Soft-photon production in proton-proton collisions in the tensor-pomeron model

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based on arXiv: 2206.03411 [hep-ph]

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Plan:

- Introduction
- $pp \to pp$ and $pp \to pp\gamma$ in the tensor-pomeron model
- Soft Photon Approximation (SPA)
- Results
- Conclusions

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Real and virtual photon production at ultra-low transverse momentum and low mass at LHC
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1. Introduction

- The emission of soft photons, that is, photons of energy $\omega \to 0$, was treated in the seminal paper
  F.E. Low, "Bremsstrahlung of very low-energy quanta in elementary particle collisions", Phys. Rev. 110 (1958) 974

  There it was shown that the term of order ω^{-1} in the amplitude for the emission reaction can be obtained from the amplitude without photon emission. To this order the emission comes exclusively from the external particles. This is a strict consequence of QFT.

  Many soft-photon approximations (SPAs) are based on this result.

- Experimental studies trying to verify Low's theorem have, in many cases, found large deviations from the SPA calculations.

- More experimental (ALICE 3) and theoretical work is needed in order to clarify this "soft photon problem".

- We started our investigations of soft-photon radiation with the processes: $\pi^- \pi^0 \to \pi^- \pi^0 \gamma$, $\pi^- \pi^+ \to \pi^- \pi^+ \gamma$
  P.L. O. Nachtmann, A. Szczurek, PRD 105 (2022) 014022, arXiv:2107.10829
  We have discussed these reactions in the tensor-pomeron model. We have determined the kinematic regions where the SPAs are a good representation of our "standard" model result.
  [O. Nachtmann, "Photon emission in pion-pion scattering and Low's theorem revised", EMMI RRTF]
  in preparatory lectures

- Recently, we have considered soft-photon radiation in the reaction: $p\, p \to p\, p\, \gamma$
  P.L. O. Nachtmann, A. Szczurek, "Soft-photon radiation in high-energy proton-proton collisions within the tensor-pomeron approach: Bremsstrahlung", arXiv: 2206.03411, in print in PRD

1. Introduction

- *P.L. O. Nachtmann, A. Szczurek, PRD 105 (2022) 014022, arXiv:2107.10829*
  Considerations concerning the amplitude for the reaction $\pi^- \pi^0 \to \pi^- \pi^0 \gamma$ in the soft-photon limit, $\omega \to 0$.

  Using only rigorous QFT methods (no model dependence is contained there) we have calculated the terms of order ω^{-1} and ω^0 in the expansion of the radiative amplitude.
  Our term of order ω^0 disagrees with that given by Low. We have analyzed this important discrepancy.
  → Low's result corresponds to the expansion of the photon emission amplitude
  of the fictitious process $\pi^- \pi^0 \to \pi^- \pi^0 \gamma$ where energy-momentum conservation is not respected

- From the theory side, we have a good model for the basic $\pi\pi \to \pi\pi$ process.
  This allowed us to construct standard amplitude for $\pi\pi \to \pi\pi\gamma$ (without anomalous terms).
  The terms ω^{-1} and ω^0 in the expansion of standard amplitude are strict results from QFT without approximations, given the on-shell $\pi\pi \to \pi\pi$ amplitudes.

  Suppose now that we have experimental measurement of photon energies ω.

  If QFT describes experiment we must have for the ratio $R_{\rm exp}(\omega) = \dfrac{d\sigma_{\rm exp}/d\omega}{d\sigma_{\rm standard}/d\omega}$

$$\lim_{\omega \to 0} R_{\rm exp}(\omega) = 1\,, \quad \lim_{\omega \to 0} \frac{dR_{\rm exp}(\omega)}{d\omega} = 0\,.$$

A violation of these relations would mean a terrible crisis for QFT!
For higher ω a value $R_{\rm exp}(\omega) \neq 1$ would mean that there are soft photons from "anomalous" terms present in experiment.

Proton-proton scattering in the tensor-pomeron approach

We consider the reaction $p(p_a) + p(p_b) \to p(p_1) + p(p_2)$

at high energies and small momentum transfer $\sqrt{s} \gg m_p, \quad \sqrt{|t|} \lesssim m_p.$

This is the kinematic region where the amplitudes are governed by the Regge exchanges.

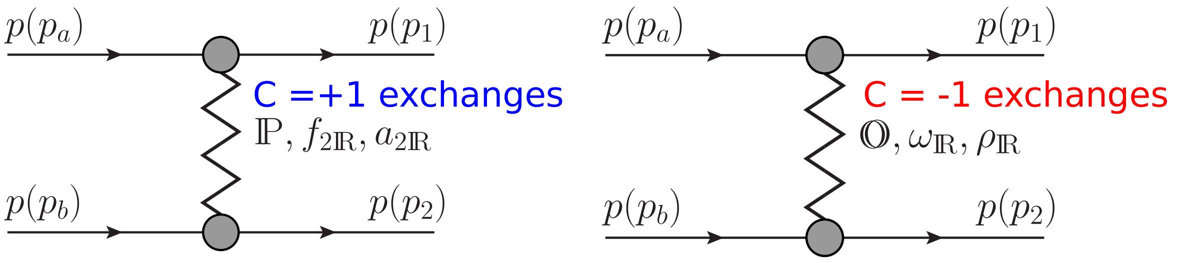
We use the model developed in C. Ewerz, M. Maniatis, O. Nachtmann, Ann. Phys. 342 (2014) 31, "A model for soft high-energy scattering: tensor pomeron and vector odderon".
This model has a good basis from nonperturbative QCD considerations
[O. Nachtmann, Ann. Phys. 209 (1991) 436].

We consider the usual Regge exchanges with charge conjugation C = +1 and C = -1:

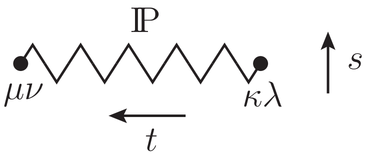
C =+1 pomeron (IP), f_2 and a_2 reggeons

C = -1 odderon (O), ω and ρ reggeons

We assume that all C = +1 exchange objects can be described as effective spin 2 symmetric tensor exchanges, all C = -1 exchanges as effective vector exchanges.



- Effective propagator for tensor-pomeron exchange

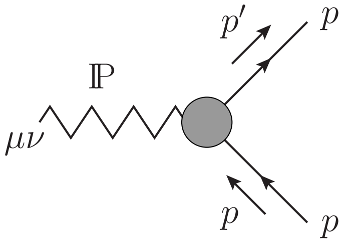


$$i\Delta^{(\mathbb{P})}_{\mu\nu,\kappa\lambda}(s,t) = \frac{1}{4s}\left(g_{\mu\kappa}g_{\nu\lambda} + g_{\mu\lambda}g_{\nu\kappa} - \frac{1}{2}g_{\mu\nu}g_{\kappa\lambda}\right)(-is\alpha'_{\mathbb{P}})^{\alpha_{\mathbb{P}}(t)-1}$$

$$\alpha_{\mathbb{P}}(t) = \alpha_{\mathbb{P}}(0) + \alpha'_{\mathbb{P}}t\,, \quad \alpha_{\mathbb{P}}(0) = 1 + \epsilon_{\mathbb{P}} = 1.0808, \quad \alpha'_{\mathbb{P}} = 0.25\,\mathrm{GeV}^{-2}$$

see e.g., A. Donnachie, H.G.Dosch, P.V.Landshoff, O.Nachtmann, “Pomeron Physics and QCD”, CUP, 2002

- Effective proton-pomeron vertex



$$i\Gamma^{(\mathbb{P}pp)}_{\mu\nu}(p',p) = -i3\beta_{\mathbb{P}pp}F_1[(p'-p)^2]\left\{\frac{1}{2}\left[\gamma_\mu(p'+p)_\nu + \gamma_\nu(p'+p)_\mu\right] - \frac{1}{4}g_{\mu\nu}(\not{p}'+\not{p})\right\}$$

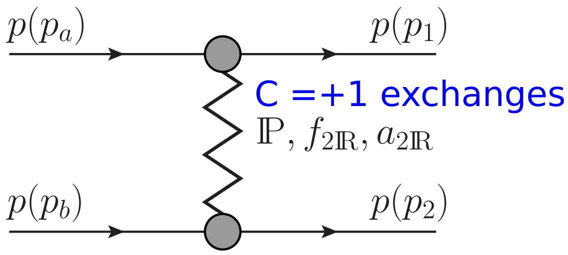
$$\beta_{\mathbb{P}pp} = 1.87\ \mathrm{GeV}^{-1}$$

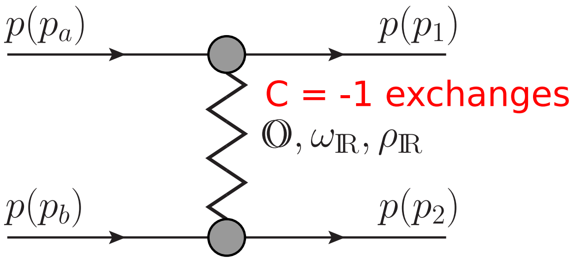
$$i\Gamma^{(\mathbb{P}\bar{p}\bar{p})}_{\mu\nu}(p',p) = i\Gamma^{(\mathbb{P}pp)}_{\mu\nu}(p',p)$$

We find from comparison to the TOTEM data:

- $\epsilon_{\mathbb{P}} = 0.0865$
- $F_1(t) \to F(t) = \exp(-b\,|t|)$ with $b = 2.95\ \mathrm{GeV}^{-2}$

- **Reaction $pp \to pp$**





We discuss the reaction

$$p(p_a, \lambda_a) + p(p_b, \lambda_b) \to p(p_1, \lambda_1) + p(p_2, \lambda_2)\,,$$

The momenta are indicated in brackets and $\lambda_a, \lambda_b, \lambda_1, \lambda_2 \in \{1/2, -1/2\}$ are the helicity indices of the protons.

The kinematic variables are

$$\begin{aligned} s &= (p_a + p_b)^2 = (p_1 + p_2)^2 \\ t &= (p_a - p_1)^2 = (p_b - p_2)^2 \\ u &= (p_a - p_2)^2 = (p_b - p_1)^2 \\ m_a^2 &= p_a^2\,, m_b^2 = p_b^2\,, m_1^2 = p_1^2\,, m_2^2 = p_2^2 \end{aligned}$$

The interchange $p_1 \leftrightarrow p_2$ implies $t \leftrightarrow u$ where $u = -s - t + m_a^2 + m_b^2 + m_1^2 + m_2^2$.

We are interested in the kinematic region

$$\sqrt{s} \gg m_p\,, \quad \sqrt{|t|} \lesssim m_p\,, \quad s \gg |m_a^2|, |m_b^2|, |m_1^2|, |m_2^2|.$$

There we can neglect the diagrams with $p_1 \leftrightarrow p_2$.

- **Off-shell** *pp* elastic scattering amplitude

We use tensor-product notation.
The first factors will always refer to the p_a-p_1 line, the second to the p_b-p_2 line.

$$\mathcal{M}^{(0)}(p_a,p_b,p_1,p_2) = \mathcal{M}^{(0)}_{\mathbb{P}} + \mathcal{M}^{(0)}_{f_{2\mathbb{R}}} + \mathcal{M}^{(0)}_{a_{2\mathbb{R}}} + \mathcal{M}^{(0)}_{\mathbb{O}} + \mathcal{M}^{(0)}_{\omega_{\mathbb{R}}} + \mathcal{M}^{(0)}_{\rho_{\mathbb{R}}}$$

$$= i\mathcal{F}_T(s,t)\left[\gamma^\mu \otimes \gamma_\mu (p_a+p_1, p_b+p_2) + (\not{p}_b + \not{p}_2) \otimes (\not{p}_a + \not{p}_1) - \frac{1}{2}(\not{p}_a + \not{p}_1) \otimes (\not{p}_b + \not{p}_2)\right] - \mathcal{F}_V(s,t)\,\gamma^\mu \otimes \gamma_\mu$$

where $\mathcal{F}_T(s,t) = \mathcal{F}_{\mathbb{P}pp}(s,t) + \mathcal{F}_{f_{2\mathbb{R}}pp}(s,t) + \mathcal{F}_{a_{2\mathbb{R}}pp}(s,t)$

$\mathcal{F}_V(s,t) = \mathcal{F}_{\mathbb{O}pp}(s,t) + \mathcal{F}_{\omega_{\mathbb{R}}pp}(s,t) + \mathcal{F}_{\rho_{\mathbb{R}}pp}(s,t)$

and $\mathcal{F}_{\mathbb{P}pp}(s,t) = \left[3\beta_{\mathbb{P}pp}F(t)\right]^2 \frac{1}{4s}(-is\alpha'_{\mathbb{P}})^{\alpha_{\mathbb{P}}(t)-1}$

- **On-shell** *pp* elastic scattering amplitude

$$\langle p(p_1,\lambda_1), p(p_2,\lambda_2)|\mathcal{T}|p(p_a,\lambda_a), p(p_b,\lambda_b)\rangle \equiv \mathcal{M}^{(\text{on shell})\,pp}(s,t)$$

$$= \bar{u}_1 \otimes \bar{u}_2 \mathcal{M}^{(0)}(p_a,p_b,p_1,p_2)\; u_a \otimes u_b\big|_{\text{on shell}}$$

$$= i\mathcal{F}_T(s,t)\left[\bar{u}_1\gamma^\mu u_a\, \bar{u}_2\gamma_\mu u_b\,(p_a+p_1, p_b+p_2) + \bar{u}_1\gamma^\mu u_a (p_b+p_2)_\mu\, \bar{u}_2\gamma^\nu u_b (p_a+p_1)_\nu - 2m_p^2\, \bar{u}_1 u_a\, \bar{u}_2 u_b\right]$$

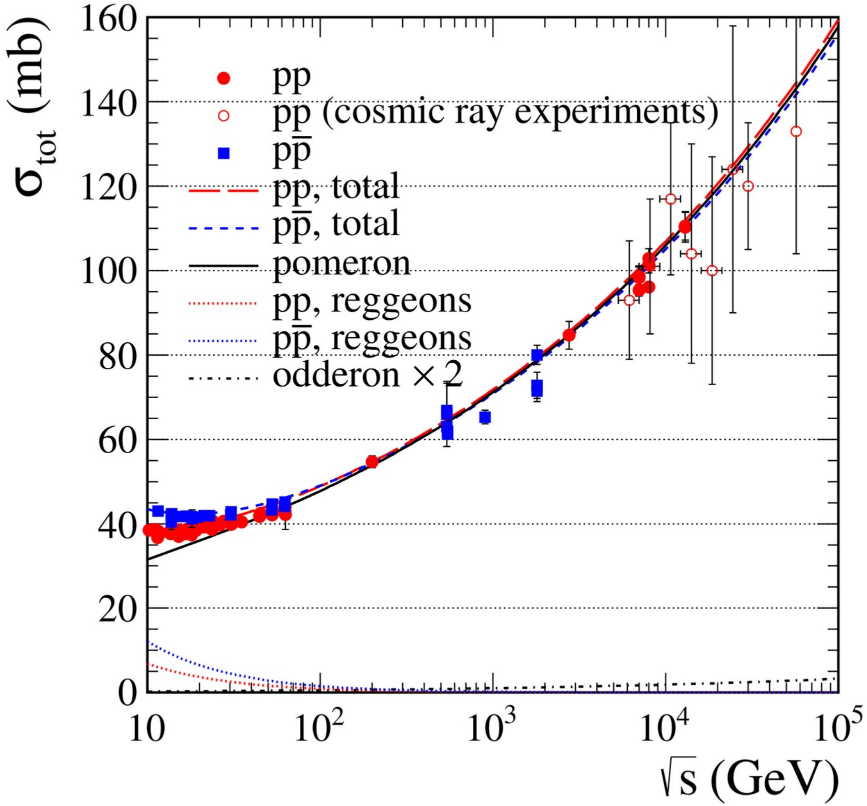
$$-\mathcal{F}_V(s,t)\, \bar{u}_1\gamma^\mu u_a\, \bar{u}_2\gamma_\mu u_b$$

where $\bar{u}_1 = \bar{u}(p_1,\lambda_1)$, $u_a = u(p_a,\lambda_a)$, etc.

- Comparison of the model with the total cross section data

The total cross section for unpolarised protons, obtained from the forward-scattering amplitudes using the optical theorem, is

$$\sigma_{\text{tot}}(pp) = \frac{1}{\sqrt{s(s-4m_p^2)}} \frac{1}{4} \sum_{\lambda_a,\lambda_b} \text{Im}\langle p(p_a,\lambda_a), p(p_b,\lambda_b)|\mathcal{T}|p(p_a,\lambda_a), p(p_b,\lambda_b)\rangle$$

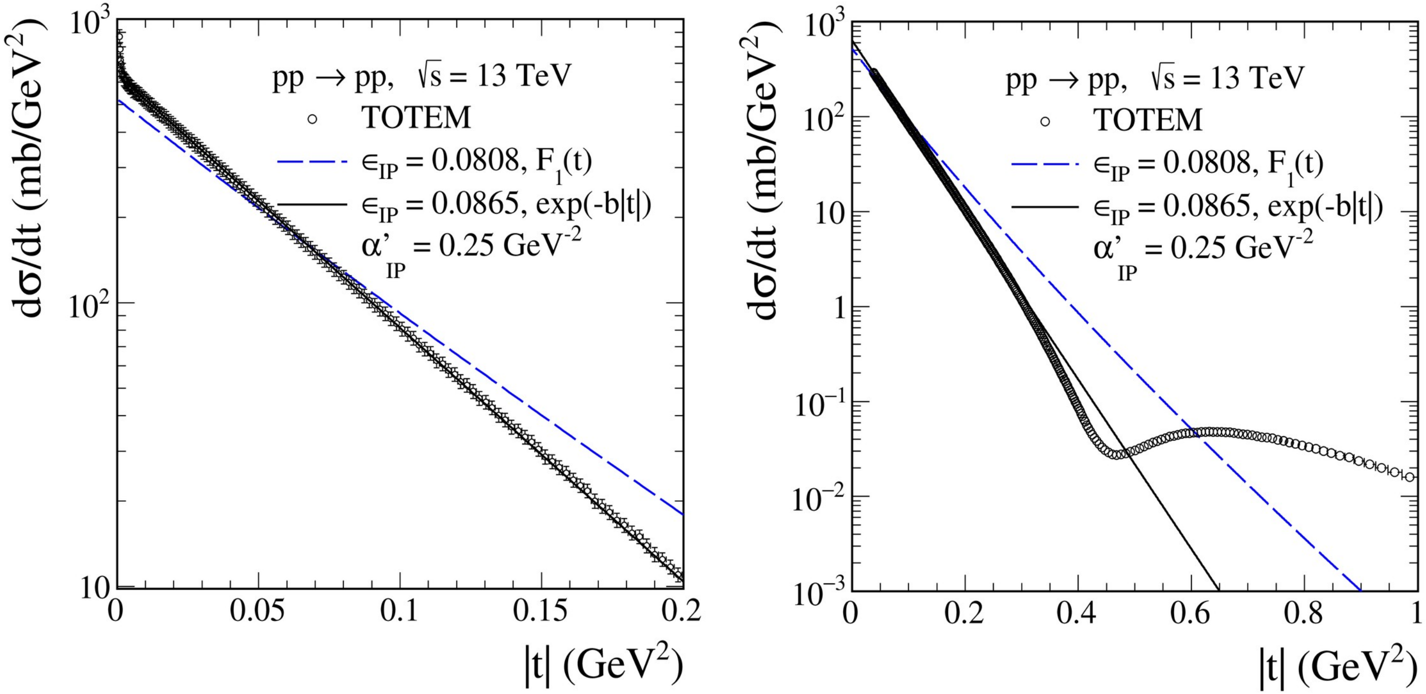


The high-energy cross section is dominated by the pomeron exchange.

The reegeon and odderon effects are very small. We get for large energies a total cross section for pp exceeding that for $p\bar{p}$ collisions, $\sigma_{\text{tot}}(pp) > \sigma_{\text{tot}}(p\bar{p})$.

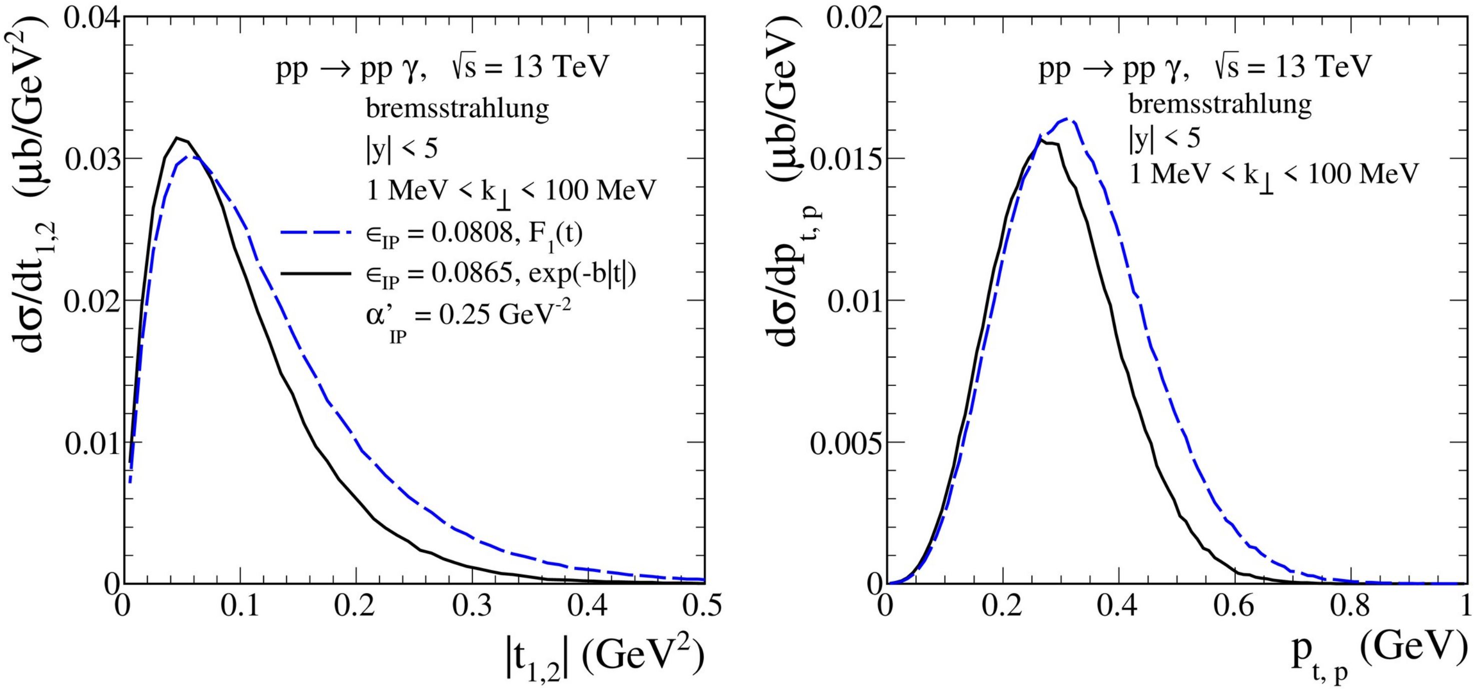
We only need a reasonable description of the data for $\sqrt{s} = 13$ TeV as a prerequisite for the calculation of photon radiation in pp collisions.

- Comparison of the model with elastic *pp* differential cross section data measured by TOTEM
  [G. Antchev et al. (TOTEM Collaboration), Eur. Phys. J. C79 (2019) 785, Eur. Phys. J. C79 (2019) 861]



- We find a good description of the data in the region $0.003\ \mathrm{GeV}^2 \leq -t \leq 0.26\ \mathrm{GeV}^2$ with our single-pomeron exchange model
- For comparison, the results for $\epsilon_{IP} = 0.0808$ and the Dirac form factor $F_1(t)$ are shown
- In order to poduce the dip one needs the interference of various terms in the amplitude, at least three terms: IP + IPIP + ggg [see, e.g., Donnachie and Landshoff, PLB 727 (2013) 500]

- Results for the ***pp* → *ppγ* reaction** (diffractive bremsstrahlung)



- The distributions in four-momentum transfer squared $|t_{1,2}|$ where $t_{1,2}$ is either t_1 or t_2 and in transverse momentum of the outgoing proton $p_{t,p}$ for the reaction $pp \to pp\gamma$

- We see that photons come predominantly from *pp* collisions with momentum transfers between the protons of order $p_{t,p} \sim \sqrt{|t_{1,2}|} \sim 0.3$ GeV

Proton-proton scattering with photon emission

We consider $p(p_a, \lambda_a) + p(p_b, \lambda_b) \to p(p'_1, \lambda_1) + p(p'_2, \lambda_2) + \gamma(k, \epsilon)$.
The momenta are denoted by $p_a, \ldots, k$, the helicities of the protons by $\lambda_a, \ldots, \lambda_2$, and ϵ is the polarisation vector of the photon.

The relevant $\mathcal{T}$-matrix element is

$$\langle p(p'_1, \lambda_1), p(p'_2, \lambda_2), \gamma(k, \epsilon)|\mathcal{T}|p(p_a, \lambda_a), p(p_b, \lambda_b)\rangle = (\epsilon^\mu)^* \mathcal{M}_\mu^{(\mathrm{total})}(p_a, \lambda_a; p_b, \lambda_b; p'_1, \lambda_1; p'_2, \lambda_2; k)\,.$$

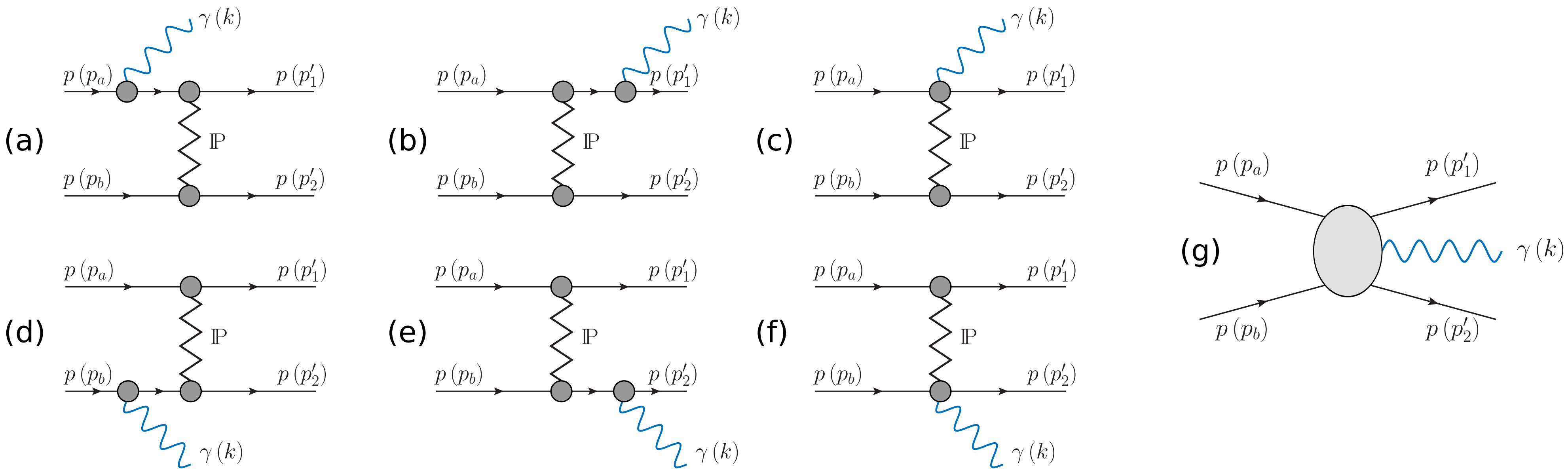
The complete amplitude is $\mathcal{M}_\mu^{(\mathrm{total})} = \mathcal{M}_\mu(p'_1, p'_2) - \mathcal{M}_\mu(p'_2, p'_1)$.
The relative minus sign here is due to the Fermi statistics, which requires the amplitude to be antisymmetric under interchange of the two final protons.
For diffractive scattering the amplitude $\mathcal{M}_\mu(p'_2, p'_1)$ can be neglected.

Therefore, we get with very accuracy, the inclusive cross section for the real-photon yield

$$\begin{aligned} d\sigma(pp \to pp\gamma) \quad = \quad & \frac{1}{2\sqrt{s(s-4m_p^2)}} \frac{d^3k}{(2\pi)^3\, 2k^0} \int \frac{d^3p'_1}{(2\pi)^3\, 2p'^0_1} \frac{d^3p'_2}{(2\pi)^3\, 2p'^0_2} (2\pi)^4 \delta^{(4)}(p'_1 + p'_2 + k - p_a - p_b) \\ & \times \frac{1}{4} \sum_{p\,\mathrm{spins}} \mathcal{M}_\mu(p'_1, p'_2) \big(\mathcal{M}_\nu(p'_1, p'_2)\big)^* (-g^{\mu\nu}) \end{aligned}$$

where $\sum_{\lambda_\gamma} (\epsilon^\mu(k, \lambda_\gamma))^* \epsilon^\nu(k, \lambda_\gamma) = -g^{\mu\nu}$

- Diagrams for the reaction *pp* → *ppγ* with exchange of the pomeron IP (a – f) and the “structure term” (g)



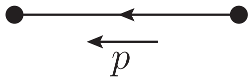
We have 7 types of diagrams. In the diagrams (a), (b), (d), and (e) the photon is emitted from the external proton lines. The diagrams (c) and (f) correspond to contact terms. We shall call the diagrams (a), (b), (d), (e), made gauge invariant by the addition of (c) and (f), the bremsstrahlung diagrams. All “anomalous” terms are subsumed in (g).

Our “standard” (diffractive photon-bremsstrahlung) amplitude is

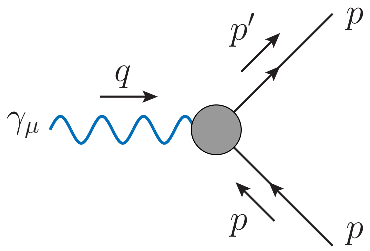
$$\mathcal{M}_\mu^{(\text{standard})} = \mathcal{M}_\mu^{(a)} + \mathcal{M}_\mu^{(b)} + \mathcal{M}_\mu^{(c)} + \mathcal{M}_\mu^{(d)} + \mathcal{M}_\mu^{(e)} + \mathcal{M}_\mu^{(f)}$$

The amplitude must satisfy the gauge-invariant relation $k^\mu \mathcal{M}_\mu^{(\text{standard})} = 0$

- We use the following standard proton propagator and γpp vertex:



$$iS_F(p) = \frac{i}{\not{p} - m_p + i\epsilon} = i\frac{\not{p} + m_p}{p^2 - m_p^2 + i\epsilon}$$



$q = p' - p$

$$i\Gamma_\mu^{(\gamma pp)}(p', p) = -ie\big[F_1(0)\gamma_\mu + \frac{i}{2m_p}\sigma_{\mu\nu}q^\nu F_2(0)\big]$$

$$F_1(0) = 1$$

$$F_2(0) = \big(\frac{\mu_p}{\mu_N} - 1\big), \quad \mu_N = \frac{e}{2m_p}, \quad \frac{\mu_p}{\mu_N} = 2.7928$$

$$e > 0, \; e = \sqrt{4\pi\alpha_{\rm em}}$$

We take the form factors at $q^2 = 0$ in order to be consistent with the Ward-Takahashi identity:

$$(p' - p)^\mu \Gamma_\mu^{(\gamma pp)}(p', p) = -e\big[S_F^{-1}(p') - S_F^{-1}(p)\big]$$

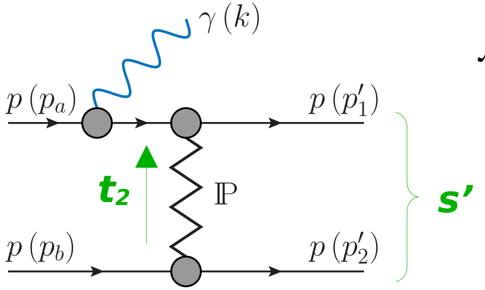
We are interested in real photon emission where $k = -q$, $k^2 = q^2 = 0$.

- The kinematic variables for the $pp \to pp\gamma$ reaction are:

$$
\begin{aligned}
s &= (p_a + p_b)^2 = (p_1' + p_2' + k)^2 \\
s' &= (p_a + p_b - k)^2 = (p_1' + p_2')^2 \\
t_1 &= (p_a - p_1')^2 = (p_b - p_2' - k)^2 \\
t_2 &= (p_b - p_2')^2 = (p_a - p_1' - k)^2
\end{aligned}
$$

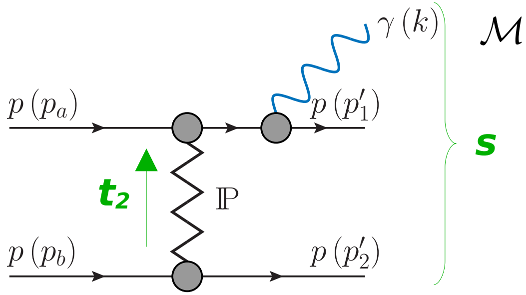
- We get with the off-shell scattering amplitudes for diagrams (a) and (b):

(a)



$$
\begin{aligned}
\mathcal{M}_\mu^{(a)} &= -\bar{u}_{1'} \otimes \bar{u}_{2'}\, \mathcal{M}^{(0)}(p_a - k, p_b, p_1', p_2')\, \big[S_F(p_a - k)\Gamma_\mu^{(\gamma pp)}(p_a - k, p_a)\, u_a\big] \otimes u_b \\
&= e\bar{u}_{1'} \otimes \bar{u}_{2'} \Big\{ i\mathcal{F}_T(s', t_2)\big[\gamma^\alpha \otimes \gamma_\alpha (p_a - k + p_1', p_b + p_2') + (\not{p}_b + \not{p}_2') \otimes (\not{p}_a - \not{k} + \not{p}_1') \\
&\quad - \frac{1}{2}(\not{p}_a - \not{k} + \not{p}_1') \otimes (\not{p}_b + \not{p}_2')\big] - \mathcal{F}_V(s', t_2)\, \gamma^\alpha \otimes \gamma_\alpha \Big\} \\
&\quad \times \Big[\frac{\not{p}_a - \not{k} + m_p}{(p_a - k)^2 - m_p^2 + i\varepsilon}\Big(\gamma_\mu - \frac{i}{2m_p}\sigma_{\mu\nu}k^\nu F_2(0)\Big) u_a\Big] \otimes u_b
\end{aligned}
$$

(b)



$$
\begin{aligned}
\mathcal{M}_\mu^{(b)} &= -\big[\bar{u}_{1'}\Gamma_\mu^{(\gamma pp)}(p_1', p_1' + k) S_F(p_1' + k)\big] \otimes \bar{u}_{2'}\, \mathcal{M}^{(0)}(p_a, p_b, p_1' + k, p_2')\, u_a \otimes u_b \\
&= e\Big[\bar{u}_{1'}\Big(\gamma_\mu - \frac{i}{2m_p}\sigma_{\mu\nu}k^\nu F_2(0)\Big)\frac{\not{p}_1' + \not{k} + m_p}{(p_1' + k)^2 - m_p^2 + i\varepsilon}\Big] \otimes \bar{u}_{2'} \\
&\quad \times \Big\{ i\mathcal{F}_T(s, t_2)\big[\gamma^\alpha \otimes \gamma_\alpha (p_a + p_1' + k, p_b + p_2') + (\not{p}_b + \not{p}_2') \otimes (\not{p}_a + \not{p}_1' + \not{k}) \\
&\quad - \frac{1}{2}(\not{p}_a + \not{p}_1' + \not{k}) \otimes (\not{p}_b + \not{p}_2')\big] - \mathcal{F}_V(s, t_2)\, \gamma^\alpha \otimes \gamma_\alpha \Big\} u_a \otimes u_b
\end{aligned}
$$

Using the Ward-Takahashi identity we find

$$
\begin{aligned}
k^\mu \mathcal{M}_\mu^{(a)} &= -e\bar{u}_{1'} \otimes \bar{u}_{2'}\, \mathcal{M}^{(0)}(p_a - k, p_b, p_1', p_2')\, u_a \otimes u_b\,, \\
k^\mu \mathcal{M}_\mu^{(b)} &= e\bar{u}_{1'} \otimes \bar{u}_{2'}\, \mathcal{M}^{(0)}(p_a, p_b, p_1' + k, p_2')\, u_a \otimes u_b\,.
\end{aligned}
$$

Now we impose the gauge invariance condition which must hold also for the photon emission from the p_a-p_1' lines in (a – c) diagrams alone:

$$
k^\mu \big(\mathcal{M}_\mu^{(a)} + \mathcal{M}_\mu^{(b)} + \mathcal{M}_\mu^{(c)}\big) = 0\,.
$$

We obtain then:

$$
\begin{aligned}
k^\mu \mathcal{M}_\mu^{(c)} &= -k^\mu \mathcal{M}_\mu^{(a)} - k^\mu \mathcal{M}_\mu^{(b)} \\
&= e\bar{u}_{1'} \otimes \bar{u}_{2'} \big[\mathcal{M}^{(0)}(p_a - k, p_b, p_1', p_2') - \mathcal{M}^{(0)}(p_a, p_b, p_1' + k, p_2')\big] u_a \otimes u_b
\end{aligned}
$$

We get

$$
\begin{aligned}
\mathcal{M}_\mu^{(c)} = e\bar{u}_{1'} \otimes \bar{u}_{2'} \Big\{ & -i\mathcal{F}_T(s,t_2)\big[2\gamma^\alpha \otimes \gamma_\alpha (p_b + p_2')_\mu + 2(\not{p}_b + \not{p}_2') \otimes \gamma_\mu - \gamma_\mu \otimes (\not{p}_b + \not{p}_2')\big] \\
& + i\frac{(2p_a + 2p_b - k)_\mu}{s} \Delta\mathcal{F}_T(s,t_2,\varkappa)\big[\gamma^\alpha \otimes \gamma_\alpha (p_a + p_1' - k, p_b + p_2') + (\not{p}_b + \not{p}_2') \otimes (\not{p}_a + \not{p}_1' - \not{k}) - \frac{1}{2}(\not{p}_a + \not{p}_1' - \not{k}) \otimes (\not{p}_b + \not{p}_2')\big] \\
& - \frac{(2p_a + 2p_b - k)_\mu}{s} \Delta\mathcal{F}_V(s,t_2,\varkappa)\,\gamma^\alpha \otimes \gamma_\alpha \Big\} u_a \otimes u_b\,.
\end{aligned}
$$

We rewrite the amplitude $\mathcal{M}_\mu^{(a+b+c)} = \mathcal{M}_\mu^{(a)} + \mathcal{M}_\mu^{(b)} + \mathcal{M}_\mu^{(c)}$, in a way which is more suitable for numerical computations.

We use the following relations:

$$\frac{\not{p}_a - \not{k} + m_p}{(p_a - k)^2 - m_p^2 + i\varepsilon}\big(\gamma_\mu - \frac{i}{2m_p}\sigma_{\mu\nu}k^\nu F_2(0)\big)u_a$$
$$= \frac{1}{-2p_a\cdot k + k^2 + i\varepsilon}\big\{2p_{a\mu} - k_\mu + (k_\mu - \not{k}\gamma_\mu) + \frac{F_2(0)}{2m_p}\big[2(p_{a\mu}\not{k} - (p_a\cdot k)\gamma_\mu) + 2m_p(k_\mu - \not{k}\gamma_\mu) - (\not{k}k_\mu - k^2\gamma_\mu)\big]\big\}u_a$$

$$\bar{u}_{1'}\big(\gamma_\mu - \frac{i}{2m_p}\sigma_{\mu\nu}k^\nu F_2(0)\big)\frac{\not{p}'_1 + \not{k} + m_p}{(p'_1 + k)^2 - m_p^2 + i\varepsilon}$$
$$= \bar{u}_{1'}\frac{1}{2p'_1\cdot k + k^2 + i\varepsilon}\big\{2p'_{1\mu} + k_\mu - (k_\mu - \gamma_\mu\not{k}) + \frac{F_2(0)}{2m_p}\big[-2(p'_{1\mu}\not{k} - (p'_1\cdot k)\gamma_\mu) - 2m_p(k_\mu - \gamma_\mu\not{k}) - (k_\mu\not{k} - k^2\gamma_\mu)\big]\big\}$$

Exploiting the properties of the Dirac spinors, $\not{p}_a u_a = m_p u_a$, $\bar{u}_{1'}\not{p}_{1'} = \bar{u}_{1'} m_p$ etc., we can write

$$\mathcal{M}_\mu^{(\text{standard})} = \sum_{j=1}^{7}\big(\mathcal{M}_{\mathrm{T},\mu}^{(a+b+c)\,j} + \mathcal{M}_{\mathrm{T},\mu}^{(d+e+f)\,j}\big) + \sum_{j'=1}^{4}\big(\mathcal{M}_{\mathrm{V},\mu}^{(a+b+c)\,j'} + \mathcal{M}_{\mathrm{V},\mu}^{(d+e+f)\,j'}\big)\,.$$

Here T and V stand for the tensor- and vector-exchange diagrams, respectively, and j and j' are just labels for the subamplitudes in the sums.

For $j = 1, 2, 4$ we have

$$\begin{aligned}
\mathcal{M}^{(a+b+c)\,1}_{\mathrm{T},\mu} &= e\bar{u}_{1'} \otimes \bar{u}_{2'} \big\{ i\mathcal{F}_T(s,t_2) \big[\gamma^\alpha \otimes \gamma_\alpha (p_a + p_1', p_b + p_2') + (\not{p}_b + \not{p}_2') \otimes (\not{p}_a + \not{p}_1') - 2m_p^2 1 \otimes 1 \big] \\
&\quad \times \big[\frac{2p_{a\mu} - k_\mu}{-2p_a \cdot k + k^2 + i\varepsilon} + \frac{2p'_{1\mu} + k_\mu}{2p_1' \cdot k + k^2 + i\varepsilon} \big] \big\} u_a \otimes u_b \\
\mathcal{M}^{(a+b+c)\,2}_{\mathrm{T},\mu} &= e\bar{u}_{1'} \otimes \bar{u}_{2'} \big\{ i\mathcal{F}_T(s',t_2) \frac{1}{-2p_a \cdot k + k^2 + i\varepsilon} \\
&\quad \times \big[\gamma^\alpha \otimes \gamma_\alpha (p_a + p_1' - k, p_b + p_2') + (\not{p}_b + \not{p}_2') \otimes (\not{p}_a + \not{p}_1' - \not{k}) \big] \\
&\quad \times \big[k_\mu - \not{k}\gamma_\mu + \frac{F_2(0)}{2m_p} \big(2p_{a\mu} \not{k} - 2(p_a \cdot k)\gamma_\mu + 2m_p (k_\mu - \not{k}\gamma_\mu) - (\not{k} k_\mu - k^2 \gamma_\mu) \big) \big] \otimes 1 \big\} u_a \otimes u_b \\
\mathcal{M}^{(a+b+c)\,4}_{\mathrm{T},\mu} &= e\bar{u}_{1'} \otimes \bar{u}_{2'} \big\{ i\mathcal{F}_T(s,t_2) \frac{1}{2p_1' \cdot k + k^2 + i\varepsilon} \\
&\quad \times \big[-(k_\mu - \gamma_\mu \not{k}) + \frac{F_2(0)}{2m_p} \big(-2p'_{1\mu} \not{k} + 2(p_1' \cdot k)\gamma_\mu - 2m_p (k_\mu - \gamma_\mu \not{k}) - (k_\mu \not{k} - k^2 \gamma_\mu) \big) \big] \otimes 1 \\
&\quad \times \big[\gamma^\alpha \otimes \gamma_\alpha (p_a + p_1' + k, p_b + p_2') + (\not{p}_b + \not{p}_2') \otimes (\not{p}_a + \not{p}_1' + \not{k}) \big] \big\} u_a \otimes u_b
\end{aligned}$$

We have

$$\mathcal{M}^{(d+e+f)\,j}_{\mathrm{T},\mu} = \mathcal{M}^{(a+b+c)\,j}_{\mathrm{T},\mu} \Big|_{\substack{(p_a,\,\lambda_a) \leftrightarrow (p_b,\,\lambda_b) \\ (p_1',\,\lambda_1) \leftrightarrow (p_2',\,\lambda_2)}} \quad \text{for } j = 1, \ldots, 7$$

where we also exchange the order of the tensor products.
All subamplitudes are separately gauge invariant: $k^\mu \mathcal{M}^{(a+b+c)\,j}_{\mathrm{T},\mu} = 0$.

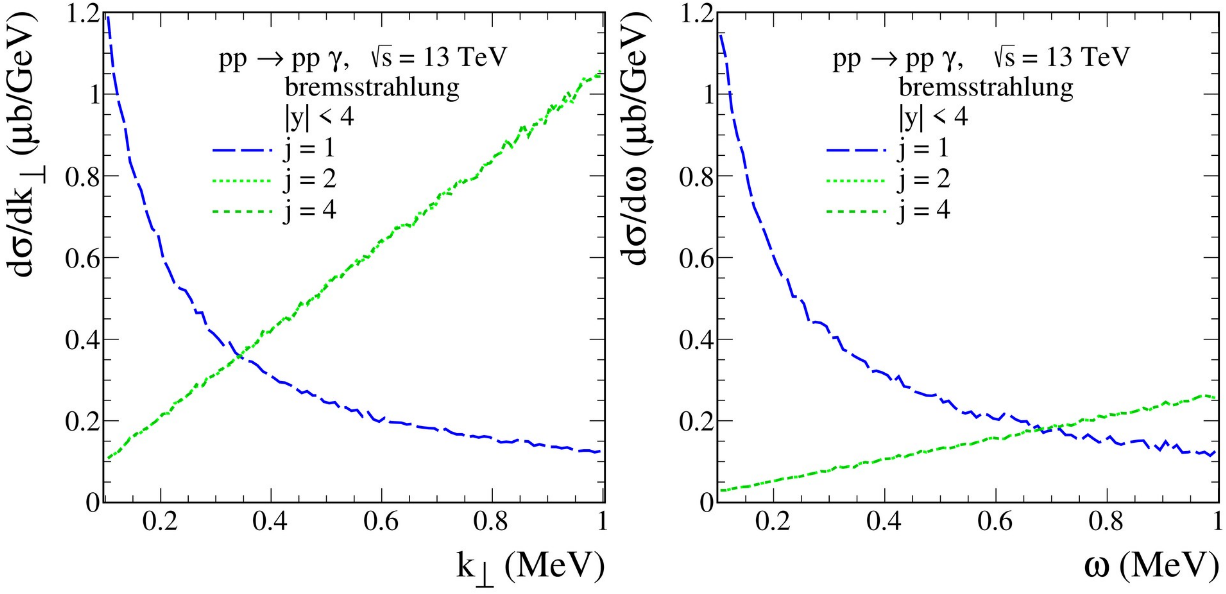
- The term j = 1 has singularity for ω → 0.

- The terms j = 2 and 4 have no singularity for ω → 0.
The main term here comes from the anomalous magnetic moment $F_2(0)$.

- Thus, the term j = 2 will win over the 2 and 4 terms individually for ω → 0.

We find that the pole term (j = 1) only dominates over these non-singular terms individually for very small $k_\perp$:

$$k_\perp \approx \omega \lesssim 2m_p^2/\sqrt{s} \cong 0.15 \text{ MeV}$$



Explicit calculations confirm the order of magnitude of this estimate.

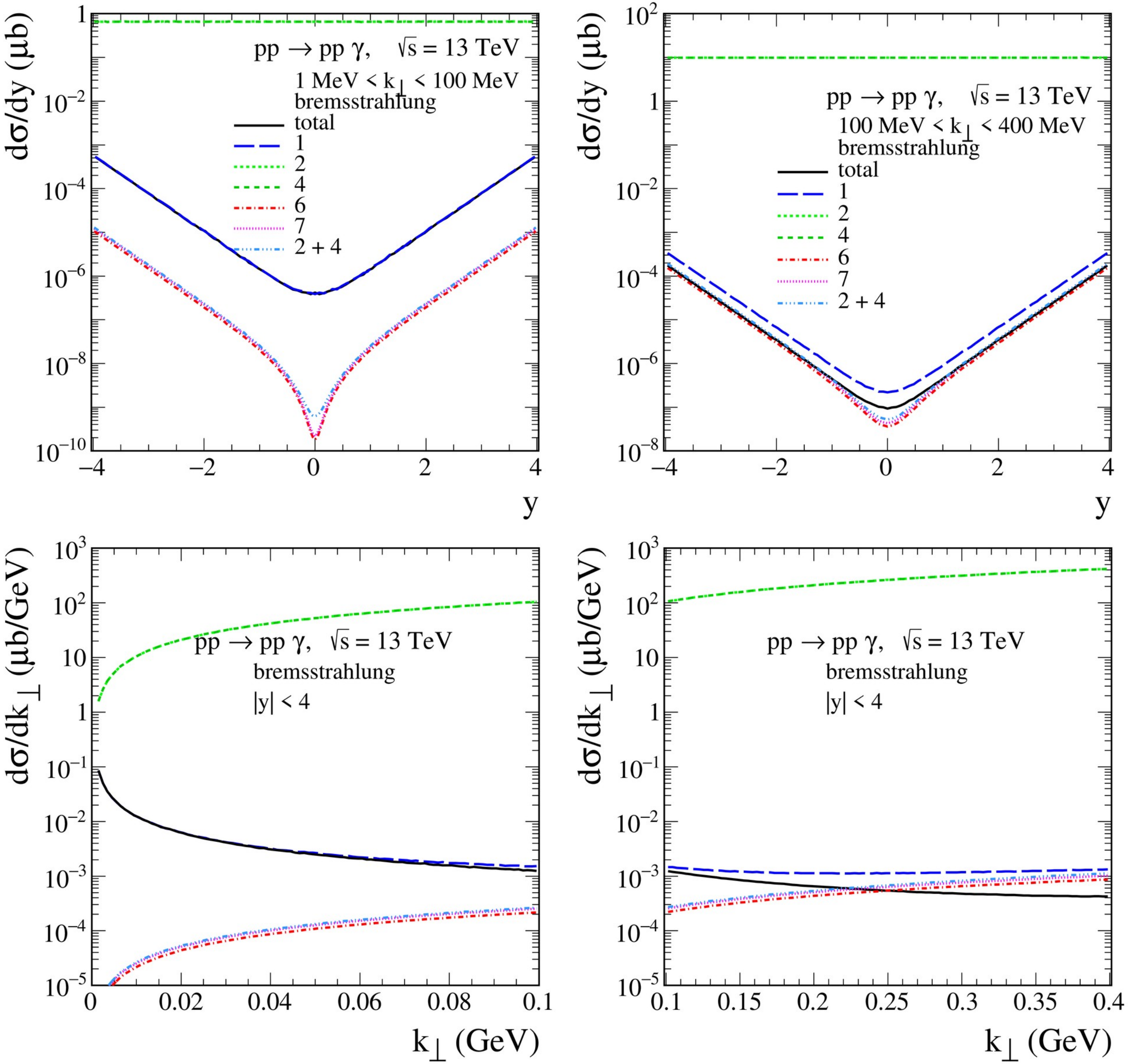
- In the literature such small values for ω as a limit for the dominance of the ω^{-1} term are mentioned:
In [V. Del Duca, High-energy bremsstrahlung theorems for soft photons, Nucl.Phys.B 345 (1990) 369]
it is argued that for hard high-energy elastic processes Low's orginal result gives a reliable representation of the radiative amplitude only in the vanishingly small region $\omega \lesssim m^2/Q$ in the limit $Q \to \infty$.
Here Q is the scale of the hard process and m is the charged particle mass.
But since there only hard processes with photon emission are considered these arguments do not apply to our case. We consider exclusive soft process $pp \to pp\gamma$ with soft photon emission.
We have, of course, to take all contributions with different labels j into account and add them coherently.

- We show the complete result (total) including interference effects and the results for individual j terms, except for j = 3 and 5 which are very small and can be neglected.
  The coherent sum of the amplitudes with j = 2 and 4 is denoted by 2 + 4.

- There is significant cancelletaion among the terms j = 2 and 4 due to destructive interference (not due to a gauge cancellation) and their sum is harmless, well below the term 1, at least for $k_\perp$ < 100 MeV and ω < 2 GeV.

  This leads to a much larger region in $k_\perp$ and ω where the pole term (j = 1) gives a good representation of the radiative amplitude.

- It is essential to add coherently all the various parts of the amplitude for soft photon emission in order not to miss important interference effects!



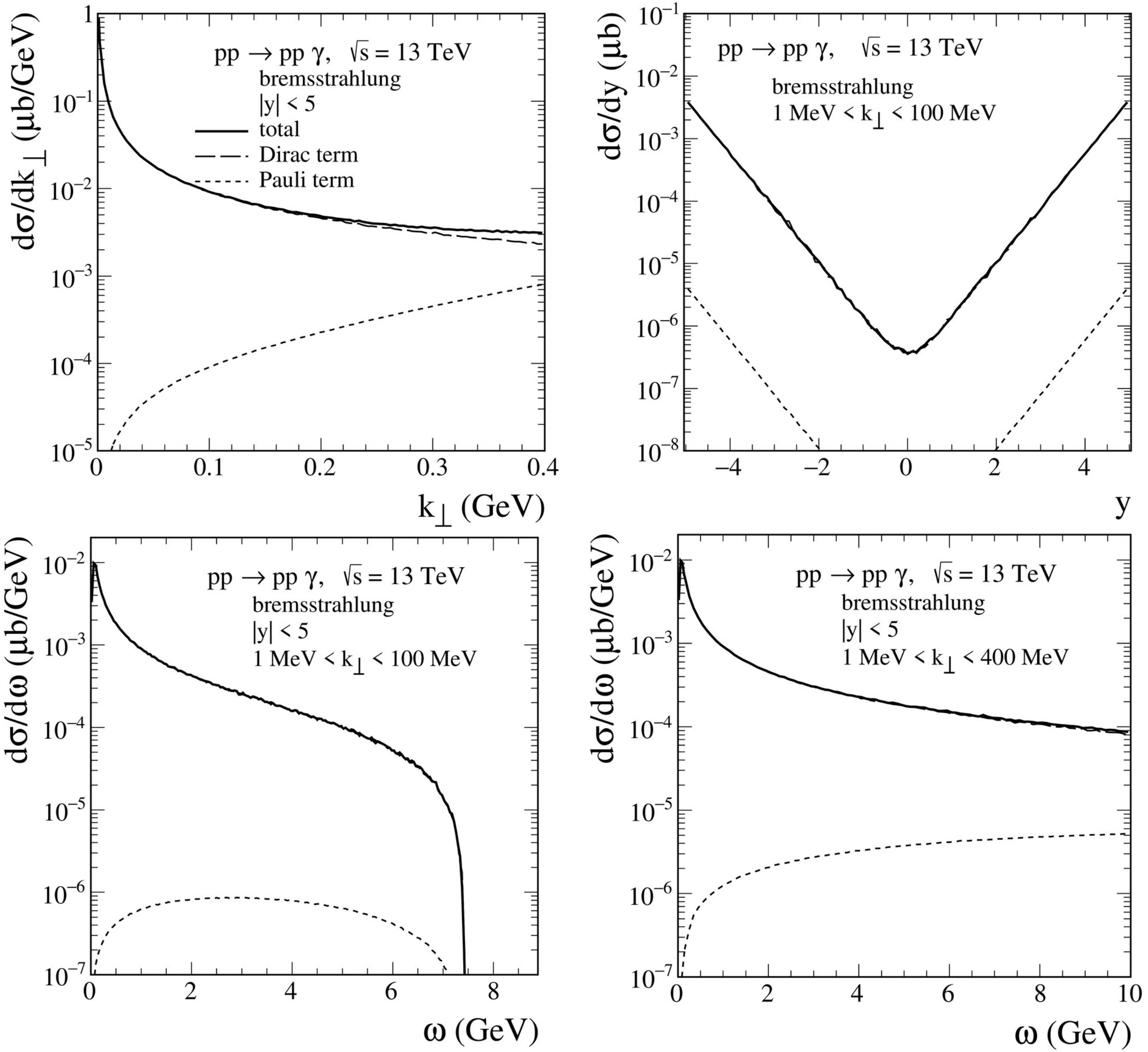
- In small $k_\perp$ and ω regions the Dirac term from *γpp* vertex function dominates while for larger values the anomalous magnetic moment of the proton (Pauli tem) plays an important role.

  For the complete result all contributions to

  $$\mathcal{M}_\mu^{(\mathrm{standard})}$$

  with Dirac and Pauli terms have to be added coherently.

- We get the integrated cross sections for $\sqrt{s} = 13$ TeV and in the $k_\perp$ range $1\,\mathrm{MeV} < k_\perp < 100\,\mathrm{MeV}$:
$\sigma = 0.21$ nb for $|\mathrm{y}| < 3.5$
and $\sigma = 4.01$ nb for $3.5 < |\mathrm{y}| < 5$.



Soft Photon Approximation (SPA)

We shall compare our “exact” model results, which we call “standard” to two SPAs.
We consider only the pomeron-exchange.

SPA 1 Here we keep only the pole terms $\propto \omega^{-1}$ for $\mathcal{M}_\mu^{(a)}, \ldots, \mathcal{M}_\mu^{(f)}$.
For real photons ($k^2 = 0$), neglecting gauge terms $\propto k_\mu$,
and with $p_1' \to p_1$, $p_2' \to p_2$ we get

$$\mathcal{M}_\mu \quad \to \quad \mathcal{M}_{\mu,\,\mathrm{SPA1}} = e\mathcal{M}^{(\mathrm{on\ shell})\,pp}(s,t)\Big[-\frac{p_{a\mu}}{(p_a\cdot k)} + \frac{p_{1\mu}}{(p_1\cdot k)} - \frac{p_{b\mu}}{(p_b\cdot k)} + \frac{p_{2\mu}}{(p_2\cdot k)}\Big]$$

In the soft photon limit, $\omega \to 0$, the SPA amplitude can be factorized into the hadron part $\mathcal{M}^{(0)}$ (on shell) and the photon emission part.

In the high-energy small-angle limit

$$\bar{u}(p_1,\lambda_1)\gamma^\mu u(p_a,\lambda_a) \cong (p_a + p_1)^\mu \delta_{\lambda_1\lambda_a}\,,$$
$$(p_a + p_1, p_b + p_2) \cong 2s\,,$$

we get $\mathcal{M}^{(\mathrm{on\ shell})\,pp}(s,t) \to i8s^2\mathcal{F}_{\mathbb{P}pp}(s,t)\delta_{\lambda_1\lambda_a}\delta_{\lambda_2\lambda_b}$, $\quad \mathcal{F}_{\mathbb{P}pp}(s,t) = \big[3\beta_{\mathbb{P}pp}F(t)\big]^2\dfrac{1}{4s}(-is\alpha'_{\mathbb{P}})^{\alpha_{\mathbb{P}}(t)-1}$

$$\widehat{\mathcal{M}}_{\mu,\,\mathrm{SPA1}} = ie8s^2\mathcal{F}_{\mathbb{P}pp}(s,t)\delta_{\lambda_1\lambda_a}\delta_{\lambda_2\lambda_b}\Big[-\frac{p_{a\mu}}{(p_a\cdot k)} + \frac{p_{1\mu}}{(p_1\cdot k)} - \frac{p_{b\mu}}{(p_b\cdot k)} + \frac{p_{2\mu}}{(p_2\cdot k)}\Big]$$

SPA 1

We get the following SPA1 result for the inclusive photon cross section where, for consistency, we neglect the photon momentum k in the energy-momentum conserving $\delta^{(4)}(.)$ function

$$
\begin{aligned}
d\sigma(pp \to pp\gamma)_{\,\mathrm{SPA1}} \quad = \quad & \frac{d^3k}{(2\pi)^3\, 2k^0} \int d^3p_1\, d^3p_2\, e^2\, \frac{d\sigma(pp \to pp)}{d^3p_1 d^3p_2} \\
& \times \Big[-\frac{p_{a\mu}}{(p_a \cdot k)} + \frac{p_{1\mu}}{(p_1 \cdot k)} - \frac{p_{b\mu}}{(p_b \cdot k)} + \frac{p_{2\mu}}{(p_2 \cdot k)} \Big] \\
& \times \Big[-\frac{p_{a\nu}}{(p_a \cdot k)} + \frac{p_{1\nu}}{(p_1 \cdot k)} - \frac{p_{b\nu}}{(p_b \cdot k)} + \frac{p_{2\nu}}{(p_2 \cdot k)} \Big] (-g^{\mu\nu})
\end{aligned}
$$

Here

$$
\begin{aligned}
\frac{d\sigma(pp \to pp)}{d^3p_1 d^3p_2} \quad = \quad & \frac{1}{2\sqrt{s(s-4m_p^2)}}\, \frac{1}{(2\pi)^3\, 2p_1^0\, (2\pi)^3\, 2p_2^0} (2\pi)^4 \delta^{(4)}(p_1 + p_2 - p_a - p_b) \\
& \times \frac{1}{4} \sum_{p\,\mathrm{spins}} |\mathcal{M}^{(\text{on shell})\, pp}(s,t)|^2
\end{aligned}
$$

SPA 2

Here we keep the correct energy-momentum conservation relation

$$p_a + p_b = p_1' + p_2' + k\,.$$

We consider again real photon emission,

$$\mathcal{M}_\mu \quad \to \quad \mathcal{M}_{\mu,\,\mathrm{SPA2}} = \mathcal{M}^{(a+b+c)\,1}_{\mathbb{P},\mu} + \mathcal{M}^{(d+e+f)\,1}_{\mathbb{P},\mu}\,.$$

The amplitudes $\mathcal{M}^{(a+b+c)\,1}_{\mathbb{P},\mu}$ and $\mathcal{M}^{(d+e+f)\,1}_{\mathbb{P},\mu}$ contain the pole terms $\propto \omega^{-1}$ for $\omega \to 0$ and are separately gauge invariant.

We examine, furthermore, the approximation

$$\begin{aligned}\widehat{\mathcal{M}}_{\mu,\,\mathrm{SPA2}} \quad &= \quad ie8s^2\mathcal{F}_{\mathbb{P}pp}(s,t_2)\delta_{\lambda_1\lambda_a}\delta_{\lambda_2\lambda_b}\Big[-\frac{p_{a\mu}}{(p_a\cdot k)} + \frac{p'_{1\mu}}{(p'_1\cdot k)}\Big] \\ &\quad +ie8s^2\mathcal{F}_{\mathbb{P}pp}(s,t_1)\delta_{\lambda_1\lambda_a}\delta_{\lambda_2\lambda_b}\Big[-\frac{p_{b\mu}}{(p_b\cdot k)} + \frac{p'_{2\mu}}{(p'_2\cdot k)}\Big]\end{aligned}$$

Comparison of our “exact” model or “standard” bremsstrahlung results with SPAs

- In the left panels we show two-dimensional differential cross sections in the ω-$k_\perp$ plane.

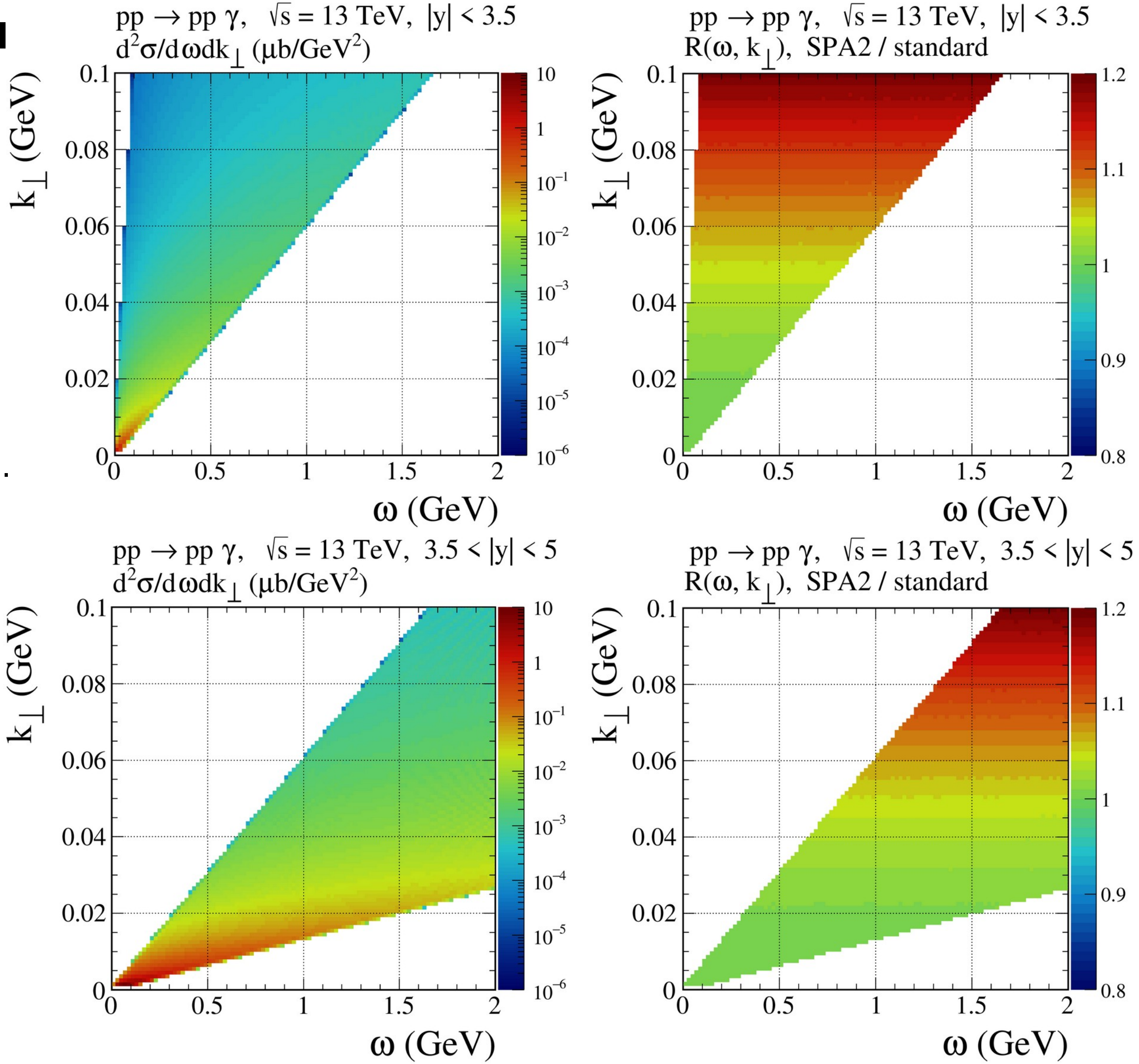
  Large y is near the ω axis and y = 0 on the $k_\perp$ axis in accordance with $\omega = k_\perp \cosh y$.

  The phase space region $\omega < k_\perp$ is forbidden.

- In the right panels we show the ratio

$$\mathrm{R}(\omega, k_\perp) = \frac{d^2\sigma_{\mathrm{SPA2}}/d\omega dk_\perp}{d^2\sigma_{\mathrm{standard}}/d\omega dk_\perp}$$

  One can see that SPA2 stays within 1% accuracy for $k_\perp$ < 22 MeV and ω < 0.35 GeV considering |y| < 3.5 and up to $\omega \simeq$ 1.7 GeV for 3.5 < |y| < 5.0.



Comparison of "standard" results to SPA 1 and SPA 2

- (top panels)
  Both SPAs follow the standard results very well.

  Surprisingly, the SPA 1 which does not have the correct energy-momentum relations fares somewhat better than SPA2.

  Where the $1/\omega$ term gives a reliable result?

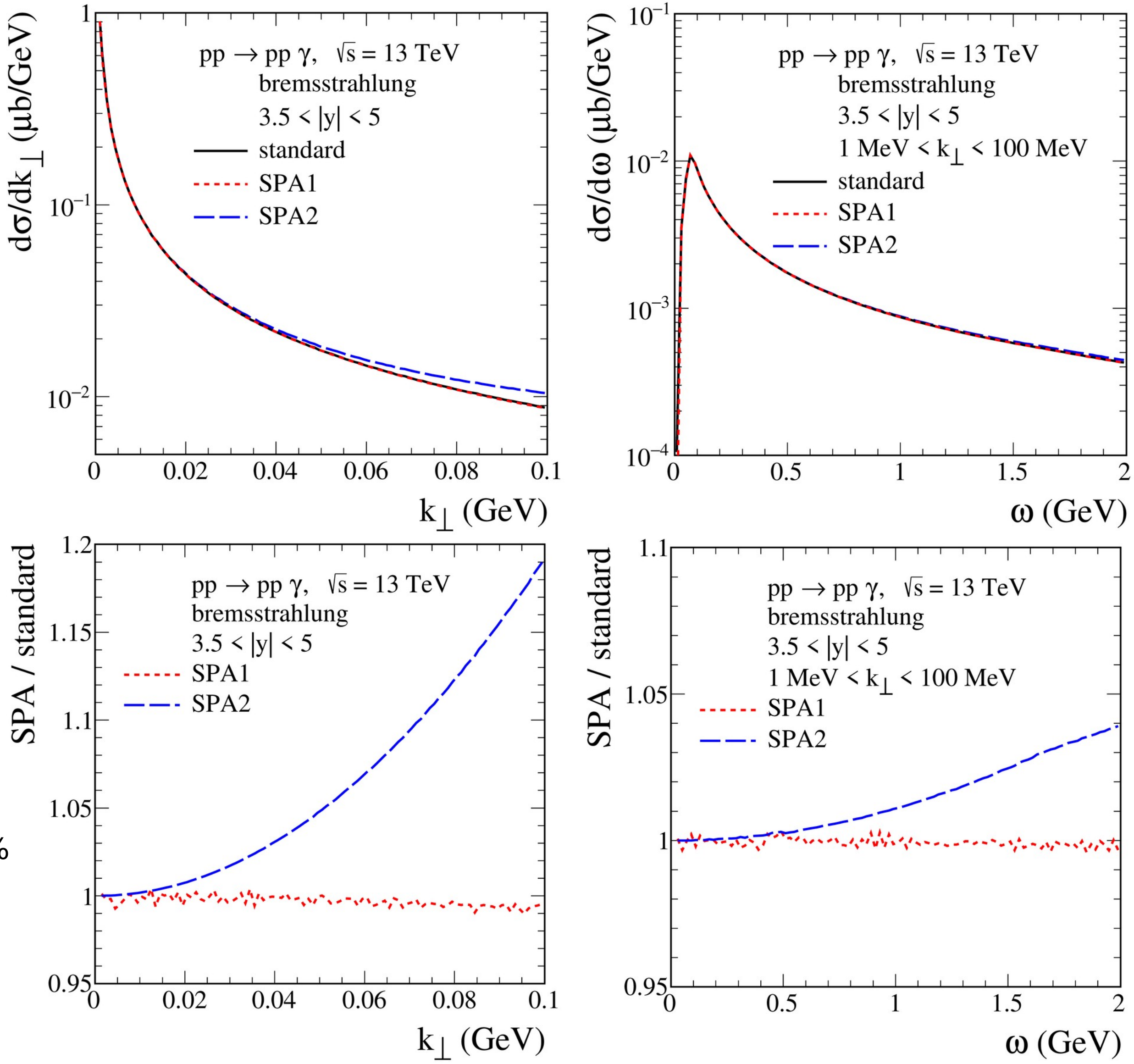
- (bottom panels)
  We show the ratios of the SPAs to the standard cross sections:

$$\frac{d\sigma_{\mathrm{SPA}}/dk_\perp}{d\sigma_{\mathrm{standard}}/dk_\perp} \quad \text{and} \quad \frac{d\sigma_{\mathrm{SPA}}/d\omega}{d\sigma_{\mathrm{standard}}/d\omega}$$

  as function of $k_\perp$ and ω, respectively.

  One can see that the deviations of the SPA1 from the standard results are up to around 1% in considered region.

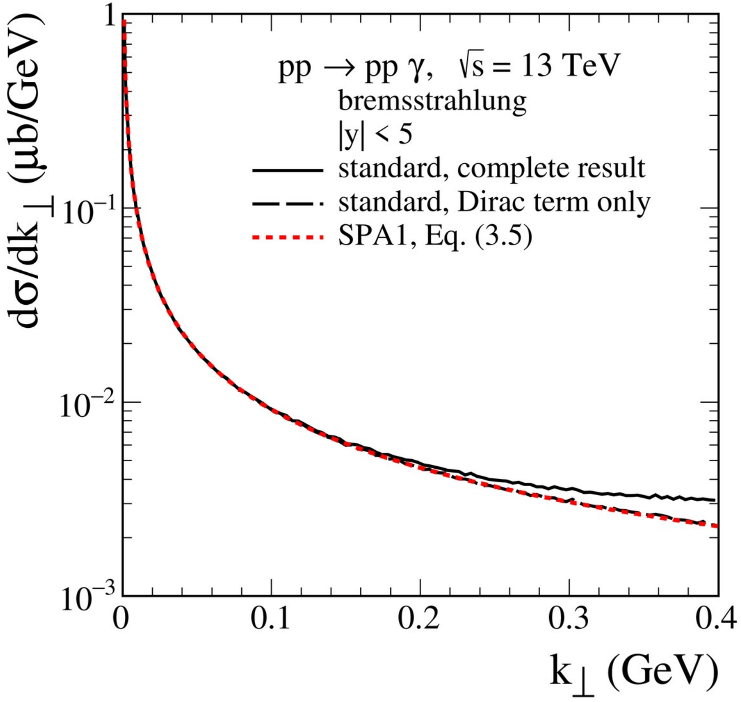
  For the SPA2 the deviations increase rapidly with growing $k_\perp$ and ω.

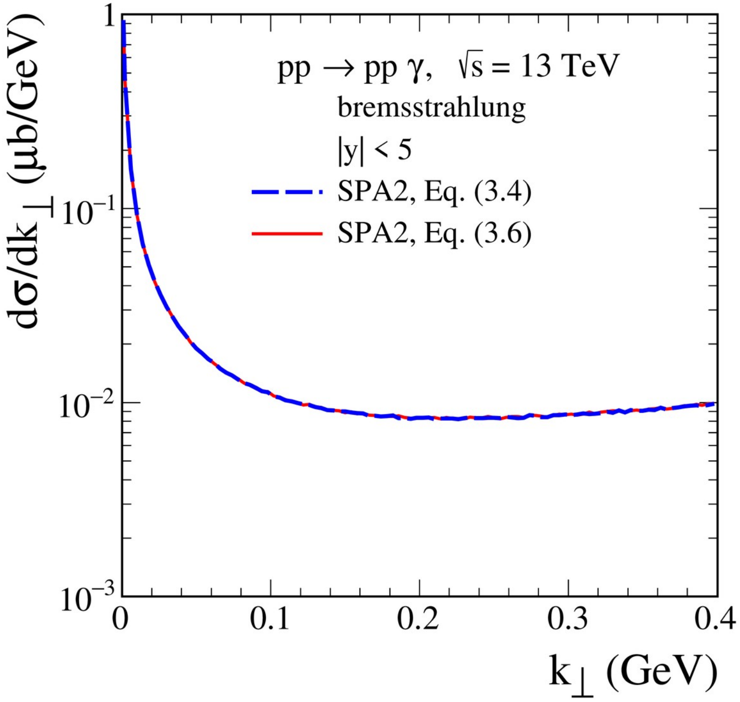


Results for SPA 1 and SPA 2 using the high-energy small-angle approximation

- From the left panel we see that the SPA 1 is in very good agreement to our standard result if we include there only the Dirac terms. We have checked numerically that both SPA1 results overlap.

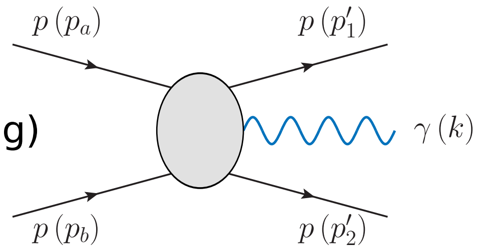
- From the right panels we see that the SPA2 results are very close to each other.





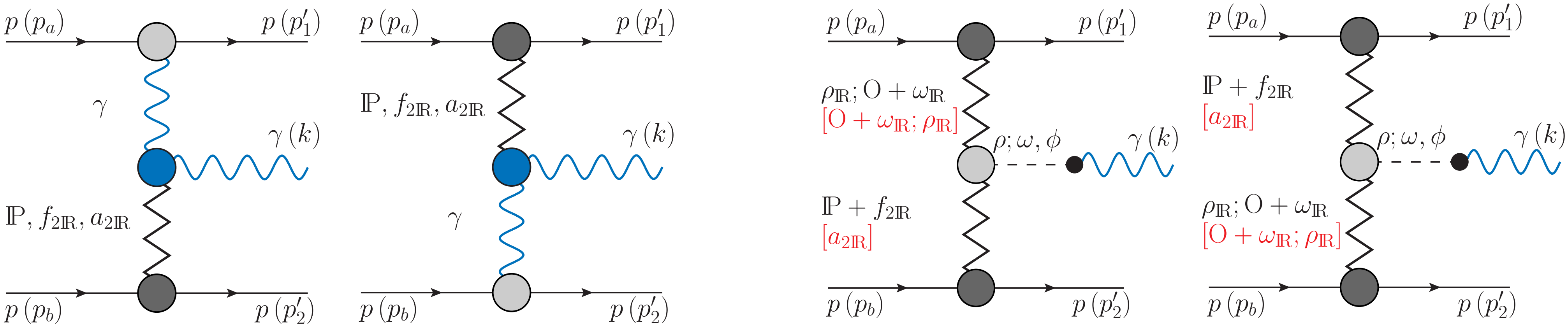
- **Other contributions**

(g)



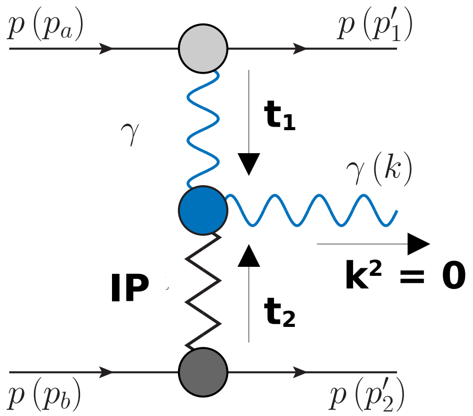
The amplitude must satisfy: $k^\mu \mathcal{M}_\mu^{(g)} = 0$
and has no singularity for $k \to 0$.

Central exclusive production (CEP), fusion processes



$$\mathcal{M}_\mu^{(\gamma-\mathbb{P}/\mathbb{R})} = \mathcal{M}_\mu^{(\gamma\mathbb{P})} + \mathcal{M}_\mu^{(\mathbb{P}\gamma)} + \mathcal{M}_\mu^{(\gamma\mathbb{R}_+)} + \mathcal{M}_\mu^{(\mathbb{R}_+\gamma)}$$

Here we assume the VMD relations in the V → γ vertices.
Vertices occuring here are discussed in:
Ewerz, Maniatis, Nachtmann, Ann. Phys. 342 (2014) 31,
PL, Nachtmann, Szczurek, PRD 101 (2020) 094012



The $\gamma\mathbb{P}$-exchange amplitude can be written as

$$\begin{aligned}
\mathcal{M}_\mu^{(\gamma\mathbb{P})} &= (-i)\,\bar{u}_{1'} i\Gamma_{\nu_1}^{(\gamma pp)}(p_1', p_a) u_a \, i\Delta^{(\gamma)\,\nu_1\nu}(q_1)\, i\Gamma^{(\mathbb{P}\gamma^*\gamma)}_{\mu\nu\kappa\rho}(k, q_1)\, i\Delta^{(\mathbb{P})\,\kappa\rho,\alpha\beta}(s_2, t_2) \\
&\quad \times \bar{u}_{2'} i\Gamma^{(\mathbb{P}pp)}_{\alpha\beta}(p_2', p_b) u_b \\
&= \bar{u}_{1'}\Gamma^{(\gamma pp)\,\nu}(p_1', p_a) u_a \, \frac{1}{t_1}\, \frac{1}{2 s_2} \left(-i s_2 \alpha'_{\mathbb{P}} \right)^{\alpha_{\mathbb{P}}(t_2) - 1} \bar{u}_{2'} \Gamma^{(\mathbb{P}pp)}_{\alpha\beta}(p_2', p_b) u_b \\
&\quad \times i \left[2 a_{\mathbb{P}\gamma^*\gamma}(t_1, k^2, t_2) \Gamma^{(0)\,\alpha\beta}_{\mu\nu}(k, -q_1) - b_{\mathbb{P}\gamma^*\gamma}(t_1, k^2, t_2) \Gamma^{(2)\,\alpha\beta}_{\mu\nu}(k, -q_1) \right]
\end{aligned}$$

dominant term: $b_{\mathbb{P}\gamma^*\gamma}(-Q_1^2, 0, t_2) = e^2 \hat{b}_{\mathbb{P}}(Q_1^2) F^{(\mathbb{P}\gamma\gamma)}(t_2)$

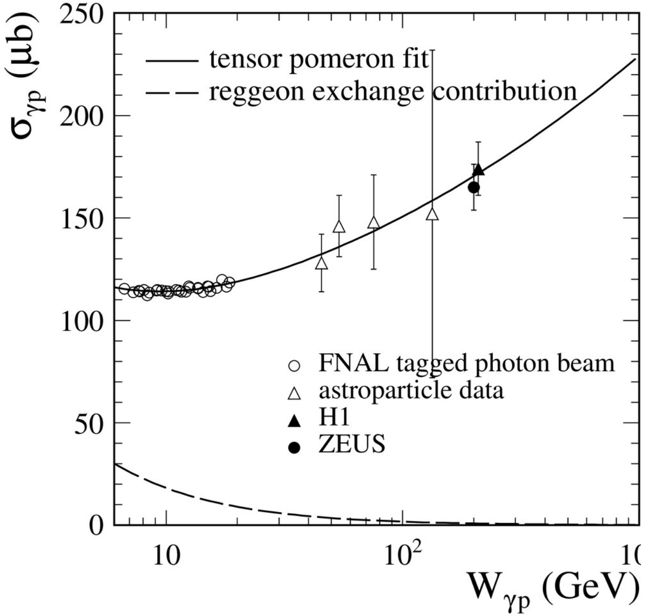
where $Q_1^2 = -t_1$ is the photon virtuality

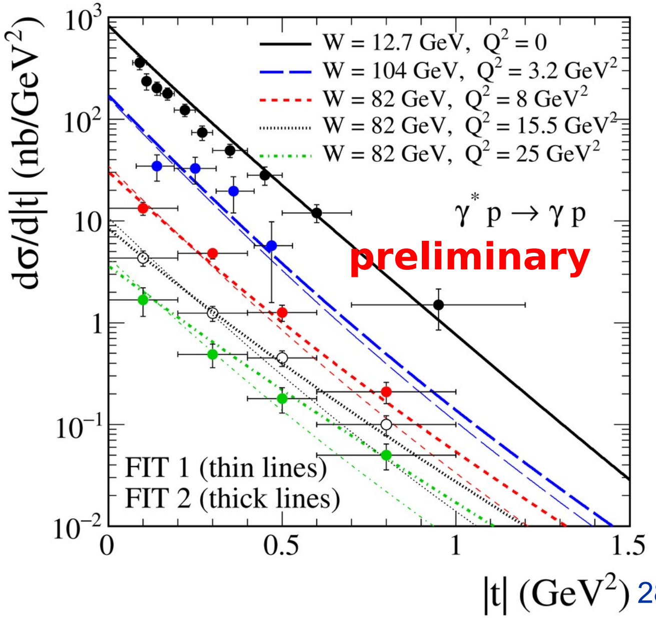
The Ansatz for the IP$\gamma\gamma$ coupling functions for both real and virtual photons is discussed in Britzger, Ewerz, Glazov, Nachtmann, Schmitt, PRD 100 (2019) 114007.

The coupling functions $\hat{a}$ and $\hat{b}$ were determined from the global fit to HERA inclusive DIS data and the total photoproduction cross section $\sigma_{\gamma p}$, and from HERA DVCS data.

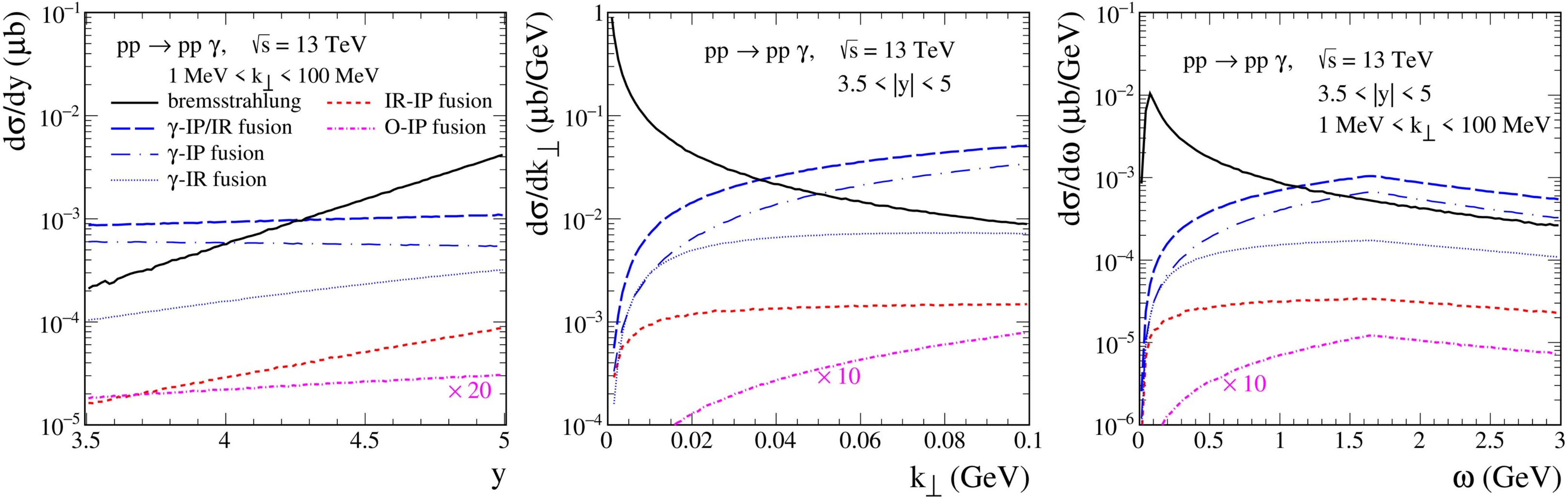
The t dependence of γp subsystem is from fit of the model to the FNAL data on real Compton scattering $\gamma p \to \gamma p$ (and also to DVCS data),

$$\begin{aligned}
F_{\text{eff}}^{(\mathbb{P})}(t) &= F^{(\mathbb{P}\gamma\gamma)}(t) \times F^{(\mathbb{P}pp)}(t) \\
&= \exp(-b_{\text{eff}}|t|/2)
\end{aligned}$$





- Comparison of diffractive bremsstrahlung to CEP fusion processes for ALICE 3 kinematics
  Preliminary results! 3.5 < |y| < 5.0 and 1 MeV < $k_\perp$ < 100 MeV

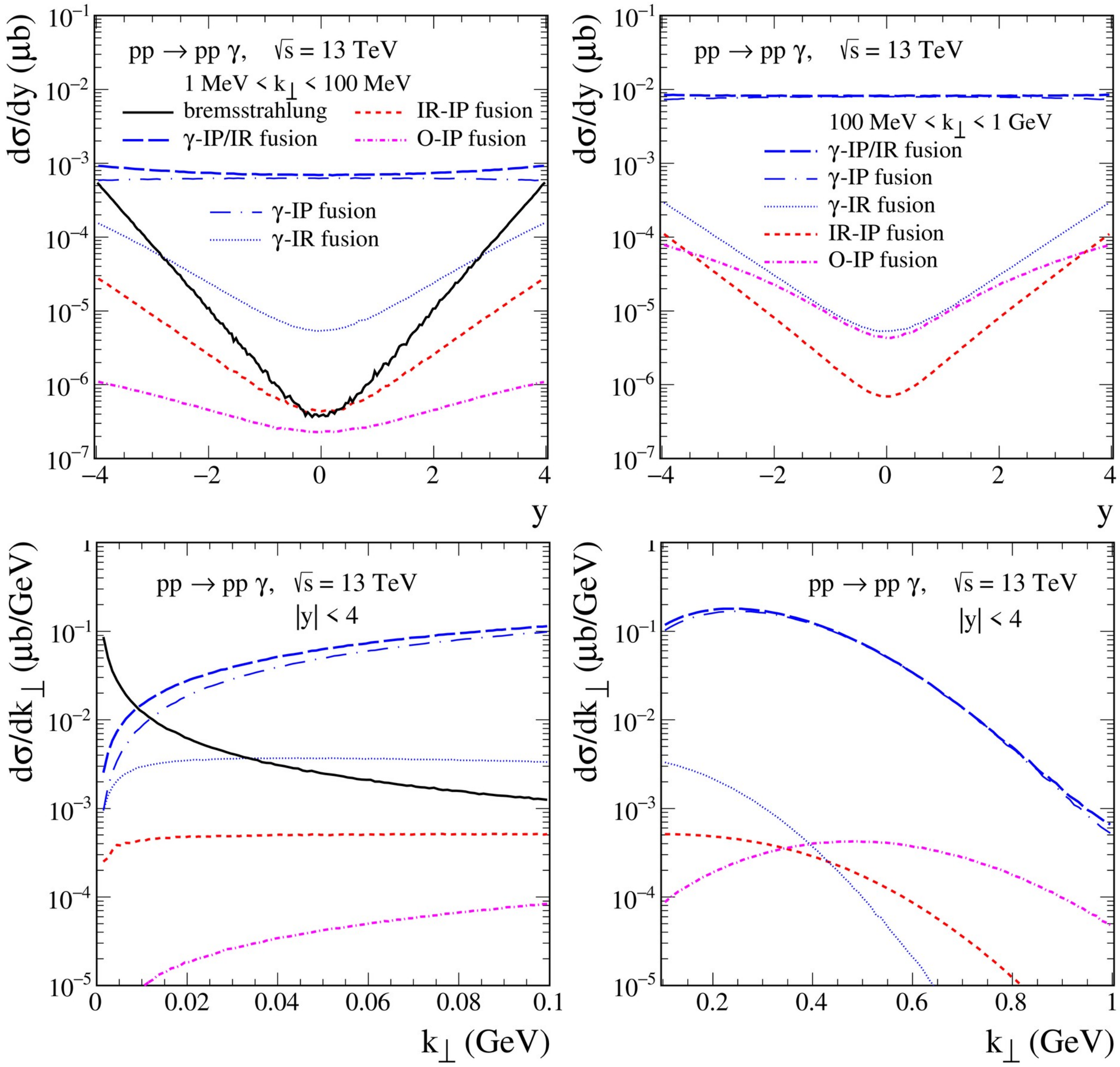


- Diffractive bremsstrahlung wins with CEP fusion processes in the soft-photon limit and large |y|
- The bremsstrahlung via the γ exchange (QED process) is about a factor 200 smaller then the diffractive one
- The γ-IP/IR contributions are important in midrapidity region, |y| < 4.3, and $k_\perp$ > 35 MeV, ω > 1 GeV.
  The purely diffractive IR-IP and O-IP contributions give much smaller cross section there.

Preliminary results!

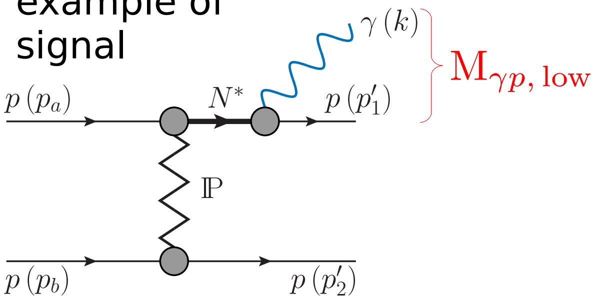
|y| < 4
1 MeV < $k_\perp$ < 100 MeV (left)
100 MeV < $k_\perp$ < 1 GeV (right)

- Photoproduction is very important at midrapidity region and large $k_\perp$
- Absorption effects due to strong proton-proton interactions and possible interference effects between various mechanisms should be included

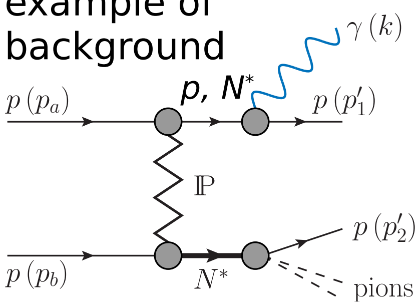


Diffractive excitations of the protons (N^* resonances)

example of signal



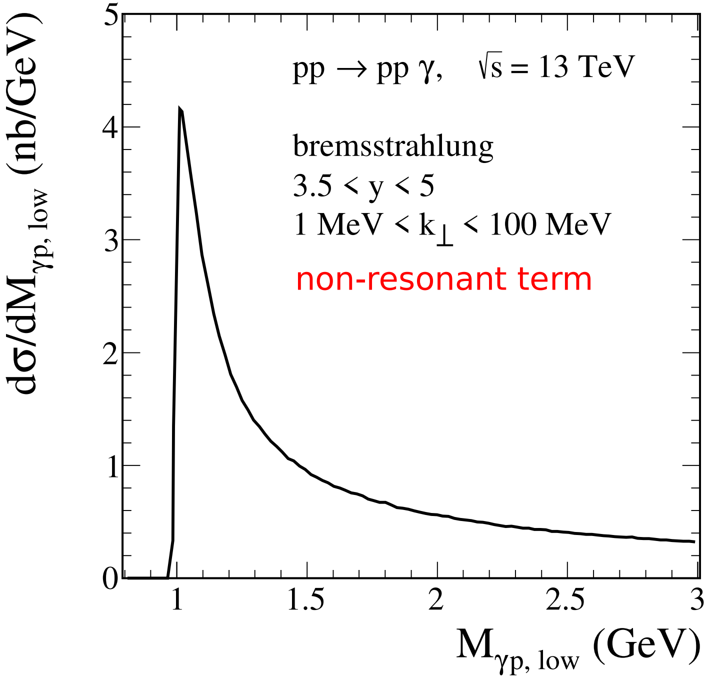
example of background

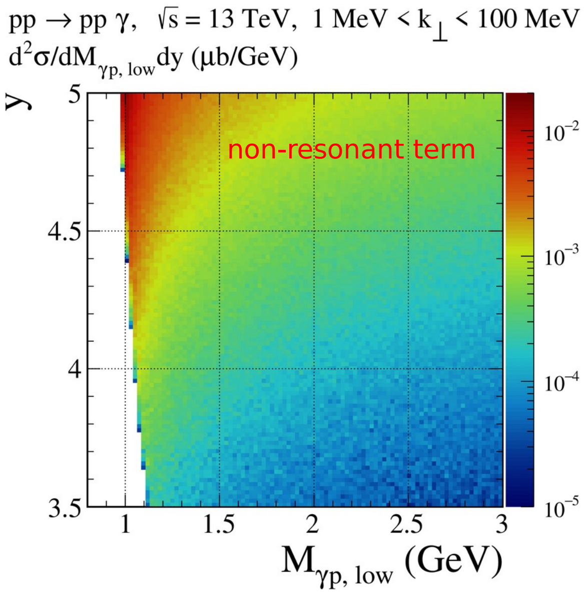


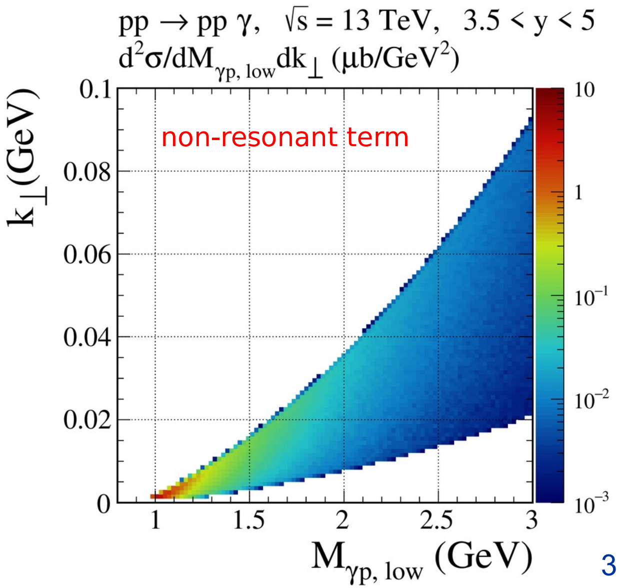
- *N** candidates are (satisfy the Gribov-Morrison rule):
  N(1440) $J^P=1/2^+$, N(1520) $J^P=3/2^-$, N(1680) $J^P=5/2^+$

  BR(N(1440) → pγ) ~0.04 %
  BR(N(1520) → pγ) ~0.3 – 0.5 %
  BR(N(1680) → pγ) ~0.2 – 0.3 % ← a sizeable cross section
  pp → *p*N(1680) was estimated at CERN ISR @ 45 GeV

- If these processes contribute significantly to our reaction then we should see them in the $M_{\gamma p,\mathrm{low}}$ distribution (possibly distorted by interference effects) as a resonance enhancement at $M_{\gamma p} = m_{N^*}$ over the non-resonant term







Conclusions

- We constructed a model for the $pp \to pp\gamma$ reaction for high energies and small momentum transfers using the tensor-pomeron approach.
  The amplitudes corresponding to photon emission from the external proton lines are determined by the off-shell $pp \to pp$ scattering amplitude.
  By constructions, the contact terms have to satisfy gauge-invariance constraints involving the previous amplitudes.

- We have taken care to write the formulas for the $pp \to pp\gamma$ amplitude in such a way that they also apply to soft-virtual photon production,
  for instance, $pp \to pp(\gamma^* \to e^+e^-)$.

- We compared our "exact" or complete model results to SPA results.
  For the region 1 MeV $< k_\perp <$ 100 MeV and $3.5 < |\mathrm{y}| < 5.0$ we find that the SPA1 ansatz with only the pole terms $\propto \omega^{-1}$ agrees at the percent level with our complete model result up to $\omega \cong 2$ GeV.

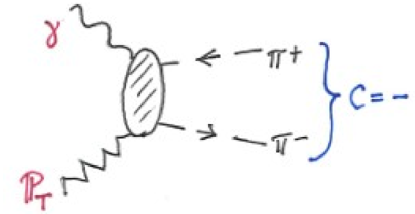
- Diffractive CEP reactions with γ emission like
  $p + p \to p + p + \gamma$ via the fusion processes (e.g. $\gamma\mathbb{P} \to \gamma$, $\mathbb{OP} \to \gamma$) and
  $p + p \to p + \pi^+ + \pi^- + p + \gamma$ could be studied at the LHC.

Thank you for your attention !

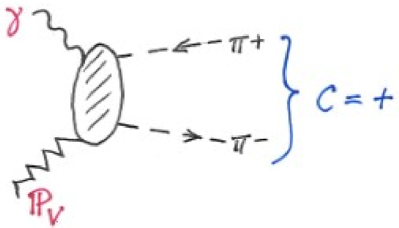
Applications of the tensor-pomeron model

- **Photoproduction and low *x* DIS** *Britzger, Ewerz, Glazov, Nachtmann, Schmitt, PRD100 (2019) 114007*
  “vector IP” decouples completely in the total photoabsorption cross section and in the structure functions of DIS

- ***$\gamma\ p \rightarrow \pi^+\ \pi^-\ p$*** *Bolz, Ewerz, Maniatis, Nachtmann, Sauter, Schöning, JHEP 01 (2015) 151*
  interference betwenn $\gamma p \rightarrow (\rho^0 \rightarrow \pi^+\pi^-)p$ (pomeron exch.) and $\gamma p \rightarrow (f_2(1270) \rightarrow \pi^+\pi^-)p$ (odderon exch.) → $\pi^+\pi^-$ charge asymmetries



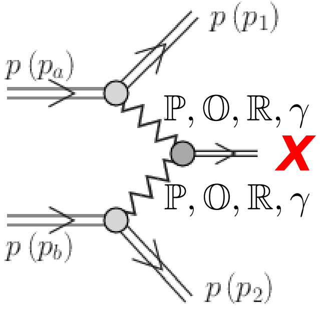
$\pi^+\pi^-$ in antisymmetric state



$\pi^+\pi^-$ in symmetric state

For a tensor (vector) pomeron the $\pi^+\pi^-$ pair is in antisymmetric (symmetric) state under the exchange $\pi^+ \longleftrightarrow \pi^-$. Since the pomeron has C = +1 the $\pi^+\pi^-$ pair must be in antisymmetric state. This gives a further clear evidence against a vector nature of the pomeron.

- **Central Exclusive Production (CEP),** *p p → p p* ***X***, *P.L., Nachtmann, Szczurek:*



X: η, η', $f_0(980)$, $f_0(1370)$, $f_0(1500)$ *Ann. Phys. 344 (2014) 301*
ρ^0 *PRD91 (2015) 074023*
$\pi^+\pi^-$ continuum, $f_2(1270) \rightarrow \pi^+\pi^-$ *PRD93 (2016) 054015, PRD101 (2020) 034008*
$\pi^+\pi^-\pi^+\pi^-$, $\rho^0\rho^0$ *PRD94 (2016) 034017*
ρ^0 with proton diss. *PRD95 (2017) 034036*
$p\bar{p}$ *PRD97 (2018) 094027*
K^+K^- *PRD98 (2018) 014001*
$\phi \rightarrow K^+K^-, \mu^+\mu^-$ *PRD101 (2020) 094012* } odderon exchange
$\phi\phi \rightarrow K^+K^-K^+K^-$ *PRD99 (2019) 094034* } odderon exchange
$f_1(1285)$, $f_1(1420)$ *P.L., Leutgeb, Nachtmann, Rebhan, Szczurek, PRD102 (2020) 114003*
$K^{*0}\bar{K}^{*0}$ continuum vs $f_2(1950)$ *P.L., PRD103 (2021) 054039*

Applications of the tensor-pomeron model

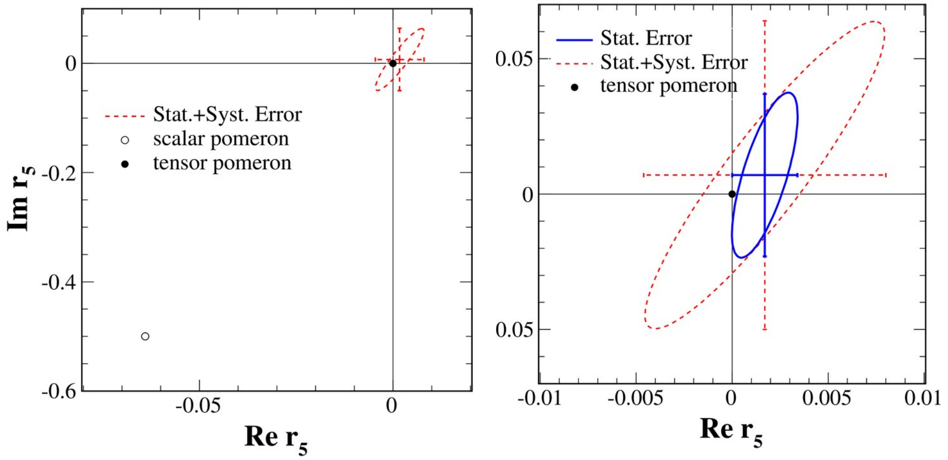
- **Helicity in proton-proton elastic scattering and the spin structure of the soft pomeron**

Ewerz, P.L., Nachtmann, Szczurek, PLB 763 (2016) 382

Studying the ratio r_5 of single-helicity-flip to non-flip amplitudes we found that the STAR data are compatible with the tensor pomeron ansatz while they exclude a scalar character of the pomeron (the scalar-pomeron result is far outside the experimental error ellipse).

STAR data: Adamczyk *et al.*, PLB 719 (2013) 62
[single spin asymmetry A_N in polarised pp → pp]

$$\sqrt{s} = 200\ \text{GeV}, \quad 0.003 \leqslant |t| \leqslant 0.035\ \text{GeV}^2$$

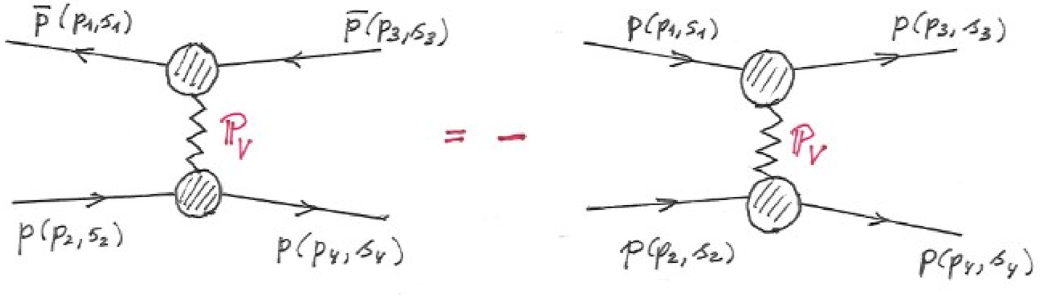


$$r_5(s,t) = \frac{2m_p\,\phi_5(s,t)}{\sqrt{-t}\,\text{Im}[\phi_1(s,t)+\phi_3(s,t)]}$$

$$r_5^{I\!P_T}(s,t) = -\frac{m_p^2}{s}\left[i + \tan\left(\frac{\pi}{2}(\alpha_{I\!P}(t)-1)\right)\right],\ r_5^{I\!P_T}(s,0) = (-0.28 - i2.20)\times 10^{-5}$$

$$r_5^{I\!P_S}(s,t) = -\frac{1}{2}\left[i + \tan\left(\frac{\pi}{2}(\alpha_{I\!P}(t)-1)\right)\right],\ r_5^{I\!P_S}(s,0) = -0.064 - i0.500$$

Problem with the vector pomeron:



$$\sigma_{tot}^{pp} = \frac{1}{2\sqrt{s(s-4m_p^2)}}\,\text{Im}\,[\phi_1(s,0)+\phi_3(s,0)]$$

Vector exchange has C = -1.

It follows

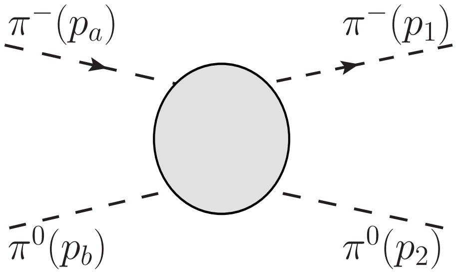
$$\sigma_{tot}^{\bar{p}p}|_{I\!P_V} = -\sigma_{tot}^{pp}|_{I\!P_V}$$

In our opinion a vector pomeron is not a viable option.

General QFT relations for pion-pion scattering without and with photon emission

- We consider the reaction, both on-shell and off-shell,

$$\pi^-(p_a) + \pi^0(p_b) \to \pi^-(p_1) + \pi^0(p_2)$$



Here p_a, p_b, p_1, p_2 are the four-momenta of the particles.

We have always energy-momentum conservation

$$p_a + p_b = p_1 + p_2$$

As kinematic variables we have the masses of the, in general off shell, pions, an energy and a momentum transfer variable:

$$m_a^2 = p_a^2\,,\;\; m_b^2 = p_b^2\,,\;\; m_1^2 = p_1^2\,,\;\; m_2^2 = p_2^2$$

$$s_L = p_a \cdot p_b + p_1 \cdot p_2$$

$$t = (p_a - p_1)^2 = (p_b - p_2)^2$$

Following Low we use here s_L instead of the more usual Mandelstam variable s:

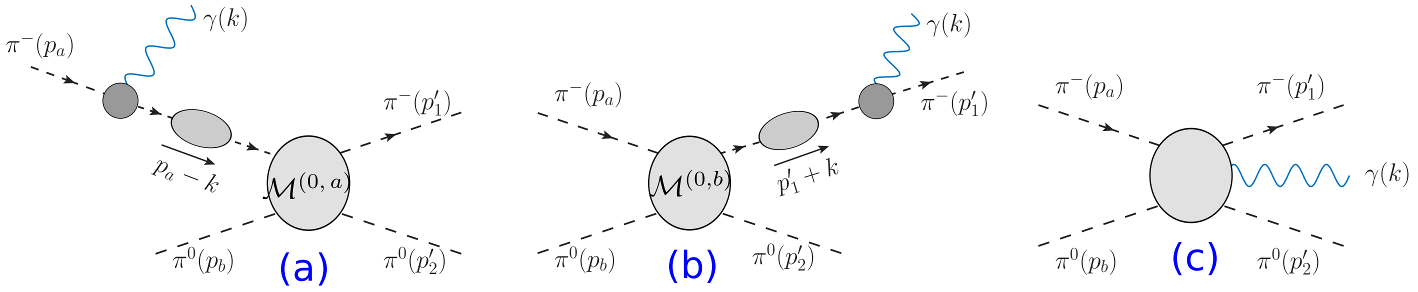
$$s = s_L + \frac{1}{2}\left(m_a^2 + m_b^2 + m_1^2 + m_2^2\right)$$

- The scattering amplitude for $\pi^- \pi^0 \to \pi^- \pi^0$ can only depend on the above variables

$$\mathcal{T}(p_a, p_b, p_1, p_2)\big|_{\text{on shell or off shell}} = \mathcal{M}^{(0)}(s_L, t, m_a^2, m_b^2, m_1^2, m_2^2)$$

For the on-shell amplitude we have $m_a^2 = m_b^2 = m_1^2 = m_2^2 = m_\pi^2$.

- For the photon-emission reaction $\pi^-(p_a) + \pi^0(p_b) \to \pi^-(p_1') + \pi^0(p_2') + \gamma(k,\epsilon)$
  we have from energy-momentum conservation: $p_a + p_b = p_1' + p_2' + k$



$$\mathcal{M}_\lambda = \mathcal{M}_\lambda^{(a)} + \mathcal{M}_\lambda^{(b)} + \mathcal{M}_\lambda^{(c)}$$

The amplitude is $\langle \pi^-(p_1'), \pi^0(p_2'), \gamma(k,\epsilon)|\mathcal{T}|\pi^-(p_a), \pi^0(p_b)\rangle = (\epsilon^\lambda)^* \mathcal{M}_\lambda$

With the $\pi\pi \to \pi\pi$ off-shell amplitude, the pion propagator Δ, and the $\gamma\pi\pi$ vertex Γ_λ we get:

for the diagram (a) $\mathcal{M}_\lambda^{(a)} = -e\,\mathcal{M}^{(0,a)}\,\Delta[(p_a-k)^2]\,\Gamma_\lambda(p_a-k, p_a)$

$$\mathcal{M}^{(0,a)} = \mathcal{T}(p_a - k, p_b, p_1', p_2')$$
$$= \mathcal{M}^{(0)}[(p_a-k, p_b) + p_1'\cdot p_2', (p_b - p_2')^2, (p_a-k)^2, m_\pi^2, m_\pi^2, m_\pi^2]$$

for the diagram (b) $\mathcal{M}_\lambda^{(b)} = -e\,\Gamma_\lambda(p_1', p_1'+k)\,\Delta[(p_1'+k)^2]\,\mathcal{M}^{(0,b)}$

$$\mathcal{M}^{(0,b)} = \mathcal{T}(p_a, p_b, p_1'+k, p_2')$$
$$= \mathcal{M}^{(0)}[p_a\cdot p_b + (p_1'+k, p_2'), (p_b-p_2')^2, m_\pi^2, m_\pi^2, (p_1'+k)^2, m_\pi^2]$$

The amplitude $\mathcal{M}_\lambda$ must satisfy the **gauge invariance** relation $k^\lambda \mathcal{M}_\lambda = k^\lambda\left(\mathcal{M}_\lambda^{(a)} + \mathcal{M}_\lambda^{(b)} + \mathcal{M}_\lambda^{(c)}\right) = 0$
and using the Ward-Takahashi identity

$(p'-p)^\lambda\,\Gamma_\lambda(p',p) = \Delta^{-1}(p'^2) - \Delta^{-1}(p^2)$ we find $\boxed{k^\lambda \mathcal{M}_\lambda^{(c)} = -e\mathcal{M}^{(0,a)} + e\mathcal{M}^{(0,b)}}$

The expansion of the photon emission amplitude

Here we discuss the expansion of the amplitude $\mathcal{M}_\lambda$ of the reaction $\pi^-(p_a) + \pi^0(p_b) \to \pi^-(p'_1) + \pi^0(p'_2) + \gamma(k)$ for small k where we set $k^0 = \omega$.

We shall in the following assume that all components of the photon momentum are proportional to ω, $k^\mu \propto \omega$, with $\omega \to 0$.

For $k = 0$ we have $p'_1 = p_1$, $p'_2 = p_2$, with p_1, p_2 corresponding to $\pi\pi \to \pi\pi$ on-shell reaction. But as we go to $k \neq 0$ we also have to change p'_1 and p'_2.

We set: $p'_1 = p_1 - l_1\,,\ p'_2 = p_2 - l_2$.

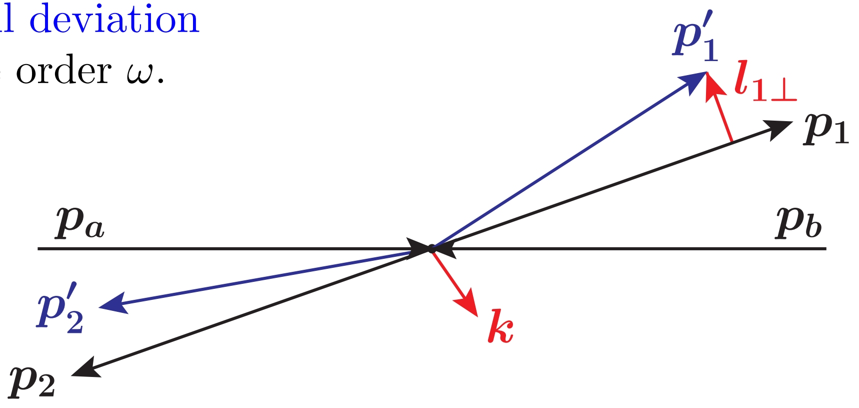
Energy-momentum conservation gives: for $k = 0$: $p_a + p_b = p_1 + p_2$

for $k \neq 0$: $p_a + p_b = p'_1 + p'_2 + k = p_1 + p_2 - l_1 - l_2 + k$

This gives the conditions: $l_1 + l_2 = k\,,\quad (p_1 - l_1)^2 = p'^2_1 = m_\pi^2\,,\quad (p_2 - l_2)^2 = p'^2_2 = m_\pi^2\,.$

Now we make the assumption that we consider only small deviation of p'_1 from p_1 and p'_2 from p_2. That is, we assume l_1, l_2 to be order ω.

Working in the overal c.m. system the above equations is solved with $\boldsymbol{l}_{1\perp}$ playing the role of the 2 free parameters.



- Expansion of the photon-emission amplitude in k alone around $k = 0$ is not a good idea, since k alone does not specify the final state completely. One possibility is to expand in k and $\boldsymbol{l}_{1\perp}$ which are independent and together specify the final state completely.

We can expand the amplitudes in powers of ω. We get with s_L and t,

$$\begin{aligned}
\mathcal{M}^{(0,a)} &= \mathcal{M}^{(0)}[(p_a-k,p_b)+p_1'\cdot p_2',(p_b-p_2')^2,(p_a-k)^2,m_\pi^2,m_\pi^2,m_\pi^2] \\
&= \mathcal{M}^{(0)}[s_L-(p_b+p_1,k)-(p_2\cdot l_1),t-2(p_a-p_1,k-l_1),m_\pi^2-2(p_a\cdot k),m_\pi^2,m_\pi^2,m_\pi^2]+\mathcal{O}(\omega^2) \\
&= \Big\{1-[(p_b+p_1,k)+(p_2\cdot l_1)]\frac{\partial}{\partial s_L}-[2(p_a-p_1,k)-2(p_a\cdot l_1)]\frac{\partial}{\partial t}-2(p_a\cdot k)\frac{\partial}{\partial m_a^2}\Big\}\,\mathcal{M}^{(0)}(s_L,t,m_a^2,m_\pi^2,m_\pi^2,m_\pi^2)\Big|_{m_a^2=m_\pi^2}+\mathcal{O}(\omega^2) \\
\mathcal{M}^{(0,b)} &= \mathcal{M}^{(0)}[p_a\cdot p_b+(p_1'+k,p_2'),(p_b-p_2')^2,m_\pi^2,m_\pi^2,(p_1'+k)^2,m_\pi^2] \\
&= \mathcal{M}^{(0)}[s_L-(p_1\cdot k),t-2(p_a-p_1,k)+2(p_a\cdot l_1),m_\pi^2,m_\pi^2,m_\pi^2+2(p_1\cdot k),m_\pi^2]+\mathcal{O}(\omega^2) \\
&= \Big\{1-(p_1\cdot k)\frac{\partial}{\partial s_L}-[2(p_a-p_1,k)-2(p_a\cdot l_1)]\frac{\partial}{\partial t}+2(p_1\cdot k)\frac{\partial}{\partial m_1^2}\Big\}\,\mathcal{M}^{(0)}(s_L,t,m_\pi^2,m_\pi^2,m_1^2,m_\pi^2)\Big|_{m_1^2=m_\pi^2}+\mathcal{O}(\omega^2)
\end{aligned}$$

To determine $\mathcal{M}_\lambda^{(c)}$ to order ω^0 we use $k^\lambda\mathcal{M}_\lambda^{(c)}=-e\,\mathcal{M}^{(0,a)}+e\,\mathcal{M}^{(0,b)}$.
To order ω we get

$$k^\lambda\mathcal{M}_\lambda^{(c)}=e\Big\{(p_b+p_2,k)\frac{\partial}{\partial s_L}+2(p_a\cdot k)\frac{\partial}{\partial m_a^2}+2(p_1\cdot k)\frac{\partial}{\partial m_1^2}\Big\}\,\mathcal{M}^{(0)}(s_L,t,m_a^2,m_\pi^2,m_1^2,m_\pi^2)\Big|_{m_a^2=m_1^2=m_\pi^2}+\mathcal{O}(\omega^2)$$

and we can read off the term of order ω^0 for $\mathcal{M}_\lambda^{(c)}$:

$$\mathcal{M}_\lambda^{(c)}=e\Big\{(p_b+p_2)_\lambda\frac{\partial}{\partial s_L}+2p_{a\lambda}\frac{\partial}{\partial m_a^2}+2p_{1\lambda}\frac{\partial}{\partial m_1^2}\Big\}\,\mathcal{M}^{(0)}(s_L,t,m_a^2,m_\pi^2,m_1^2,m_\pi^2)\Big|_{m_a^2=m_1^2=m_\pi^2}+\mathcal{O}(\omega)$$

When adding the amplitudes $\mathcal{M}_\lambda^{(a)}$, $\mathcal{M}_\lambda^{(b)}$, and $\mathcal{M}_\lambda^{(c)}$ the off-mass-shell contributions vanish up to $\mathcal{O}(\omega^0)$.

$$\Delta[(p_a - k)^2]\,\Gamma_\lambda(p_a - k, p_a) = \frac{(2p_a - k)_\lambda}{-2(p_a \cdot k) + k^2} + \mathcal{O}(\omega)$$

$$\Gamma_\lambda(p_1', p_1' + k)\,\Delta[(p_1' + k)^2] = \frac{(2p_1' + k)_\lambda}{2(p_1' \cdot k) + k^2} + \mathcal{O}(\omega)$$

Now we collect everything together and we obtain

$$\begin{aligned}
\mathcal{M}_\lambda &= \mathcal{M}_\lambda^{(a)} + \mathcal{M}_\lambda^{(b)} + \mathcal{M}_\lambda^{(c)} \\
&= e\mathcal{M}^{(0)}(s_L, t, m_\pi^2, m_\pi^2, m_\pi^2, m_\pi^2)\left[\frac{(2p_a - k)_\lambda}{2(p_a \cdot k) - k^2} - \frac{(2p_1' + k)_\lambda}{2(p_1' \cdot k) + k^2}\right] \\
&\quad +2e\frac{\partial}{\partial s_L}\mathcal{M}^{(0)}(s_L, t, m_\pi^2, m_\pi^2, m_\pi^2, m_\pi^2)\left[-(p_b \cdot k)\frac{p_{a\lambda}}{(p_a \cdot k)} + p_{b\lambda}\right] \\
&\quad -2e\frac{\partial}{\partial t}\mathcal{M}^{(0)}(s_L, t, m_\pi^2, m_\pi^2, m_\pi^2, m_\pi^2)\left[(p_a - p_1, k) - (p_a \cdot l_1)\right]\left[\frac{p_{a\lambda}}{(p_a \cdot k)} - \frac{p_{1\lambda}}{(p_1 \cdot k)}\right] + \mathcal{O}(\omega)
\end{aligned}$$

In the first term we should, for consistency of the expansion in ω up to ω^0, make the following replacements:

$$\frac{(2p_a - k)_\lambda}{2(p_a \cdot k) - k^2} \to \frac{p_{a\lambda}}{(p_a \cdot k)} + \frac{1}{2(p_a \cdot k)^2}[p_{a\lambda}k^2 - k_\lambda(p_a \cdot k)]$$

$$\frac{(2p_1' + k)_\lambda}{2(p_1' \cdot k) + k^2} \to \frac{p_{1\lambda}}{(p_1 \cdot k)} + \frac{1}{2(p_1 \cdot k)^2}[p_{1\lambda}(2(l_1 \cdot k) - k^2) - (2l_{1\lambda} - k_\lambda)(p_1 \cdot k)]$$

For real photons ($k^2 = 0$) and dropping gauge terms $\propto k_\lambda$ we get:

$$\mathcal{M}_\lambda = e\Big[\frac{p_{a\lambda}}{(p_a\cdot k)} - \frac{p_{1\lambda}}{(p_1\cdot k)}\Big]\mathcal{M}^{(0)}(s_L, t, m_\pi^2, m_\pi^2, m_\pi^2, m_\pi^2) \qquad \mathcal{O}(\omega^{-1})$$

$$+e\Big\{-\frac{1}{(p_1\cdot k)^2}\big[p_{1\lambda}(l_1\cdot k) - l_{1\lambda}(p_1\cdot k)\big] + 2\Big[-p_{a\lambda}\frac{(p_b\cdot k)}{(p_a\cdot k)} + p_{b\lambda}\Big]\frac{\partial}{\partial s_L}$$
$$-2\big[(p_a - p_1, k) - (p_a\cdot l_1)\big]\Big[\frac{p_{a\lambda}}{(p_a\cdot k)} - \frac{p_{1\lambda}}{(p_1\cdot k)}\Big]\frac{\partial}{\partial t}\Big\}\mathcal{M}^{(0)}(s_L, t, m_\pi^2, m_\pi^2, m_\pi^2, m_\pi^2) \qquad \mathcal{O}(\omega^0)$$
$$+\,\mathcal{O}(\omega)$$

The terms of order ω^{-1} and ω^0 are determined by the on-shell amplitude $\mathcal{M}^{(0)}$.

Low's result reads:

$$\mathcal{M}_\lambda^{\text{Low}} = e\Big[\frac{p_{a\lambda}}{(p_a\cdot k)} - \frac{p_{1\lambda}}{(p_1\cdot k)}\Big]\mathcal{M}^{(0)}(s_L, t, m_\pi^2, m_\pi^2, m_\pi^2, m_\pi^2) \qquad \mathcal{O}(\omega^{-1})$$

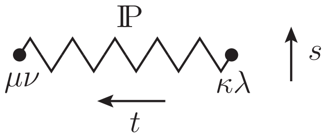
$$+e\Big[-\frac{(p_b\cdot k)}{(p_a\cdot k)}p_{a\lambda} - \frac{(p_2\cdot k)}{(p_1\cdot k)}p_{1\lambda} + p_{b\lambda} + p_{2\lambda}\Big]\frac{\partial}{\partial s_L}\mathcal{M}^{(0)}(s_L, t, m_\pi^2, m_\pi^2, m_\pi^2, m_\pi^2) \qquad \mathcal{O}(\omega^0)$$
$$+\,\mathcal{O}(\omega)$$

We agree with the ω^{-1} term but disagree with the ω^0 term.
What is the origin of this discrepancy?

- Low's result corresponds to the expansion of the fictitious process $\pi^-(p_a)\pi^0(p_b) \to \pi^-(p_1)\pi^0(p_2)\gamma(k)$ where energy-momentum conservation is not respected.

- Tensor-pomeron model

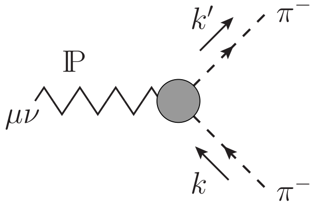
Propagator for tensor-pomeron exchange



$$i\Delta^{(\mathbb{P})}_{\mu\nu,\kappa\lambda}(s,t) = \frac{1}{4s}\left(g_{\mu\kappa}g_{\nu\lambda} + g_{\mu\lambda}g_{\nu\kappa} - \frac{1}{2}g_{\mu\nu}g_{\kappa\lambda}\right)(-is\alpha'_{\mathbb{P}})^{\alpha_{\mathbb{P}}(t)-1}$$

pomeron trajectory: $\alpha(t) = \alpha(0) + \alpha' t\,, \quad \alpha(0) = 1.0808, \quad \alpha' = 0.25\,\mathrm{GeV}^{-2}$

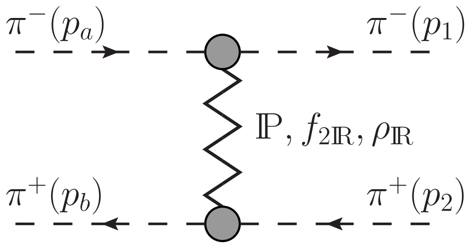
Pomeron-pion coupling



$$i\Gamma^{(\mathbb{P}\pi\pi)}_{\mu\nu}(k',k) = -i\,2\beta_{\mathbb{P}\pi\pi}F_M[(k'-k)^2]\left[(k'+k)_\mu(k'+k)_\nu - \frac{1}{4}g_{\mu\nu}(k'+k)^2\right]$$

$$\beta_{\mathbb{P}\pi\pi} = 1.76\ \mathrm{GeV}^{-1}\,, \quad F_M(t) = \frac{m_0^2}{m_0^2 - t}\,, \quad m_0^2 = 0.50\ \mathrm{GeV}^2$$

$\pi^-\pi^+$ scattering



The general off-shell $\pi^-\pi^+$ scattering amplitude is

$$\mathcal{M}^{(0)\pi^-\pi^+}(s_L, t, m_a^2, m_b^2, m_1^2, m_2^2) = \mathcal{M}^{(0)}_{\mathbb{P}} + \mathcal{M}^{(0)}_{f_{2\mathbb{R}}} + \mathcal{M}^{(0)}_{\rho_{\mathbb{R}}}$$

$$\mathcal{M}^{(0)}_{\mathbb{P}} = i\mathcal{F}_{\mathbb{P}}(s,t)\left[2(p_a+p_1, p_b+p_2)^2 - \frac{1}{2}(p_a+p_1)^2(p_b+p_2)^2\right]$$

$$= i\mathcal{F}_{\mathbb{P}}(s,t)\left[2(2s_L+t)^2 - \frac{1}{2}(-t+2m_a^2+2m_1^2)(-t+2m_b^2+2m_2^2)\right]$$

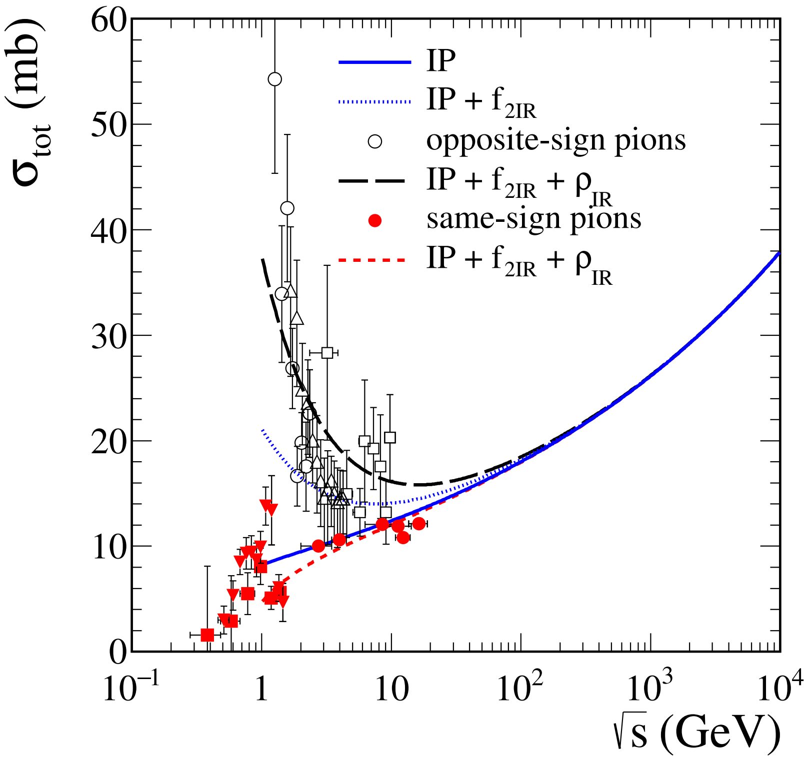
where $\mathcal{F}_{\mathbb{P}}(s,t) = [2\beta_{\mathbb{P}\pi\pi}F_M(t)]^2\,\frac{1}{4s}(-is\alpha'_{\mathbb{P}})^{\alpha_{\mathbb{P}}(t)-1}$

For the on-shell amplitude we get

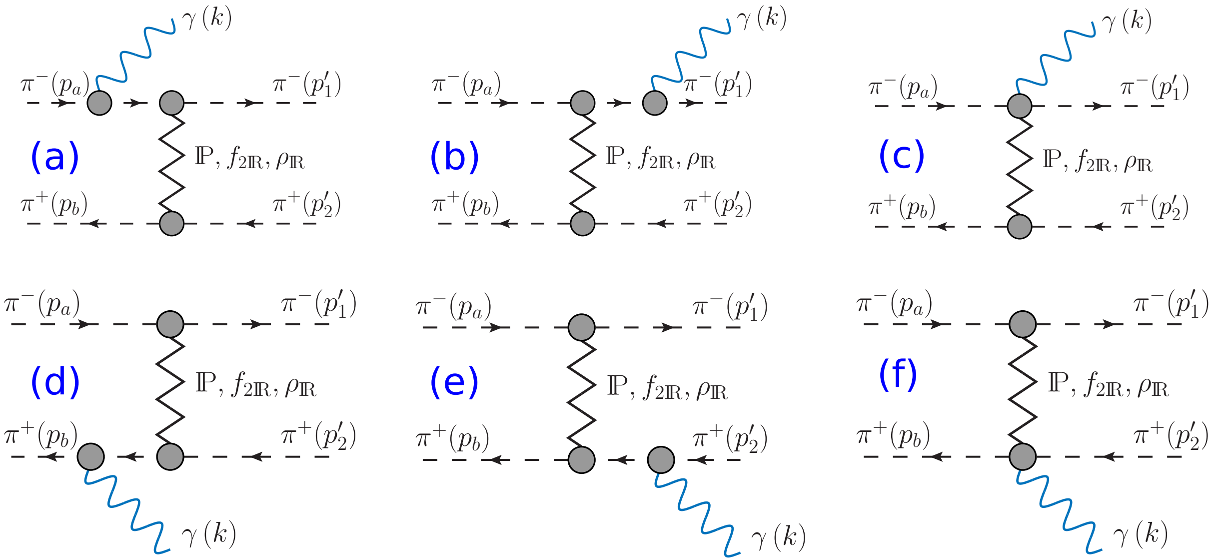
$$\mathcal{M}^{(0)\pi^-\pi^+}(s_L, t, m_\pi^2, m_\pi^2, m_\pi^2, m_\pi^2) \equiv \mathcal{M}^{(0)\pi^-\pi^+}(s,t)$$
$$= i\Big[\mathcal{F}_{\mathbb{P}}(s,t) + \mathcal{F}_{f_{2\mathbb{R}}}(s,t)\Big]\Big[2(p_a+p_1,p_b+p_2)^2 - \frac{1}{2}(p_a+p_1)^2(p_b+p_2)^2\Big]$$
$$+\mathcal{F}_{\rho_{\mathbb{R}}}(s,t)(p_a+p_1,p_b+p_2)$$
$$= 8is^2\Big[\mathcal{F}_{\mathbb{P}}(s,t) + \mathcal{F}_{f_{2\mathbb{R}}}(s,t)\Big]\Big[1 - \frac{4m_\pi^2 - t}{s} + \frac{3}{16s^2}(4m_\pi^2 - t)^2\Big]$$
$$+2s\mathcal{F}_{\rho_{\mathbb{R}}}(s,t)\Big[1 - \frac{4m_\pi^2 - t}{2s}\Big]$$

The total cross section for $\pi\pi$ scattering is obtained from the forward-scattering amplitude using the optical theorem:

$$\sigma_{\mathrm{tot}\,,\pi^-\pi^+}(s) = \frac{1}{\sqrt{s(s-4m_\pi^2)}}\,\mathrm{Im}\mathcal{M}^{(0)\pi^-\pi^+}(s,0)$$



- Photon emission process $\pi^-(p_a) + \pi^+(p_b) \to \pi^-(p_1') + \pi^+(p_2') + \gamma(k,\epsilon)$



$$s = (p_a + p_b)^2 = (p_1' + p_2' + k)^2$$
$$t_1 = (p_a - p_1')^2 = (p_b - p_2' - k)^2$$
$$t_2 = (p_b - p_2')^2 = (p_a - p_1' - k)^2$$

$$\mathcal{M}_\lambda^{(a)} = \mathcal{M}_{\lambda\mathbb{P}}^{(a)} + \mathcal{M}_{\lambda f_{2\mathbb{R}}}^{(a)} + \mathcal{M}_{\lambda\rho_{\mathbb{R}}}^{(a)}$$

and similarly for $\mathcal{M}_\lambda^{(b)}, \dots, \mathcal{M}_\lambda^{(f)}$

$$\mathcal{M}_\lambda^{(d)} = -\mathcal{M}_\lambda^{(a)}\Big|_{p_a, p_1' \leftrightarrow p_b, p_2'}$$
$$\mathcal{M}_\lambda^{(e)} = -\mathcal{M}_\lambda^{(b)}\Big|_{p_a, p_1' \leftrightarrow p_b, p_2'}$$
$$\mathcal{M}_\lambda^{(f)} = -\mathcal{M}_\lambda^{(c)}\Big|_{p_a, p_1' \leftrightarrow p_b, p_2'}$$

$$k^\lambda \left(\mathcal{M}_\lambda^{(a)} + \mathcal{M}_\lambda^{(b)} + \mathcal{M}_\lambda^{(c)}\right) = 0$$
$$k^\lambda \left(\mathcal{M}_\lambda^{(d)} + \mathcal{M}_\lambda^{(e)} + \mathcal{M}_\lambda^{(f)}\right) = 0$$

$$\langle \pi^-(p_1'), \pi^+(p_2'), \gamma(k,\epsilon) | \mathcal{T} | \pi^-(p_a), \pi^+(p_b) \rangle = (\epsilon^\lambda)^* \mathcal{M}_\lambda^{(\pi^-\pi^+ \to \pi^-\pi^+\gamma)}$$

$$\mathcal{M}_\lambda^{(\pi^-\pi^+ \to \pi^-\pi^+\gamma)} = \mathcal{M}_\lambda^{(a)} + \mathcal{M}_\lambda^{(b)} + \mathcal{M}_\lambda^{(c)} + \mathcal{M}_\lambda^{(d)} + \mathcal{M}_\lambda^{(e)} + \mathcal{M}_\lambda^{(f)}$$

The inclusive cross section for the real-photon yield:

$$d\sigma(\pi^-\pi^+ \to \pi^-\pi^+\gamma(k)) = \frac{1}{2\sqrt{s(s-4m_\pi^2)}} \frac{d^3k}{(2\pi)^3\, 2k^0} \int \frac{d^3p_1'}{(2\pi)^3\, 2p_1'^0} \frac{d^3p_2'}{(2\pi)^3\, 2p_2'^0}$$
$$\times (2\pi)^4 \delta^{(4)}(p_1' + p_2' + k - p_a - p_b) \mathcal{M}_\lambda^{(\pi^-\pi^+ \to \pi^-\pi^+\gamma)} \left(\mathcal{M}_\rho^{(\pi^-\pi^+ \to \pi^-\pi^+\gamma)}\right)^* (-g^{\lambda\rho})$$

We use the standard pion propagator and the standard $\gamma\pi\pi$ vertex function.

This gives

$$\left.\begin{aligned} \Delta[(p_a-k)^2]\,\Gamma_\lambda(p_a-k,p_a) &= \frac{(2p_a-k)_\lambda}{-2(p_a\cdot k)+k^2} \\ \Gamma_\lambda(p_1',p_1'+k)\,\Delta[(p_1'+k)^2] &= \frac{(2p_1'+k)_\lambda}{2(p_1'\cdot k)+k^2} \end{aligned}\right\}$$

These relations are exact for $\omega \to 0$ up to corrections of order ω.

Now we can calculate amplitudes $\mathcal{M}_\lambda^{(a)}$ and $\mathcal{M}_\lambda^{(b)}$ in terms of off-shell amplitudes $\mathcal{M}^{(0,a)}$ and $\mathcal{M}^{(0,b)}$ with the $\mathbb{P}$ exchange:

$$\mathcal{M}_{\lambda\mathbb{P}}^{(a)} = -e\,\mathcal{M}_{\mathbb{P}}^{(0,\,a)}\,\frac{(2p_a-k)_\lambda}{-2(p_a\cdot k)+k^2},$$

$$\mathcal{M}_{\lambda\mathbb{P}}^{(b)} = -e\,\mathcal{M}_{\mathbb{P}}^{(0,\,b)}\,\frac{(2p_1'+k)_\lambda}{2(p_1'\cdot k)+k^2},$$

$$\mathcal{M}_{\mathbb{P}}^{(0,\,a)} = i\mathcal{F}_{\mathbb{P}}[(p_a+p_b-k)^2,t_2]\left[2(p_a+p_1'-k,p_b+p_2')^2-\frac{1}{2}(p_a+p_1'-k)^2(p_b+p_2')^2\right]$$

$$\mathcal{M}_{\mathbb{P}}^{(0,\,b)} = i\mathcal{F}_{\mathbb{P}}(s,t_2)\left[2(p_a+p_1'+k,p_b+p_2')^2-\frac{1}{2}(p_a+p_1'+k)^2(p_b+p_2')^2\right]$$

For $\mathcal{M}_{\lambda\mathbb{P}}^{(c)}$ we have the QFT relation: $k^\lambda\mathcal{M}_{\lambda\mathbb{P}}^{(c)} = -e\,\mathcal{M}_{\mathbb{P}}^{(0,\,a)} + e\,\mathcal{M}_{\mathbb{P}}^{(0,\,b)}$. To order ω^0 this eqution determined $\mathcal{M}_{\lambda\mathbb{P}}^{(c)}$ uniquely.

$$\begin{aligned} \mathcal{M}_{\lambda\mathbb{P}}^{(c)} = -ie\mathcal{F}_{\mathbb{P}}(s,t_2)\big\{ &-8(p_b+p_2')_\lambda(p_a+p_1',p_b+p_2') + 2(p_a+p_1')_\lambda(p_b+p_2')^2 \\ &+(2p_a+2p_b-k)_\lambda\,(2-\alpha_{\mathbb{P}}(t_2))\,g_{\mathbb{P}}(\varkappa,t_2)\,\frac{1}{s}\left[2(p_a+p_1'-k,p_b+p_2')^2-\frac{1}{2}(p_a+p_1'-k)^2(p_b+p_2')^2\right]\big\} \end{aligned}$$

$$\varkappa = \frac{(2p_a+2p_b-k,k)}{s}, \quad g_{\mathbb{P}}(\varkappa,t_2) = \frac{1}{(2-\alpha_{\mathbb{P}}(t_2))\,\varkappa}\left[(1-\varkappa)^{\alpha_{\mathbb{P}}(t_2)-2}-1\right]$$

We choose for our standard model the simplest solution. But other solutions are possible. There "anomalous" terms, not directly related to the $\pi\pi\to\pi\pi$ amplitude, could come up.

Soft Photon Approximation (SPA)

We shall compare our "exact" model results for photon emission in $\pi^+ \pi^-$ scattering, which we call "standard" results, to a frequently used SPA (what we call SPA1).

Here we keep only the pole terms $\propto \omega^{-1}$ for $\mathcal{M}_\lambda^{(a)} \cdots \mathcal{M}_\lambda^{(f)}$.

This amounts to the following replacements, using $k^2 = 0$, $p_1' \to p_1$, $p_2' \to p_2$:

$$\mathcal{M}_\lambda^{(a)} \to e\mathcal{M}^{(0)\pi^-\pi^+}(s,t)\frac{p_{a\lambda}}{(p_a \cdot k)} \qquad \mathcal{M}_\lambda^{(d)} \to -e\mathcal{M}^{(0)\pi^-\pi^+}(s,t)\frac{p_{b\lambda}}{(p_b \cdot k)}$$

$$\mathcal{M}_\lambda^{(b)} \to -e\mathcal{M}^{(0)\pi^-\pi^+}(s,t)\frac{p_{1\lambda}}{(p_1 \cdot k)} \qquad \mathcal{M}_\lambda^{(e)} \to e\mathcal{M}^{(0)\pi^-\pi^+}(s,t)\frac{p_{2\lambda}}{(p_2 \cdot k)}$$

$$\mathcal{M}_\lambda^{(c)} \to 0 \qquad \mathcal{M}_\lambda^{(f)} \to 0$$

We get

$$\mathcal{M}_{\lambda,\,\mathrm{SPA1}}^{(\pi^-\pi^+\to\pi^-\pi^+\gamma)} = e\mathcal{M}^{(0)\pi^-\pi^+}(s,t)\Big[\frac{p_{a\lambda}}{(p_a \cdot k)} - \frac{p_{1\lambda}}{(p_1 \cdot k)} - \frac{p_{b\lambda}}{(p_b \cdot k)} + \frac{p_{2\lambda}}{(p_2 \cdot k)}\Big]$$

In the soft photon limit, $\omega \to 0$, the SPA amplitude can be factorized into the hadron part $\mathcal{M}^{(0)}$ (on shell) and the photon emission part.

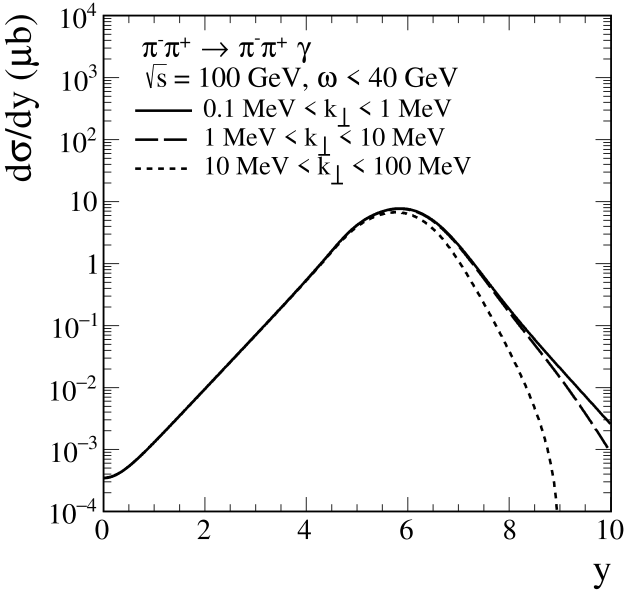
$$d\sigma_{\,\mathrm{SPA1}} = \frac{d^3k}{(2\pi)^3\, 2k^0}\int d^3p_1\, d^3p_2\, e^2 \times\Big[\frac{p_{a\lambda}}{(p_a \cdot k)} - \frac{p_{1\lambda}}{(p_1 \cdot k)} - \frac{p_{b\lambda}}{(p_b \cdot k)} + \frac{p_{2\lambda}}{(p_2 \cdot k)}\Big]\Big[\frac{p_{a\rho}}{(p_a \cdot k)} - \frac{p_{1\rho}}{(p_1 \cdot k)} - \frac{p_{b\rho}}{(p_b \cdot k)} + \frac{p_{2\rho}}{(p_2 \cdot k)}\Big](-g^{\lambda\rho})\frac{d\sigma(\pi^-\pi^+ \to \pi^-\pi^+)}{d^3p_1 d^3p_2}$$

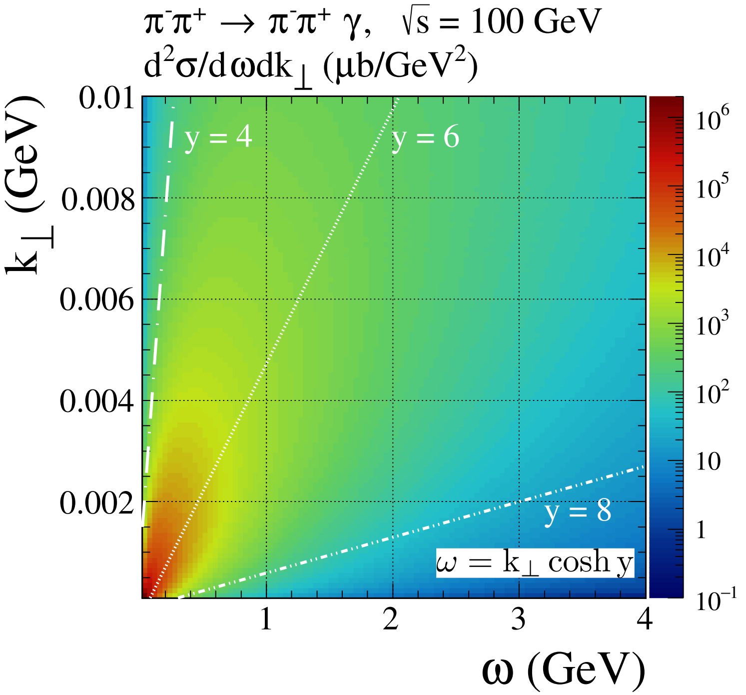
where

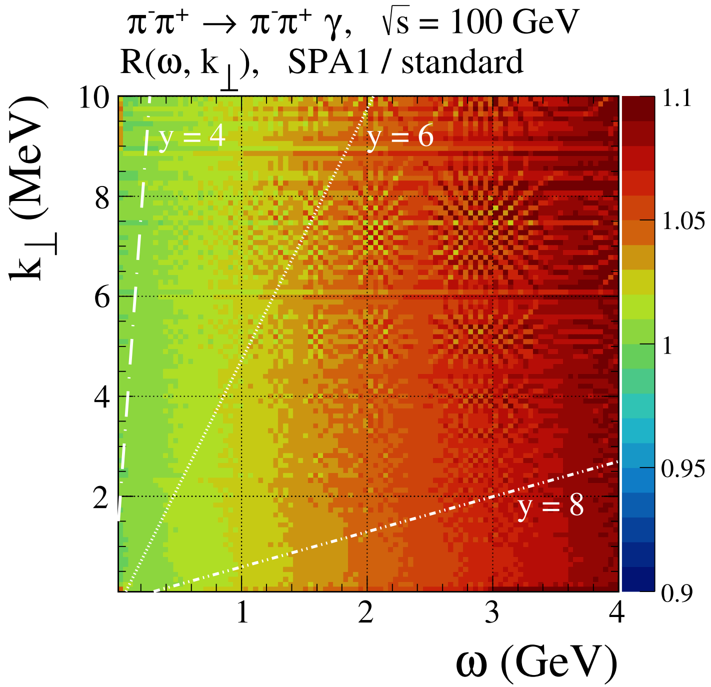
the photon momentum k was, on purpose, omitted in $\delta^{(4)}(.)$

$$\frac{d\sigma(\pi^-\pi^+ \to \pi^-\pi^+)}{d^3p_1 d^3p_2} = \frac{1}{2\sqrt{s(s-4m_\pi^2)}}\,\frac{1}{(2\pi)^3\, 2p_1^0\,(2\pi)^3\, 2p_2^0}(2\pi)^4\delta^{(4)}(p_1+p_2-p_a-p_b)\,|\mathcal{M}^{(0)\pi^-\pi^+}(s,t)|^2$$

Results







- The distribution in rapidity of the photon in the reaction $\pi^-\pi^+ \to \pi^-\pi^+\gamma$ for different $k_\perp$ intervals. Plotted are the results only for positive y since the distributions are symmetric under y → -y
- Two-dimensional distribution in the (ω, $k_\perp$) plane with different rapidity ranges.
- The ratio:

$$\mathrm{R}(\omega, k_\perp) = \frac{d^2\sigma_{\mathrm{SPA1}}/d\omega dk_\perp}{d^2\sigma_{\mathrm{standard}}/d\omega dk_\perp}$$

The accuracies of SPA1 have been determined.
We find reasonable agreement for $k_\perp \lesssim 10$ MeV and $\omega \lesssim 0.5$ GeV.
For larger values of $k_\perp$ and ω the discrepancies between the standard result and SPA result increase rapidly.