2021.01.21 meeting with Tohoku and Mainz group
J. Yoshida

- How does the B_Λ calculation using the old Λ mass deviate from the true value?
- Is it possible to re-calibrate the past value of B_Λ?
- How accurately can we measure the B_Λ of single Λ hypernuclei with E07 emulsion sheets?

Nuclear Physics B52 (1973) 1-30.
A NEW DETERMINATION OF THE BINDING-ENERGY VALUES OF THE LIGHT HYPERNUCLEI (A ~< 15)
M. JURIC, et al.

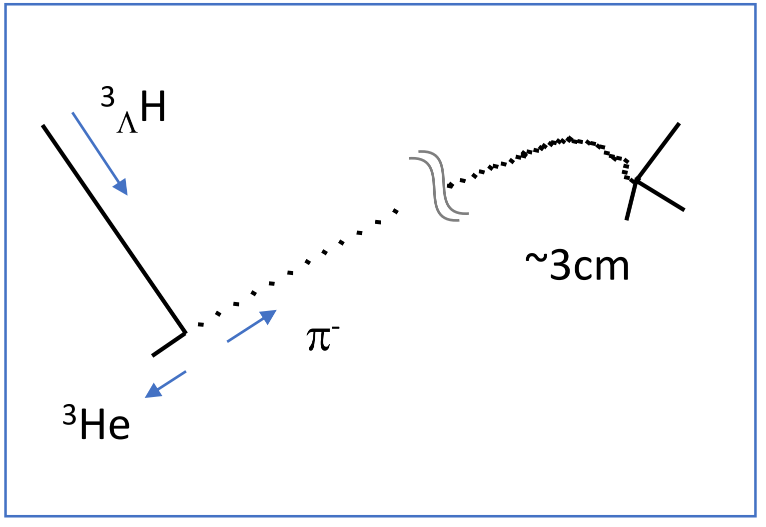
Nuclear Physics B4 (1968) 511-526.
A DETERMINATION OF THE BINDING-ENERGY VALUES OF LIGHT HYPERNUCLEI
G. Bohm, et al.

Procedure of B_Λ measurement of hypernucleus

- Event search
- Event selection
- Measurement of range and angles of the tracks
- Measurement of emulsion density
- Measurement of Λ mass
- Kinematical analysis and Identification of nuclide

Event selection and measurement

Example: ${}^{3}_{\Lambda}H \rightarrow {}^{3}He + \pi^{-}$



K.E._daughter = Barkas_formula(range, nuclide, emulsion_density)

Mass_hypernucleus = $\sum_{\text{daughters}}$ (Mass + K.E.) ← to be updated with kinematic fitting

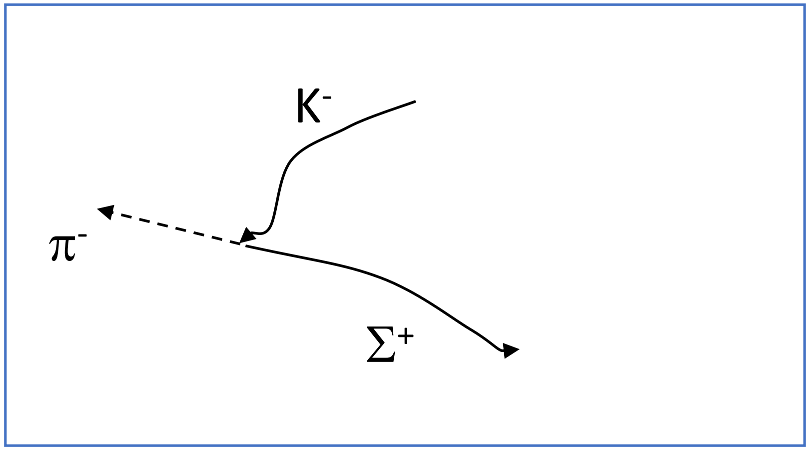
B_{Λ} = Mass_core + Mass_Λ – Mass_hypernucleus

Measurement of emulsion density and Σ^+ mass

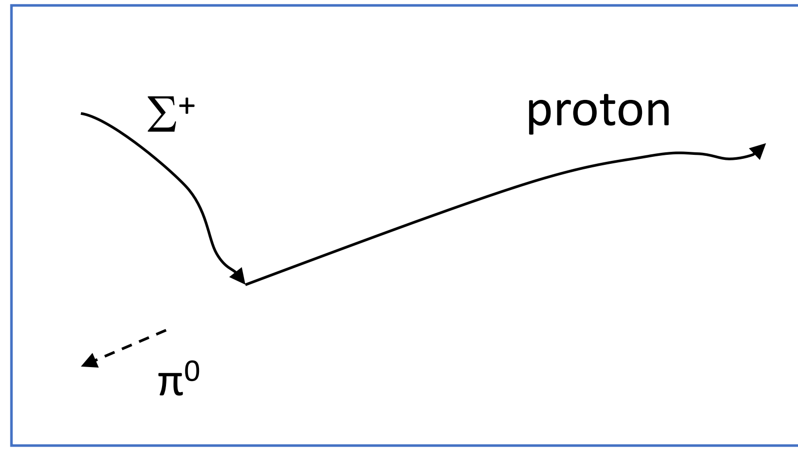
Bohm, et al., Nuclear Physics B48 (1972) 1-12.

https://doi.org/10.1016/0550-3213(72)90047-8

Reaction(2): $K^- + p \rightarrow \Sigma^+ + \pi^-$



Reaction(1): $\Sigma^+ \rightarrow p + \pi^0$



The values of the mass of the Σ^+ hyperon, m_{Σ^+}, and the density of the emulsion, d, were obtained by equating the kinetic energies of the Σ^+ hyperon from reaction (2) and the proton from reaction (1), $T^{\mathrm{R}}_{\Sigma^+}$ and $T^{\mathrm{R}}_{\mathrm{p}}$ respectively, derived from their measured ranges in emulsion with those obtained from the kinematics of these reactions $T^{\mathrm{K}}_{\Sigma^+}$ and $T^{\mathrm{K}}_{\mathrm{p}}$. The problem reduces to solving the two simultaneous equations,

$$T^{\mathrm{R}}_{\mathrm{p}}(R_{\mathrm{p}}, m_{\mathrm{p}}, A_i, r; d) - T^{\mathrm{K}}_{\mathrm{p}}(m_{\mathrm{p}}, m_{\pi^0}; m_{\Sigma^+}) = 0\,, \tag{10}$$

$$T^{\mathrm{R}}_{\Sigma^+}(R_{\Sigma^+}, m_{\mathrm{p}}, A_i, r; m_{\Sigma^+}, d) - T^{\mathrm{K}}_{\Sigma^+}(m_{\pi^-}, m_{\mathrm{K}^-}, m_{\mathrm{p}}; m_{\Sigma^+}) = 0 \tag{11}$$

to determine the unknowns m_{Σ^+} and d, all the other quantities being known. The

Bohm, et al., Nuclear Physics B48 (1972) 1-12.
https://doi.org/10.1016/0550-3213(72)90047-8

Via simultaneous equations,

$M_{\Sigma+}$ = 1189.39 +- 0.06 MeV/c^2

Emulsion density = 3.843 + 0.003 g/cm^3

"slightly higher than the standard value, 3.815"

PDG2020

$M_{\Sigma+}$ = 1189.37 +- 0.07 MeV/c^2

Σ^+ MASS

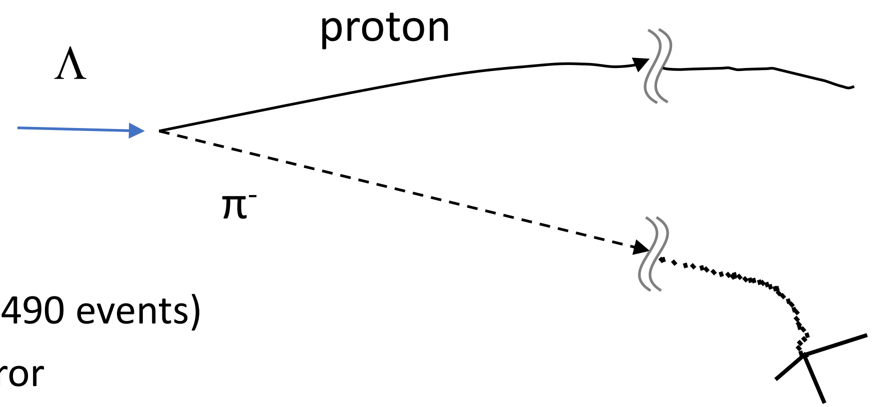
The fit uses Σ^+, Σ^0, Σ^-, and Λ mass and mass-difference measurements.

VALUE (MeV)	*EVTS*	*DOCUMENT ID*		*TECN*	*COMMENT*
1189.37±0.07 OUR FIT Error includes scale factor of 2.2.					
1189.37±0.06 OUR AVERAGE Error includes scale factor of 1.8. See the ideogram below.					
1189.33±0.04	607	[1] BOHM	72	EMUL	
1189.16±0.12		HYMAN	67	HEBC	
1189.61±0.08	4205	SCHMIDT	65	HBC	See note with Λ mass
1189.48±0.22	58	[2] BHOWMIK	64	EMUL	
1189.38±0.15	144	[2] BARKAS	63	EMUL	

[1] BOHM 72 is updated with our 1973 K^-, π^-, and π^0 masses (Reviews of Modern Physics **45** S1 (1973)).

Measurement of Λ mass in emulsion

B. Bhowmik et al., Il Nuovo Cimento **22**, 296-303 (1961)
https://link.springer.com/article/10.1007/BF02783020



M_{Λ} = 1115.57 +- 0.03 MeV/c^2 (490 events)

Only statistical error

PDG2020

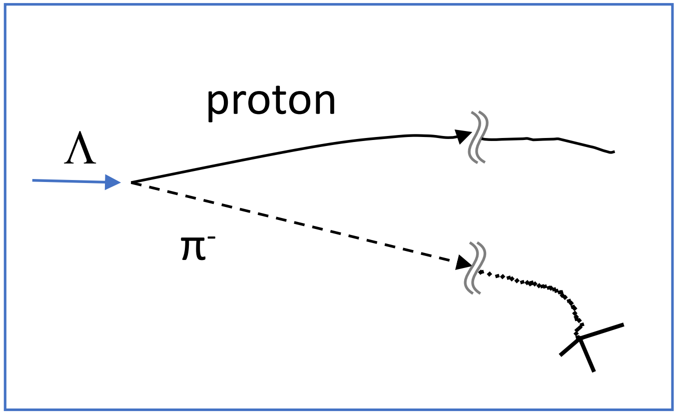
M_{Λ} = 1115.683 $\pm$ 0.006 MeV/c^2

Λ MASS

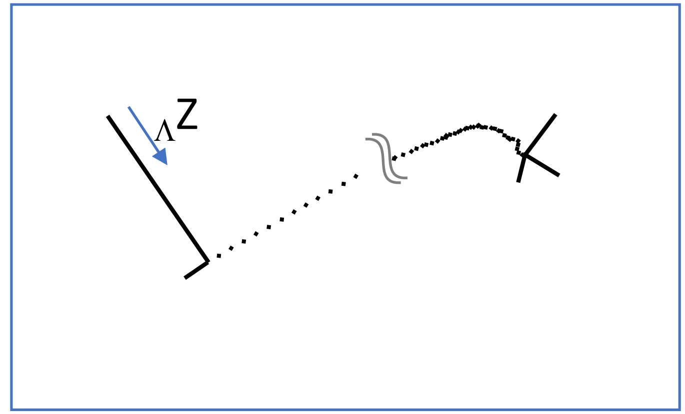
The fit uses Λ, Σ^+, Σ^0, Σ^- mass and mass-difference measurements.

VALUE (MeV)	*EVTS*	*DOCUMENT ID*		*TECN*	*COMMENT*
1115.683±0.006 OUR FIT					
1115.683±0.006 OUR AVERAGE					
1115.678±0.006±0.006	20k	HARTOUNI	94	SPEC	pp 27.5 GeV/c
1115.690±0.008±0.006	18k	[1] HARTOUNI	94	SPEC	pp 27.5 GeV/c

Cancellation of systematic deviation



Measured_Mass(Λ) = True_Mass(Λ) + $\varepsilon 1$



Measured_Mass($_{\Lambda}$Z) = True_Mass($_{\Lambda}$Z) + $\varepsilon 2$

Due to the error of the Range-Energy relation

B_{Λ} = Mass(Core) + Measured_Mass(Λ) - Measured_Mass($_{\Lambda}$Z)

If $\varepsilon 1$ and $\varepsilon 2$ is the same, They will cancel out.
However, evidence is necessary.

Identification of nuclide

The conditions for an event to be considered as uniquely identified

(i) A fit exists for which the resultant momentum of the decay products is zero with a C.L. > 10% (χ^2 < 6.3, D.O.F=3)

(ii) |The_fit_B_Λ – known_B_Λ | < 5 MeV

(iii) there exists no other fit to a known decay mode for which the resultant momentum is zero with a C.L. > 1% (χ^2 < 11.3, D.O.F=3)

Condition(i) in Bohm (1968)

514 G. BOHM et al.

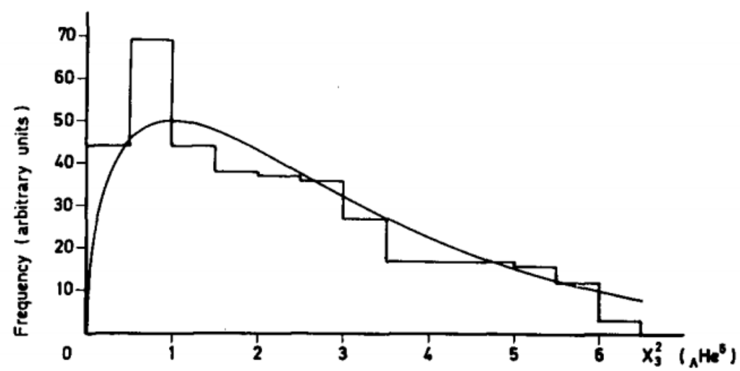


Fig. 1. χ_3^2 distribution for $^5_\Lambda He \rightarrow \pi^- {}^1H^4He$. The curve represents the expected distribution for three degrees of freedom.

Table of B_Λ

M. JURIC, et al. (1973)

Table 1
Binding energies for the s-shell hypernuclei.

In 2008 events

Hypernucleus	Decay mode	No of events	$B_\Lambda \pm \Delta B_\Lambda$ (MeV)
${}^{3}_{\Lambda}H$	$\pi^- + {}^1H + {}^2H$	24	0.23 ± 0.11
	$\pi^- + {}^3He$	58	0.06 ± 0.11
	total	82	0.15 ± 0.08
${}^{4}_{\Lambda}H$	$\pi^- + {}^1H + {}^3H$	56	2.14 ± 0.07
	$\pi^- + {}^2H + {}^2H$	11	1.92 ± 0.12
	total	67	2.08 ± 0.06
${}^{4}_{\Lambda}He$	$\pi^- + {}^1H + {}^3He$	83	2.42 ± 0.05
	$\pi^- + {}^1H + {}^1H + {}^2H$	15	2.44 ± 0.09
	total	98	2.42 ± 0.04
${}^{5}_{\Lambda}He$	$\pi^- + {}^1H + {}^4He$	798	3.19 ± 0.02
	$\pi^- + {}^1H + {}^1H + {}^3H$	8	2.95 ± 0.07
	$\pi^- + {}^2H + {}^3He$	15	3.04 ± 0.06
	$\pi^- + {}^1H + {}^2H + {}^2H$	1	3.49 ± 0.14
	total	822	3.17 ± 0.02

Large dependency on decay mode

G. Bohm, et al. (1968)

Table 3

In 2237 events

Hypernuclide	Decay mode	Number of events	$B_\Lambda \pm \Delta B_\Lambda$ (MeV)
${}_{\Lambda}{}^{3}H$	$\pi^-\ {}^3He$	86	+0.05 ± 0.08
	$\pi^-\ {}^1H{}^2H$	16	-0.11 ± 0.13
	total	102	+0.01 ± 0.07
${}_{\Lambda}{}^{4}H$	$\pi^-\ {}^4He$	552	2.29 ± 0.04
	$\pi^-\ {}^1H{}^3H$	63	2.08 ± 0.06
	$\pi^-\ {}^2H{}^2H$	7	
	not averaged, see text		
${}_{\Lambda}{}^{4}He$	$\pi^-\ {}^1H{}^3He$	127	2.36 ± 0.04
	$\pi^-\ {}^1H{}^1H{}^2H$	3	
${}_{\Lambda}{}^{5}He$	$\pi^-\ {}^1H{}^4He$	724	
	$\pi^-\ {}^2H{}^3He$	10	3.08 ± 0.02
	$\pi^-\ {}^1H{}^1H{}^3H$	1	

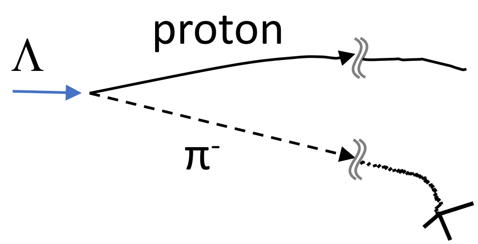
Large dependency on decay mode

Not used

Why 2-body decay of ${}^{4}_{\Lambda}H$ is not used?

G. Bohm et al., Il Nuovo Cimento A 70, 384–390 (1970)
https://link.springer.com/article/10.1007/BF02725382

- Discrepancy between the B_Λ by [${}^{4}_{\Lambda}H \rightarrow {}^{4}He + \pi^-$] and [${}^{4}_{\Lambda}H \rightarrow {}^{1}H + {}^{3}H + \pi^-$]
- Due to systematic error in the range-energy relation for particle velocities > 0.6c
- Lambda mass with π^- 4cm range is deviate from the average.

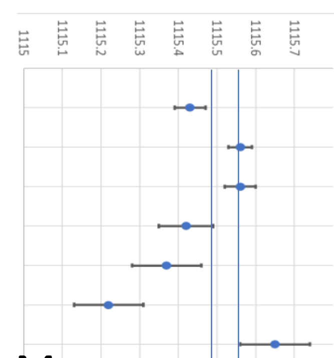


AN INVESTIGATION OF THE RANGE-ENERGY RELATION IN EMULSION 387

TABLE I. – *Variation of the observed value for* $\bar{M}_\Lambda$ *with the range of the* π^-*-meson.* The quoted errors are statistical only.

Limits of pion range (μm)	No. of events	$\bar{M}_\Lambda \pm \Delta\bar{M}_\Lambda$ (MeV)
$R \leqslant 10\,000$	181	1115.43 ± 0.04
$10\,000 < R \leqslant 20\,000$	594	1115.56 ± 0.03
$20\,000 < R \leqslant 30\,000$	371	1115.56 ± 0.04
$30\,000 < R \leqslant 35\,000$	133	1115.42 ± 0.07
$35\,000 < R \leqslant 40\,000$	82	1115.37 ± 0.09
$40\,000 < R \leqslant 45\,000$	77	1115.22 ± 0.09
$R > 45\,000$	86	1115.65 ± 0.09

Λ Mass



Mean
1115.52 +- 0.03

Combine

The binding energy of the ${}^{5}_{\Lambda}$He hypernucleus has been measured also, as a further point of calibration (see subsect. 3. l).

Table 2
Comparison of the B_Λ values for the s-shell hypernuclei obtained by Bohm et al. [2] and in this work

	$B_\Lambda \pm \Delta B_\Lambda$ (MeV)		δB_Λ (MeV)
	Bohm et al.[a)]	This work	
${}^{3}_{\Lambda}$H	0.01 ± 0.07	0.15 ± 0.08	0.14 ± 0.11
${}^{4}_{\Lambda}$H [b)]	2.09 ± 0.06	2.08 ± 0.06	–0.01 ± 0.09
${}^{4}_{\Lambda}$He	2.39 ± 0.04	2.42 ± 0.04	0.03 ± 0.06
${}^{5}_{\Lambda}$He	3.08 ± 0.02	3.17 ± 0.02	0.09 ± 0.03

a) The small difference appearing between some of the quoted values and those reported by Bohm et al. (see table 3 of ref. [2]) come from the procedure used in calculating the mean values. In Bohm et al. a cut based on both the momentum and energy balances was applied. The value quoted here were obtained by the iterative procedure based on a cut at 3 standard deviations from the mean B_Λ as in this experiment.
b) Excluding π-recoil decays.

“the results of both works are consistent and may thus be combined. “

Combined data

4042 uniquely identified events, 37000 mesonic decay

M. Jurić et al., Hypernuclei binding energies 25

Table 8
B_Λ compilation

Hypernuclide	Number of events	$B_\Lambda \pm \Delta B_\Lambda$ (MeV)
${}^3_\Lambda$H	204	0.13 ± 0.05
${}^4_\Lambda$H	155	2.04 ± 0.04
${}^4_\Lambda$He	279	2.39 ± 0.03
${}^5_\Lambda$He	1784	3.12 ± 0.02

	Juric	Bohm	Others	total
${}^3_\Lambda$H	82	102	20	204
${}^4_\Lambda$H	67	70	18	155
${}^4_\Lambda$He	98	130	51	279
${}^5_\Lambda$He	822	735	227	1784

“Recalibration” of B_Λ of ${}^{3}_{\Lambda}H$ by STAR group

arXiv:1904.10520v1 [hep-ex] 23 Apr 2019

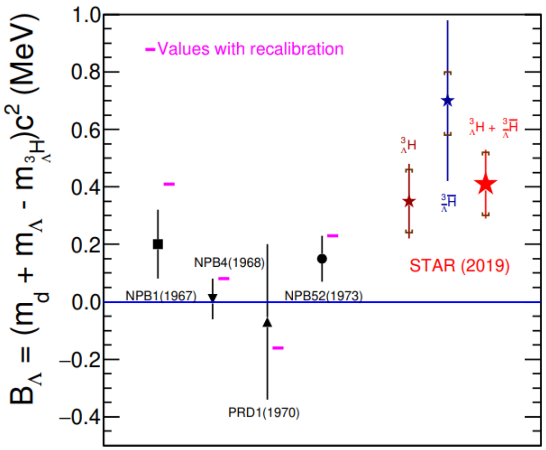


Table 1 | Assumed masses in past and present determinations of hypertriton binding energy B_Λ. All masses are in units of MeV/c^2.

Measurements	Λ mass	π^- mass	p mass	d mass	^{3}He mass
Gajewski *et al.* (1967)[31]	1115.44[32]	139.59[41]	938.26[41]	1875.50[40,45,46]	2808.22[40,45,46]
Bohm *et al.* (1968)[32]	1115.57[32]	139.58[42]	938.26[42]	1875.50[40,45,46]	2808.22[40,45,46]
Keyes *et al.* (1970)[33]	1115.67[33]	139.58[43]	938.26[43]	1875.58[33]	2808.22[40,45,46]
Juric ~~Bohm~~ *et al.* (1973)[4]	1115.57[4]	139.58[44]	938.26[44]	1875.50[40,45,46]	2808.22[40,45,46]
Present study	1115.68[18]	139.57[18]	938.27[18]	1875.61[30]	2808.39[30]

Table 2 | The previous measurements of B_Λ for hypertriton and its corresponding recalibration results. B_Λ is in units of MeV. The uncertainties are the reported statistical uncertianties.

Measurements	Original		Recalibrated	
	B_Λ	Combined B_Λ	B_Λ	Combined B_Λ
Gajewski *et al.* (1967)[31]	0.13 ± 0.15 (2-body) 0.33 ± 0.21 (3-body)	0.20 ± 0.12	0.33 ± 0.15 (2-body) 0.58 ± 0.21 (3-body)	0.41 ± 0.12
Bohm *et al.* (1968)[32]	0.05 ± 0.08 (2-body) -0.11 ± 0.13 (3-body)	0.01 ± 0.07	0.11 ± 0.08 (2-body) 0.00 ± 0.13 (3-body)	0.08 ± 0.07
Keyes *et al.* (1970)[33]	0.25 ± 0.31 (2-body) -0.74 ± 0.43 (3-body)	-0.07 ± 0.27	0.13 ± 0.31 (2-body) -0.73 ± 0.43 (3-body)	-0.16 ± 0.27
Juric ~~Bohm~~ *et al.* (1973)[4]	0.06 ± 0.11 (2-body) 0.23 ± 0.11 (3-body)	0.15 ± 0.08	0.12 ± 0.11 (2-body) 0.34 ± 0.11 (3-body)	0.23 ± 0.08

The paper on the recalibration

Peng Liu *et al.*, 2019 *Chinese Phys. C* **43** 124001
https://arxiv.org/pdf/1908.03134v2.pdf

"We note that the early emulsion measurements in 1968 and 1973 benefited from a compensating effect in normalizing the B_Λ values via measuring the mass of the Λ hyperon with the decay daughter π^- range of 1-2 cm in the same emulsion stack."

- They recognized the benefit of the calculation of early measurement.

"This difference in π^- range can also yield a difference in the measured Q value as large as 0.43 +- 0.13 (stat.) MeV, and cannot ensure the deviations of measured Q value for Λ decay and hypernuclear decay are in the same direction."

- I agree with this statement.

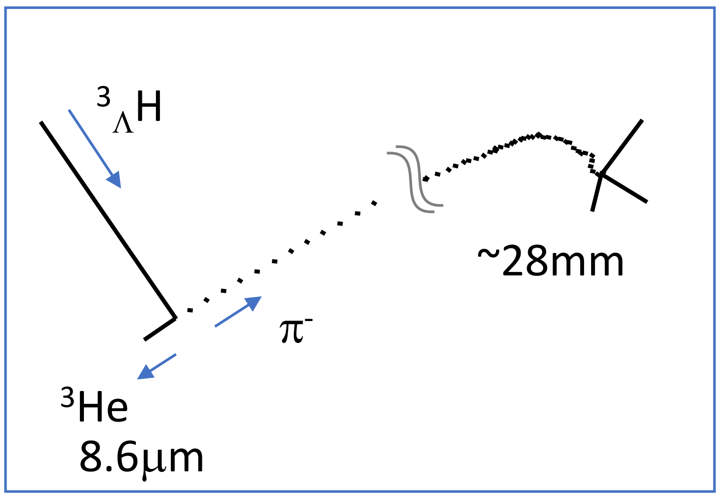
We will evaluate how the error or shift of the inputs affect B_Λs using MC simulation.

This study is ongoing and to be published by A. Kasagi and E. Liu.

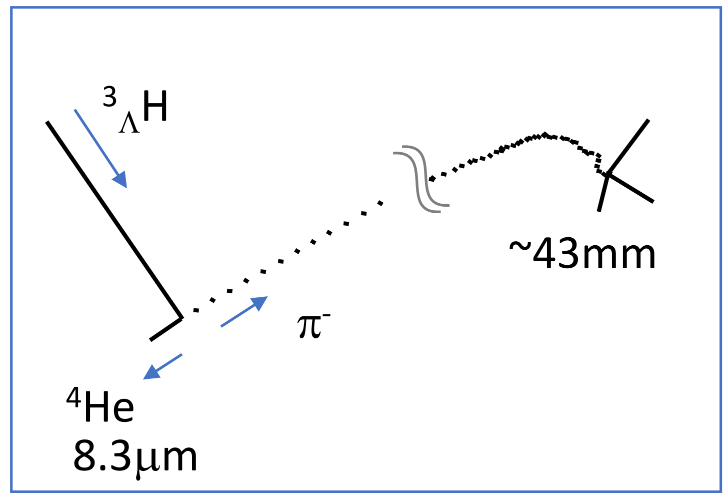
How accurately can we measure the B_Λ of single Λ hypernuclei with E07 emulsion sheets?

- We are trying re-measurement of the hypertriton mass.
- Collaboration between High Energy Nuclear Physics Lab at RIKEN and Gifu-U.
- Machine learning based object detection.
- 2-body decay of ${}^{3}_{\Lambda}H$ and ${}^{4}_{\Lambda}H$.

${}^{3}_{\Lambda}H \rightarrow {}^{3}He + \pi^-$



${}^{4}_{\Lambda}H \rightarrow {}^{4}He + \pi^-$



If emulsion density = 3.500 g/cm^3

- Typical B_Λ error of an event is ~0.5 MeV
- The B_Λ of ${}^{4}_{\Lambda}H$ will be compared to that of MAMI's experiment

Event search

2.1. Exposure, processing and scanning method

Paper	G. Bohm, et al. (1968)	M. JURIC, et al. (1973)
Experiment	CERN P.S	AGS
Beam	K- 700MeV/c -> degraded	K- 760MeV/c -> degraded
Emulsion		
Type	Illford K5	Illford K5
Amount	20 litters	6 litters
Sheet	pellicles	pellicles
		15cm*20cm*600μm*363
Optics		
area-scan	x300	x300
meas.	?	x600
Mesonic decay		
found	7000	27000
identified	2008	2237