

# Status and Prospects: Experimental tests of Low's theorem

EMMI Rapid Reaction Task Force (RRTF):

Real and virtual photon production at ultra-low transverse momentum and low mass at LHC

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## A long-standing puzzle

Peter Braun-Munzinger

- EMMI seminar, Feb. 2021, "[Soft photons, the Low theorem, and ALICE 3](#)"

- [Quark Matter 2022 student lecture](#)

In 1958, Francis Low wrote a seminal paper on how to relate hadron production in a high energy collision to the production of soft photons. Francis E. Low, Phys.Rev.Lett. 110 (1958) 468

Striking discrepancies were found between predictions and experimental measurements.

**No agreement exists on their possible origin**, despite > 40 years of research.

Low's theorem is an example of a soft theorem in QFT. Considerable theoretical interest as soft theorems are related to **symmetries reflecting infrared structure of gravity and gauge theory**.\*

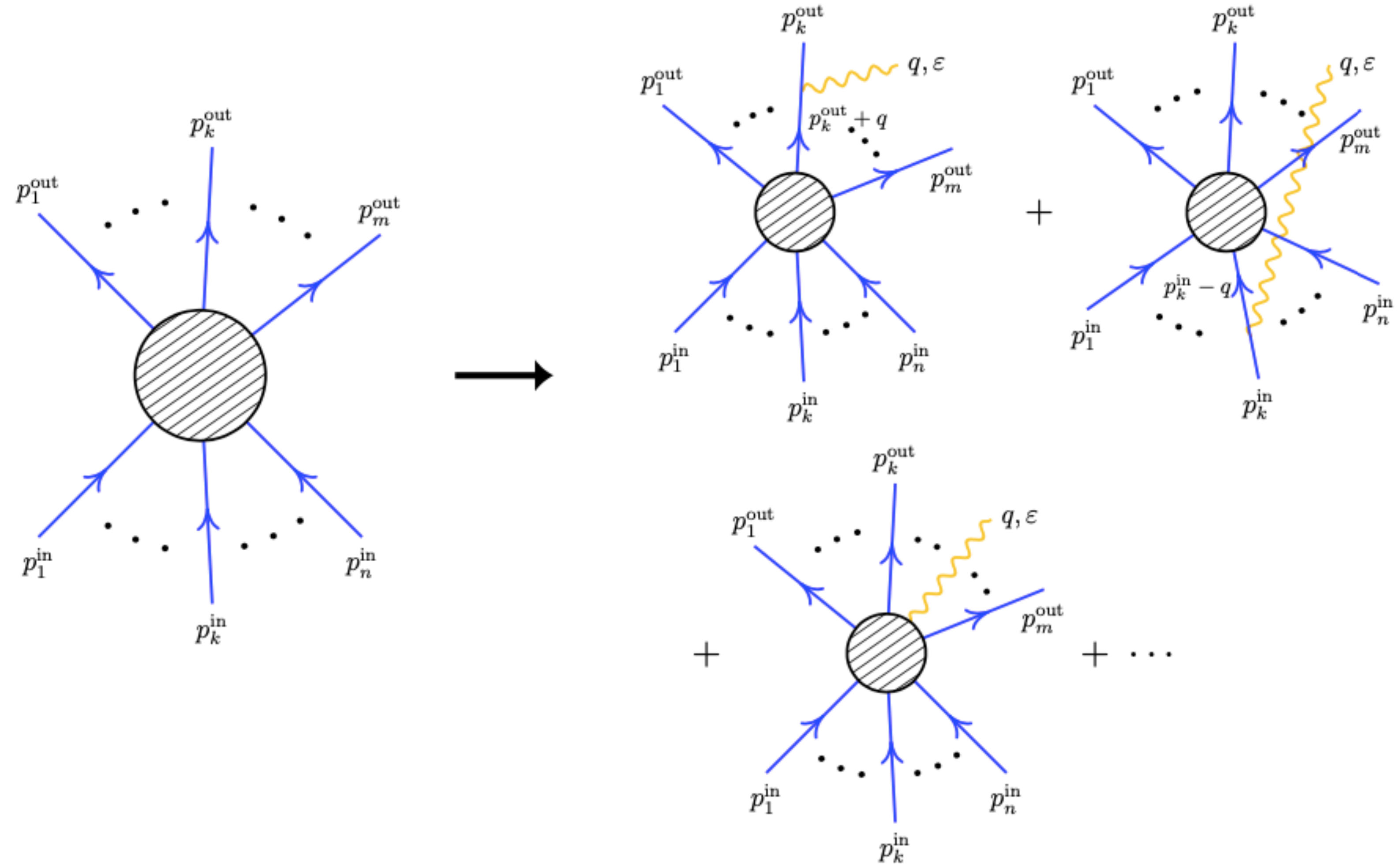
V. Lysov, S. Pasterski and A. Strominger, Phys. Rev. Lett. 113 (2014)

With the **ALICE 3 Forward Conversion Tracker** there is concept for a detector that can provide a soft-photon measurement in the range  $p_T < 10$  MeV/c to test Low's theorem.

\* "These theorems tell us that a surprisingly large — in fact, infinite — number of soft particles are produced in any physical process, but in a highly controlled manner that is central to the consistency of quantum field theory", A. Strominger, arXiv:1703.05448

# Soft photon theorem

$n \rightarrow m$  scattering with an additional soft photon



Andrew Strominger, Lectures on the Infrared Structure of Gravity and Gauge Theory arXiv:1703.05448

Add a vertex and propagator for the particle to whose external leg the photon is added

Propagator for a scalar particle of mass  $m$  gives a factor

$$\frac{-i}{(p+q)^2 + m^2} = \frac{-i}{p^2 + 2p \cdot q + q^2 + m^2} = \frac{-i}{2p \cdot q}$$

$q^\mu$  : momentum vector of the photon  
polarization:  $\varepsilon^\mu q_\mu = 0$

Here we used that all the external lines must be on-shell:  $q^2 = 0$  and  $p^2 = -m^2$

(-, +, +, +) signature:  $p \cdot q = \vec{p} \cdot \vec{q} - p^0 q^0$

Total contribution to scattering amplitude:

$$ie\varepsilon^\mu (2Qp_\mu) \frac{-i}{(p+q)^2 + m^2} \rightarrow \frac{eQ\varepsilon \cdot p}{q \cdot p}$$



## Amplitude = original amplitude multiplied by the soft factor

There is one such term for every outgoing particle, while for the incoming particles there is an additional minus sign:

$$\sum_{k=1}^m \frac{e Q_k^{\text{out}} p_k^{\text{out}} \cdot \varepsilon}{p_k^{\text{out}} \cdot q} - \sum_{k=1}^n \frac{e Q_k^{\text{in}} p_k^{\text{in}} \cdot \varepsilon}{p_k^{\text{in}} \cdot q}$$

For diagrams in which the photon attaches to an external leg, we simply multiply the matrix element by this factor.

Internal propagators are never on-shell and one then never has the cancellation between  $p^2$  and  $m^2$ , so if we take  $q_\mu \rightarrow 0$ , the difference between  $p^2$  and  $m^2$  will dominate, and we will not get a pole.

$$\langle \text{out} | a_+^{\text{out}}(\vec{q}) \mathcal{S} | \text{in} \rangle = e \left[ \sum_{k=1}^m \frac{Q_k^{\text{out}} p_k^{\text{out}} \cdot \varepsilon^+}{p_k^{\text{out}} \cdot q} - \sum_{k=1}^n \frac{Q_k^{\text{in}} p_k^{\text{in}} \cdot \varepsilon^+}{p_k^{\text{in}} \cdot q} \right] \langle \text{out} | \mathcal{S} | \text{in} \rangle + \mathcal{O}(q^0)$$

This is the **soft photon theorem**: any S-matrix element with an additional soft ( $q_\mu \rightarrow 0$ ) photon is equal to the original matrix element multiplied by the soft factor plus corrections of order  $q_0$ .

## Low's theorem and its range of validity

### Strict range of validity rather limited

Francis E. Low, Phys.Rev.Lett. 110 (1958) 468  
S. Weinberg, Phys. Rev. 140, B516 (1965)  
Vittorio Del Duca, Nucl.Phys.B 345 (1990) 369-388  
Domenico Bonocore, Anna Kulesza, 2112.08329

### Low's theorem

Both the leading ( $\omega_k^{-1}$ ) and next-to-leading ( $\omega_k^0$ ) behavior in the photon energy of a general bremsstrahlung amplitude may be expressed in terms of the corresponding nonradiative amplitude.

Leading power result is universal and does not receive loop corrections.

### Range of validity

Low theorem holds if photon energy  $\omega_k$  is smaller than any other momentum scale in the amplitude.

At next-to-leading power Low's theorem is only valid for  $\omega_k \ll \frac{m^2}{Q}$ .

$\omega_k$ : photon energy  
 $m$ : mass of the lightest charged particle  
 $Q$ : typical energy  $Q$  of the process

## Low's theorem as used by experiments

# Leading Power (LP) term of order $1/E_\gamma$ in Low's soft theorem

Photon momentum spectrum ("inner bremsstrahlung"):

$$\frac{dN_\gamma}{d^3\vec{k}} = \frac{\alpha}{(2\pi)^2} \frac{-1}{E_\gamma} \int d^3\vec{p}_1 \dots d^3\vec{p}_N \left( \sum_i \frac{\eta_i e_i P_i}{P_i K} \right)^2 \frac{dN_{\text{hadrons}}}{d^3\vec{p}_1 \dots d^3\vec{p}_N}$$

$\sum_i$  : sum over  $N + 2$  particles (2 incoming,  $N$  outgoing)

$K, \vec{k}$  : photon four- and three momentum ( $E_\gamma \equiv |\vec{k}|$ )

$P_i, \vec{p}_i$  : four- and three momentum of particle  $i$

$e_i = 1$  for positive particle,  $e_i = -1$  for negative particle

$\eta_i = 1$  for outgoing particle,  $\eta_i = -1$  for incoming particle

First explicitly shown in

Goshaw et al.,

Phys. Rev. Lett. 43, 1065 (1979)

Francis E. Low, Phys.Rev.Lett. 110 (1958) 468

see also

Belogianni et al. (WA102), Phys. Lett. B 548, 129 (2002)

DELPHI, Eur. Phys. J. C 47, 273 (2006)



# Anomalous soft photon production: excess above inner bremsstrahlung

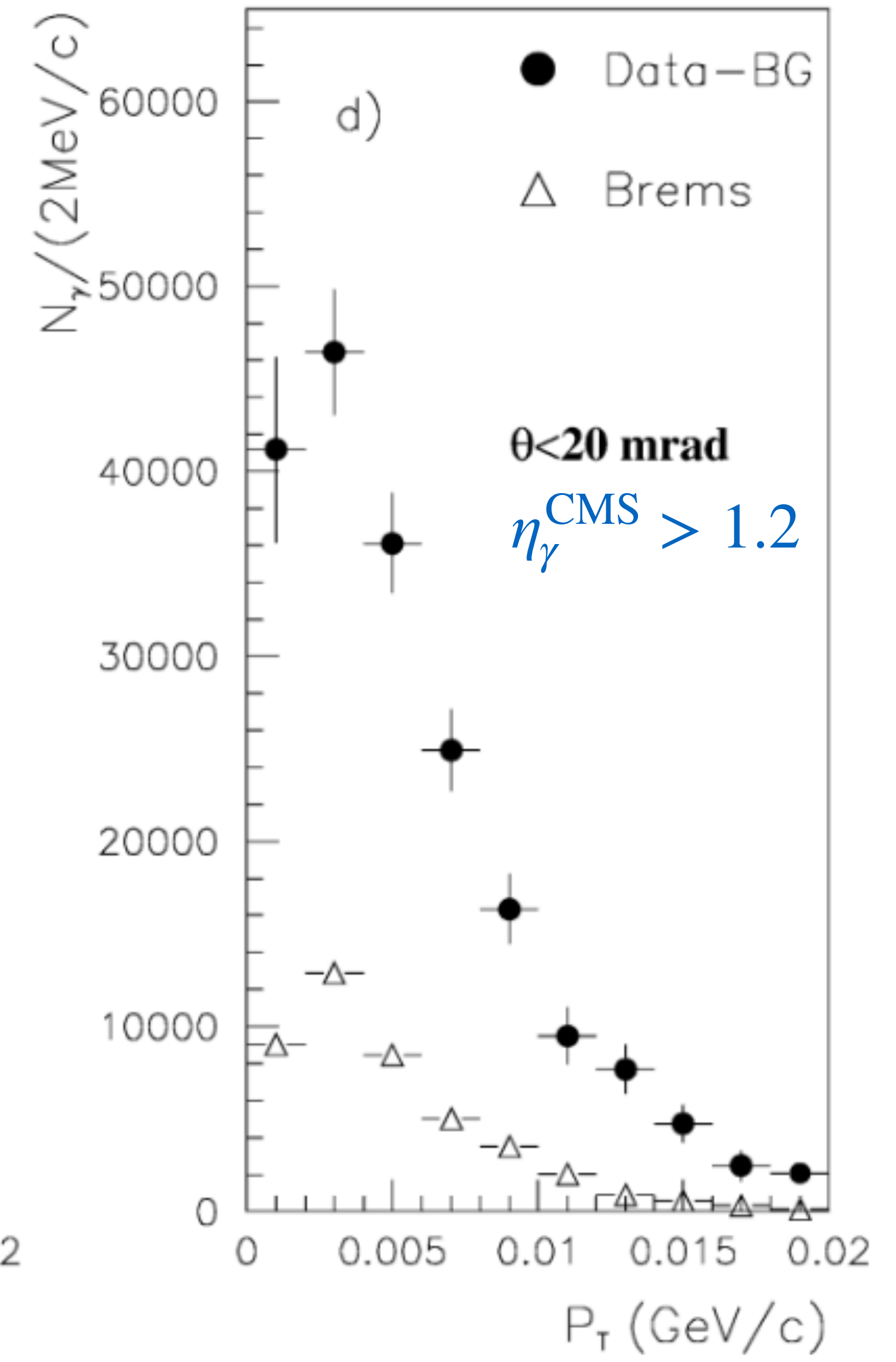
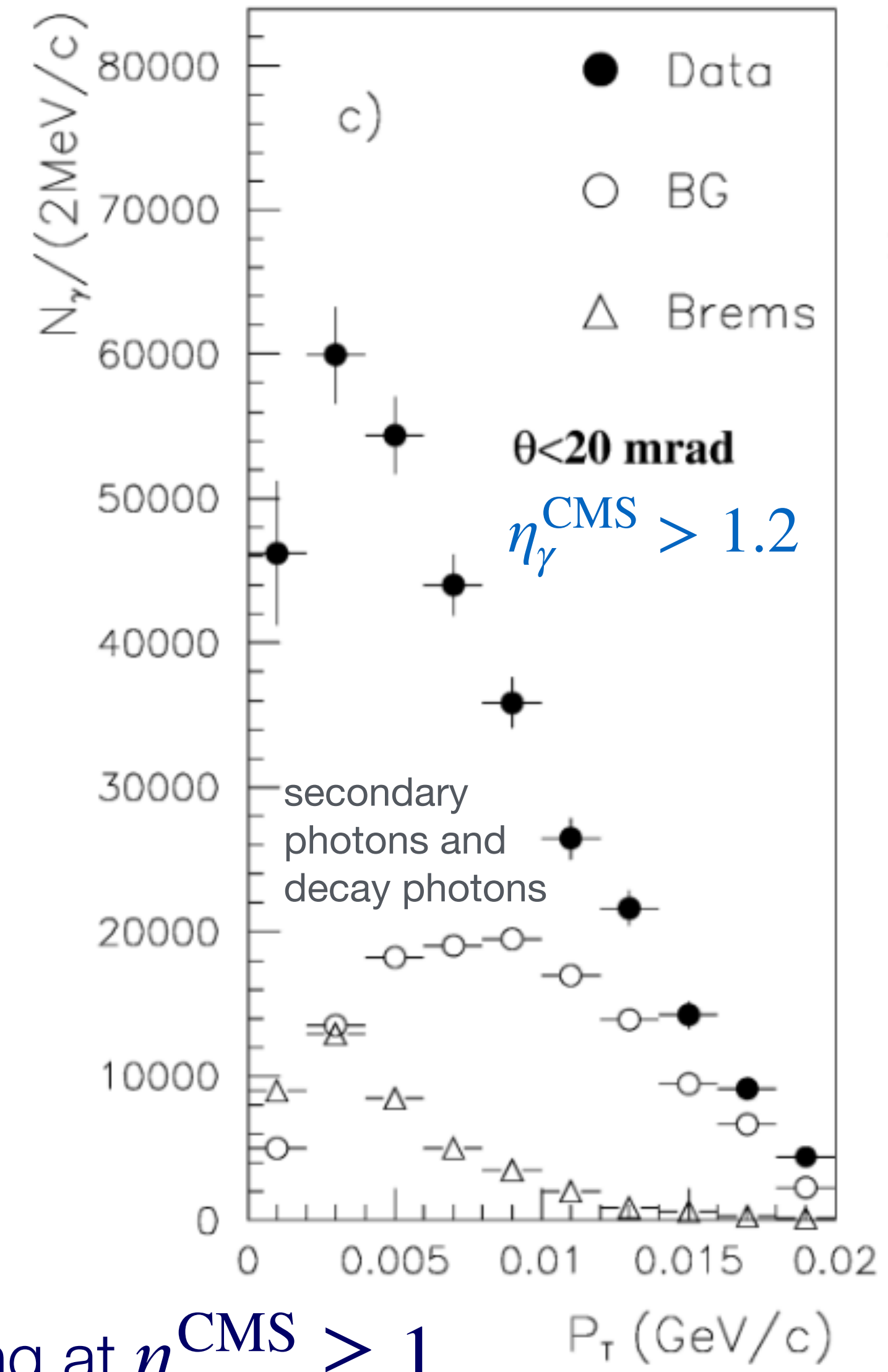
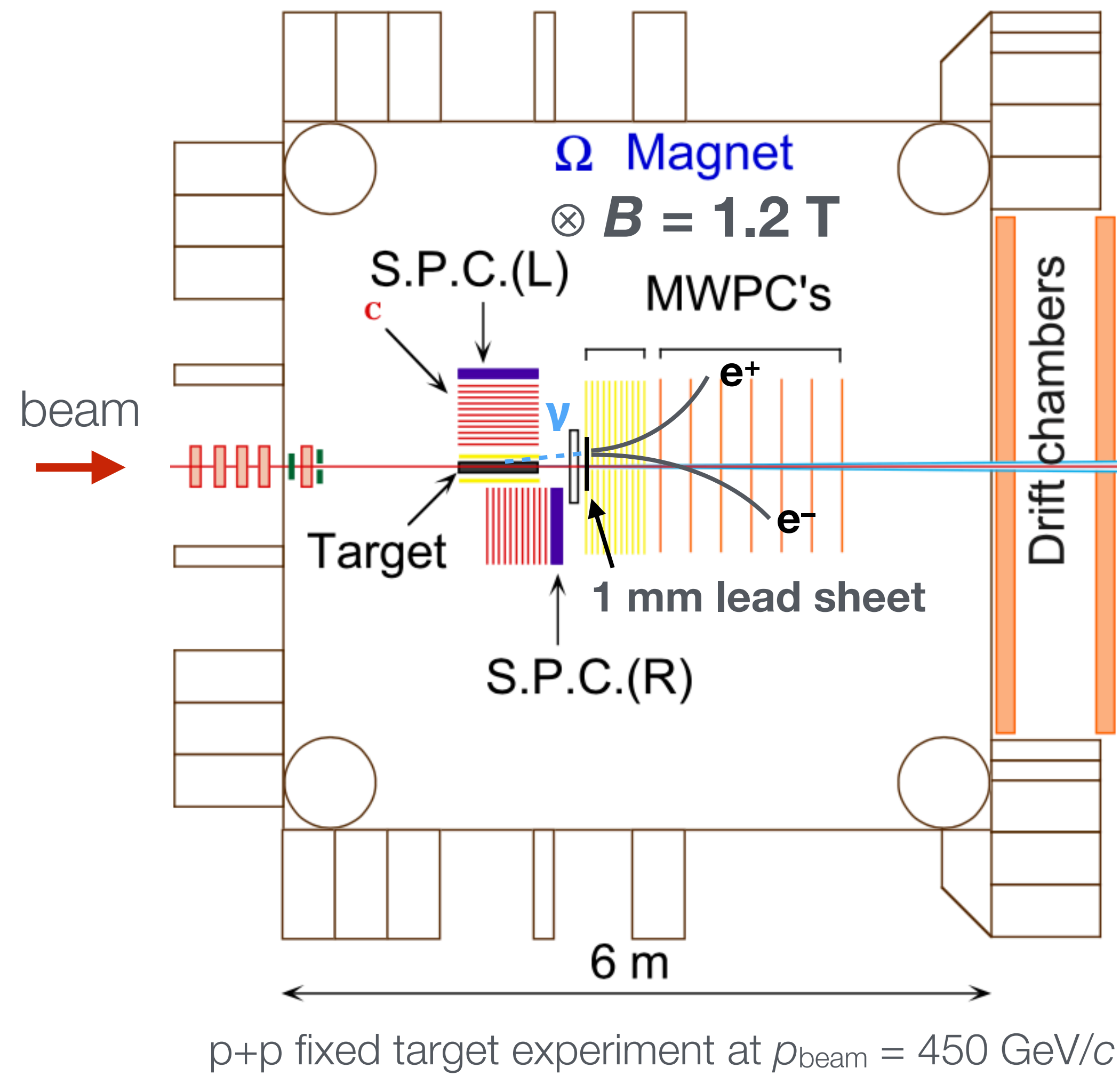
Most experiments show excess by a factor in the range 4 – 8

| Experiment  | Year | Collision energy | Photon $p_T$   | Photon / Brems Ratio  | Detection method                | Reference<br>(click to go to paper)                               |
|---|------|------------------|--|-----------------------|---------------------------------|---|
| $\pi^+p$  | 1979 | 10.5 GeV         | $p_T < 30 \text{ MeV}/c$   | $1.25 \pm 0.25$       | bubble chamber                  | <a href="#">Goshaw et al., Phys. Rev. Lett. 43, 1065 (1979)</a>   |
| $K^+p$<br>WA27, CERN                                | 1984 | 70 GeV           | $p_T < 60 \text{ MeV}/c$   | $4.0 \pm 0.8$         | bubble chamber<br>(BEBC)        | <a href="#">Chliapnikov et al., Phys. Lett. B 141, 276 (1984)</a> |
| $\pi^+p$<br>CERN, EHS, NA22                         | 1991 | 250 GeV          | $p_T < 40 \text{ MeV}/c$   | $6.4 \pm 1.6$         | bubble chamber<br>(RCBC)        | <a href="#">Botterweck et al., Z. Phys. C 51, 541 (1991)</a>      |
| $K^+p$<br>CERN, EHS, NA22                           | 1991 | 250 GeV          | $p_T < 40 \text{ MeV}/c$   | $6.9 \pm 1.3$         | bubble chamber<br>(RCBC)        | <a href="#">Botterweck et al., Z. Phys. C 51, 541 (1991)</a>      |
| $\pi^-p$ ,<br>CERN, WA83, OMEGA                     | 1993 | 280 GeV          | $p_T < 10 \text{ MeV}/c$<br>( $0.2 < E_\gamma < 1 \text{ GeV}$ ) | $7.9 \pm 1.4$         | calorimeter                     | <a href="#">Banerjee et al., Phys. Lett. B 305, 182 (1993)</a>    |
| p-Be  | 1993 | 450 GeV          | $p_T < 20 \text{ MeV}/c$   | $< 2$                 | pair conversion,<br>calorimeter | <a href="#">Antos et al., Z. Phys. C 59, 547 (1993)</a>           |
| p-Be, p-W   | 1996 | 18 GeV           | $p_T < 50 \text{ MeV}/c$   | $< 2.65$              | calorimeter                     | <a href="#">Lissauer et al., Phys.Rev. C54 (1996) 1918</a>        |
| $\pi^-p$ ,<br>CERN, WA91, OMEGA                     | 1997 | 280 GeV          | $p_T < 20 \text{ MeV}/c$<br>( $0.2 < E_\gamma < 1 \text{ GeV}$ ) | $7.8 \pm 1.5$         | pair conversion                 | <a href="#">Belogianni et al., Phys. Lett. B 408, 487 (1997)</a>  |
| $\pi^-p$ ,<br>CERN, WA91, OMEGA                     | 2002 | 280 GeV          | $p_T < 20 \text{ MeV}/c$<br>( $0.2 < E_\gamma < 1 \text{ GeV}$ ) | $5.3 \pm 1.0$         | pair conversion                 | <a href="#">Belogianni et al., Phys. Lett. B 548, 122 (2002)</a>  |
| pp,<br>CERN, WA102,                                 | 2002 | 450 GeV          | $p_T < 20 \text{ MeV}/c$<br>( $0.2 < E_\gamma < 1 \text{ GeV}$ ) | $4.1 \pm 0.8$         | pair conversion                 | <a href="#">Belogianni et al., Phys. Lett. B 548, 129 (2002)</a>  |
| $e^+e^- \rightarrow 2 \text{ jets}$<br>CERN, DELPHI | 2006 | 91 GeV (CM)      | $p_T < 80 \text{ MeV}/c$<br>( $0.2 < E_\gamma < 1 \text{ GeV}$ ) | $4.0 \pm 0.3 \pm 1.0$ | pair conversion                 | <a href="#">DELPHI, Eur. Phys. J. C 47, 273 (2006)</a>            |
| $e^+e^- \rightarrow \mu^+\mu^-$<br>CERN, DELPHI     | 2008 | 91 GeV (CM)      | $p_T < 80 \text{ MeV}/c$<br>( $0.2 < E_\gamma < 1 \text{ GeV}$ ) | $\sim 1$              | pair conversion                 | <a href="#">DELPHI, Eur. Phys. J. C57, 499 (2008)</a>             |



# Anomalous soft photon production in pp collisions

$4 \times 10^6$  events (with less than 8 charged tracks)



Most of the excess above inner bremsstrahlung at  $\eta_\gamma^{\text{CMS}} \gtrsim 1$

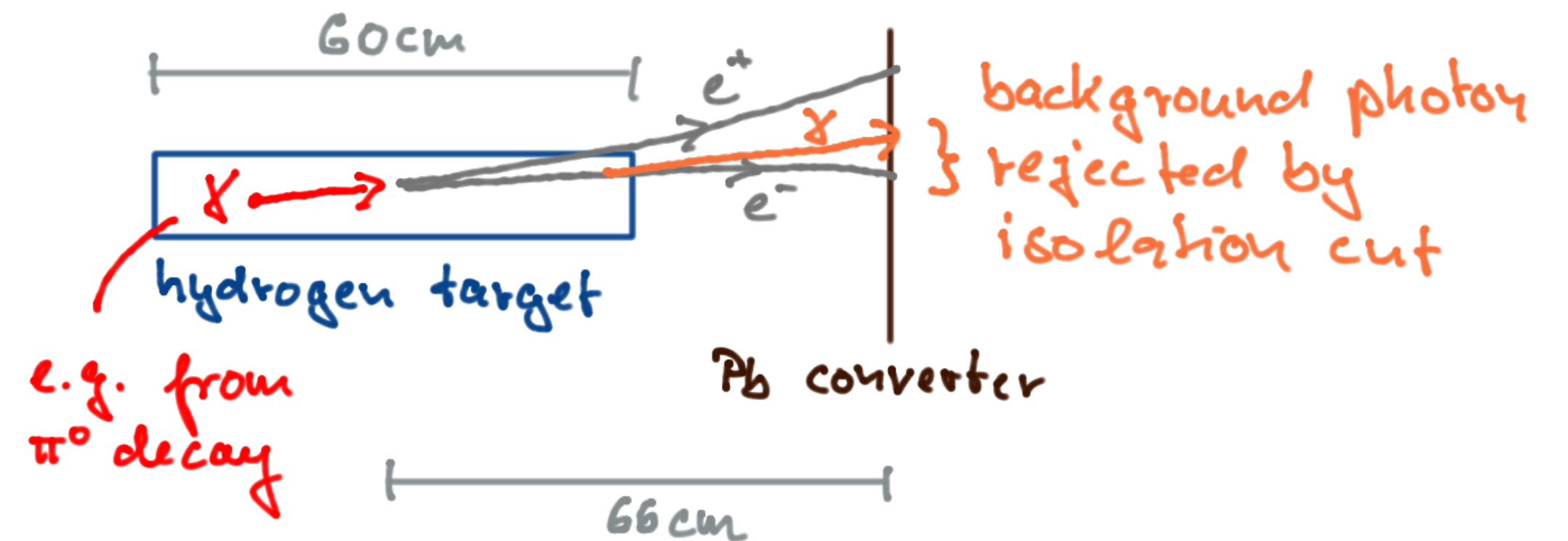
## External bremsstrahlung reduced by isolation cut

Table 1

Calculated soft gamma yields ( $0.2 < E_\gamma < 1$  GeV,  $\theta < 20$  mrad). The quoted errors are statistical. The systematic errors for background calculations are estimated to be 10%, and for inner bremsstrahlung 5%

| Source<br>(see text for definitions)                               | Number of $\gamma$ per event (%)<br>corrected for conversion<br>probability |
|--|---|
| (a) Inner bremsstrahlung   | $1.16 \pm 0.02$   |
| (b) $\gamma$ 's from hadronic decays                               | $1.70 \pm 0.03$   |
| (c) Dalitz pairs   | $0.05 \pm 0.01$   |
| (d) Knock-on electrons   | $0.008 \pm 0.003$   |
| (e) Spurious $\gamma$ 's   | $(1 \pm 1) \times 10^{-3}$  |
| (f) Secondary $\gamma$ 's  | $1.25 \pm 0.03$   |
| (g) Beam halo $\gamma$ 's  | $0.08 \pm 0.08$   |
| (h) $\gamma$ 's from secondary interactions                        | $< 0.7$ (95% C.L.)  |
| Sum over sources (b) to (g)<br>(full non-direct photon background) | $3.09 \pm 0.09$   |

Secondary  $\gamma$ 's:



“Only those photons were taken where the spatial separation of their  $e^+e^-$  vertex from any charged track at the lead sheet was greater than 3 mm in the  $x$ - $y$  plane (isolation cut). This was necessary to avoid including soft photons which were produced in the lead sheet by electrons or positrons originating upstream of the sheet. **Moreover, this cut suppressed a large fraction of the bremsstrahlung photons radiated by these particles upstream of the lead sheet, since both the parent particle and its radiation arrive on the lead sheet with a small separation.**”



# Anomalous soft photon production in $e^+e^- \rightarrow Z^0 \rightarrow 2 \text{ jets}$

## Factor of four more photons than expected from inner bremsstrahlung

Photon range:

$$0.2 < E_\gamma < 1 \text{ GeV}$$

$$p_T < 80 \text{ MeV}/c$$

Expected from inner bremsstrahlung:

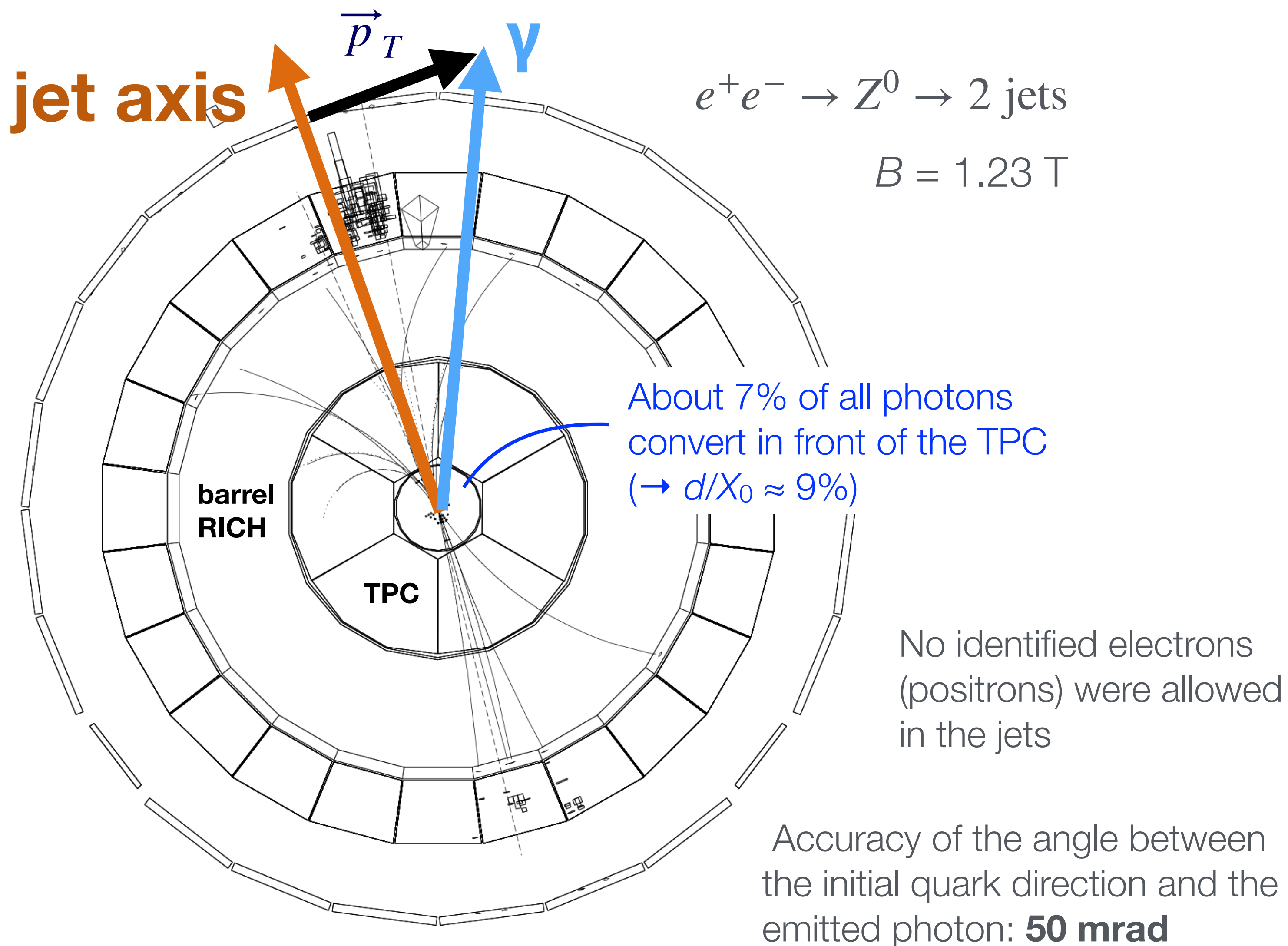
$$(17.1 \pm 0.01 \pm 1.21) \times 10^{-3} \gamma/\text{jet}$$

Observation:

$$(69.1 \pm 4.5 \pm 15.7) \times 10^{-3} \gamma/\text{jet}$$

Ratio:

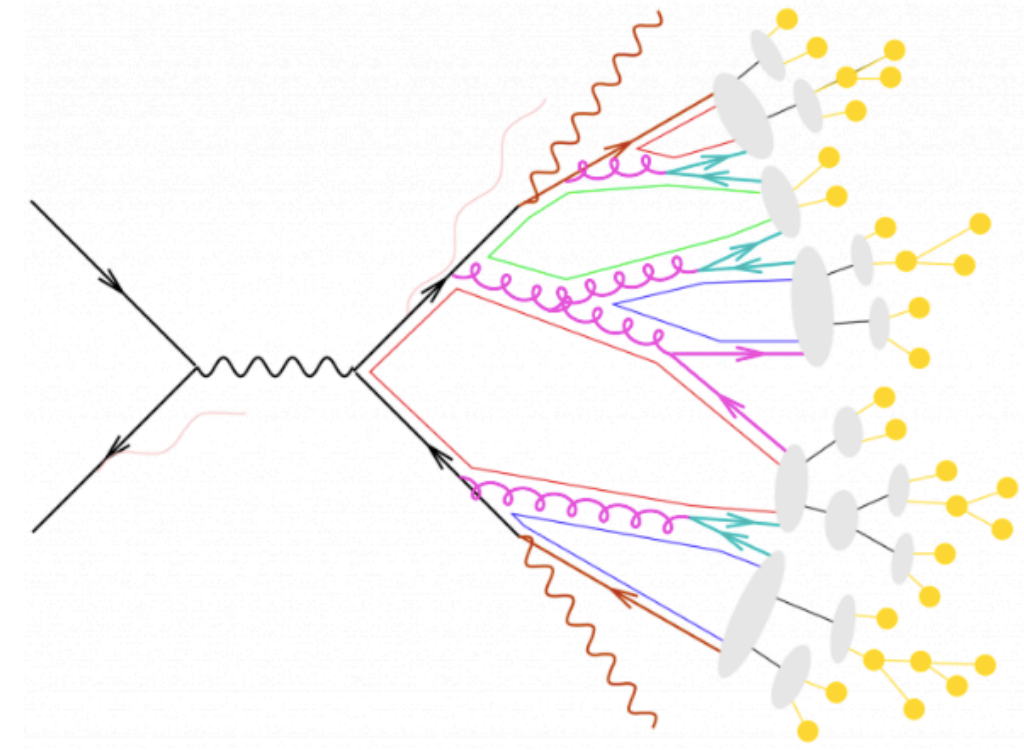
$$4.0 \pm 0.3 \pm 1.0$$



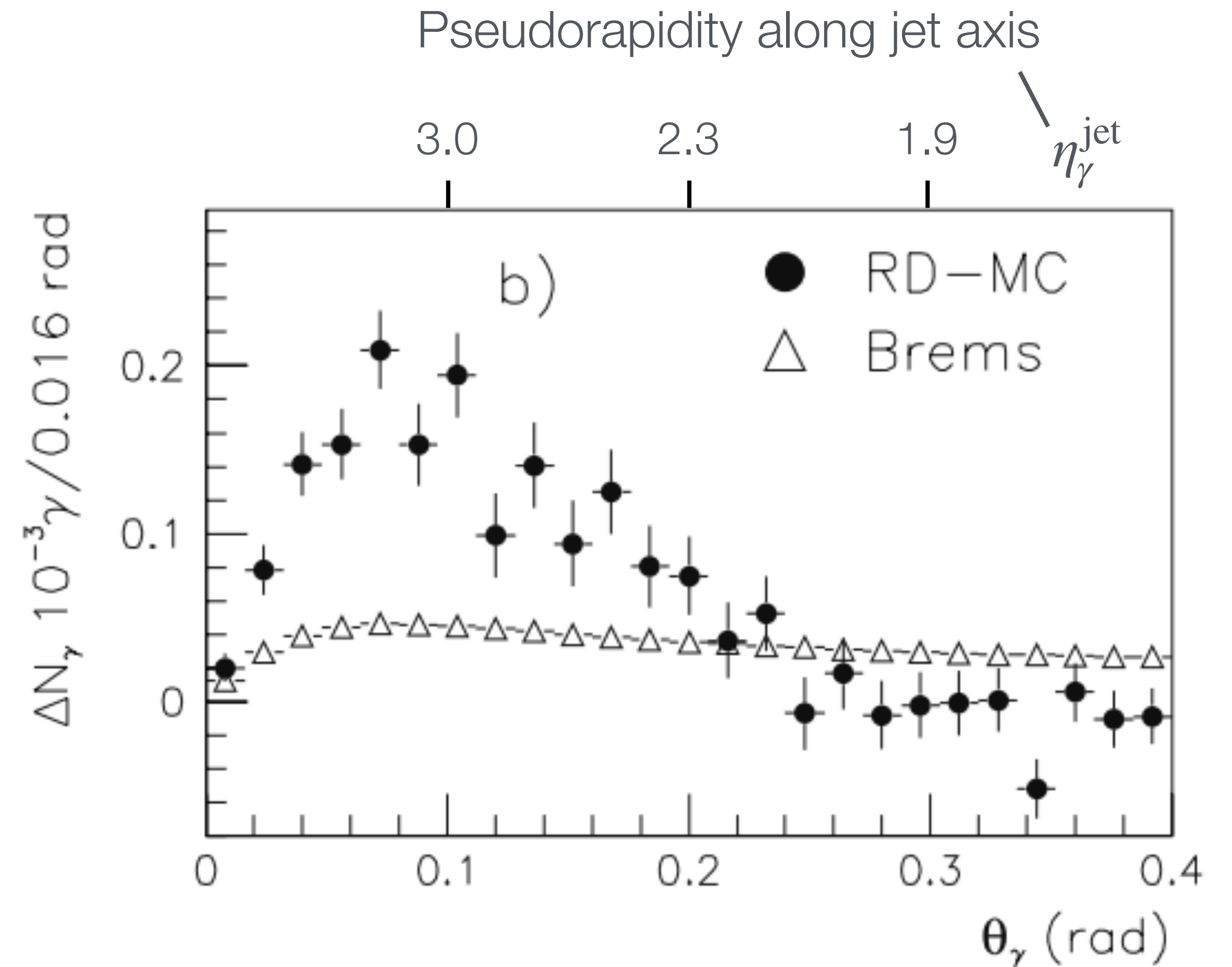
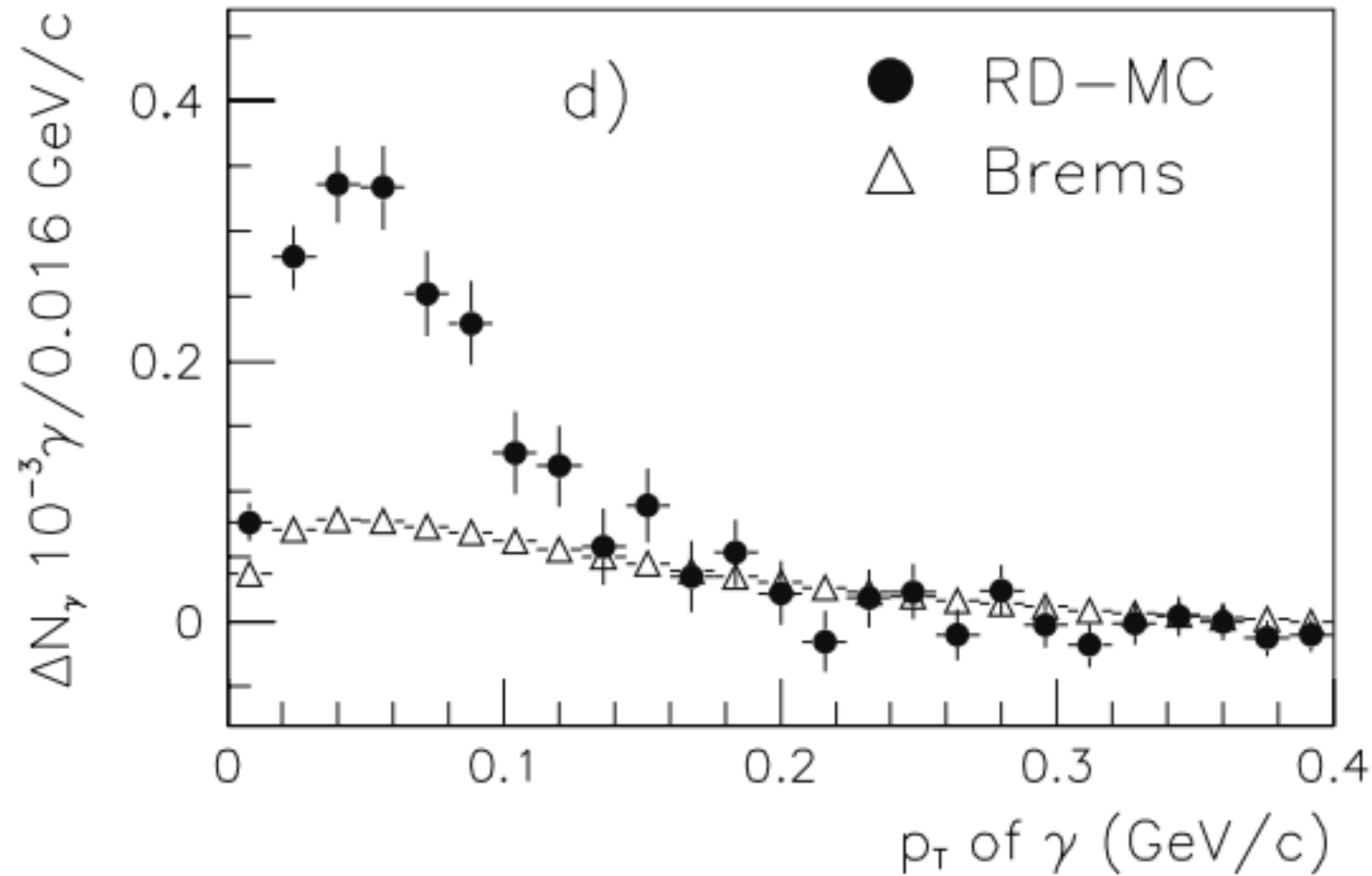
# Transverse momentum and angular dependence of the excess above inner bremsstrahlung

Excess for  $p_T < 100 \text{ MeV}/c$  and  $\theta < 0.2 \text{ rad}$

Does Low's theorem apply to parton showers?

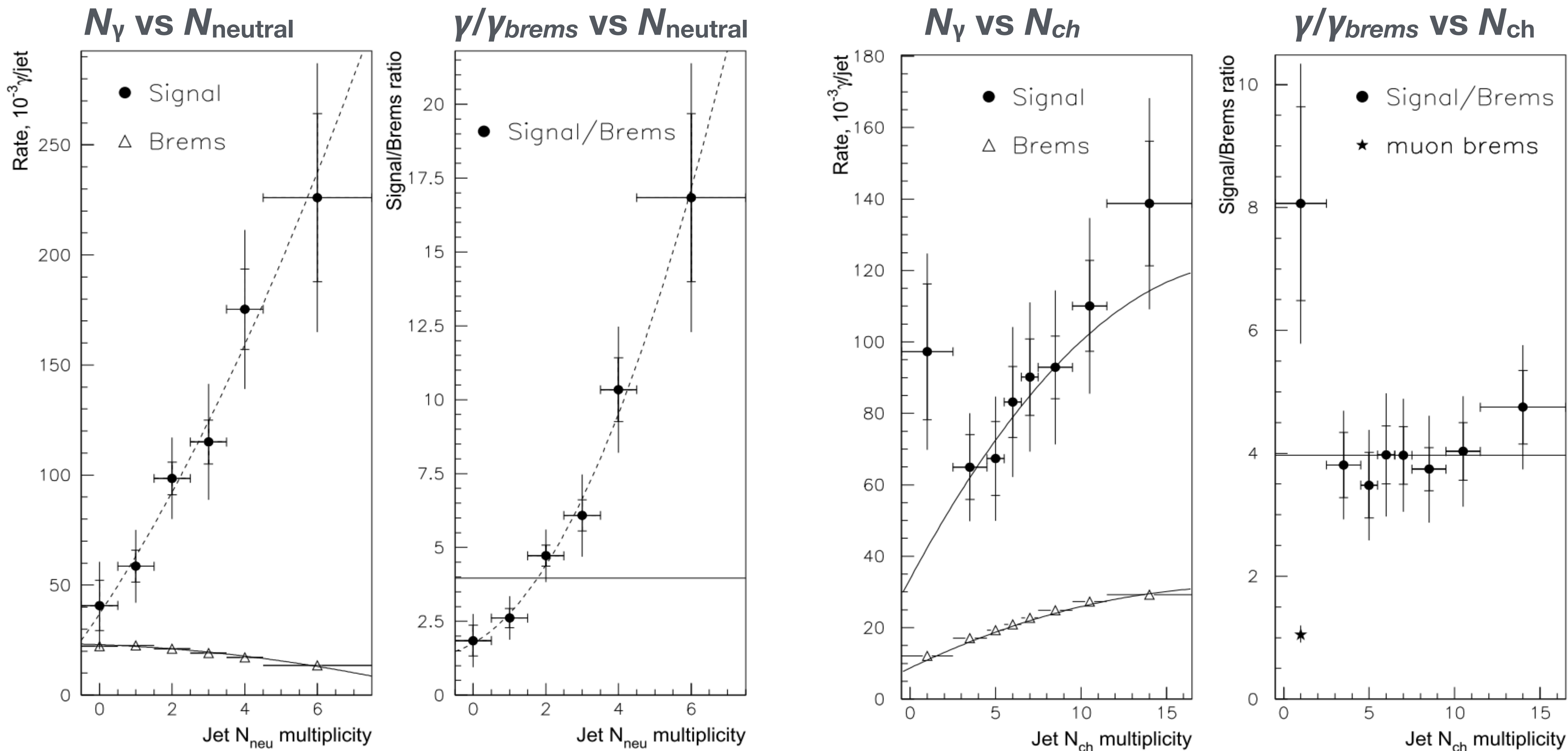


Real data – MC,  
MC = event generator (jetset) + detector sim.





Soft photon yield increases much faster with neutral particle multiplicity than with charged particle multiplicity → Very puzzling

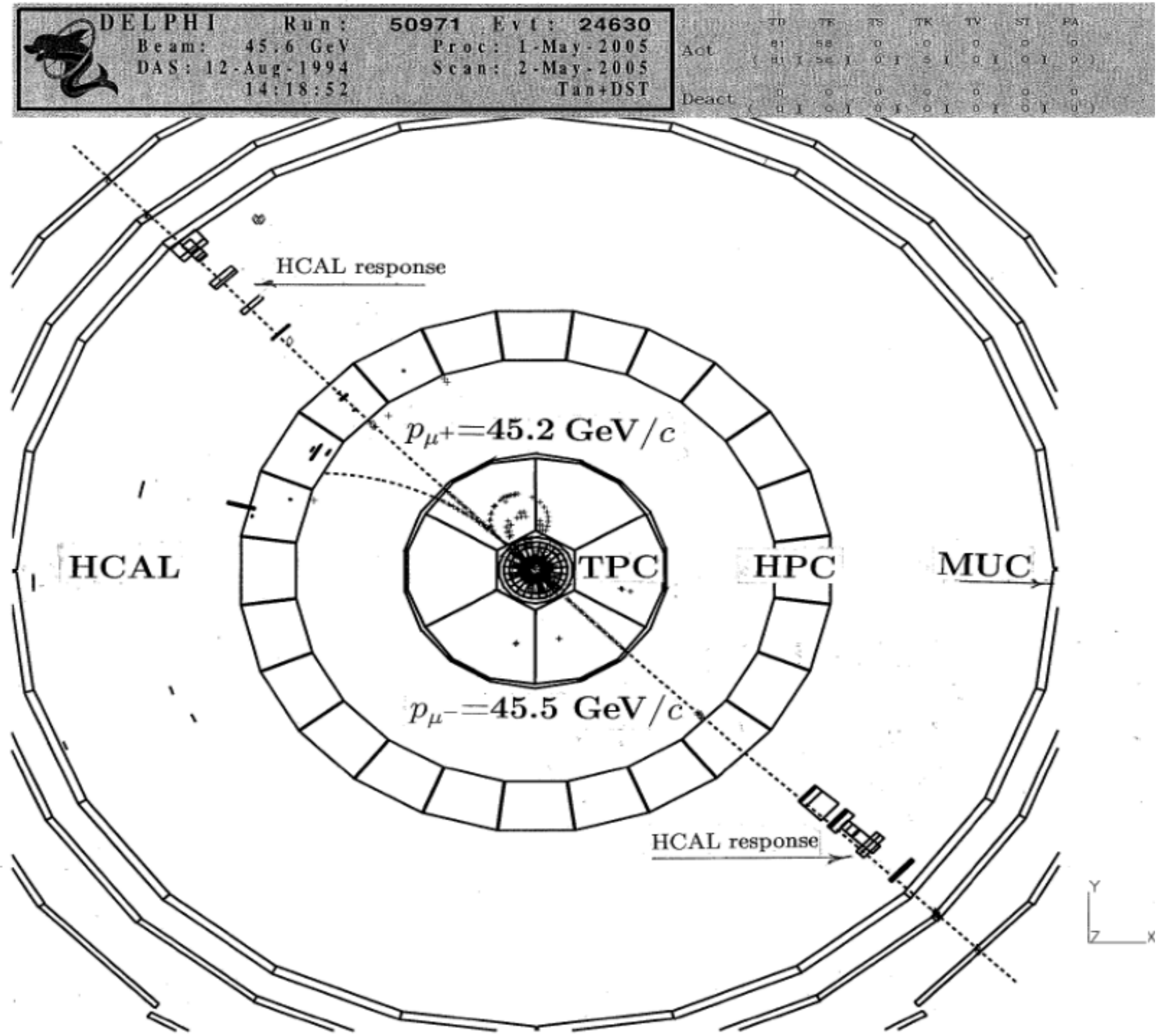
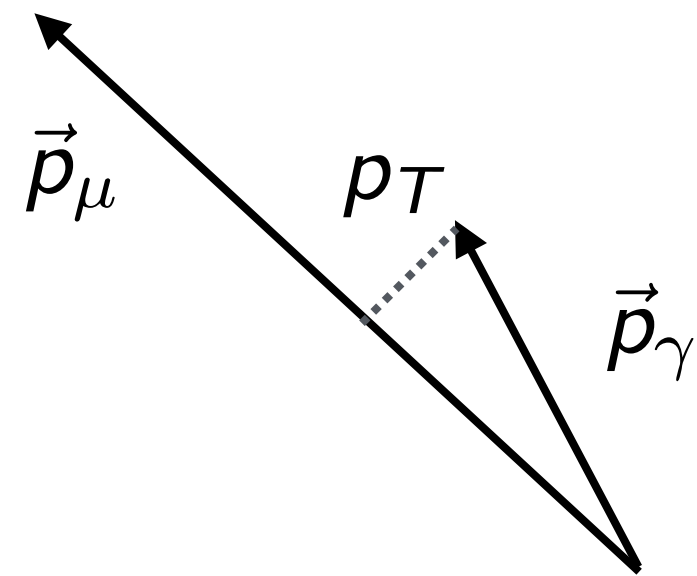


DELPHI soft photon measurement in  $e^+e^- \rightarrow \mu^+\mu^-$

# DELPHI dimuon event with a photon

$$e^+e^- \rightarrow Z^0 \rightarrow \mu^+\mu^-n\gamma, \quad n \geq 1$$

Photon  $p_T$  defined w.r.t. muons direction:





Measured signal agrees with inner bremsstrahlung expectation

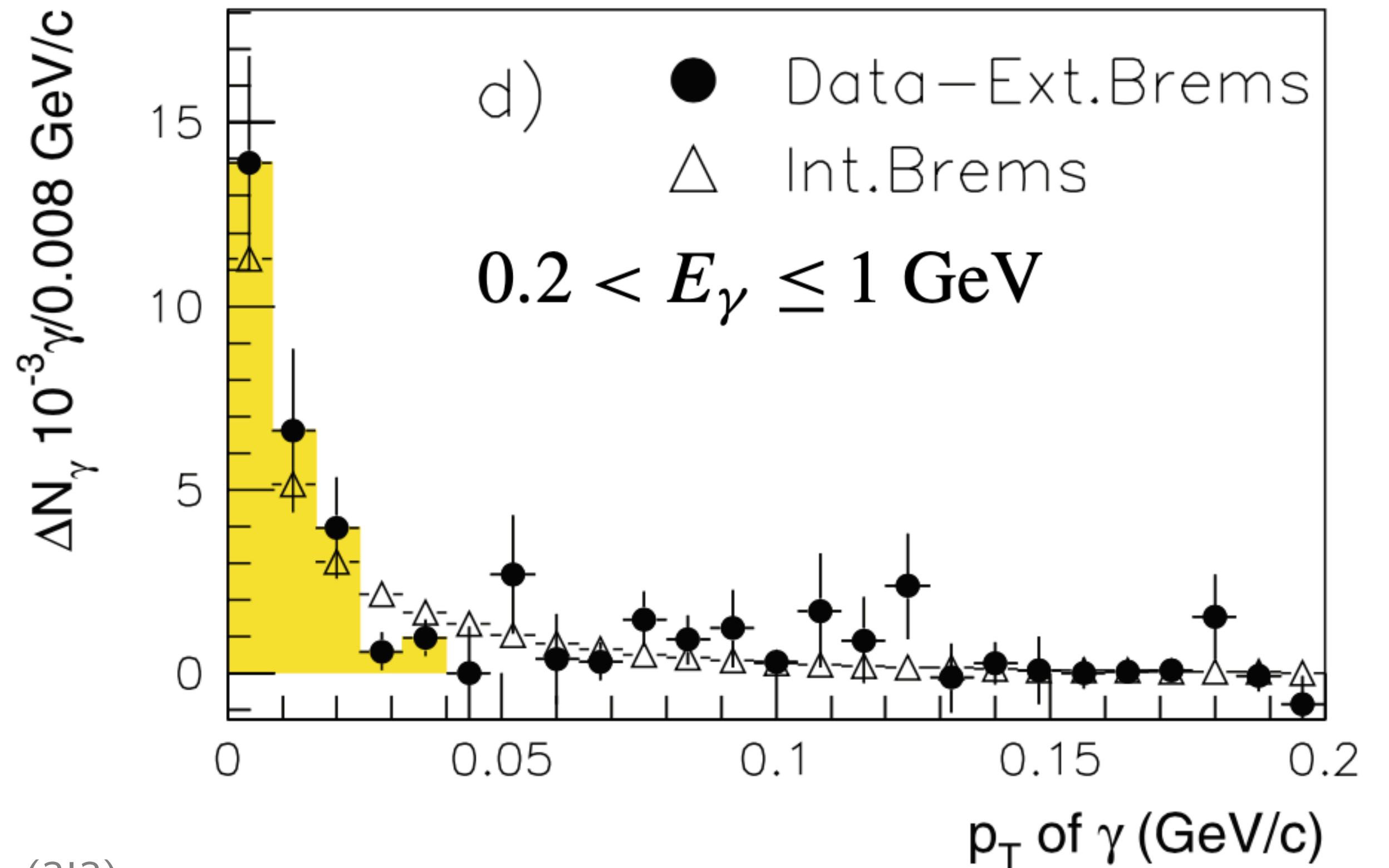
Photons per 1000 muons corrected for detection efficiency:

|              |                           |
|--------------|---------------------------|
| signal       | $25.9 \pm 4.0 \pm 1.4$    |
| inner brems. | $23.30 \pm 0.01 \pm 0.93$ |
| signal / IB  | $1.11 \pm 0.17 \pm 0.07$  |

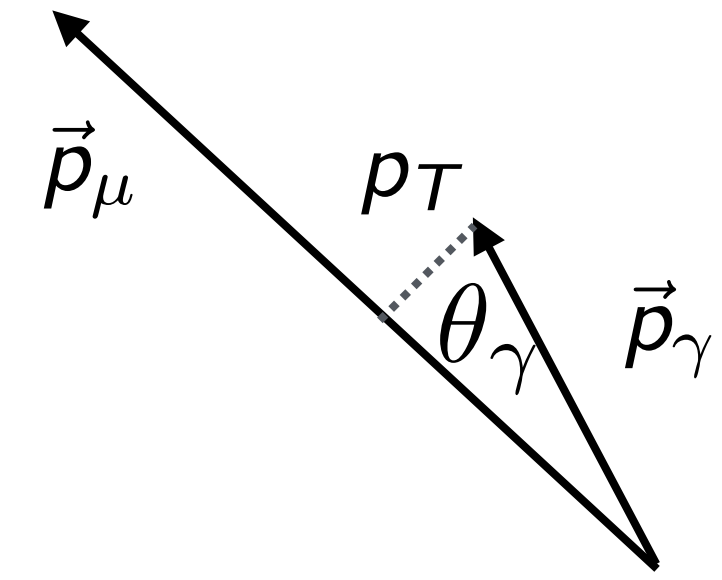
Range of validity of Low's theorem at NLP:

$$m \approx 100 \text{ MeV}, Q \approx 100 \text{ GeV} \rightarrow \omega_k \ll \frac{m^2}{Q} = 0.1 \text{ MeV} \quad (?!?)$$

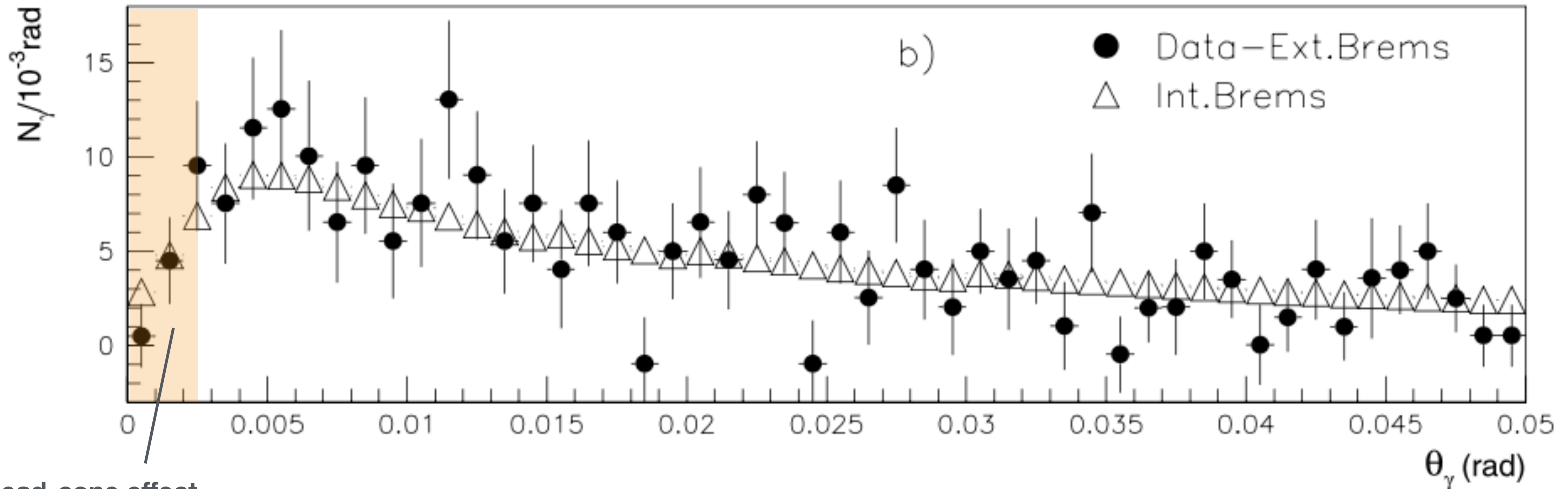
$$e^+e^- \rightarrow Z^0 \rightarrow \mu^+\mu^-n\gamma, \quad n \geq 1$$



Angular distribution confirms dead-cone effect



$$e^+e^- \rightarrow Z^0 \rightarrow \mu^+\mu^-n\gamma, \quad n \geq 1$$



dead-cone effect

radiation suppressed for angles  $\theta_\gamma \lesssim m_\mu/E_\mu = 0.0023$

Dead-cone effect in QCD:

[https://alice-collaboration.web.cern.ch/ALICE\\_deadcone](https://alice-collaboration.web.cern.ch/ALICE_deadcone) (ALICE, Nature 605, pages 440–446 (2022))



# Properties of anomalous soft photons

1. They are produced only in association with hadrons.  
They are not produced in  $e^+e^- \rightarrow \mu^+\mu^-$ . ] excess a hadronization effect?
2.  $p_T \approx 10$  MeV/c in fixed-target experiments. Larger  $p_T$  in  $e^+e^- \rightarrow 2$  jets w.r.t. to jet axis presumably due to limited accuracy in reconstruction initial quark direction.
3. Total anomalous soft photon yield is proportional to total hadron yield.
4. Anomalous soft photon yield increase faster with increasing neutral hadron multiplicity  $N_{\text{neu}}$  than with charged hadron multiplicity  $N_{\text{ch}}$ .

see also C.-Y. Wong,

<https://indico.in2p3.fr/event/7399/contributions/41877/attachments/33559/41348/wongphoton2013.pdf>

Attempts at explaining the anomalous soft photons production

**No generally accepted explanation exists to this date**

Cold quark-gluon plasma (glob model, *van Hove, Lichard*)

van Hove, Ann. Phys. 192, 66 (1989),  
van Hove & Lichard, PLB 245, 605 (1990)

Partons at end of virtuality evolution form a glob of cold quark-gluon system of low temperature of  $T \sim 10 - 30$  MeV

Quark synchrotron radiation (*Nachtmann et al.*)

Nachtmann, Reiter, Z. Phys. C, 24, 283-296 (1984)  
Botz, Haberl, Nachtmann, Z. Phys. C 67, 143-158 (1995)

Synchrotron radiation of energetic quarks traveling through the QCD vacuum

String fragmentation (*Czyz & Florkowski*)

Czyz & Florkowski, Z. Phys. C 61,171 (1994)

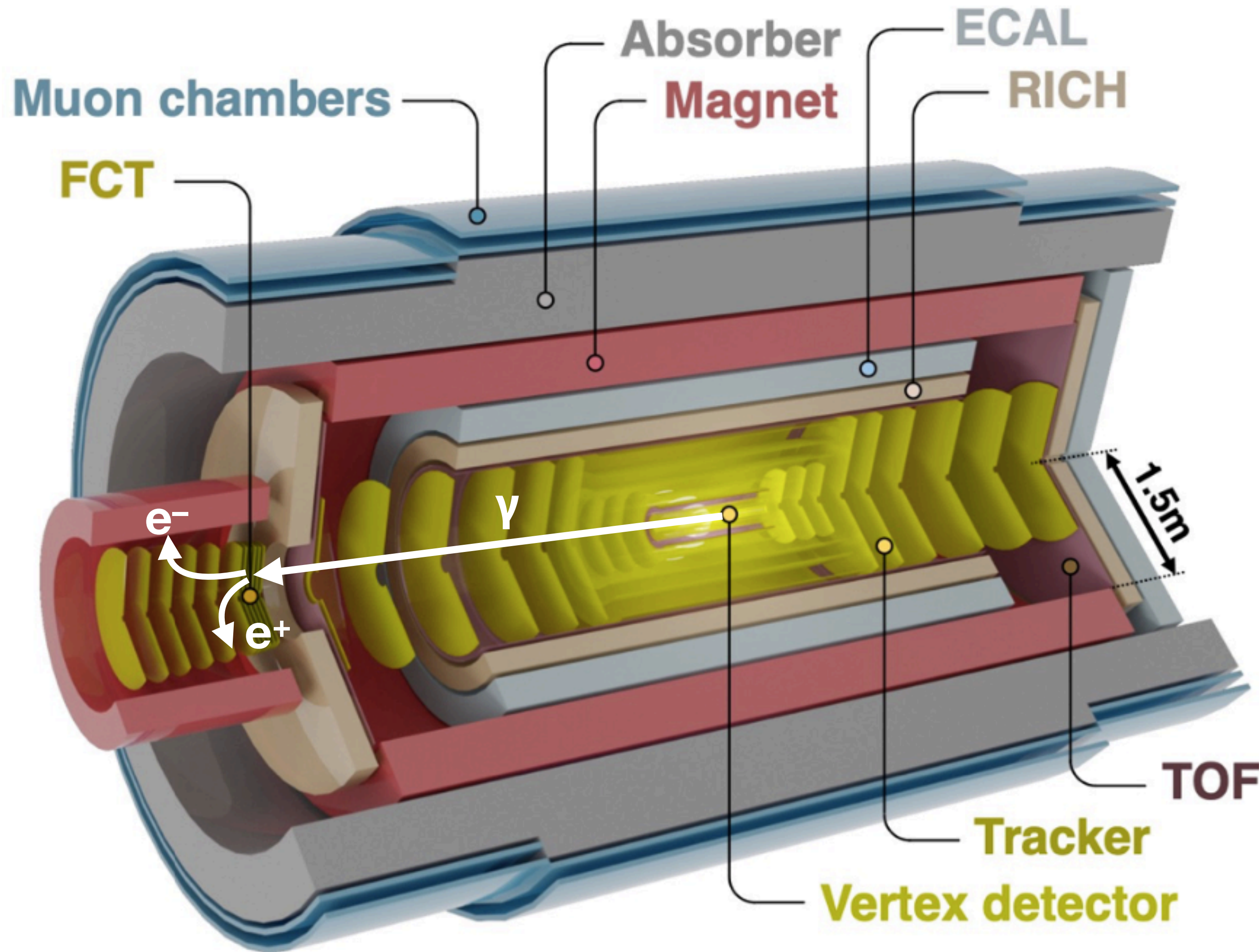
Bremsstrahlung of partons with parton trajectories following string breaking

More complete overview: C.-Y. Wong,

<https://indico.in2p3.fr/event/7399/contributions/41877/attachments/33559/41348/wongphoton2013.pdf>



# Addressing the soft-photon puzzle at the LHC



## Ultra-soft photons:

Forward Conversion Tracker (FCT),  
 $3 < \eta < 5$ ,  $1 < p_T < 50 \text{ MeV}/c$

$$p_T = \frac{E_\gamma}{\cosh \eta}$$

$E_\gamma = 100 \text{ MeV}$ :

| $\eta$                | 3  | 4   | 5   |
|-----------------------|----|-----|-----|
| $p_T \text{ (MeV}/c)$ | 10 | 3.7 | 1.3 |

## ALICE 3 FCT team

P. Braun-Munzinger, H. Büsching,  
 K. Reygers, T. Rogoschinski,  
 K. Schweda, G. Skorodumovs,  
 J. Stachel, C. van Veen, M. Völkl

# Go to higher multiplicities starting from simple exclusive reactions

Establish a baseline by studying a “clean” exclusive process:  $pp \rightarrow pp \pi^+ \pi^- \gamma$

Precise models for  $pp \rightarrow pp \pi^+ \pi^-$  exist. We expect  $\lim_{\omega \rightarrow 0} \frac{d\sigma_{\text{exp}}/d\omega}{d\sigma_{\text{exact}}/d\omega} = 1$ .

“A violation of these relations would mean a terrible crisis for QFT!”  
(Lebiedowicz, Nachtmann, Szczurek, arXiv:2107.10829, see also arXiv:2206.03411)

Another interesting channel:  $pp \rightarrow pp J/\psi \gamma$  with  $J/\psi \rightarrow e^+ e^-, \mu^+ \mu^-$

## Study soft-photon production in inelastic (non-diffractive) pp collisions

Requirements in terms of statistics are moderate:

1% stat. uncertainty in  $5 < p_T < 6$  MeV bin for  $3 < \eta < 5$  with 1% conversion probability  
obtained with  $160 \times 10^6$  pp collisions @ 13 TeV

## Extend study to reactions/systems with higher charged particle multiplicities



# Resolving the soft-photon puzzle

Soft-photon puzzle:

Most (but not all) experiments find a significant excess of a factor 4 - 8 above inner bremsstrahlung exceptions.

Should one take the puzzle seriously? I would say yes!

Serious checks and discussions of systematic effects in the experimental publication (WA102, DELPHI, ...). Experimenters seemed to be well aware of possible experimental pitfalls.

Currently not understood aspect of parton shower evolution / hadronization? Connection to fundamental aspects of quantum-field theories?

Studying different collisions systems with the Forward Conversion Tracker of ALICE 3 likely the only way to address the puzzle in the near future.