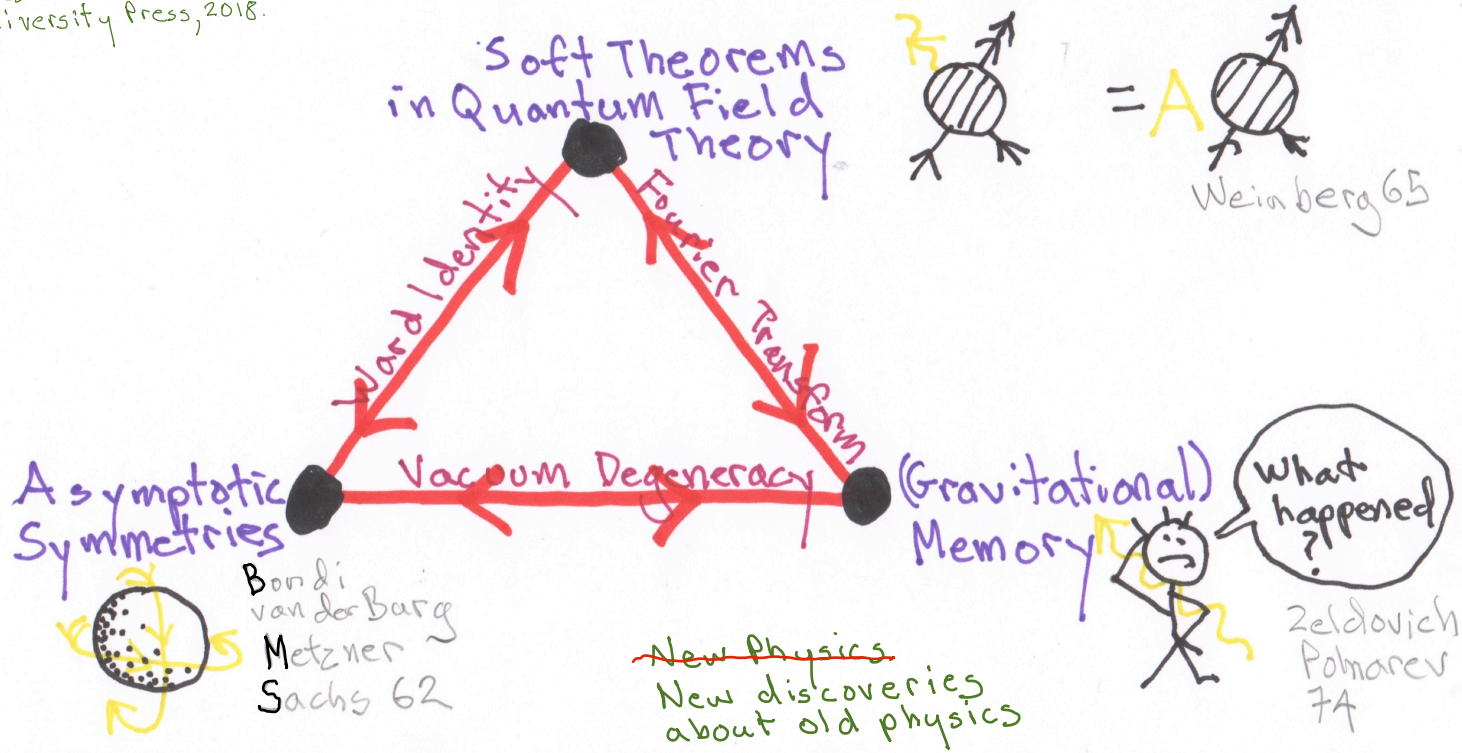
Infrared Structure & Memory in Gauge Theory

EMM | Rapid Reaction Task Force
November 2020

Andy Strominger, Harvard

Over the last 7 years, an exact mathematical equivalence has been discovered of 3 previously disparate phenomena, each studied for half a century:

A. S. "Lectures on the Infrared Structure of Gauge Theory & Gravity" Princeton University Press, 2018.

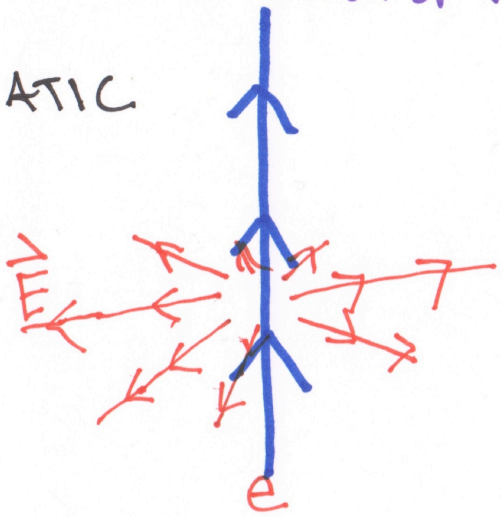


This has led to surprising new insights into the low-energy structure of gravitational & electromagnetic theories, implications for quantum black hole information & novel experiments at hadron accelerators, the EIC, LIGO, LISE & Elsewhere.

∞ of conserved charges in E&M *

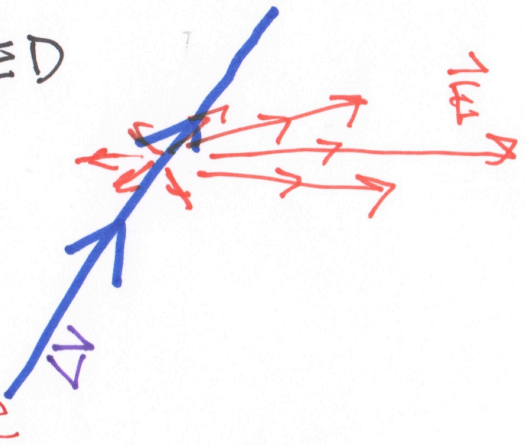
Lienard-Wiechert radial electric field

STATIC



$$F_{rt} = \frac{e}{4\pi r^2}$$

BOOSTED

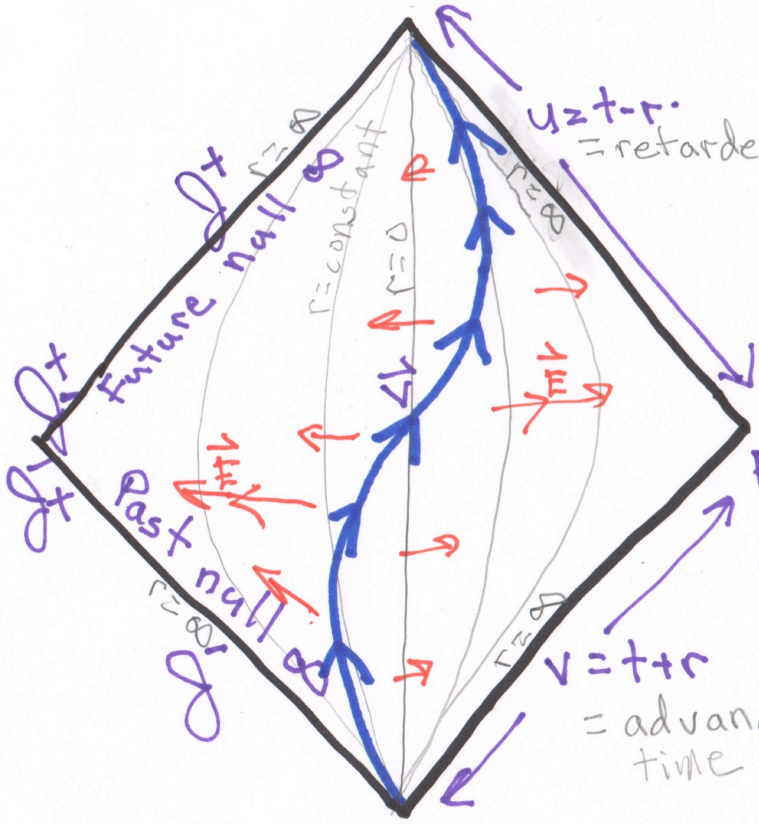


$$F_{rt} = \frac{e\sqrt{1-v^2}\,(r^2 - t\,\vec{x}\cdot\vec{v})}{4\pi r\left[(1-v^2)(t-\vec{x}\cdot\vec{v})^2 - t^2 + r^2\right]^{3/2}}$$

The BOOSTED field is <u>not</u> single-valued near spatial ∞. We need to understand this carefully.

* This talk ignores gravity, concentrating on QED.

Penrose diagram for a moving charge in Minkowski space



$$F_{ru} = \frac{e}{4\pi} \frac{(1-v^2)}{(r-\vec{v}\cdot\vec{x})^2}$$

$$F_{rv} = \frac{e}{4\pi} \frac{(1-v^2)}{(r+\vec{v}\cdot\vec{x})^2}$$

different sign

The general Lorentz-invariant boundary condition is antipodal:

$$r^2 F_{ru}(\theta^A)\Big|_{\mathcal{I}^+_-} = r^2 F_{rv}(\tilde{\theta}^A)\Big|_{\mathcal{I}^-_+}$$

$A = (\theta, \phi)$ angle on sphere

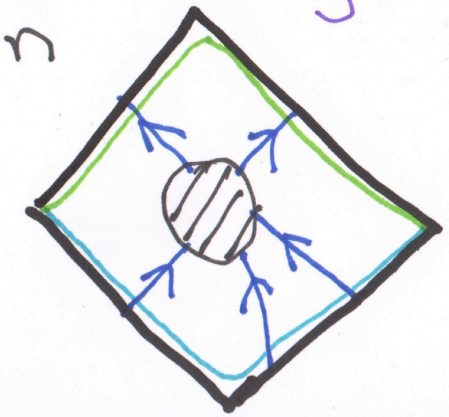
antipodal angle on sphere

This implies an ∞ of 'antipodal' conservation laws

$$Q^+_\varepsilon \equiv \int_{\mathcal{J}^+_-} d^2\Omega\, r^2 F_{ru}\, \varepsilon(\theta^A)$$

$$= Q^-_\varepsilon \equiv \int_{\mathcal{J}^-_+} d^2\Omega\, r^2 F_{rv}\, \varepsilon(\theta^A)$$

where $\varepsilon(\theta^A)|_{\mathcal{J}^+_-} = \varepsilon(\tilde{\theta}^A)|_{\mathcal{J}^-_+}$ is <u>any</u> function on the sphere. For the special case $\varepsilon = 1$ using Gauss's law $\nabla^\mu F_{\mu\nu} = j^M_\nu$ this is global charge conservation



$$\sum_K e^{in}_K = \sum_K e^{out}_K$$

AS hep-th 1308.0589
He, Mitra, Porfyriadis, AS hep-th 1407.3789
Campiglia & Laddha hep-th 1505.05346
Kapec, Pate, AS hep-th 1506.02906

But what are the conservation laws when $\partial_A \varepsilon(\theta^A) \neq 0$???

Integrating by parts and using Gauss's law, the conservation laws are

$$\sum_K \varepsilon(\theta_K^A) e_K^{in} + \text{  } = \sum_K \varepsilon(\theta_K^A) e_K^{out} + \text{  }$$

incoming charges weighted by angle

strange duck

$$\text{  } = \int d^2\Omega \int_{-\infty}^{\infty} dv\, \partial_A \varepsilon F^A{}_v = \underline{\text{soft photon w/ polarization } \partial_A \varepsilon}$$

soft = zero-energy

At the classical level, this cons. law equates the sum of a zero mode of the incoming EM field and a moment of the incoming charge distribution to its antipodal outgoing counterparts.

Quantum Conservation Laws

In QM $|out\rangle = S|in\rangle$ ← S-matrix

the ∞ of conservation laws are

$$\langle out|Q^+_\varepsilon S - S Q^-_\varepsilon |in\rangle = 0$$

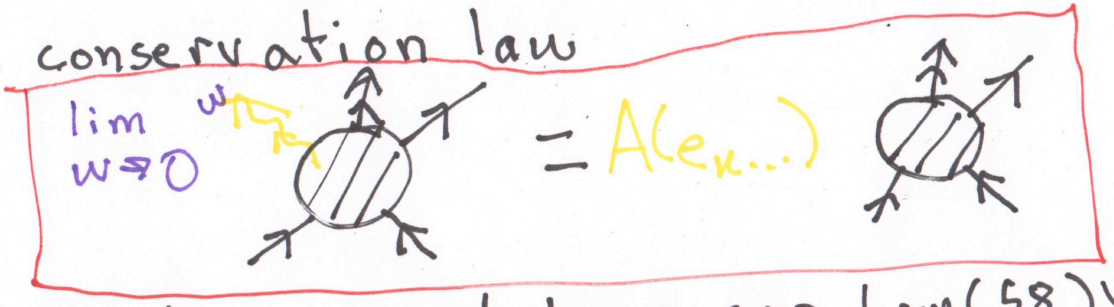
He Mitra Porfyriadis Rapec Pate Campiglia Laddha AS

for any pair $|in\rangle, |out\rangle$ and any ε.



$= \lim_{w\to 0} \int d^2z \Pi \int_{-\infty}^{\infty} du e^{iwu} \partial_A \varepsilon F^A{}_u$ creates/annihilates soft photons.

Bizzare conservation law



but it was discovered long ago Low(58) Weinberg(65) and = SOFT PHOTON THEOREM all EM theories e.g. QED

Can reverse logic: soft photon theorem $\Rightarrow Q^+_\varepsilon = Q^-_\varepsilon$

Conservation Laws ⇒ Symmetries

$$[G^{+}_{\varepsilon}, A_B]_{gf} = i\partial_B \varepsilon$$

are "large gauge transformations" that go to angle-dependent constants at null infinity. They act non-trivially on physical states & can be measured via the electromagnetic memory effect: EM analog of well-known gravitational memory effect.

Bieri, Garfinkle gr-qc 1307.5098
Pasterski hep-th 1505.00716
Susskind hep-th 1507.02584

∞ Vacuum Degeneracy

$H|0\rangle = 0 \qquad H Q^+_\varepsilon |0\rangle = 0$

$\langle 0 | Q^+_\varepsilon |0\rangle = 0$

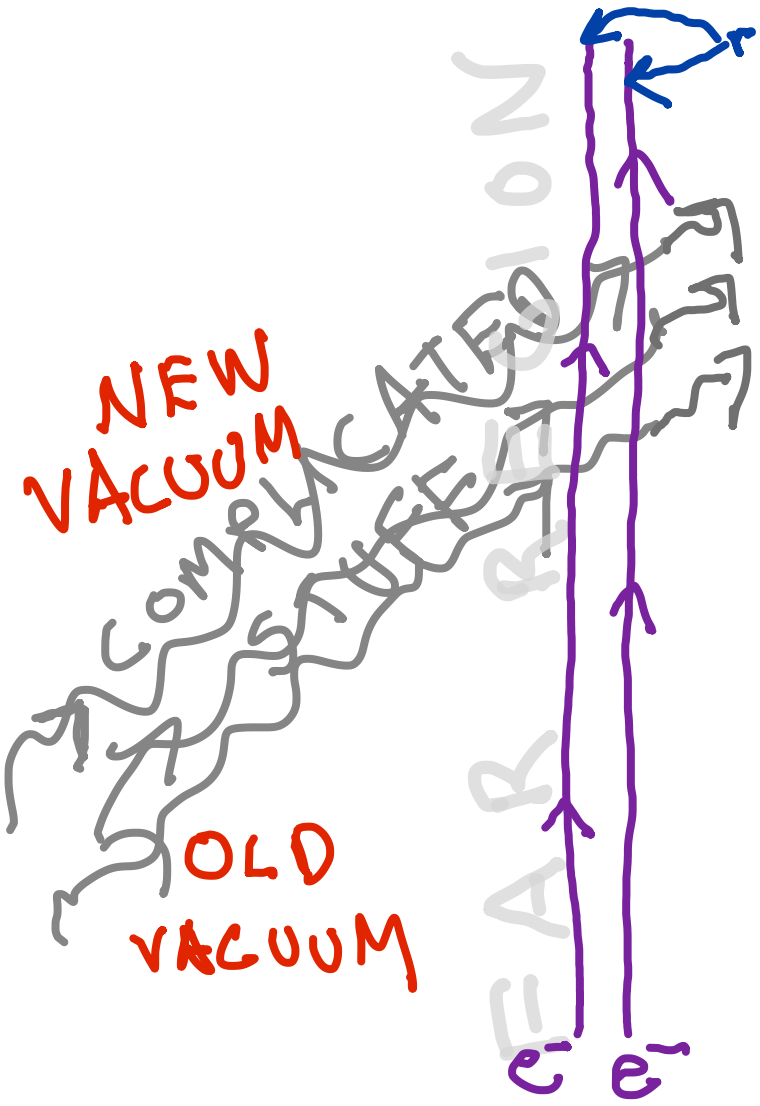
$\Rightarrow Q^+_\varepsilon |0\rangle \neq |0\rangle$

= an additional soft photon on $|0\rangle$

$\exists$ ∞ many degenerate vacua w/ different angular momenta: quantum vacuum has 'soft hair'

Large gauge symmetries are spontaneously broken. Soft photon = Goldstone boson

Memory Effect



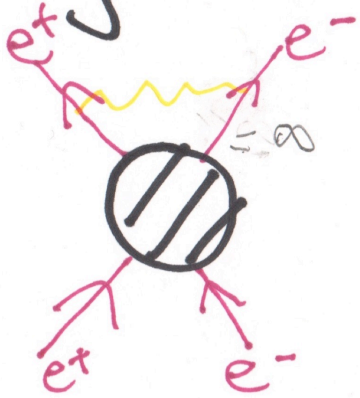
Force	Symmetry	Experiment
Gravity	Diffeo	Displacement
YM	NAG Gauge Trans	Color Rotation
QED	A Gauge Trans	Quantum Phase





Will return...

This is related to the problem of IR divergences in QED, which set all amplitudes to zero as they should be because vacuum shift is not accounted for.



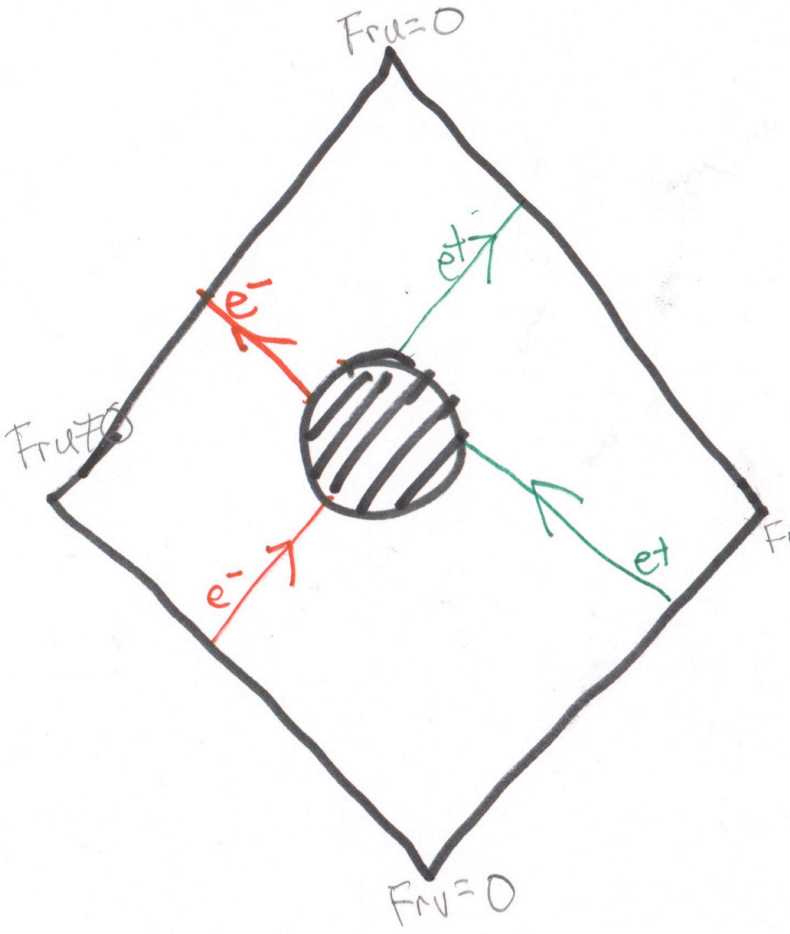
Incorporating vacuum shift → IR finite amplitudes. This is Kapec, Perry, Raclariu AS 2017

in some sense a generalization of Faddeev & Kulish (1970).

Also Choi, Kol, Akhoury

Bhaba scattering $e^+e^- \to e^+e^-$

take $m_e \to 0$ for simplicity



Constraints

On $\mathcal{I}^+$ (no radiative modes)

$$\partial_u F_{ru} + D^2 F_{uz} + D^2 F_{u\bar{z}} = -e^2 g_u$$

On $\mathcal{I}^-$

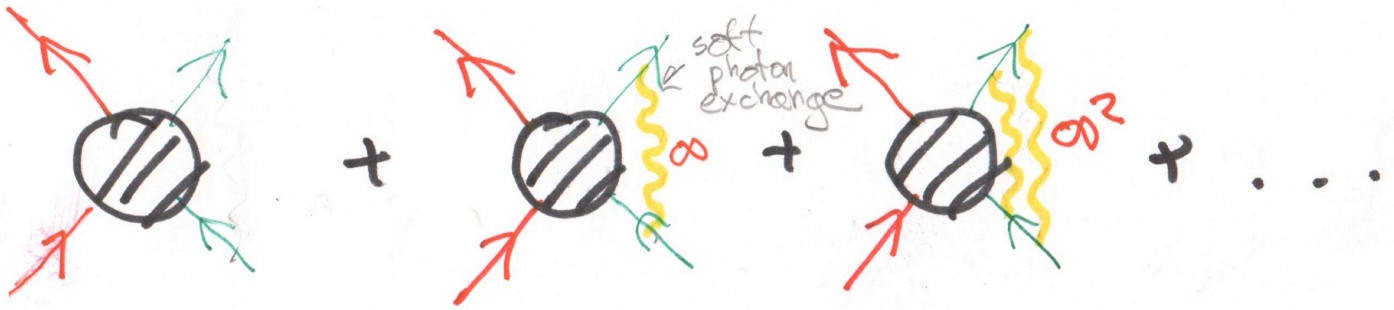
$$\partial_v F_{rv} - D^2 F_{vz} - D^2 F_{v\bar{z}} = -e^2 g_v$$

$$\Rightarrow F_{ru}(z,\bar{z})|_{\mathcal{I}^+_-} \neq F_{rv}(z,\bar{z})|_{\mathcal{I}^-_+}$$

Antipodal match violated

$$\Rightarrow \boxed{a(e^+e^- \to e^+e^-) = 0}$$

This is well known, and usually attributed to IR divergences:



$$= e^{-\infty} = 0.$$

Here we see the role of IR divergences as a clever trick by QFT to set to zero conservation-law-violating amplitudes.

No 'real' IR divergences.

But we need to compute something other than 0. Lets solve the constraints differently:

$$\cancel{\partial_u F_{rv}}^{\to 0} + D^2 F_{uz} + D^{\bar z} F_{u\bar z} = -e^2 j_u = -e^2 \delta(u-u_0)\delta^2(z-z_0)$$

Radiative solution:

$$A_z = -\frac{\theta(u-u_0)e^2}{4\pi(z-z_0)}$$ has pole for $w \to 0$

Note $A_z|_{j_+} - A_z|_{j_-} \neq 0 \Rightarrow$ vacuum shift

Quantum state

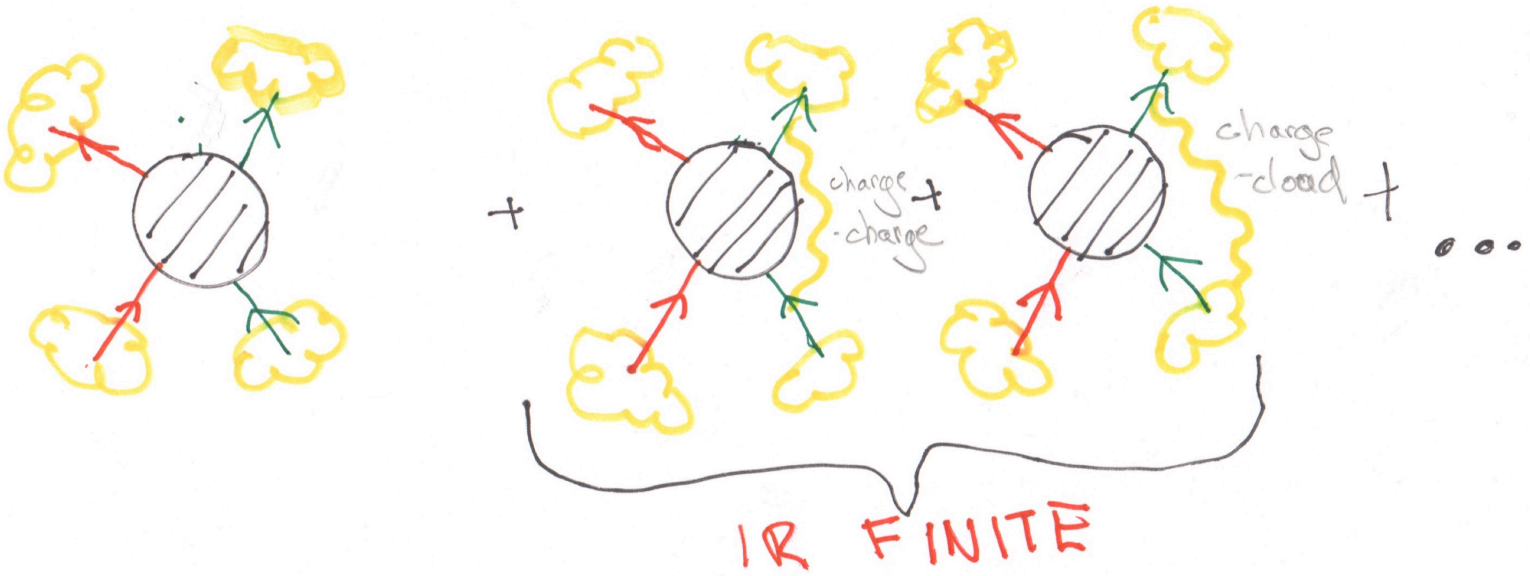
$$|\psi\rangle_{\text{dressed}} = e^{-\frac{e^2}{2\pi}\int \frac{d^2z}{(z-z_0)} A_{\bar z}(u_0,z,\bar z) + \text{h.c.}} |u_0,z_0\rangle$$

obeys $[D^2 F_{uz} + D^{\bar z} F_{u\bar z} + e^2 j_u]\,|\psi\rangle_{\text{dressed}} = 0$

Coulomb field is 'shielded'. Dressing all particles $\Rightarrow F_{ru}|_{j_-} = F_{rv}|_{j_+} = 0$ (EXCEPT ZERO MODE ABSENT IN BHABA)

& conservation laws trivially satisfied.

Scattering of these dressed state is IR finite!!! (Faddeev & Kulish 1970)



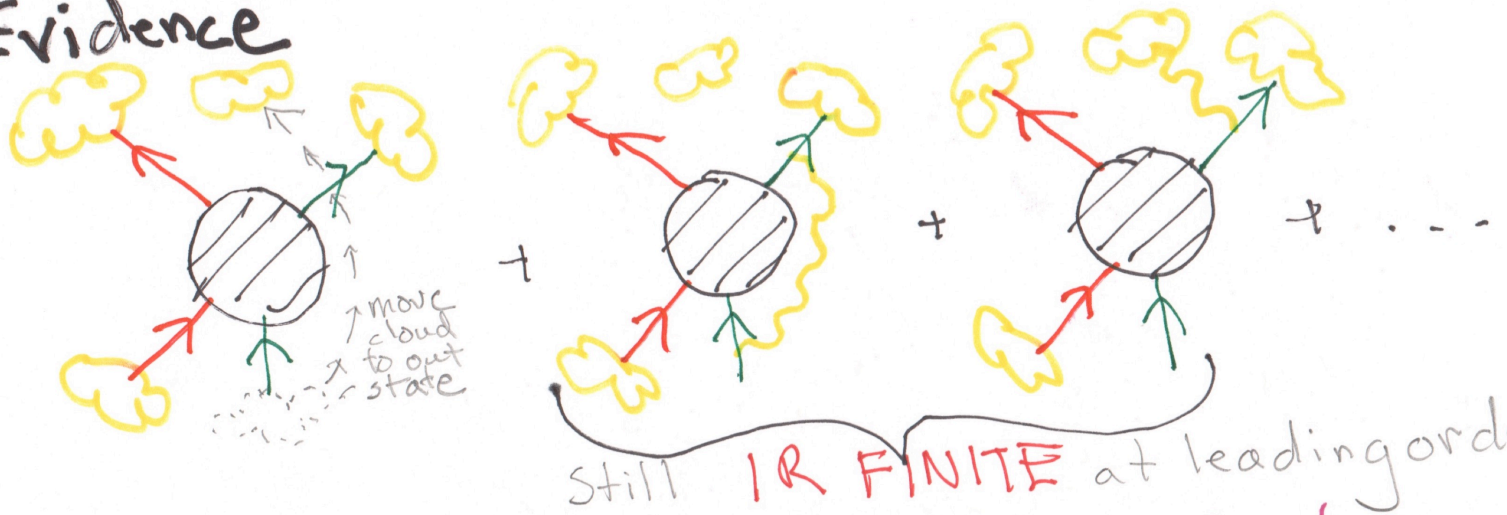
Reinterpretation of FK dressing: solves constraints radiatively, ensures conservation laws, implements required vacuum shift.

N.B. Soft clouds do not really 'surround' asymptotic hard particle.

All scattering amplitudes obeying ∞ of conservation laws are IR finite.

FK states are highly unphysical. We do not follow up LHC protons with finely-tuned charge-cancelling soft photon clouds!

Evidence



FK soft clouds are just the soft radiation produced in scattering as dictated by soft theorem = conservation laws.

All orders proof Akhoury & Choi ...

Conjecture for (unconfined & unhiggsed) nonabelian gauge theory:

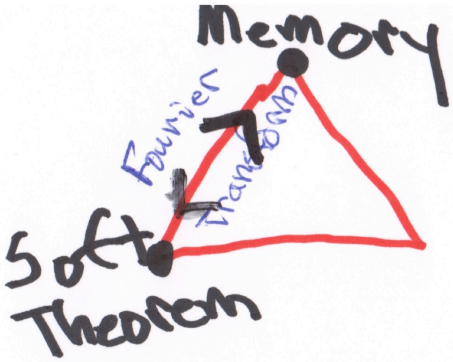
Ditto.

Charge conservation $\Rightarrow$ IR finite

Also ditto for gravity.

Recent generalization of KLN to gravity: Wang, Saotome, Akhoury

Return to MEMORY EFFECT



"Infrared Photons & Gravitons"
S. Weinberg, 1965

"Gravitational Wave-Burst with Memory and Experimental Prospects"
V. Braginsky and K.S. Thorne 1987

The dominance of the $1/(p\cdot q)$ pole in (2.5) implies that the effect of attaching one soft-graviton line to an arbitrary diagram is to supply a factor equal to the sum of (2.5) over all external lines in the diagram

$$(8\pi G)^{1/2}\sum_n \eta_n p_n^\mu p_n^\nu/[p_n\cdot q - i\eta_n\epsilon]. \qquad (2.7)$$

permanent change in the gravitational-wave field (the burst's memory) δh_{ij}^{TT} is equal to the 'transverse, traceless (TT) part'[36] of the time-independent, Coulomb-type, $1/r$ field of the final system minus that of the initial system. If $\mathbf{P}^A$ is the 4-momentum of mass A of the system and P_i^A is a spatial component of that 4-momentum in the rest frame of the distant observer, and if $\mathbf{k}$ is the past-directed null 4-vector from observer to source, then δh_{ij}^{TT} has the following form:

$$\delta h_{ij}^{TT} = \delta\left(\sum_A \frac{4P_i^A P_j^A}{\mathbf{k}\cdot\mathbf{P}^A}\right)^{TT} \qquad (1)$$

Here we use units with $G = c = 1$. In the observer's local Car-

$\int dw e^{iwv}$ = !!!

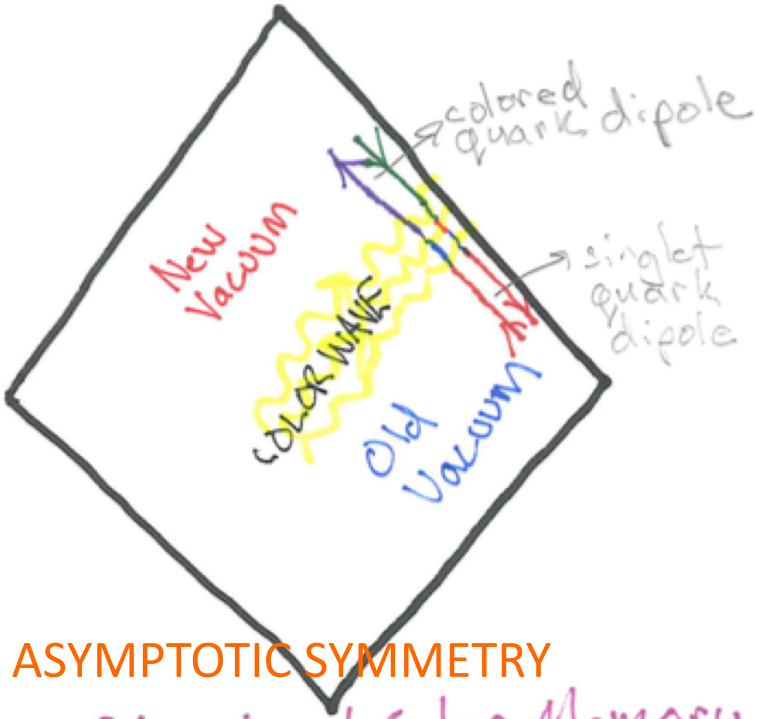
Changing notations, using $\theta(u) = \int \frac{dw}{w} \frac{e^{iwu}}{2\pi i\delta}$ and elementary particles → black holes, stars!

Memory effect provides physical method to measure soft gravitons.

Likely to be measured at LIGO/LISE, even spin memory Nichols, !..

Color Memory at Electron-Ion Colliders





MEMORY = SOFT THEOREM = ASYMPTOTIC SYMMETRY

Classical Gravitational Memory

$$\frac{\Delta S}{S}(z,\bar{z}) = \int d^2w\, G(z,\bar{z};w,\bar{w}) \int du\, T_{uu}$$

= soft graviton theorem

likely to be measured LIGO

Classical Color Memory

$$W(C) = \mathrm{Tr}\left(P\, e^{i g_{ym} \oint A} \right)$$

closed $q\bar{q}$ worldline

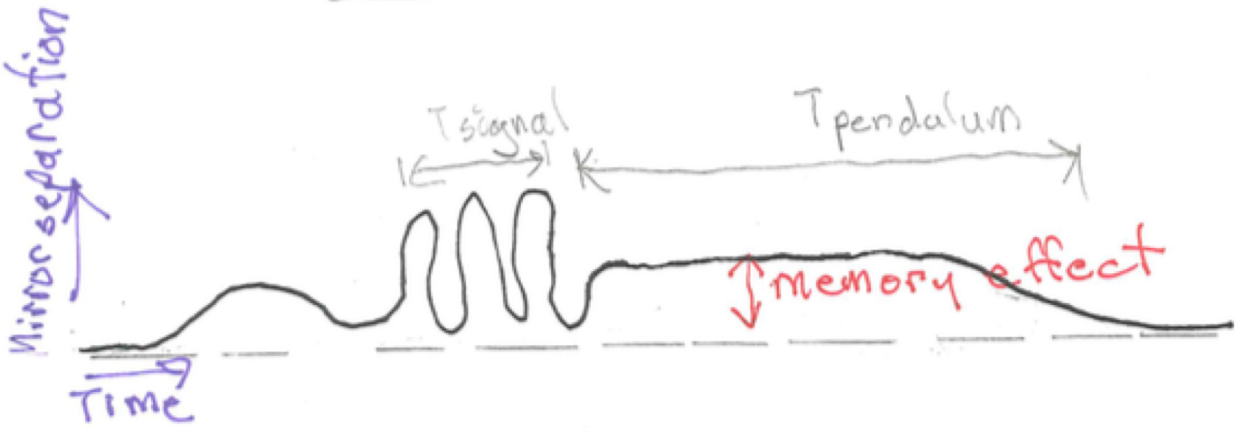
$\nabla \cdot A \sim j_{color\ flux}$

= soft gluon theorem

Why measurable in QCD?

Pate, Raclariu, AS 170708016

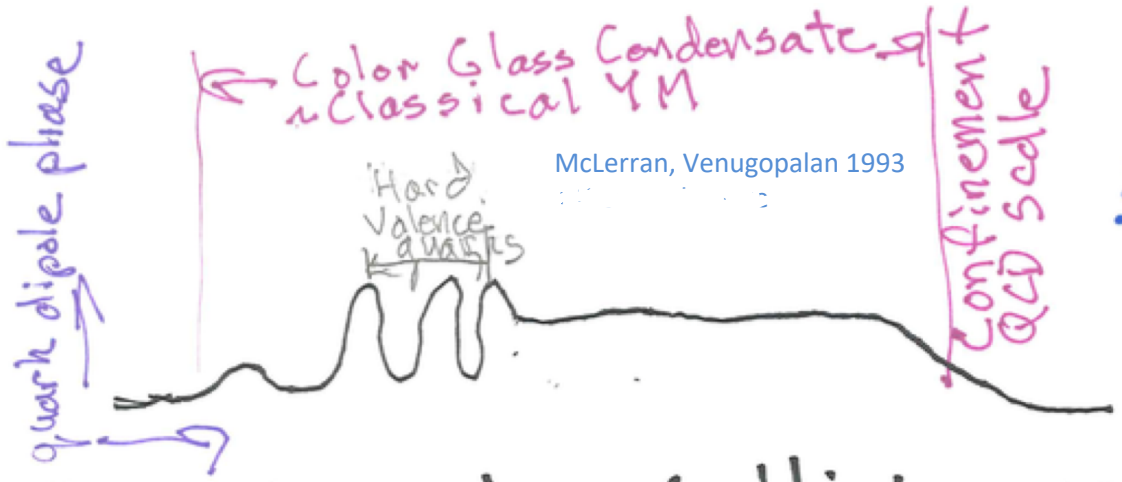
Scale separation



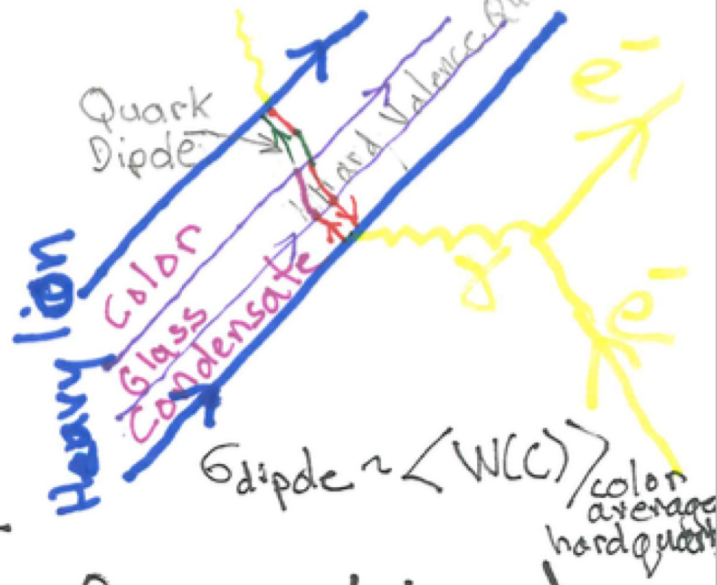
GW150914

To measure memory need $\frac{T_{signal}}{T_{pend}} < 1$

Not so small at h160 but still hope to measure.



McLerran, Venugopalan 1993



Heavy Ion Collision in Regge limit

Ball, Pate, Raclariu, AS, Venugopalan 180512224

In the Regge limit, it is argued that the inclusive virtual photon-ion cross section factorizes

$\sigma_{\gamma^* \text{Ion}} \sim \sigma_{\gamma^* \to q\bar{q}} \otimes \sigma_{\text{dipole}} \propto 1 - \langle W(C) \rangle_{\text{color averaged}}$

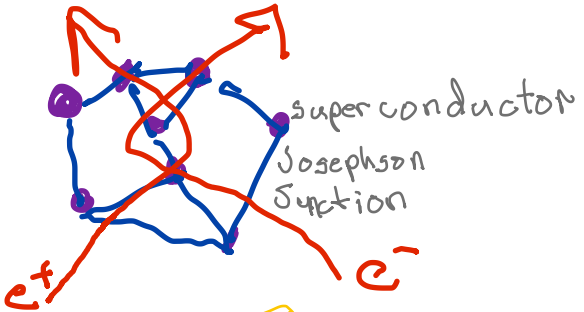
and is proportional to the memory effect. The current situation is fluid as to whether or not color memory has been observed. Definitive measurement should be possible at the currently planned Future Electron-Ion collider! ALICE?

Measuring EM Memory

- No concrete proposal that I know of!
- Similar to measuring soft photons, also to electric Aharonov Bohm.

Susskind 1507.02584
Directly measure phase change at nodes

Vague discussions
Rai Weiss, Monica Pate
Ana Raclariu AS
Neutron Interferometry
Pate Raclariu AS Thaler
Encircle beam dump
with interferometer





IDEAS WANTED!

Grav. memory ~ $\Delta \dot{Q}_{ij}$ EM memory ~ $\Delta \dot{D}_i$

Conclusions

There is much yet-to-be understood about the deep IR in Minkowski space. It is relevant both to ongoing experiments and to fundamental issues in holography, black holes and quantum gravity.


THANK YOU!