JLU
NEUE WEGE. SEIT 1607.

JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

HFHF
Helmholtz Forschungsakademie Hessen für FAIR

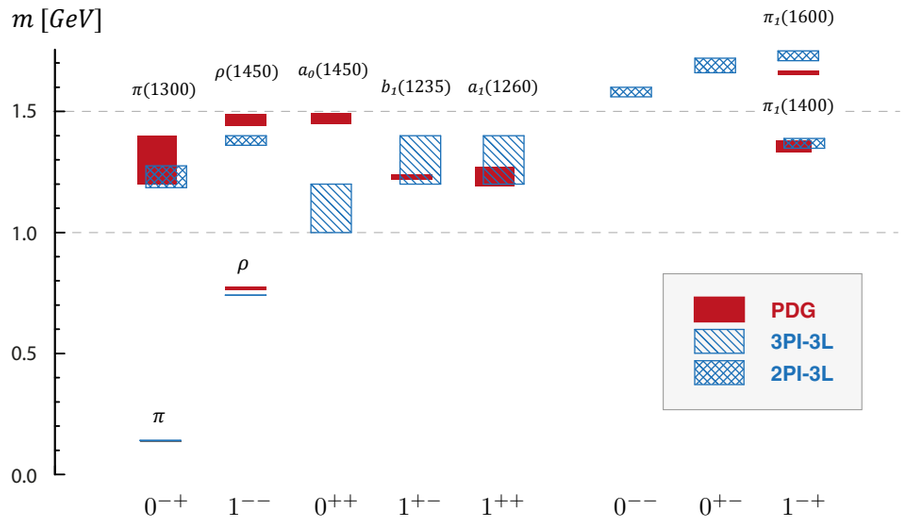
Dissecting XYZ-states with functional methods

Mini-Review: Eichmann, CF, Heupel, Santowsky, Wallbott, FBS 61 (2020) [2008.10240]

Overview

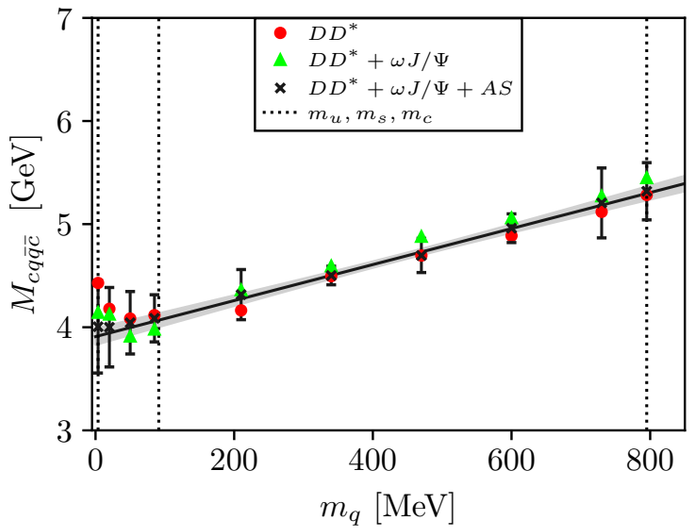
1. Light two- and four-quark states: the f0(500)

Williams, CF, Heupel, PRD 93 (2016) 034026 [arXiv:1512.00455]
Eichmann, CF, Heupel, PLB 753 (2016) 282 [arXiv:1508.07178]
Heupel, Eichmann, CF, PLB 718 (2021) 545 [arXiv:1206.5129]

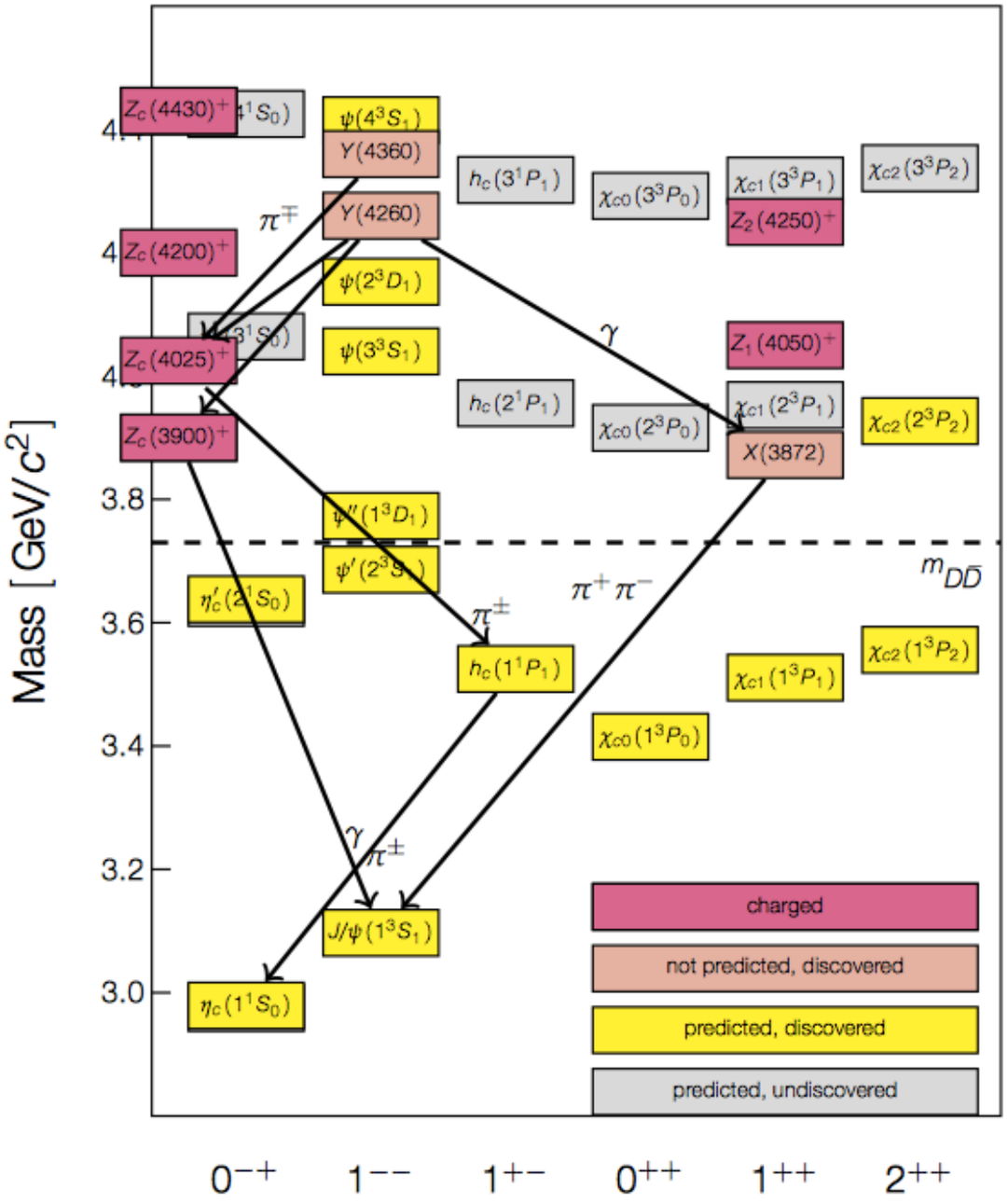


2. Heavy-light four-quark states: X(3872) and more…

Wallbott, Eichmann and CF, PRD100 (2019) no.1, 014033, [arXiv:1905.02615]
Wallbott, Eichmann and CF, PRD102 (2020) no.5, 051501, [arXiv:2003.12407]



Tetraquark candidates with $cq\bar{q}\bar{c}$-content



Many new unexpected states found: Belle, BABAR, BES, LHCb …

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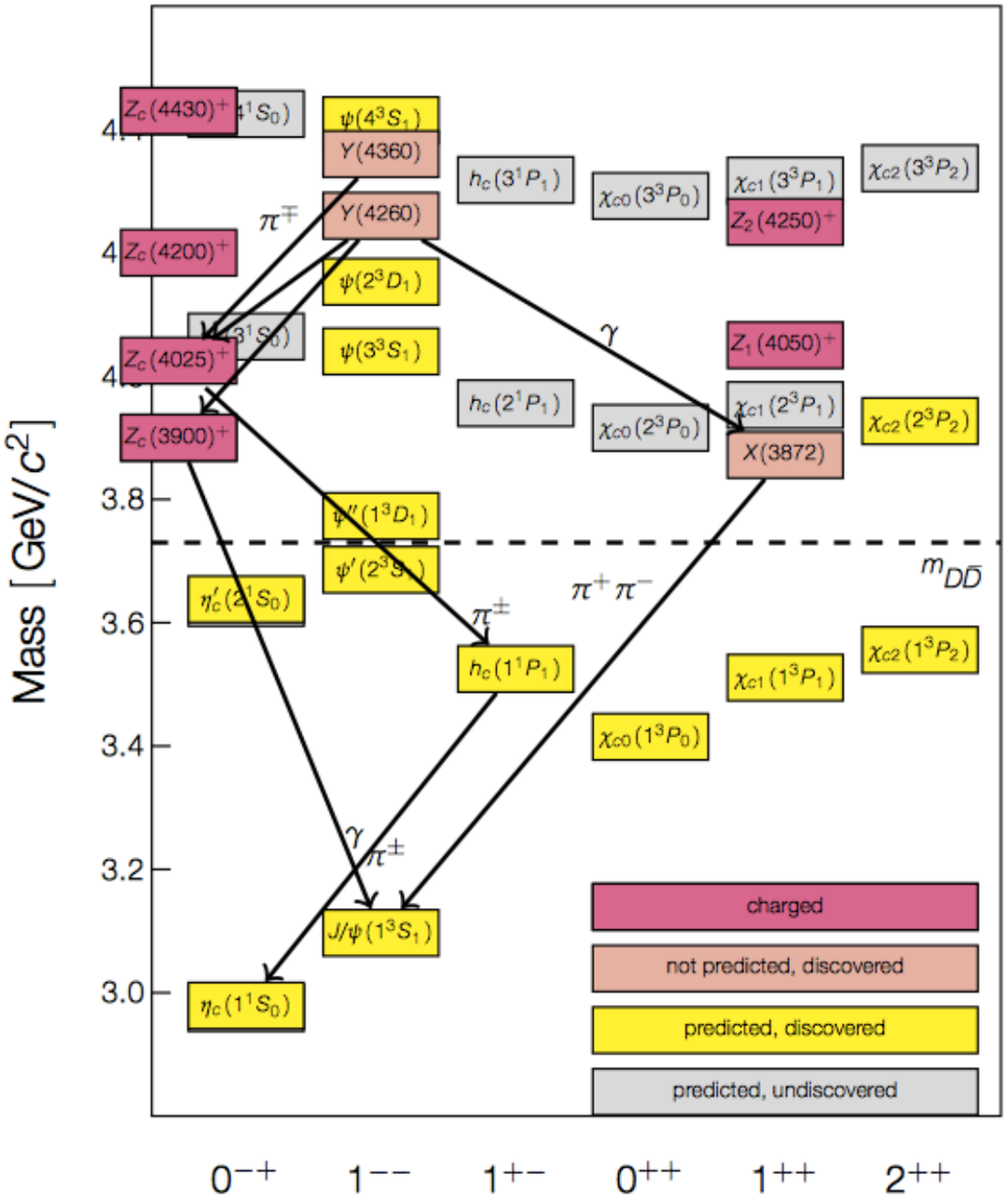

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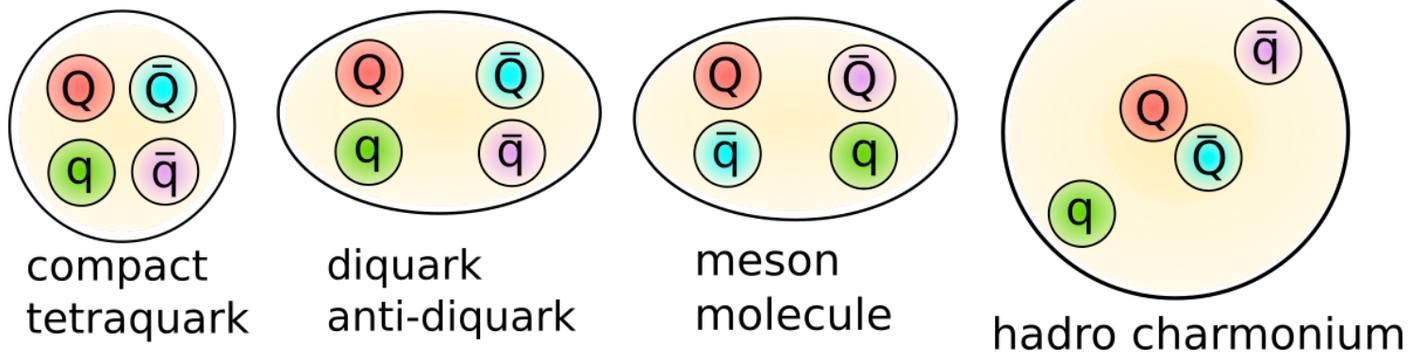
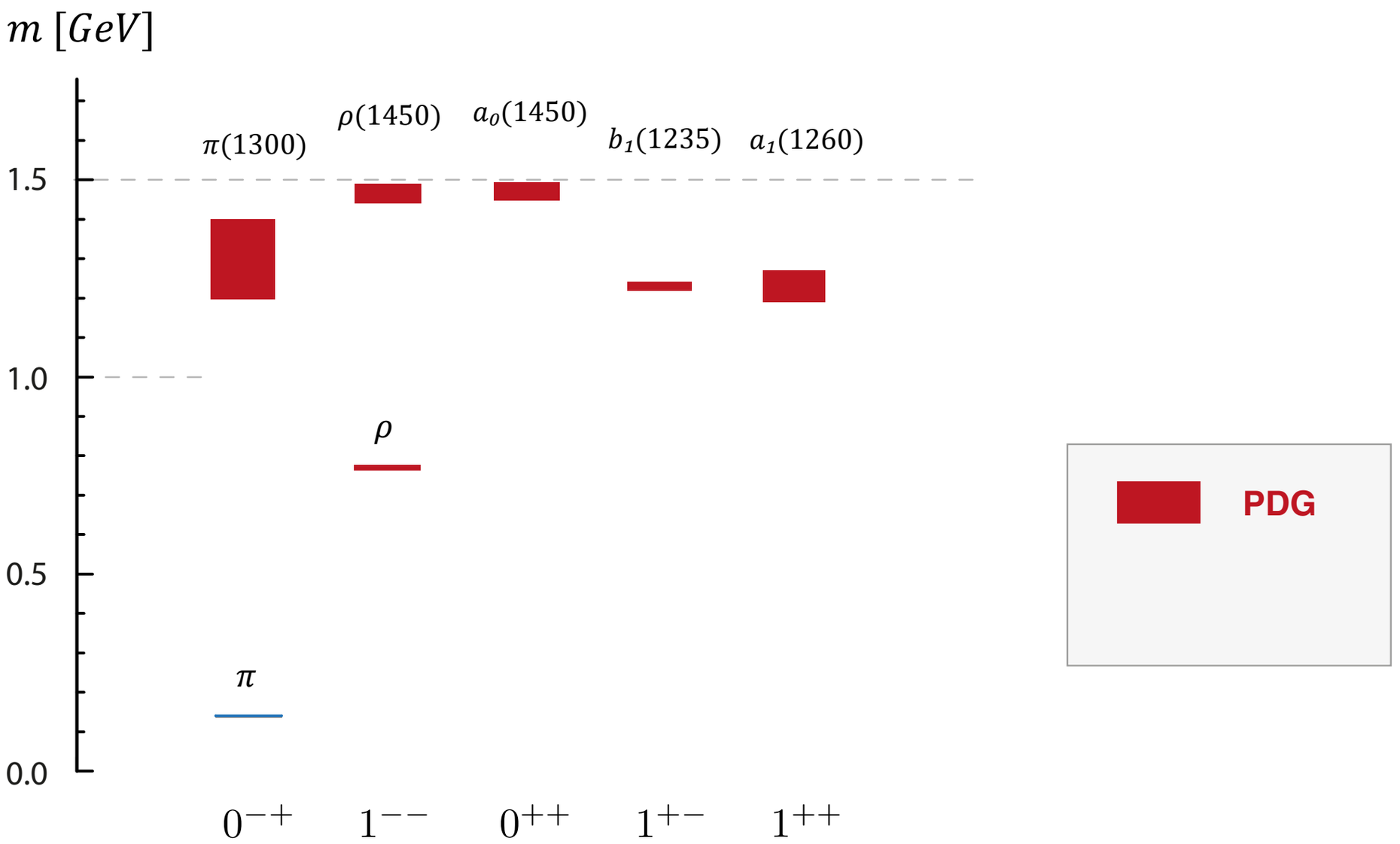
Internal structure ??



Related to details of underlying QCD forces between quarks and gluons

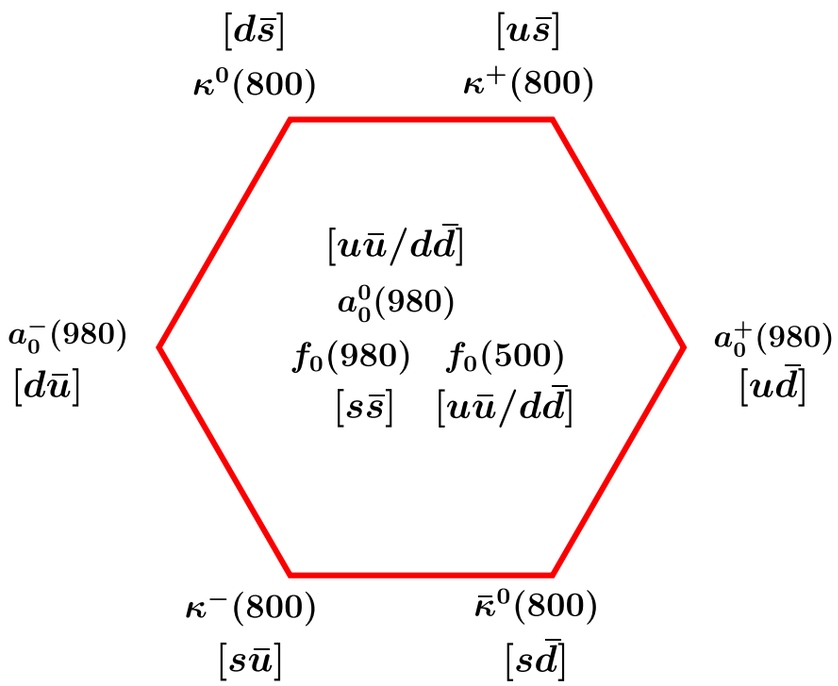
Light meson spectrum



- < 1 GeV: light nonet
- > 1GeV: 'heavy' nonet, glueball…

Light mesons with $\bar{q}\bar{q}qq$-content

Light scalar mesons:



wrong level ordering

R. L. Jaffe, Phys. Rev. D 15, 267 (1977)

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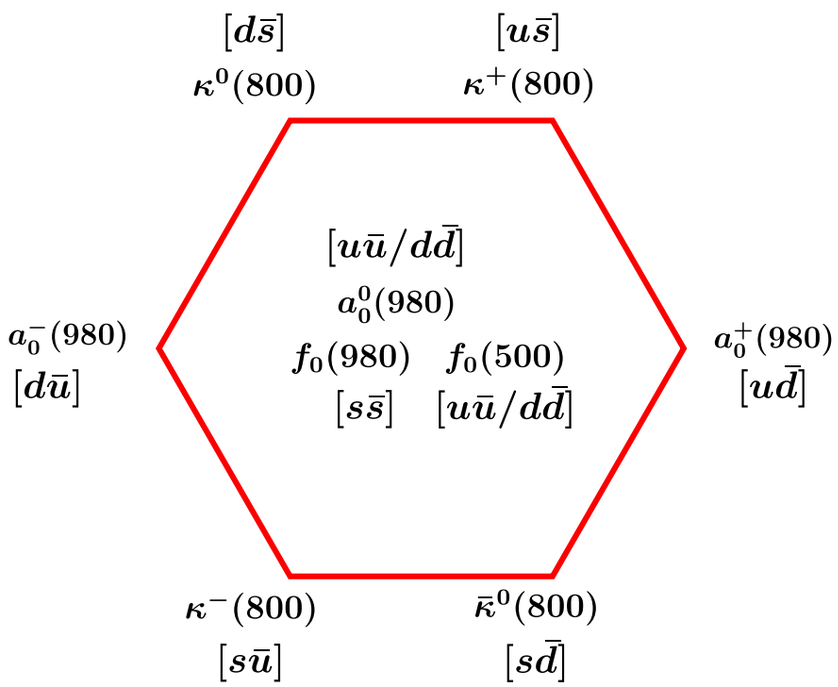

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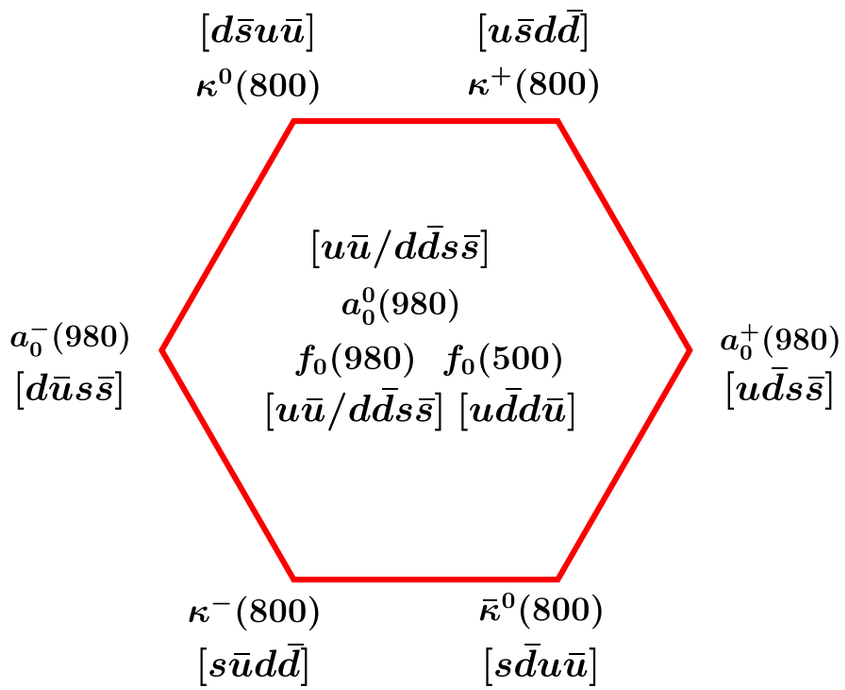

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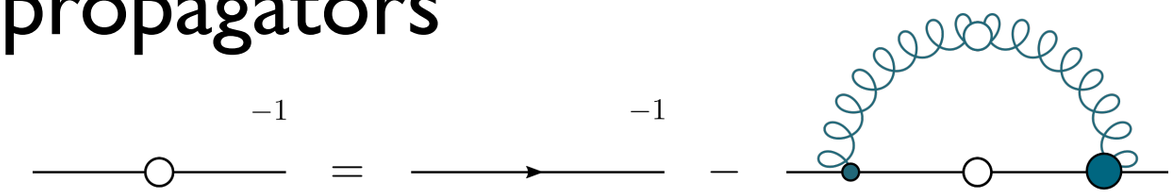
Dyson-Schwinger equations - "3PI vs RL"

$$\mathcal{Z}_{QCD} = \int \mathcal{D}[\Psi, A] \exp\left\{ - \int d^4x \left(\bar{\Psi} \left(i \not{D} - m \right) \Psi - \frac{1}{4} \left(F^a_{\mu\nu} \right)^2 \right) \right\}$$

propagators



CF, Alkofer, PRD67 (2003) 094020
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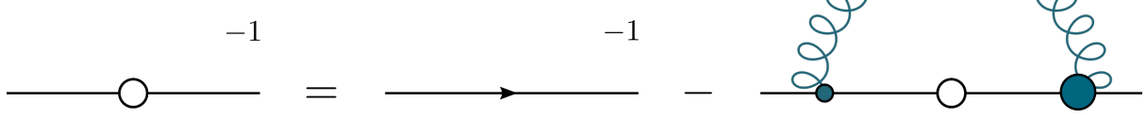

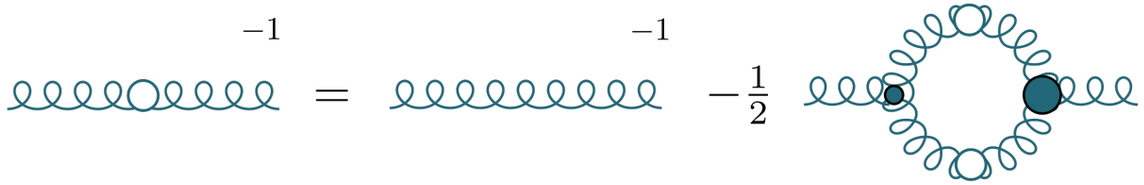

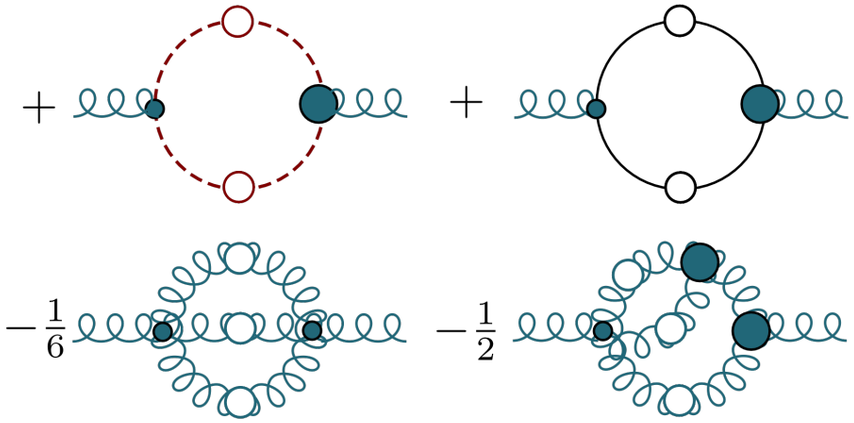

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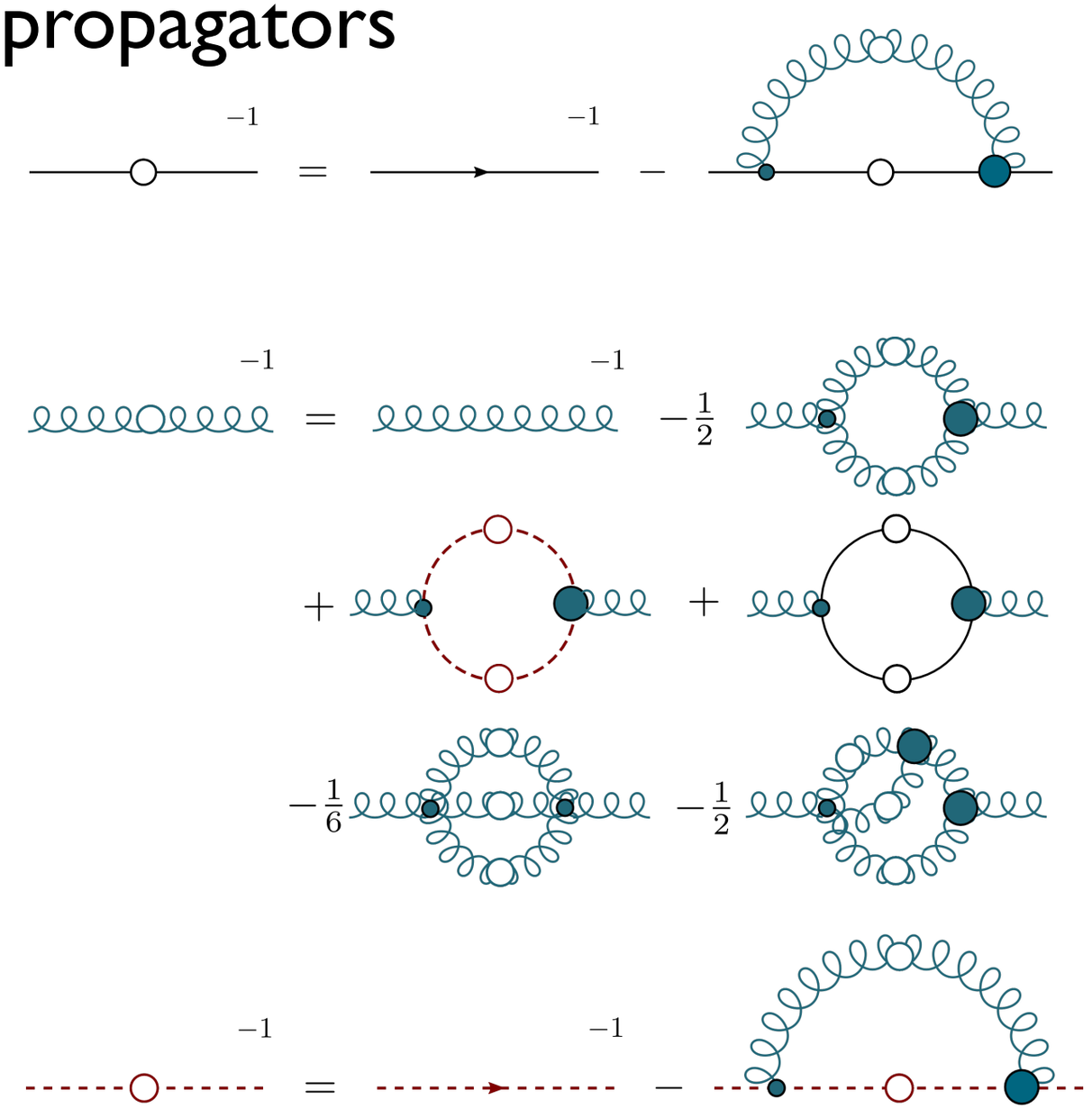
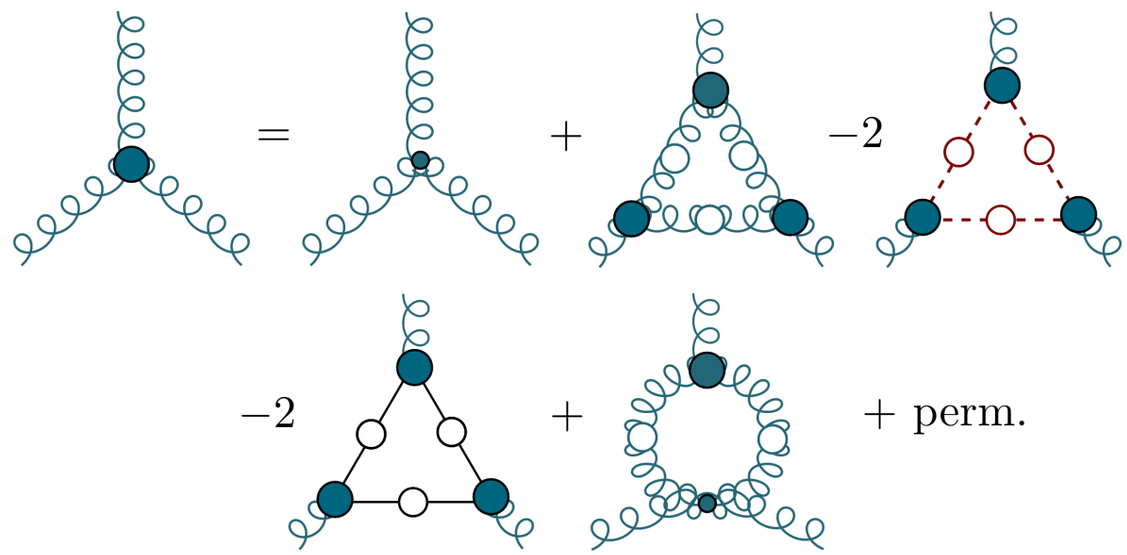


vertices



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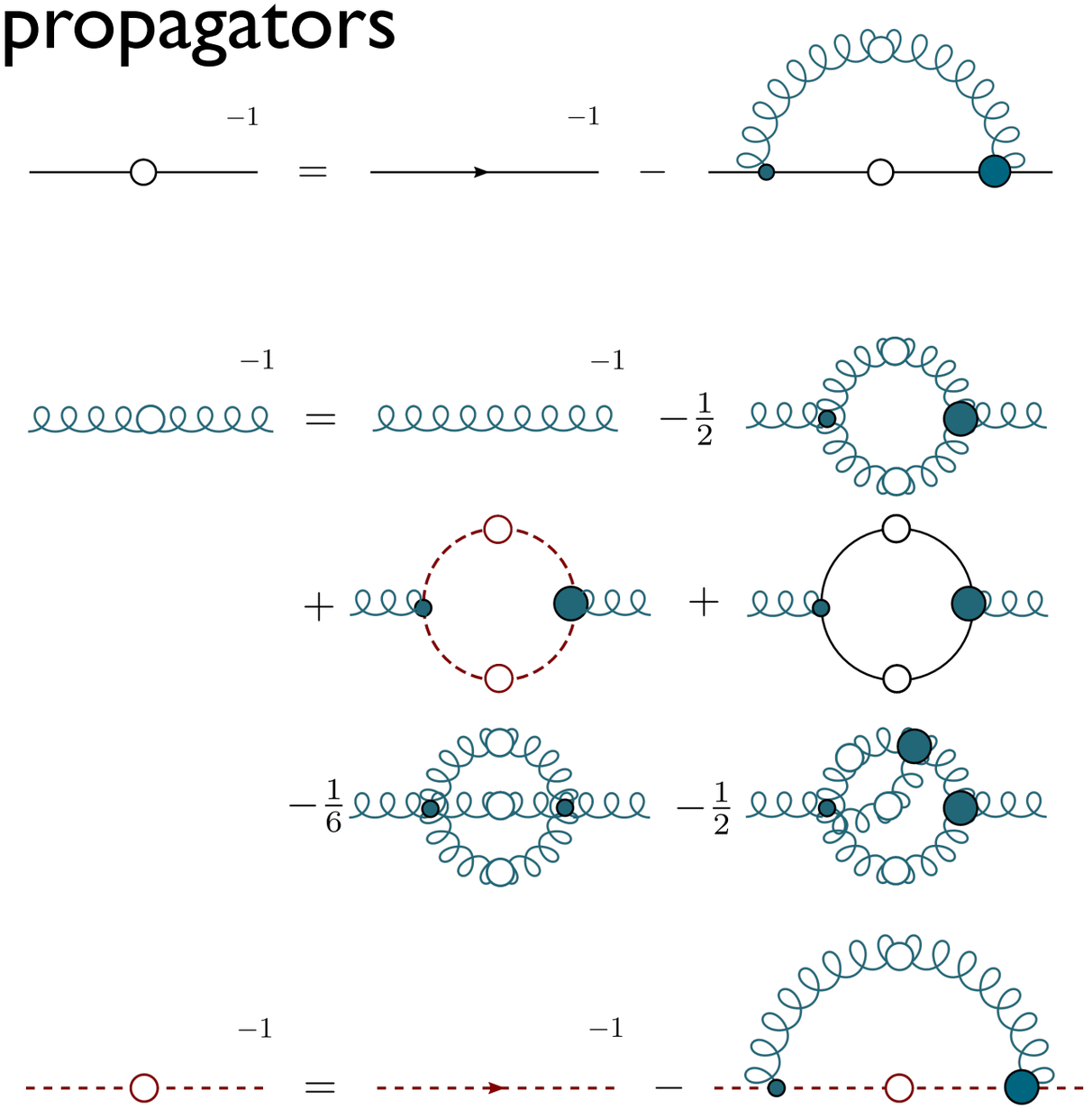

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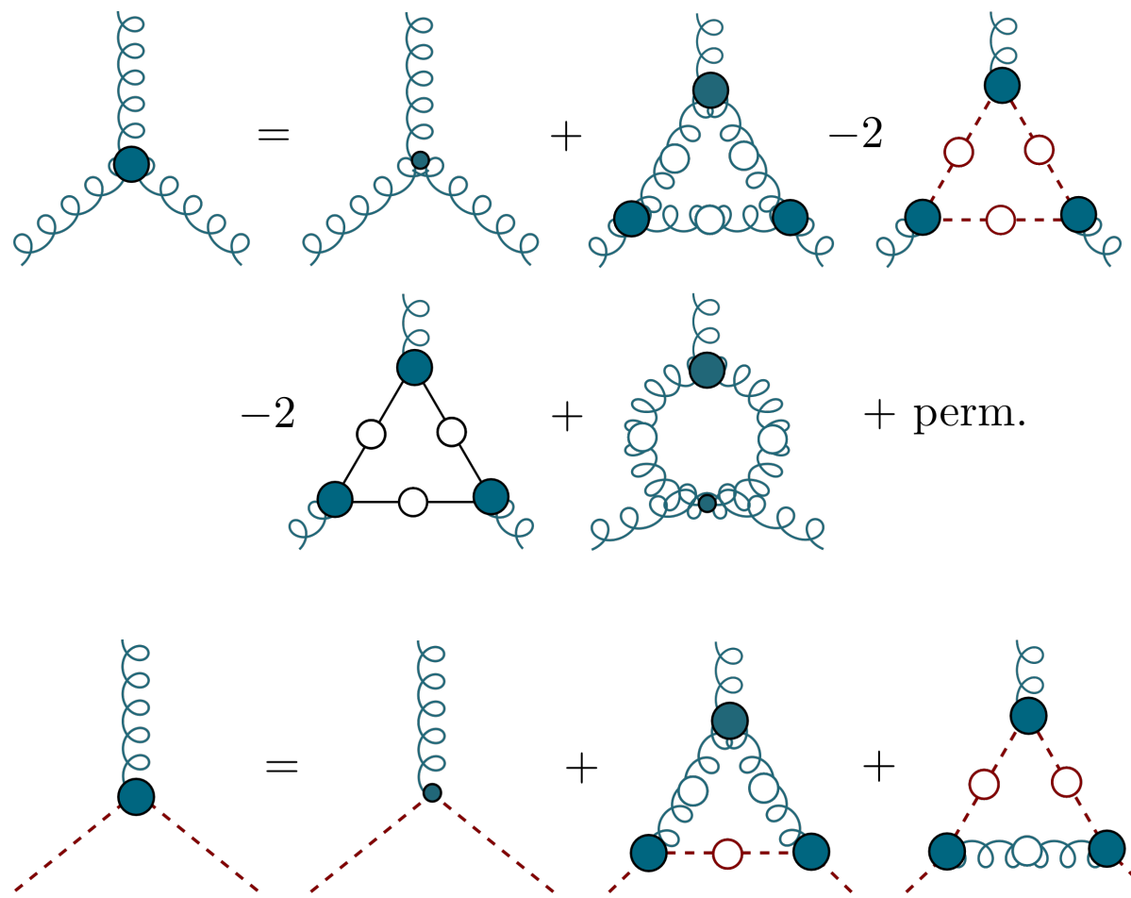

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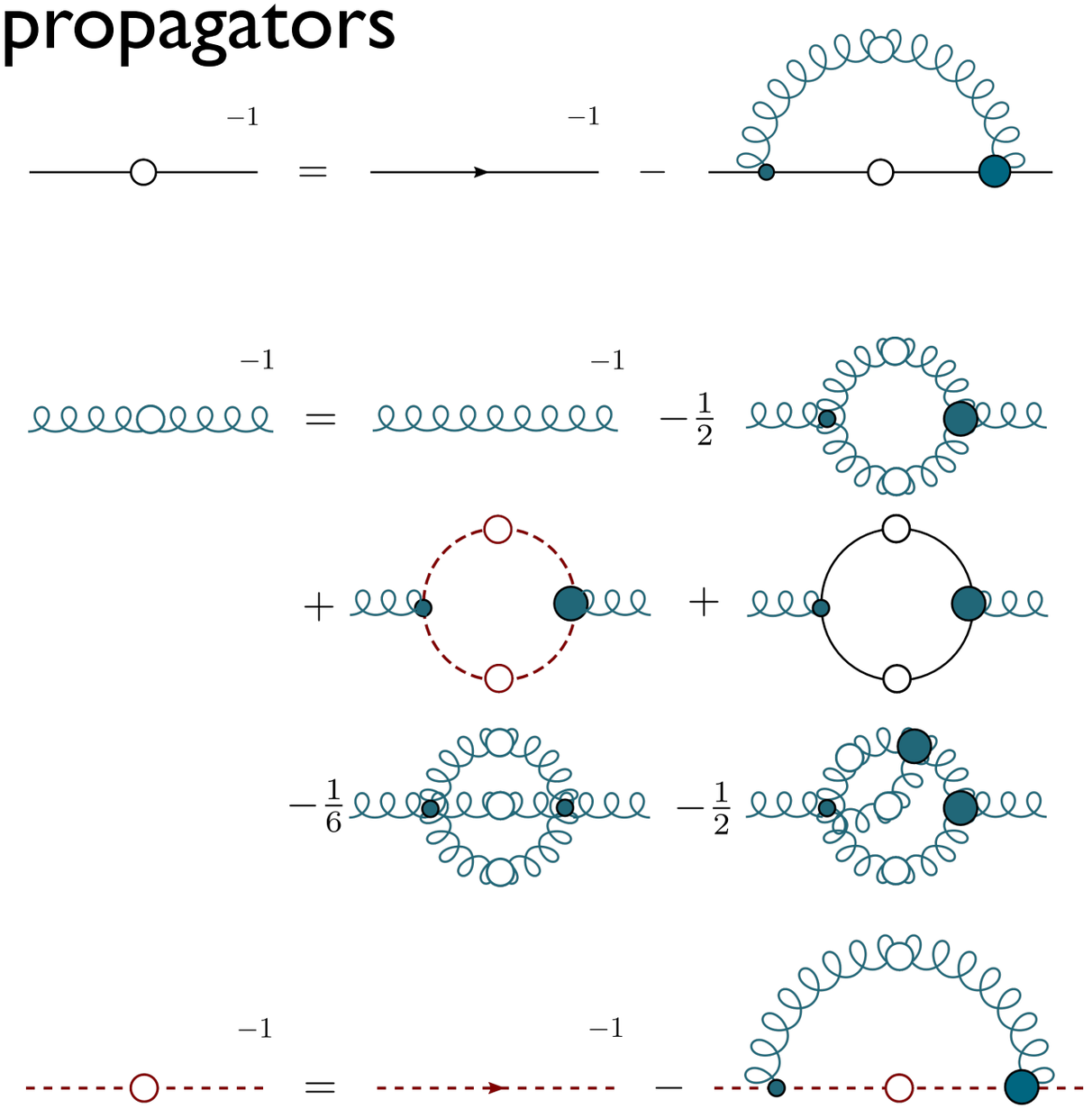

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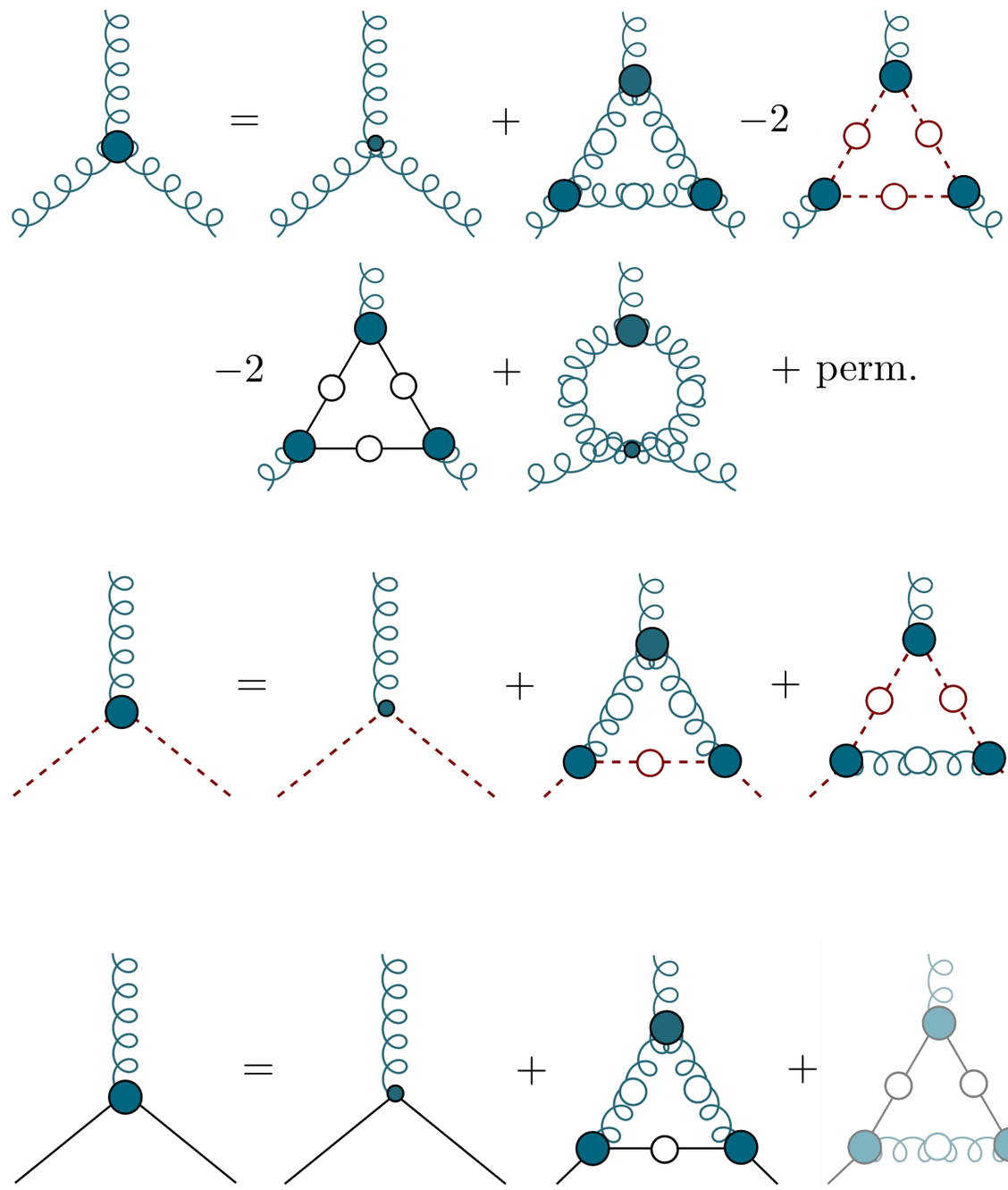

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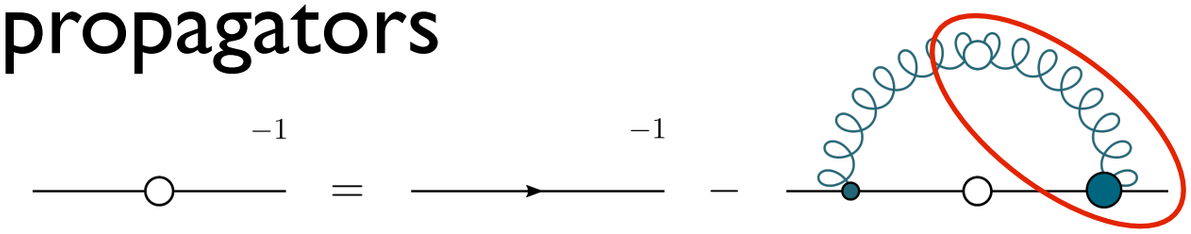

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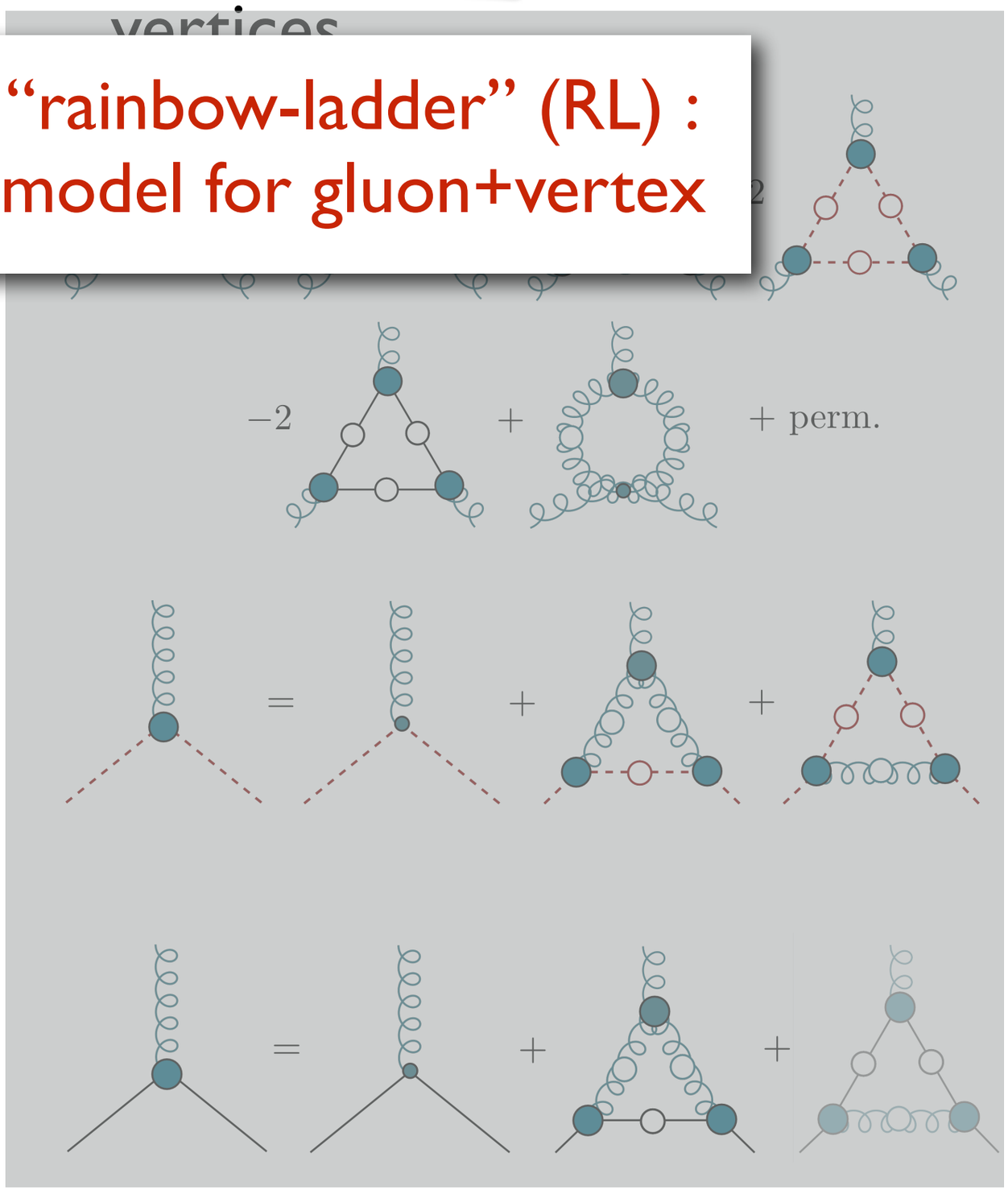
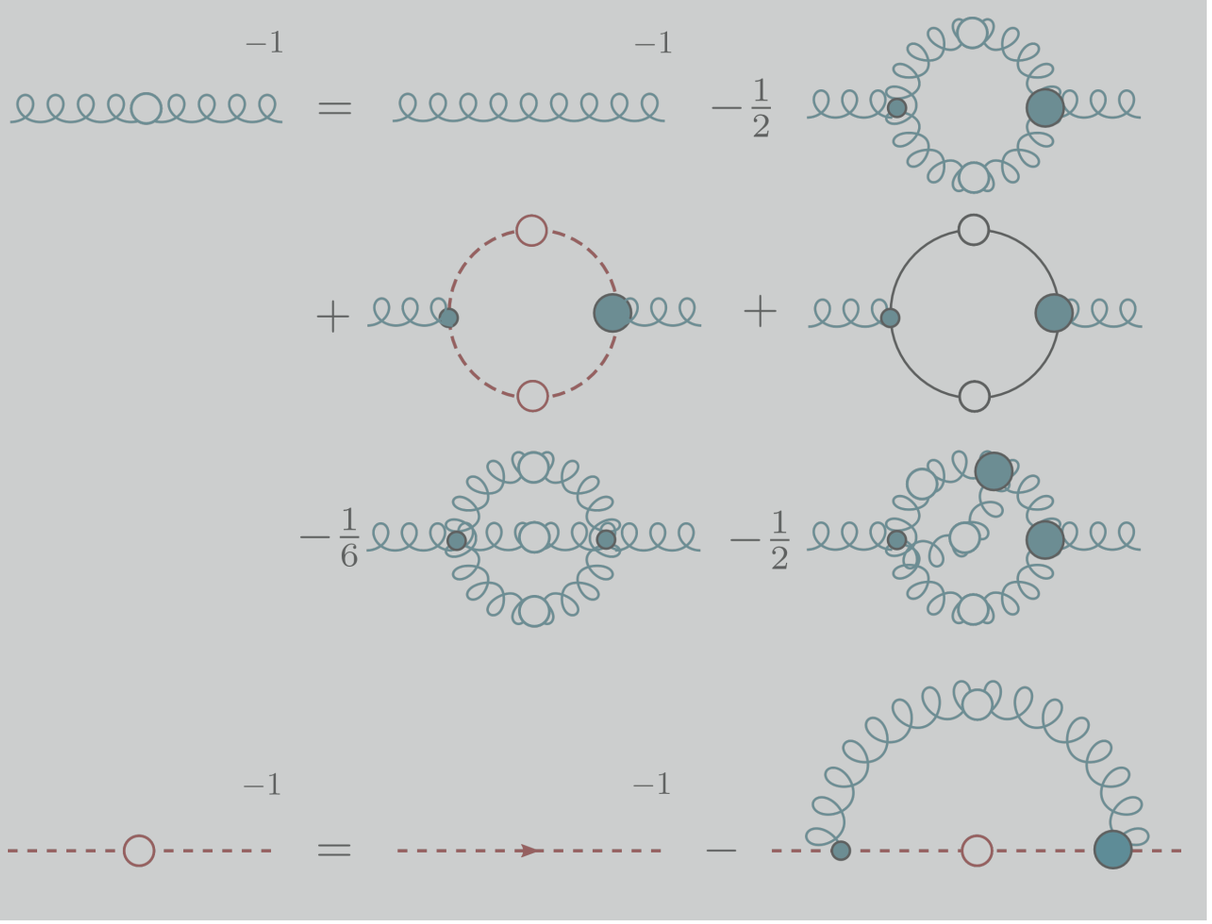


"rainbow-ladder" (RL) : model for gluon+vertex



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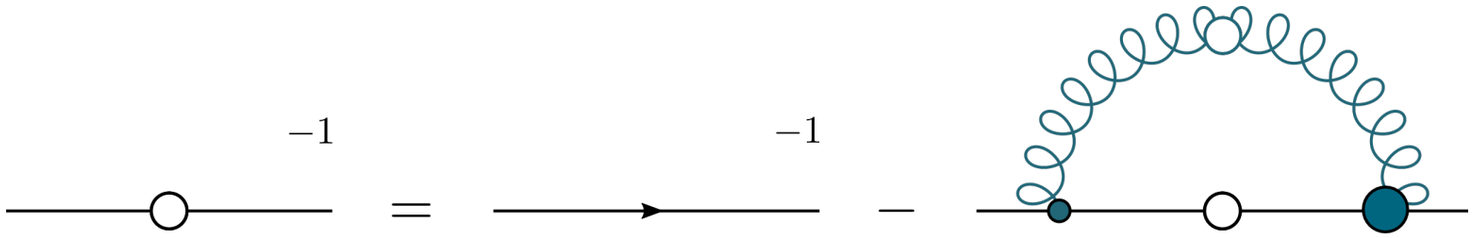
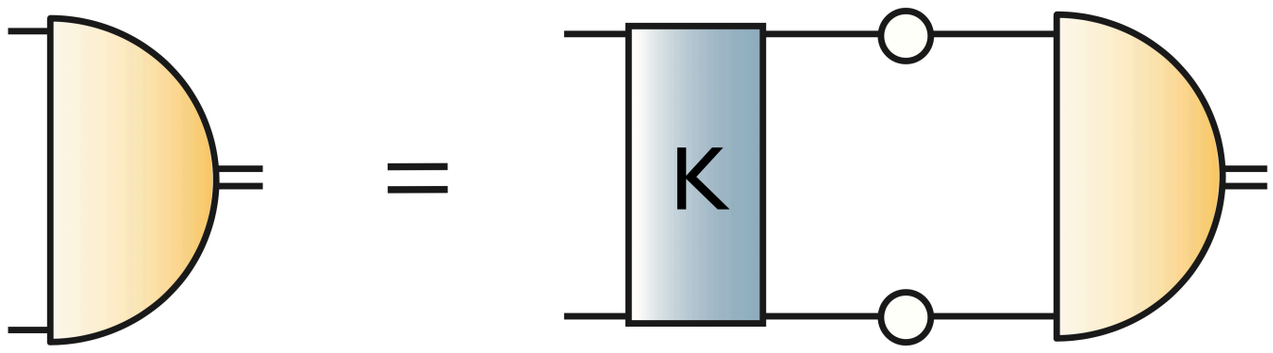
Mesons as quark-antiquark states





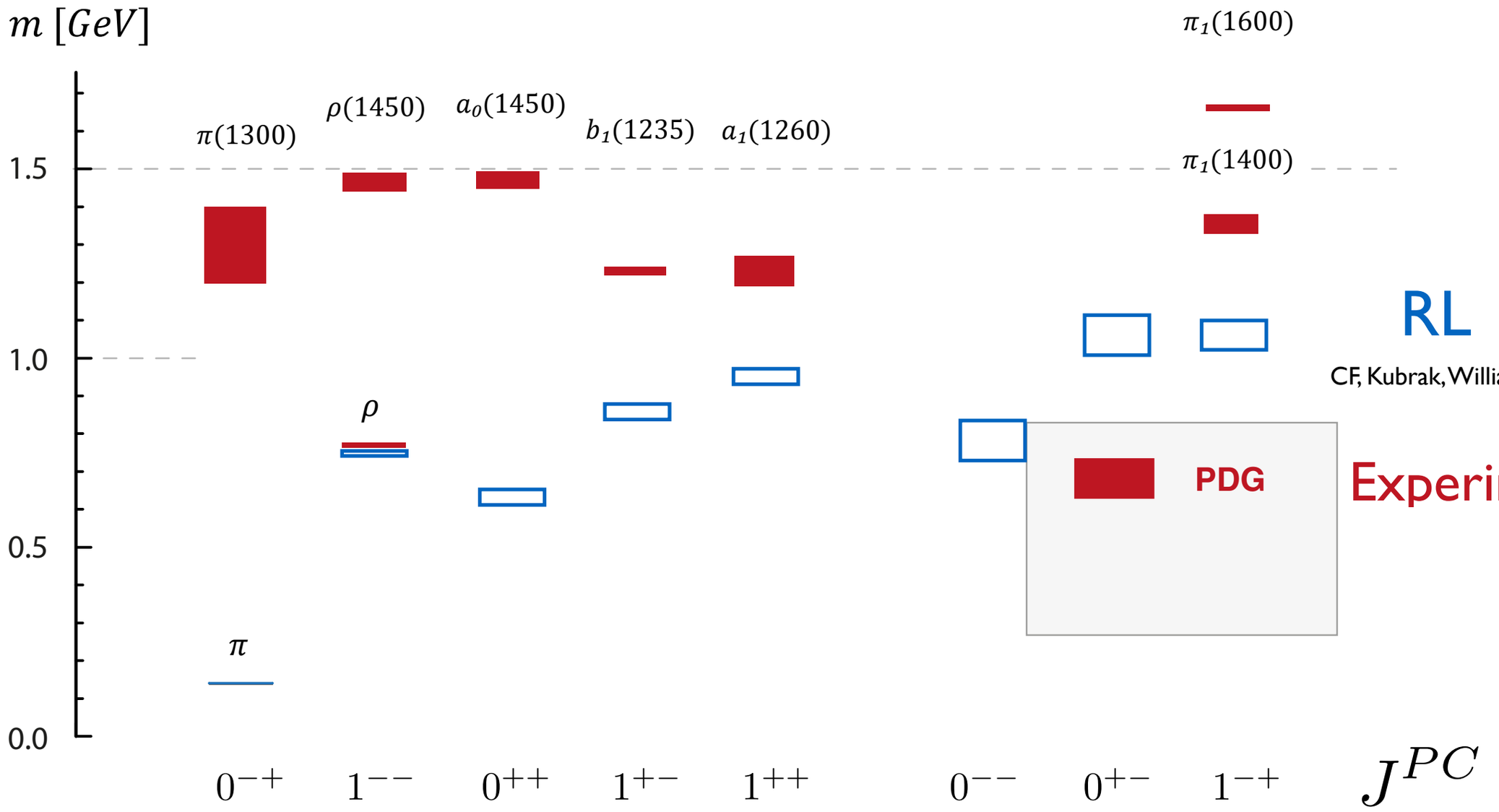
Kernel K uniquely related to quark-DSE via axWTI

→Pion is bound state **and** Goldstone boson

Maris, Roberts, Tandy, PLB 420 (1998) 267

- Determine gauge invariant spectrum from underlying gauge dependent quark/gluon dynamics

Light meson spectrum - full 3PI-calculation



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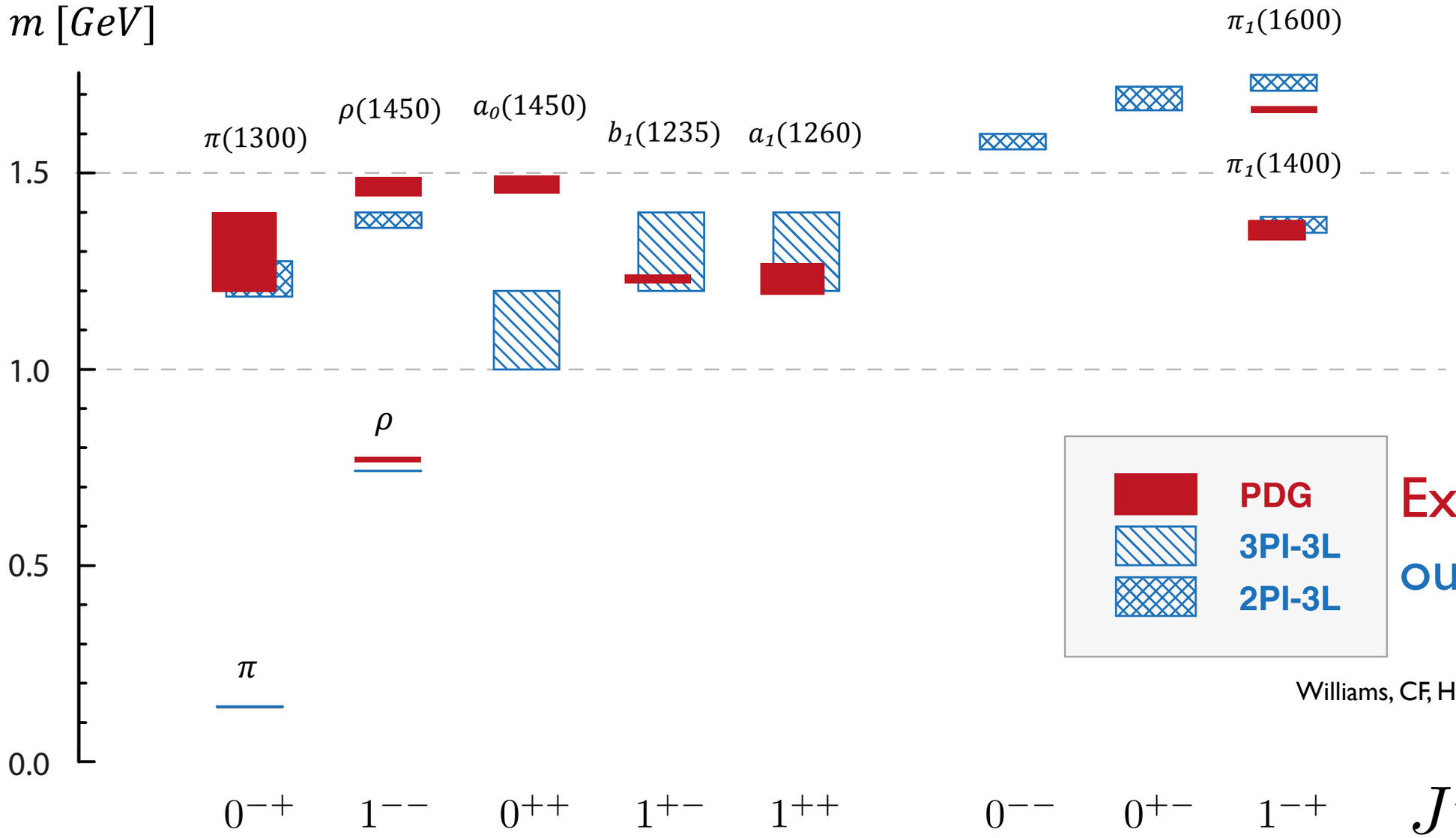
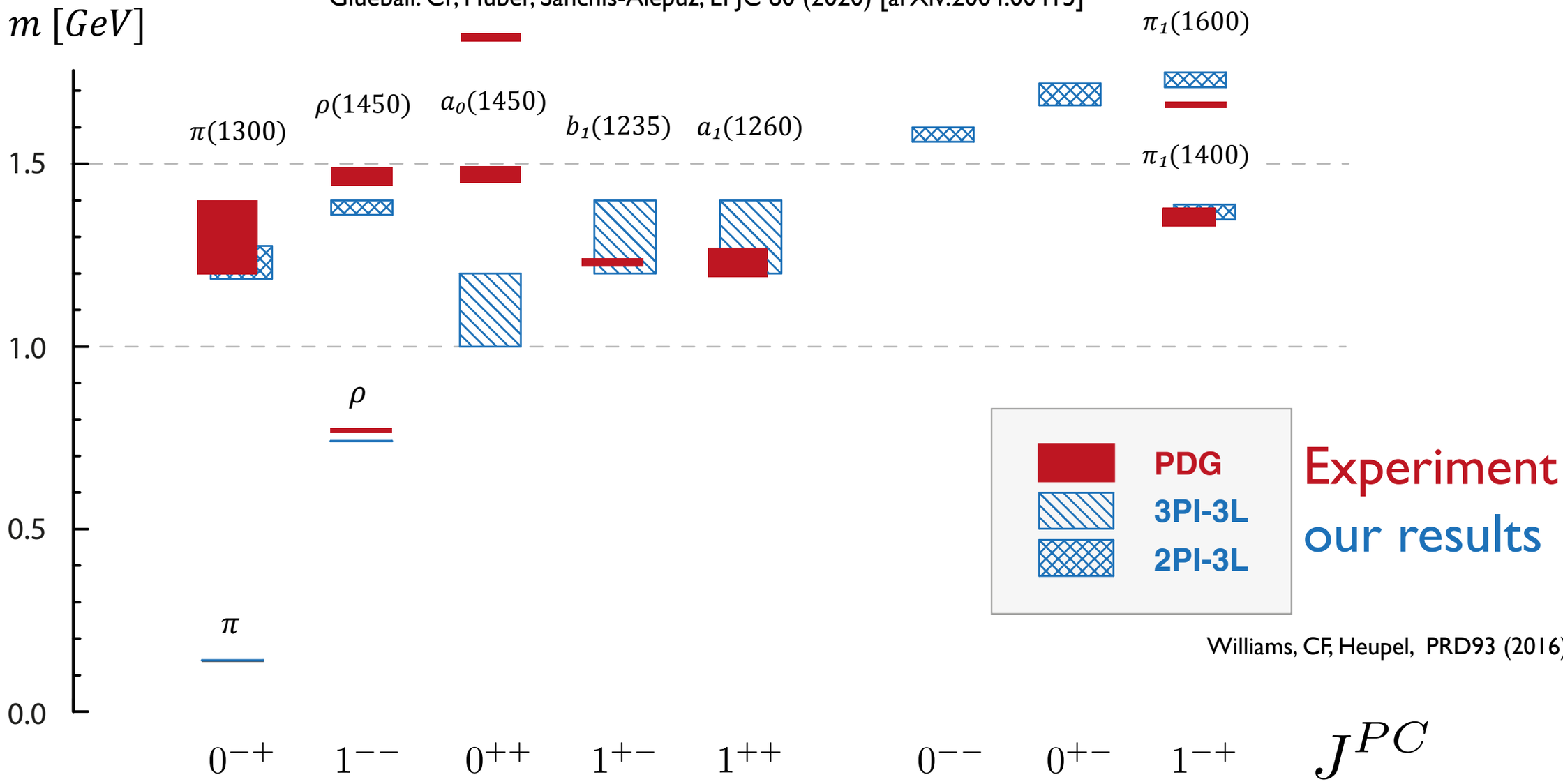


Williams, CF, Heupel, PRD93 (2016) 034026

- good agreement with experiment in most channels
- special channels:
  pseudoscalar 0^{-+} : (pseudo-) Goldstone bosons
  scalar 0^{++} : complicated channel…

Light meson spectrum - full 3PI-calculation

Glueball: CF, Huber, Sanchis-Alepuz, EPJC 80 (2020) [arXiv:2004.00415]



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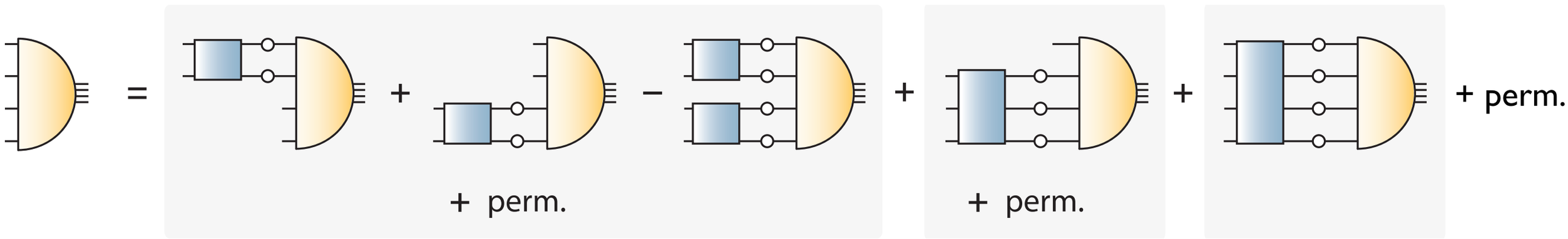
Tetraquarks from the four-body equation

Exact equation:



Two-body interactions

Three- and four-body interactions

Kvinikhidze & Khvedelidze, Theor. Math. Phys. 90 (1992)

Heupel, Eichmann, CF, PLB 718 (2012) 545-549

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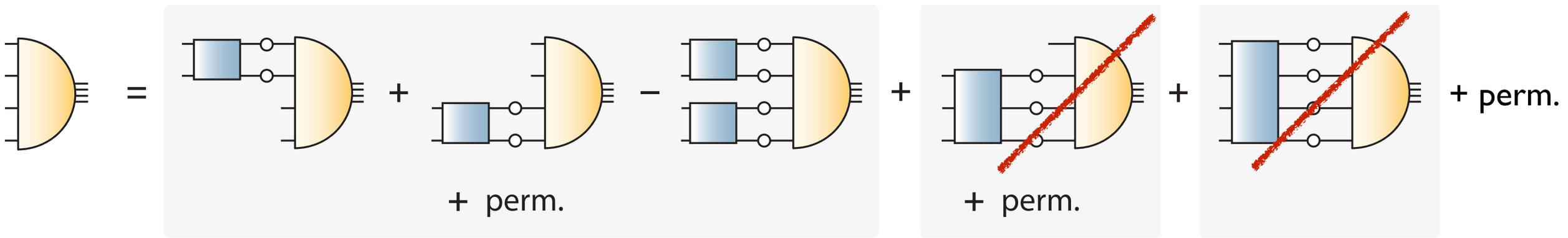

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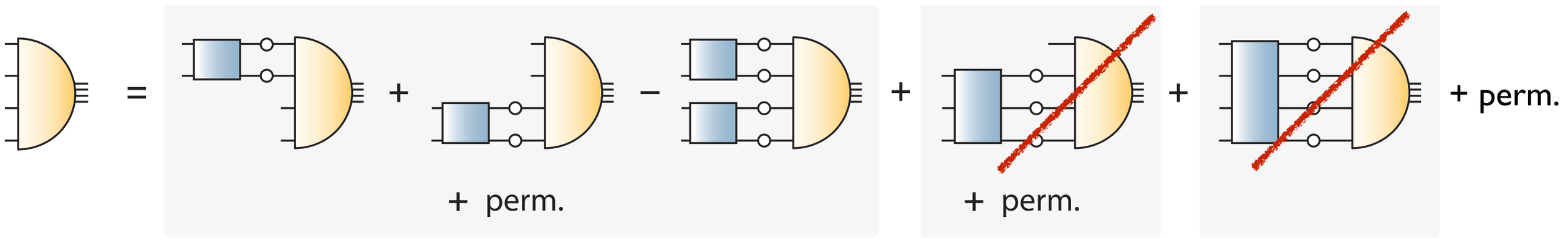

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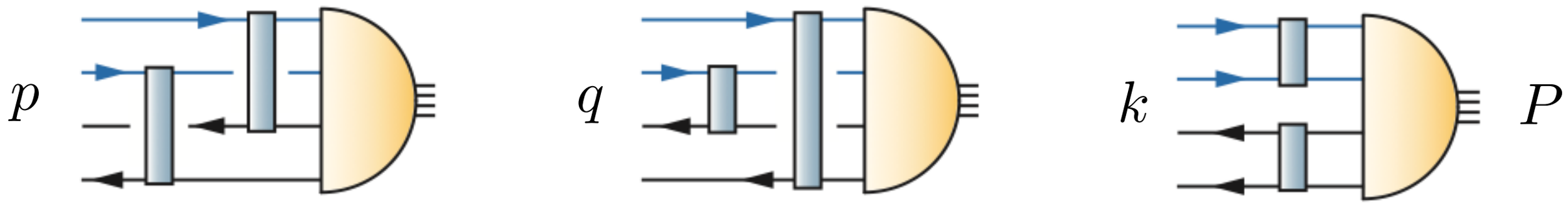
- Two-body interactions: allow for internal clustering
- use rainbow-ladder approximation…

Structure of the amplitude

Scalar tetraquark:



$$\Gamma(P,p,q,k) = \sum_i f_i(s_1,\cdots,s_9) \times \tau_i(P,p,q,k) \times color \times flavor$$

9 Lorentz scalars (built from P,p,q,k)

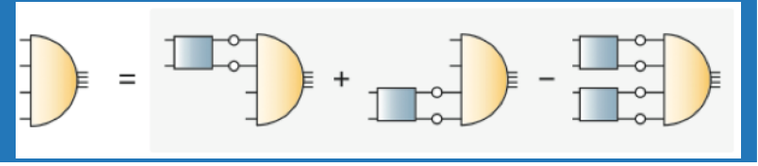
256 tensor structures (scalar tetra)

$3 \otimes \bar{3}, 6 \otimes \bar{6}$ or $1 \otimes 1, 8 \otimes 8$

- reduce # tensor structures guided by physics:

  → ~20 tensor structures

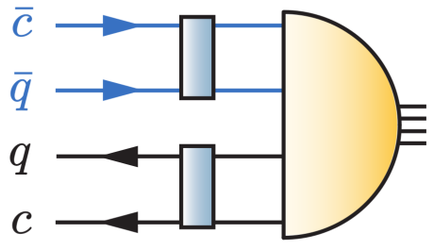
Four-body equation: permutations



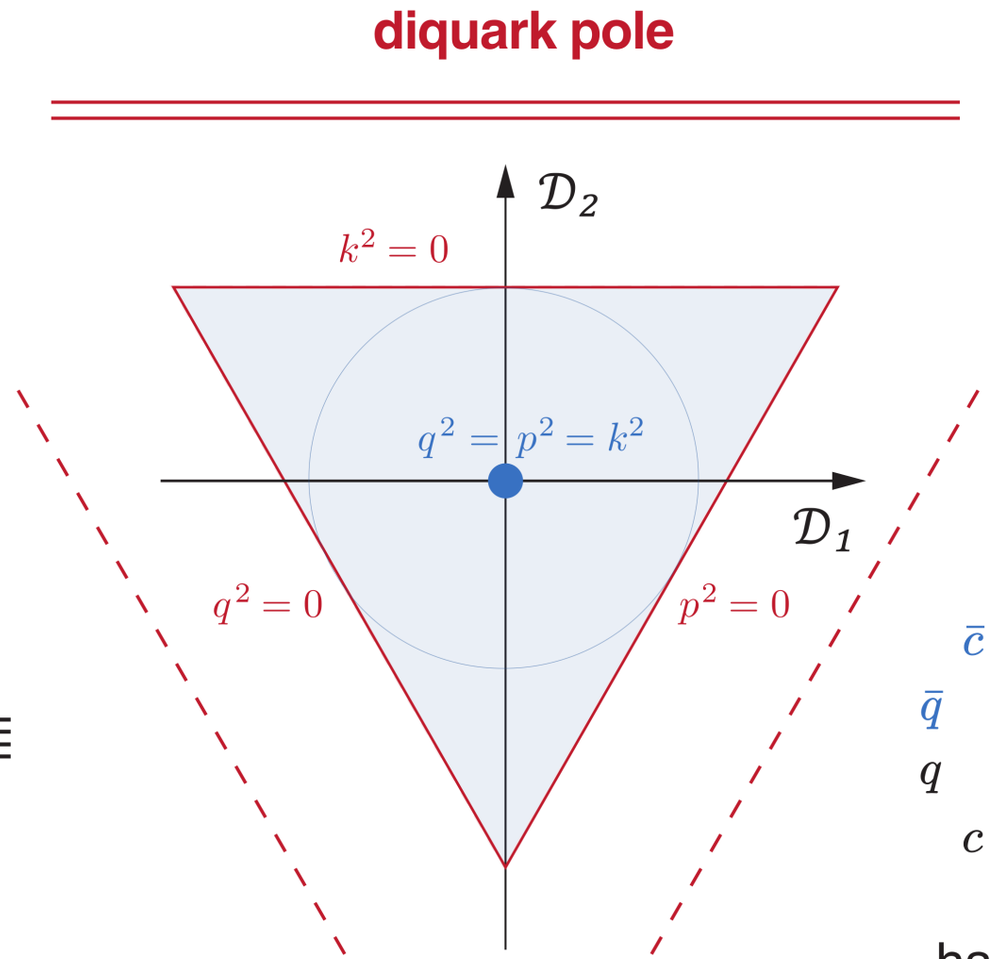
- **Singlet:** $S_0 = (p^2 + q^2 + k^2)/4$
- **Doublet:** $\mathcal{D}_1 \sim p^2 + q^2 - 2k^2$

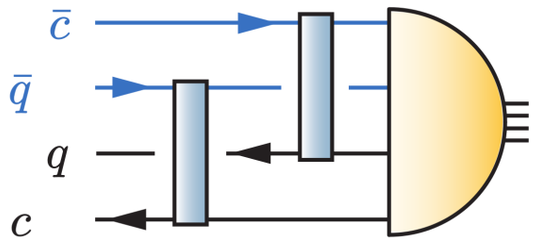
  $\mathcal{D}_2 \sim q^2 - p^2$

p, q, k : relative momenta

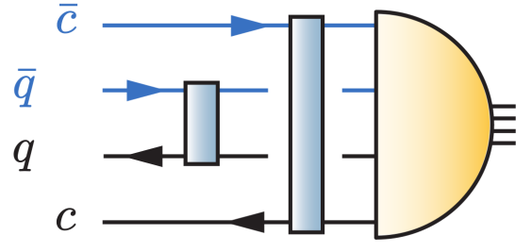


diquark-antidiquark



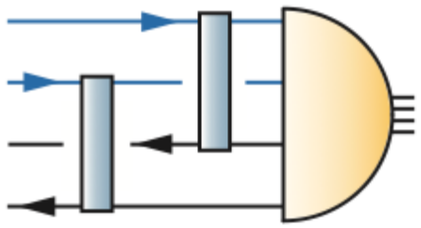


heavy-light mesons



hadrocharmonium

Bound state vs resonance: scalar tetraquarks



$$\Gamma(S_0, \cancel{s, a,} \ldots)$$

without π-clustering

0

1200

$M_{\rm Tetra}[MeV]$

Bound state of four massive quarks

$$m_\kappa \sim 750\,\mathrm{MeV}$$

$$m_{a_0, f_0} \sim 1080\,\mathrm{MeV}$$

Eichmann, CF, Heupel, PLB 753 (2016) 282-287

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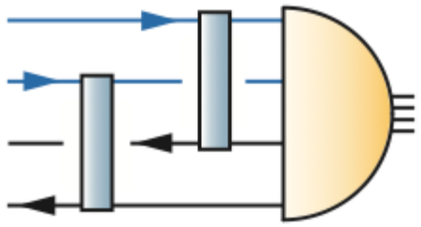

$$\Gamma(S_0, s, a, ...)$$

without π-clustering

0 — 400 — 1200 — $M_{\rm Tetra}[MeV]$

with π-clustering

Two-pion resonance

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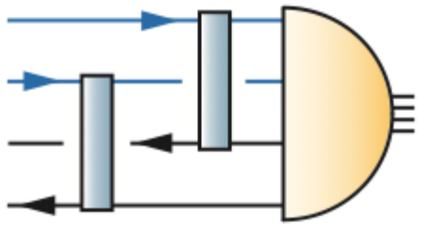

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⟶ identify with $f_0(500)$ ('σ-meson')

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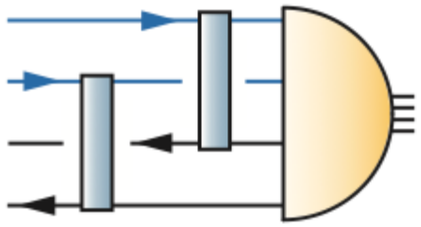

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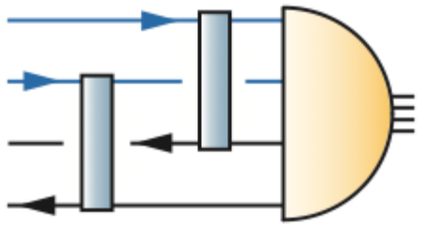
with strange quarks: $m_\kappa \sim 750\,\mathrm{MeV}$

$m_{a_0, f_0} \sim 1080\,\mathrm{MeV}$

Eichmann, CF, Heupel, PLB 753 (2016) 282-287

Bound state vs resonance: scalar tetraquarks



$$\Gamma(S_0, s, a, ...)$$

without π-clustering

0 — 400 — 1200 — $M_{\rm Tetra}[MeV]$

with π-clustering

Two-pion resonance

Bound state of four massive quarks

⟶ identify with $f_0(500)$ ('σ-meson')

with strange quarks: $m_\kappa \sim 750\,\mathrm{MeV}$

$m_{a_0, f_0} \sim 1080\,\mathrm{MeV}$

Eichmann, CF, Heupel, PLB 753 (2016) 282-287

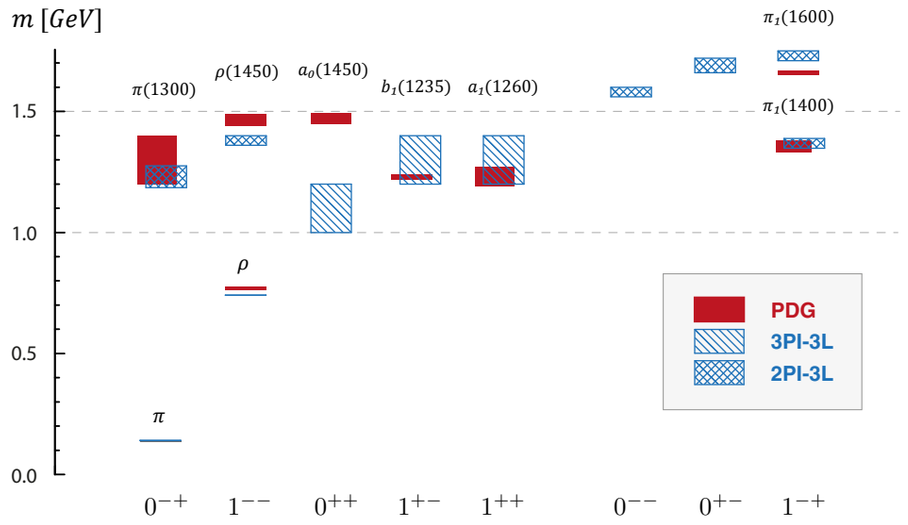
Mixing with $q\bar{q}$: small effect

Santowsky, Eichmann, CF, Wallbott and Williams, PRD 102 (2020) no.5, 056014, arXiv:2007.06495.

Overview

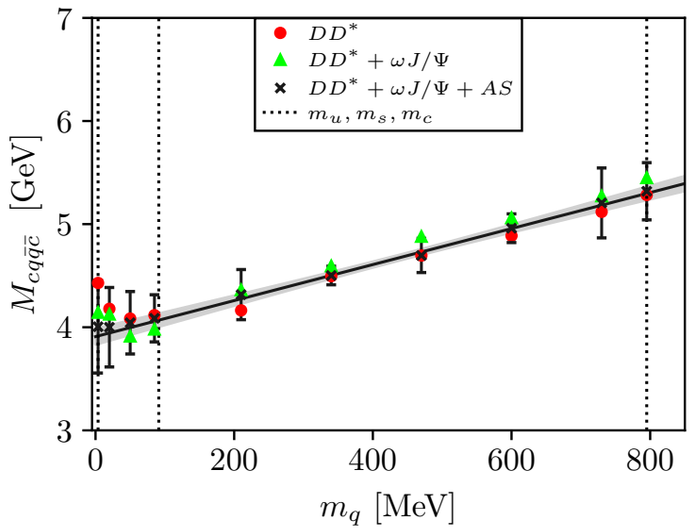
1. Light two- and four-quark states: the f0(500)

Williams, CF, Heupel, PRD 93 (2016) 034026 [arXiv:1512.00455]
Eichmann, CF, Heupel, PLB 753 (2016) 282 [arXiv:1508.07178]
Heupel, Eichmann, CF, PLB 718 (2021) 545 [arXiv:1206.5129]

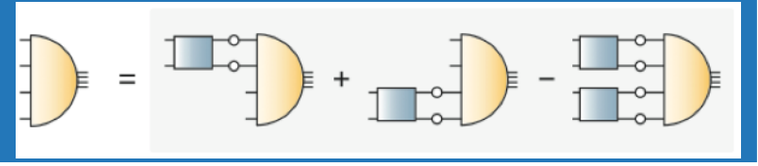


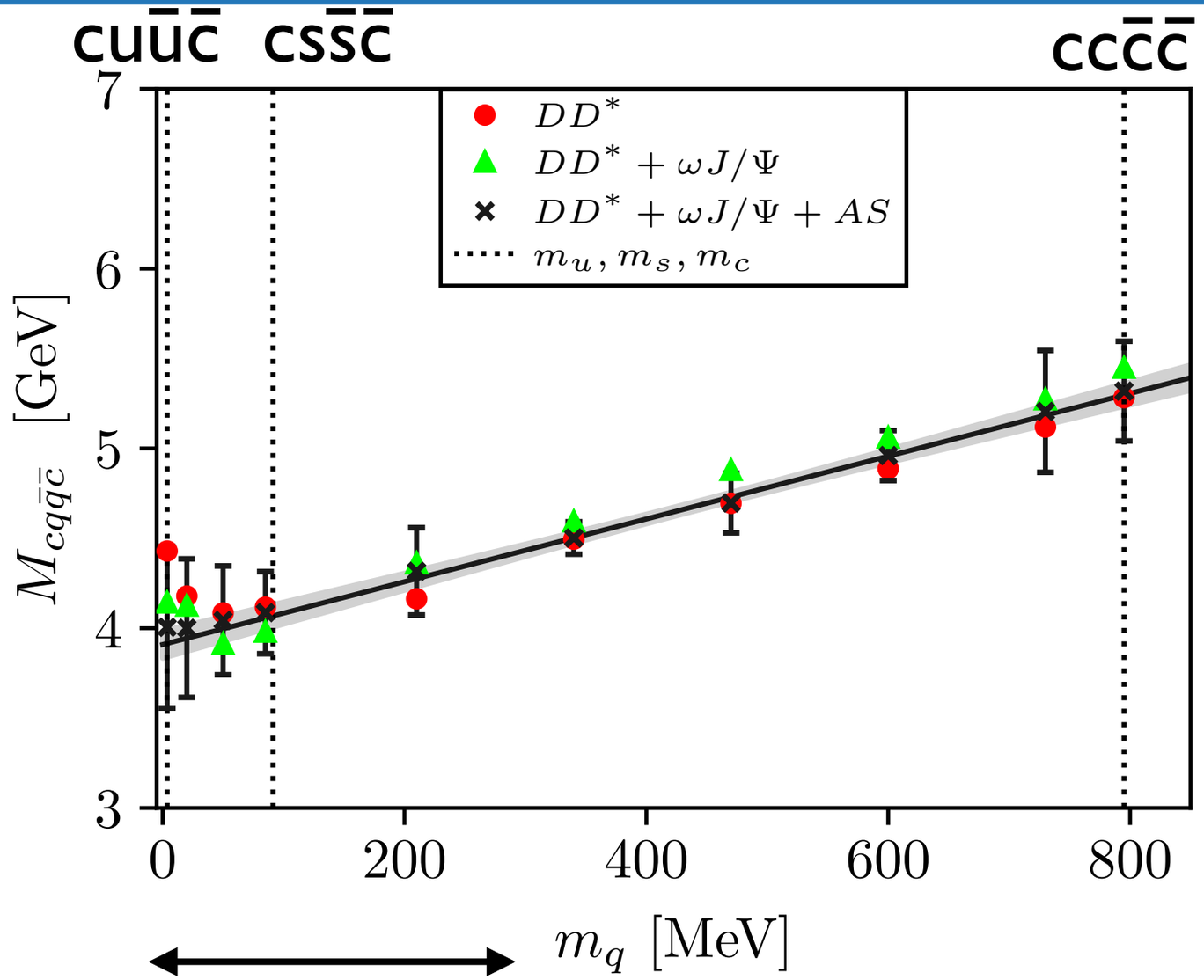
2. Heavy-light four-quark states: X(3872) and more…

Wallbott, Eichmann and CF, PRD100 (2019) no.1, 014033, [arXiv:1905.02615]
Wallbott, Eichmann and CF, PRD102 (2020) no.5, 051501, [arXiv:2003.12407]



$J^{PC} = 1^{++}$





Extrapolation in BSE necessary…

m_c fixed

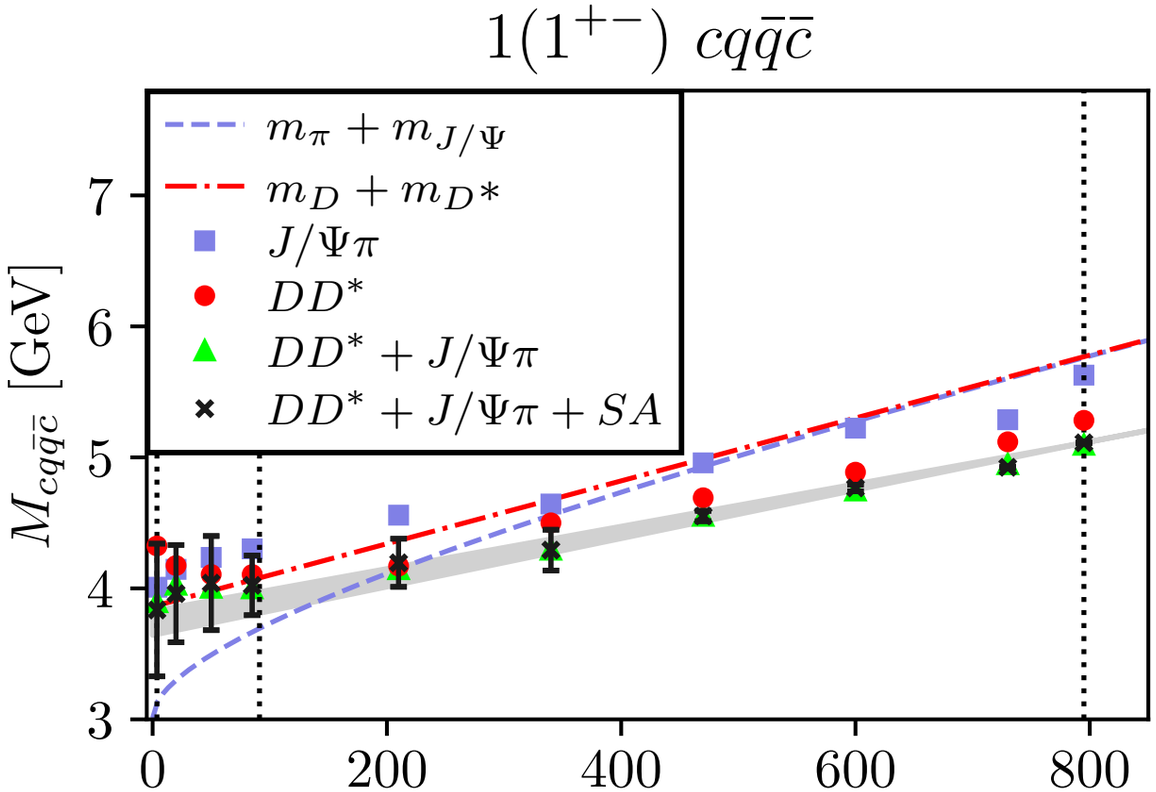
m_q varied

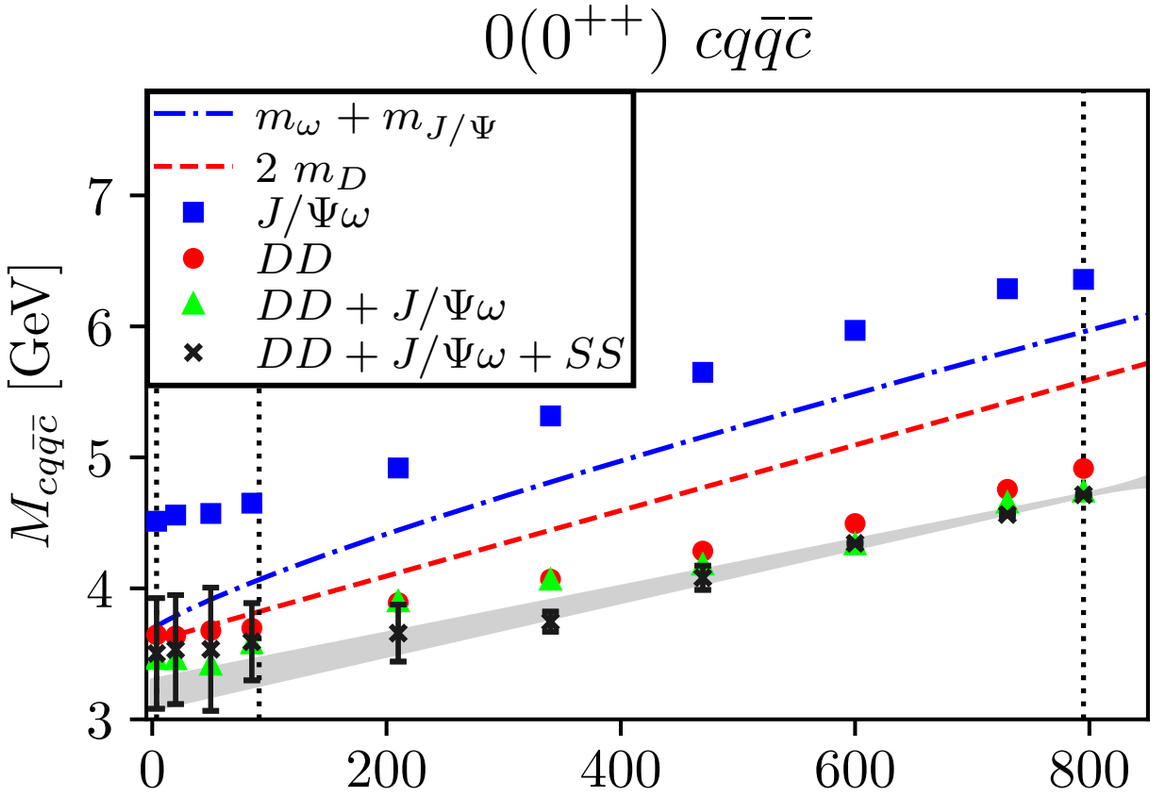
- DD* components dominate !

$$M_{1^{++}}^{cq\bar{q}\bar{c}} = 3916(74)\,\mathrm{MeV} \longrightarrow X(3872)$$

Wallbott, Eichmann and CF, PRD100 (2019) 014033, [1905.02615]

$J^{PC} = 1^{+-}$ and 0^{++}

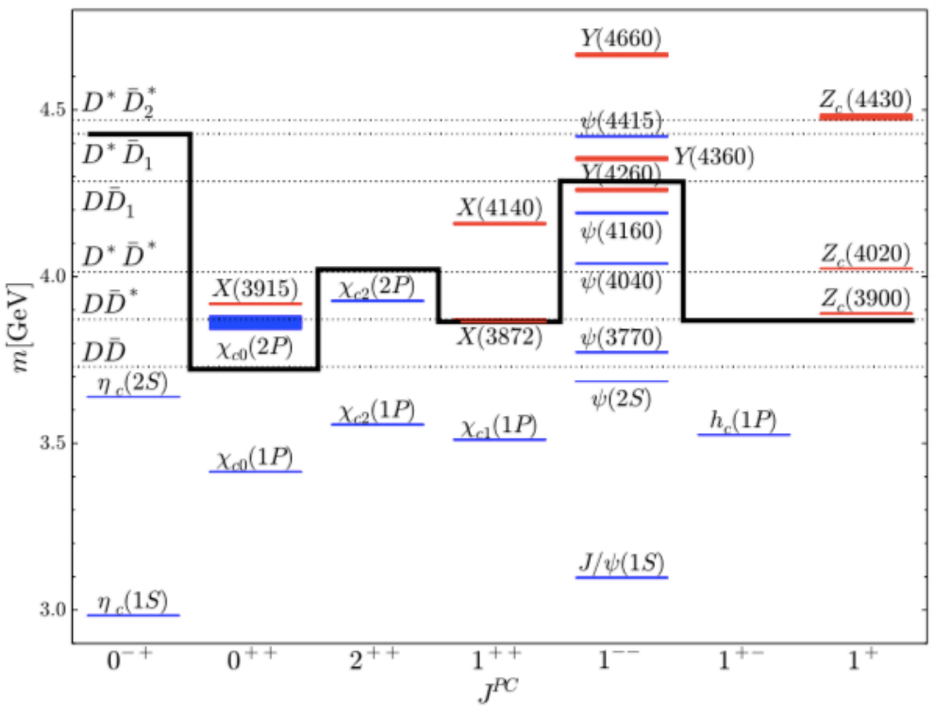




Wallbott, Eichmann and CF, PRD102 (2020)no.5, 051501, arXiv:2003.12407

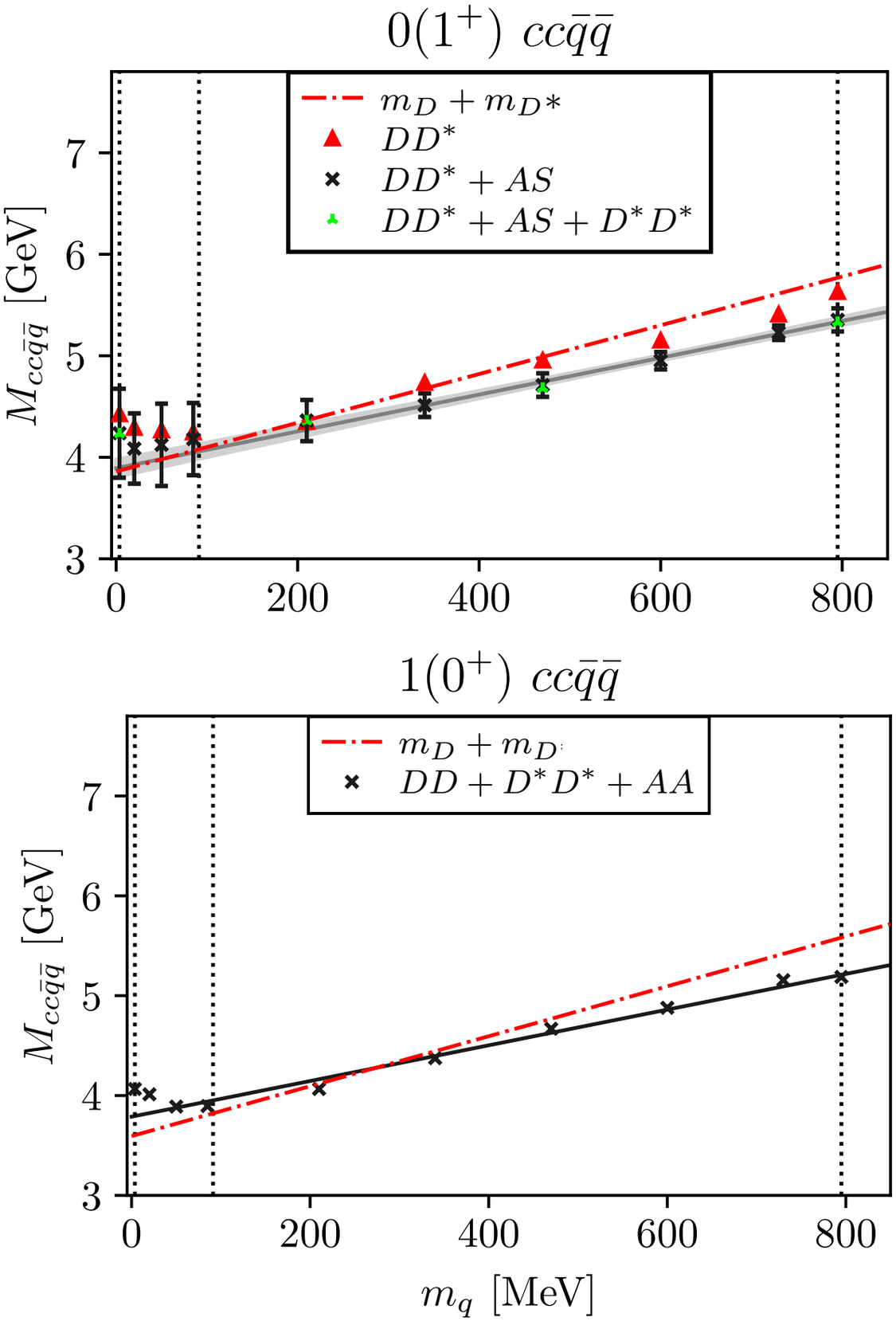
$$M^{cq\bar{q}\bar{c}}_{1^{+-}} = 3741(91) \ \rightarrow \ Z(3900)$$

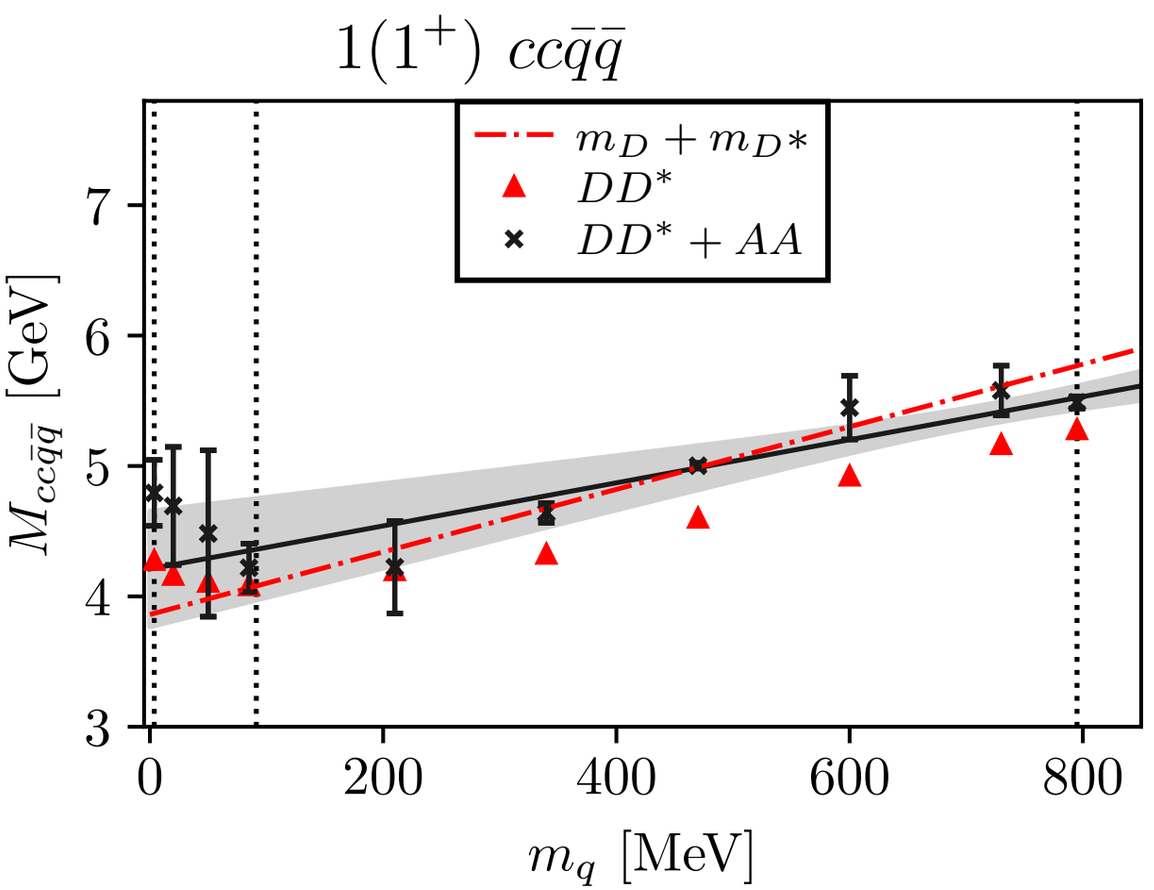
$$M^{cq\bar{q}\bar{c}}_{0^{++}} = 3195(107) \ \rightarrow \ ?$$



mass pattern matches molecule picture of Cleven et al. PRD 92 (2015) 014005:

Open charm four-quark states





- DD(*) and diquarks important!

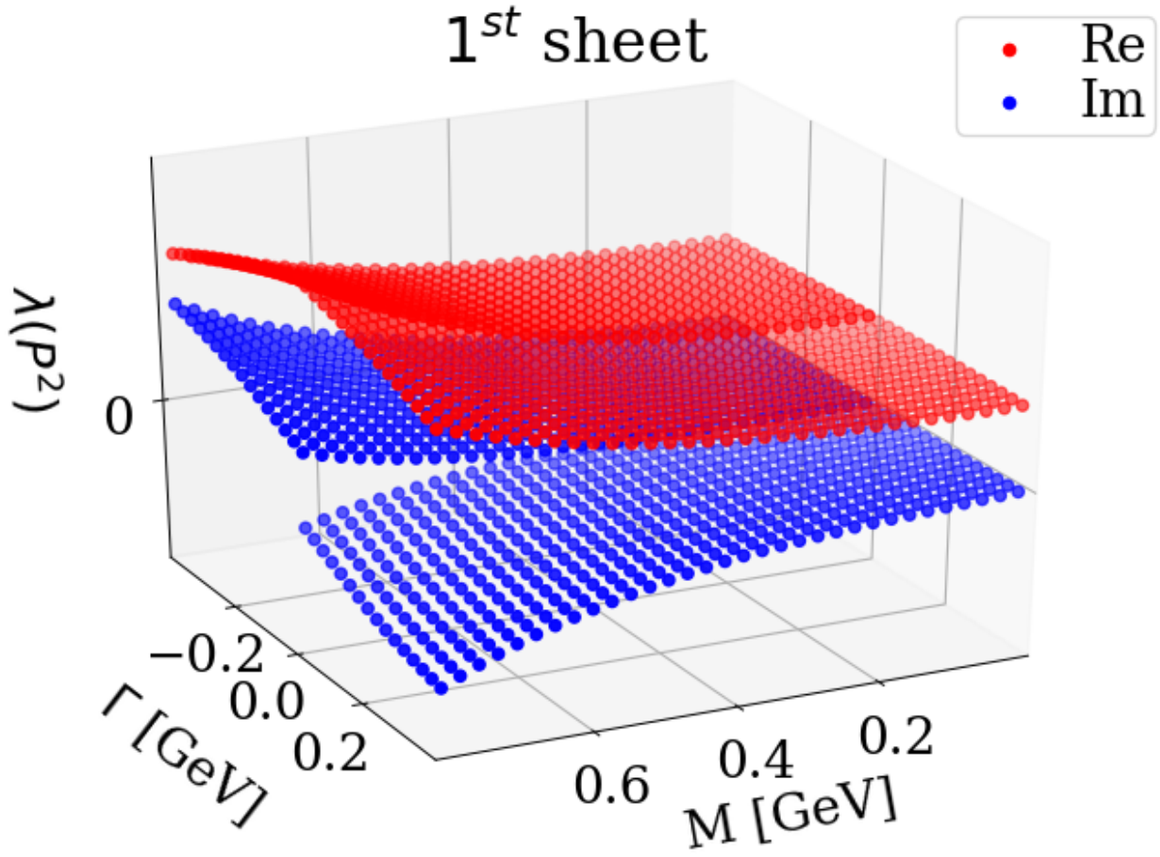
Wallbott, Eichmann and CF, PRD102 (2020)no.5, 051501, arXiv:2003.12407

Work to do…

- improve two-body interactions
- further study mixing with $q\bar{q}$ in I=0 sector

Santowsky, Eichmann, CF, Wallbott and Williams, PRD 102 (2020) no.5, 056014, arXiv:2007.06495.

- solve four-body BSE in the complex momentum plane



successful for ρ-meson:

Williams, PLB 798 (2019) 134943, [arXiv:1804.11161]

Summary

Internal dynamics very important !!

Glueballs:

- First quantitatively reliable results using very involved truncation

CF, Huber, Sanchis-Alepuz, EPJC 80 (2020) [arXiv:2004.00415]

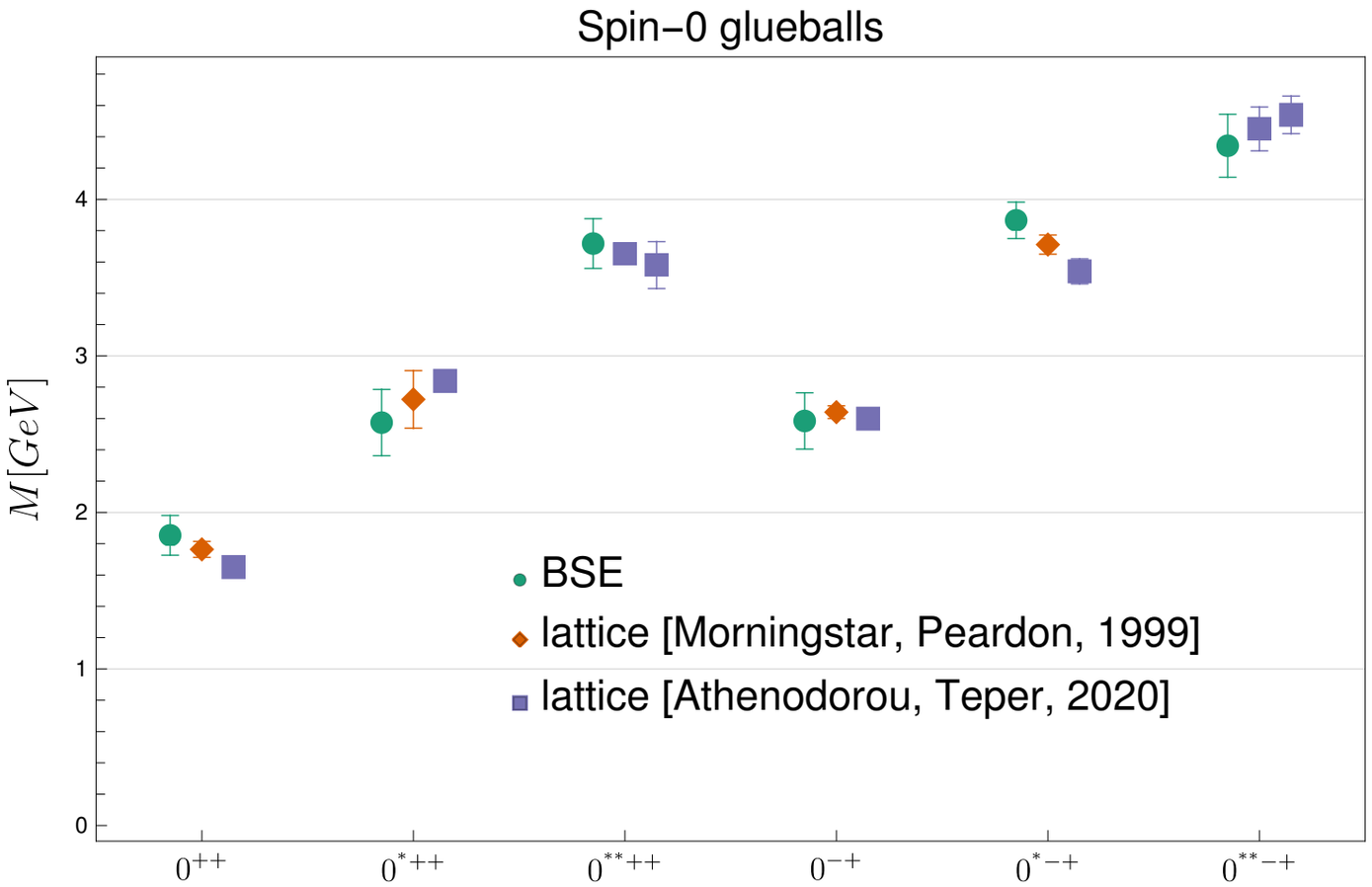
Four-quark states:

- Dynamical description of σ: π-π resonance — Eichmann, CF, Heupel, PLB 753 (2016) 282-287
- Dynamical description of X(3872) and Z(3900): DD* dominated
- First results in open charm channels — Wallbott, Eichmann and CF, PRD100 (2019) 014033, [1905.02615]; Wallbott, Eichmann and CF, PRD102 (2020) 051501, [2003.12407]
- Mixing with $q\bar{q}$ studied for light mesons — Santowsky, Eichmann, CF, Wallbott and Williams, PRD 102 (2020) no.5, 056014, [2007.06495].

Mini-Review: Eichmann, CF, Heupel, Santowsky, Wallbott, FBS 61 (2020) 4 38, [2008.10240]

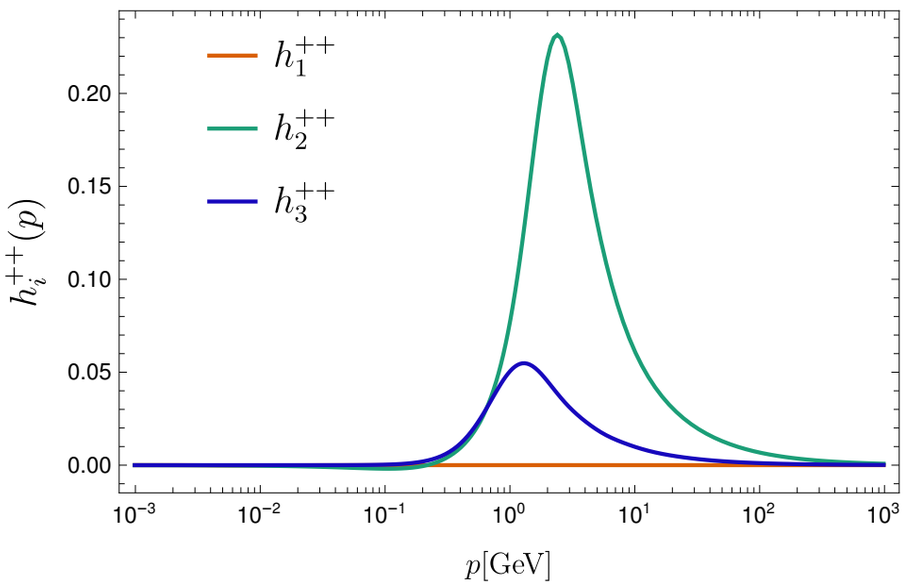
Backup Slides

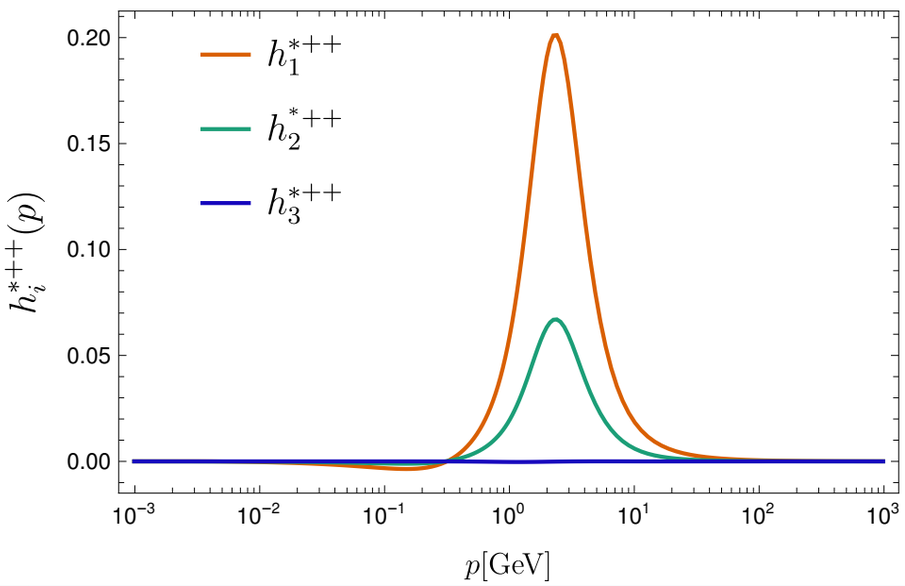
Glueballs: results



- spectrum:
  very good agreement
  lattice vs. DSE/BSE

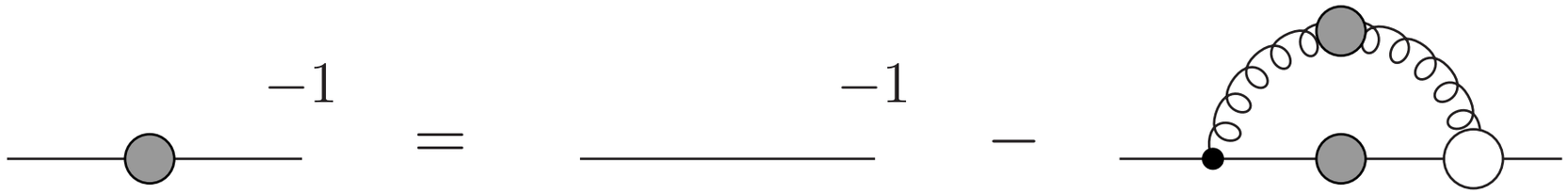
CF, Huber, Sanchis-Alepuz, EPJC 80 (2020) [arXiv:2004.00415]





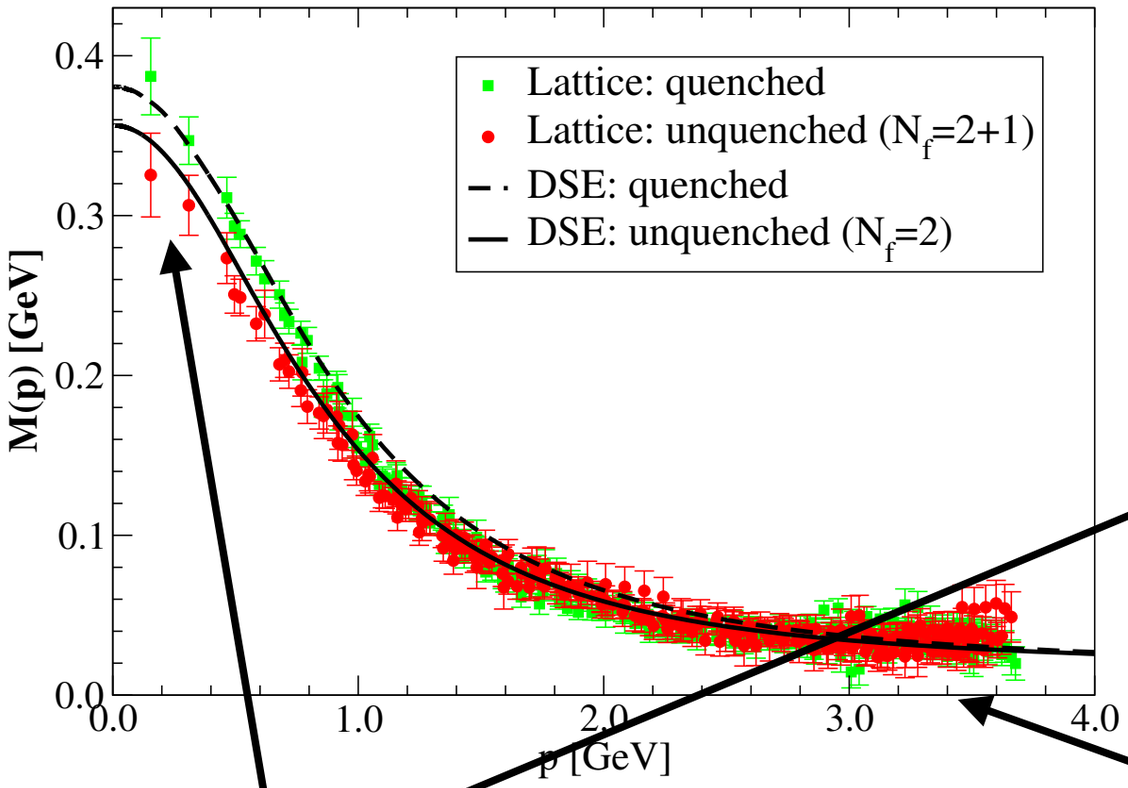
- excited states:
  different internal
  structure

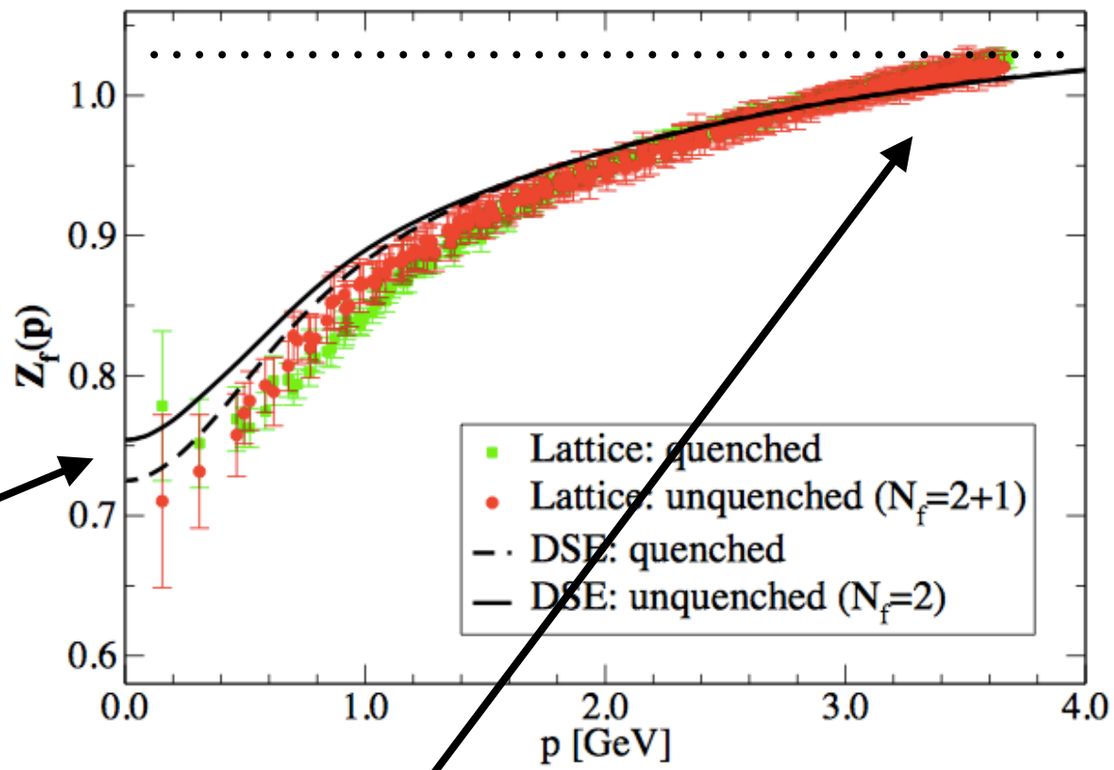
Quarks: mass from interaction



$$S(p) = Z_f(p^2)\frac{-i\not{p} + M(p^2)}{p^2 + M^2(p^2)}$$

DSE: CF, Nickel, Williams, EPJ C 60 (2009) 47
Williams, CF, Heupel, PRD 93 (2016) 034026
Lattice: P. O. Bowman, et al PRD 71 (2005) 054507





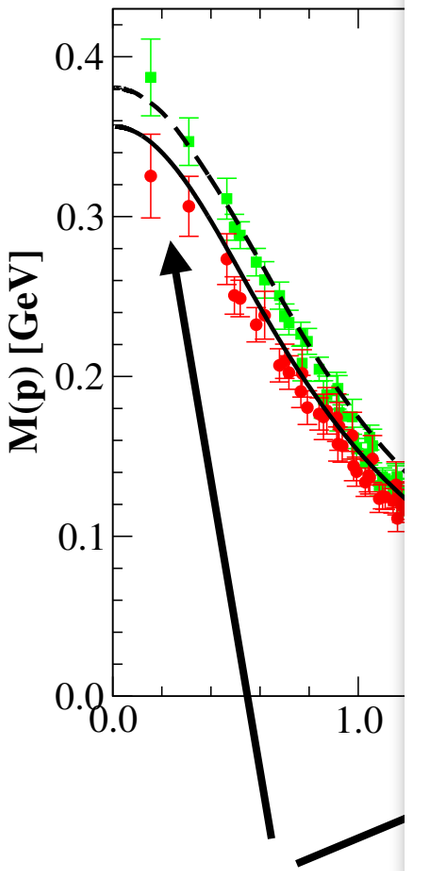
‘constituent quark’:
large mass; very composite

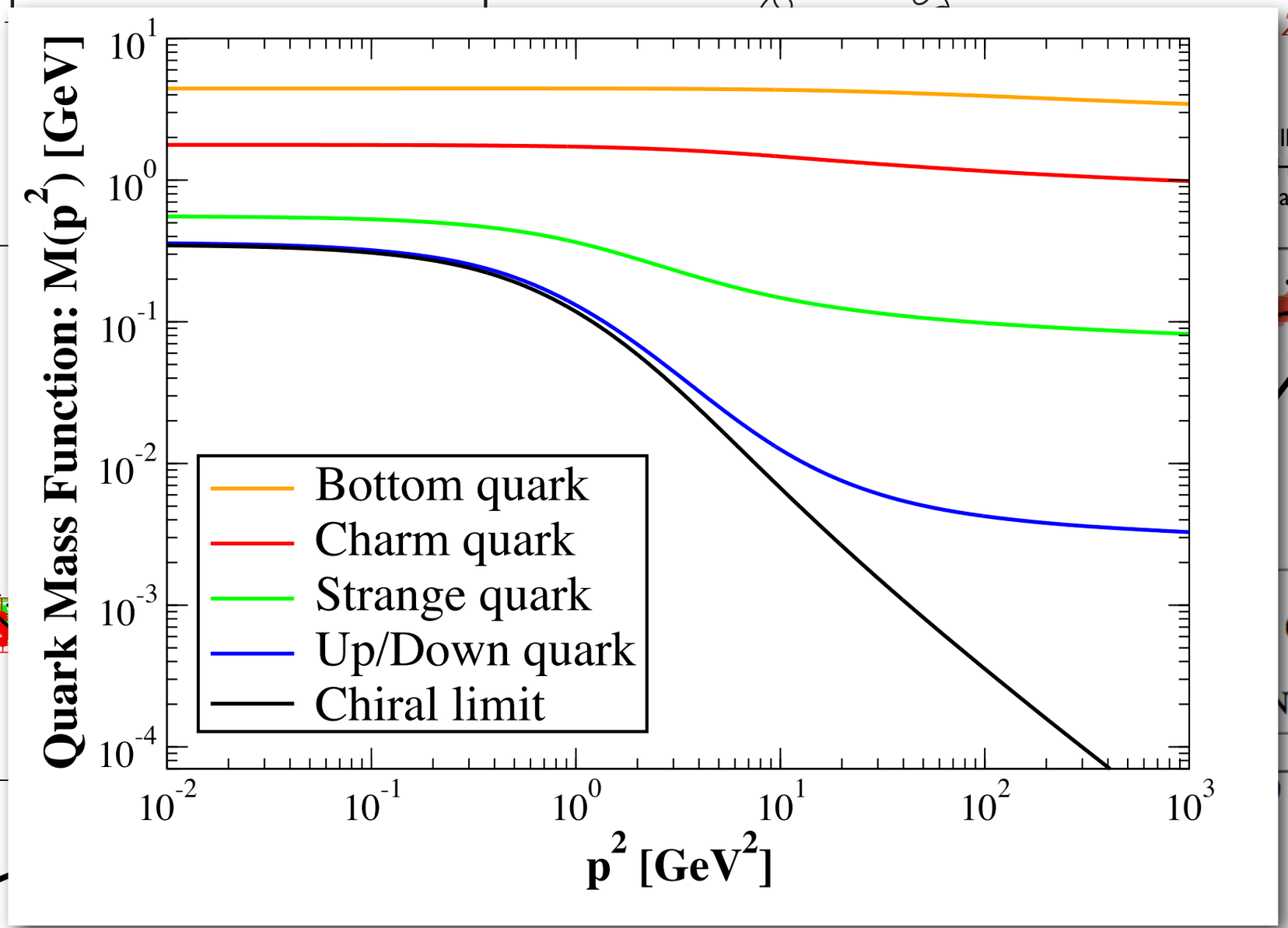
‘current quark’:
- small mass; non-composite

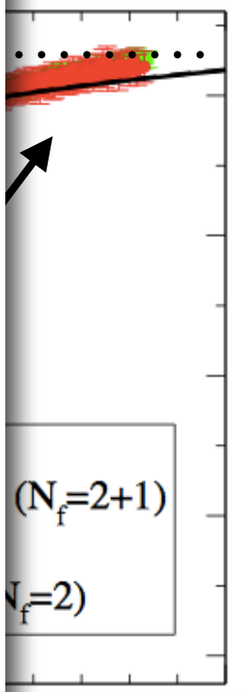
Quarks: mass from interaction

$$Z_f(p^2)\,\frac{-i\not{p} + M(p^2)}{p^2 + M^2(p^2)}$$

lliams, EPJ C 60 (2009) 47
Heupel, PRD 93 (2016) 034026
an, et al PRD 71 (2005) 054507







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