Can one measure electric conductivity of the quark-gluon plasma?

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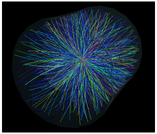
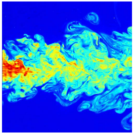
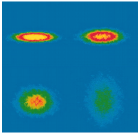
Fluid dynamics

- long distances, long times or strong enough interactions
- quantum fields form a fluid!
- needs **macroscopic** fluid properties
  - thermodynamic equation of state $p(T, \mu)$
  - shear + bulk viscosity $\eta(T, \mu)$, $\zeta(T, \mu)$
  - heat conductivity $\kappa(T, \mu)$, $\ldots$
  - relaxation times, ...
  - electrical conductivity $\sigma(T, \mu)$
- fixed by **microscopic** properties encoded in Lagrangian $\mathscr{L}_{\mathrm{QCD}}$

Relativistic fluid dynamics

Energy-momentum tensor and conserved current

$$T^{\mu\nu} = \epsilon\, u^\mu u^\nu + (p + \pi_{\text{bulk}})\Delta^{\mu\nu} + \pi^{\mu\nu}$$
$$J^\mu = n\, u^\mu + \nu^\mu$$

- tensor decomposition using fluid velocity u^μ, $\Delta^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$
- thermodynamic equation of state $p = p(T, \mu)$

Covariant **conservation laws** $\nabla_\mu T^{\mu\nu} = 0$ and $\nabla_\mu N^\mu = 0$ imply

- equation for energy density ϵ
- equation for fluid velocity u^μ
- equation for particle number density or charge density n

Need **further evolution equations** [e.g Israel & Stewart]

- equation for shear stress $\pi^{\mu\nu}$
- equation for bulk viscous pressure π_{bulk}

$$\tau_{\text{bulk}}\, u^\mu \partial_\mu \pi_{\text{bulk}} + \ldots + \pi_{\text{bulk}} = -\zeta\, \nabla_\mu u^\mu$$

- equation for diffusion current ν^μ
- non-hydrodynamic degrees of freedom are needed for relativistic causality!

Electric current

- quarks carry electric charge
- electromagnetic current

$$J^\mu = nu^\mu + \nu^\mu$$

- conservation law for electromagntic current

$$\nabla_\mu J^\mu = u^\mu \partial_\mu n + n \nabla_\mu u^\mu + \nabla_\mu \nu^\mu = 0$$

- supplemented by evolution equation for diffusion current

$$\nu^\alpha + \tau \Delta^\alpha{}_\beta u^\mu \nabla_\mu \nu^\beta = \sigma \Delta^{\alpha\nu} E_\nu - D \Delta^{\alpha\nu} \partial_\nu n$$

- electric conductivity σ
- diffusion coefficient $D = \sigma/\chi$
- charge susceptibility $\chi = (\partial n/\partial \mu)|_T$
- relaxation time τ constrained by causality

$$\tau > D = \frac{\sigma}{\chi}$$

Spectral function from fluid dynamics

- Retarded response function

$$G_R^{\mu\nu}(x-y) = i\theta(x^0 - y^0)\langle [J^\mu(x), J^\nu(y)]\rangle$$

- determines current response

$$\delta\langle J^\mu(x)\rangle = \int_y G_R^{\mu\nu}(x-y)\,\delta A_\nu(y)$$

- Inverse propagator encodes equations of motion

$$\underbrace{\begin{pmatrix} -iN_1\omega & iN_1\mathbf{p} \\ iN_2 D\mathbf{p} & N_2(1-i\omega\tau)\mathbb{1} \end{pmatrix}}_{=P(\omega,\mathbf{p})=G_{\mathsf{R}}^{-1}(\omega,\mathbf{p})} \begin{pmatrix} \rho(\omega,\mathbf{p}) \\ \mathbf{J}(\omega,\mathbf{p}) \end{pmatrix} = \begin{pmatrix} A_0(\omega,\mathbf{p}) \\ \mathbf{A}(\omega,\mathbf{p}) \end{pmatrix}$$

- coefficients $N_1 = 1/(i\chi\omega)$ and $N_2 = 1/(i\sigma\omega)$ can be determined from homogeneous and static limit $\mathbf{p} = 0$, $\omega \to 0$
- spectral function obtains from

$$\rho(\omega,\mathbf{p}) = g_{\mu\nu}\,\mathrm{Im}(G_R^{\mu\nu}(\omega,\mathbf{p}))$$

Photon production rate in local thermal equilibrium

- photon production rate per unit volume and time

$$p^0 \frac{dR}{d^3p} = \frac{1}{(2\pi)^3} n_{\mathsf{B}}(\omega)\rho(\omega),$$

- electromagnetic spectral function $\rho(\omega)$
- frequency in the fluid rest frame

$$\omega = -u_\mu p^\mu$$

- Bose-Einstein distribution factor

$$n_{\mathsf{B}}(\omega) = \frac{1}{e^{\omega/T} - 1}$$

Dilepton production rate in local thermal equilibrium

- thermal dilepton production rate per unit volume and time

$$\frac{dR}{d^4p} = \frac{\alpha}{12\pi^4}\frac{1}{M^2} n_{\mathsf{B}}(\omega)\,\rho(\omega, M) \times \left(1 + \frac{2m^2}{M^2}\right)\sqrt{1 - \frac{4m^2}{M^2}}\,\Theta(M^2 - 4m^2),$$

- momentum of the dilepton pair $p^\mu = p_1^\mu + p_2^\mu$
- lepton mass m
- electromagnetic fine structure constant $\alpha = e^2/(4\pi)$

Electric conductivity

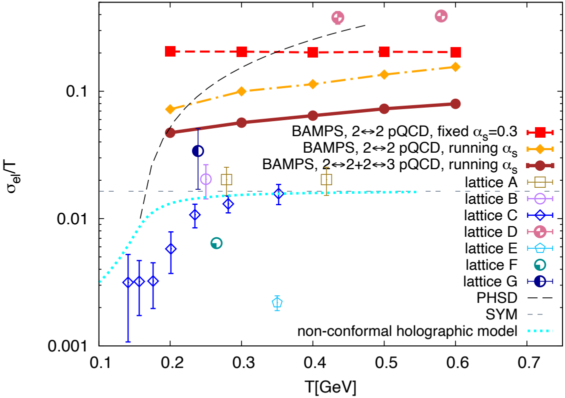
- Kubo relation for electric conductivity

$$\sigma = \frac{1}{2} \lim_{\omega \to 0} \frac{1}{\omega} \rho(\omega, \mathbf{p}) \Big|_{\mathbf{p}^2 = \omega^2} = \frac{1}{3} \lim_{\omega \to 0} \frac{1}{\omega} \rho(\omega, \mathbf{p}) \Big|_{\mathbf{p} = 0}$$

- small frequency limit either at $\mathbf{p}^2 = \omega^2$ or at $\mathbf{p} = 0$
- ratio ρ/ω has transport peak at small frequency

Predictions of electrical conductivity

- many predictions of electric conductivity in the literature
- perturbative predictions [Arnold, Moore & Yaffe] $0.19 < \sigma/T < 2$
- lattice estimates vary
- would be great to have some experimental constraints



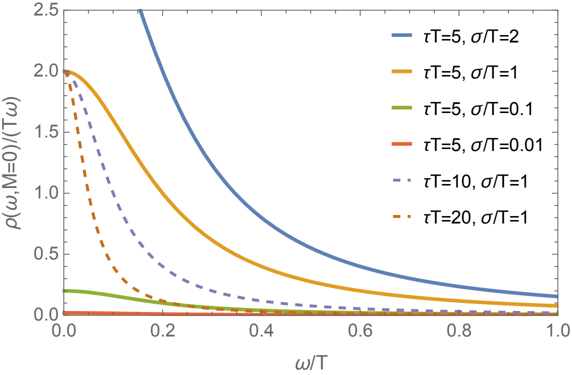
[figure compiled by Greif et al. (2014)]

Electric current spectral function

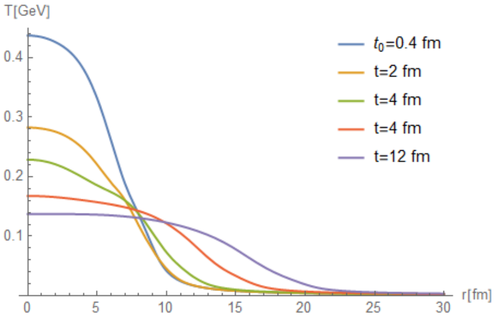
- from equations of motion we find the spectral function

$$\rho(\omega, \mathbf{p}) = \frac{\sigma\omega(\omega^2 - \mathbf{p}^2)}{(\tau\omega^2 - D\mathbf{p}^2)^2 + \omega^2} + 2\frac{\sigma\omega}{\tau^2\omega^2 + 1}.$$

- height of peak proportional to conductivity
- decay governed by width $\sim 1/\tau$

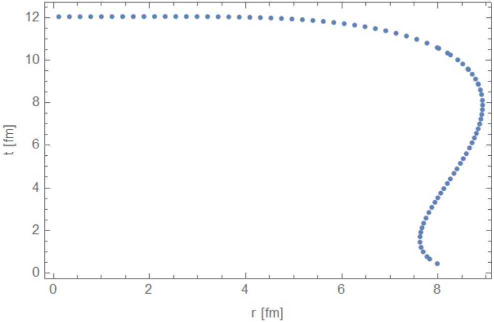


Fluid dynamics



- integrate over the QGP fire ball using $T(r,t)$ and $u(r,t)$ from FluiduM [Floerchinger et al. 2019]
- Pb-Pb-collisions at $\sqrt{s_{\rm NN}} = 5.02$ TeV
- centrality class 0-5%

Freeze-out surface



- kinetic freeze out surface: hypersurface after which particle momenta don't change any more
- integrate photon and dielectorn production rate up to this freeze-out surface
- electromagnetic currents freeze in, no radiation afterwards
- take here $T_{\text{fo}} = 140$ MeV

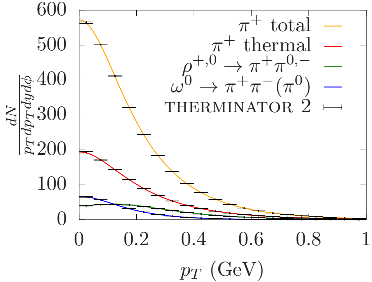
Decay contributions

- Calculate also photons from resonance decays with FastReso
  [Mazeliauskas, Floerchinger, Grossi, Teaney, EPJC 79, 284 (2019)]
- Cooper-Frye with resonance decays

$$E_p \frac{dN_a}{d^3p} = -\frac{1}{(2\pi)^3}\int d\Sigma_\mu\, g_a^\mu(x,p), \qquad g_b^\mu(x,p) = \int_q D_b^a(p,q) f_a(x,q) q^\mu$$

- decay map relates spectra before and after resonance decays

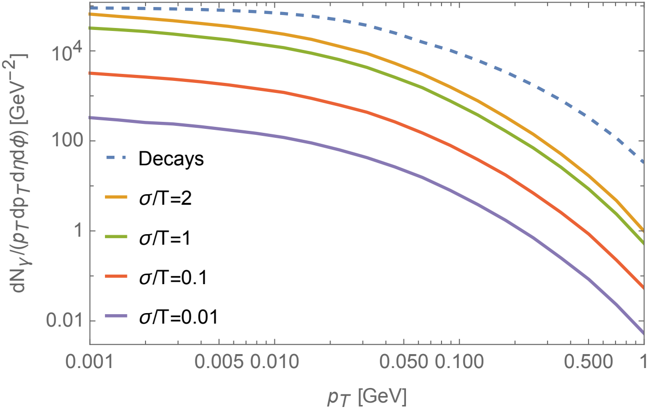
$$E_p \frac{dN_b}{d^3p} = \int_q D_b^a(p,q)\; E_q \frac{dN_a}{d^3q}$$



- dielectron from resonances calculated with PYTHIA

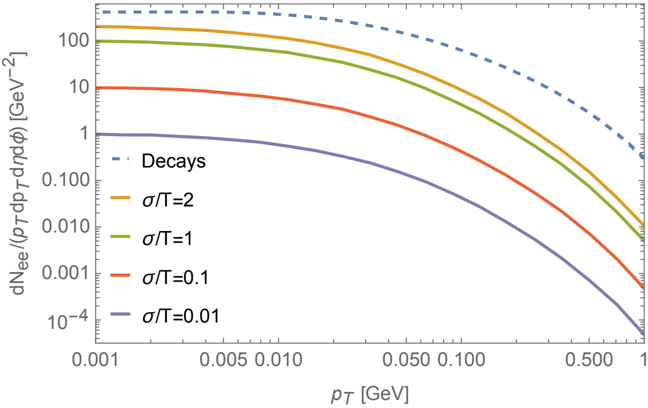
Photon spectrum

- Transverse momentum spectrum of photons
- Four possible choices for electic conductivity to temperature ratio
- Photons from hadronic resonance decays also shown



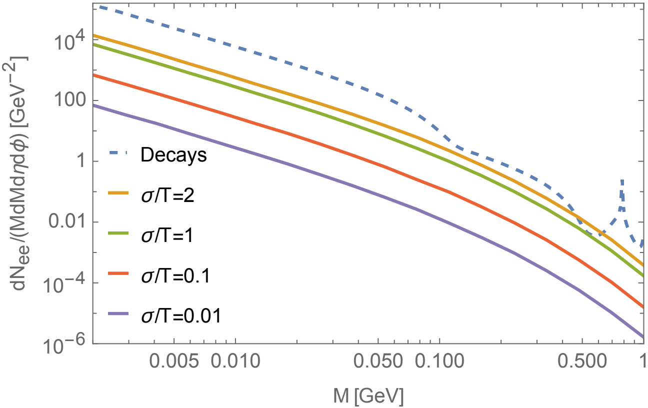
Dielectron spectrum

- Transverse momentum spectrum of electron-positon pairs
- Four possible choices for electic conductivity to temperature ratio
- Dielectrons from hadronic resonance decays also shown



Dielectron mass spectrum

- Invariant mass spectrum of electron-positon pairs
- Four possible choices for electic conductivity to temperature ratio
- Dielectrons from hadronic resonance decays also shown



How to deal with resonance decays?

- for dielectrons it helps to accept only pairs at $M > 100$ MeV to reduce the decay background
- for photons one could use Hanbury-Brown-Twiss interferometry to disentangle contributions from resonance decays and thermal photons
- could one use Hanbury-Brown-Twiss methods also for dielectrons?

Conclusion

- Electric current spectral function at small frequencies and momenta determined by fluid dynamics
- Electric conductivity can be constrained experimentally
- Background from resonance decays must be subtracted (e. g. with Hanbury-Brown-Twiss method)