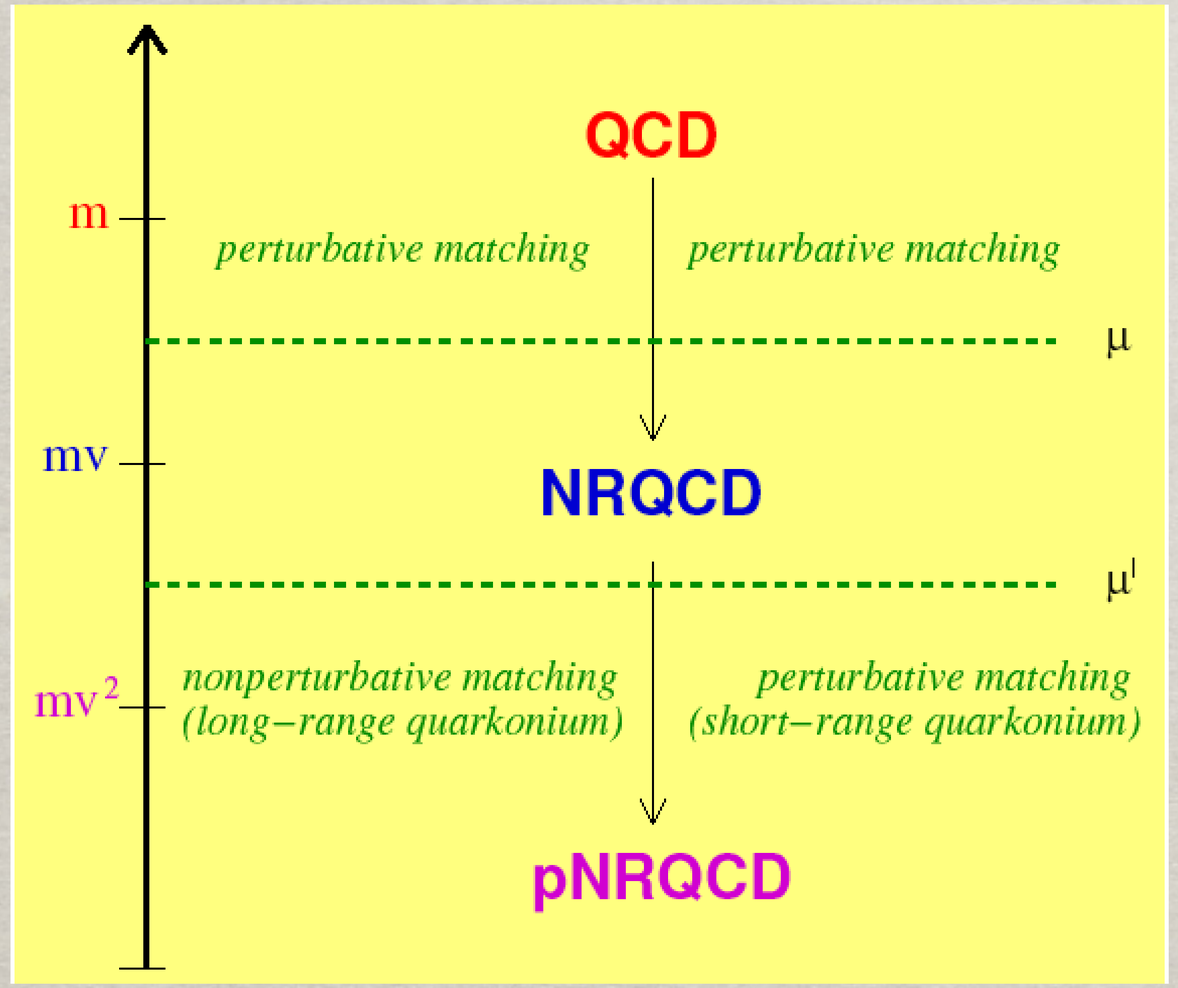
Ultrasoft
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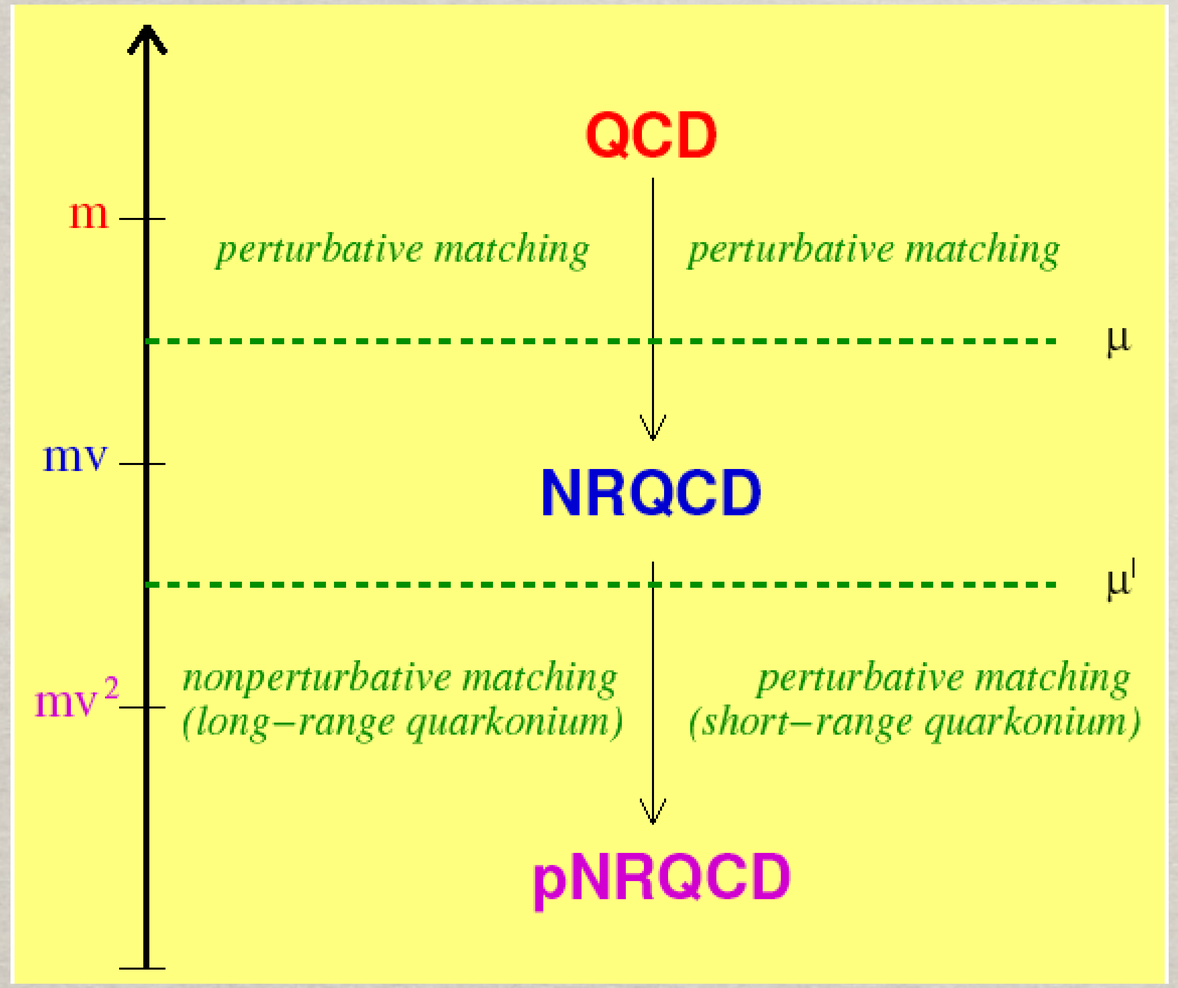
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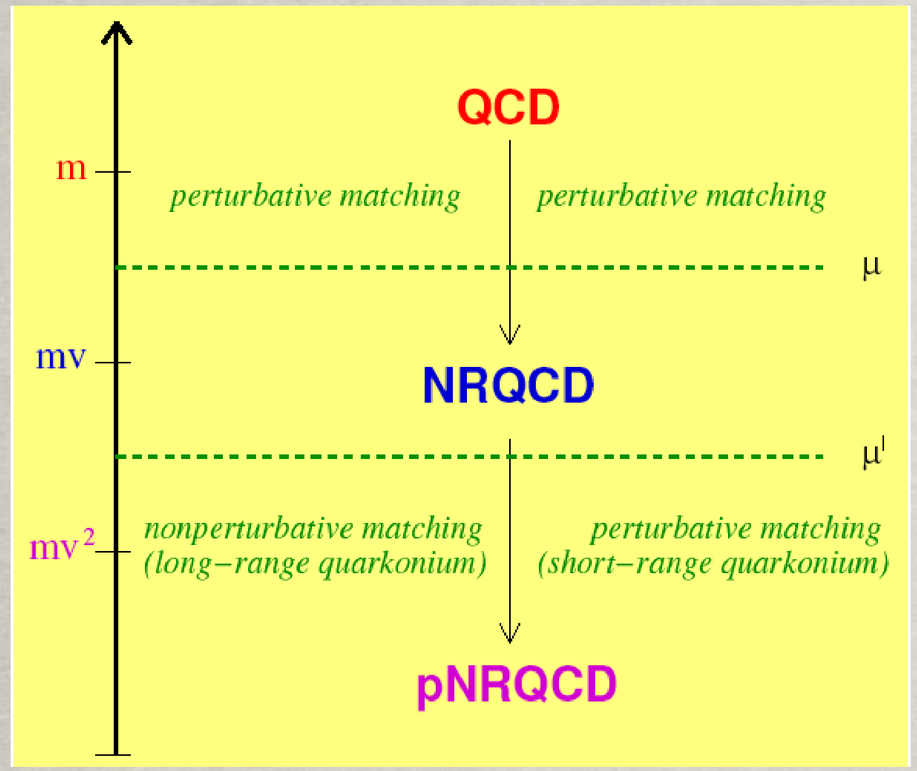
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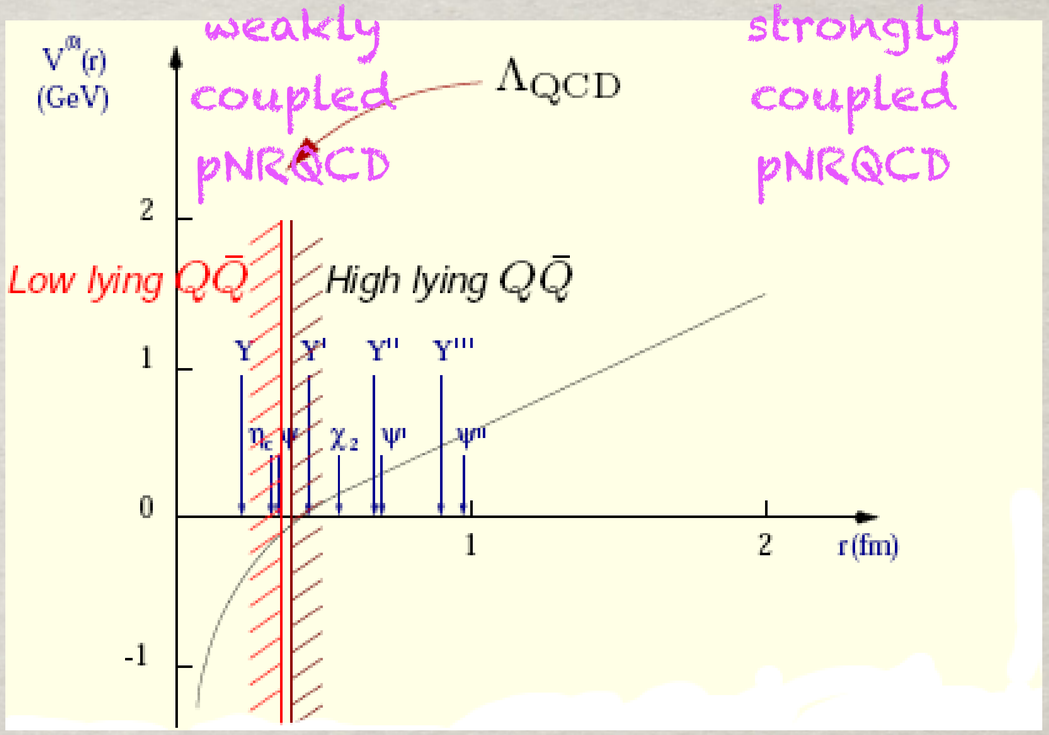
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Quarkonium with NR EFT: pNRQCD

Pineda, Soto 97, N.B., Pineda, Soto, Vairo 99
N.B. Vairo, Pineda, Soto 00--014
N.B., Pineda, Soto, Vairo Review of Modern Physis 77(2005) 1423





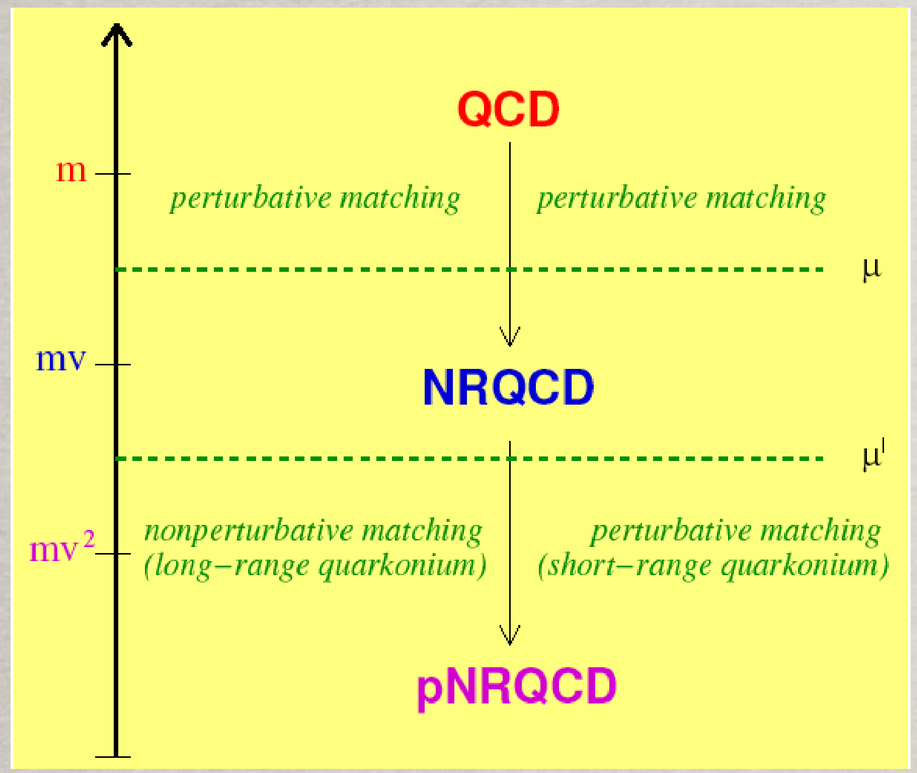
In QCD another scale is relevant $\Lambda_{\rm QCD}$

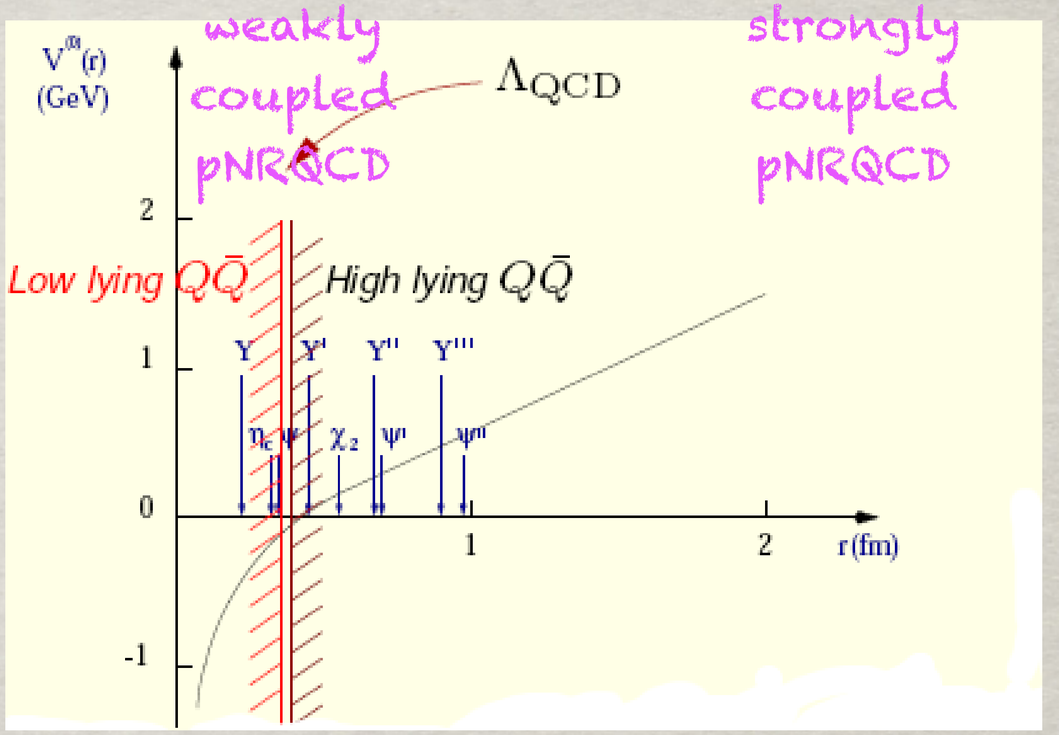
A potential picture arises at the level of pNRQCD:

- the potential is perturbative if $mv \gg \Lambda_{\rm QCD}$
- the potential is non-perturbative if $mv \sim \Lambda_{\rm QCD}$

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The matching coefficients of pNRQCD are the potentials
Non potential effects are there as correction to the leading picture
A predcction of pNRQCD is prediction of QCD

NRQCD: Enables spectrum lattice calculation (relevant also for X Y Z see talk of Pflaumer)

Gives a framework for production at colliders (relevant for X Y Z , see production studies of X(6900)
see e.g. eprints 2104.03887, 2009.08376

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pNRQCD at finite temperature : defines the quark-antiquark potential at finite temperature; with open quantum systems allows to describe the nonequilibrium evolution of quarkonium inside the quark gluon plasma in heavy ion collisions —> may be relevant for the description of X Y Z in heavy ion collisions

N. Brambilla, M. Escobedo, J. Ghiglieri. P. Petreczky, J. Soto,
M. Strickland, A. Vairo, P. Vandergriend,J. Weber, eprints 2012.01240, 1903.08 063, 1711.04515, 0804.0993

Weakly coupled pNRQCD

- Pineda Soto NP PS 64 (1998) 428
  Brambilla Pineda Soto Vairo NPB 566 (2000) 275

- If $mv \gg \Lambda_{\rm QCD}$, the matching is perturbative

  Non-analytic behaviour in $r \to$ matching coefficients V

The gauge fields are multipole expanded:

$$A(R,r,t) = A(R,t) + \mathbf{r}\cdot\boldsymbol{\nabla} A(R,t) + \dots$$

$\mathbf{R}$ = center of mass

$\mathbf{r} = Q\bar{Q}$ distance

$$\mathcal{L}^{\rm pNRQCD} = \int d^3r\, {\rm Tr}\,\{S^\dagger(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_S + \cdots)S + O^\dagger(iD_0 - \frac{\mathbf{p}^2}{m} - V_O + \cdots)O + \quad \text{LO in r}$$

$$+V_A(S^\dagger \mathbf{r}\cdot g\mathbf{E} O + O^\dagger \mathbf{r}\cdot g\mathbf{E} S) + \frac{V_B}{2}(O^\dagger \mathbf{r}\cdot g\mathbf{E} O + O^\dagger O \mathbf{r}\cdot g\mathbf{E})\} + \dots \quad \text{NLO in r}$$

$$-\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu\,a} + \sum_{i=1}^{n_f}\bar{q}_i\, i\not{D} q_i$$

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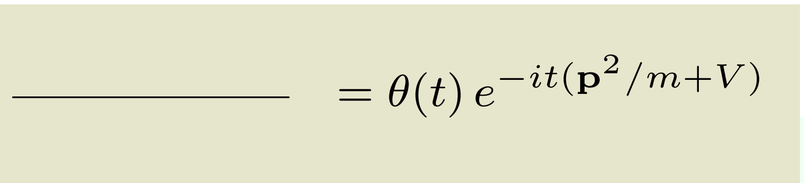
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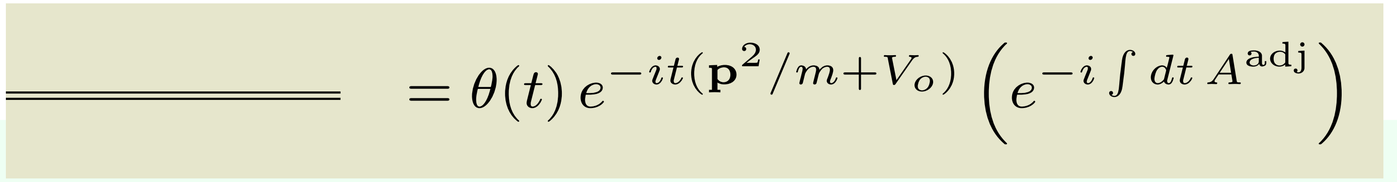
The matching coefficients are the Coulomb potential

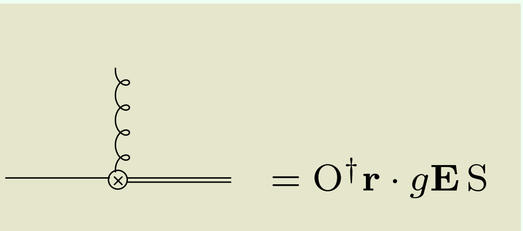
$$V_S(r) = -C_F\frac{\alpha_{\rm s}}{r} + \dots, \qquad V_o(r) = \frac{1}{2N}\frac{\alpha_{\rm s}}{r} + \dots,$$

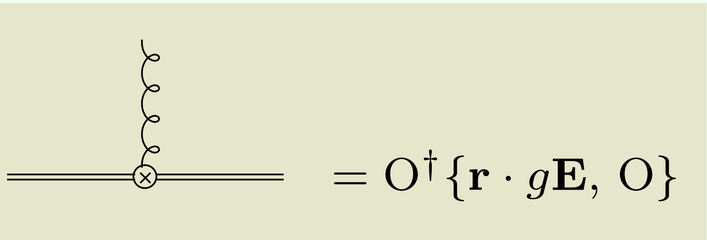
$$V_A = 1 + \mathcal{O}(\alpha_{\rm s}^2),\ V_B = 1 + \mathcal{O}(\alpha_{\rm s}^2).$$

Feynman rules









pNRQCD

- pNRQCD provides a QM description from field theory: the Schroedinger equation and the potentials appear once all scales above the binding energy have been integrated out: this provides a clear definition of the potential

- The EFT accounts for non-potential terms as well. They provide loop corrections to the leading potential picture. Retardation effects are typically related to the nonperturbative physics

- The Quantum Mechanical divergences are cancelled by the NRQCD matching coefficients.

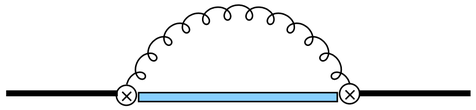
- Poincare' invariance is intact and is realized via exact relations among the matching coefficients (potentials)

These characteristics will be common to the BOEFT

Energies at order m alpha^5 (NNNLO)

$m\alpha_s^5 \ln \alpha_s$ Brambilla Pineda Soto Vairo 99, Kniehl Penin 99
$m\alpha_s^5$ Kniehl Penin Smirnov Steinhauser 02 NNLL Pineda 02

NNNLL Peset Pineda et al 2018,2019, Kiyo Sumino 2014, Beneke, Kiyo Schuler 05,08

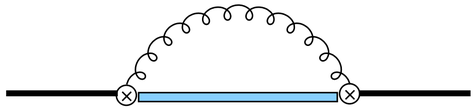
$$E_n = 2m + \langle n|\frac{p^2}{m} + V_s|n\rangle + \langle n| \quad \text{} \quad |n\rangle$$

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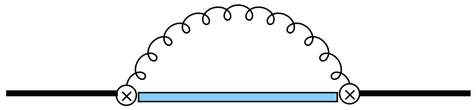
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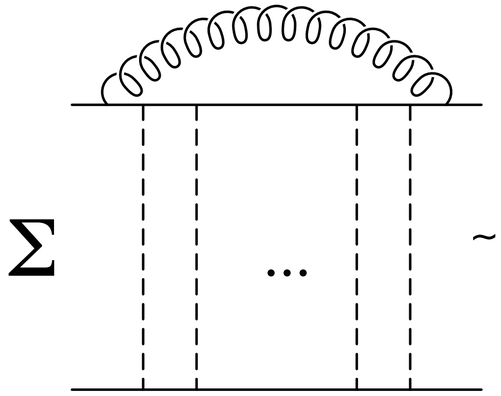
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local condensates as predicted in a paper by Misha Voloshin in 1979

—>used to extract precise (NNNLO) determination of m_c and m_b

$E_n^{(0)} - H_o \sim \Lambda_{\rm QCD} \Rightarrow$ no expansion possible, non-local condensates, analogous to the Lamb shift in QED

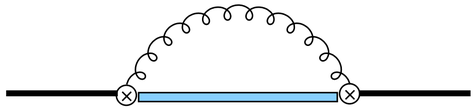


$\sim \alpha_s^5 \ln \alpha_s$ (Lamb-shift)

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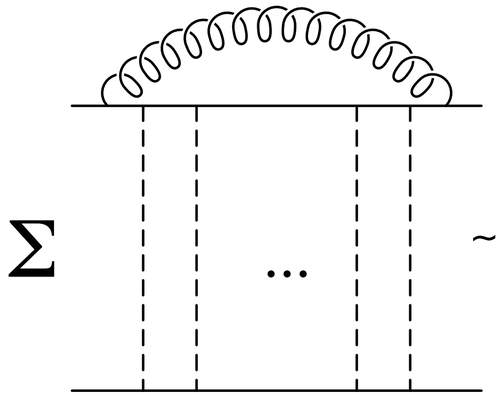
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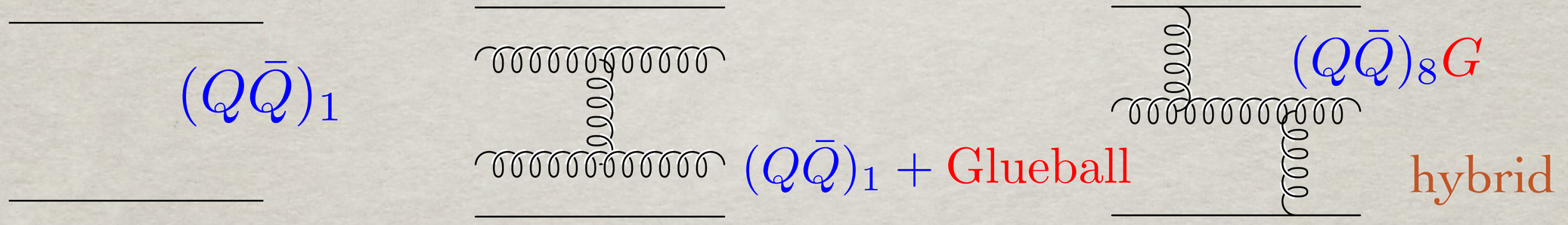


$\sim \alpha_s^5 \ln \alpha_s$ (Lamb-shift)

Applications of weakly coupled pNRQCD include:
ttbar production, quarkonia spectra, decays, E1 and M1 transitions, QQq and QQQ energies, thermal masses and potentials

Strongly coupled pNRQCD: Hitting the scale $\Lambda_{\rm QCD}$ $r \sim \Lambda_{QCD}^{-1}$

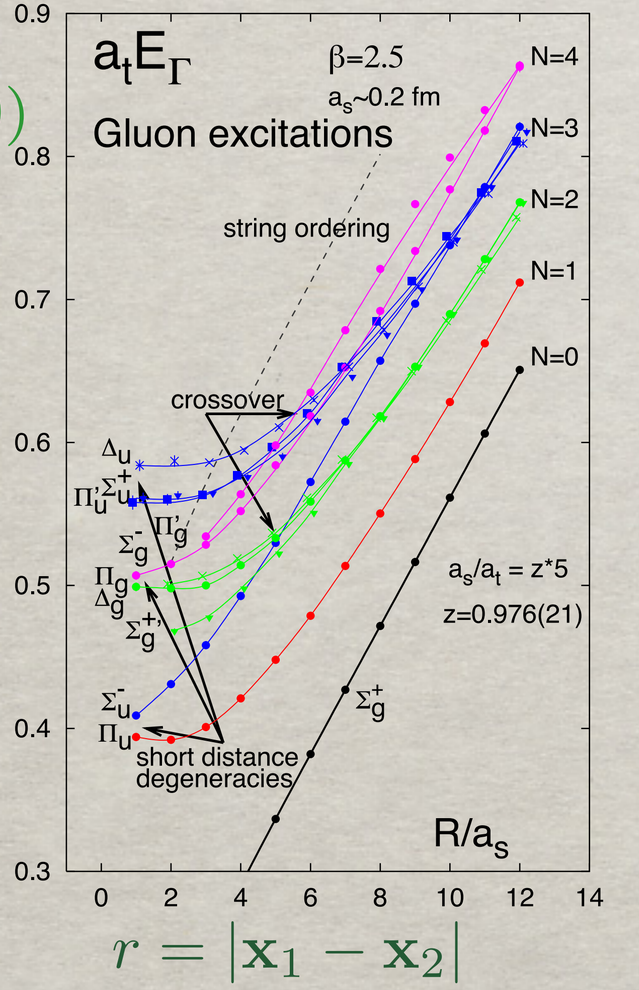
The degrees of freedoms now are



with gluons at the scale $\Lambda_{\rm QCD}$

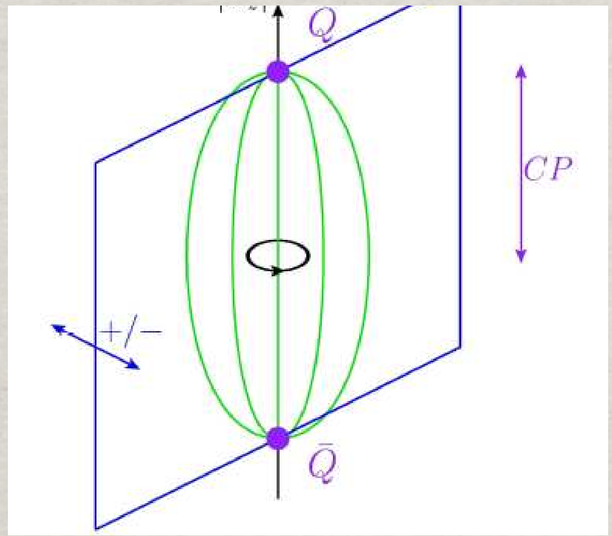
Strongly coupled pNRQCD: Static quenched NRQCD spectrum from lattice QCD

Hitting the scale $\Lambda_{\rm QCD}$ $r \sim \Lambda_{QCD}^{-1}$



Juge Kuti Mornigstar 98-06

Λ



Static states classified by symmetry group $D_{\infty h}$
Representations labeled $\Lambda_\eta^\sigma --> n$

- Λ rotational quantum number
  $|\lambda| = |\hat{\mathbf{n}} \cdot \mathbf{K}| = 0, 1, 2 \ldots$ corresponds to $\Lambda = \Sigma, \Pi, \Delta \ldots$
- η eigenvalue of CP: $g \hat{=} +1$ (gerade), $u \hat{=} -1$ (ungerade)
- σ eigenvalue of reflections
- σ label only displayed on Σ states (others are degenerate)

K is the angular momentum of the light degrees of freedom;same symmetry as the diatomic molecule

NRQCD

$$\mathcal{H}^{(0)} |\underline{n}; \mathbf{x}_1, \mathbf{x}_2\rangle^{(0)} = E_n^{(0)}(\mathbf{x}_1, \mathbf{x}_2) |\underline{n}; \mathbf{x}_1, \mathbf{x}_2\rangle^{(0)}$$

$$|\underline{n}; \mathbf{x}_1, \mathbf{x}_2\rangle^{(0)} = \psi^\dagger(\mathbf{x}_1)\chi(\mathbf{x}_2)|n; \mathbf{x}_1, \mathbf{x}_2\rangle^{(0)}$$

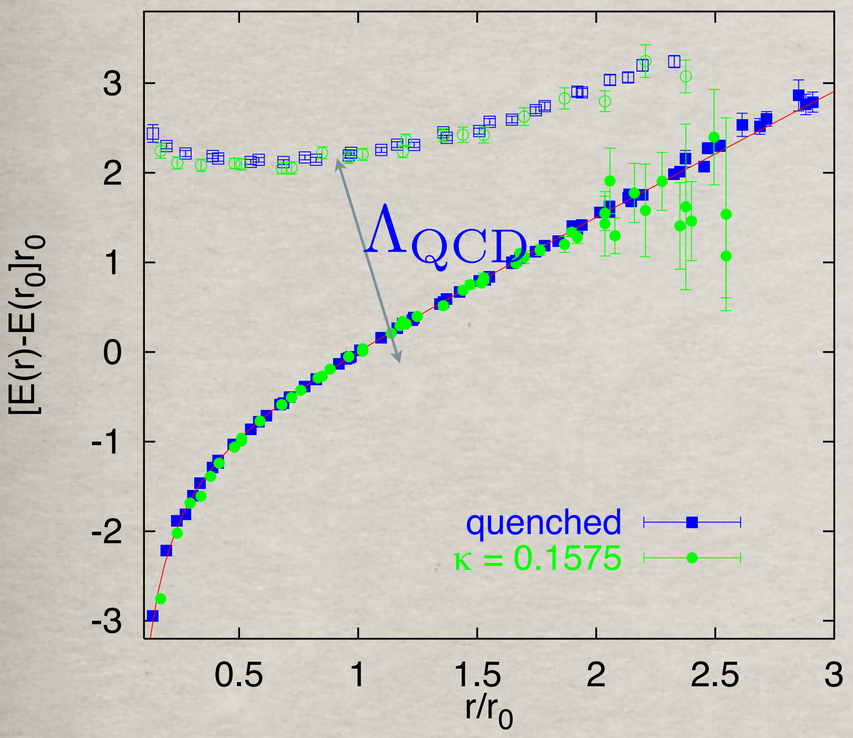
NRQCD states

$|\underline{0}; \mathbf{x}_1\mathbf{x}_2\rangle -->$ $|(Q\bar{Q})_1\rangle \rightarrow$ Quarkonium Singlet

$|\underline{n} > 0; \mathbf{x}_1\mathbf{x}_2\rangle -->$ $|(Q\bar{Q})g^{(n)}\rangle \rightarrow$ Higher Gluonic Excitations

$mv \sim \Lambda_{QCD}$ • pNRQCD and the potentials come from integrating out all scales up to mv^2

Bali et al. 98

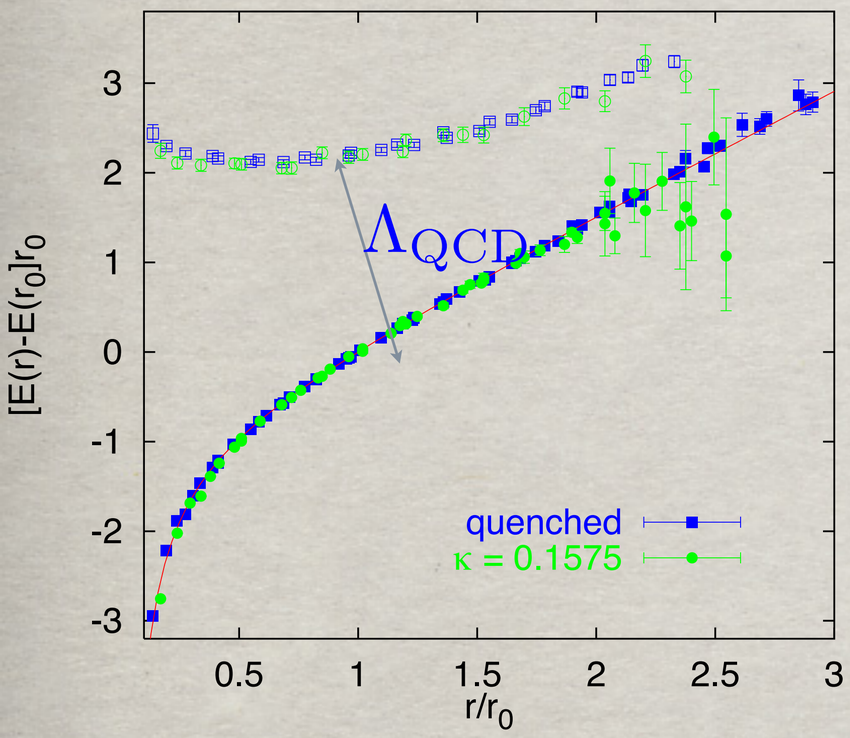


- gluonic excitations develop a gap $\Lambda_{\rm QCD}$ and are integrated out

Brambilla Pineda Soto Vairo 00

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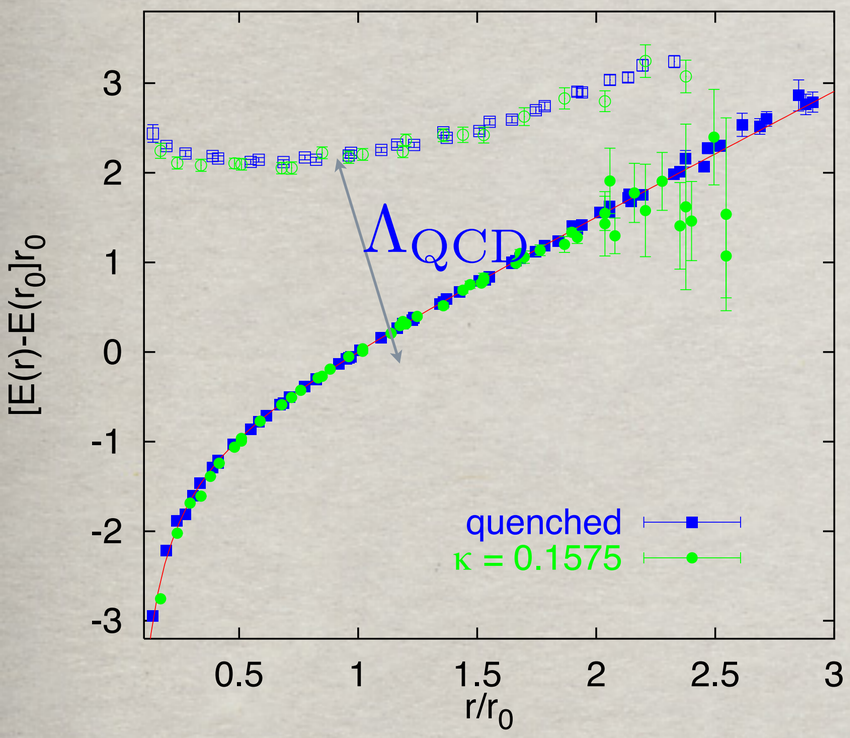
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⇒ The singlet quarkonium field S of energy mv^2 is the only the degree of freedom of pNRQCD (up to ultrasoft light quarks, e.g. pions).

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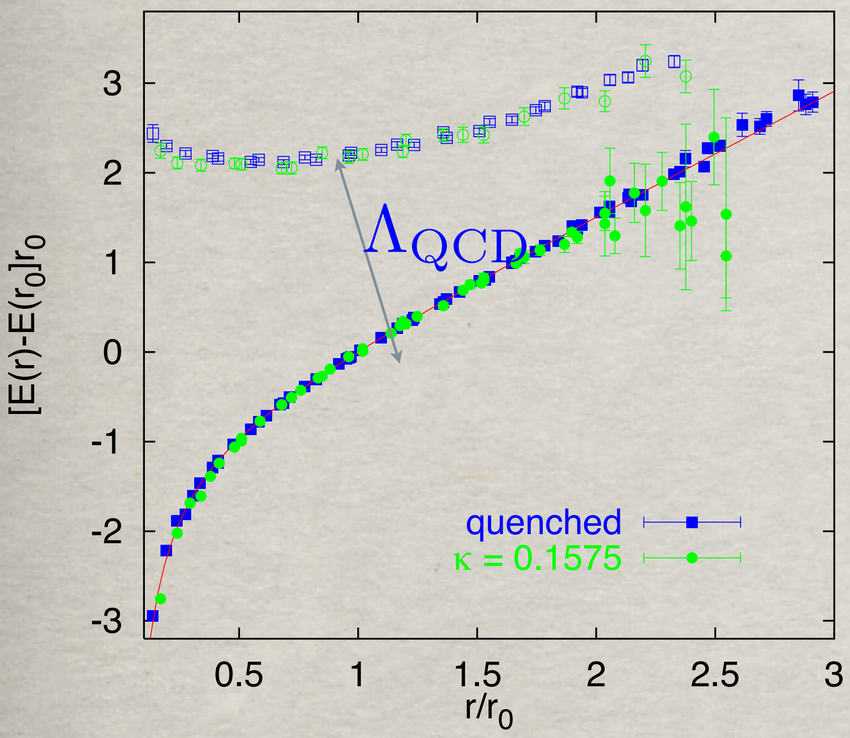
$\Rightarrow$ The singlet quarkonium field S of energy mv^2 is the only the degree of freedom of pNRQCD (up to ultrasoft light quarks, e.g. pions).

$$\mathcal{L} = \mathrm{Tr}\left\{ \mathrm{S}^\dagger \left(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_s \right) \mathrm{S} \right\} + \Delta\mathcal{L}(\text{US light quarks})$$

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- A pure potential description emerges from the EFT
- The potentials $V = \mathrm{Re}V + ImV$ from QCD in the matching: get spectra and decays
- We obtain the form of the nonperturbative potentials V in terms of generalized Wilson loops (static wilson loops with insertion of chromoelectric and chromagnetic fields), that are low energy pure gluonic correlators: all the flavour dependence is pulled out

Brambilla Pineda Soto Vairo 00

The singlet potential has the general structure

the fact that spin dependent corrections appear at order 1/m^2 is called Heavy Quark Spin Symmetry

$$V = V_0 + \frac{1}{m} V_1 + \frac{1}{m^2}(V_{SD} + V_{VD})$$

static

spin dependent

velocity dependent

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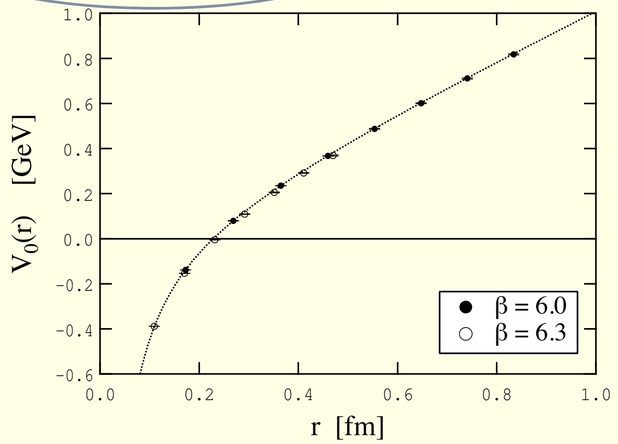
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$$V_s^{(0)} = \lim_{T\to\infty} \frac{i}{T} \ln\langle W(r \times T)\rangle = \lim_{T\to\infty} \frac{i}{T} \ln\langle \square \rangle$$

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○ Koma Koma NPB 769(07)79

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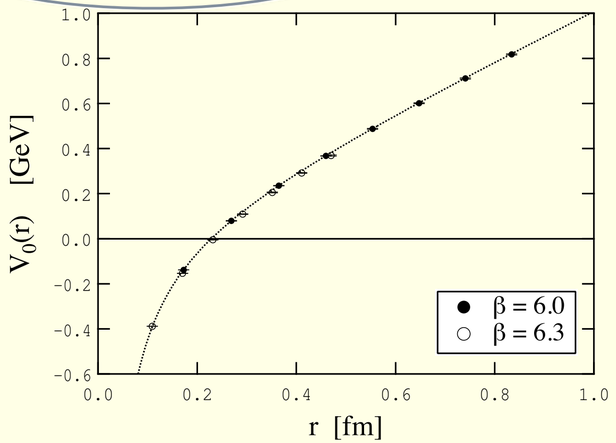
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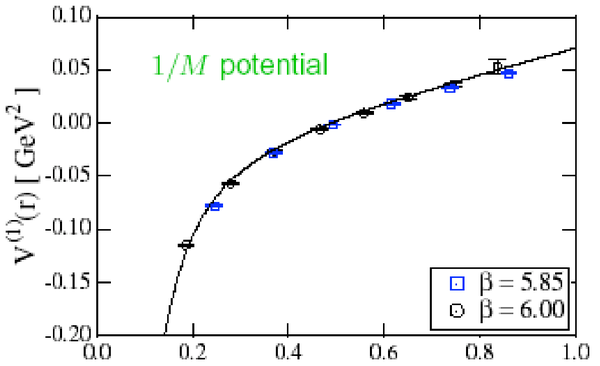
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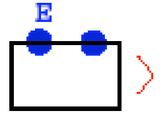


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Koma Koma Wittig PoS LAT2007(07)111

$$\frac{V_s^{(1)}}{m} = -\frac{1}{2m}\int_0^\infty dt\, t \langle \; \rangle$$



The singlet potential has the general structure

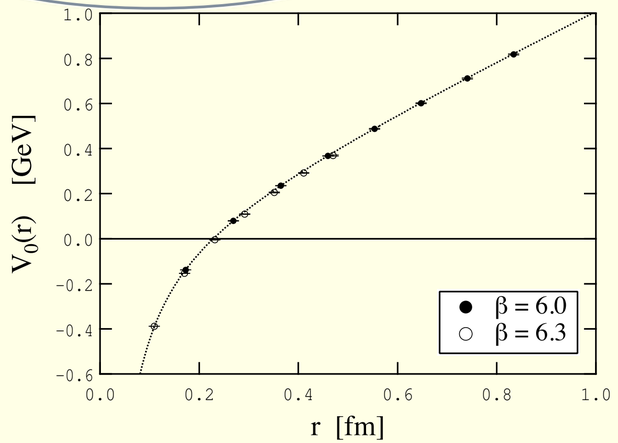
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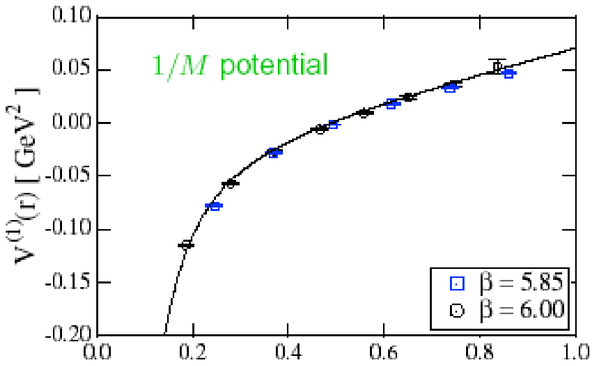
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By inserting the potentials in the Schroedinger eq. one can calculate all energy levels. The imaginary parts describe decays



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$$\frac{V_s^{(1)}}{m} = -\frac{1}{2m}\int_0^\infty dt\, t \langle \square \rangle$$

E

X Y Z : close or above the quarkonium strong decay threshold

X Y Z : close or above the quarkonium strong decay threshold

the situation is much more complicate

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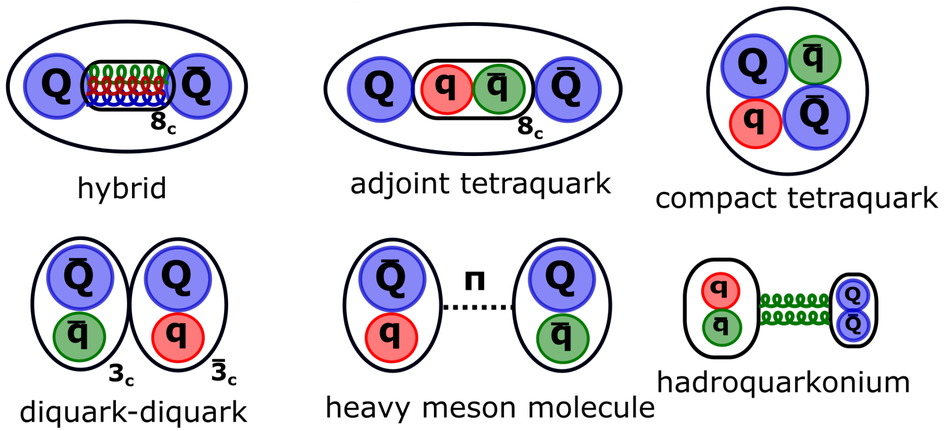
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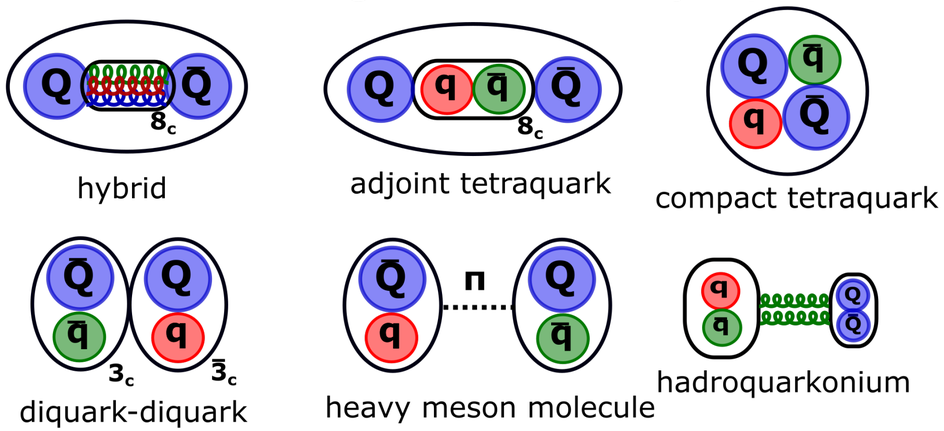
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Still: m is the bigger scale —> NRQCD is still valid

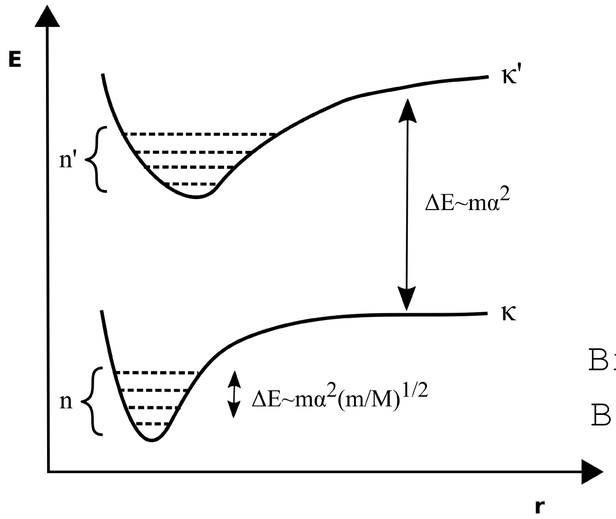
Separation of scales allows to construct an EFT —> BOEFT

BOEFT: EFT for nonrelativistic pairs and light d.o.f.

Consider bound states of two nonrelativistic particles and some light d.o.f., e.g., molecules/quarkonium hybrids ($Q\bar{Q}g$ states)or tetraquarks ($Q\bar{Q}q\bar{q}$ states):

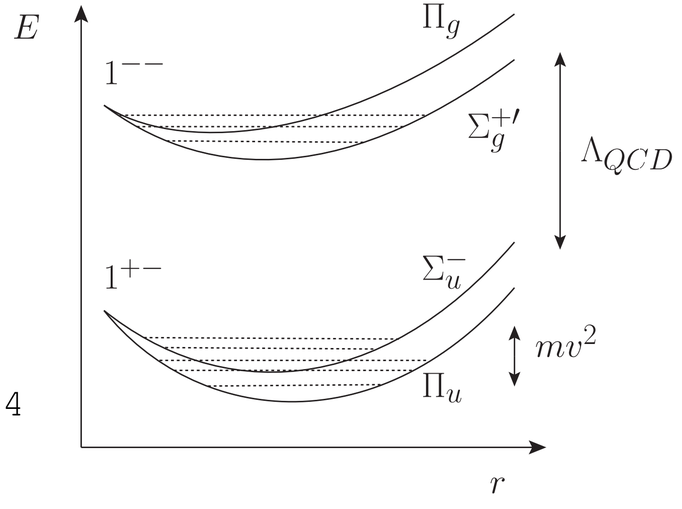
- electron/gluon fields change adiabatically in the presence of heavy quarks/nuclei. The heavy quarks/nuclei interaction may be described at leading order in the non-relativistic expansion by an effective potential V_κ between static sources, where κ labels different excitations of the light d.o.f.
- a plethora of states can be built on each of the potentials V_κ by solving the corresponding Schrödinger equation.

This picture goes also under the name of Born-Oppenheimer approximation. Starting from pNRQED/pNRQCD the Born-Oppenheimer approximation can be made rigorous and cast into a suitable nonrelativistic EFT called Born–Oppenheimer EFT (BOEFT).



Michael et al. 1983,
Juge, Kuti, Mornigstar 1997, 1998,

Braaten PRL 111 (2013) 162003
Braaten Langmack Smith PRD 90 (2014) 014044



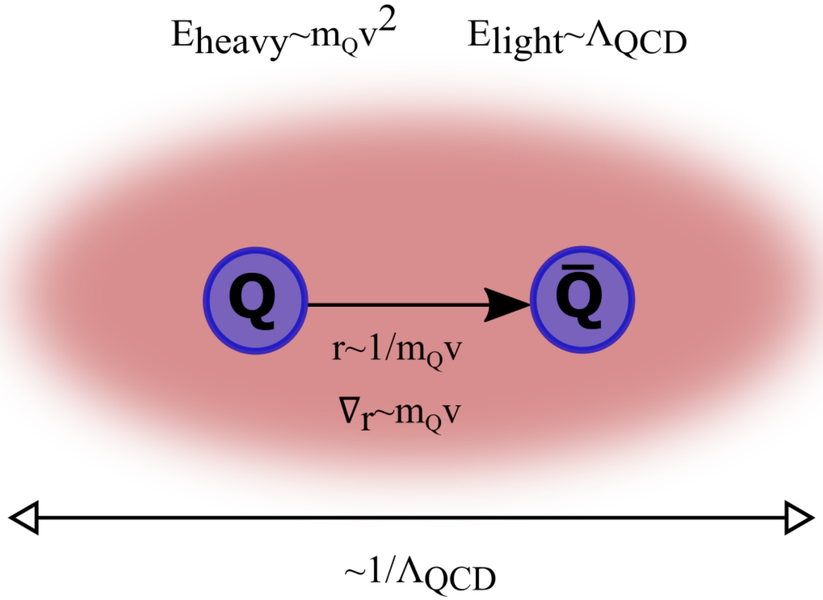
Focus on hybrids

two different scales

$$\Lambda_{\rm QCD} \gg m v^2$$

we proceed to integrate out 1/r and then $\Lambda_{\rm QCD}$

(or simultaneously see talk Tarrus)



analogous to

$$E_{electrons} \gg E_{nuclei}$$

in QED

$\Lambda_{\rm QCD}$

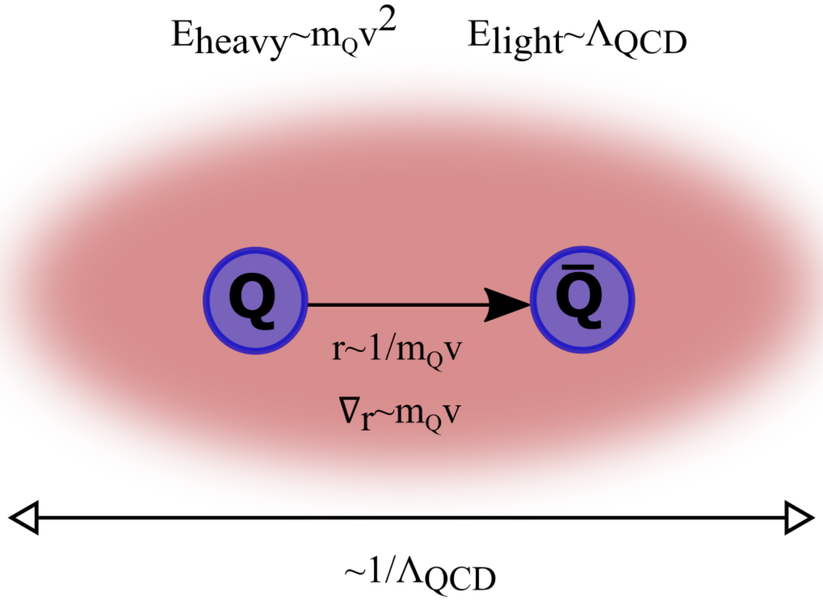
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$\Lambda_{\rm QCD}$ is nonperturbative but we can use the lattice to calculate the appropriate gluonic static energies (corresponding in molecular physics to the electronic static energies)

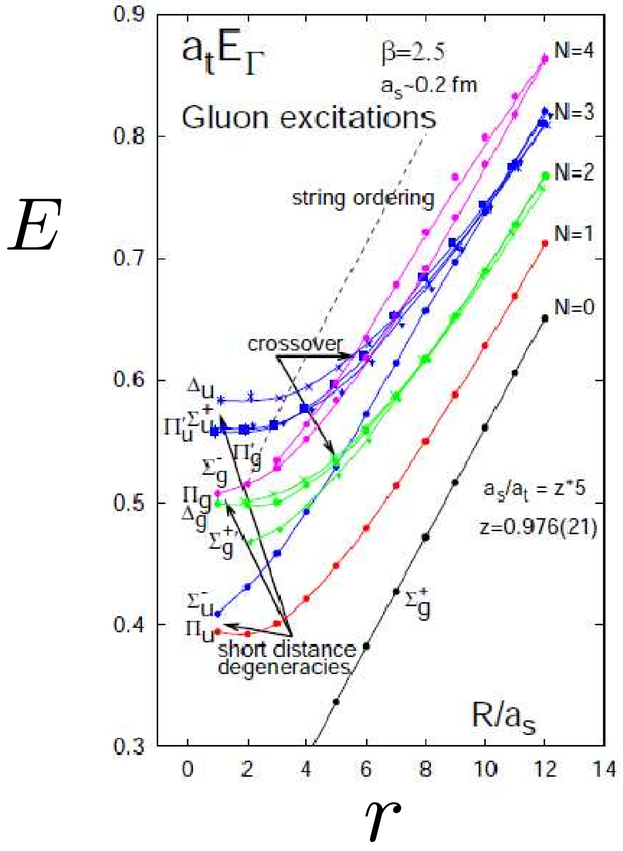
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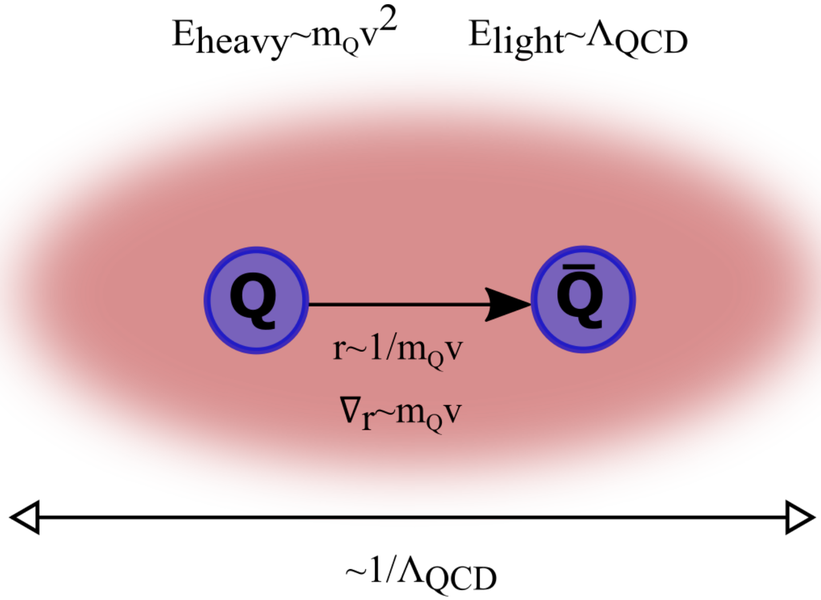
wilson loop

$$E_n^{(0)}(r) = \lim_{T\to\infty} \frac{i}{T} \log\langle X_n,\, T/2 | X_n,\, -T/2\rangle$$

$$|X_n\rangle = \chi(\mathbf{x_2})\phi(\mathbf{x_2},\mathbf{R})T^a H^a(\mathbf{R})\phi(\mathbf{R},\mathbf{x}_1)\psi^\dagger(\mathbf{x_1})|\mathrm{vac}\rangle$$

Phi wilson lines and H gluonic operator with the correct quantum numbers





- Σ_g^+ is the ground state potential that generates the standard quarkonium states.
- The rest of the static energies correspond to excited gluonic states that generate hybrids.
- The two lowest hybrid static energies are Π_u and Σ_u^-, they are nearly degenerate at short distances.

- Juge Kuti Morningstar PRL 90 (2003) 161601
  Capitani Philipsen Reisinger Riehl Wagner PRD 99 (2019) 03450

Bali Pineda PRD69 (2004) 094001

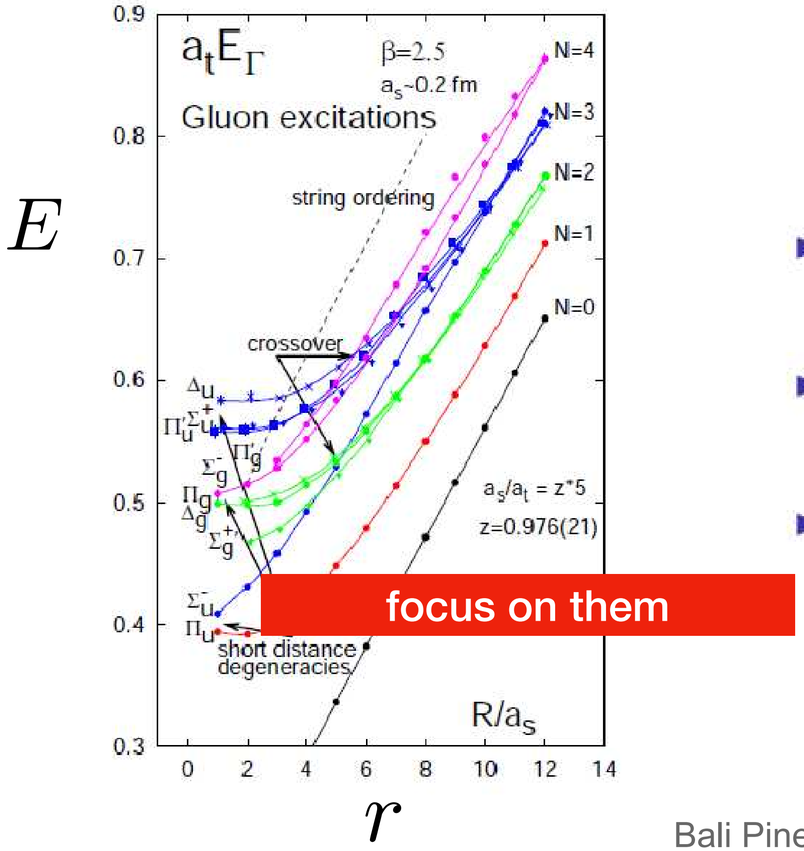
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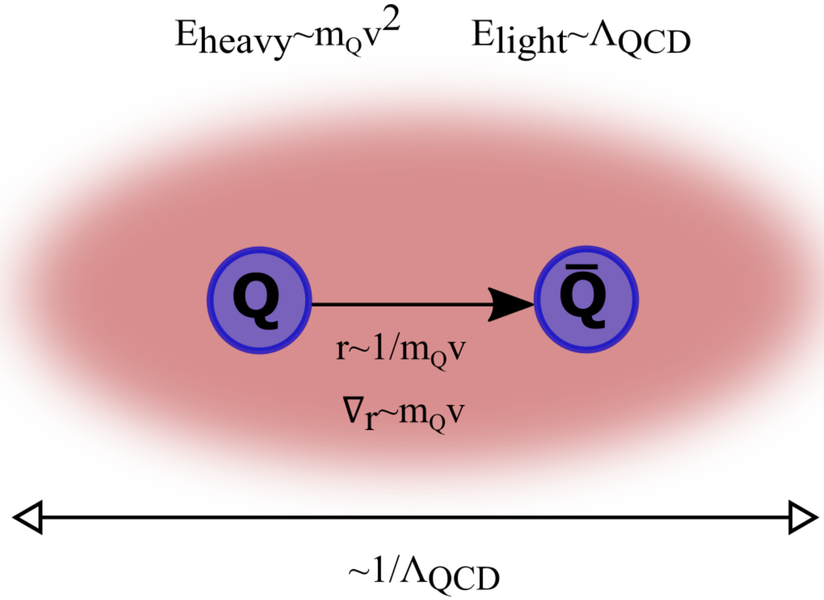
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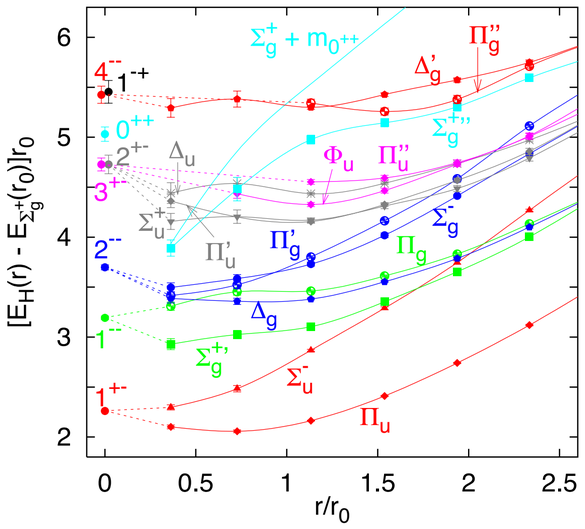
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The BOEFT characterises the hybrids static energy for short distance



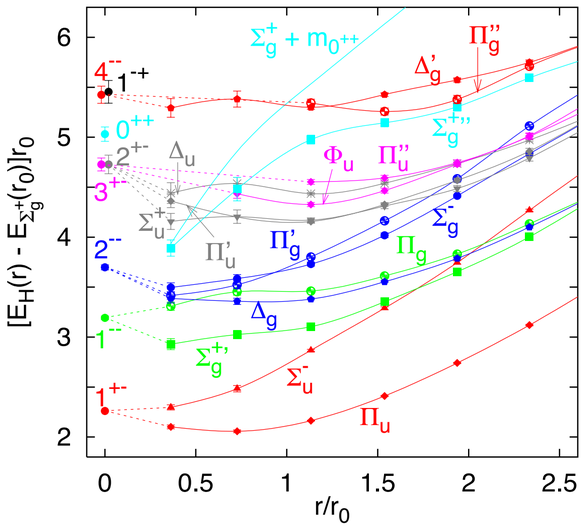
In the short-range hybrids become gluelumps, i.e., quark-antiquark octets, O^a, in the presence of a gluonic field, H^a: $H(R,r,t) = H^a(R,t)O^a(R,r,t)$.

the hybrid ' static energy can be written as a (multipole) expansion in r:

octet potential

$$E_g = \frac{\alpha_s}{6r} + \Lambda_g + a_g r^2 + \ldots$$

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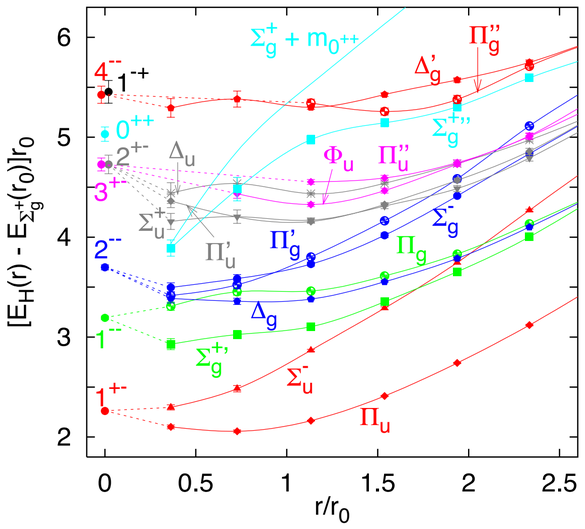
Λ_g is the gluelump mass: $\Lambda_g = \lim_{T\to\infty} \frac{i}{T} \ln\langle H^a(T/2)\phi^{\mathrm{adj}}_{ab}(T/2,-T/2)H^b(-T/2)\rangle$

calculated on the lattice

Foster Michael PRD 59 (1999) 094509
Bali Pineda PRD 69 (2004) 094001
Lewis Marsh PRD 89 (2014) 014502

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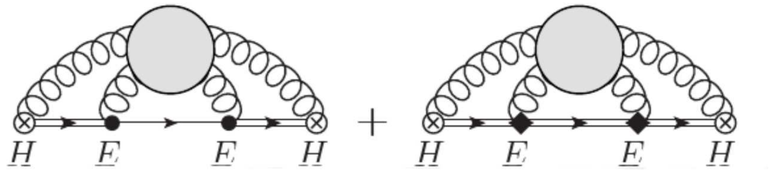
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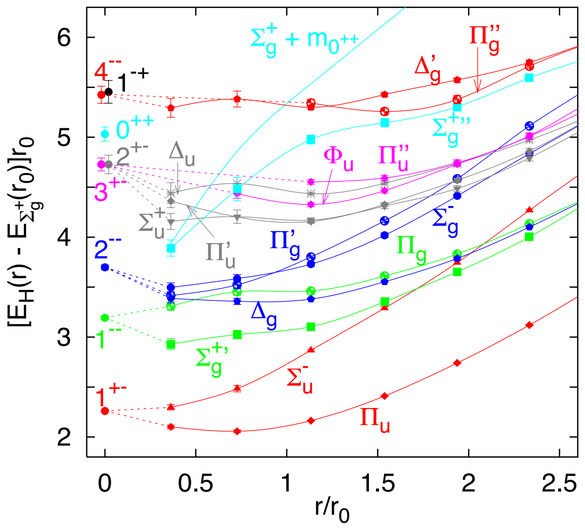
Foster Michael PRD 59 (1999) 094509
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a_g can be expressed as field correlators (single line = singlet, double line = octet), e.g.,



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the hybrid ' static energy can be written as a (multipole) expansion in r:

octet potential

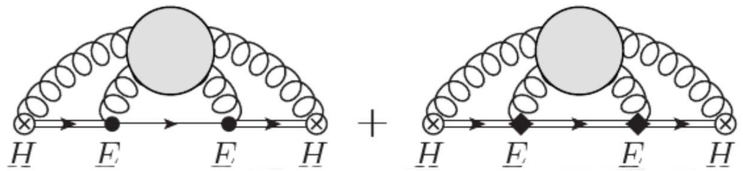
$$E_g = \frac{\alpha_s}{6r} + \Lambda_g + a_g r^2 + \ldots$$

Λ_g is the gluelump mass: $\Lambda_g = \lim_{T\to\infty} \frac{i}{T} \ln\langle H^a(T/2)\phi^{\rm adj}_{ab}(T/2,-T/2)H^b(-T/2)\rangle$

calculated on the lattice

Foster Michael PRD 59 (1999) 094509
Bali Pineda PRD 69 (2004) 094001
Lewis Marsh PRD 89 (2014) 014502

a_g can be expressed as field correlators (single line = singlet, double line = octet), e.g.,



In the limit $r \to 0$ more symmetry: $D_{\infty h} \to O(3) \times C$

- Several Λ_η^σ representations contained in one J^{PC} representation:
- Static energies in these multiplets have same $r \to 0$ limit.

The gluelump multiplets Σ_u^-, Π_u; $\Sigma_g^{+\prime}$, Π_g; Σ_g^-, Π_g', Δ_g; Σ_u^+, Π_u', Δ_u are degenerate.

Gluonic excitation operators up to dim 3

Λ_η^σ	K^{PC}	H^a
Σ_u^-	1^{+-}	$\mathbf{r}\cdot\mathbf{B}$, $\mathbf{r}\cdot(\mathbf{D}\times\mathbf{E})$
Π_u	1^{+-}	$\mathbf{r}\times\mathbf{B}$, $\mathbf{r}\times(\mathbf{D}\times\mathbf{E})$
$\Sigma_g^{+\prime}$	1^{--}	$\mathbf{r}\cdot\mathbf{E}$, $\mathbf{r}\cdot(\mathbf{D}\times\mathbf{B})$
Π_g	1^{--}	$\mathbf{r}\times\mathbf{E}$, $\mathbf{r}\times(\mathbf{D}\times\mathbf{B})$
Σ_g^-	2^{--}	$(\mathbf{r}\cdot\mathbf{D})(\mathbf{r}\cdot\mathbf{B})$
Π_g'	2^{--}	$\mathbf{r}\times((\mathbf{r}\cdot\mathbf{D})\mathbf{B}+\mathbf{D}(\mathbf{r}\cdot\mathbf{B}))$
Δ_g	2^{--}	$(\mathbf{r}\times\mathbf{D})^i(\mathbf{r}\times\mathbf{B})^j+(\mathbf{r}\times\mathbf{D})^j(\mathbf{r}\times\mathbf{B})^i$
Σ_u^+	2^{+-}	$(\mathbf{r}\cdot\mathbf{D})(\mathbf{r}\cdot\mathbf{E})$
Π_u'	2^{+-}	$\mathbf{r}\times((\mathbf{r}\cdot\mathbf{D})\mathbf{E}+\mathbf{D}(\mathbf{r}\cdot\mathbf{E}))$
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The BOEFT gives the set of coupled Schroedinger equation and the recipe to construct multiplets

- The hybrid energy levels are obtained by solving at the level of BOEFT the corresponding Schrödinger equation with the Hamiltonian $H_{\rm kin} + E_g$.
- In BOEFT one may also compute the decay width into lower states, e.g., hybrid $\to$ quarkonium transitions.
- $H_{\rm kin}$ acts also on the gluon wave function and may mix different gluonic excitations. Mixing may be relevant if the gluonic excitations are not well separated.

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$(\Lambda_\eta^\sigma = \Sigma_u^-,\, \Pi_u)$

Hybrids Multiplets

We consider hybrids that are excitations of the lowest lying static energies Π_u and Σ_u^-. In the $r \to 0$ limit Π_u and Σ_u^- are degenerate and correspond to a gluonic operator with quantum numbers 1^{+-}.

Multiplet	T	$J^{PC}(S=0)$	$J^{PC}(S=1)$	E_Γ
H_1	1	1^{--}	$(0,1,2)^{-+}$	$E_{\Sigma_u^-}$, E_{Π_u}
H_2	1	1^{++}	$(0,1,2)^{+-}$	E_{Π_u}
H_3	0	0^{++}	1^{+-}	$E_{\Sigma_u^-}$
H_4	2	2^{++}	$(1,2,3)^{+-}$	$E_{\Sigma_u^-}$, E_{Π_u}

T is the sum of the orbital angular momentum of the quark-antiquark pair and the gluonic angular momentum; $T=0$ state turns out not to be the lowest mass state.

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different models have a different multiplets structure

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BOEFT for E_{Π_u} and $E_{\Sigma_u^-}$ hybrids

- Berwein Brambilla Tarrus Vairo PRD 92 (2015) 114019
  Oncala Soto PRD 96 (2017) 014004
  Brambilla Krein Tarrus Vairo PRD 97 (2018) 016016

$$\mathcal{L}_{\text{BOEFT for } 1^{+-}} = \int d^3r \sum_{\lambda\lambda'} \mathrm{Tr}\left\{ \Psi^\dagger_{1^{+-}\lambda} \left(i\partial_0 - V_{1^{+-}\lambda\lambda'}(r) + \hat{r}^{i\dagger}_\lambda \frac{\boldsymbol{\nabla}_r^2}{m} \hat{r}^i_{\lambda'} \right) \Psi_{1^{+-}\lambda'} \right\}$$

- $\lambda = \pm 1, 0; \quad \hat{r}^i_0 = \hat{r}^i$ and $\hat{r}^i_{\pm 1} = \mp \left(\hat{\theta}^i \pm i\hat{\phi}^i \right)/\sqrt{2}$.
- $V_{1^{+-}\lambda\lambda'} = V^{(0)}_{1^{+-}\lambda\lambda'} + \dfrac{V^{(1)}_{1^{+-}\lambda\lambda'}}{m} + \dfrac{V^{(2)}_{1^{+-}\lambda\lambda'}}{m^2} + \cdots$
- For the static potential: $V^{(0)}_{1^{+-}\lambda\lambda'} = \delta_{\lambda\lambda'} V^{(0)}_{1^{+-}\lambda}$, with $V^{(0)}_{1^{+-}0} = E_{\Sigma_u^-}$, $V^{(0)}_{1^{+-}\pm 1} = E_{\Pi_u}$

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The LO e.o.m. for the fields $\Psi^\dagger_{1^{+-}\lambda}$ are a set of coupled Schrödinger equations:

$$i\partial_0 \Psi_{1^{+-}\lambda} = \left[\left(-\frac{\boldsymbol{\nabla}_r^2}{m} + V^{(0)}_{1^{+-}\lambda} \right) \delta_{\lambda\lambda'} - \sum_{\lambda'} C^{\text{nad}}_{1^{+-}\lambda\lambda'} \right] \Psi_{\kappa\lambda'}$$

The eigenvalues $\mathcal{E}_N$ give the masses M_N of the states as $M_N = 2m + \mathcal{E}_N$.

$$\hat{r}^{i\dagger}_\lambda \left(\frac{\boldsymbol{\nabla}_r^2}{m} \right) \hat{r}^i_{\lambda'} = \delta_{\lambda\lambda'} \frac{\boldsymbol{\nabla}_r^2}{m} + C^{\text{nad}}_{1^{+-}\lambda\lambda'}$$

with $C^{\text{nad}}_{1^{+-}\lambda\lambda'} = \hat{r}^{i\dagger}_\lambda \left[\dfrac{\boldsymbol{\nabla}_r^2}{m}, \hat{r}^i_{\lambda'} \right]$ called the nonadiabatic coupling.

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fitted from the lattice hybrids static energies

$$\left[-\frac{1}{mr^2}\partial_r r^2 \partial_r + \frac{1}{mr^2} \begin{pmatrix} l(l+1)+2 & 2\sqrt{l(l+1)} \\ 2\sqrt{l(l+1)} & l(l+1) \end{pmatrix} + \begin{pmatrix} E^{(0)}_\Sigma & 0 \\ 0 & E^{(0)}_\Pi \end{pmatrix} \right] \begin{pmatrix} \psi^{(N)}_\Sigma \\ \psi^{(N)}_{-\Pi} \end{pmatrix} = \mathcal{E}_N \begin{pmatrix} \psi^{(N)}_\Sigma \\ \psi^{(N)}_{-\Pi} \end{pmatrix}$$

$$\left[-\frac{1}{mr^2}\,\partial_r\, r^2\, \partial_r + \frac{l(l+1)}{mr^2} + E^{(0)}_\Pi \right] \psi^{(N)}_{+\Pi} = \mathcal{E}_N\, \psi^{(N)}_{+\Pi}$$

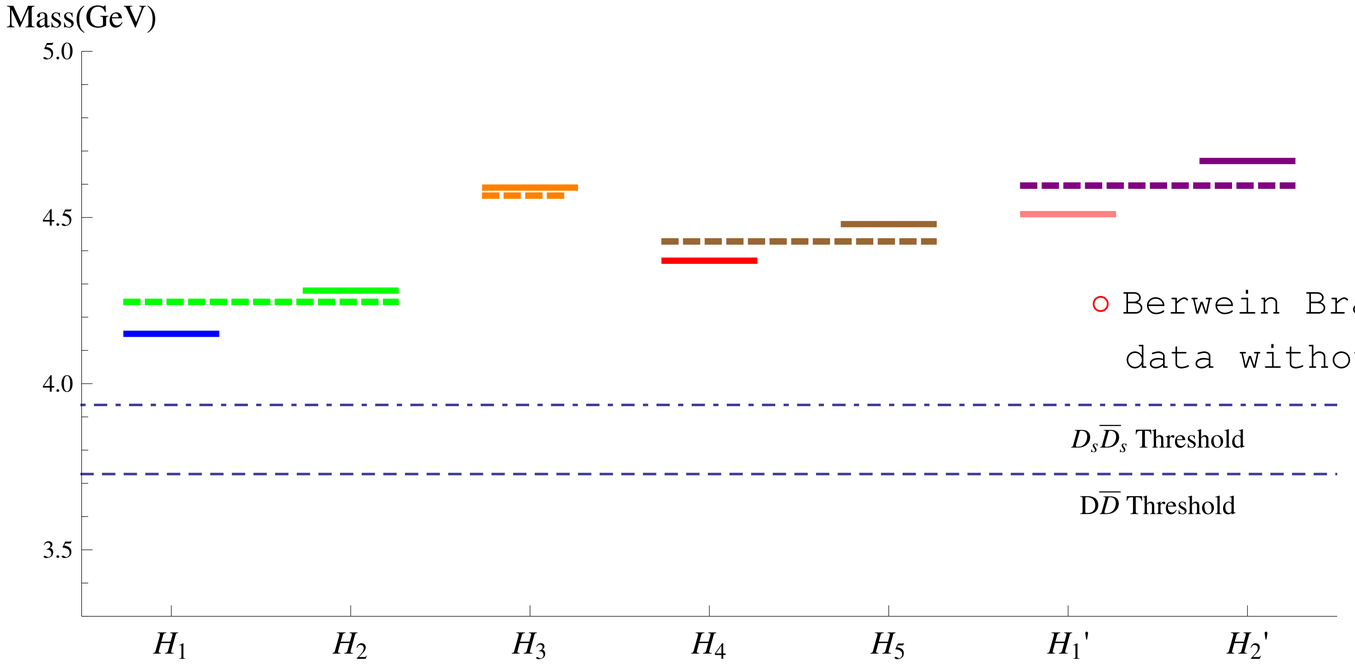
- $l(l+1)$ is the eigenvalue of angular momentum $\boldsymbol{L}^2 = \left(\boldsymbol{L}_{Q\bar{Q}} + \boldsymbol{L}_g \right)^2$
- the two solutions correspond to **opposite parity** states: $(-1)^l$ and $(-1)^{l+1}$
- corresponding eigenvalues under charge conjugation: $(-1)^{l+s}$ and $(-1)^{l+s+1}$

Spectrum: general consideration

- The Schrödinger equation mixes states with the same parity.
  A consequence is Λ-doubling, i.e., the lifting of degeneracy between states with opposite parity. This happens also in molecular physics, however, there Λ-doubling is a subleading effect, while it is a LO effect in the quarkonium hybrid spectrum.
- The eigenstates are organized in the multiplets H_1, H_2, ... Neglecting off-diagonal terms, the multiplets H_1 and H_2 would be degenerate.
- We compute the spectrum using quark masses in the renormalon subtraction (RS) scheme: $m_{c\,\mathrm{RS}} = 1.477(40)$ GeV and $m_{b\,\mathrm{RS}} = 4.863(55)$ GeV.

The gluelump masses, which enter in the normalization of the hybrid potentials, have been computed in the same scheme and assigned an uncertainty of
he largest source of uncertainty in the hybrid masses.

Spectrum: with mixing and Λ-doubling



charmonium hybrids

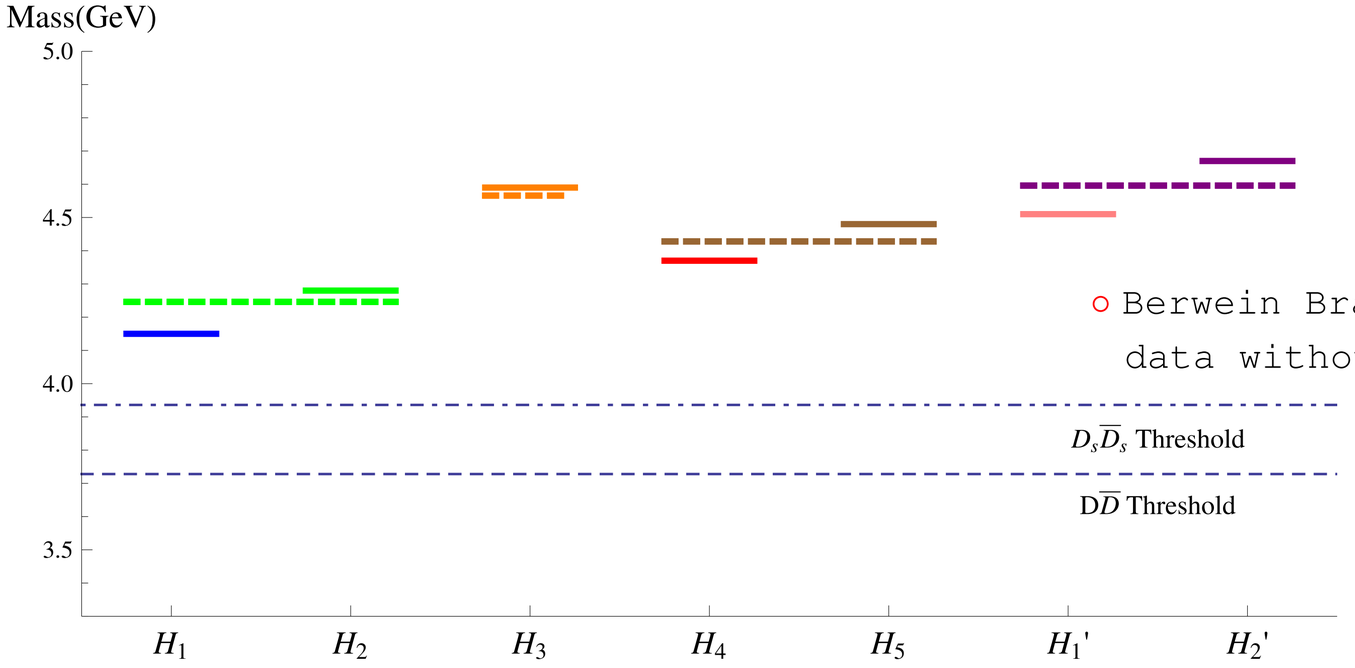
- Berwein Brambilla Tarrus Vairo PRD 92 (2015) 114019
  data without mixing (dashed) from Braaten et al PRD 90 (2014)

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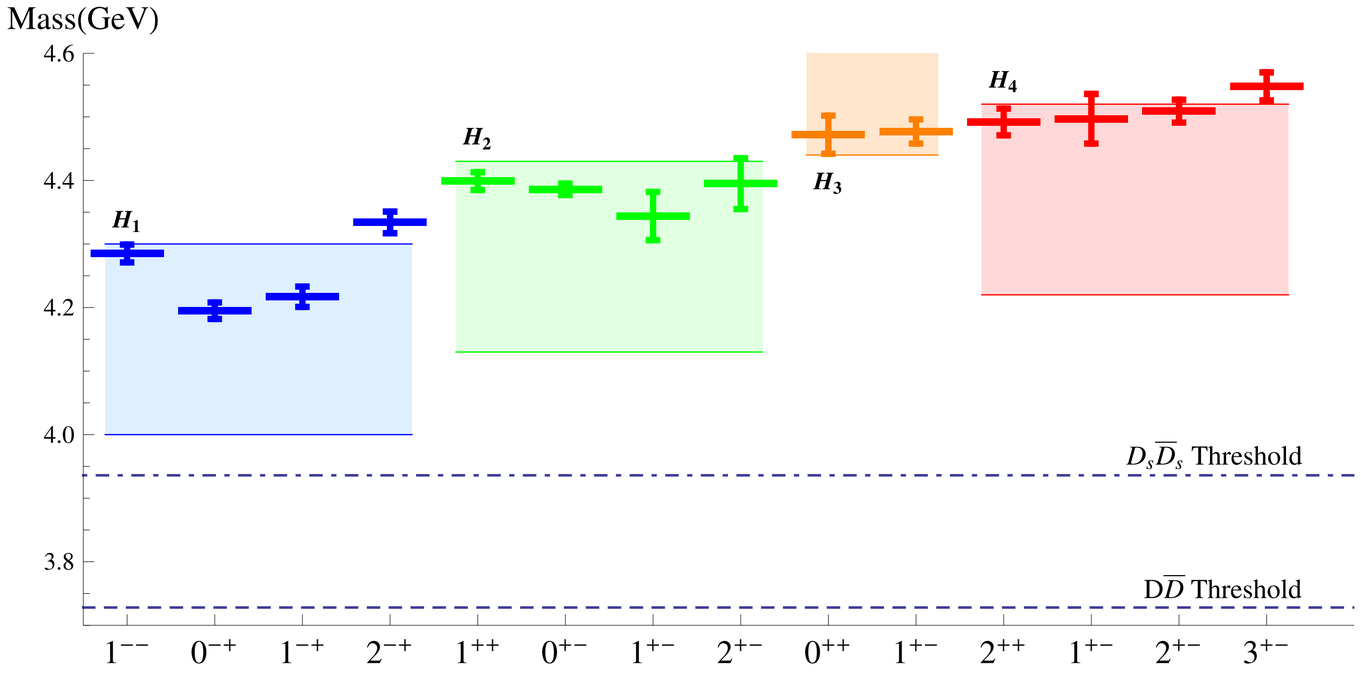


charmonium hybrids

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  data without mixing (dashed) from Braaten et al PRD 90 (2014)

in all existing BO papers
without the BOEFT
masses of opposite parity
states are degenerate

Charmonium hybrid states vs direct lattice data



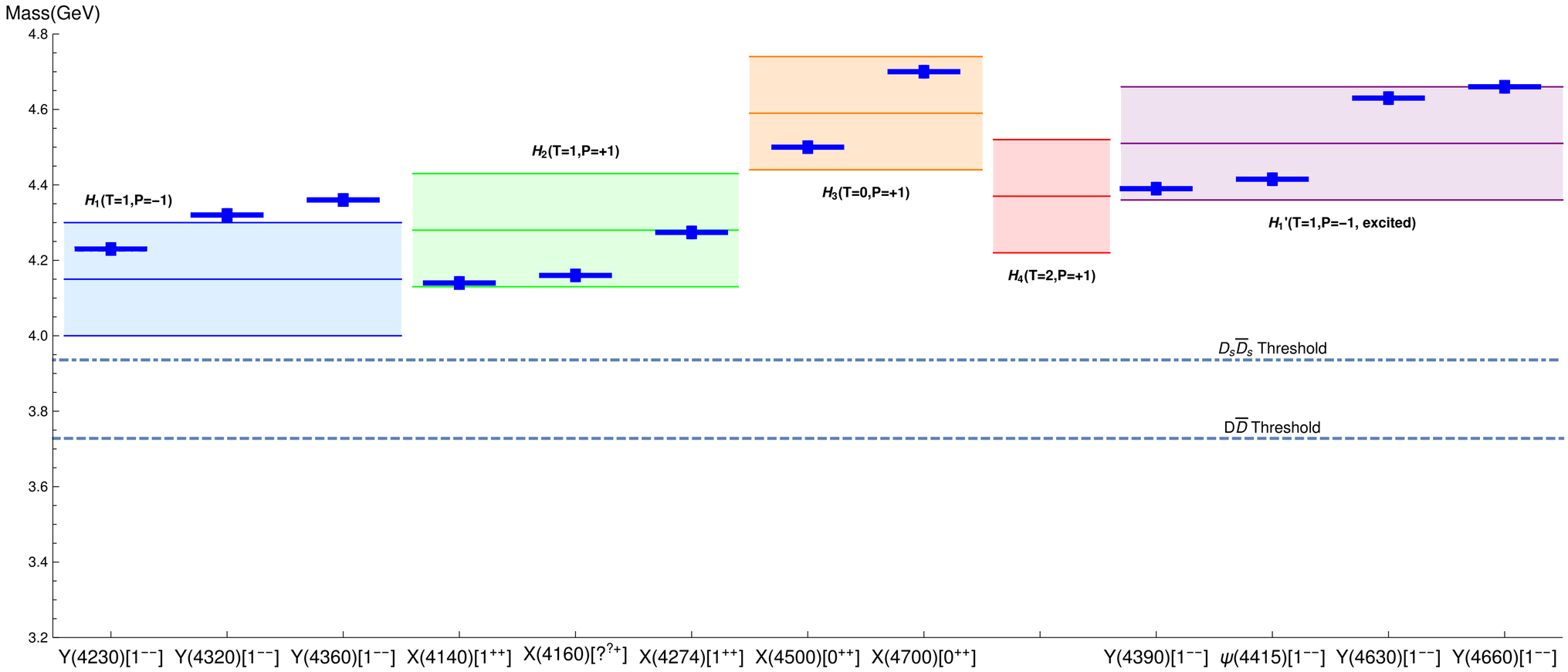
Lattice (crosses) confirms Lambda doubling (H_1 not degenerate with H_2)

Bands BOEFT predict -uncertainty comes from the uncertainty on the mass of the gluelump

- Berwein Brambilla Tarrus Vairo PRD 92 (2015) 114019
  lattice data from the Hadron Spectrum coll JHEP 1207 (2012) 126
  [2+1 flavors, $m_\pi = 400$ MeV]

Quarkonium hybrid states vs experiments I

neutral isoscalar states in charmonium with matching quantum numbers



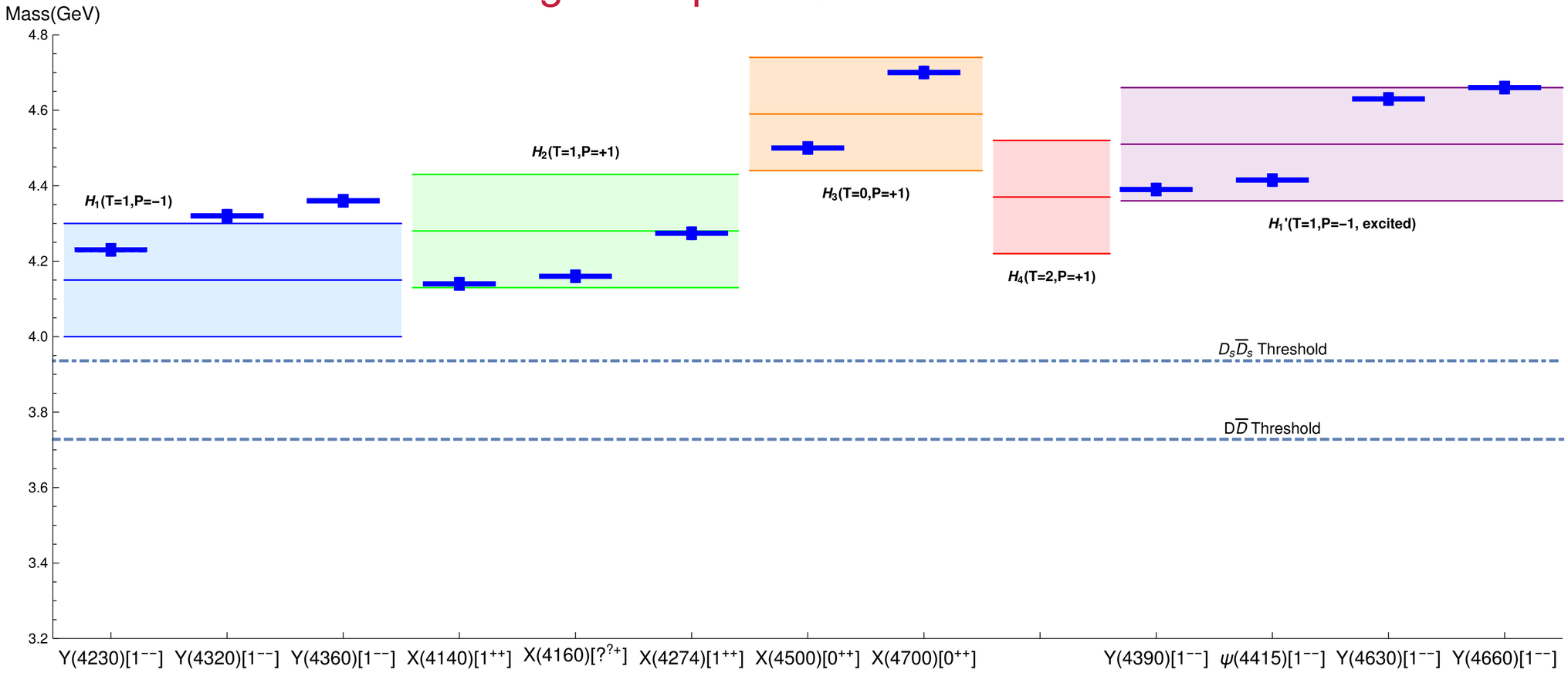
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  updated in Brambilla Eidelman Hanhart Nefediev Shen Thomas Vairo Yuan arXiv:1907.11747

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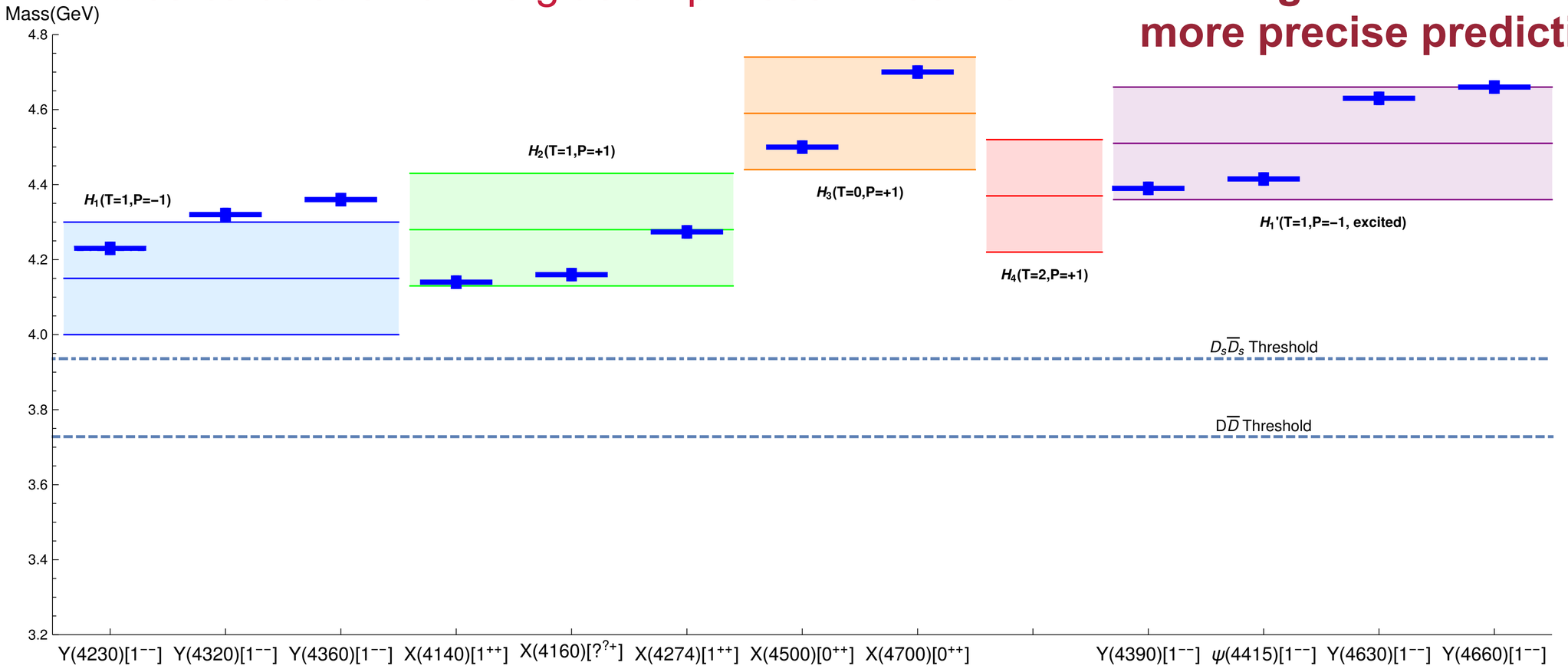
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Quarkonium hybrid states vs experiments II

- Promising candidates for charmonium hybrids or for states with a large hybrid component are the $Y(4230)$ and $Y(4390)$ because of their significant width into $\pi^+\pi^- h_c$. This decay does not need spin flipping of the heavy quark-antiquark pair, which is in a spin zero state. Spin-flipping terms are suppressed in the heavy quark limit. Nevertheless, mixing with spin one quarkonium states happens already at order $\Lambda^2_{\rm QCD}/m_h$. This possibly large mixing may allow for significant widths also into final states with spin one quarkonia, in particular $\pi^+\pi^- J/\psi$.
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- From the experimental side, candidate states of bottomonium hybrids in the H_1 or H_1' multiplets are the $\Upsilon(10860)$ $[1^{--}]$, with a mass of $M_{\Upsilon(10860)} = (10891.1 \pm 3.2^{+0.6}_{-1.7})$ MeV and the $\Upsilon(11020)$ $[1^{--}]$, with a mass of $M_{\Upsilon(11020)} = (10987.5^{+6.4\ +9.0}_{-2.5\ -2.1})$ MeV
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to understand the data
we should consider
spin corrections,
include mixing
and calculate decays

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The **BOEFT** gives a prescription to calculate the **hybrids spin dependent potentials at order 1/m and 1/m^2**

1/m

$$V^{(1)}_{1^{+-}\lambda\lambda'\,\mathrm{SD}}(\boldsymbol{r}) = V_{SK}(r)\left(\hat{r}^{i\dagger}_{\lambda}\boldsymbol{K}^{ij}\hat{r}^{j}_{\lambda'}\right)\cdot\boldsymbol{S} + V_{SK\,b}(r)\left[\left(\boldsymbol{r}\cdot\hat{\boldsymbol{r}}^{\dagger}_{\lambda}\right)\left(r^{i}\boldsymbol{K}^{ij}\hat{r}^{j}_{\lambda'}\right)\cdot\boldsymbol{S} + \left(r^{i}\boldsymbol{K}^{ij}\hat{r}^{j\dagger}_{\lambda}\right)\cdot\boldsymbol{S}\left(\boldsymbol{r}\cdot\hat{\boldsymbol{r}}_{\lambda'}\right)\right]$$

$\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2$

$S_{12} = 12(\mathbf{S}_1\cdot\hat{\mathbf{r}})(\mathbf{S}_2\cdot\hat{\mathbf{r}}) - 4(\mathbf{S}_1\cdot\mathbf{S}_2)$

1/m^2

$$V^{(2)}_{1^{+-}\lambda\lambda'\,\mathrm{SD}}(\boldsymbol{r}) = V^{(2)}_{LS\,a}(r)\left(\hat{r}^{i\dagger}_{\lambda}\boldsymbol{L}\,\hat{r}^{i}_{\lambda'}\right)\cdot\boldsymbol{S} + V^{(2)}_{LS\,b}(r)\hat{r}^{i\dagger}_{\lambda}\left(L^{i}S^{j}+S^{i}L^{j}\right)\hat{r}^{j}_{\lambda'} + V^{(2)}_{S^2}(r)\boldsymbol{S}^2\delta_{\lambda\lambda'} + V^{(2)}_{S_{12}\,a}(r)S_{12}\delta_{\lambda\lambda'} + V^{(2)}_{S_{12}\,b}(r)\hat{r}^{i\dagger}_{\lambda}\hat{r}^{j}_{\lambda'}\left(S^{i}_{1}S^{j}_{2}+S^{i}_{2}S^{j}_{1}\right)$$

$\left(K^{ij}\right)^{k} = i\epsilon^{ikj}$ is the angular momentum of the spin one gluons

$\boldsymbol{L}$ is the orbital angular momentum of the heavy-quark-antiquark pair.

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$\boldsymbol{L}$ is the orbital angular momentum of the heavy-quark-antiquark pair.

Features:

- New spin structures with respect to the quarkonium case: all terms at order 1/m and two terms at order 1/m^2

Differently from the quarkonium case, the hybrid potential gets a first contribution already at order $\Lambda^2_{\mathrm{QCD}}/m_h$. The corresponding operator does not contribute at LO to matrix elements of quarkonium states as its projection on quark-antiquark color singlet states vanishes. Hence, spin splittings are remarkably less suppressed in heavy quarkonium hybrids than in heavy quarkonia.

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Mixing via spin is also enhanced and decay to different spin states are enhanced

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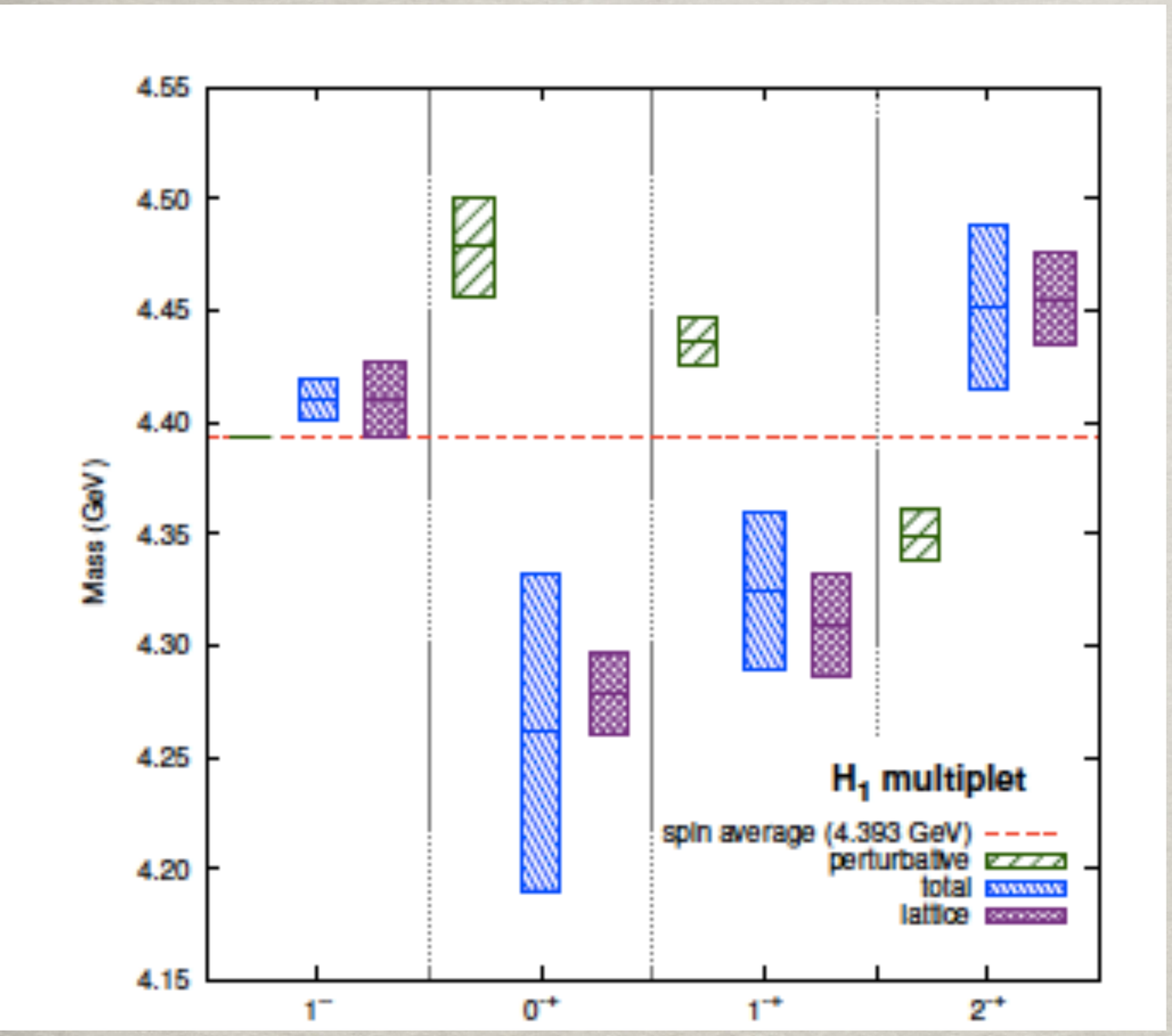
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USE LATTICE CALCULATION OF THE CHARMONIUM SPIN MULTIPLETS TO EXTRACT the 6 UNKNOWNs and PREDICT THE BOTTOMONIUM SPIN MULTIPLETS, learn also about the DYNAMICS

Charmonium Hybrids Multiplets H_1



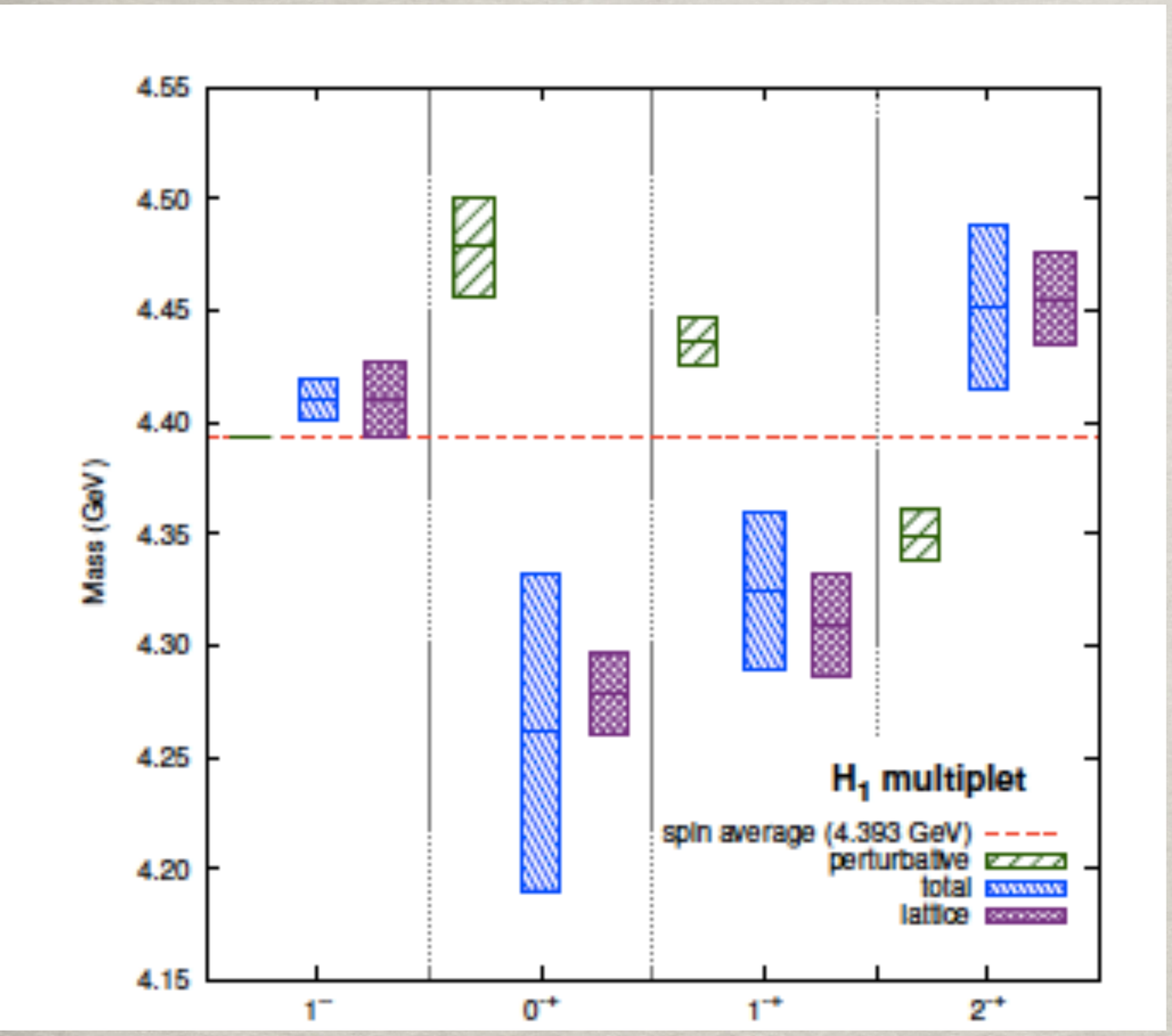
lattice data from (violet) from

G. K. C. Cheung, C. O'Hara, G. Moir, M. Peardon, S. M. Ryan, C. E. Thomas, and D. Tims (Hadron Spectrum), JHEP **12**, 089 (2016), arXiv:1610.01073 [hep-lat].

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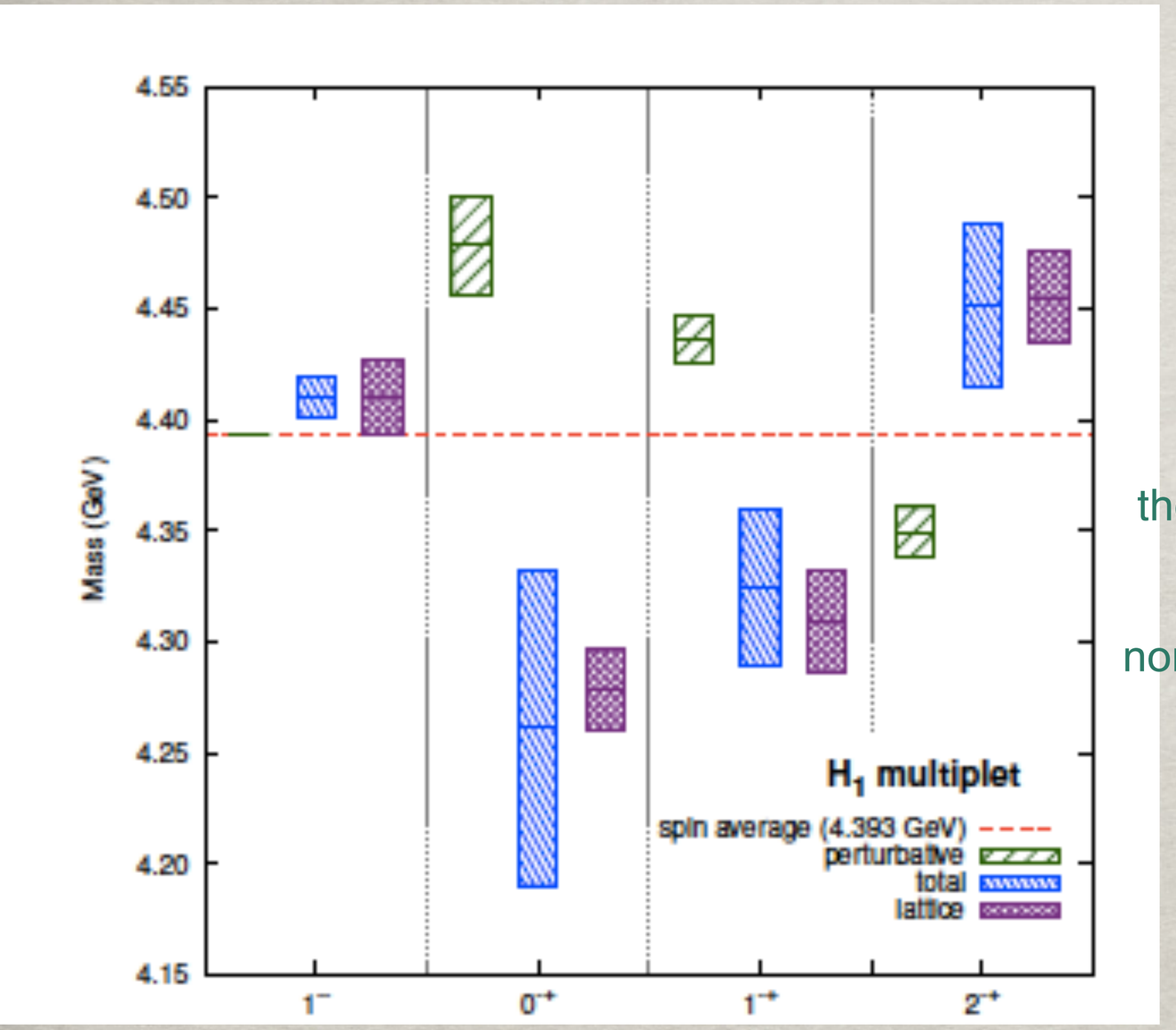
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height of the boxes is an estimate of the uncertainty:
estimated by the parametric size of higher order corrections, m alpha_s^5 for the perturbative part, powers of Lambda_qcd/m for the nonperturbative part, plus the statistical error on the fit

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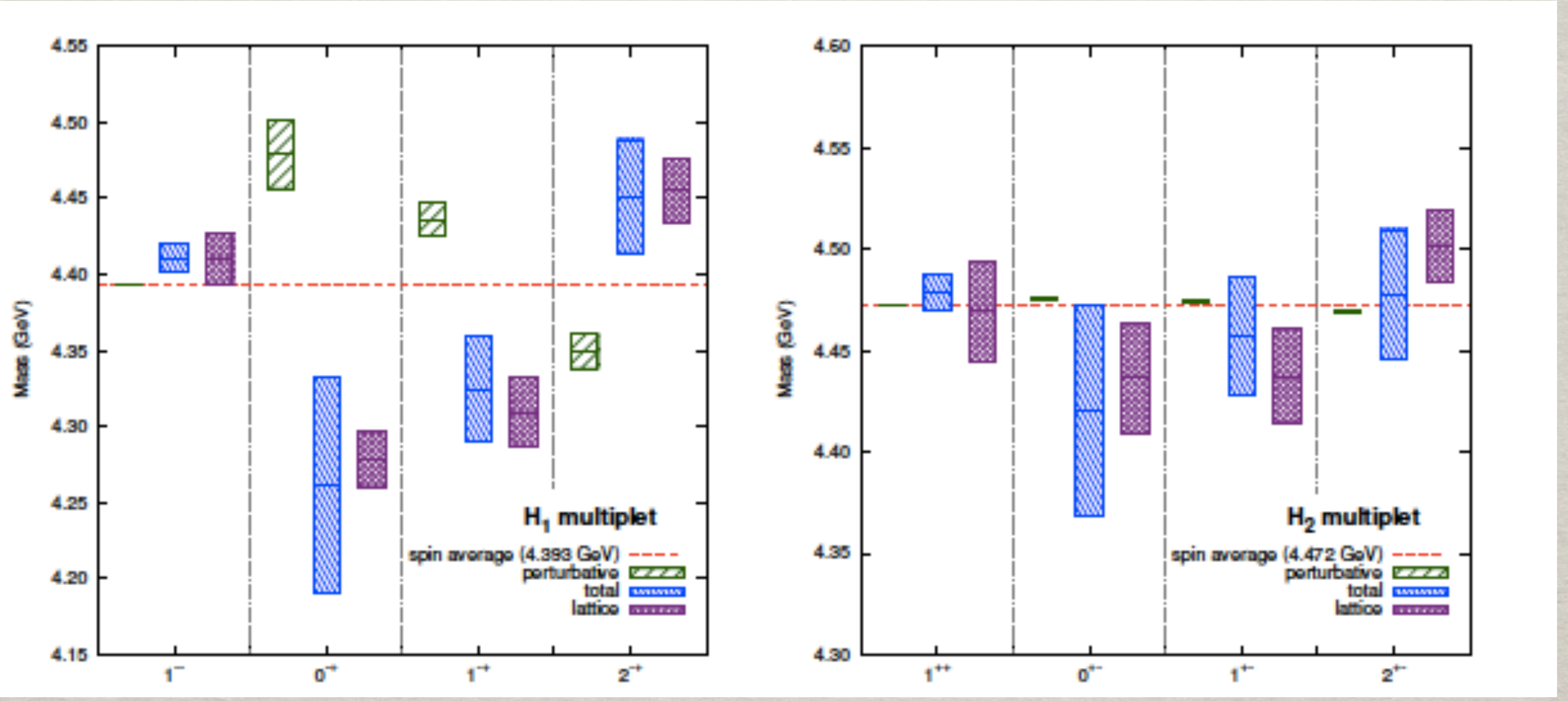
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the perturbative part produces a pattern opposite to the lattice and to ordinary quarkonia —> discrepancy can be reconciled thanks to the nonperturbative parts, especially the one at order 1/m which goes like Lambda^2/m and is parametrically larger than the perturbative contribution at order m v^4

which is interesting as some models are taking the spin interaction from perturbation theory with a constituent gluon

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Charmonium Hybrids Multiplets H_1 and H_2



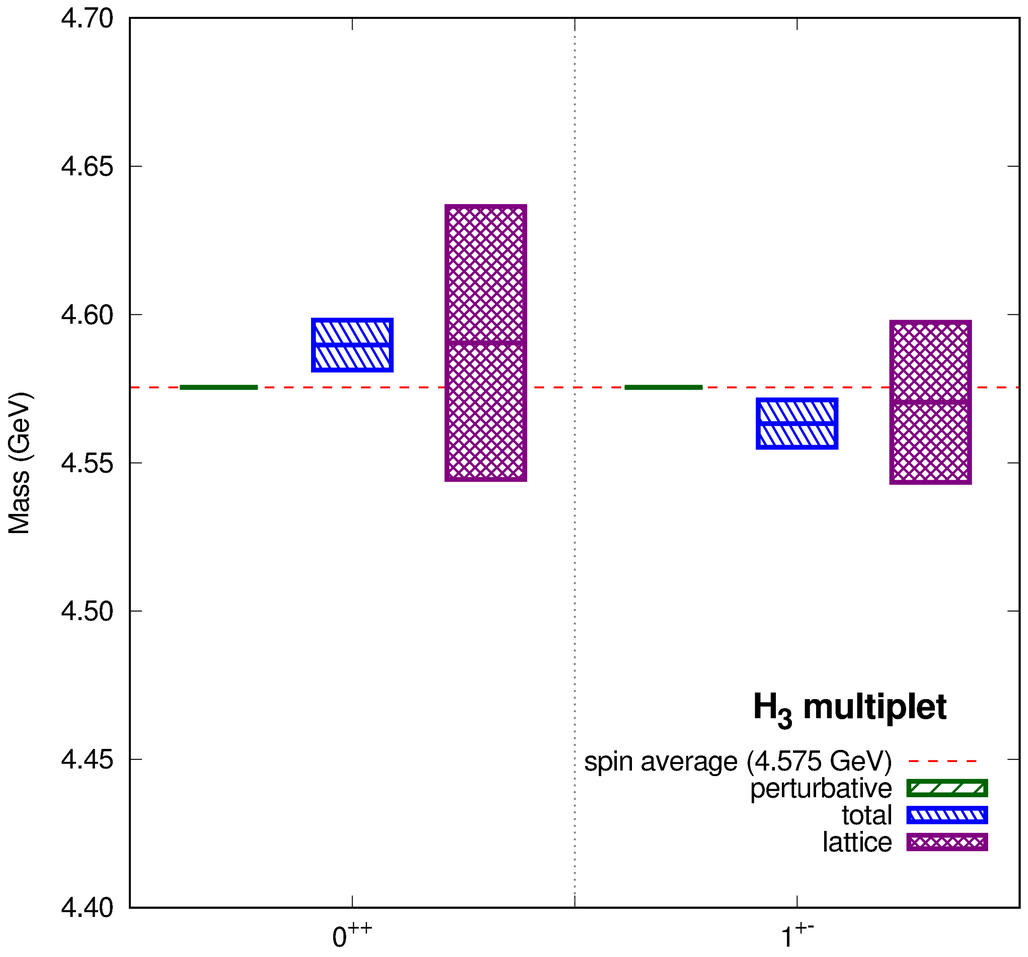
H_1 and H_2 corresponds to I=1 and are negative and positive parity resp. The mass splitting between H_1 and H_2 is a result of lambda-doubling

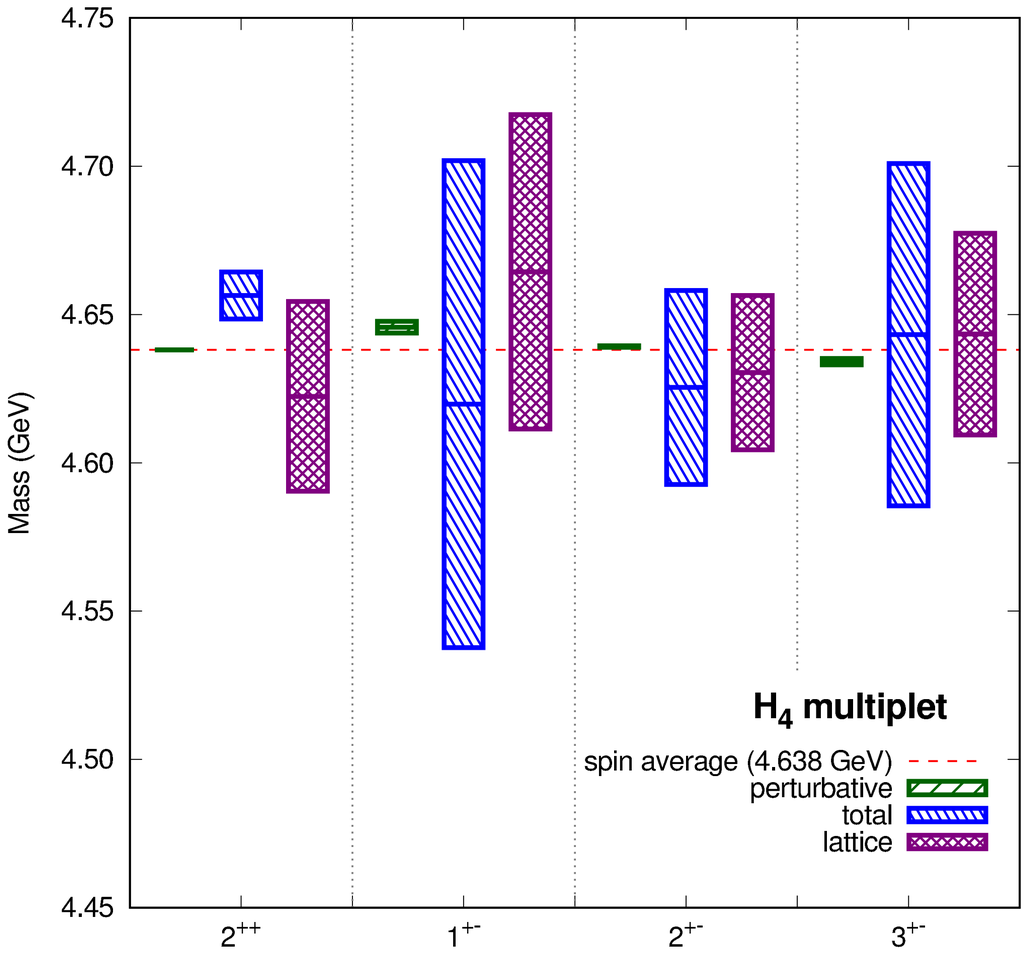
H_3 and H_4 are also calculated

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here you find predictions for all H multiplets

Charmonium Hybrids Multiplets H_3 and H_4





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  [2+1 flavors, $m_\pi = 240$ MeV]

Charmonium H_1 hybrid spin splittings

thanks to the BOEFT factorization
fix the nonperturbative unknowns from
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the nonperturbative low energy unknowns
do not depend on the flavor: they can fixed
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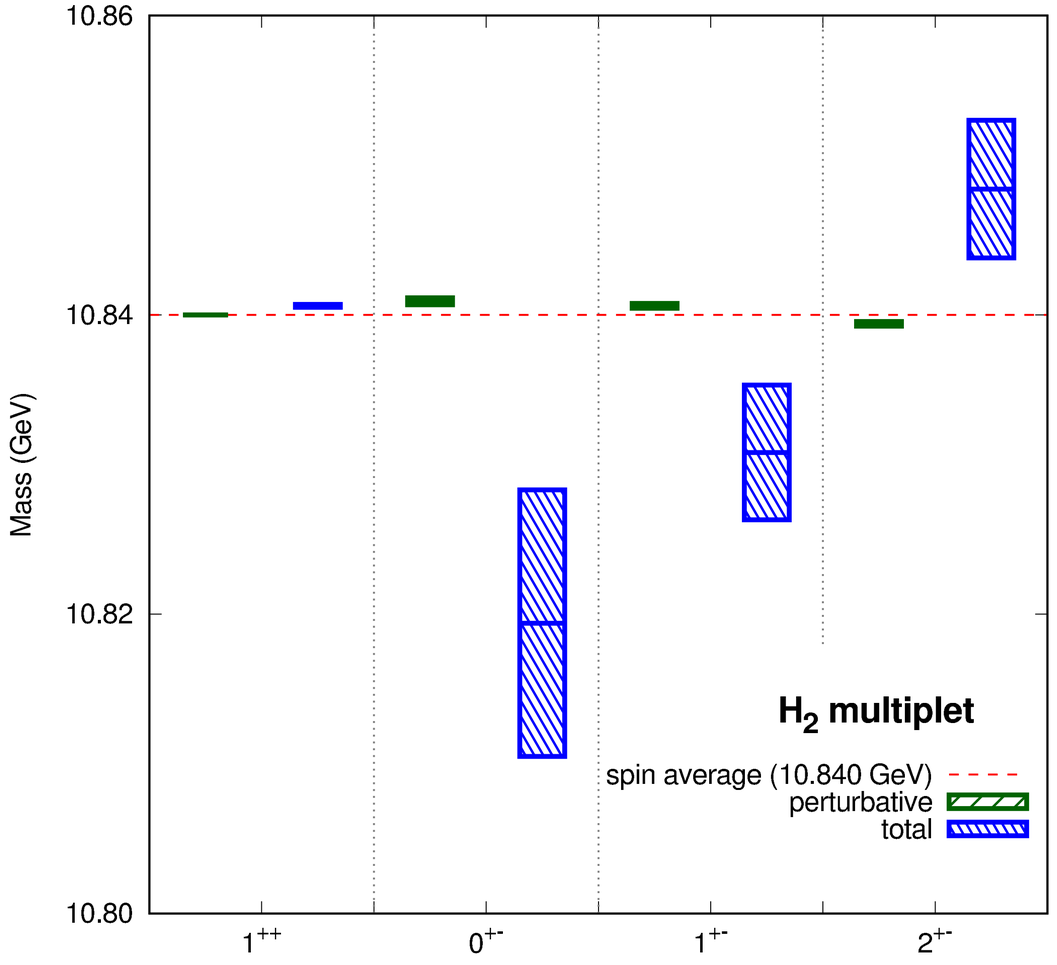
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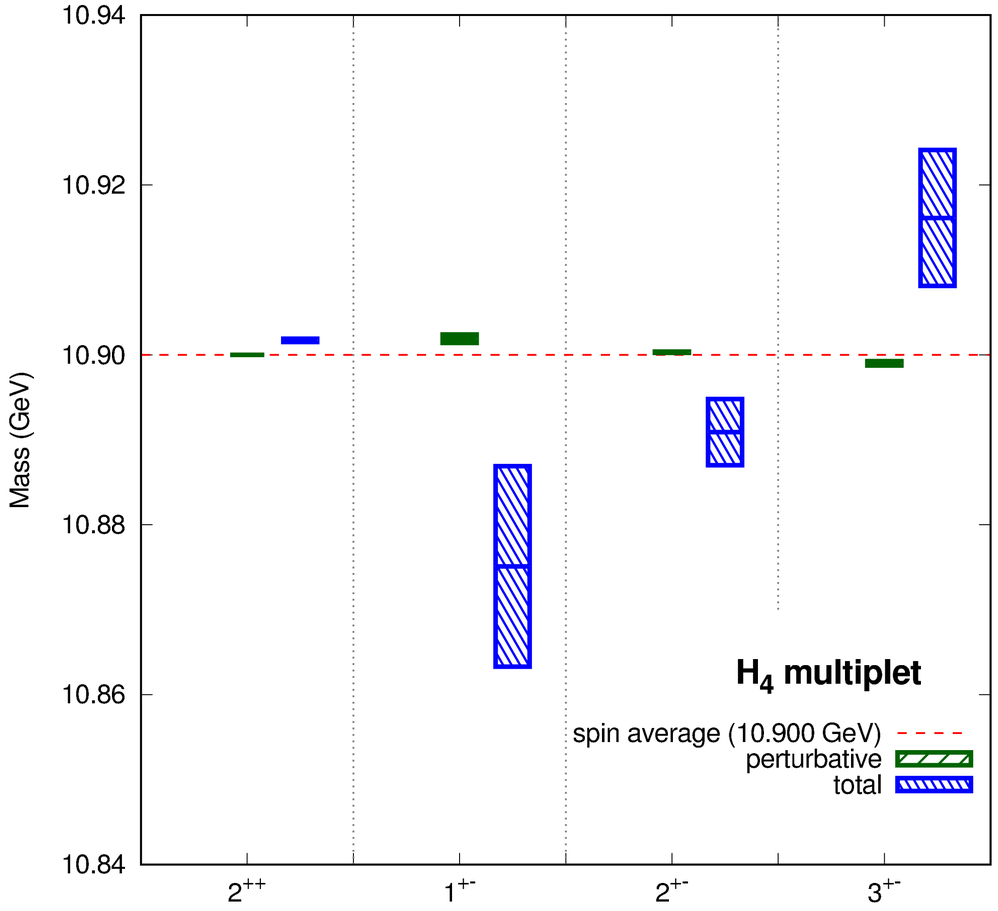
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Bottomonium hybrid spin splittings



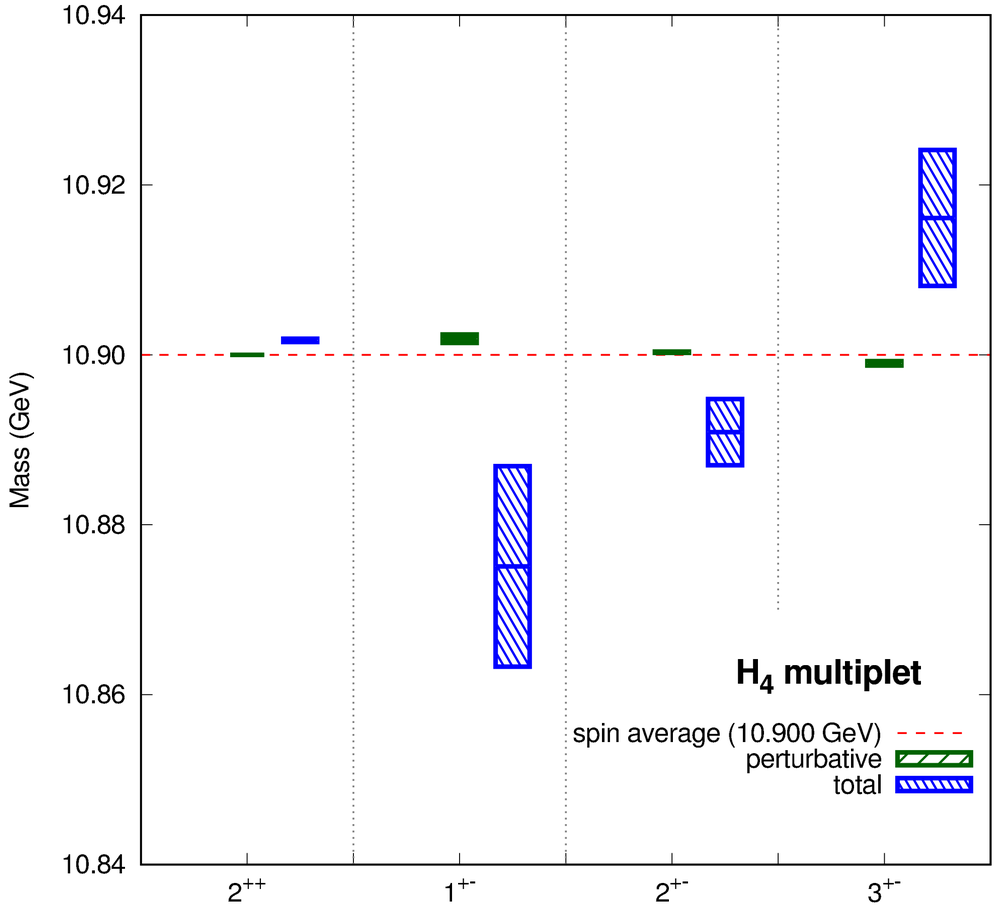
and also the other H multiplets

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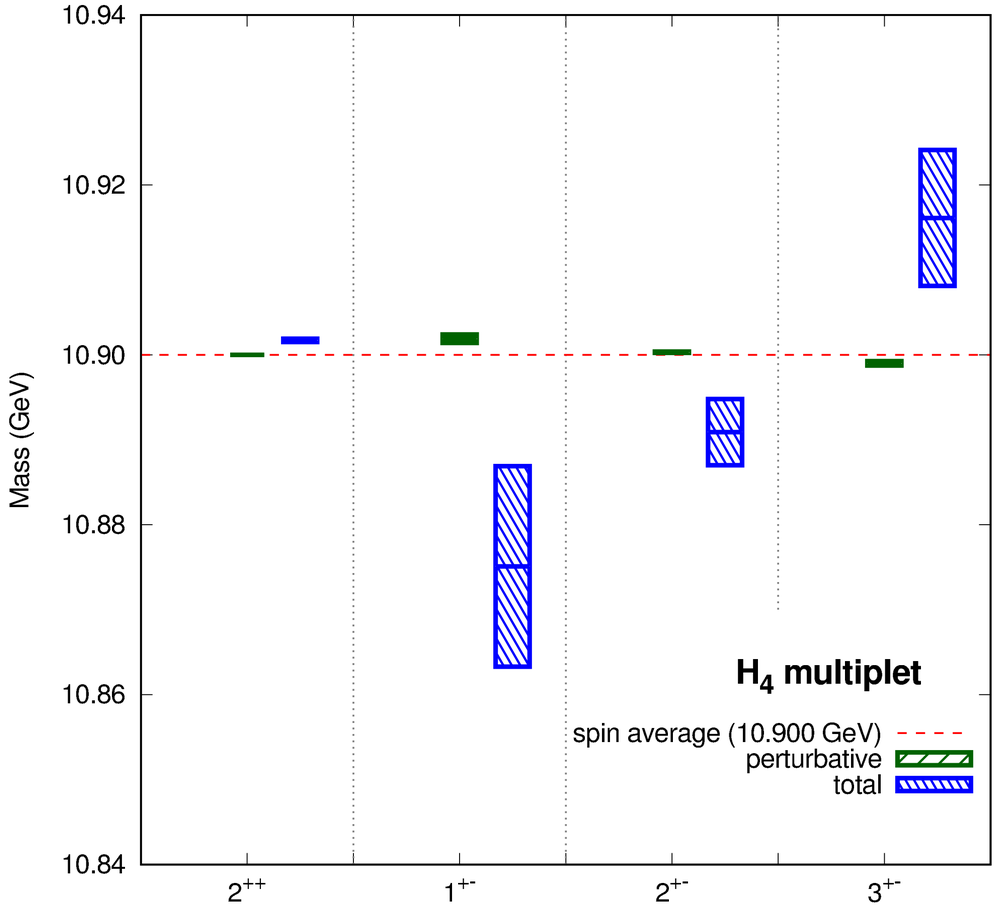
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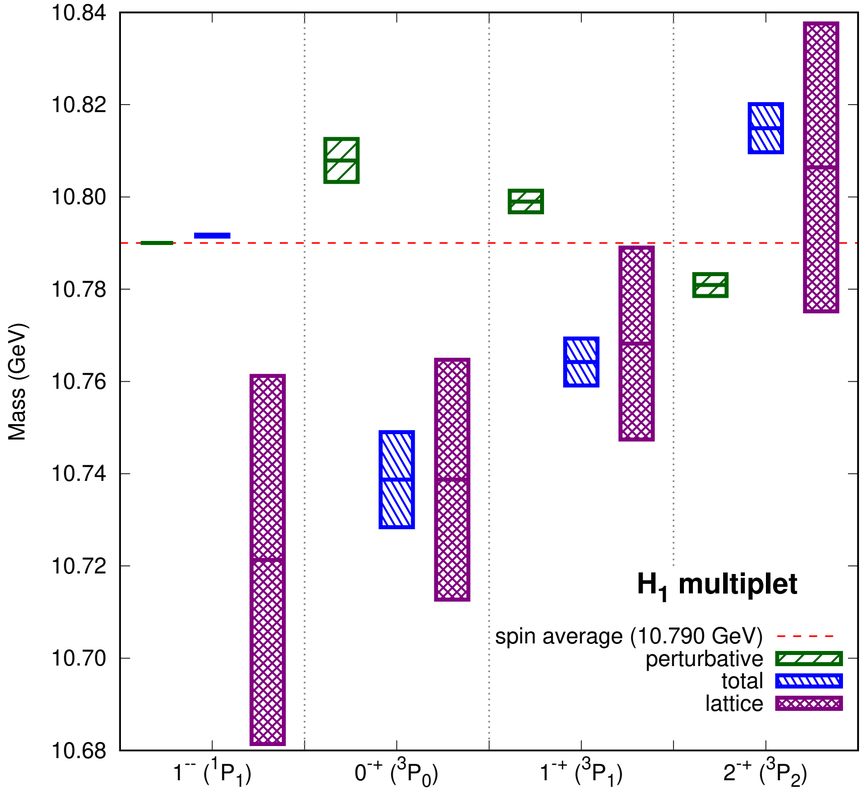
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blue BOEFT predictions (more precise),
violet actual lattice calculation

- Ryan et al arXiv:2008.02656 [2+1 flavors, $m_\pi = 400$ MeV]
  unpublished plot by J. Segovia

Mixing

Oncala Soto 1702.03900

The 1/m operator giving origin to the 1/m spin potential in heavy hybrids is also responsible for a mixing between spin 0 (1) hybrids and spin 1 (0) quarkonia

This may explain

in dependence of the strength of the mixing, which is of order $\Lambda_{\rm QCD}^2/m_h$ and non-perturbative, why some hybrid candidates appear to decay both into $\pi^+\pi^- J/\psi$ and $\pi^+\pi^- h_c$.

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Decays from hybrids to quarkonium

	$nL_T \to n'L'$	ΔE (MeV)	Γ (MeV)
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we are currently studying features of mixing and decays

N. B., A. Mohapatra, W.K. Lai, A. Vairo

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Tetraquarks and pentaquarks

BOEFT can be used to describe any system made by two heavy quarks bound adiabatically with some light degrees of freedom: glue (hybrids) or light quarks (tetraquarks, pentaquark)

in case of light quarks isospin quantum numbers should be added

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steps go as before:

identify the symmetries, identify the interpolating operators O_n and define the static energies

$$\mathcal{O}_n(t,\boldsymbol{r},\boldsymbol{R}) = \chi(t,\boldsymbol{R}-\boldsymbol{r}/2)\phi(t,\boldsymbol{R}-\boldsymbol{r}/2,\boldsymbol{R})H_n(t,\boldsymbol{R})\phi(t,\boldsymbol{R},\boldsymbol{R}+\boldsymbol{r}/2)\psi^\dagger(t,\boldsymbol{R}+\boldsymbol{r}/2)$$

$$E_n^{(0)}(r) = \lim_{T\to\infty}\frac{i}{T}\log\langle\mathcal{O}_n(T,\boldsymbol{r},\boldsymbol{R})|\mathcal{O}_n(0,\boldsymbol{r},\boldsymbol{R})\rangle$$

see the talk of J. Tarrus

.. Examples of gluonic operators and light-quark operators for quarkonium hybrids and tetraquarks respectively, $\boldsymbol{q}=(u,d)$ and τ^a are isospin Pauli matrices.

Λ_η^σ	κ	H	$H = H^a T^a(I=0, I=1)$
Σ_g^+	0^{++}	$\mathbb{1}$	$\bar{\boldsymbol{q}}T^a(\mathbb{1},\boldsymbol{\tau})\boldsymbol{q}$
Σ_u^-	1^{+-}	$\hat{\mathbf{r}}\cdot\mathbf{B}$	$\bar{\boldsymbol{q}}\,[(\hat{\boldsymbol{r}}\times\boldsymbol{\gamma})\cdot,\boldsymbol{\gamma}]\,T^a(\mathbb{1},\boldsymbol{\tau})\boldsymbol{q}$
Π_u	1^{+-}	$\hat{\mathbf{r}}\times\mathbf{B}$	$\bar{\boldsymbol{q}}\,[\hat{\boldsymbol{r}}\cdot\boldsymbol{\gamma},\boldsymbol{\gamma}]\,T^a(\mathbb{1},\boldsymbol{\tau})\boldsymbol{q}$
$\Sigma_g^{+\prime}$	1^{--}	$\hat{\mathbf{r}}\cdot\mathbf{E}$	$\bar{\boldsymbol{q}}\,(\hat{\boldsymbol{r}}\cdot\boldsymbol{\gamma})T^a(\mathbb{1},\boldsymbol{\tau})\boldsymbol{q}$
Π_g	1^{--}	$\hat{\mathbf{r}}\times\mathbf{E}$	$\bar{\boldsymbol{q}}\,(\hat{\boldsymbol{r}}\times\boldsymbol{\gamma})T^a(\mathbb{1},\boldsymbol{\tau})\boldsymbol{q}$

N.B. G. Krein, J. Tarrus, A. Vairo 1707.09647. J. Tarrus 1901. 09761, J. Soto, J. Tarrus 20005.00552

BOEFT for $I = 1$ tetraquarks

Tarrus arXiv:1901.09761

$$\Gamma_\mu = \left(u^\dagger \partial_\mu u + u \partial_\mu u^\dagger\right)/2 \text{ and } u = \exp(i\boldsymbol{\pi}\cdot\boldsymbol{\tau}/$$

$$D_\mu \boldsymbol{Z} = \partial_\mu + [\Gamma_\mu, \boldsymbol{Z}]$$

$$\begin{aligned}
\mathcal{L}_{\text{BOEFT for } I=1} = & \int d^3r \ \text{Tr}\left\{ Z^\dagger_{0+-}\left(iD_0 - V^{\text{tetra}}_{\Sigma_g^+}(r) + \frac{\boldsymbol{\nabla}_r^2}{m_h}\right) Z_{0+-}\right\} \\
& + \int d^3r \ \sum_{\lambda\lambda'} \text{Tr}\left\{ Z^\dagger_{1+-\lambda}\left(iD_0 - V^{\text{tetra}}_{1^{+-}\lambda\lambda'}(r) + \hat{r}^{i\dagger}_\lambda \frac{\boldsymbol{\nabla}_r^2}{m_h}\hat{r}^i_{\lambda'}\right) Z_{1+-\lambda'}\right\} \\
& + \int d^3r \ \sum_{\lambda\lambda'} \text{Tr}\left\{ Z^\dagger_{1--\lambda}\left(iD_0 - V^{\text{tetra}}_{1^{--}\lambda\lambda'}(r) + \hat{r}^{i\dagger}_\lambda \frac{\boldsymbol{\nabla}_r^2}{m_h}\hat{r}^i_{\lambda'}\right) Z_{1--\lambda'}\right\} \\
& + \text{terms with higher orbital momentum and mixing of states}
\end{aligned}$$

with the isovector field

$$Z_\kappa = Z^i_\kappa \sigma^i = \begin{pmatrix} Z^0_\kappa & \sqrt{2}Z^+_\kappa \\ \sqrt{2}Z^-_\kappa & -Z^0_\kappa \end{pmatrix}$$

needs lattice calculations of tetraquarks static energies

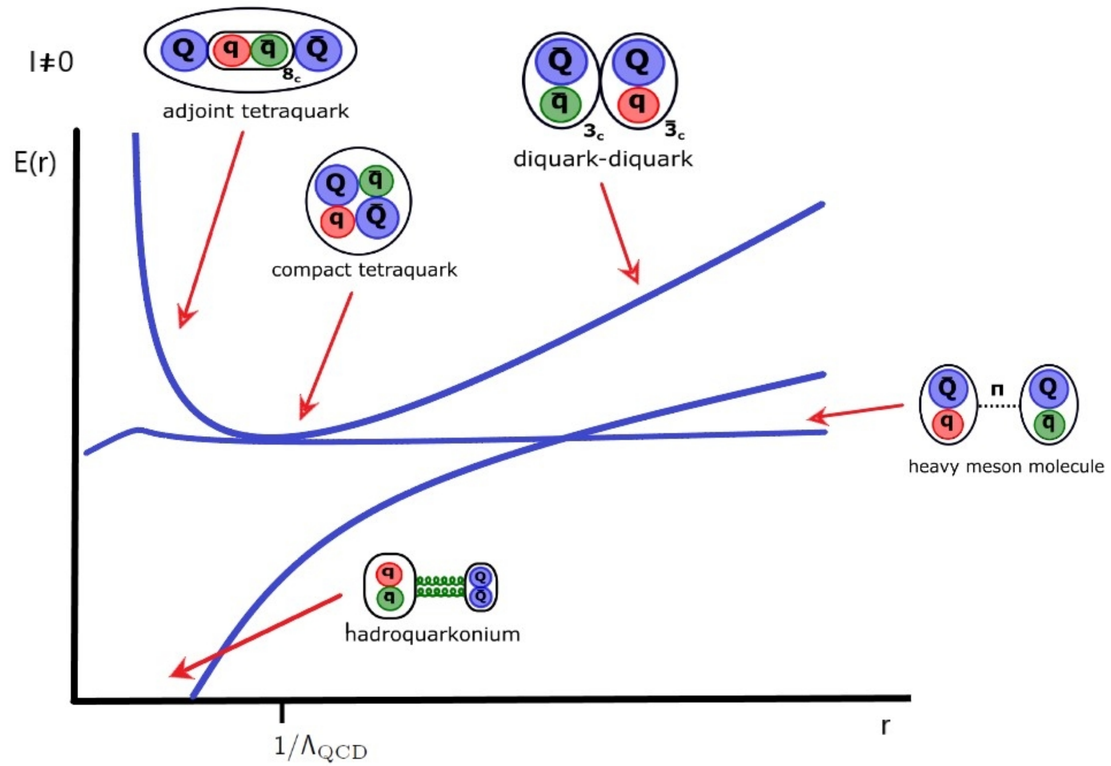
The direct use of the $I = 1$ BO effective Lagrangian is limited by the fact that the potentials have not, even in their static limit, been measured on the lattice. Hence, the situation is different from the hybrid case, where static hybrid energies are known since long time.

I=1 S. Prevlosek, H. Bahtiyar, J. Petrovich eprint: 1912.02656

I=0 Bicudo Cichy Peters Wagner PRD 93 (2016) 034501

The BOEFT contains all configurations: what dominates and where depends on the QCD dynamics

Static energies for $I \neq 0$ (schematic):

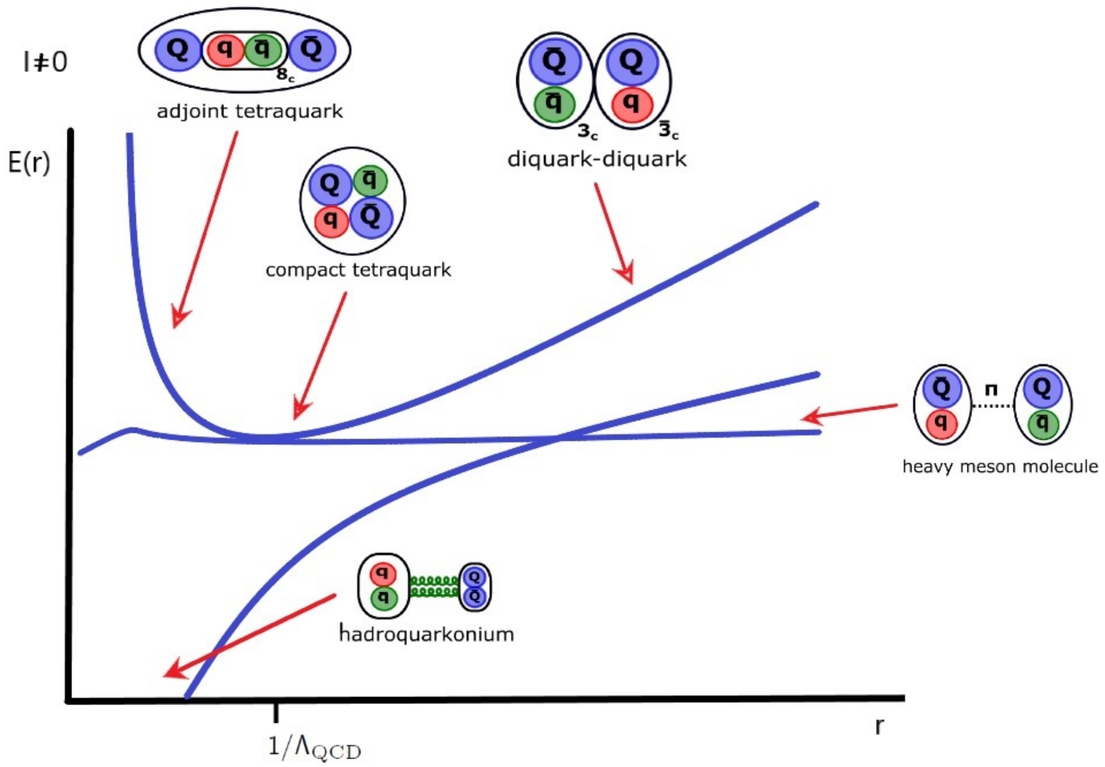


The static energies are defined in BOEFT that gives the appropriate set of operators to be used and could describe the short distance limit.
Being nonperturbative objects E(r) should be calculated on the lattice (or in QCD vacuum models)

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Figure from J. Tarrus

$\bar{b}\,b\,\bar{d}\,u$ Z_b channel $\vec{S}_h = \vec{S}_b + \vec{S}_{\bar{b}}$

Eigen-energies $E_n(r)$: channel S_h=1, CP=-1, ε=-1

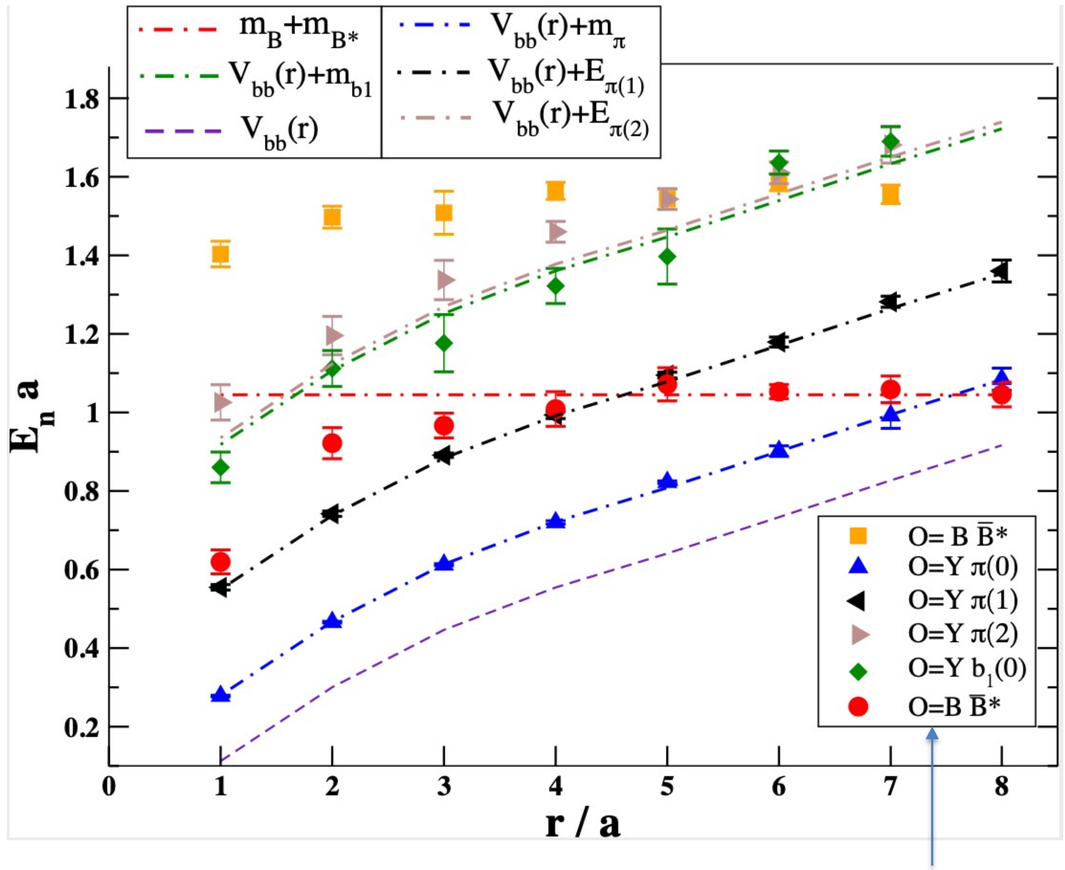
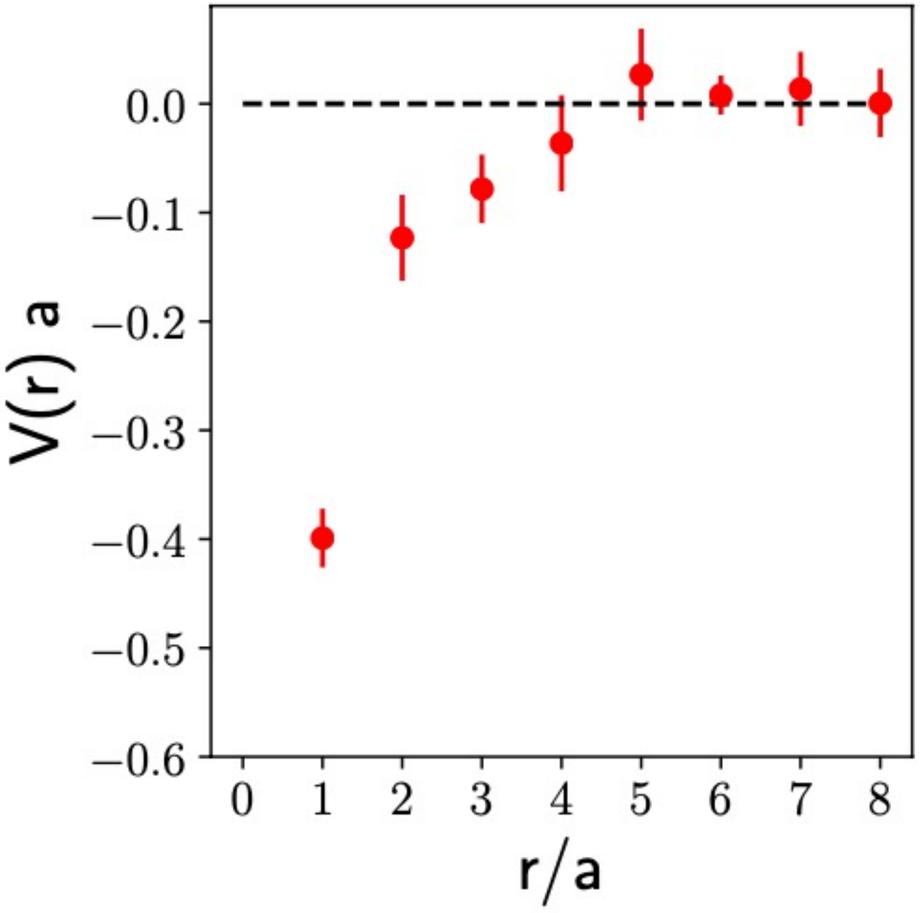


Figure from S. Prevlosek

The BOEFT contains all configurations: what dominates and where depends on the QCD dynamics

Binding configuration found on the lattice



S. Prevlosek, H. Bahtiyar, J. Petrovich eprint: 1912.02656

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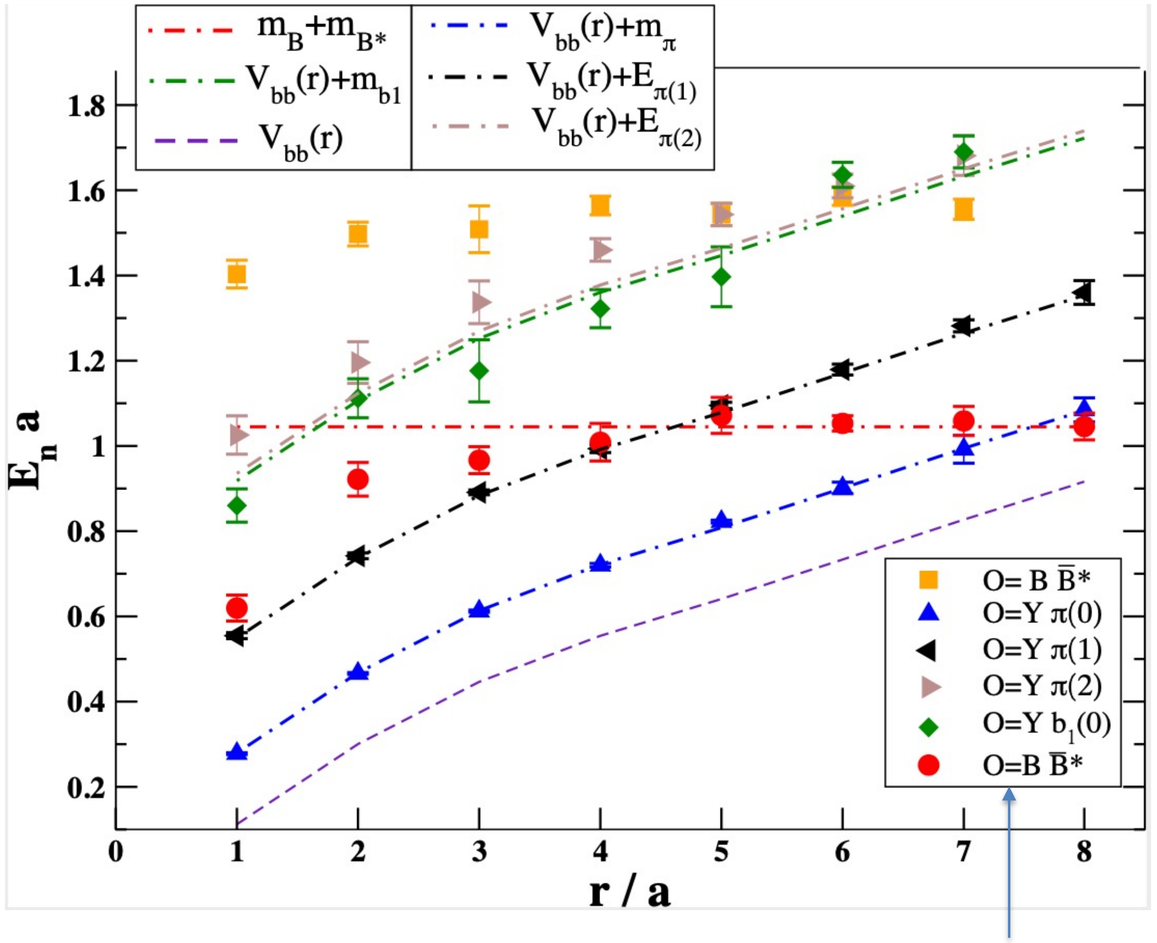


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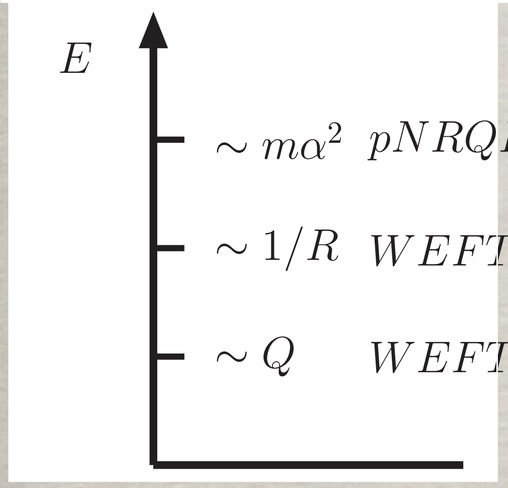
Notice: pNREFT is the lowest energy EFT for a single NR system but in the interaction between two NR systems more energy scales can be integrated out giving interaction potentials:

WEFT the EFT for bound-state—bound state-interaction

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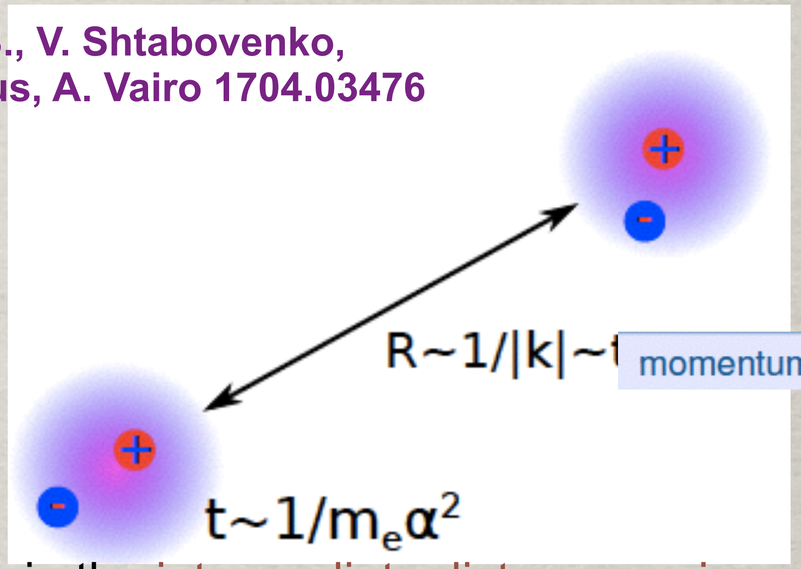
- Consider van der Waals interactions between two hydrogen atoms in the ground state at the distance R



WEFT is matched to pNREFT

Van der Walls EFT

N.B., V. Shtabovenko,
J. Tarrus, A. Vairo 1704.03476



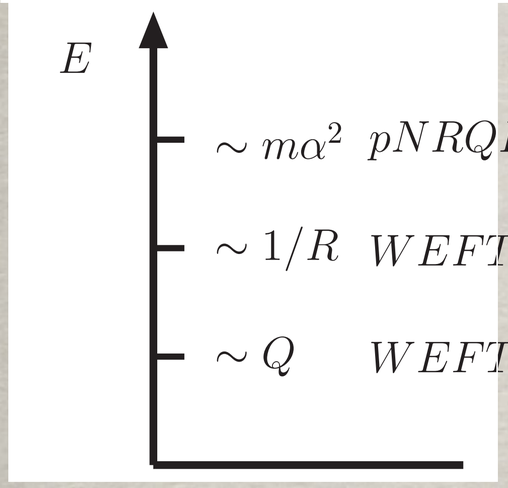
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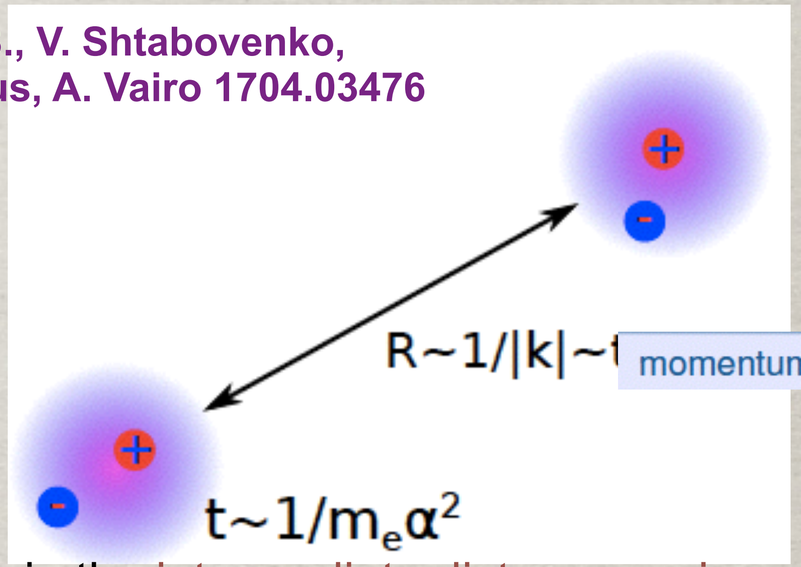
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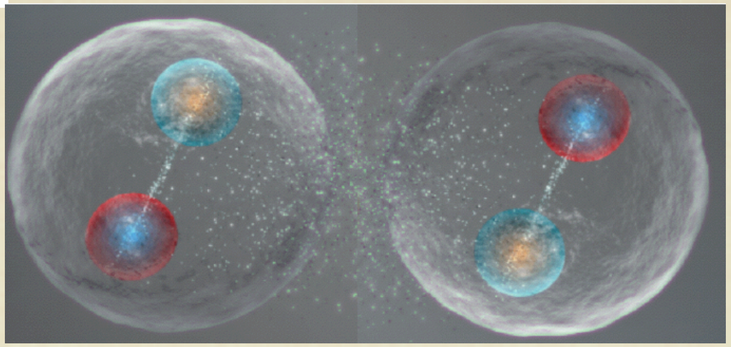
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In relation to the X(6900) it becomes interesting to explore the interaction of two heavy quarkonia we focus on the etab-etab interaction and possible bound state formation

Chromopolarizability & color van der Waals forces

$$\eta_b - \eta_b$$



Interactions between color neutral objects:

Via creation of instantaneous color dipole moments & gluon transitions in virtual color-octet intermediate state

— Polarizability—

— Chromopolarizability of 1S bottomonium;
use pNRQC (potential Nonrelativistic QCD)

N. Brambilla, GK, J. Tarrús-Castellà, A. Vairo, PRD 93, 054002 (2016)

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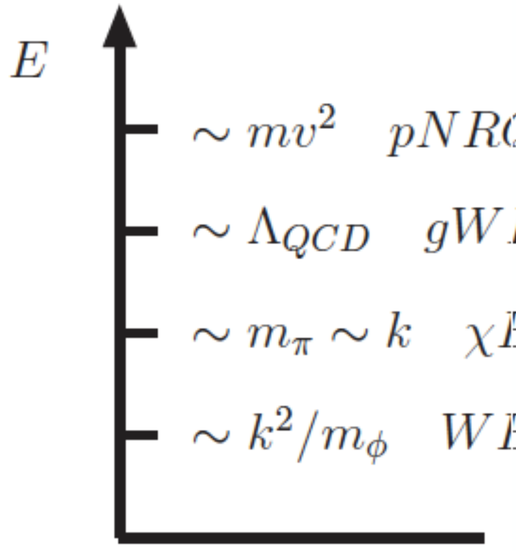
— van der Waals force between two bottomonia;
use QCD trace anomaly to match pNRQC to a chiral EFT

QCD ⟶ NRQCD ⟶ pNRQCD

↓

gWEFT ⟶ χEFT ⟶ WEFT

Polarizability van der Waals



m : bottom mass, v : relative velocity

$$m \gg mv \gg mv^2 \gg \Lambda_{\rm QCD}$$

m_ϕ : mass bottomonium, $r_{\phi\phi} \sim 1/m_\pi$: relative distance

$$\mathbf{k}^2_{\phi\phi}/m_\phi = m^2_\pi/m_\phi \ll m_\pi$$

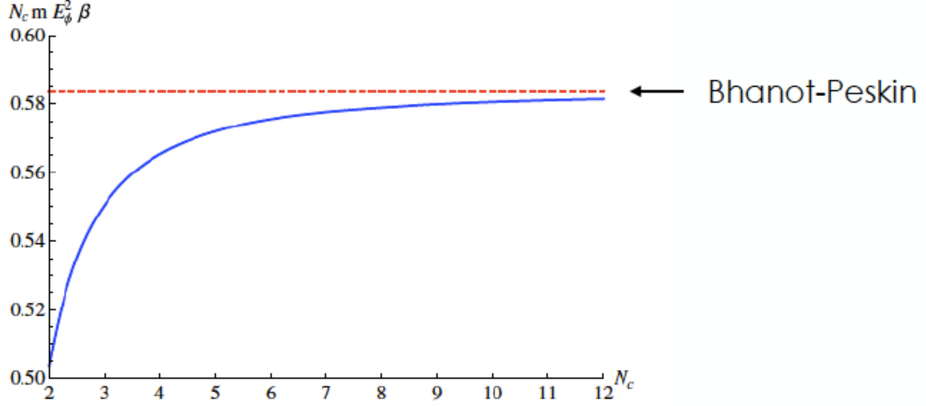


FIG. 3. The dependence of the polarizability on the number of colors. The dashed line at the constant value 7/12 corresponds to the large-N_c limit computed in Ref. [13].

Chromopolarizability



pNRQCD ⟶ gWEFT

$$L_{\rm gWEFT} = \int d^3\boldsymbol{R}\left\{\phi^\dagger(t,\boldsymbol{R})\left[i\partial_0 + E_\phi - \frac{\boldsymbol{\nabla}_{\boldsymbol{R}}^2}{4m} + \frac{1}{2}\beta g^2 \boldsymbol{E}_a^2 + \cdots\right]\phi(t,\boldsymbol{R})\right\} + \mathcal{L}_{\rm light}$$

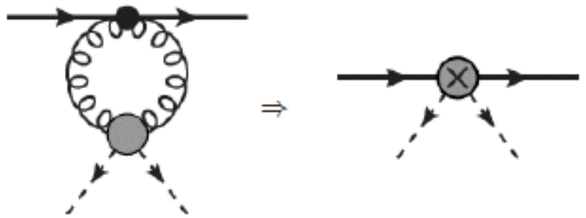
Chromopolarizability

$$\beta = -\frac{2V_A^2 T_F}{3N_c}\langle\phi|\boldsymbol{r}\frac{1}{E_\phi - h_o}\boldsymbol{r}|\phi\rangle$$

$$= -\frac{2V_A^2 T_F}{3N_c}\sum_l \int \frac{d^3p}{(2\pi)^3}|\langle\phi|\boldsymbol{r}|\boldsymbol{p}\,l\rangle|^2 \frac{1}{E_\phi - \frac{p^2}{m}}$$

van der Waals force

gWEFT ⟶ χEFT



QCD trace anomaly

$$g^2\langle\pi^+(p_1)\pi^-(p_2)|\boldsymbol{E}_a^2|0\rangle = \frac{8\pi^2}{3b}\left((p_1+p_2)^2\kappa_1 + m_\pi^2\kappa_2\right)$$

$\kappa_1 = 1-9\kappa/4,\ \kappa_2 = 1-9\kappa/2$

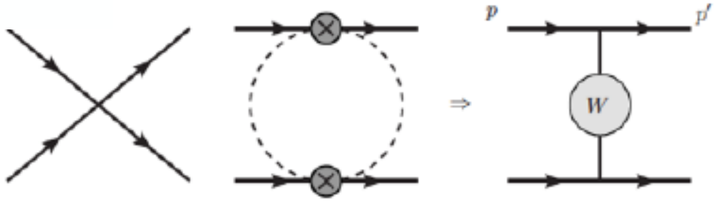
$b = \frac{11}{3}N_c - \frac{2}{3}N_f$

$\kappa = 0.186 \pm 0.003 \pm 0.006$ ⟵ $\psi' \to J/\psi\pi^+\pi^-$ – integrate out the pion

χEFT ⟶ WEFT

$r_{\phi\phi} \sim 1/m_\pi$

$\mathbf{k}_{\phi\phi}^2/m_\phi = m_\pi^2/m_\phi \ll m_\pi$



— vdW potential

$$W(r) = \frac{1}{2\pi^2 r}\int_{2m_\pi}^{\infty} d\mu\,\mu\, e^{-\mu r}\,\mathrm{Im}\left[\widetilde{W}(\epsilon - i\mu)\right]$$

$$= -\frac{3\pi\beta^2 m_\pi^2}{8b^2 r^5}\left[\left(4\left(\kappa_2+3\right)^2\left(m_\pi r\right)^3 + \left(3\kappa_1^2+43\kappa_2^2+14\kappa_1\kappa_2\right)m_\pi r\right)K_1(2m_\pi r)\right.$$

$$\left.+2\left(2\left(\kappa_2+3\right)\left(\kappa_1+5\kappa_2\right)\left(m_\pi r\right)^2 + \left(3\kappa_1^2+43\kappa_2^2+14\kappa_1\kappa_2\right)\right)K_2(2m_\pi r)\right]$$

↓ asymptotic

$$W(r) = -\frac{3(3+\kappa_2)^2\pi^{3/2}\beta^2}{4b^2}\frac{m_\pi^{9/2}}{r^{5/2}}e^{-2m_\pi r}$$

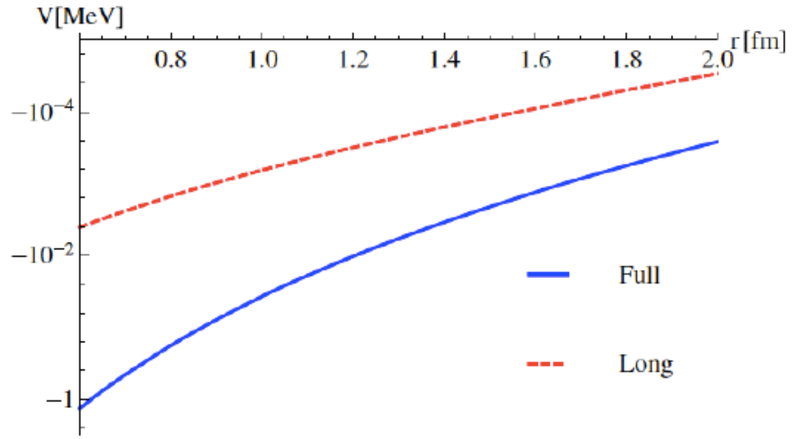


FIG. 9. Comparison of the van der Waals potential (40) (blue line) with its long-range expansion (41) (red line) for $\beta = 0.92$ GeV^{-3} and other parameters like in Fig. 8.

Are there $\eta_b\,\eta_b$ bound-states?

It is likely, but depends somewhat on the medium- and short-range pieces

a similar description could be used to study the J/psi — J/psi interaction -> relevant for the X(6900)

however likely one cannot use a multipole expansion and need a different definition of the polarisability

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Quarkonium polarisability has been studied by Misha Voloshin as any aspect of quarkonia and exotics, his pioneering papers on quarkonium in OPE expansion as been inspirational for the construction of pNRQCD:
in summary we owe you a lot Misha and we do miss you a lot!

Outlook

NREFTs and lattice allows us to describe the physics of quarkonium away from the strong decay threshold in quantum field theory: higher order perturbative calculation can be performed and quarkonium can be used for precision physics/ factorisation allows to systematically studyconfinement

NREFTs and lattice and open quantum system allows us to describe the nonequilibrium evolution quarkonium in the quark gluon plasma

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Combining BOEFT + open quantum systems one can attempt to study the X Y Z in heavy ion collisions

Strongly coupled pNRQCD: quantum mechanical matching

$$\langle H|\overset{\text{NRQCD}}{\mathcal{H}}|H\rangle = \langle nljs|\,\frac{\mathbf{p}^2}{m} + \sum_n \frac{V_s^{(n)}}{m^n}\,|nljs\rangle$$

the matching conditions are :

$$|\underline{0};\mathbf{x}_1\mathbf{x}_2\rangle - > S^\dagger(\mathbf{x}_1\mathbf{x}_2)|\text{vac}\rangle$$

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expand quantomechanically NRQCD states and energies in 1/m around the zero order and identify the QCD potentials

$$\overset{\text{NRQCD}}{\mathcal{H}} = \mathcal{H}^{(0)} + \frac{\delta\mathcal{H}^{(1)}}{m} + \frac{\delta\mathcal{H}^{(2)}}{m^2} + \frac{\delta\mathcal{H}^{(3)}}{m^3} + \frac{\delta\mathcal{H}^{(4)}}{m^4} + \ldots$$

$$\mathcal{H}^{(0)} = \int d^3\mathbf{x}\,\frac{1}{2}\left(\mathbf{\Pi}^a\mathbf{\Pi}^a + \mathbf{B}^a\mathbf{B}^a\right) - \sum_{n_f}\bar{q}\,i\mathbf{D}\cdot\boldsymbol{\gamma}\,q$$

$$\delta\mathcal{H}^{(1)} = -\int d^3\mathbf{x}\,\psi^\dagger\left(\frac{\mathbf{D}^2}{2} + c_F\,g\,\mathbf{S}\cdot\mathbf{B}\right)\psi + \text{antip.}$$

$$|H\rangle \rightarrow |\underline{0};\mathbf{x}_1,\mathbf{x}_2\rangle \otimes |nljs\rangle$$

$$|\underline{0};\mathbf{x}_1,\mathbf{x}_2\rangle = |\underline{0};\mathbf{x}_1,\mathbf{x}_2\rangle^{(0)} + \sum_{n\neq 0}\int d^3z_1\,d^3z_2|\underline{n};\mathbf{z}_1,\mathbf{z}_2\rangle^{(0)}$$

$$\times\frac{{}^{(0)}\langle\underline{n};\mathbf{z}_1,\mathbf{z}_2|\delta\mathcal{H}^{(1)}|\underline{0};\mathbf{x}_1,\mathbf{x}_2\rangle^{(0)}}{E_0^{(0)}(z) - E_n^{(0)}(x)} + \ldots$$

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$$\mathcal{H}^{(0)} = \int d^3\mathbf{x}\,\frac{1}{2}\left(\mathbf{\Pi}^a\mathbf{\Pi}^a + \mathbf{B}^a\mathbf{B}^a\right) - \sum \bar{q}\,i\mathbf{D}\cdot\boldsymbol{\gamma}\,q$$

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$$\times\frac{{}^{(0)}\langle\underline{n};\mathbf{z}_1,\mathbf{z}_2|\delta\mathcal{H}^{(1)}|\underline{0};\mathbf{x}_1,\mathbf{x}_2\rangle^{(0)}}{E_0^{(0)}(z) - E_n^{(0)}(x)} + \dots$$

$$V = V^{(0)} + \frac{V^{(1)}}{m} + \frac{V^{(2)}}{m^2} + \dots$$

$$V^{(0)}(r) = \lim_{T\to\infty}\frac{i}{T}\ln\langle\ \square\ \rangle$$

given in terms of gauge invariant Wilson loops

$$\square = \exp\left\{ig\oint_{r\times T} dz^\mu A_\mu\right\}$$