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Soft photon bremsstrahlung at higher orders

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Outline

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NLP with shifted kinematics

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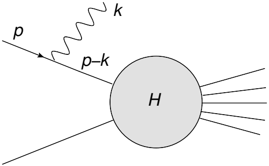
Introduction

AN EXPERIMENTAL CONUNDRUM



Theoretical descriptions of soft photon emission spectra typically rely on a formula based on the **Leading Power (LP)** eikonal approximation, where the photon momentum $k \to 0$:

$$\frac{d\sigma_{\mathrm{LP}}}{d^3k} = \frac{\alpha}{(2\pi)^2}\frac{1}{\omega_k}\int d^3p_3 \cdots \int d^3p_n \left(\sum_{i,j=1}^{n} -\eta_i\eta_j \frac{p_i\cdot p_j}{(p_i\cdot k)(p_j\cdot k)}\right) d\sigma_H(p_1,\ldots,p_n)$$

Eikonal factor is

- universal
- insensitive to spin of the hard emitter
- insensitive to recoil of the hard emitter
- in agreement with classical power spectrum $\frac{d\sigma}{d\omega_k} \sim \frac{1}{\omega_k} \implies I(\omega_k) = \frac{d\sigma}{d\omega_k}\hbar\omega_k \sim \text{const.}$

An experimental conundrum

Tension between data and predicted LP bremsstrahlung spectrum:

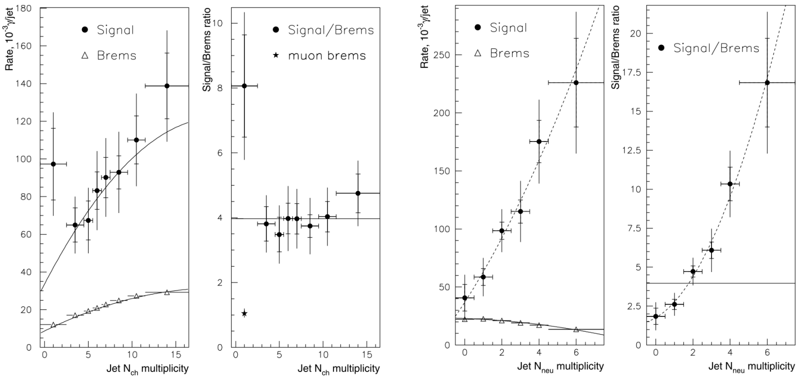
Experiment	Collision Energy	Photon k_T	Photon/Brem Ratio
K^+p, CERN,WA27, BEBC (1984)	70 GeV/c	$k_T < 60$ MeV/c	4.0 $\pm$0.8
K^+p, CERN,NA22, EHS (1993)	250 GeV/c	$k_T < 40$ MeV/c	6.4 $\pm$1.6
π^+p, CERN,NA22, EHS (1997)	250 GeV/c	$k_T < 40$ MeV/c	6.9 $\pm$1.3
π^-p, CERN,WA83,OMEGA (1997)	280 GeV/c	$k_T < 10$ MeV/c	7.9 $\pm$1.4
π^+p, CERN,WA91,OMEGA (2002)	280 GeV/c	$k_T <$20 MeV/c	5.3 $\pm$0.9
pp, CERN,WA102,OMEGA (2002)	450 GeV/c	$k_T <$20 MeV/c	4.1 $\pm$0.8
$e^+e^-\to$hadrons, CERN,DELPHI with hadron production (2010)	$\sim$91 GeV(CM)	$k_T <$60 MeV/c	4.0
$e^+e^-\to\mu^+\mu^-$, CERN,DELPHI with no hadron production (2008)	$\sim$91 GeV(CM)	$k_T <$60 MeV/c	1.0

[Table taken from Cheuk-Yin Wong, arXiv:1404.0040. See also Martha Spyropoulus-Stassinaki, CF 2002, V. Perepelitsa, for the DELPHI Collaboration, Nonlin. Phenom. Complex Syst. 12, 343 (2009)]

An experimental conundrum

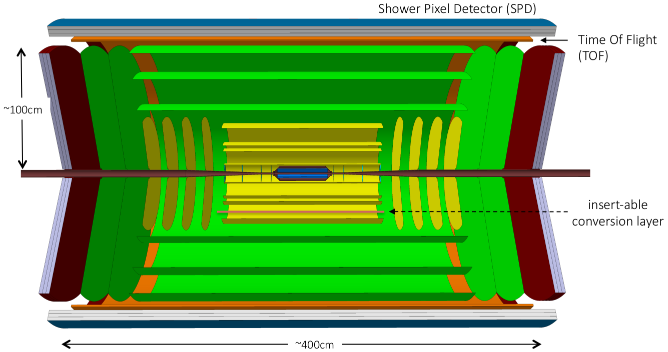
DELPHI data for hadronic Z decays

Photon range: 200 MeV $< \omega_k <$ 1 GeV, p_t (w.r.t. jet) $<$ 80 MeV



[Abdallah et al. Eur. Phys. J. C (2010) 67: 343–366]

Planned upgrade of the ALICE detector



[Credits: D. Adamová et al., “A next-generation LHC heavy-ion experiment,” 1902.01211.]

- possibility to measure ultra soft photons at very low transverse momentum

How can we improve the theoretical predictions?

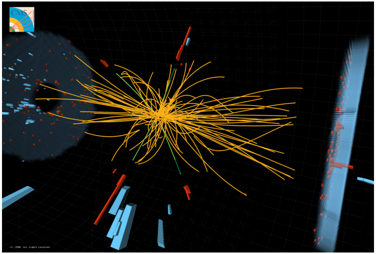
How can we improve the theoretical predictions?

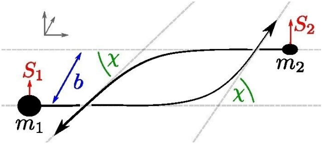
A possibility within the realm of **perturbation theory** and parton model: go beyond the soft approximation $k \to 0$ at Next-to-Leading Power

Disclaimer: every time I will mention a hadron I have in mind a **massless particle** (a quark or a gluon). The corresponding partonic cross section (which contain collinear divergences) must be convoluted with the non-perturbative PDFs/FFs

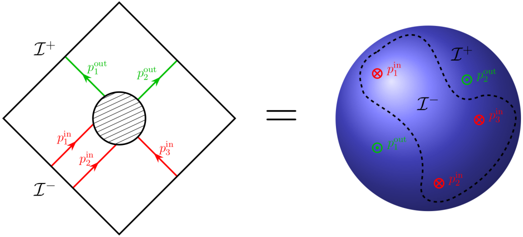
$$\sigma = \int \phi_1 \, \phi_2 \, \hat{\sigma} \; + \; \text{non pert. corr.}$$

NLP: AN INTERDISCIPLINARY AREA





NLP in k as perturbation theory in G/b
(corrections to Newton)



[Image credits: CMS (cds.cern.ch/record/1406073), Antonelli, Kavanagh, Khalil, Steinhoff, Vines PRL 125, 011103, Strominger arXiv:1703.05448]

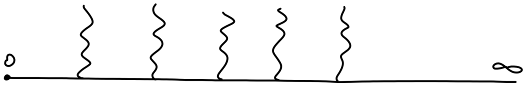
A worldline approach: the GWL

- **Wilson Line** on a straight trajectory is well-known:

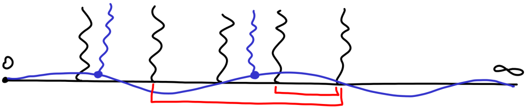
$$\text{QCD: } W_p(0,\infty) = \exp\left\{ ig\int_0^\infty dt\, p^\mu A_\mu(pt)\right\} = e^{ig\int \frac{d^dk}{(2\pi)^d}\frac{p_\mu}{p\cdot k}\tilde{A}^\mu(k)}$$

$$\text{grav: } W_p(0,\infty) = \exp\left\{ \frac{-i\kappa}{2}\int_0^\infty dt\, p_\mu p_\nu\, h^{\mu\nu}(pt)\right\} = e^{\frac{-\kappa}{2}\int \frac{d^dk}{(2\pi)^d}\frac{p_\mu p_\nu}{p\cdot k}\tilde{h}^{\mu\nu}(k)}$$

WL generates **an infinite number of soft emissions** along direction p^μ (soft resummation)



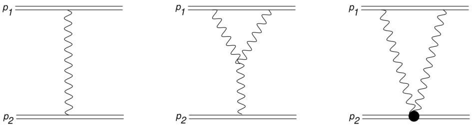
- **Generalized Wilson Line** $\widetilde{W}_p(0,\infty)$ [Laenen, Stavenga, White 2008, White 2011, DB 2020]: generalize this procedure to subleading powers in k (i.e. include fluctuations and correlations) .



A worldline approach: the GWL

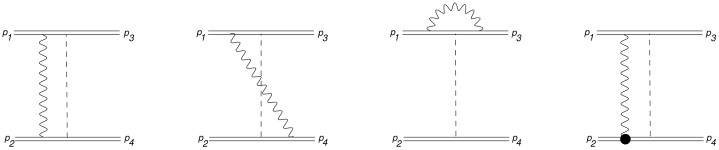
[Luna,Melville,Naculich,White 2017, DB, Kulesza, Pirsch 2021]

- **classical** terms: deflection angle θ from the eikonal phase χ_{NE}



$$e^{i\chi_{\mathrm{NE}}} = \langle 0|\widetilde{W}^{\mathrm{cl}}_{p_1}(0,-\infty,\infty)\widetilde{W}^{\mathrm{cl}}_{p_2}(z,-\infty,\infty)|0\rangle\ , \qquad \theta \sim \frac{\partial \chi_{\mathrm{NE}}}{\partial z}$$

- inclusion of **quantum** terms: Reggeization

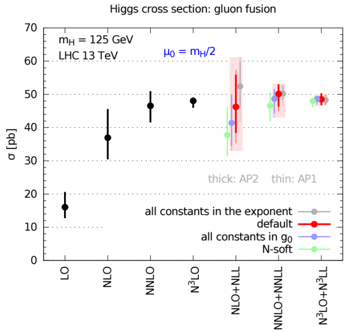


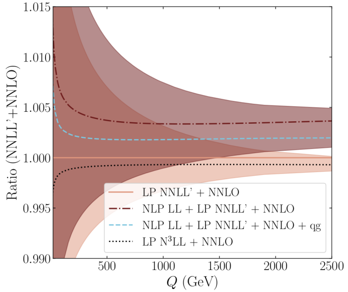
SOFT GLUON RESUMMATION AT NLP

Goal is to extend traditional threshold resummation ($\xi \to 0$) at NLP

[Abbas, DB, Damsté, Laenen, Magnea, Vernazza, vanBeekveld, White, Beneke, Broggio, Garny, Jaskiewicz, Szafron, Wang, Moult, Stewart, Tackmann, Vita, Zhu, Liu, Neubert,... 2015-2022]

$$\frac{d\sigma}{d\xi} = \sum_{n=0}^{\infty} \sum_{m=0}^{2n-1} \Bigg(\underbrace{c_{nm}^{(-1)} \left(\frac{\log^m \xi}{\xi} \right)_+}_{\text{LP (LL,NLL,NNLL,...)}} + \underbrace{c_{nm}^{(0)} \log^m \xi}_{\text{NLP (LL,NLL,NNLL,...)}} + \dots \Bigg)$$





[Image credits: (left) Bonvini, Marzani, Muselli, Rottoli, 1603.08000, (right) van Beekveld, Laenen, Sinninghe Damsté, Vernazza 2101.07270]

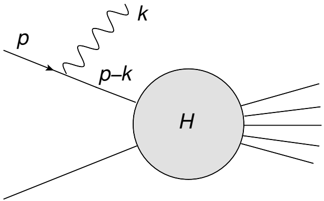
In the light of this recent progress in NLP techniques in different areas of physics, it is natural to ask:

can we apply these techniques to the photon bremsstrahlung? How?

Review of Low-Burnett-Kroll theorem at NLP

LBK THEOREM (LP)

Consider the **Feynman diagram** for a photon emitted from an initial state fermion of charge Q. The interactions of the fermion with the other hard particles can be collected into the sub-diagram $\mathcal{H}$.



$$\mathcal{A}^\mu = \mathcal{H}(p-k)\,\frac{(\not{p}-\not{k})}{(p-k)^2}\,(Q\,\gamma^\mu)\,u(p)$$

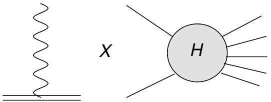
Then take the leading term for $k \to 0$ (**eikonal** approximation):

$$\mathcal{H}(p-k) = \mathcal{H}(p) + \mathcal{O}(k^0)\,, \qquad (p-k)^2 = -2p\cdot k + \mathcal{O}(k^0)\,, \tag{1}$$

$$\not{p}\gamma^\mu\, u(p) = (2p^\mu - \gamma^\mu \not{p})\, u(p) = 2p^\mu\, u(p) \quad \text{(Dirac eq.)}\ . \tag{2}$$

Then the radiative amplitude $\mathcal{A}^\mu$ becomes proportional to the non-radiative amplitude $\mathcal{A}$:

$$\mathcal{A}^\mu = \left(-Q\,\frac{p^\mu}{p\cdot k}\right)\underbrace{\bar{u}(p)\,\mathcal{H}(p)}_{\mathcal{A}} + \mathcal{O}(k^0) \quad =$$



LBK THEOREM (LP)

Summing over the n charged external legs of the amplitude $\mathcal{A}_n$ (inserting a factor $\eta = \pm 1$ for incoming-outgoing particles), we get the **leading soft theorem**:

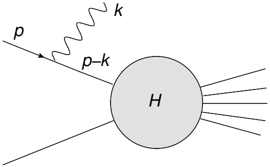
$$\epsilon^*_\mu(k)\mathcal{A}^\mu = \mathcal{S}_{LP}\mathcal{A}_n \;, \qquad \mathcal{S}_{LP} = \sum_{i=1}^{n} Q_i \eta_i \frac{\epsilon^*(k)\cdot p_i^\mu}{p_i \cdot k} \tag{3}$$

- The photon interacts only via the eikonal rule $\frac{p^\mu}{p\cdot k}$: we lost information on the spin of the external legs (i.e. the charged emitters)
- hard particles do not recoil, because photon momentum $k \to 0$
- soft factor has decoupled from the hard dynamics, thus is insensitive to the short distance physics, i.e. soft photons cannot resolve details of the non radiative amplitude $\mathcal{A}_n$

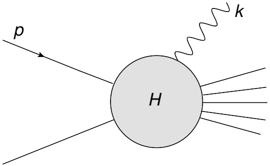
LBK theorem (NLP)

From NLP the photon starts to be sensitive the hard dynamics, hence we allow the possibility that it is emitted from a virtual particle inside the subdiagram $\mathcal{H}$.

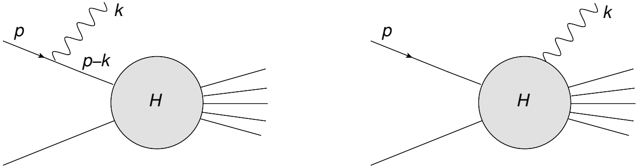
Therefore, we classify the diagrams in **external emissions** (already present at LP)



and **internal emissions**:



LBK theorem (NLP)



- External emission: expand up to $\mathcal{O}(k)$

$$\mathcal{A}^{\mu}_{\text{ext}}(p) = \mathcal{H}(p-k)\,\frac{(\not{p}-\not{k})}{(p-k)^2}\,(Q\,\gamma^{\mu})u(p) \tag{4}$$

$$= Q\,\mathcal{H}(p)\left(\frac{p^{\mu}}{p\cdot k} + \frac{k^{\mu}}{2p\cdot k} - \frac{k^2 p^{\mu}}{2(p\cdot k)^2} - \frac{i k_{\nu}\sigma^{\mu\nu}}{p\cdot k}\right)u(p)$$

$$+\; Q\,\frac{p^{\mu}}{p\cdot k}k^{\nu}\underbrace{\frac{\partial\mathcal{H}(p-k)}{\partial k_{\nu}}\Big|_{k=0}}_{-\frac{\partial\mathcal{H}(p)}{\partial p_{\nu}}}u(p) \;+\; \mathcal{O}(k)$$

Here we used $\gamma^{\mu}\gamma^{\nu} = g^{\mu\nu} - i\sigma^{\mu\nu}$ where $\sigma^{\mu\nu} = \frac{i}{4}[\gamma^{\mu},\gamma^{\nu}]$ is the Lorentz generator for particles of spin $\frac{1}{2}$. Then sum over all external legs $\mathcal{A}^{\mu}_{\text{ext}} = \sum_i \mathcal{A}^{\mu}_{\text{ext}}(p_i)$

LBK theorem (NLP)

- Internal emission: use Ward identity $k_\mu(\mathcal{A}^\mu_{\text{ext}} + \mathcal{A}^\mu_{\text{int}}) = 0$

$$\mathcal{A}^\mu_{\text{int}} = \sum_i Q_i \frac{\partial \mathcal{H}(p^i)}{\partial p^i_\mu} u(p^i) \tag{5}$$

- Adding $\mathcal{A}^\mu_{\text{ext}}$ and $\mathcal{A}^\mu_{\text{int}}$:

$$\begin{aligned}\mathcal{A}^\mu &= \sum_i Q_i \frac{p_i^\mu}{p_i \cdot k} \mathcal{A}(p_1 ... p_n) \\ &+ \sum_i Q_i \left(\frac{k^\mu}{2p \cdot k} - \frac{k^2 p^\mu}{2(p \cdot k)^2} - \frac{i k_\nu \sigma^{\mu\nu}}{p \cdot k} \right) \mathcal{A}(p_1 ... p_n) \\ &+ \sum_i Q_i \underbrace{\left(-\frac{p_i^\mu k^\nu}{p_i \cdot k} \frac{\partial}{\partial p_i^\nu} + \frac{\partial}{\partial p_i^\mu} \right)}_{-\frac{k_\nu}{p_i \cdot k} \underbrace{\left(p_i^\mu \frac{\partial}{\partial p_i^\nu} - p_i^\nu \frac{\partial}{\partial p_i^\mu} \right)}_{\equiv L^{\mu\nu}}} \mathcal{A}(p_1 ... p_n) \end{aligned} \tag{6}$$

$L^{\mu\nu}$ is the angular momentum generator of the Lorentz group

LBK THEOREM (NLP)

This is the **sub-leading** soft theorem, known as **Low-Burnett-Kroll theorem**:
[Low 1958 (scalar emitters), Burnett-Kroll 1968 (spin $\frac{1}{2}$ emitters, conjecture for generic spin), Bell-VanRoyen 1969 (generic spin)]
For a real photon ($k^2 = 0,\ \epsilon(k) \cdot k = 0$) it has the more compact form

$$\epsilon^*_\mu(k)\mathcal{A}^\mu = \left(\mathcal{S}_{LP} + \mathcal{S}_{NLP-tree}\right)\mathcal{A}_n\ , \tag{7}$$

$$\mathcal{S}_{LP} = \sum_{i=1}^{n} Q_i \frac{\epsilon^*(k) \cdot p_i}{p_i \cdot k}\ , \quad \mathcal{S}_{NLP-tree} = \sum_{i=1}^{n} Q_i \frac{\epsilon^*_\mu(k) k_\nu (\sigma^{\mu\nu} + L^{\mu\nu})}{p_i \cdot k} \tag{8}$$

- corrections to the strict limit $k \to 0$: small recoil of the emitter taken into account
- sensitive to the spin of the emitter (e.g. $\sigma^{\mu\nu} = 0$ for scalars, $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$for spin 1/2, etc.)
- orbital angular momentum $L^{\mu\nu}$ is sensitive to the short distance interactions in $\mathcal{A}$ (hard lines do not start from a pointlike vertex)
- NLP corrections here are valid only at the tree-level

NLP (tree-level) with shifted kinematics

NLP BREMSSTRAHLUNG WITH SHIFTED KINEMATICS

Squaring amplitude and summing over polarizations

$$\sum_{\text{pol}} |\mathcal{A}(p_1,\ldots,p_n,k)|^2 = \sum_{ij}(-\eta_i\eta_j)\frac{p_i\cdot p_j}{p_i\cdot k\, p_j\cdot k}|\mathcal{A}(p_1,\ldots,p_n)|^2 \quad \to \textbf{LP}$$

$$+\sum_{ij}(-\eta_i\eta_j)\frac{p^i_\mu}{p_i\cdot k}G_j^{\mu\nu}\frac{\partial}{\partial p_j^\nu}|\mathcal{A}(p_1,\ldots,p_n)|^2 \quad \to \textbf{NLP}$$

where

$$G_i^{\mu\nu} = g^{\mu\nu} - \frac{(2p_i-k)^\mu k^\nu}{2p_i\cdot k} = g^{\mu\nu} - \frac{p_i^\mu k^\nu}{p_i\cdot k} + \mathcal{O}(k)$$

Some drawbacks:

- How to efficiently implement NLP corrections in cross-sections?
- Derivatives might give instabilities in numerical implementations
- Violation of momentum conservation in non-radiative amplitude
- Similar interest in QCD-resummation program ($\to$ need for alternative analytic forms of LBK theorem)

$\to$ rewrite NLP contribution in terms of shifted kinematics

LBK theorem becomes

$$|\mathcal{A}(p_1,\ldots,p_n,k)|^2 = \Bigg(\sum_{i,j=1}^{n} \underbrace{-\eta_i\eta_j \frac{p_i\cdot p_j}{p_i\cdot k\, p_j\cdot k}}_{\textbf{LP}\text{ factor!}}\Bigg) |\mathcal{A}(p_1+\delta p_1,\ldots,p_n+\delta p_n)|^2$$

where shifts are of order $\mathcal{O}(k)$ and are defined as

$$\delta p_\ell^\mu = \left(\sum_{i,j=1}^{n} \eta_i\eta_j \frac{p_i\cdot p_j}{p_i\cdot k\, p_j\cdot k}\right)^{-1} \sum_{m=1}^{n}\left(\eta_m\eta_\ell \frac{(p_m)_\nu G_\ell^{\mu\nu}}{p_m\cdot k}\right)$$

Note that

$$\delta p_1^\mu + \cdots + \delta p_n^\mu = -k^\mu$$

$$\delta p_i\cdot p_i = 0 \qquad\qquad \delta p_i\cdot k = -\frac{k^2}{2} = 0$$

hence momentum conservation is restored in the non-radiating amplitude.

NLP BREMSSTRAHLUNG WITH SHIFTED KINEMATICS

Kinematical invariants $s_{ij} = (p_i + p_j)^2$ are shifted according to

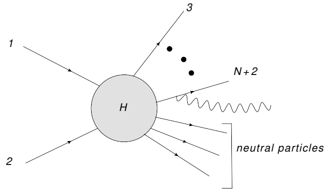
$$s_{ij} \to s_{ij}\left(1 - \frac{2(p_i + p_j)\cdot k}{s_{ij}} R_{ij}\right)$$

$$R_{ij} = \left(\sum_{a,b=1}^{n} \eta_a \eta_b \frac{p_a \cdot p_b}{p_a \cdot k\, p_b \cdot k}\right)^{-1} \left(\eta_i \frac{(p_i)_\mu}{p_i \cdot k} + \eta_j \frac{(p_j)_\mu}{p_j \cdot k}\right) \sum_{c=1}^{n} \eta_c \frac{p_c^\mu}{p_c \cdot k}$$

e.g.

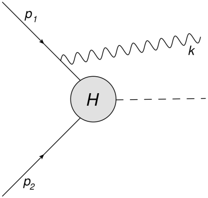
$$s = (p_1 + p_2)^2 \to s\,(1 - (1-z)R_{12})$$

where $z = Q^2/s$. Q^2 is the invariant mass of the charged final states.



NLP BREMSSTRAHLUNG WITH SHIFTED KINEMATICS

A simple case: $n = 2$ [DelDuca, Laenen, Mangea, Vernazza, White 2017]



$$|\mathcal{A}(p_1, p_2, k)|^2 = \Bigg(\sum_{i,j=2}^{n} \underbrace{-\eta_i \eta_j \frac{p_i \cdot p_j}{p_i \cdot k \, p_j \cdot k}}_{\textbf{LP}\text{ factor!}} \Bigg) |\mathcal{A}(p_1 + \delta p_1, p_2 + \delta p_2)|^2$$

$$\delta p_1^\mu = -\frac{1}{2} \left(\frac{p_2 \cdot k}{p_1 \cdot p_2} p_1^\mu - \frac{p_1 \cdot k}{p_1 \cdot p_2} p_2^\mu + k^\mu \right) \tag{9}$$

$$\delta p_2^\mu = -\frac{1}{2} \left(-\frac{p_2 \cdot k}{p_1 \cdot p_2} p_1^\mu + \frac{p_1 \cdot k}{p_1 \cdot p_2} p_2^\mu + k^\mu \right)$$

Also $R_{12} = 1$, thus

$$s \to s\, z = Q^2$$

NLP BREMSSTRAHLUNG WITH SHIFTED KINEMATICS

Shifted kinematics allows to efficiently implement LBK th. in the bremsstrahlung cross-section

$$\frac{d\sigma_{\text{LP+(NLP-tree)}}}{d^3k} = \frac{\alpha}{(2\pi)^2}\frac{1}{\omega_k}\int d^3p_3 \cdots \int d^3p_n \left(\sum_{i,j=1}^{n} -\eta_i\eta_j \frac{p_i \cdot p_j}{(p_i \cdot k)(p_j \cdot k)}\right)$$
$$(1-(1-z)R_{12})\; d\sigma_H(p_1+\delta p_1, ..., p_n+\delta p_n)$$

- $(\omega_k)^0 \sim 1$ correction to LP soft photon bremsstrahlung ($\sim \frac{1}{\omega_k}$)
- NLP shifts take into account spin and recoil of the hard emitter
- for which value of ω_k and p_t can we test NLP-tree level effects?

NLP with QCD loop corrections

- at LP, soft theorems **do not** receive **loop corrections**.

$$\epsilon^*_\mu(k)\mathcal{A}^\mu = \mathcal{S}_{LP}\ \mathcal{A}_n\ , \qquad \mathcal{A}_n = \mathcal{A}_n^{(0)}, \mathcal{A}_n^{(1)}, \mathcal{A}_n^{(2)}, \ldots$$

$$\mathcal{S}_{LP} = \sum_{i=1}^{n} Q_i \frac{\epsilon^*(k)\cdot p_i}{p_i \cdot k}\ ,$$

- at NLP, soft theorems **do** receive **loop corrections**.[Bern,Davies,Nohle 2014, He,Huang,Wen 2014, Larkoski,Neill, Stewart 2014, DB,Laenen,Magnea,Vernazza,White 2014]

$$\epsilon^*_\mu(k)\mathcal{A}^{\mu(0)} = \left(\mathcal{S}_{LP} + \mathcal{S}_{NLP-tree}\right)\mathcal{A}_n^{(0)}\ ,$$

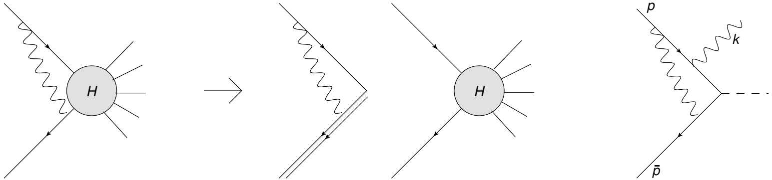
$$\epsilon^*_\mu(k)\mathcal{A}^{\mu(1)} = \left(\mathcal{S}_{LP} + \mathcal{S}_{NLP-tree}\right)\mathcal{A}_n^{(1)} + \mathbf{?}\ ,$$

$$\mathcal{S}_{LP} = \sum_{i=1}^{n} Q_i \frac{\epsilon^*(k)\cdot p_i}{p_i \cdot k}\ , \qquad \mathcal{S}_{NLP-tree} = \sum_{i=1}^{n} Q_i \frac{\epsilon^*_\mu(k)k_\nu(\sigma^{\mu\nu} + L^{\mu\nu})}{p_i \cdot k}$$

Various sources of correction. E.g. soft region in the massive case [Engel,Signer,Ulrich 2021]. In the high energy limit, it is interesting to look at the **massless limit** (crucial for the massless parton model) and the **collinear region**

NLP bremsstrahlung with QCD corrections

Virtual collinear effects are captured by **radiative jet functions** J^μ [DelDuca 1990, DB, Laenen, Magnea, Vernazza, White 2014, Gervais 2017, Beneke, Garny, Szafron, Wang 2018, Laenen, Damste, Vernazza, Waalewijn, Zoppi 2020, Liu, Neubert, Schnubel, Wang 2021].



In particular, the one-loop quark radiative jet function in dimensional regularization (with $d = 4 - 2\epsilon$ and $\bar{\mu}$ the $\overline{\text{MS}}$ scale) reads
[DB,Laenen,Magnea,Melville,Vernazza,White,2015]

$$J^{\mu(1)} = \left(\frac{\bar{\mu}^2}{2p\cdot k}\right)^{\epsilon} \left[\left(\frac{2}{\epsilon} + 4 + 8\epsilon\right) \left(\frac{n\cdot k}{p\cdot k}\frac{p^\mu}{p\cdot n} - \frac{n^\mu}{p\cdot n}\right) - (1+2\epsilon)\frac{ik_\alpha S^{\alpha\mu}}{p\cdot k} + \left(\frac{1}{\epsilon} - \frac{1}{2} - 3\epsilon\right)\frac{k^\mu}{p\cdot k} + (1+3\epsilon)\left(\frac{\gamma^\mu \not{n}}{p\cdot n} - \frac{p^\mu}{p\cdot k}\frac{\not{k}\not{n}}{p\cdot n}\right)\right] + \mathcal{O}(\epsilon^2, k)$$

NLP bremsstrahlung with QCD corrections

Thus, the next-to-soft theorem (i.e. LBK theorem) receives a **logarithmic correction**:

$$\epsilon^*_\mu(k)\mathcal{A}^{\mu(0)} = \left(\mathcal{S}_{LP} + \mathcal{S}_{NLP-tree}\right)\mathcal{A}_n^{(0)} \,,$$

$$\epsilon^*_\mu(k)\mathcal{A}^{\mu(1)} = \left(\mathcal{S}_{LP} + \mathcal{S}_{NLP-tree}\right)\mathcal{A}_n^{(1)} + \left(\sum_i \epsilon^*_\mu(k)\, q_i\, J_i^{\mu(1)}\right)\mathcal{A}_n^{(0)} \,,$$

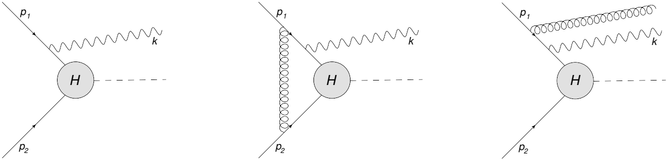
$$\mathcal{S}_{LP} = \sum_{i=1}^{n} Q_i \frac{\epsilon^*(k)\cdot p_i}{p_i\cdot k} \,, \qquad \mathcal{S}_{NLP-tree} = \sum_{i=1}^{n} Q_i \frac{\epsilon^*_\mu(k)k_\nu(\sigma^{\mu\nu} + L^{\mu\nu})}{p_i\cdot k}$$

$$\left(\sum_i \epsilon^*_\mu(k)\, q_i\, J_i^{\mu(1)}\right)\mathcal{A}_n^{(0)} = \frac{2}{p_1\cdot p_2}\left[\sum_{ij}\left(\frac{1}{\epsilon} + \log\left(\frac{\bar{\mu}^2}{2p_i\cdot k}\right)\right) q_j\, p_i\cdot k\, \frac{p_j\cdot\epsilon}{p_j\cdot k}\right]\mathcal{A}_n^{(0)}$$

- Note that amplitude is IR divergent $\epsilon \to 0$
- $\log(\omega_k)$ corrections to soft theorems in QED also discussed (mainly classically) by Laddha-Sahoo-Sen. Here however more standard approach (i.e. dim.reg.) to regularization of soft and collinear divergences, which allows implementation in the massless limit required in QCD partonic calculations.

NLP BREMSSTRAHLUNG WITH QCD CORRECTIONS

IR divergences ($1/\epsilon$) cancel by adding real emission diagram:



The **soft photon** emission from the loop with a **collinear gluon** is captured by the radiative jet function J^μ (note here the **mixed QED-QCD** effect)
The corresponding contribution is what is needed for a process with a single quark-antiquark pair in the **massless limit** such as

- $e^+e^- \to q\,\bar{q}\,\gamma$
- $p\,p \to \mu^+\mu^-\gamma$
- ...

For processes with more than two colored particles situation more subtle (but structure is similar)

NLP BREMSSTRAHLUNG WITH QCD CORRECTIONS

The soft photon bremsstrahlung at $\mathcal{O}(\alpha_s)$ becomes

$$\frac{d\sigma_{\text{NLP}}}{d^3k} = \frac{d\sigma_{\text{LP+(NLP-tree)}}}{d^3k} + \frac{\alpha_s}{4\pi}\frac{d\sigma_{\text{NLP-}J}}{d^3k} \,,$$

where

$$\frac{d\sigma_{\text{NLP-}J}}{d^3k} = \frac{\alpha}{(2\pi)^2}\frac{1}{\omega_k}\int d^3p_3\ldots d^3p_n \left(\sum_{i=1}^{2}\eta_i \frac{8\log\left(\frac{\bar{\mu}^2}{2p_i\cdot k}\right)}{p_i\cdot k}\right) d\sigma_H(p_1,\ldots,p_n)$$

- **Correction of order** $\boxed{\alpha_s \log\left(\frac{\bar{\mu}^2}{2p_i\cdot k}\right)}$ **to LP spectrum** $\frac{d\sigma}{d\omega_k}$
  hence particularly enhanced for small ω_k **and small** k_t
- **relevant only for hadrons (i.e. negligible for leptons -** $\alpha \ll \alpha_s$, $m \to 0$**)**
- Open issues: generic number of QCD-charged particles? More than one-photon emission? How many detected photons in experimental setup? Resummation?

CONDITIONS ON ω_k FOR VALIDITY OF LBK/LOOP CORRECTIONS

TREE LEVEL (relevant scales: soft energy ω_k and hard scale Q)

- when is LP valid? $\omega_k \ll Q$
- when is NLP-tree valid? $\omega_k \ll Q$

1-LOOP LEVEL (loops (i.e. virtual particles) generate new scales!)

- when is LP valid? $\omega_k \ll Q$
- when is NLP valid?
  1. NLP-J ($\sim \log(Q/(p_i \cdot k))$) for $\omega_k \ll Q$ but also $\omega_k > m$ (i.e. $m \to 0$ is safe).
  2. NLP-S ($\sim \log(m/Q), \log(m/(p_i \cdot k))$) for $\omega_k \ll Q$ but also $\omega_k \ll m, Q$

     (fermion mass not negligible)

Need for a **precise** estimate of the cuts on photon energy/p_t. This aspect (which is process-dependent) can be resolved numerically (work in progress)

Conclusions and outlook

Demand for accurate theoretical predictions from experiments:

- Discrepancy th/exp in the soft photon bremsstrahlung $\sim \frac{1}{\omega_k}$
- Plans for precise measurements of ultrasoft photons at LHC

To this aim, tools available from recent progress in QCD resummation.

Correct LP formula with

- NLP-tree-level $\sim (\omega_k)^0 \sim 1$ with kinematical shifts (can we measure the constant shift? at which soft energy/p_t? access to spin of the emitter?)
- NLP-radiative jets $\sim \log \omega_k$ (enhanced for very small energies-high rapidity. Relevant with final state hadrons? jet sub-structure?)

Generalizations, with open issues (check more QCD charged particles, many photons (detected and undetected), resummation, numerics,...)