Test of a Kinematic Fitting Procedure for the Λ Decay in the $pK^+\Lambda$ Final State at HADES

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Hyperon Session

Outline

- **Motivation**
- **Constraints for Fitter**
- **Tests**
- **Upcoming Features**
- **Outlook**

Why Kinematic refit?
- Λ Polarization in pp reactions

Previous study:

- Polarization of Λ Hyperons In Proton-Proton Reactions At 3.5 GeV Measured With Hades, PoS(INPC2016)275

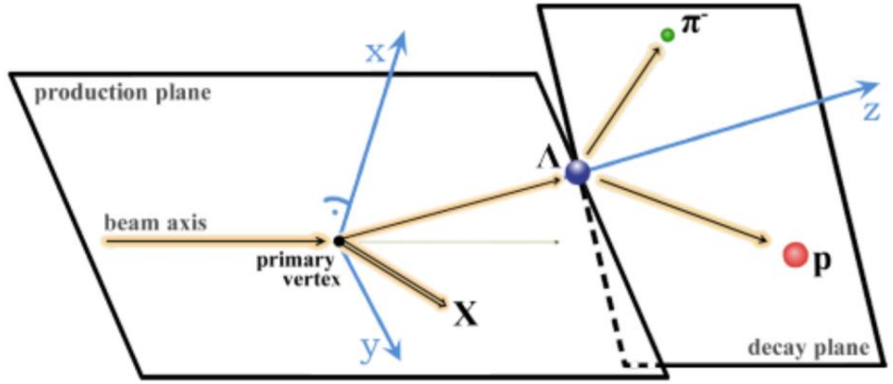
$$\frac{dN}{dcos(\zeta)} = C(1 + \alpha P cos(\zeta))$$

P-polarization
C-constant
α-decay asymmetry parameter of Λ decay

- Difference between generated and reconstructed polarization angle show large uncertainty
    - **Kinematic refit might improve resolutions and hence results**

Coordinate System

$$\vec{n}_x = \frac{\vec{p}_{beam} \times \vec{p}_\Lambda}{|\vec{p}_{beam} \times \vec{p}_\Lambda|}, \quad \vec{n}_y = \vec{n}_x \times \vec{n}_z, \quad \vec{n}_z = \frac{\vec{p}_\Lambda}{|\vec{p}_\Lambda|}$$



Constraints

- Kinematic fitting based on Lagrange multipliers has been implemented in Hydra (HADES Software)
- Constraints:
  - Invariant mass
  - Missing mass
  - Primary Vertex
  - Decay Vertex with

  Waleed Esmail

  4 Momentum Conservation (currently being implemented) Jenny + Jana

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Waleed Esmail

Jenny + Jana

Procedure, Lagrange Multiplier Technique

Equations

$$\chi^2 = (y - \eta)^T V^{-1} (y - \eta) = minimum$$

$$f_K\,(\eta_1, \eta_2, \ldots, \eta_N, \xi_1, \xi_2, \ldots, \xi_J) = 0$$

$$f\,(\eta, \xi) = 0$$

f- constraint function
η – set of measured quantities
ξ – set of unmeasured quantities
y- vector of fitted quantities
λ- Lagrange multiplier

$$\chi^2 = (y - \eta)^T V^{-1} (y - \eta) + 2\lambda^T f\,(\eta, \xi) = minimum$$

Finding parameters minimizing equations

$$\nabla_\eta \chi^2 = -2V^{-1}(y - \eta) + 2\, F_\eta^T \lambda = 0$$

$$\nabla_\xi \chi^2 = 2F_\xi^T \lambda = 0$$

$$\nabla_\lambda \chi^2 = 2\, f\,(\eta, \xi) = 0$$

$$(F_\eta)_{ki} = \frac{\partial f_k}{\partial \eta_i} \qquad (F_\xi)_{kj} = \frac{\partial f_k}{\partial \xi_j}$$

Procedure

Solution can be found iteratively

1. $\xi^{\nu+1} = \xi^{\nu} - (F_{\xi}^{T} S^{-1} F_{\xi})^{-1} F_{\xi}^{T} S^{-1} \mathrm{r}$

2. $\lambda^{\nu+1} = S^{-1}[\mathrm{r} + F_{\xi} (\xi^{\nu+1} - \xi^{\nu})]$

3. $\eta^{\nu+1} = y - V F_{\lambda}^{T} \lambda^{\nu+1}$

4. $V^{\nu+1} = V^{\nu} - V^{\nu}[F_{\eta}^{T} S^{-1} F_{\eta} - ((F_{\eta}^{T} S^{-1} F_{\xi})(F_{\xi}^{T} S^{-1} F_{\xi})^{-1} (F_{\eta}^{T} S^{-1} F_{\xi})^{T})] V^{\nu}$

where

$$r = f^{\nu} + F_{\eta}^{\nu}(y - \eta^{\nu}) \qquad S = F_{\eta}^{\nu} S^{-1} (F_{\eta}^{T})^{\nu}$$

Track Representation and Constraints

Track Representation

$$\left(\frac{1}{p}, \theta, \varphi, R, Z\right)$$

p – particle momentum
θ – polar angle
ϕ – azimuthal angle
R- closest **distance** of track to beam line
Z- closest **point** along beamline

Invariant Mass Constraint, 1C fit

d = $E^2 - P_x^2 - P_y^2 - P_z^2 - M^2$

$$P_x = \mathrm{P} \cdot \sin(\theta) \cdot \cos(\varphi)$$

$$P_y = \mathrm{P} \cdot \sin(\theta) \cdot \sin(\varphi)$$

$$P_z = \mathrm{P} \cdot \cos(\theta)$$

$$E = \sqrt{P^2 - M^2}$$

Track Representation and Constraints

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Primary Vertex Constraint, 1C fit

$$d = (d_1 \times d_2) \cdot (b_1 - b_2)$$

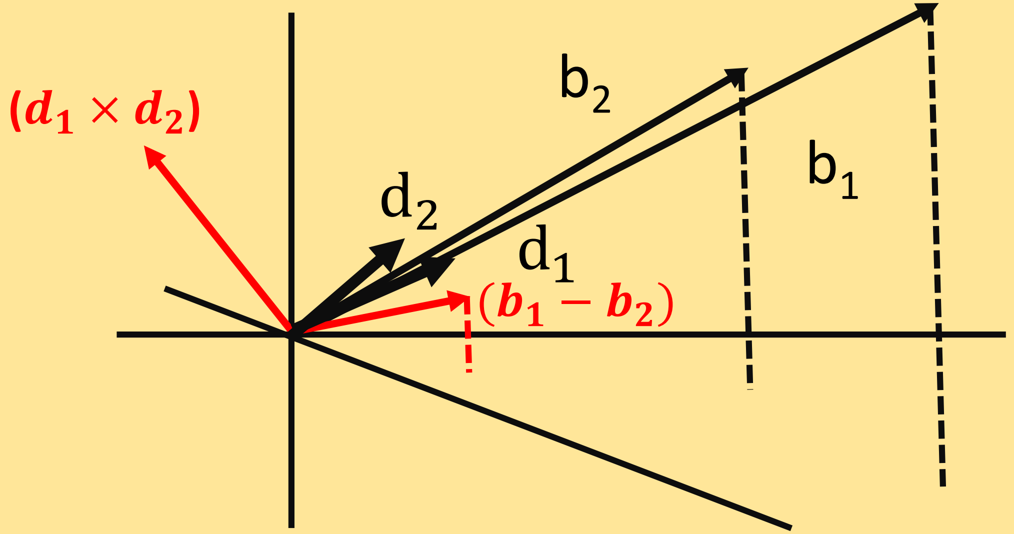
$|d|/|d_1 \times d_2|$ – closest distance between tracks

Direction Vector

$$\begin{cases} d_x = \sin(\theta) \cdot \cos(\varphi) \\ d_y = \sin(\theta) \cdot \sin(\varphi) \\ d_z = \cos(\theta) \end{cases}$$

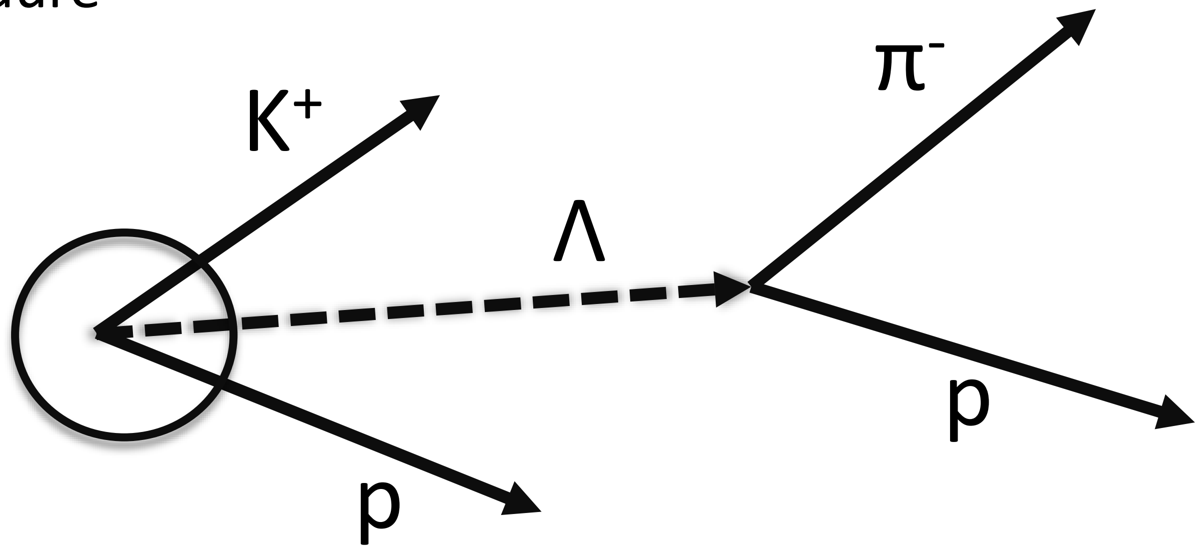
Base Vector

$$\begin{cases} b_x = R \cdot \cos\left(\varphi + \frac{\pi}{2}\right) \\ b_y = R \cdot \sin\left(\varphi + \frac{\pi}{2}\right) \\ b_z = z \end{cases}$$

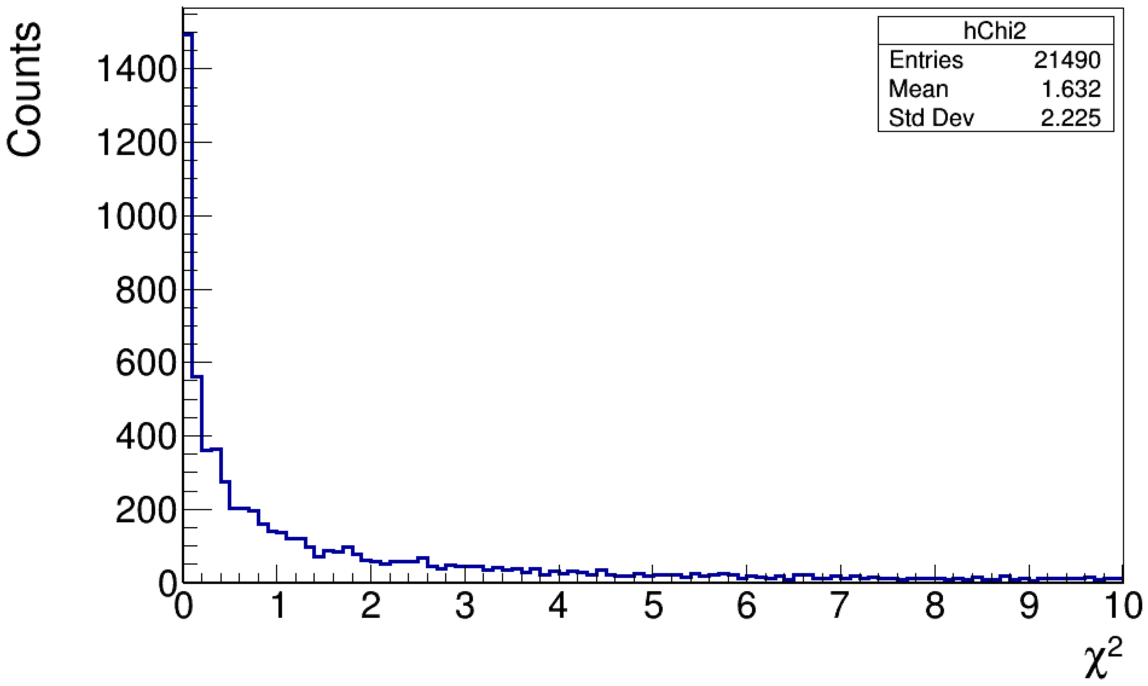


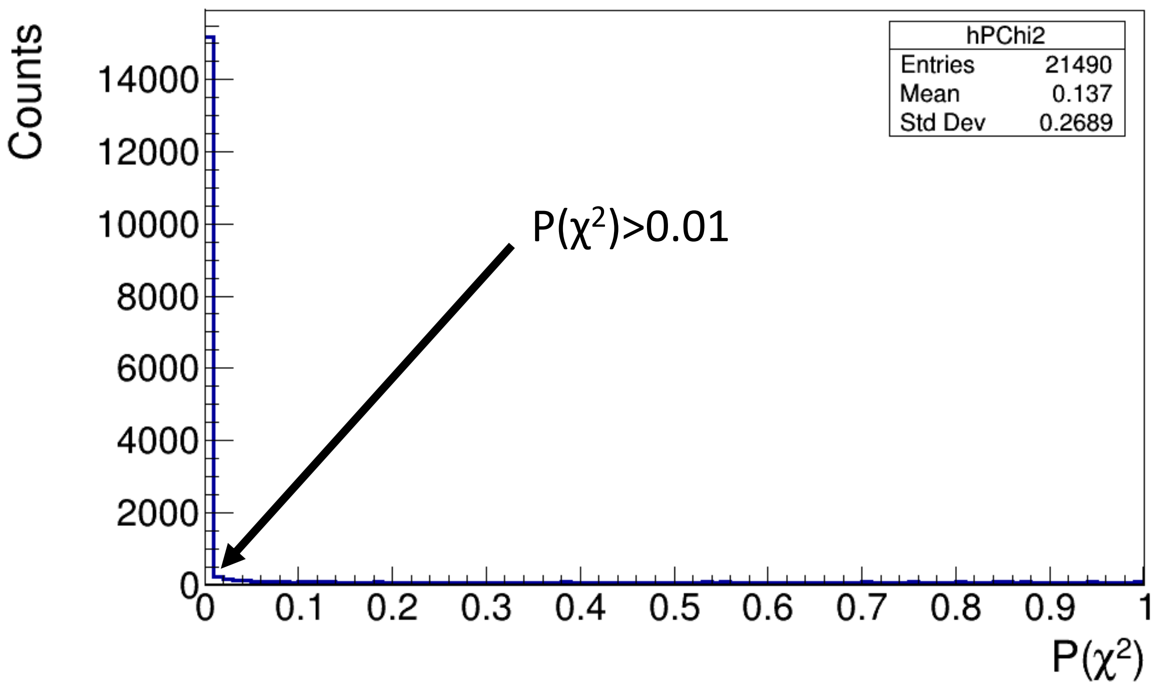
p(3.5GeV)p->pK$^+$Λ, Λ-> pπ$^-$

- 100 000 Pluto events
- Geant Particle ID used to identify p, π$^-$ and K$^+$
  - Only combinatorial background
- Mass constraint on p and π$^-$
- Primary Vertex Constraint on p and K$^+$
- One iteration in Fitting procedure



χ^2 Invariant Mass Constraint



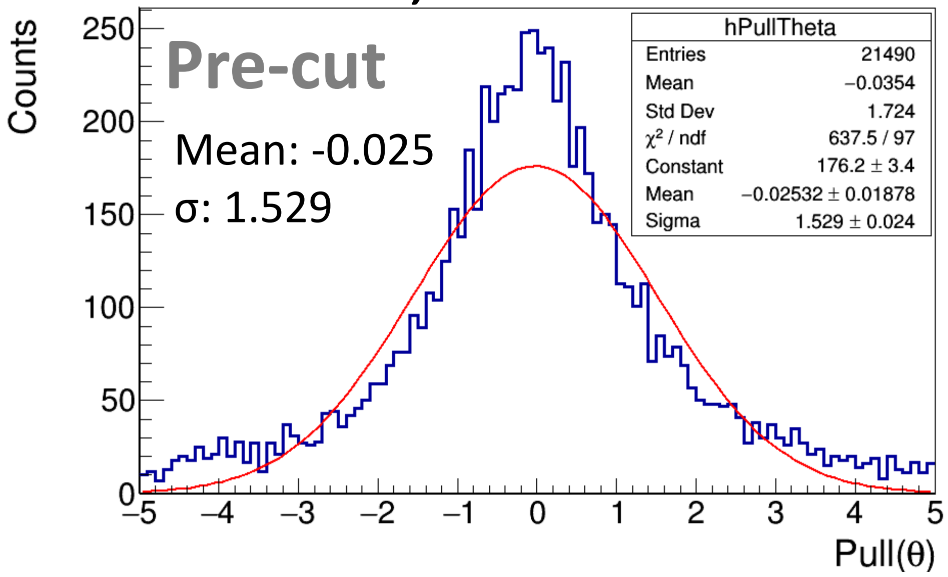


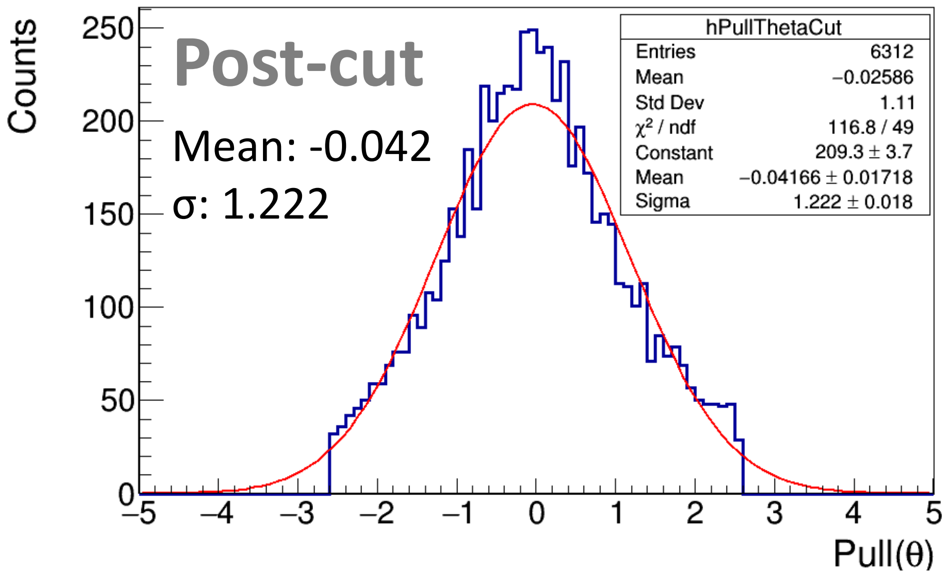
Pull Distributions, Mass Constraint

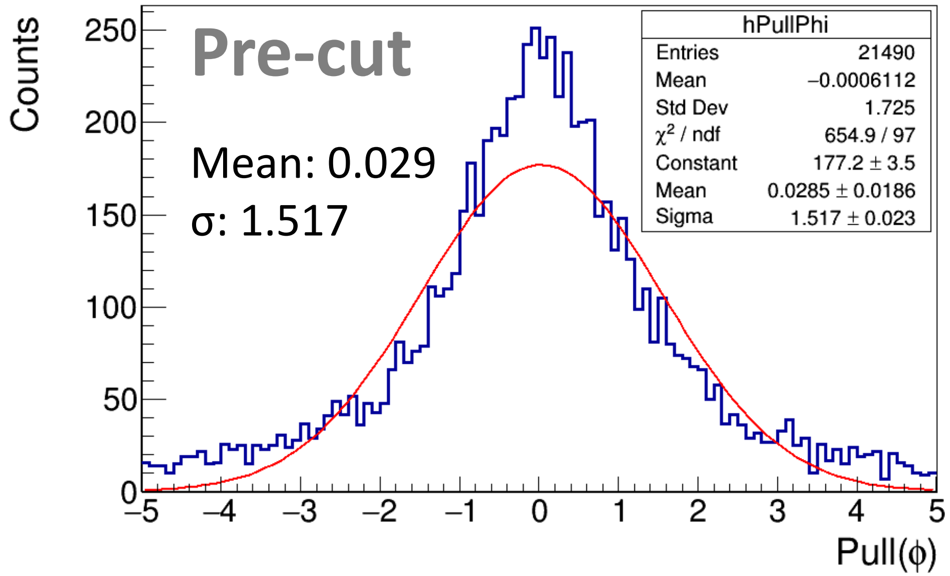
$$z_i = \frac{y_i - \eta_i}{\sqrt{\sigma^2(y_i) - \sigma^2(\eta_i)}}$$

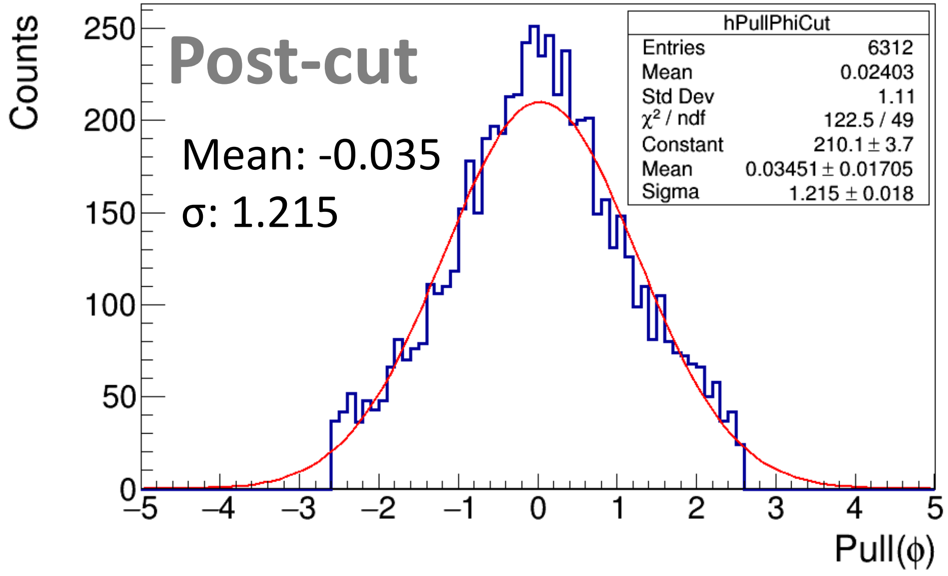
Ideally N(0,1)

Effects of probability cut.
Eff. loss: 71%









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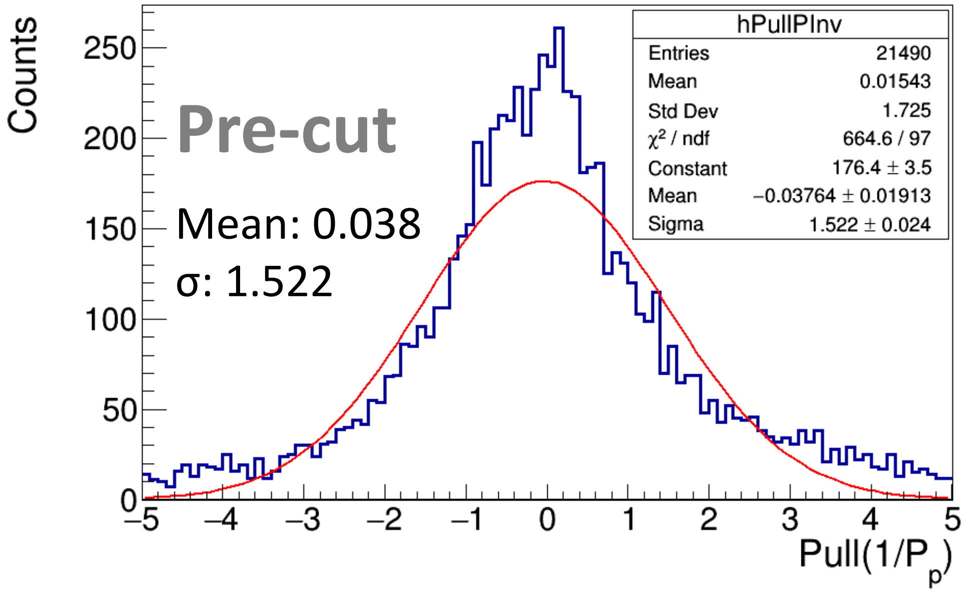
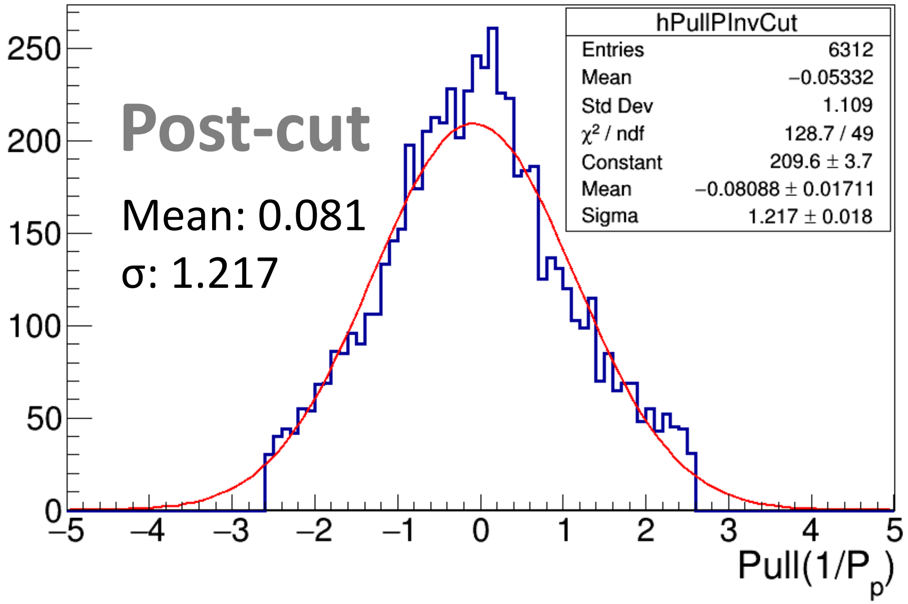




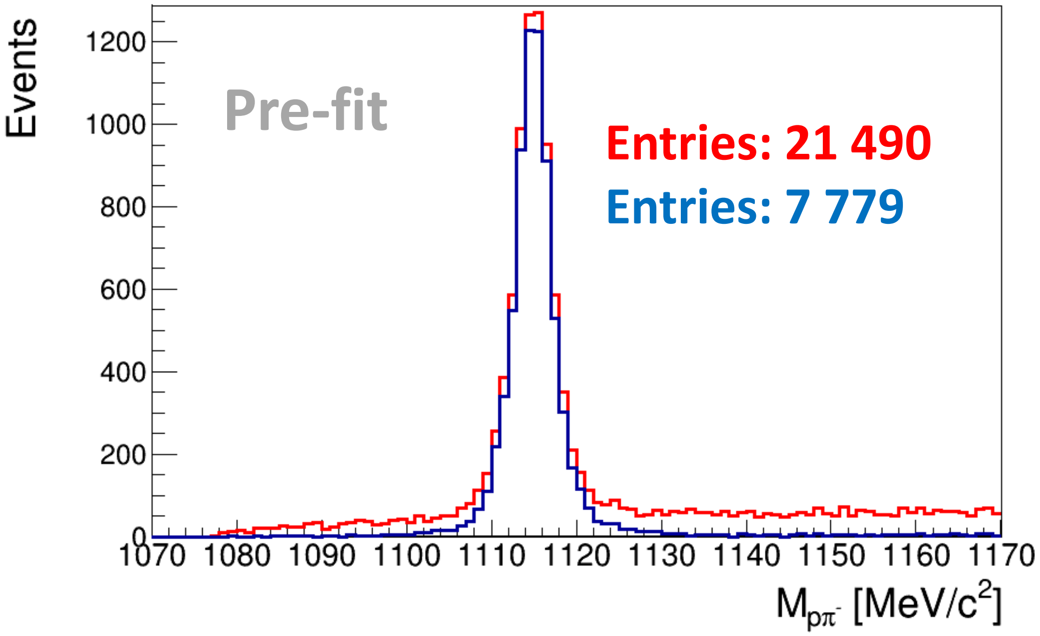
- Pull distributions have slightly larger σ than expected
- Applying $P(\chi^2)$ cut reduces σ but brings mean further away from 0

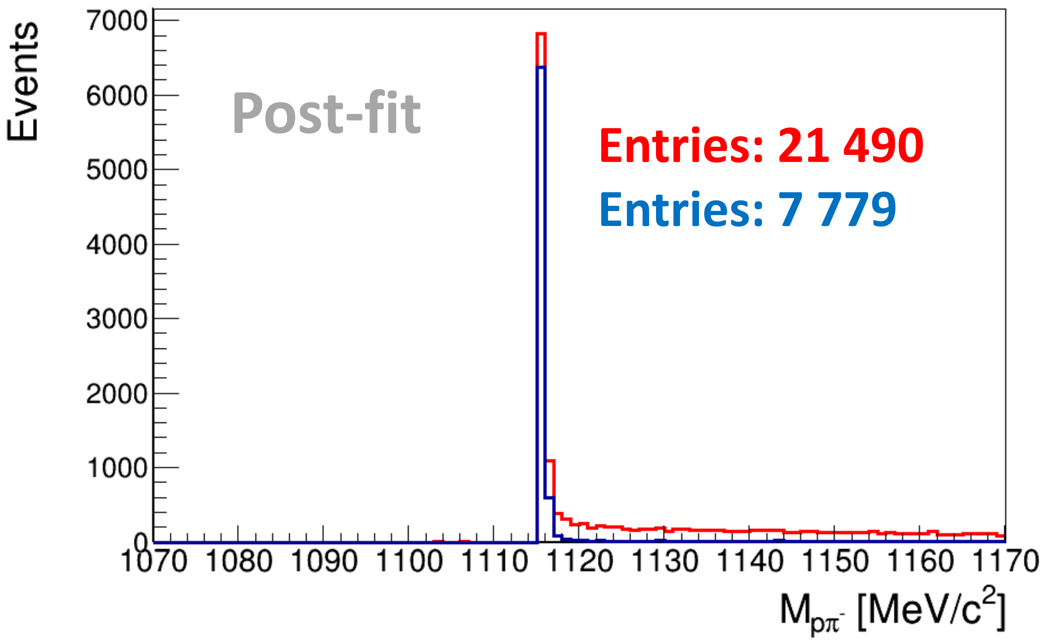
Invariant Mass

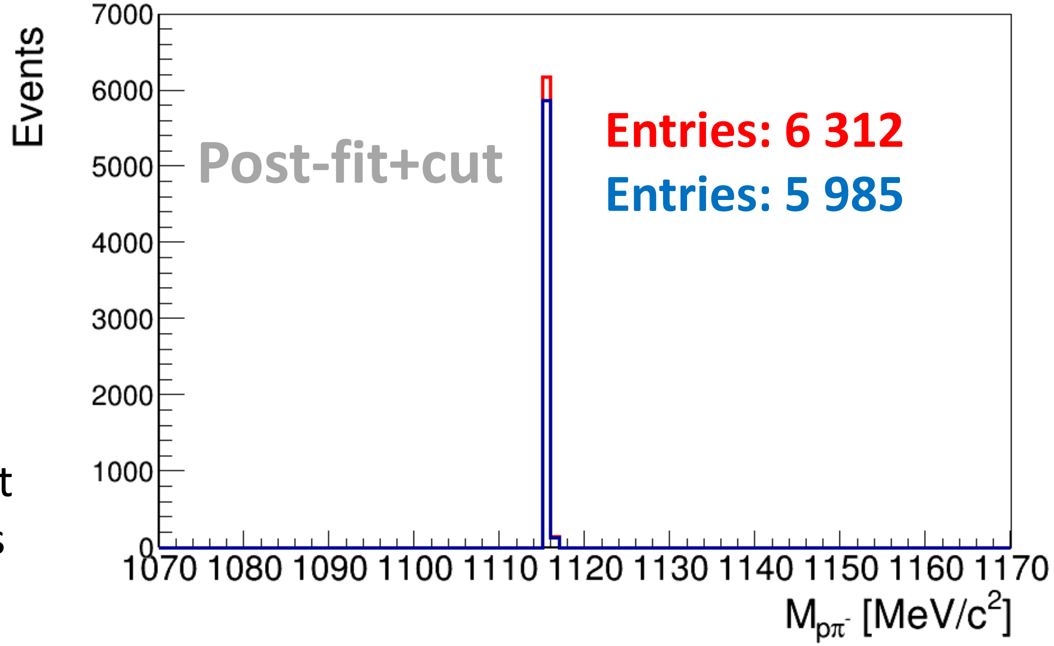
In Histograms:

Red line: Combinatorial Background

Blue line: No Combinatorial Background

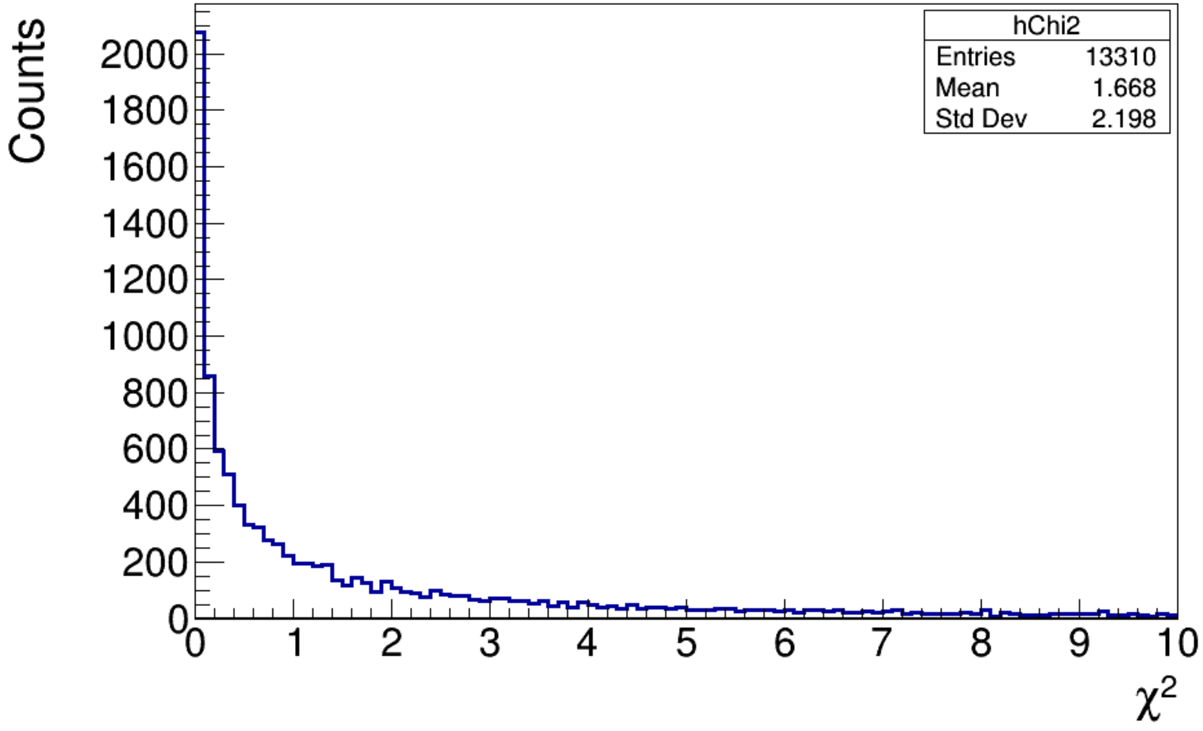


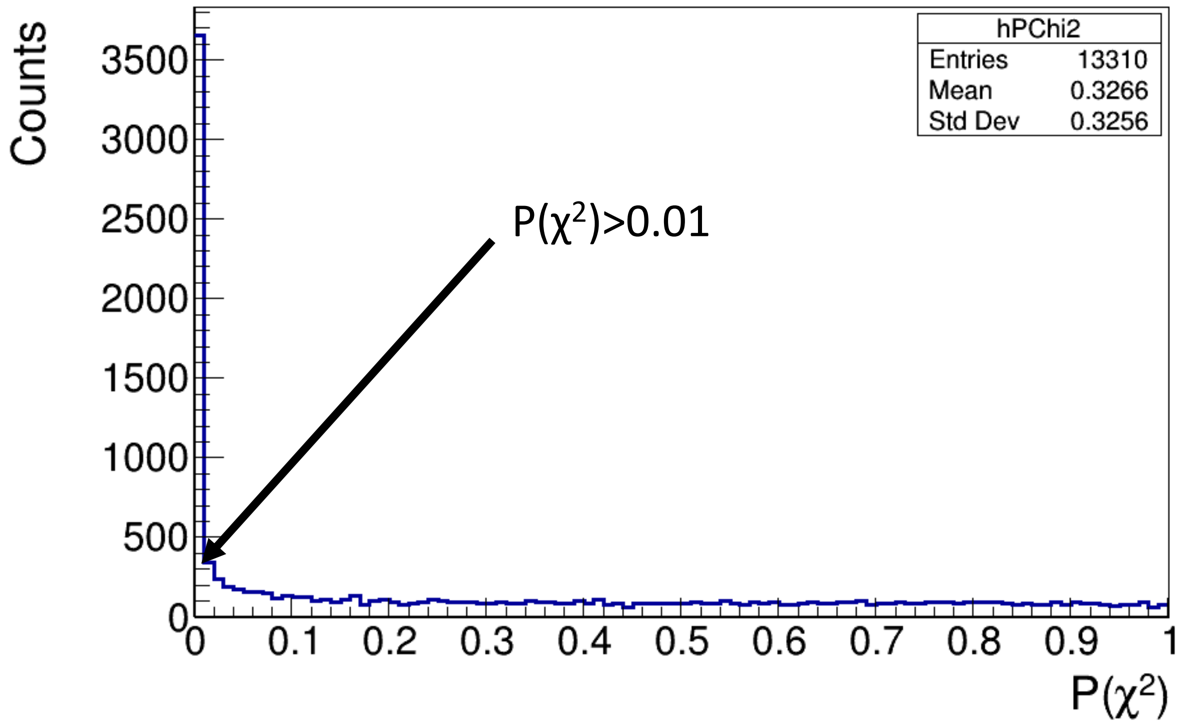




- Removing combinatorial background reduces σ of Pull Distr.
- Invariant mass spectra behaves as expected after refit and cut
- A large part of the events cut away by probability cut appears to be combinatorial background

χ^2 Primary Vertex Constraint





Pull Distributions

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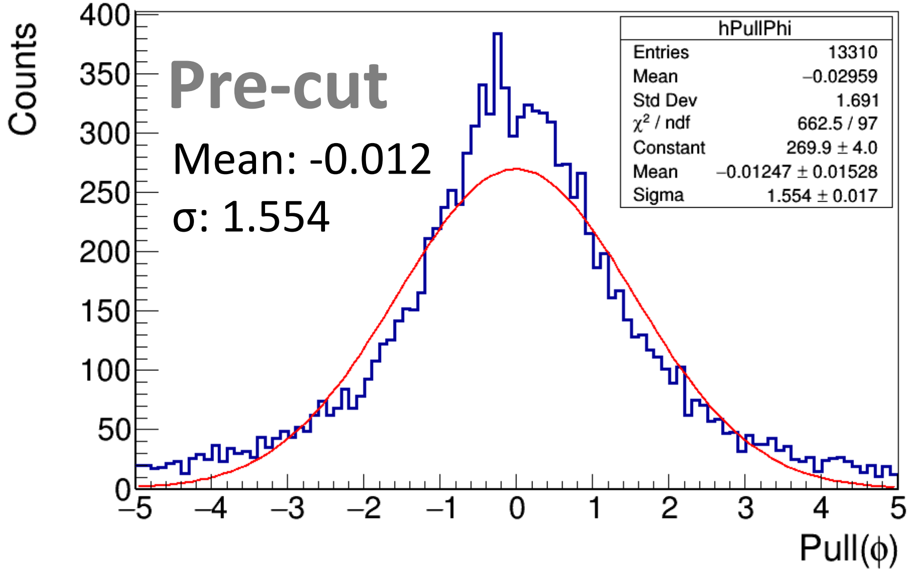
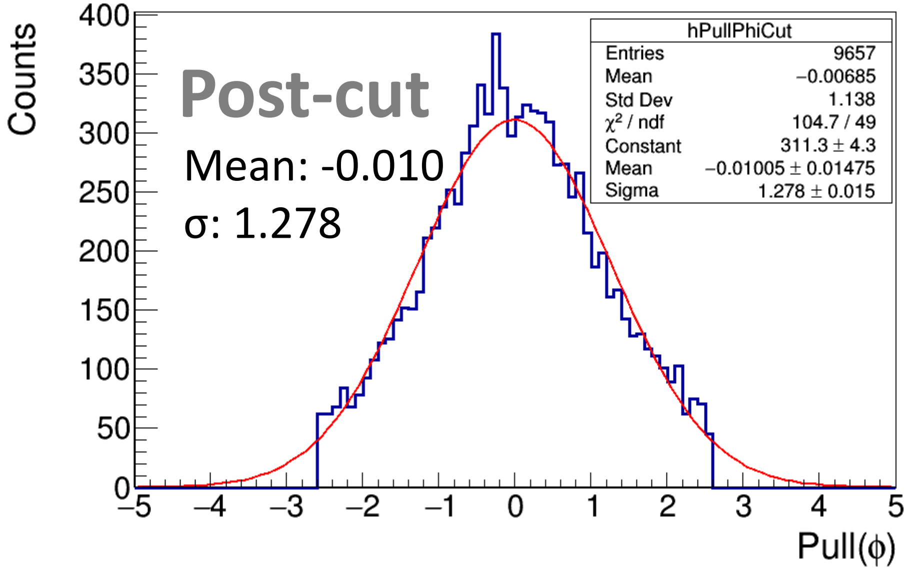
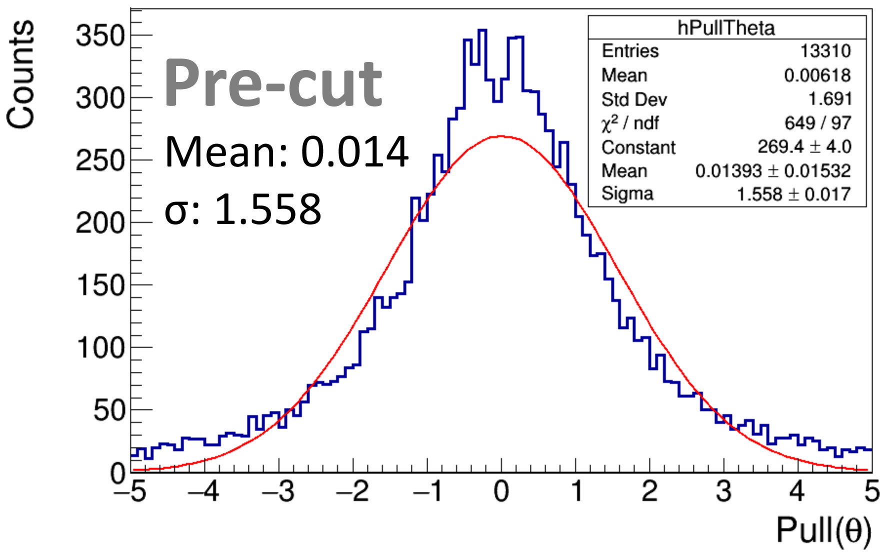
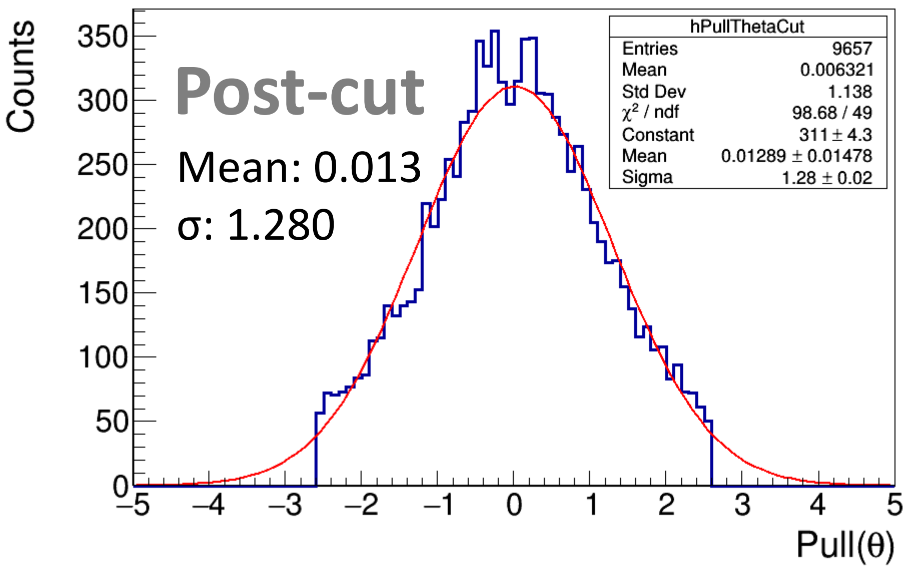
Ideally N(0,1)





Effects of probability cut.
Eff. loss: 27%





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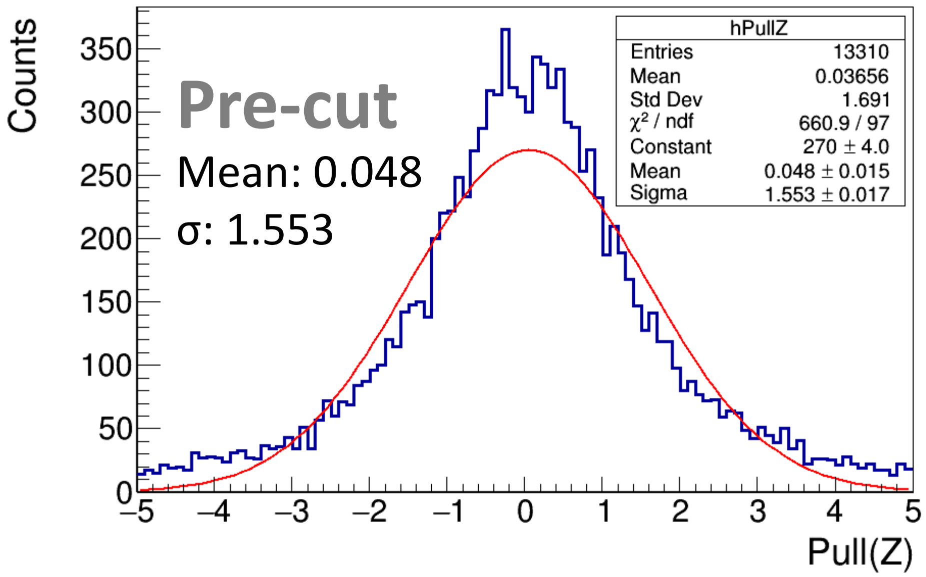
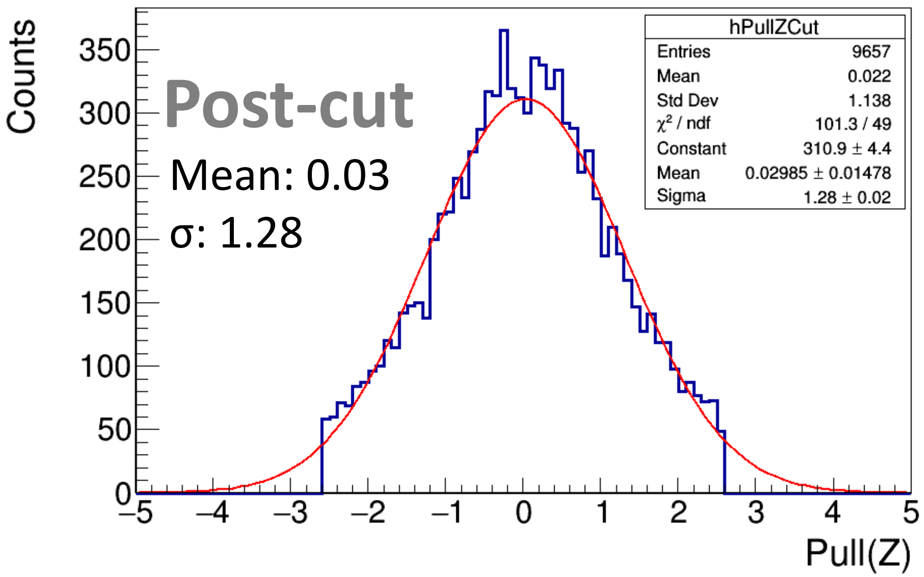
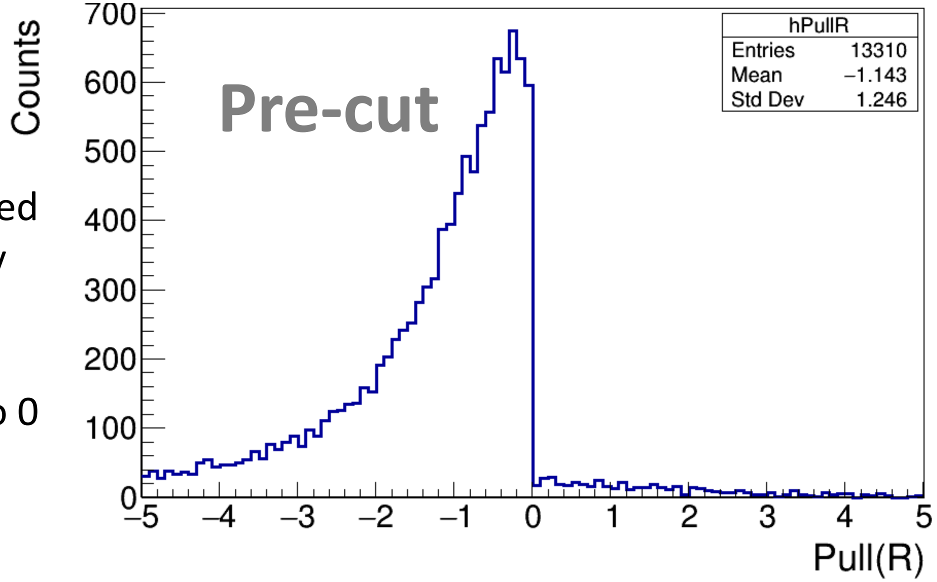
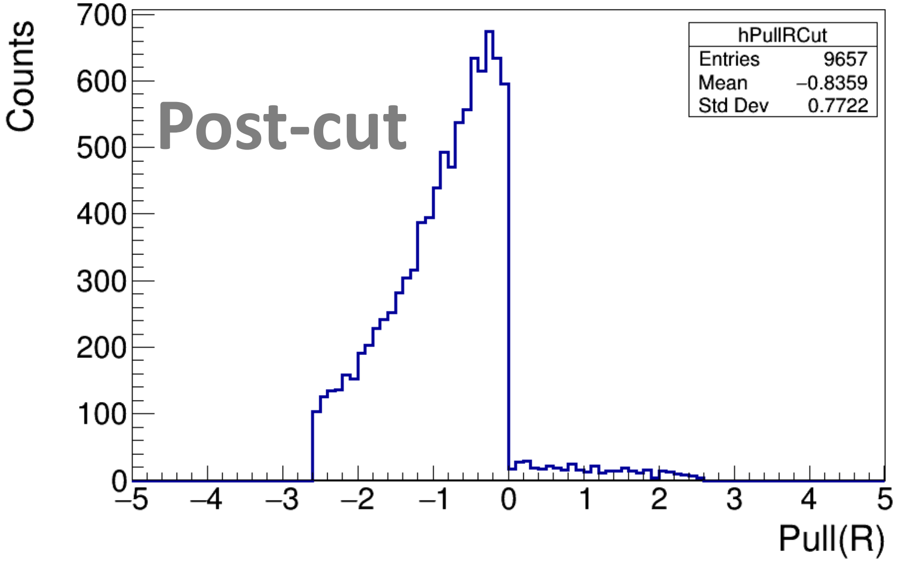








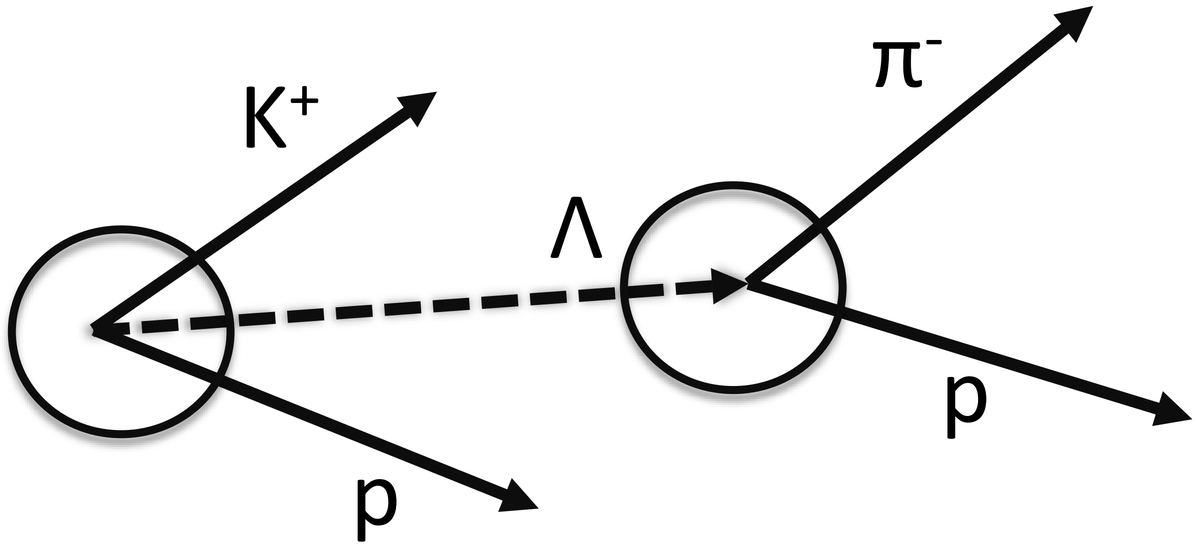
- Pull for R needs to be explained
- Pull distributions have slightly larger σ than expected
- Applying $P(\chi^2)$ cut reduces σ and brings the mean closer to 0

p(3.5GeV)p->pK$^+$Λ, Λ-> pπ$^-$

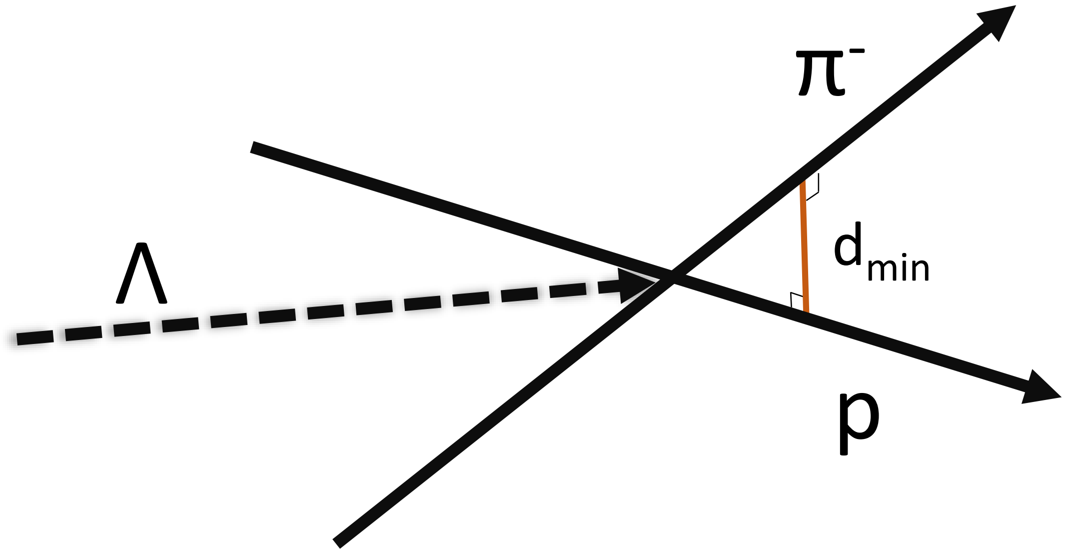
Next Steps:

Apply decay vertex and Momentum Conservation constraint for p, π$^-$ and Λ

Use together with additional constraints



Decay Vertex Handling

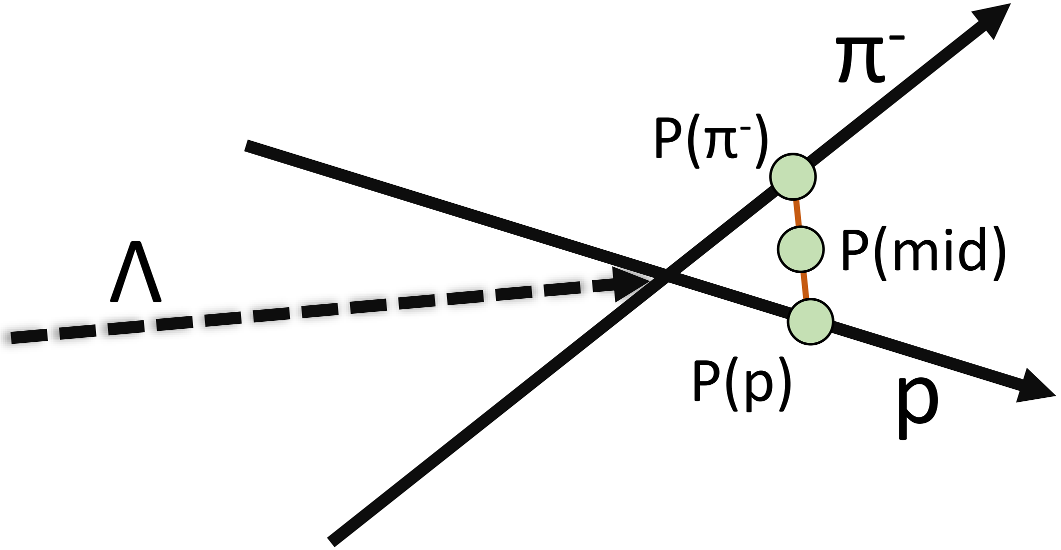


Presently: **P**oint **O**f **C**losest **A**pproach

- Calculate minimum distance between charged tracks
  - Charged tracks rejected as pairs if too far from each other

Question: could a decay vertex, utilizing momentum conservation, fit improve the momentum resolutions and hence the analysis results?

Track Representation and Constraints



Evaluate θ and φ at P(π⁻) and P(p)
Or Evaluate θ and φ at P(mid)

Work In Progress!

Primary Vertex Constraint, 1C fit

$$d = (d_1 \times d_2) \cdot (b_1 - b_2)$$

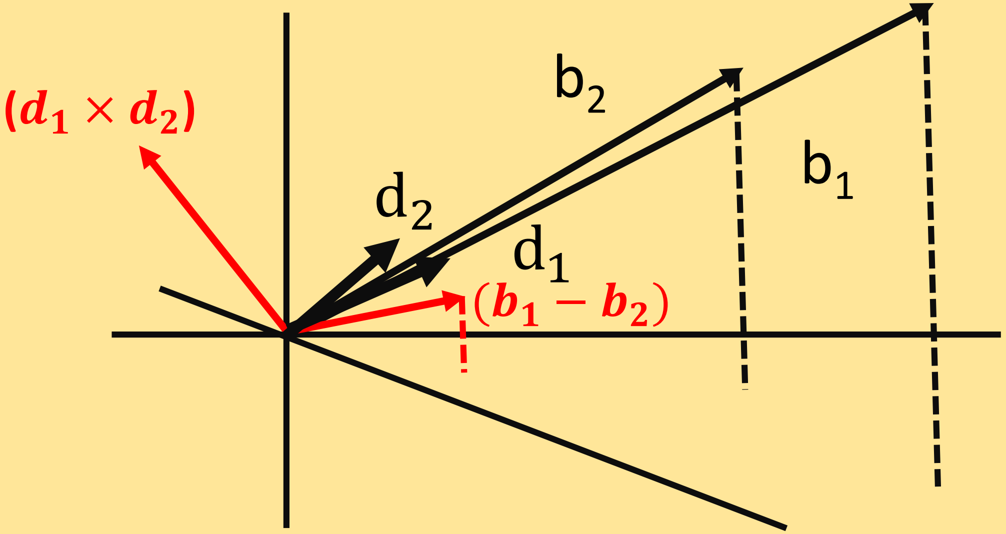
$|d|/|d_1 \times d_2|$ – closest distance between tracks

Direction Vector

$$\begin{cases} d_x = \sin(\theta) \cdot \cos(\varphi) \\ d_y = \sin(\theta) \cdot \sin(\varphi) \\ d_z = \cos(\theta) \end{cases}$$

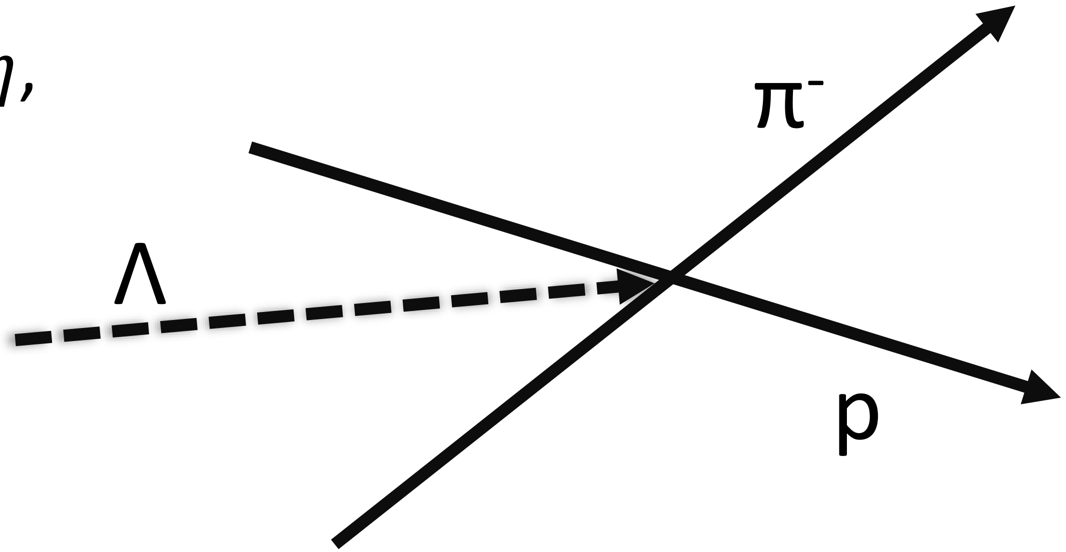
Base Vector

$$\begin{cases} b_x = R \cdot \cos\left(\varphi + \frac{\pi}{2}\right) \\ b_y = R \cdot \sin\left(\varphi + \frac{\pi}{2}\right) \\ b_z = z \end{cases}$$



4 Momentum Conservation In Decay Vertex

Constraint Eqs. f, with measured, η, and unmeasured, ξ , quantities:



$$f_K\ (\eta_1, \eta_2, \dots, \eta_N\ , \xi_1, \xi_2, \dots, \xi_J) = 0$$

where

$$\vec{\eta} = (P_{\pi^-}, \theta_{\pi^-}\ , \varphi_{\pi^-}, P_p, \theta_p\ , \varphi_p\ , \theta_\Lambda\ , \varphi_\Lambda)$$

$\vec{\xi} = (P_\Lambda\ )$ $\quad P_\Lambda$ - need start value for iterations

θ_Λ=arccos$\left(\frac{z}{r}\right)$

φ_Λ=arctan$\left(\frac{y}{x}\right)$

where

r=$\sqrt{(x^2 + y^2 + z^2)}$

x,y,z – coordinates of calculated decay vertex

4C fit:

$$f_1 = -p_\Lambda sin\theta_\Lambda cos\varphi_\Lambda + p_{\pi^-} sin\theta_{\pi^-} cos\varphi_{\pi^-} + p_p sin\theta_p cos\varphi_p = 0 \quad (p_x)$$

$$f_2 = -p_\Lambda sin\theta_\Lambda sin\varphi_\Lambda + p_{\pi^-} sin\theta_{\pi^-} sin\varphi_{\pi^-} + p_p sin\theta_p sin\varphi_p = 0 \quad (p_y)$$

$$f_3 = -p_\Lambda cos\theta_\Lambda + p_{\pi^-} cos\theta_{\pi^-} + p_p cos\theta_p = 0 \quad (p_z)$$

$$f_4 = -\sqrt{p_\Lambda^2 + m_\Lambda^2} + \sqrt{p_{\pi^-}^2 + m_{\pi^-}^2} + \sqrt{p_p^2 + m_p^2} = 0 \quad (E).$$

Comparison to PandaRoot

4C Fit

PandaRoot

Constrain the final state particles to pbar-p system

Benefit: pbar-p system known basically without errors

Hydra

Constrain final state particles to intermediate state, *e.g.* Λ

Benefit: do not need all final state particles but only Λ decay products

Summary

- Kinematic fitting procedure based on Lagrange multipliers exist
  - Constraints*: invariant mass, missing mass, primary vertex, invariant mass + primary vertex*
- Results of mass and primary vertex constraint look promising for the channel p(3.5GeV)p->pK^+Λ, Λ-> pπ^-

Outlook

- Finalize implementation and testing of decay vertex fitting procedure and 4 Momentum constraints
- Covariance matrix need further optimization
- Optimization needed for stopping criteria for number of iterations
- Apply for spin observables measurement in decay Λ→pπ^-

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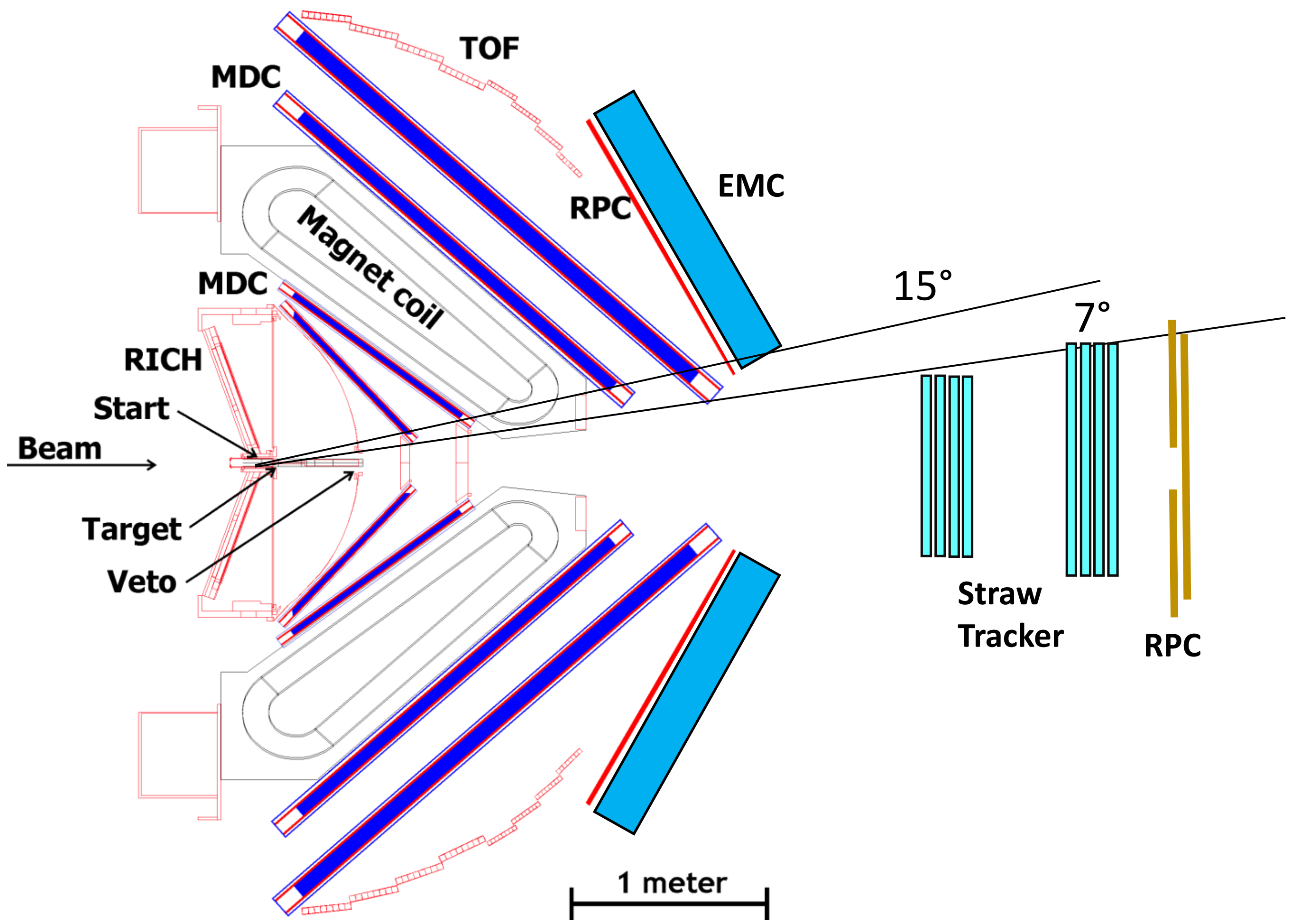
Thank You for your attention!

Questions?

Backup

HADES Setup



FAIR Phase-0 Upgrade

EMC:
improved energy information for electrons and leptons

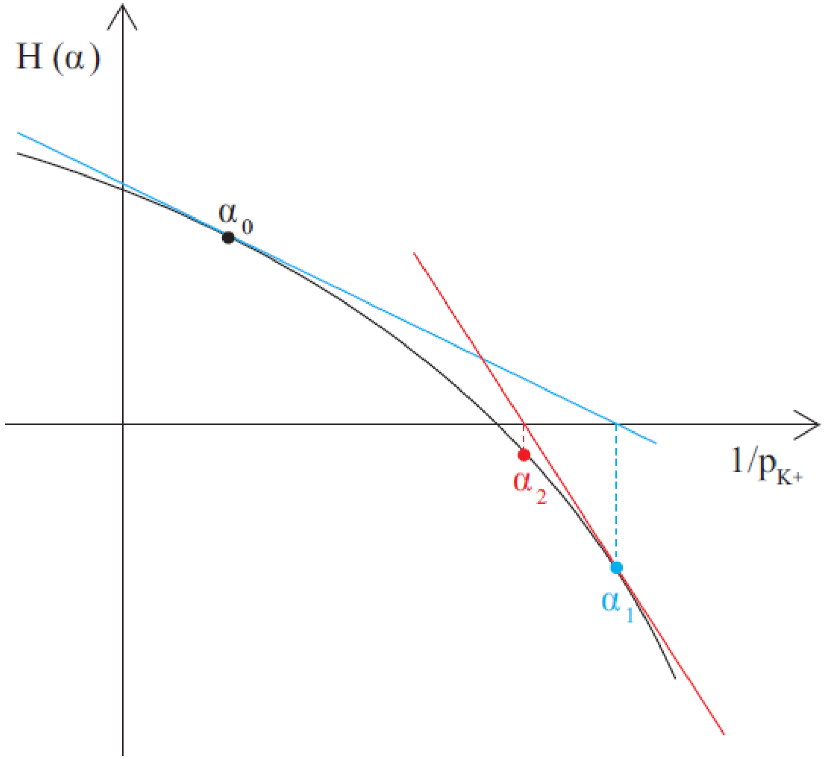
Straw Tracker:
Based on PADNA ST

RPC:
Resistive Plate Chambers
TOF information

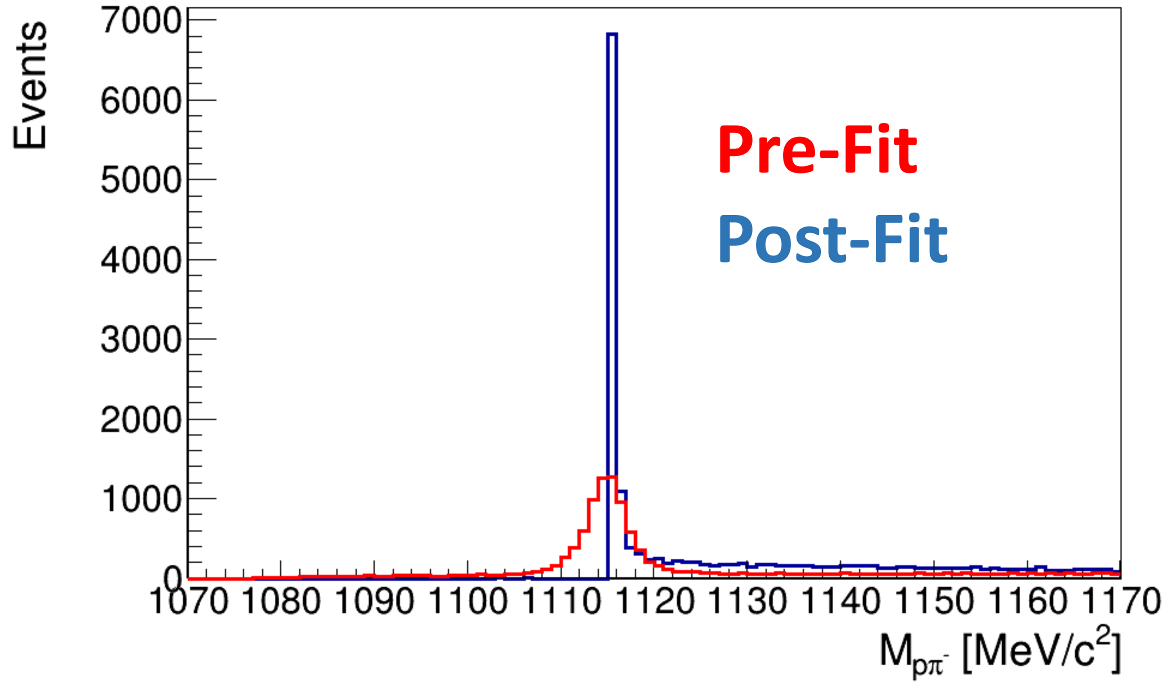
Example Constraint Function

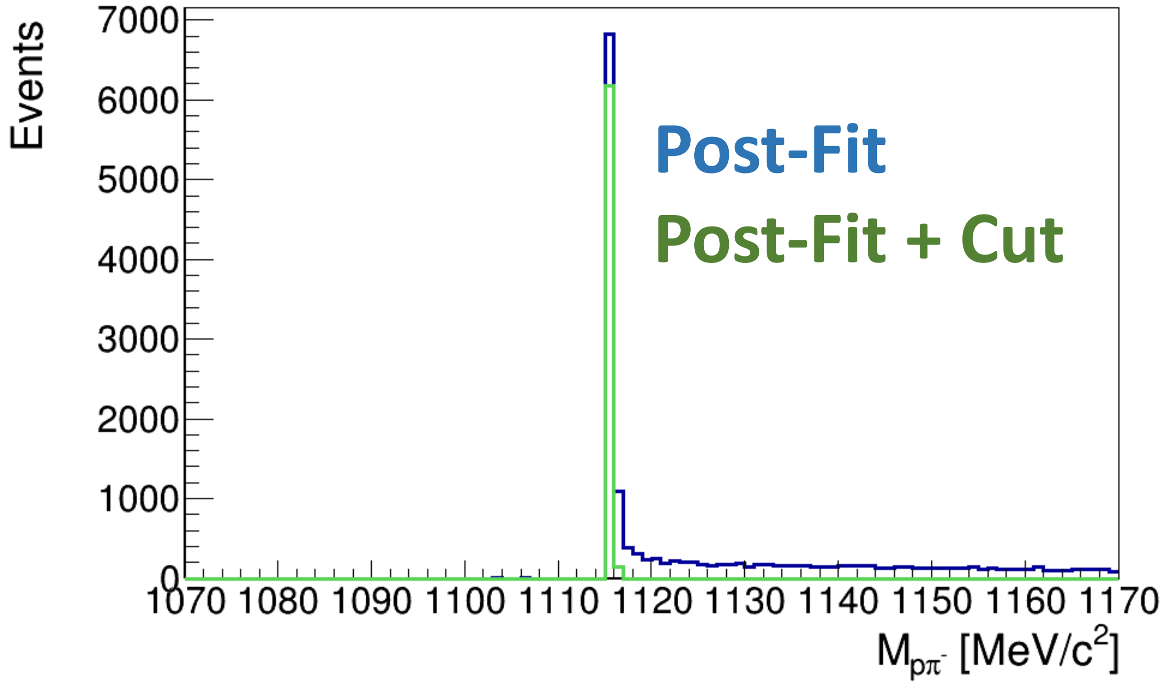
From:

https://hades.gsi.de/sites/default/files/web/media/documents/thesis/Diploma/Exclusive_analysis_of_the_Lambda(1405)_resonance_in_the_charged_Sigma-pi_decay_channels_in_proton_proton_reactions_with_HADES_Johannes_Siebenson_2011-Jan.pdf



Invariant Mass





Entries

Pre-fit: 21 490

Post-fit: 21 490

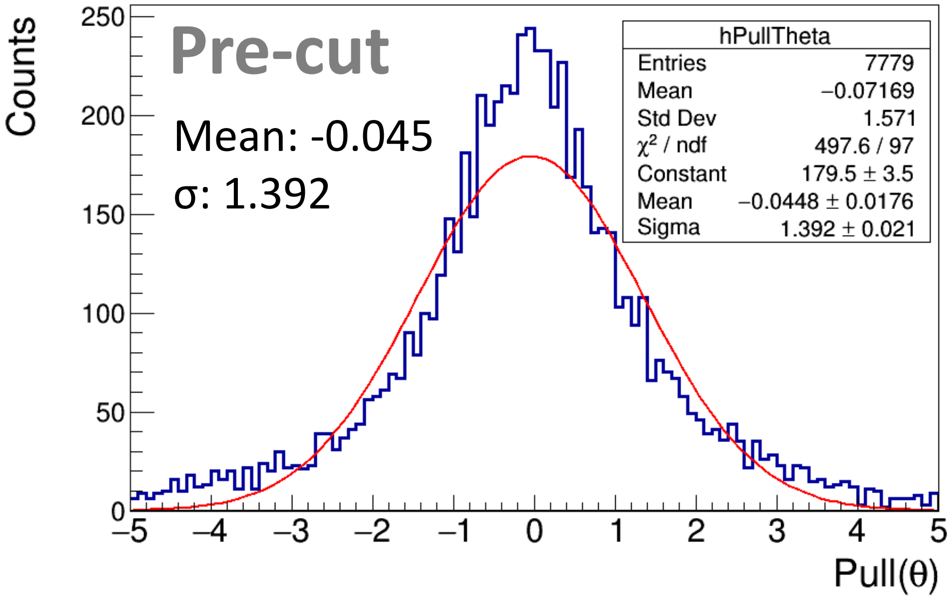
Post-fit+cut: 6 312

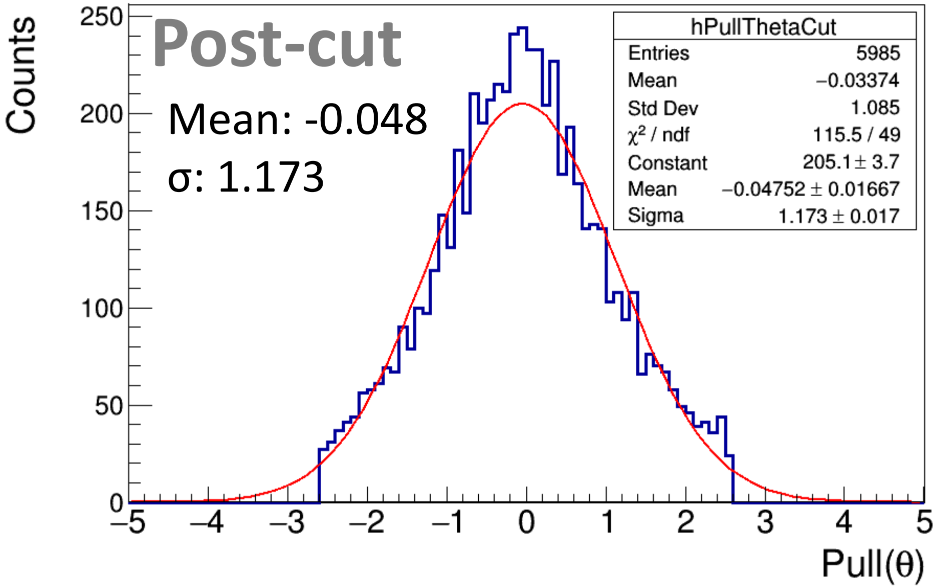
Pull Distributions, No Combinatorial Bkg.

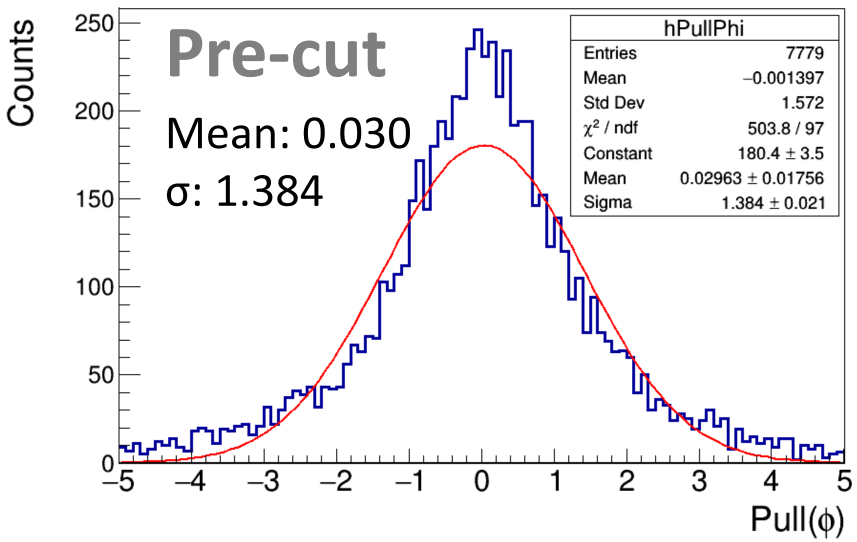
$$z_i = \frac{y_i - \eta_i}{\sqrt{\sigma^2(y_i) - \sigma^2(\eta_i)}}$$

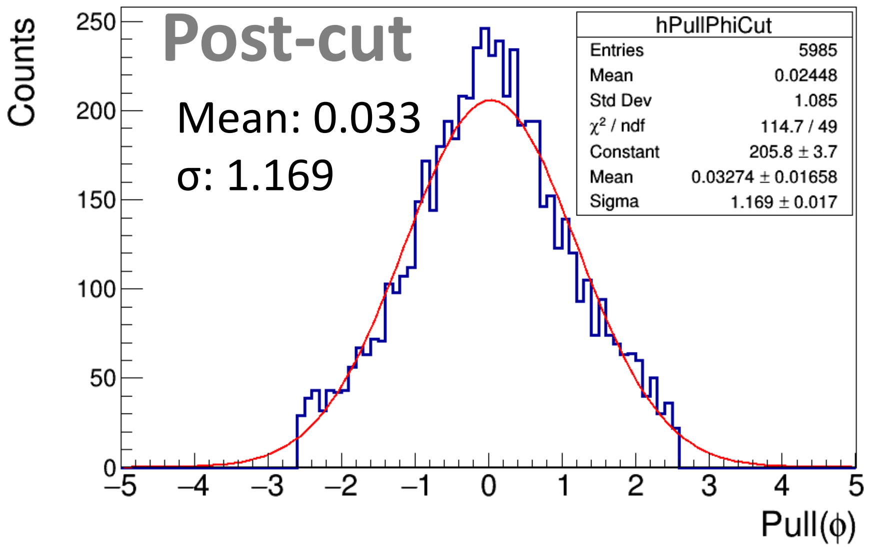
Ideally N(0,1)

Effects of probability cut.
Eff. loss: 23%









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