Longitudinal Dynamics

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4/11/2020

Motion of particles

I. Newton

$$F = \frac{dp}{dt}$$

$$F = qE + qv \times B$$

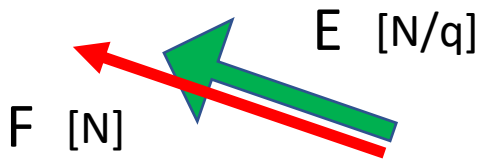
J.C. Maxwell

Even for the longitudinal dynamics!

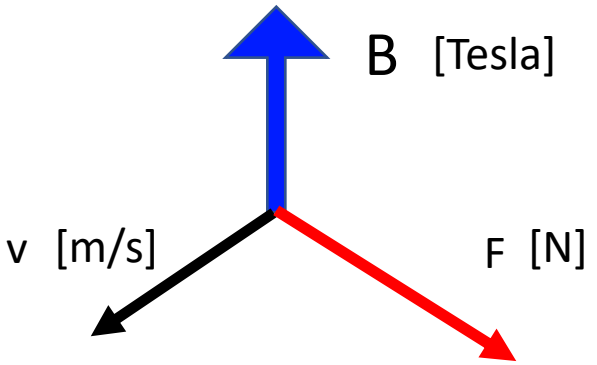
The Lorentz force

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

Electric field



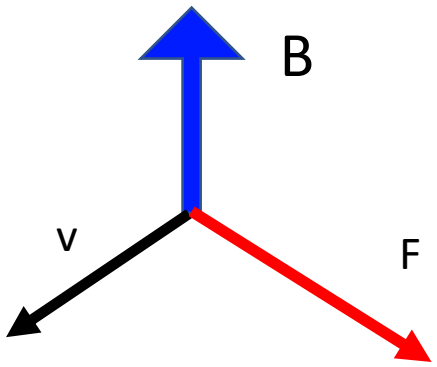
Magnetic field

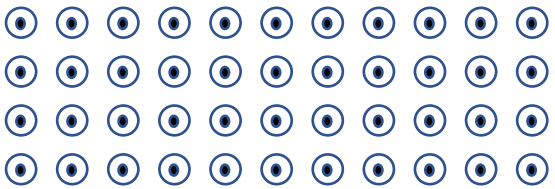


Magnetic field does not make work

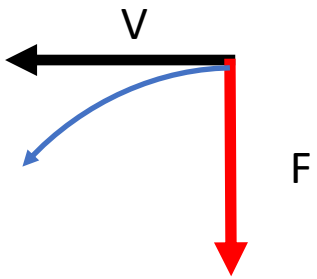
The Lorentz force

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

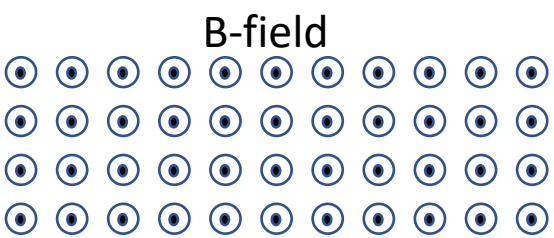


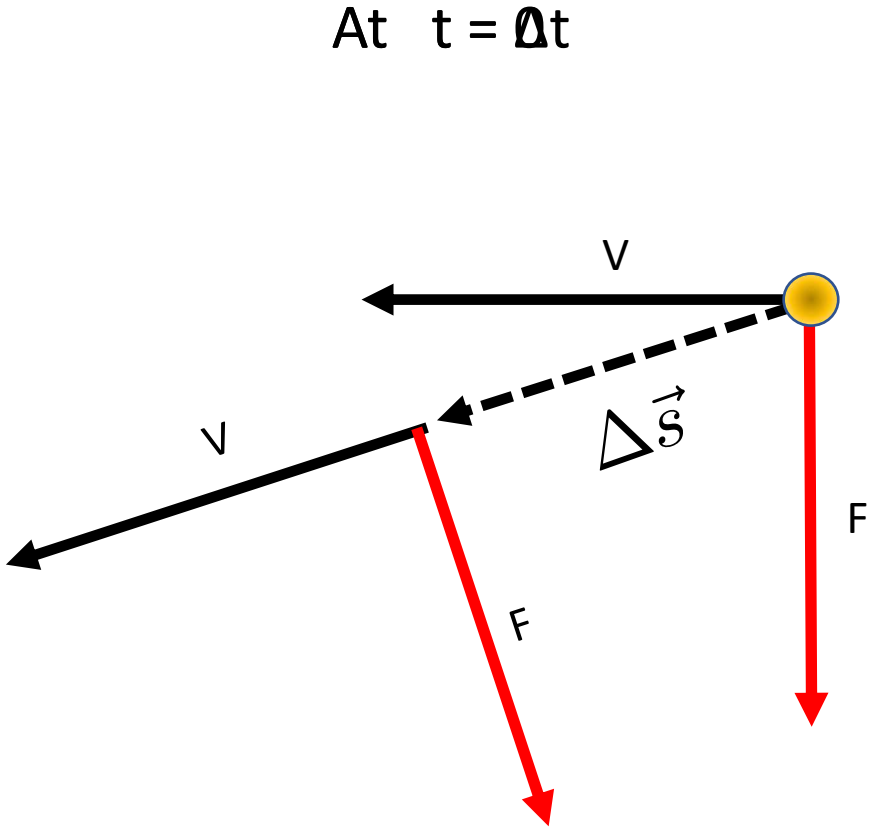


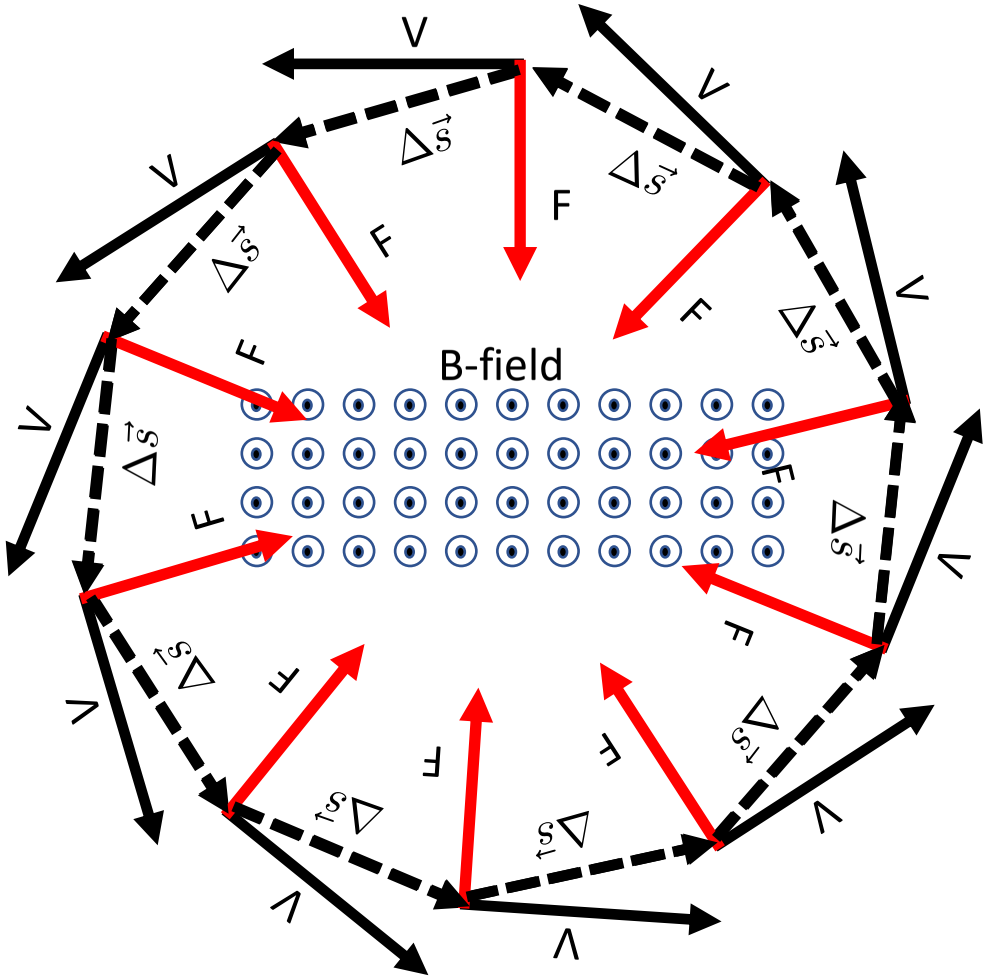
B-field

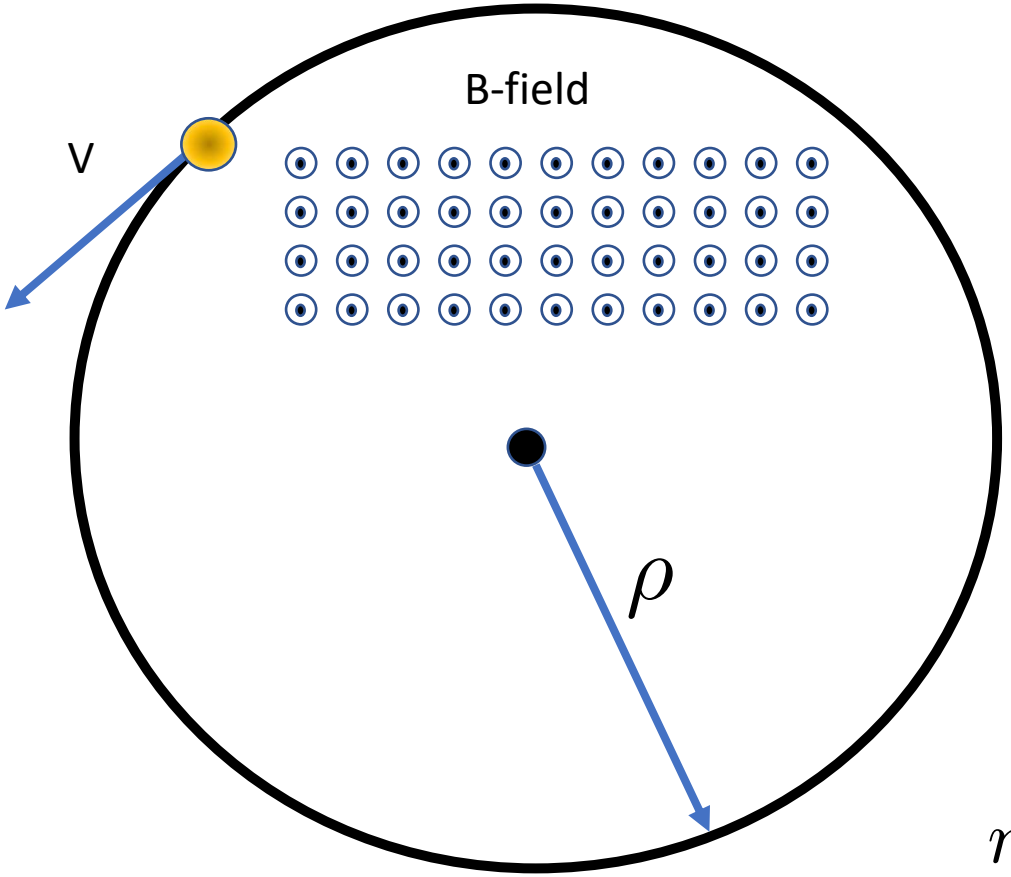


The Lorentz force





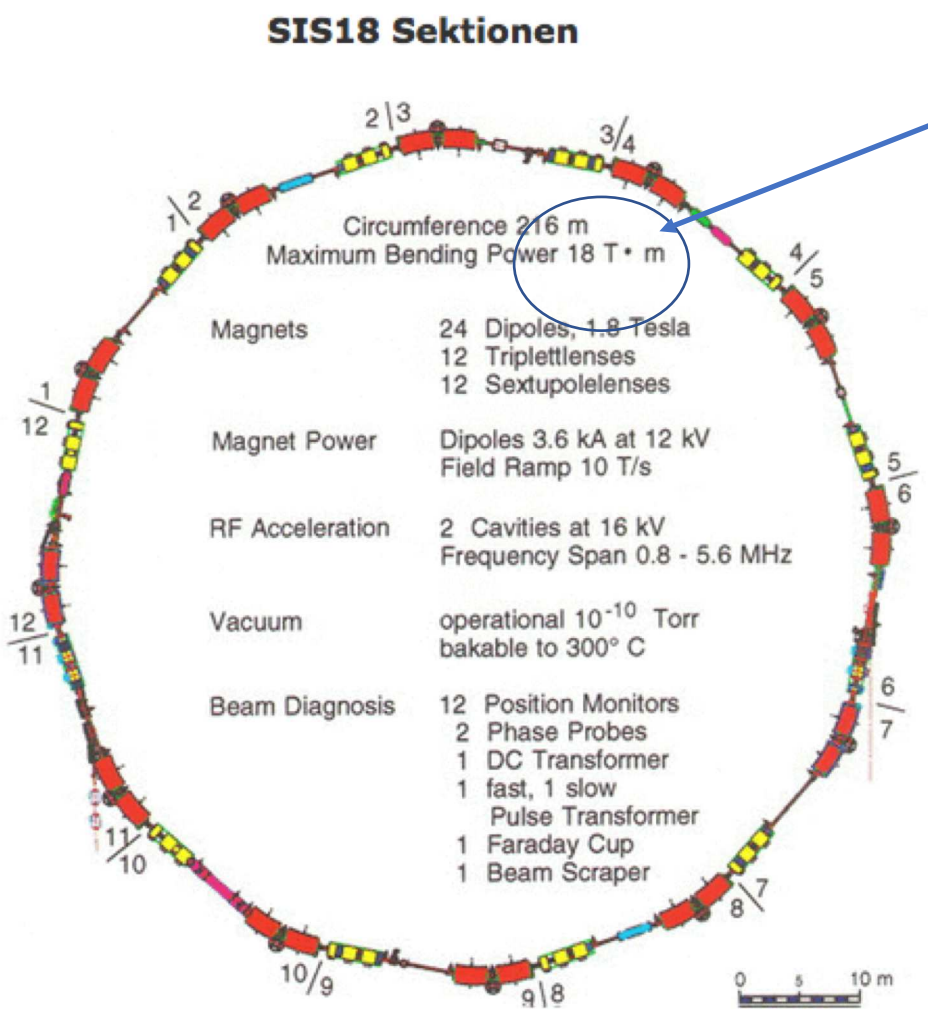

V
F
B-field



$$m\gamma v = q\rho B$$

$$\frac{p}{q} = \rho B$$

Example: SIS18



maximum

$$B\rho = 18\ Tm$$

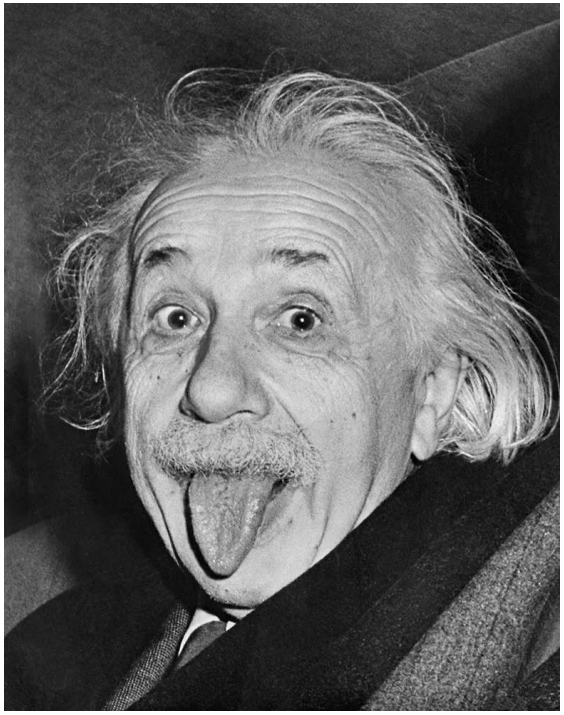
B = 1.8 Tesla

Bending radius = 10 m

But this for the correct particle energy !

Energy ←→ Momentum

$$E = mc^2$$



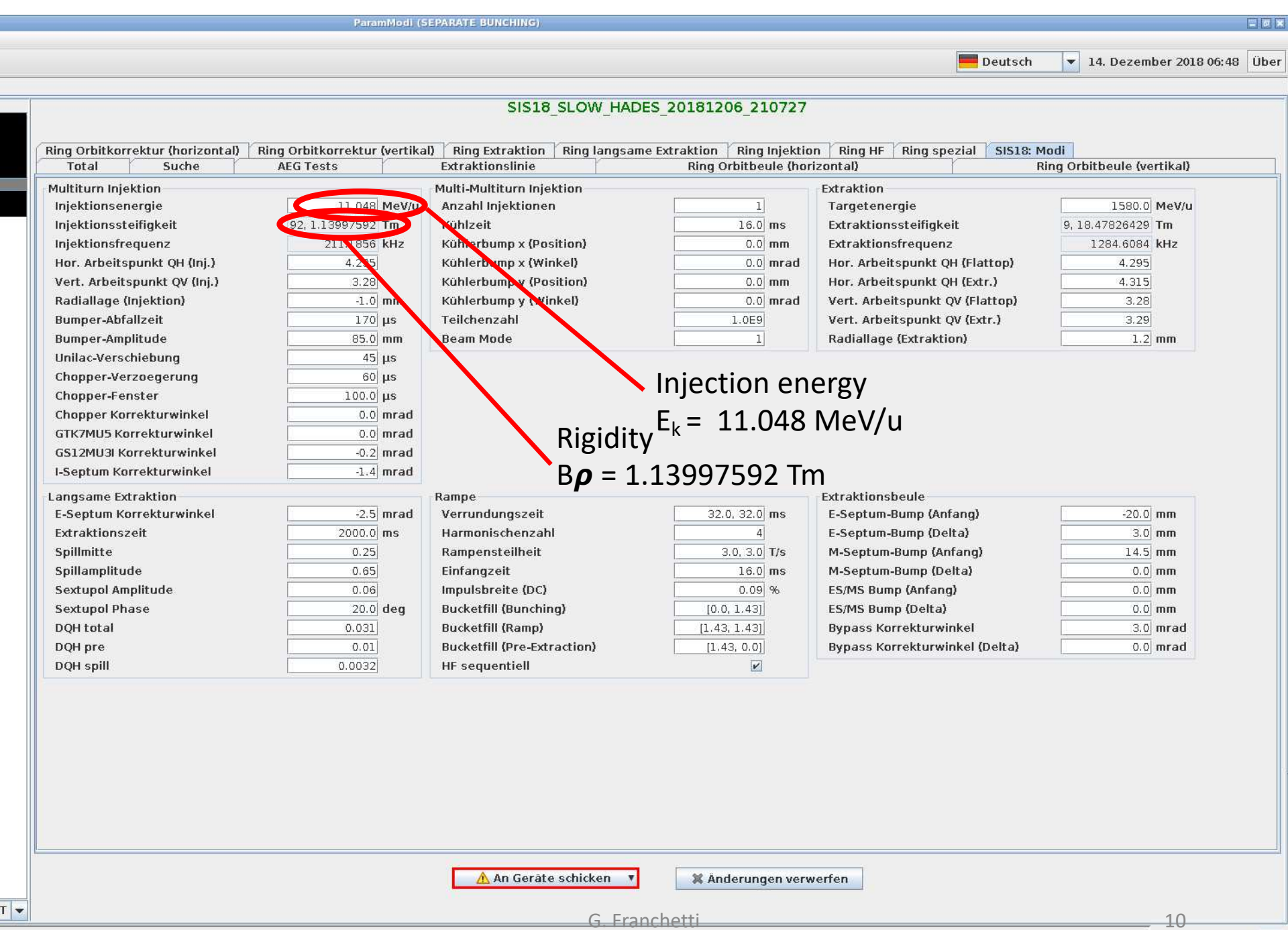
$$E = mc^2 + E_k$$

Kinetic energy
of the ion

$$E^2 = (mc^2)^2 + (pc)^2$$

energy

momentum



Injection energy

E_k = 11.048 MeV/u

Rigidity

$B\rho$ = 1.13997592 Tm

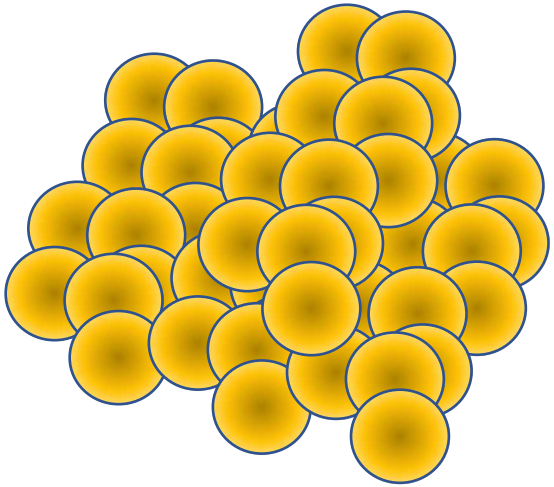
Notation

Energy is per nucleon

1 Nucleon  11.048 MeV/u

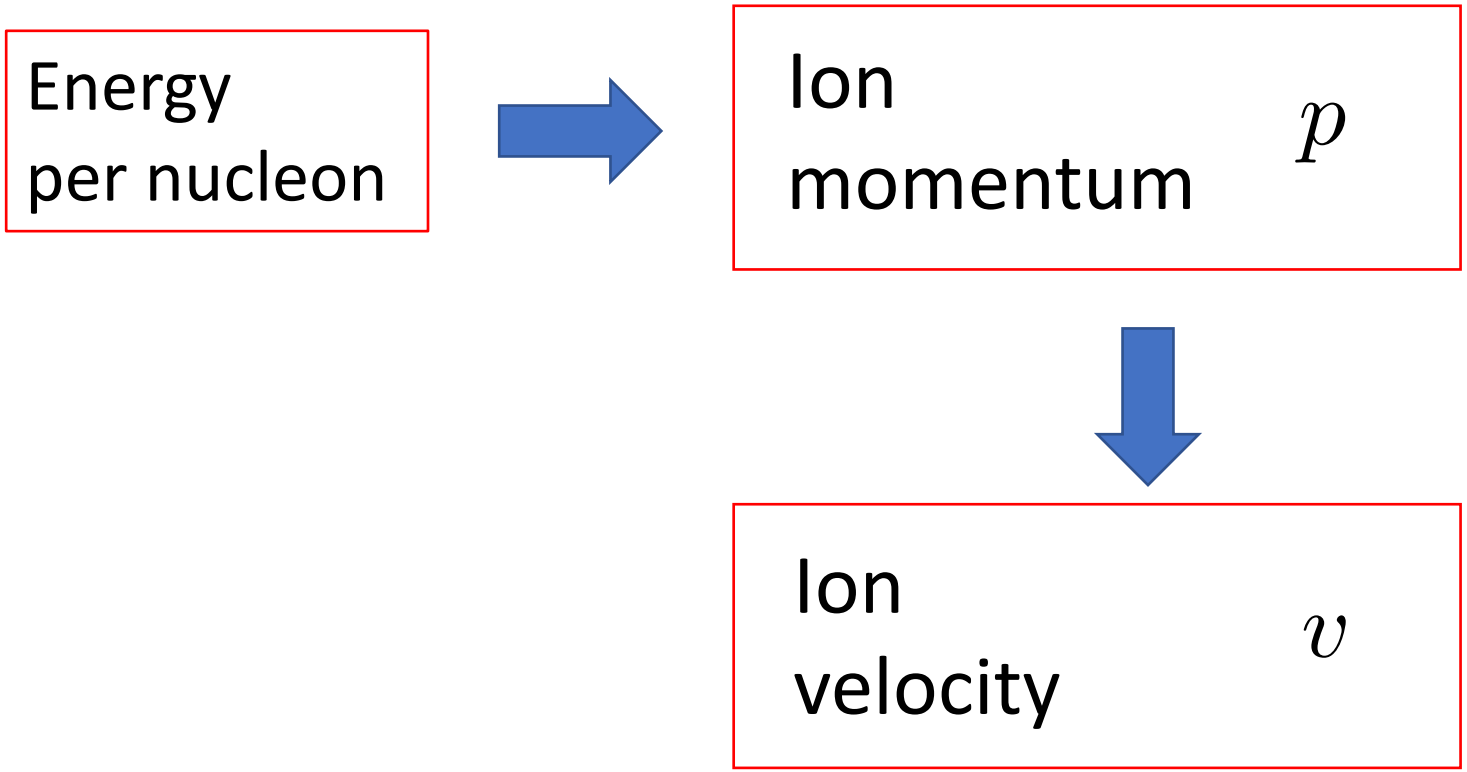
Silver

107 nucleons



Energy → 11.048 x 107 MeV

Ion property



Accelerator settings

Ion momentum p_0

Ion charge q

Accelerator radius ρ_0

$$\frac{p_0}{q} = \rho_0 B_0 \quad \Rightarrow \quad B_0$$

Example: Silver 107, Charge +45

Injection energy

E_k = 11.048 MeV/u

= 11.048 x 10^6 e/u

= 1.77008 x 10^{-12} J/u

m_u = 1.66056 x 10^{-27} Kg

E = 107 m_u c^2 + E_k 107

= 1.59688 x 10^{-8} + 1.89399 x 10^{-10} J

= 1.61582 x 10^{-8} J

Ion momentum

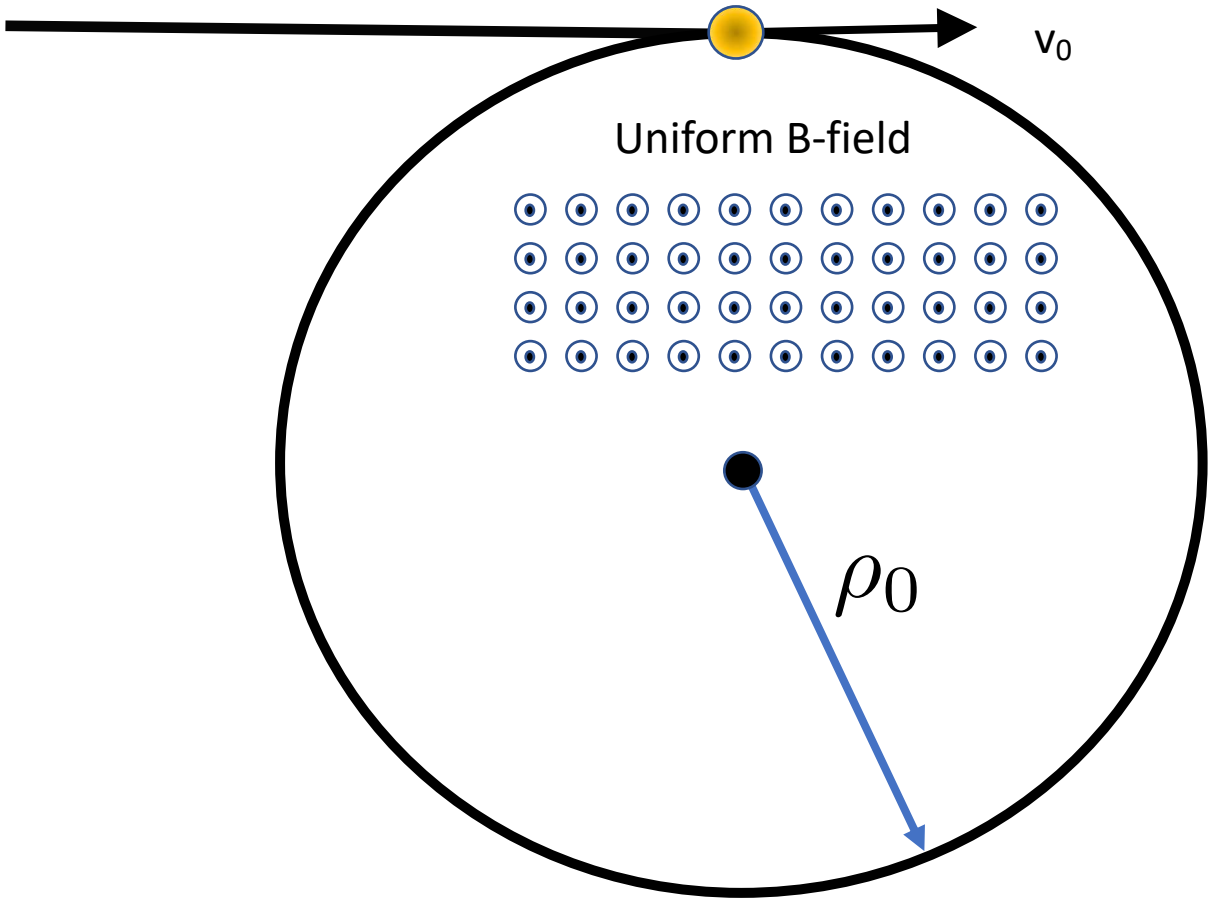
$$E^2 = (mc^2)^2 + (pc)^2$$

p = 8.22818 x 10^{-18} Kg m / s

Ion velocity

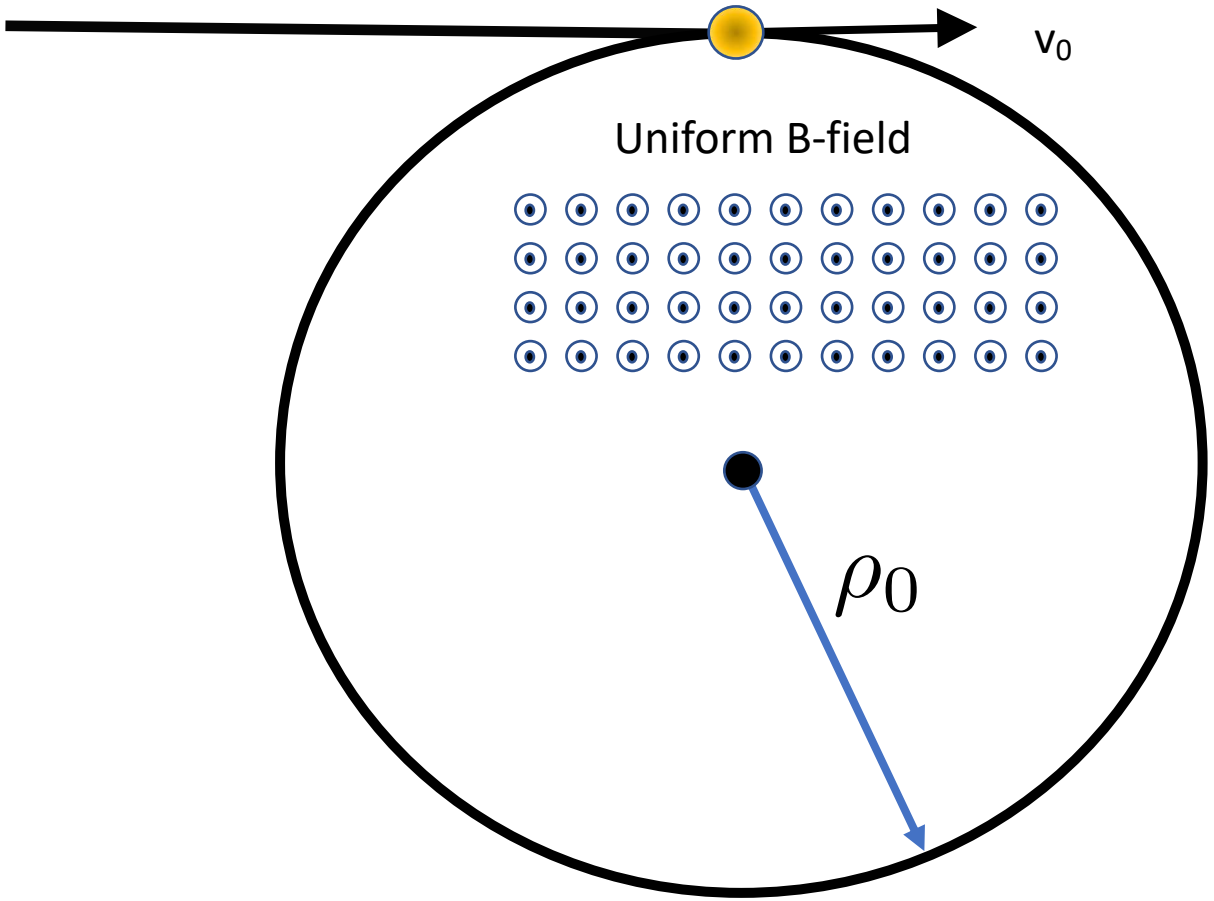
v = 45766823.10 m/s

Energy matching



$$\frac{p_0}{q} = \rho_0 B_0$$

Revolution time



$$\tau_0 = \frac{2\pi\rho_0}{v_0}$$

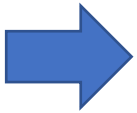
Silver: Mass 107, Charge +45

At injection energy

Circumference = 216.72 m

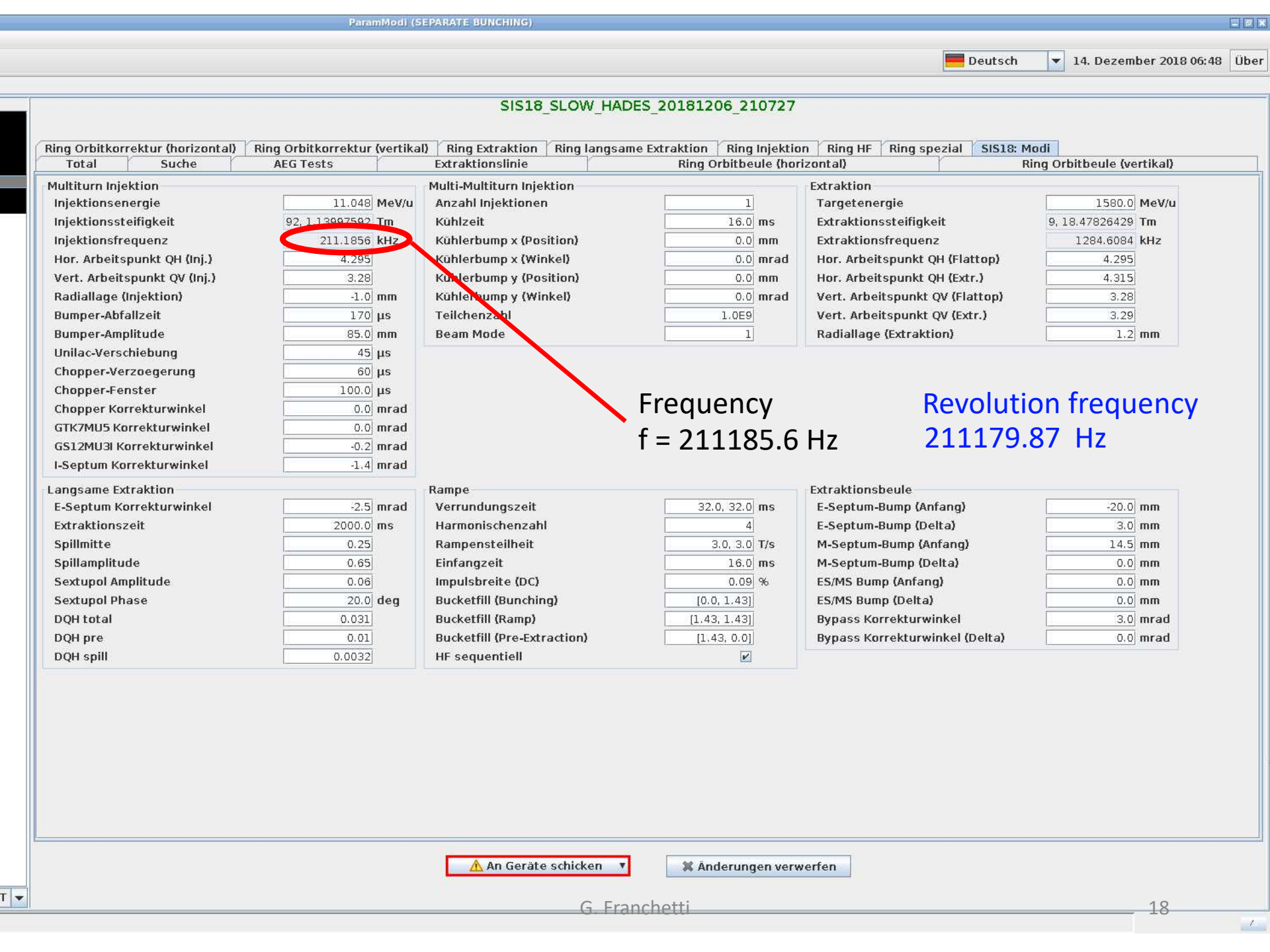
Ion velocity

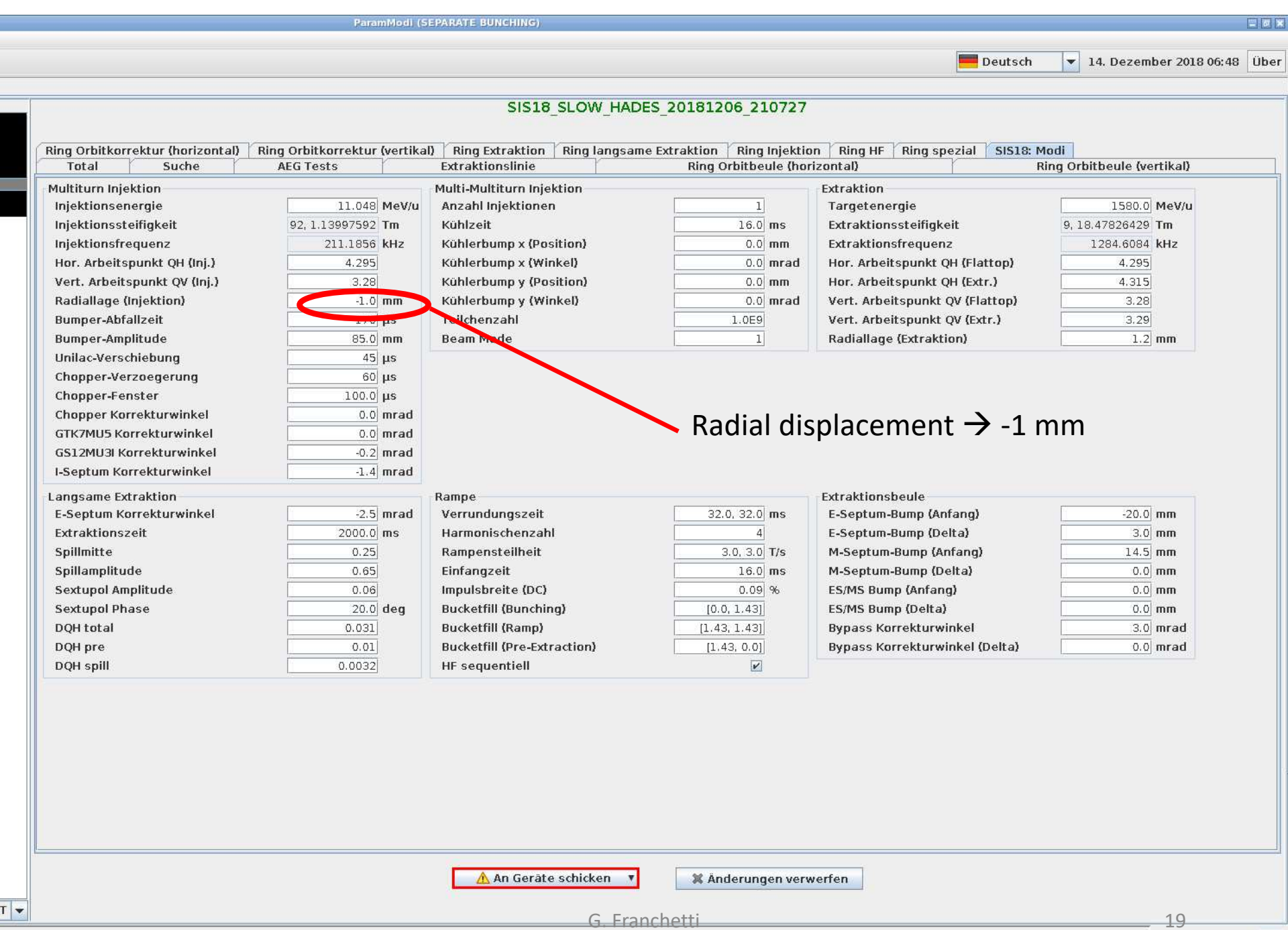
v = 45766823.10 m/s



$\tau = 4.7227$ microseconds

Revolution frequency → 211179.87 Hz


ParamModi (SEPARATE BUNCHING)
Deutsch
14. Dezember 2018 06:48
Über
SIS18_SLOW_HADES_20181206_210727
Ring Orbitkorrektur (horizontal)
Ring Orbitkorrektur (vertikal)
Ring Extraktion
Ring langsame Extraktion
Ring Injektion
Ring HF
Ring spezial
SIS18: Modi
Total
Suche
AEG Tests
Extraktionslinie
Ring Orbitbeule (horizontal)
Ring Orbitbeule (vertikal)
Multiturn Injektion
Injektionsenergie 11.048 MeV/u
Injektionssteifigkeit 92, 1.13997592 Tm
Injektionsfrequenz 211.1856 kHz
Hor. Arbeitspunkt QH (Inj.) 4.295
Vert. Arbeitspunkt QV (Inj.) 3.28
Radiallage (Injektion) -1.0 mm
Bumper-Abfallzeit 170 µs
Bumper-Amplitude 85.0 mm
Unilac-Verschiebung 45 µs
Chopper-Verzoegerung 60 µs
Chopper-Fenster 100.0 µs
Chopper Korrekturwinkel 0.0 mrad
GTK7MU5 Korrekturwinkel 0.0 mrad
GS12MU3I Korrekturwinkel -0.2 mrad
I-Septum Korrekturwinkel -1.4 mrad
Multi-Multiturn Injektion
Anzahl Injektionen 1
Kühlzeit 16.0 ms
Kühlerbump x (Position) 0.0 mm
Kühlerbump x (Winkel) 0.0 mrad
Kühlerbump y (Position) 0.0 mm
Kühlerbump y (Winkel) 0.0 mrad
Teilchenzahl 1.0E9
Beam Mode 1
Extraktion
Targetenergie 1580.0 MeV/u
Extraktionssteifigkeit 9, 18.47826429 Tm
Extraktionsfrequenz 1284.6084 kHz
Hor. Arbeitspunkt QH (Flattop) 4.295
Hor. Arbeitspunkt QH (Extr.) 4.315
Vert. Arbeitspunkt QV (Flattop) 3.28
Vert. Arbeitspunkt QV (Extr.) 3.29
Radiallage (Extraktion) 1.2 mm
Frequency
f = 211185.6 Hz
Revolution frequency
211179.87 Hz
Langsame Extraktion
E-Septum Korrekturwinkel -2.5 mrad
Extraktionszeit 2000.0 ms
Spillmitte 0.25
Spillamplitude 0.65
Sextupol Amplitude 0.06
Sextupol Phase 20.0 deg
DQH total 0.031
DQH pre 0.01
DQH spill 0.0032
Rampe
Verrundungszeit 32.0, 32.0 ms
Harmonischenzahl 4
Rampensteilheit 3.0, 3.0 T/s
Einfangzeit 16.0 ms
Impulsbreite (DC) 0.09 %
Bucketfill (Bunching) [0.0, 1.43]
Bucketfill (Ramp) [1.43, 1.43]
Bucketfill (Pre-Extraction) [1.43, 0.0]
HF sequentiell
Extraktionsbeule
E-Septum-Bump (Anfang) -20.0 mm
E-Septum-Bump (Delta) 3.0 mm
M-Septum-Bump (Anfang) 14.5 mm
M-Septum-Bump (Delta) 0.0 mm
ES/MS Bump (Anfang) 0.0 mm
ES/MS Bump (Delta) 0.0 mm
Bypass Korrekturwinkel 3.0 mrad
Bypass Korrekturwinkel (Delta) 0.0 mrad
An Geräte schicken
Änderungen verwerfen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Bumper-Amplitude 85.0 mm
Unilac-Verschiebung 45 µs
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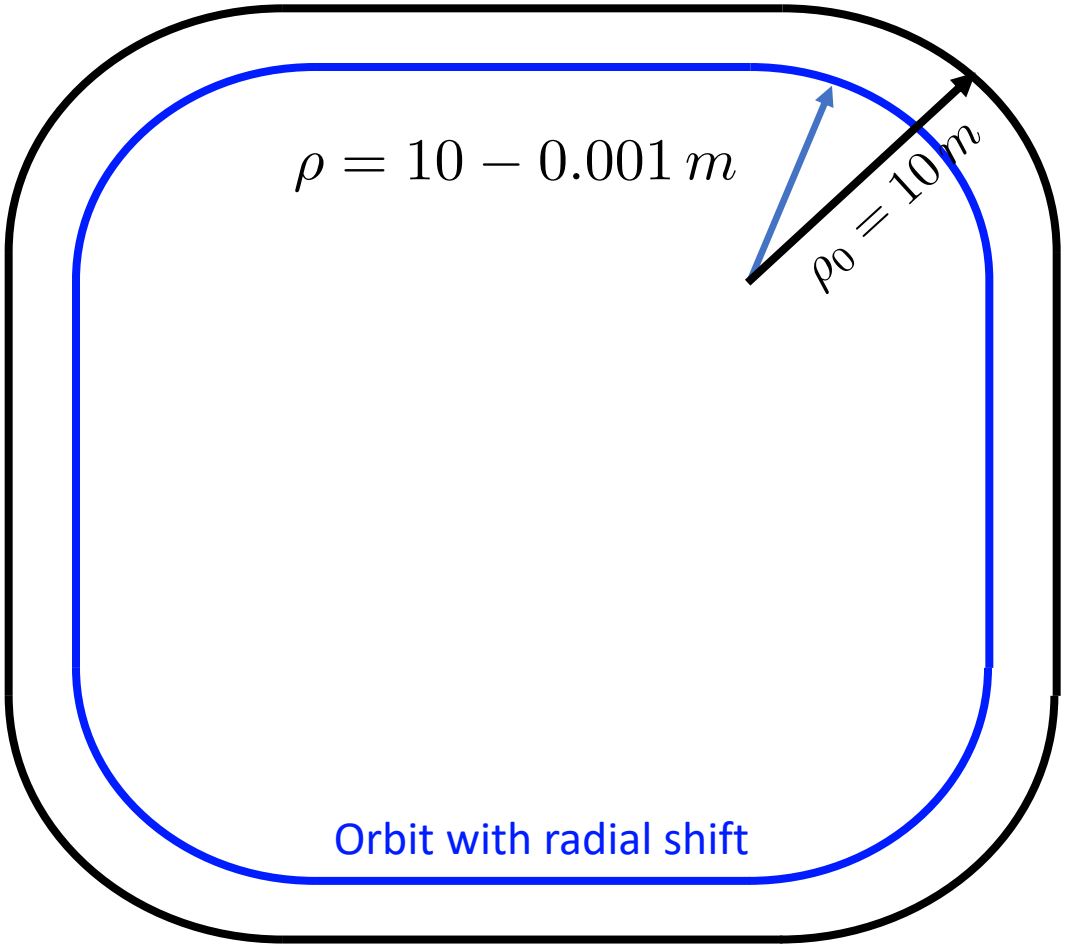
Chopper Korrekturwinkel 0.0 mrad
GTK7MU5 Korrekturwinkel 0.0 mrad
GS12MU3I Korrekturwinkel -0.2 mrad
I-Septum Korrekturwinkel -1.4 mrad
Multi-Multiturn Injektion
Anzahl Injektionen 1
Kühlzeit 16.0 ms
Kühlerbump x (Position) 0.0 mm
Kühlerbump x (Winkel) 0.0 mrad
Kühlerbump y (Position) 0.0 mm
Kühlerbump y (Winkel) 0.0 mrad
Teilchenzahl 1.0E9
Beam Mode 1
Extraktion
Targetenergie 1580.0 MeV/u
Extraktionssteifigkeit 9, 18.47826429 Tm
Extraktionsfrequenz 1284.6084 kHz
Hor. Arbeitspunkt QH (Flattop) 4.295
Hor. Arbeitspunkt QH (Extr.) 4.315
Vert. Arbeitspunkt QV (Flattop) 3.28
Vert. Arbeitspunkt QV (Extr.) 3.29
Radiallage (Extraktion) 1.2 mm
Radial displacement → -1 mm
Langsame Extraktion
E-Septum Korrekturwinkel -2.5 mrad
Extraktionszeit 2000.0 ms
Spillmitte 0.25
Spillamplitude 0.65
Sextupol Amplitude 0.06
Sextupol Phase 20.0 deg
DQH total 0.031
DQH pre 0.01
DQH spill 0.0032
Rampe
Verrundungszeit 32.0, 32.0 ms
Harmonischenzahl 4
Rampensteilheit 3.0, 3.0 T/s
Einfangzeit 16.0 ms
Impulsbreite (DC) 0.09 %
Bucketfill (Bunching) [0.0, 1.43]
Bucketfill (Ramp) [1.43, 1.43]
Bucketfill (Pre-Extraction) [1.43, 0.0]
HF sequentiell
Extraktionsbeule
E-Septum-Bump (Anfang) -20.0 mm
E-Septum-Bump (Delta) 3.0 mm
M-Septum-Bump (Anfang) 14.5 mm
M-Septum-Bump (Delta) 0.0 mm
ES/MS Bump (Anfang) 0.0 mm
ES/MS Bump (Delta) 0.0 mm
Bypass Korrekturwinkel 3.0 mrad
Bypass Korrekturwinkel (Delta) 0.0 mrad
An Geräte schicken
Änderungen verwerfen



L_0 = 216.72 m

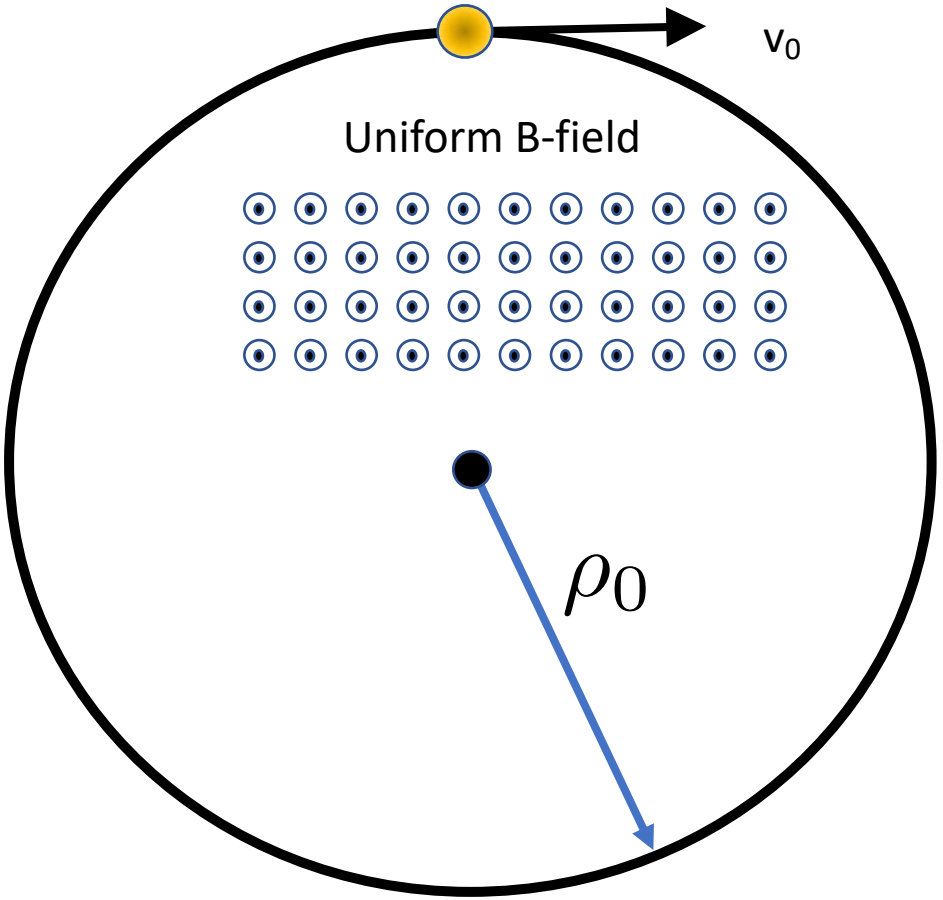
Change in circumference

DL = – 2 $\boldsymbol{\pi}$ 10^{-3} m

$f = f_0 (1 + DL/L_0)$

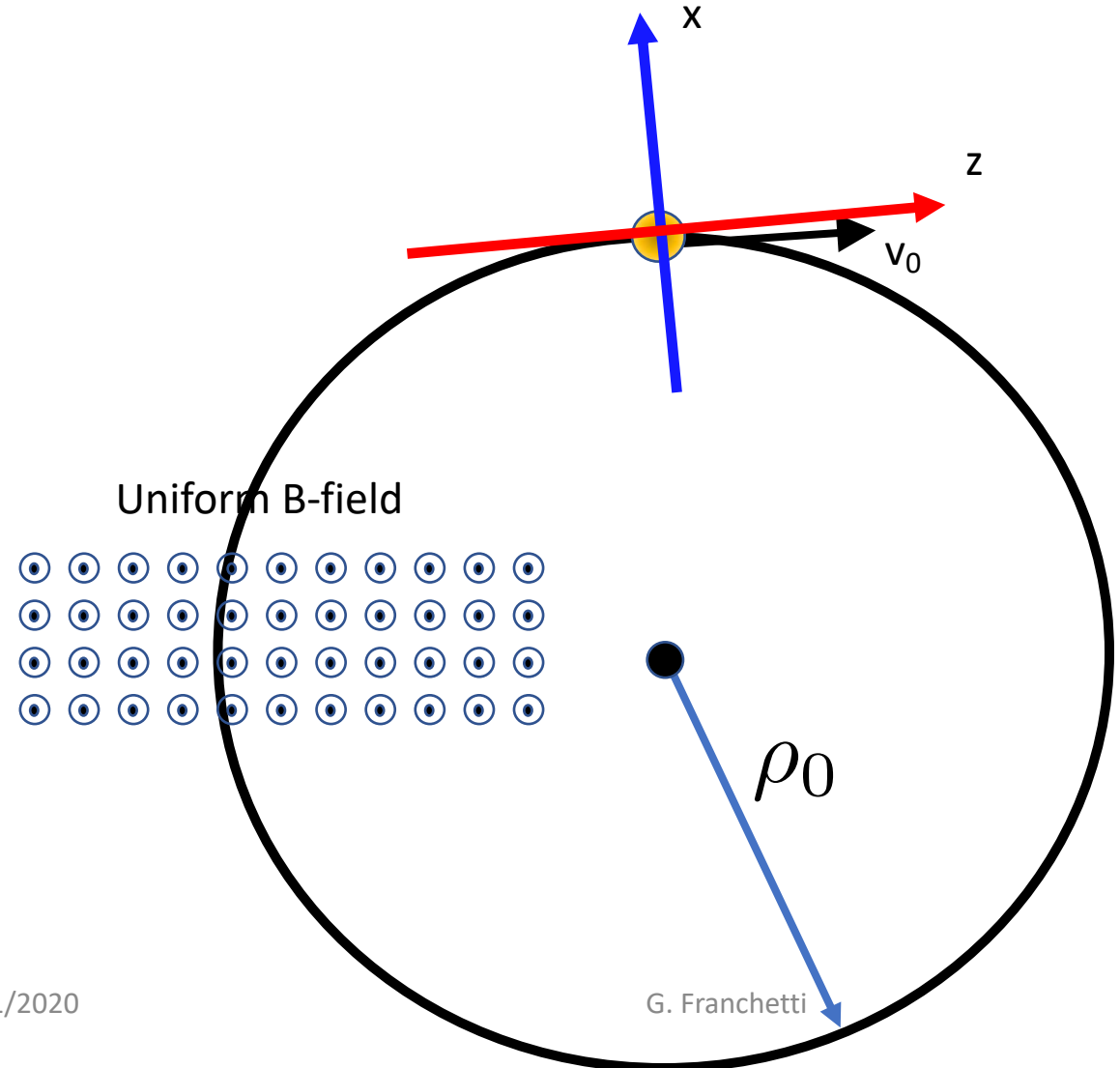
f = 211185.99 Hz

Reference particle



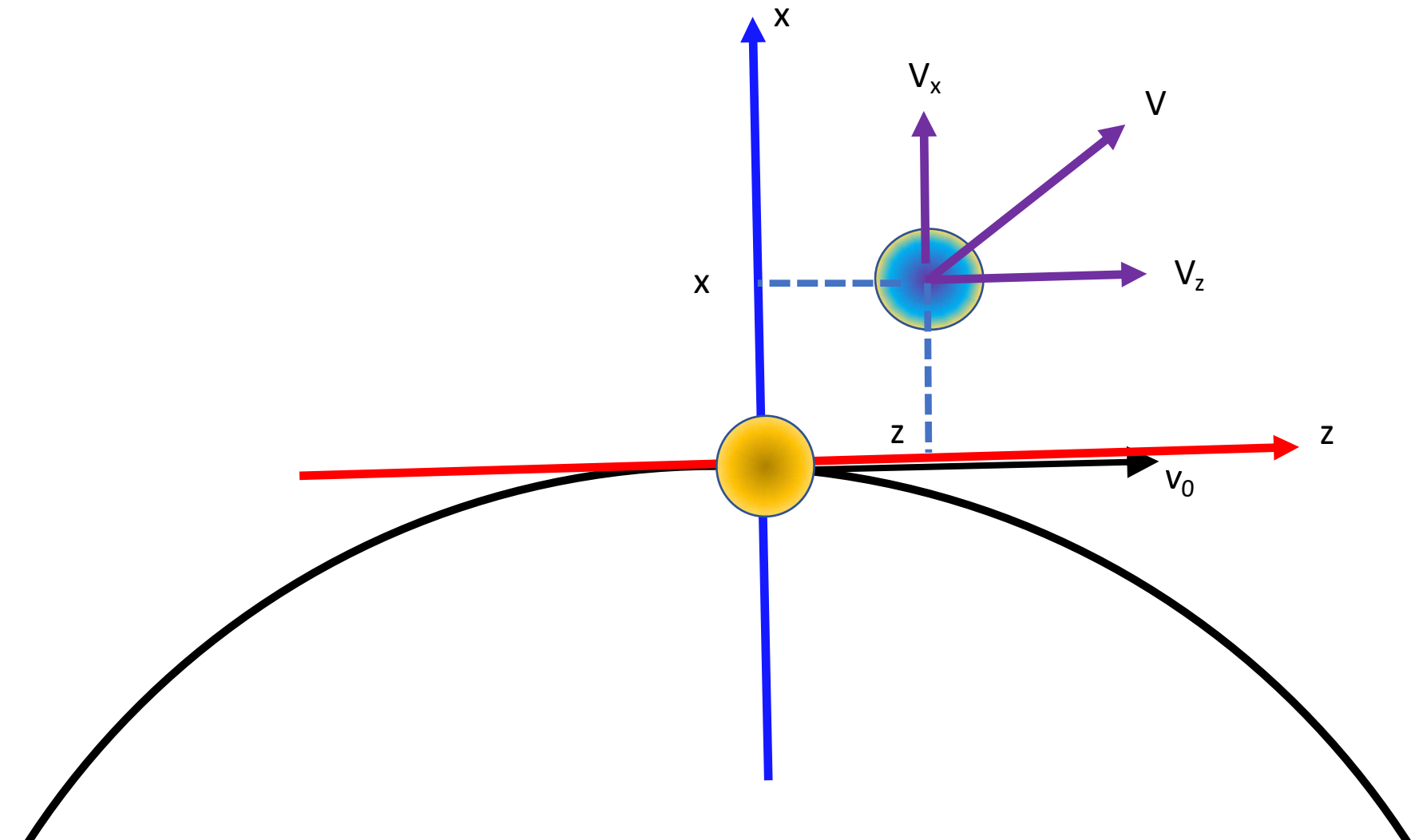
$$\frac{p_0}{q} = \rho_0 B_0$$

Reference frame attached to the reference particle

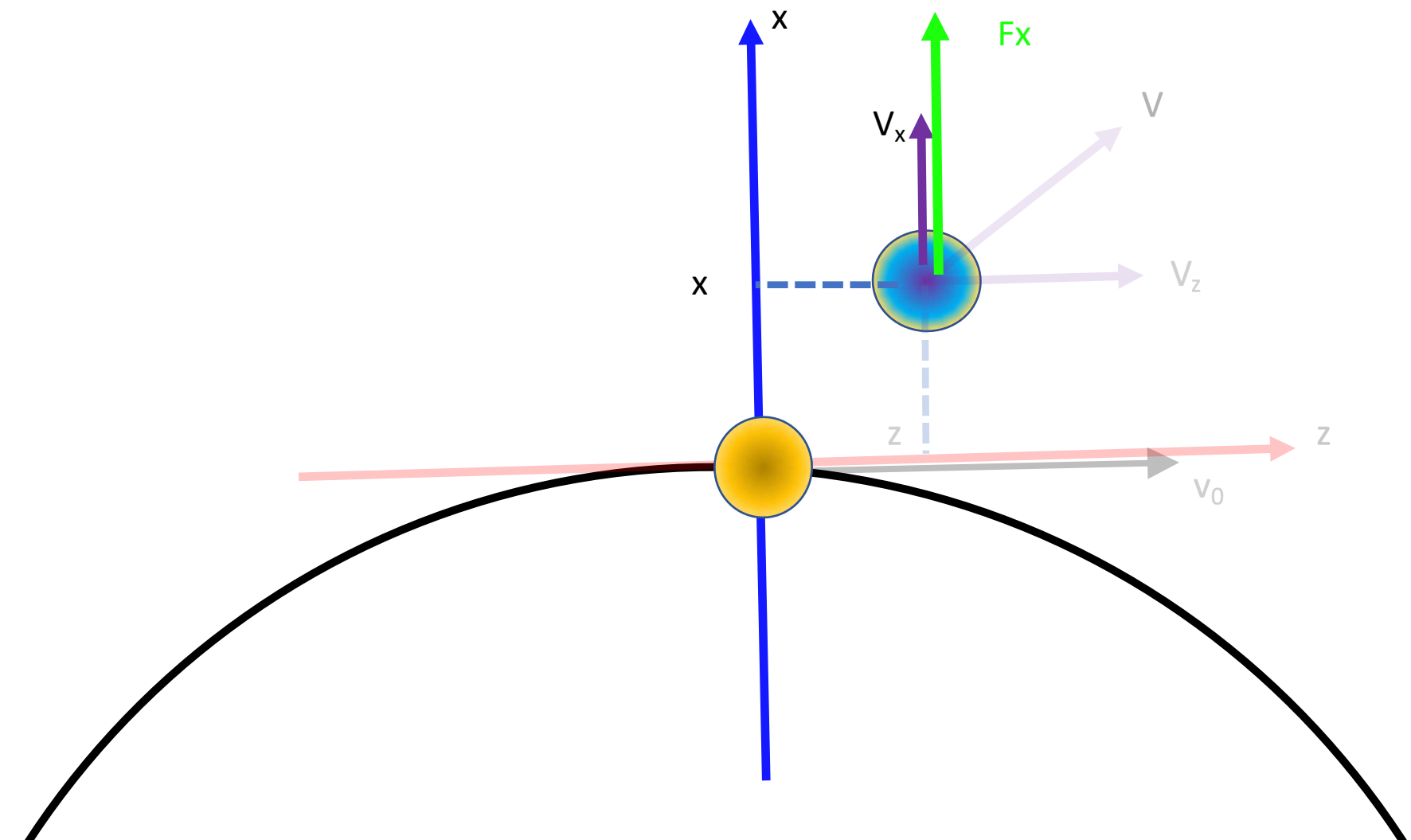


$$\frac{p_0}{q} = \rho_0 B_0$$

Particles near the reference particle can be described in the local reference frame

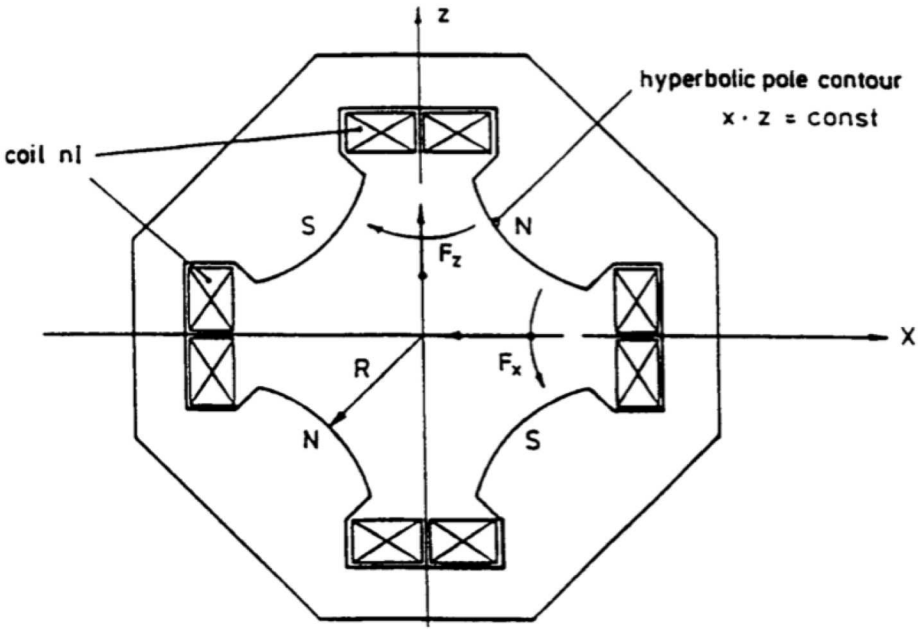


Transverse dynamics



Transverse forces → from magnets

$$F = qv \times B$$



$$B_x = \alpha y$$

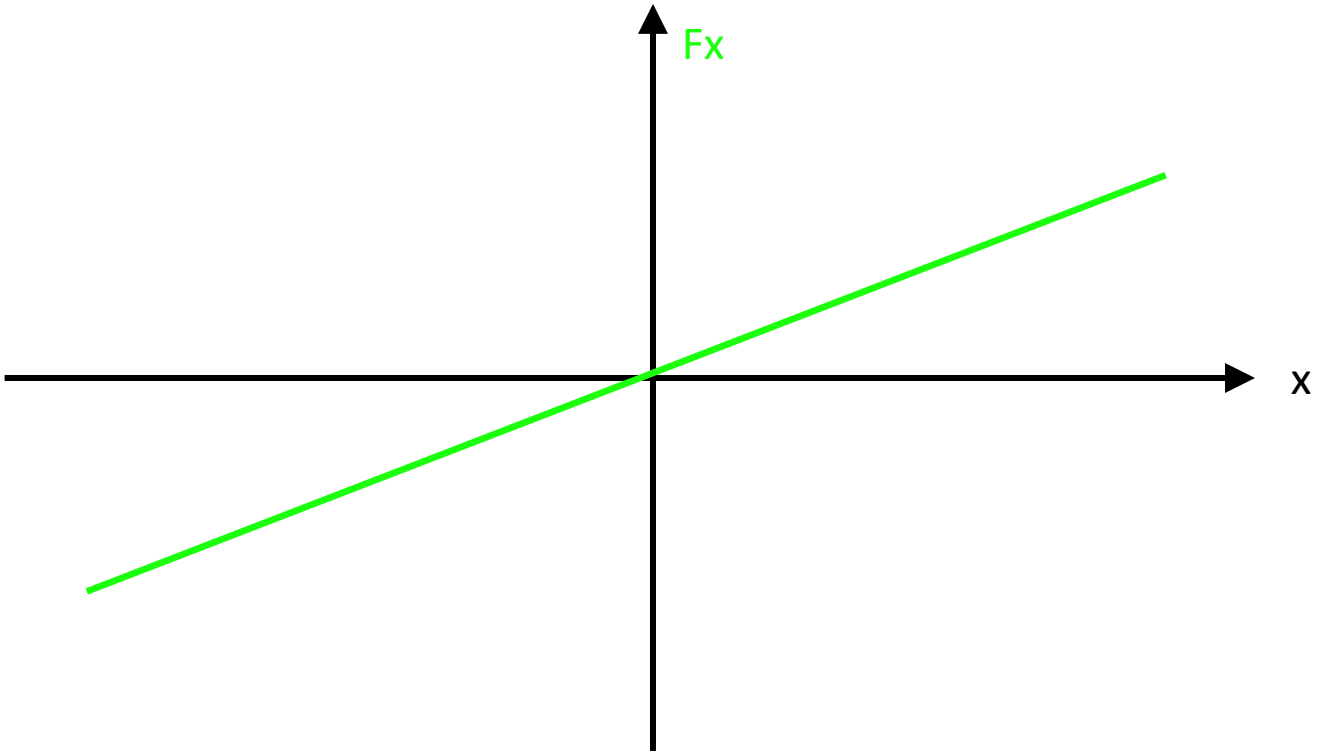
$$B_y = -\alpha x$$

The bending angle depends on the quantity

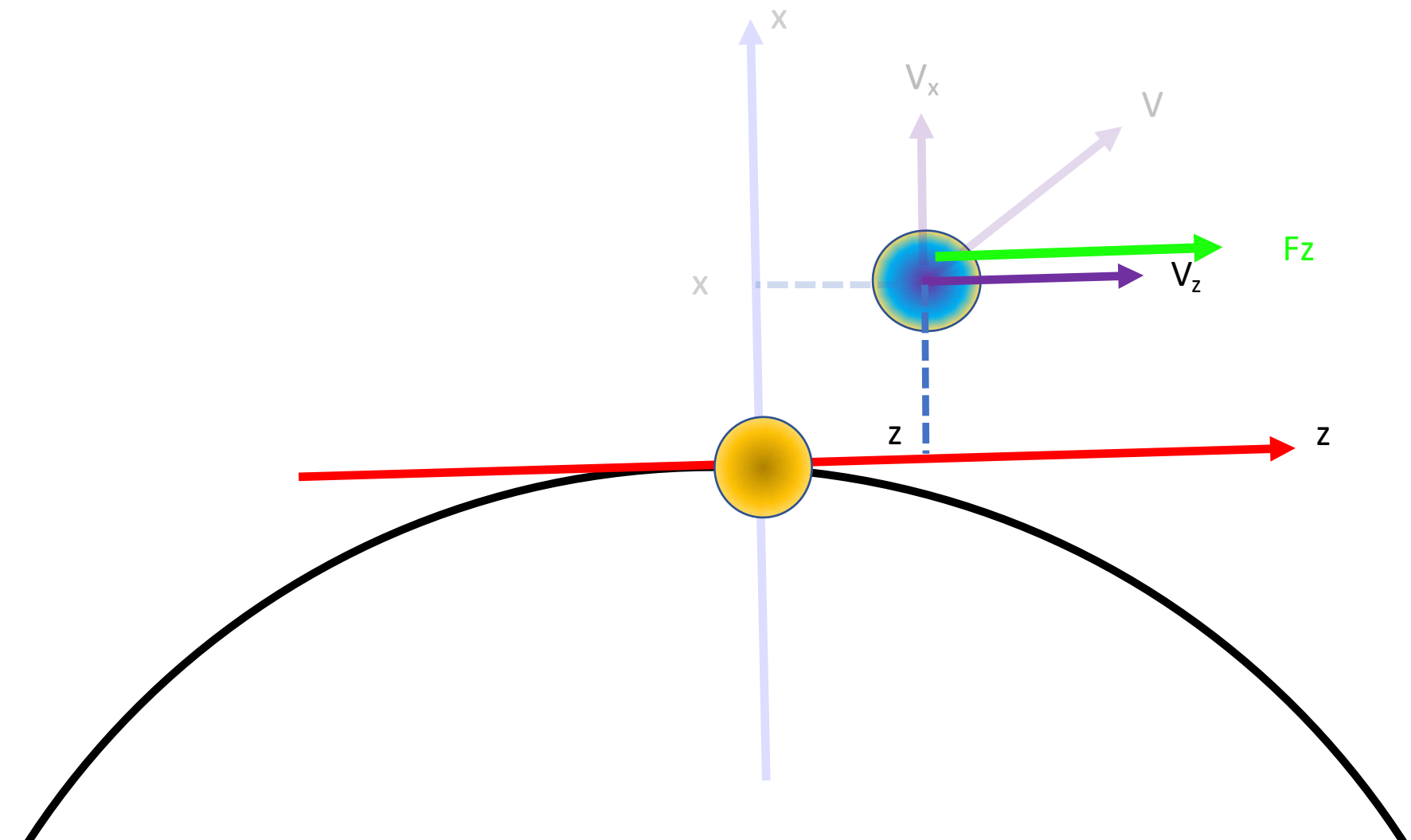
$$k = \frac{\alpha}{B\rho}$$

Transverse forces → from magnets

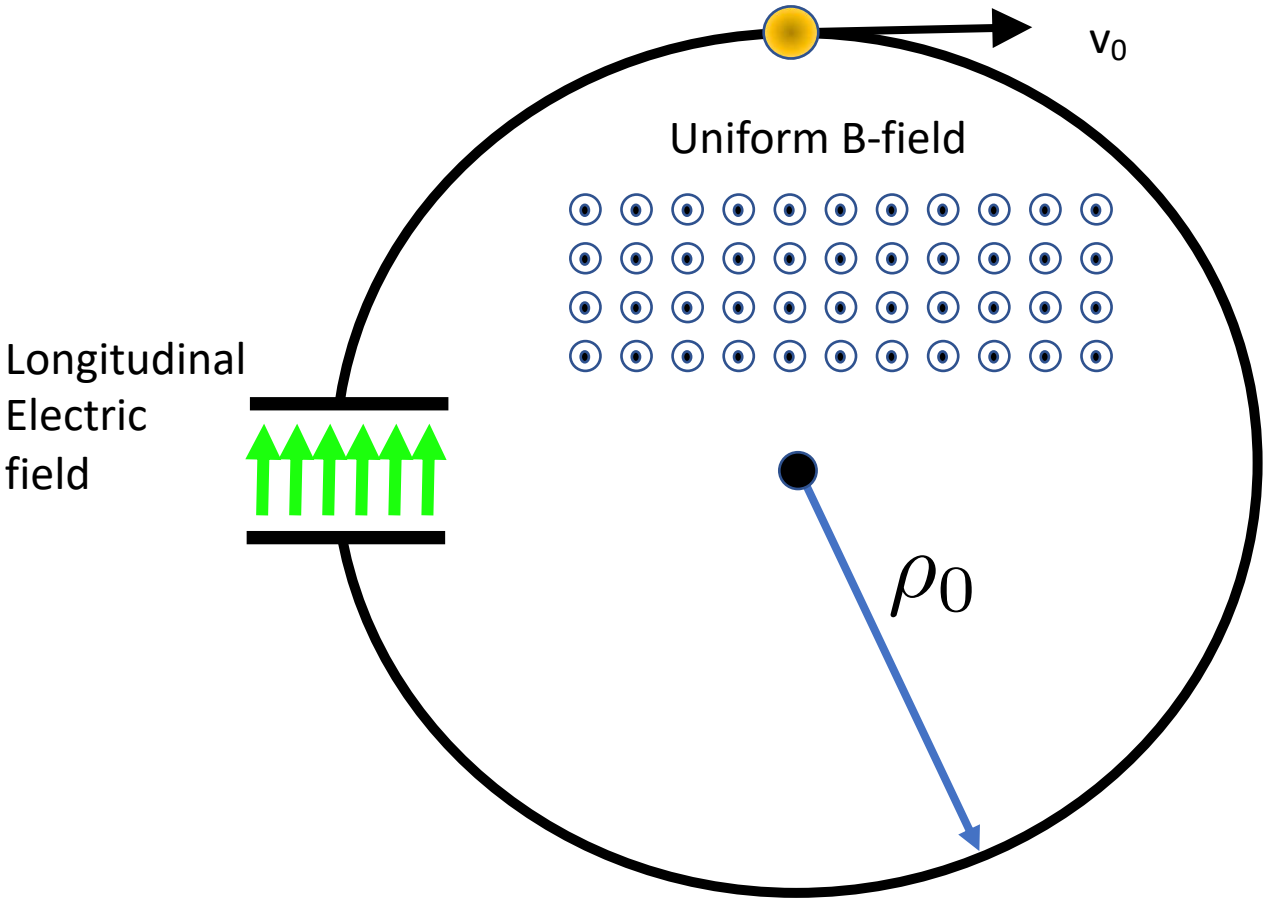
Forces depends on the particle **“position”**



Longitudinal dynamics



Longitudinal forces → from cavities



$$\frac{p_0}{q} = \rho_0 B_0$$

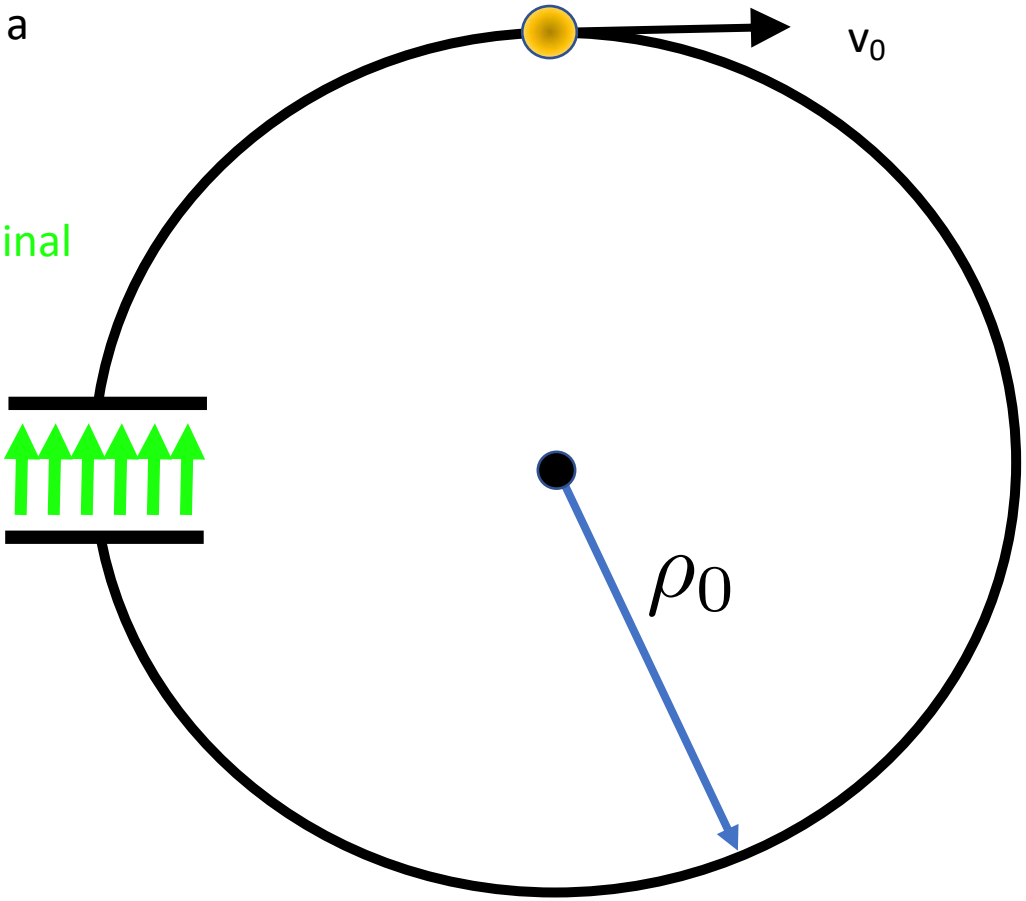
Longitudinal forces → from cavities

Forces depends on the "**time**" when a particle goes through the cavity

Longitudinal Electric field

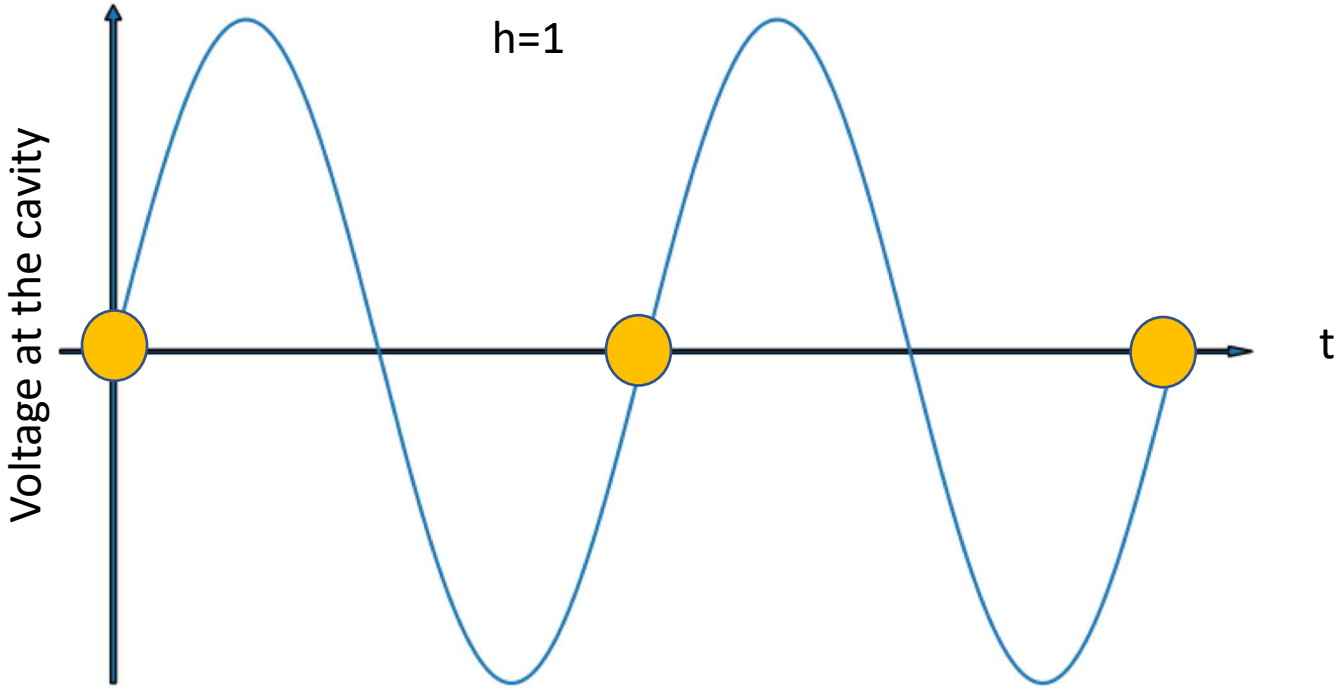
$$E_z = V \sin(2\pi f t)$$

The field oscillates with frequency f



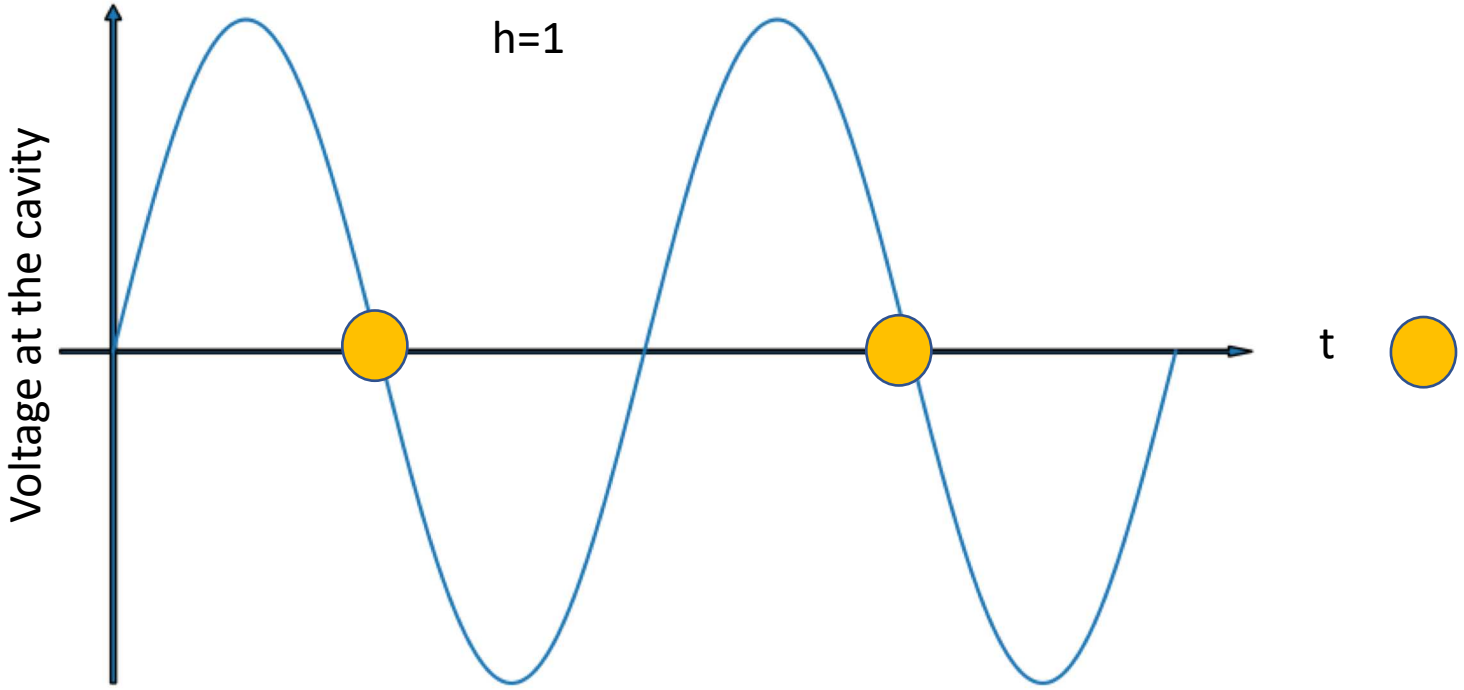
Energy matching

For a matched particle the cavity does not make any effect $f = h\frac{1}{\tau_0}$



Energy matching: second possibility

For a matched particle the cavity does not make any effect $f = h\frac{1}{\tau_0}$



Energy mismatch / Momentum mismatch

Matched particle

Momentum → p_0

Energy → E_0

Velocity → v_0

Mis-matched particle

Momentum → $p = p_0 + \delta p$

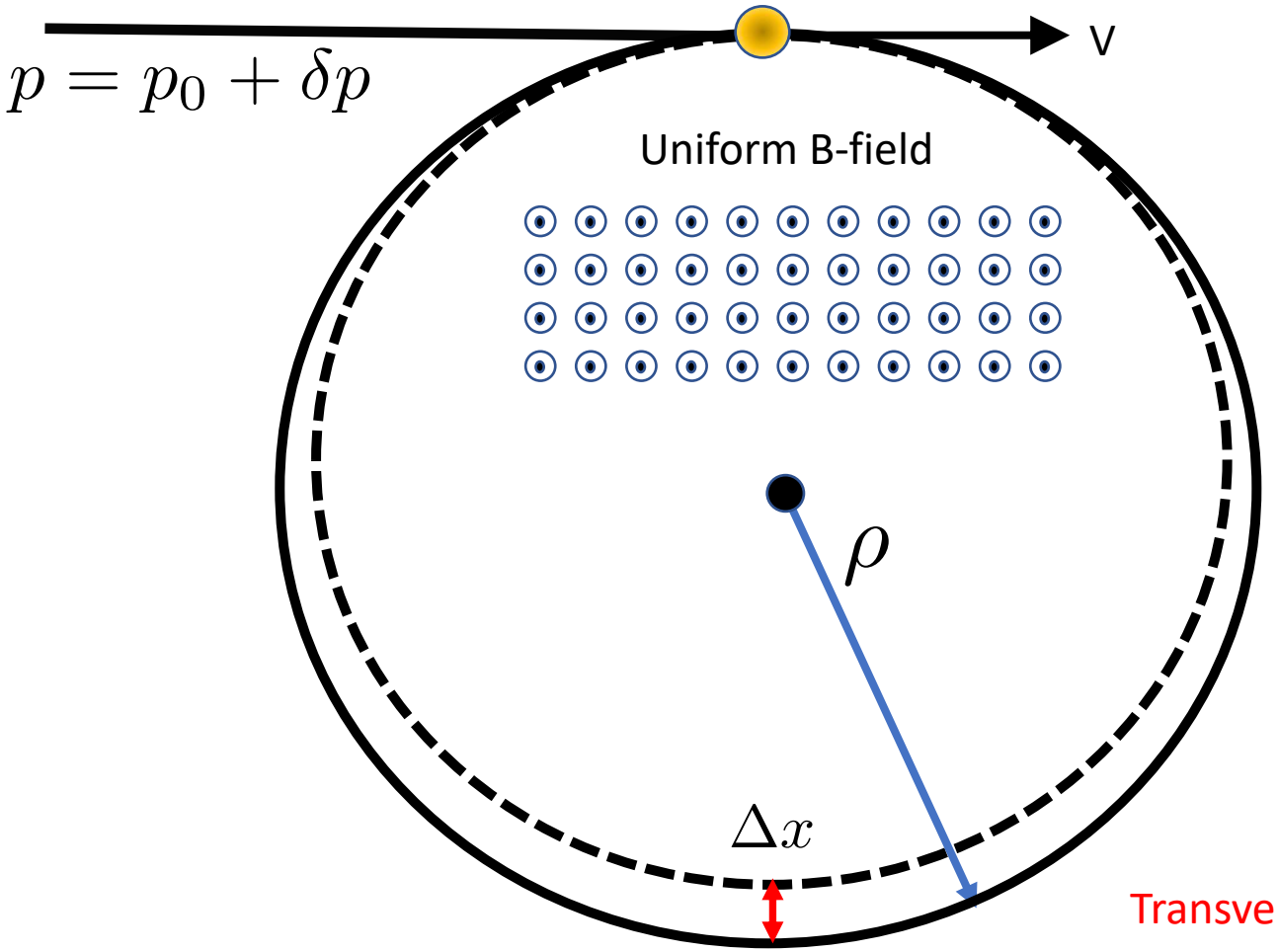
Energy → $E = E_0 + \delta E$

Velocity → $v = v_0 + \delta v$

Only one "variation" is necessary → the other two automatically follow

We take → $\boldsymbol{\delta}$p "off-momentum"

Energy mismatch: simple example



$$\frac{p}{q} = \rho B_0$$

Transverse displacement Δx

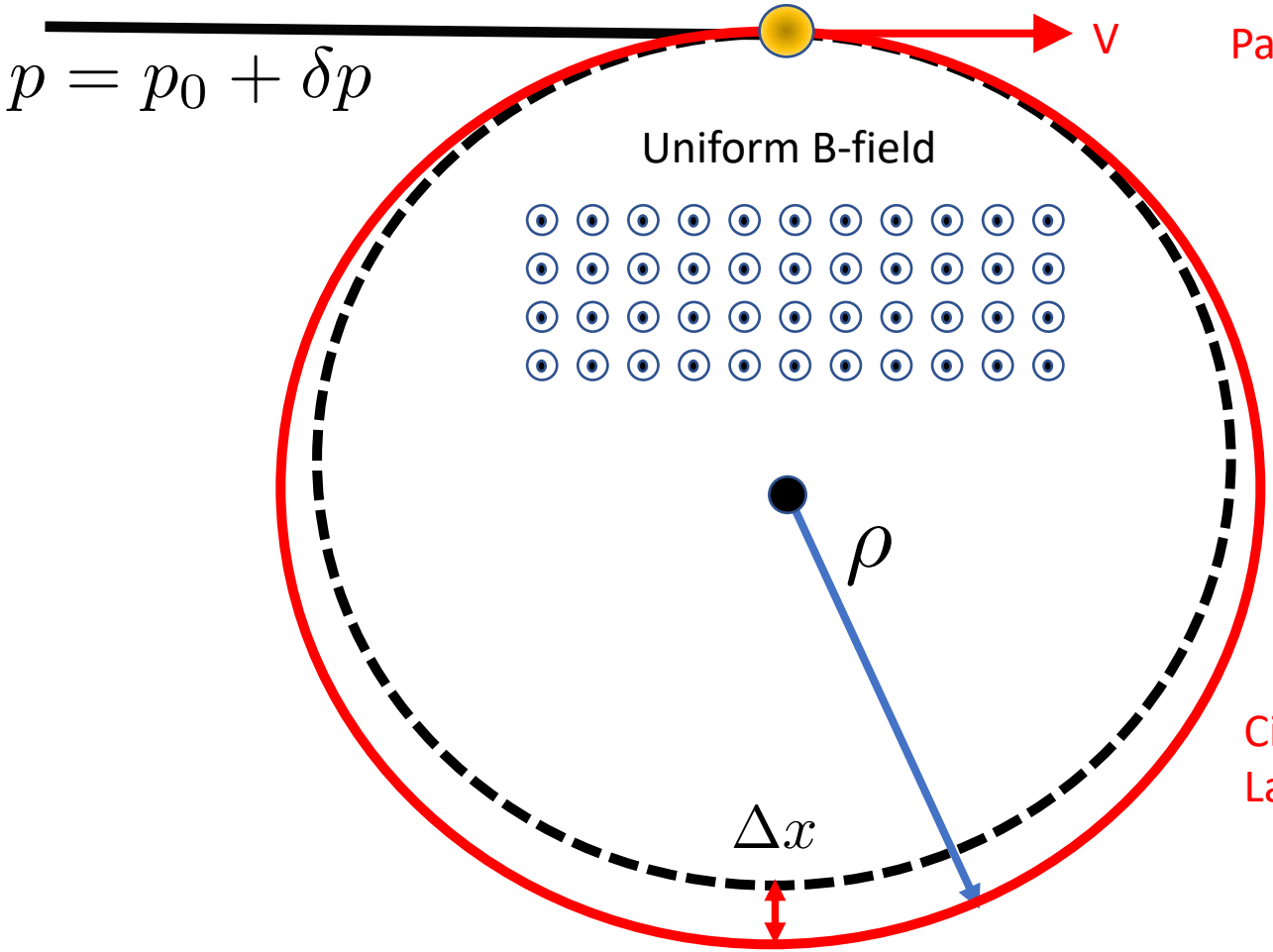
Simple estimate

$$\frac{p}{q} = \rho B_0 \quad \Rightarrow \quad \frac{\delta\rho}{\rho} = \frac{\delta p}{p} \quad \Rightarrow \quad \Delta x = 2\rho\frac{\delta p}{p}$$

For SIS18 type of machine

Mismatch $\frac{\delta p}{p} = 10^{-3} \Rightarrow \Delta x = 2\ cm$ (big!)

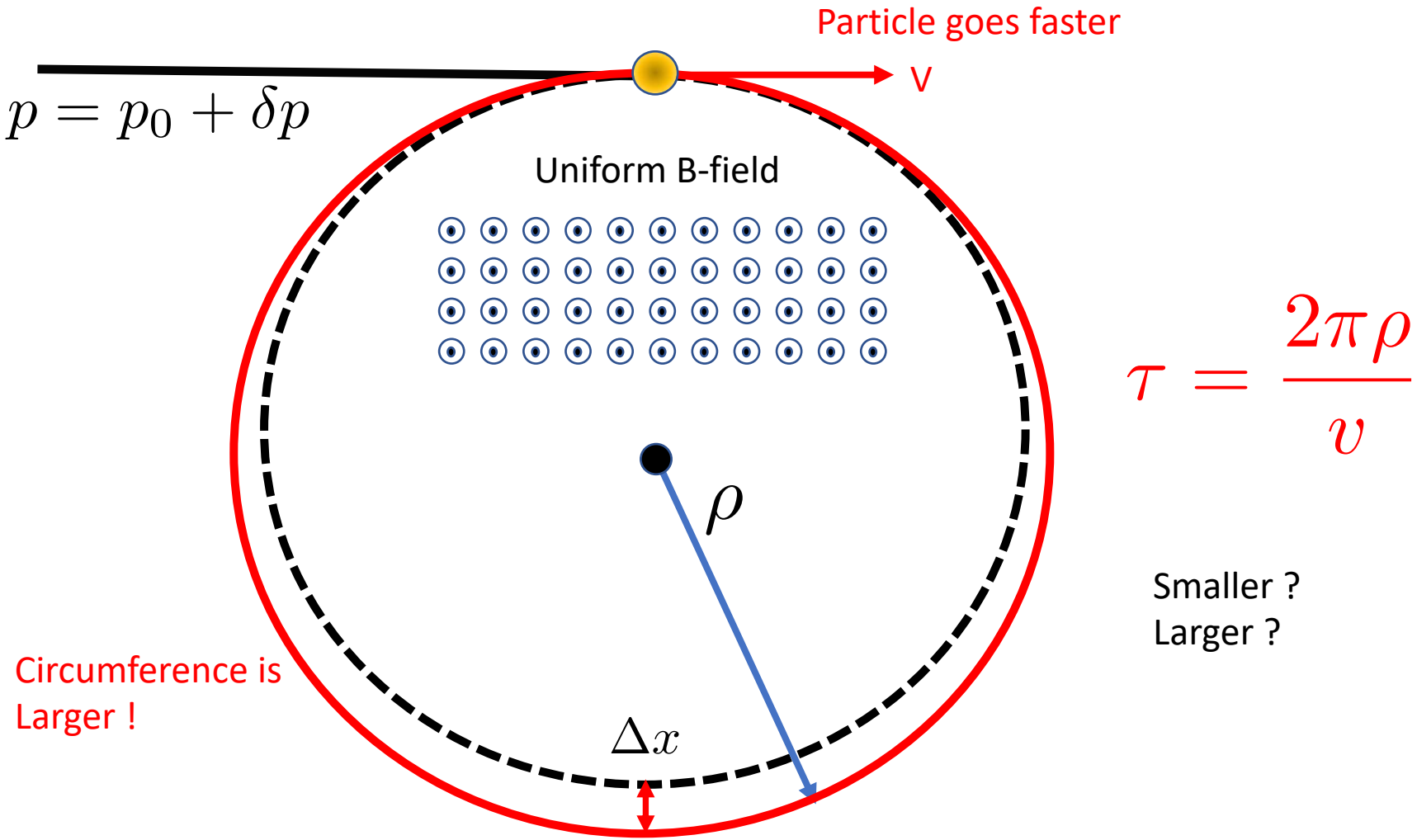
Revolution time ?



Particle goes faster

Circumference is Larger !

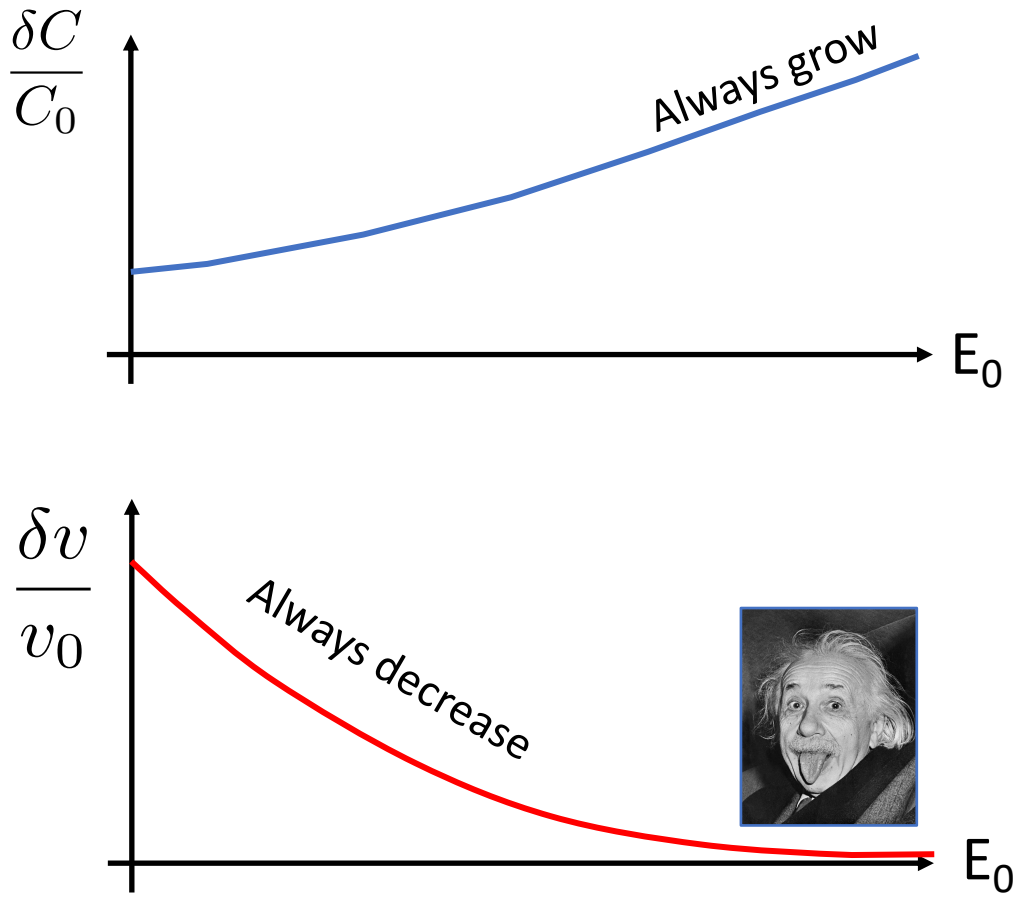
Revolution time ?



$$\tau = \frac{2\pi\rho}{v}$$

Smaller ?
Larger ?

At energy E_0 change particle momentum → $\delta p/p_0$



Change of revolution time

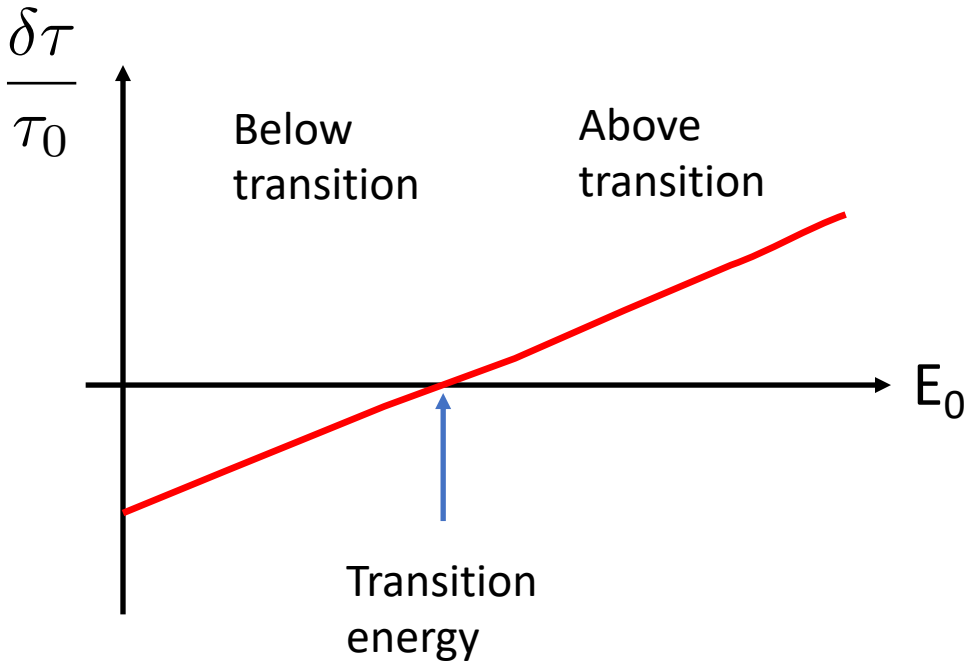
$$\frac{\delta\tau}{\tau_0} = \frac{\delta C}{C_0} - \frac{\delta v}{v_0}$$

C = circumference

Transition energy

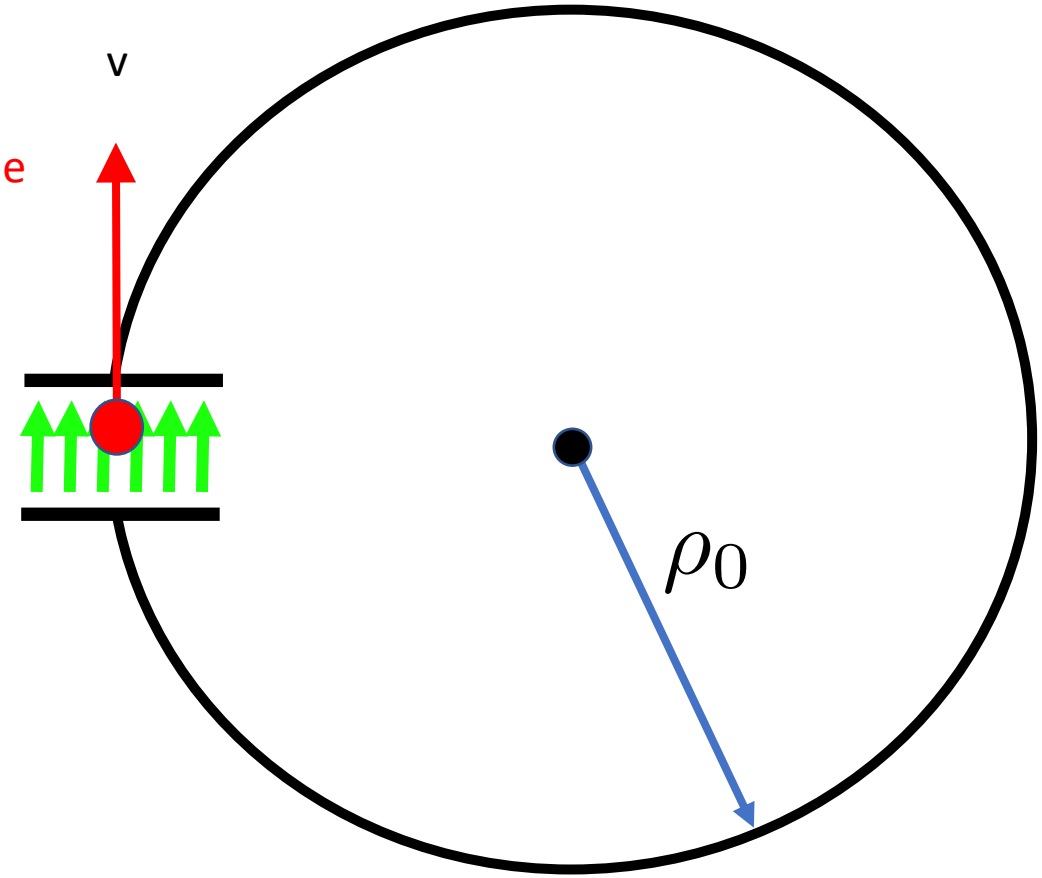
Change of $\boldsymbol{\delta}p/p_0$ → Change of revolution time

$$\frac{\delta\tau}{\tau_0} = \frac{\delta C}{C_0} - \frac{\delta v}{v_0}$$



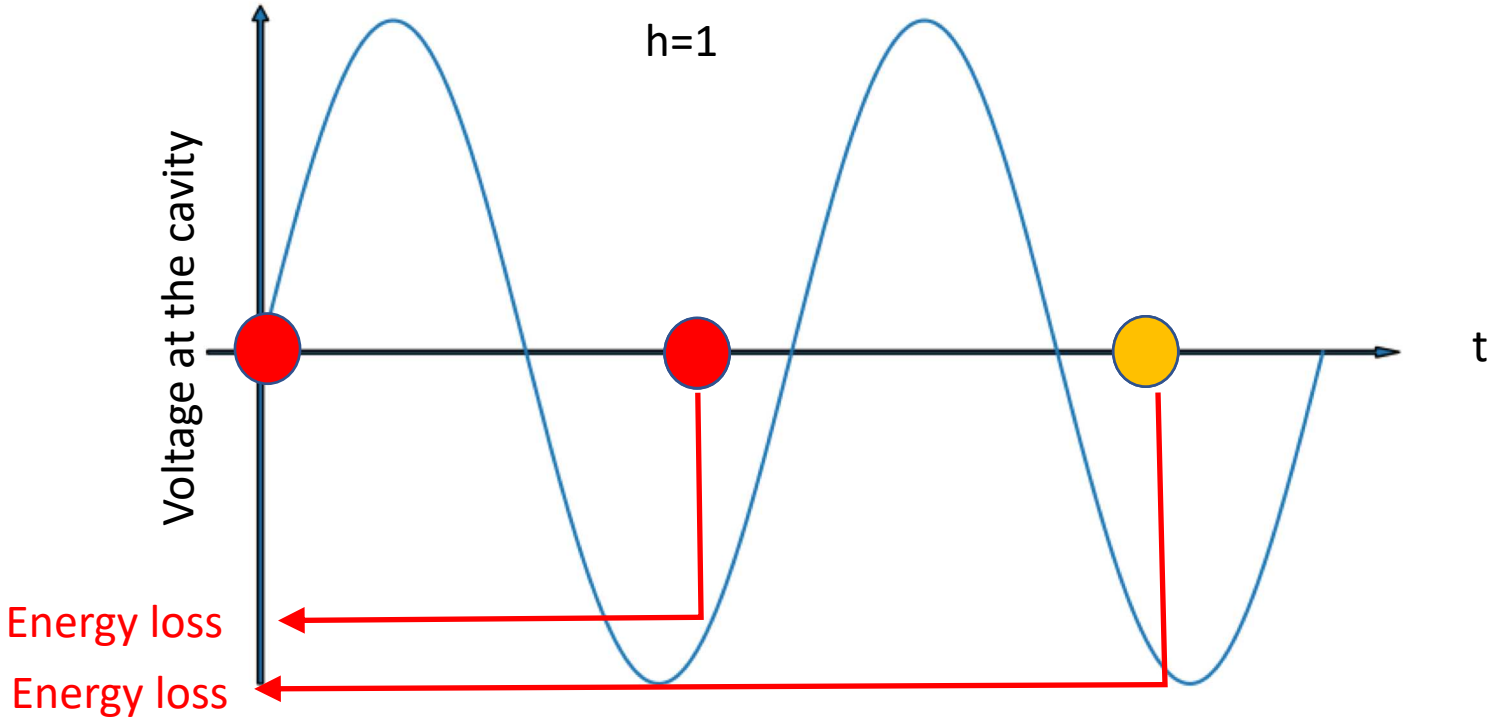
What happen at the cavity

$$E_z = V \sin(2\pi f t)$$

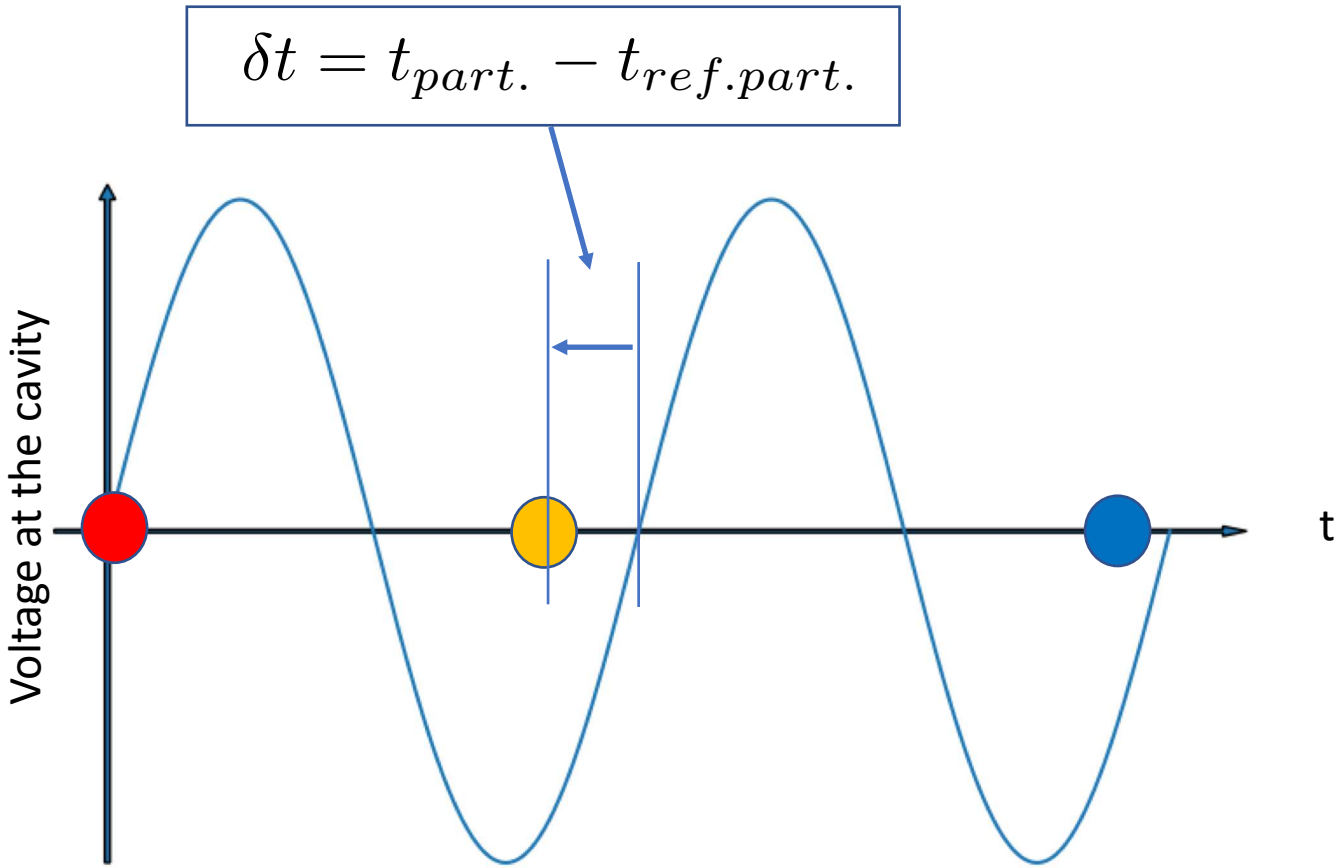


Below transition: energy mismatch

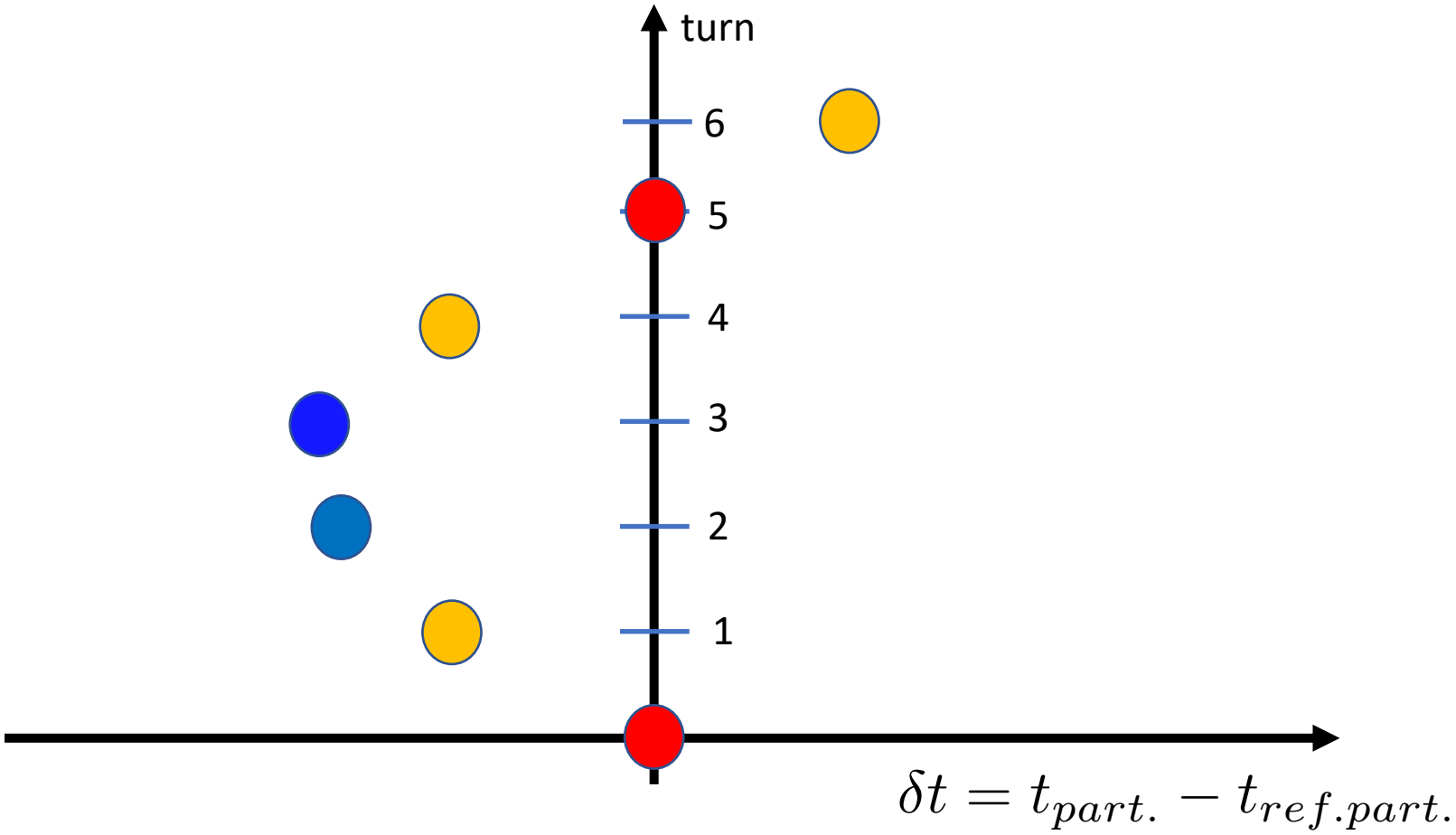
Take a particle “faster” than the reference particle



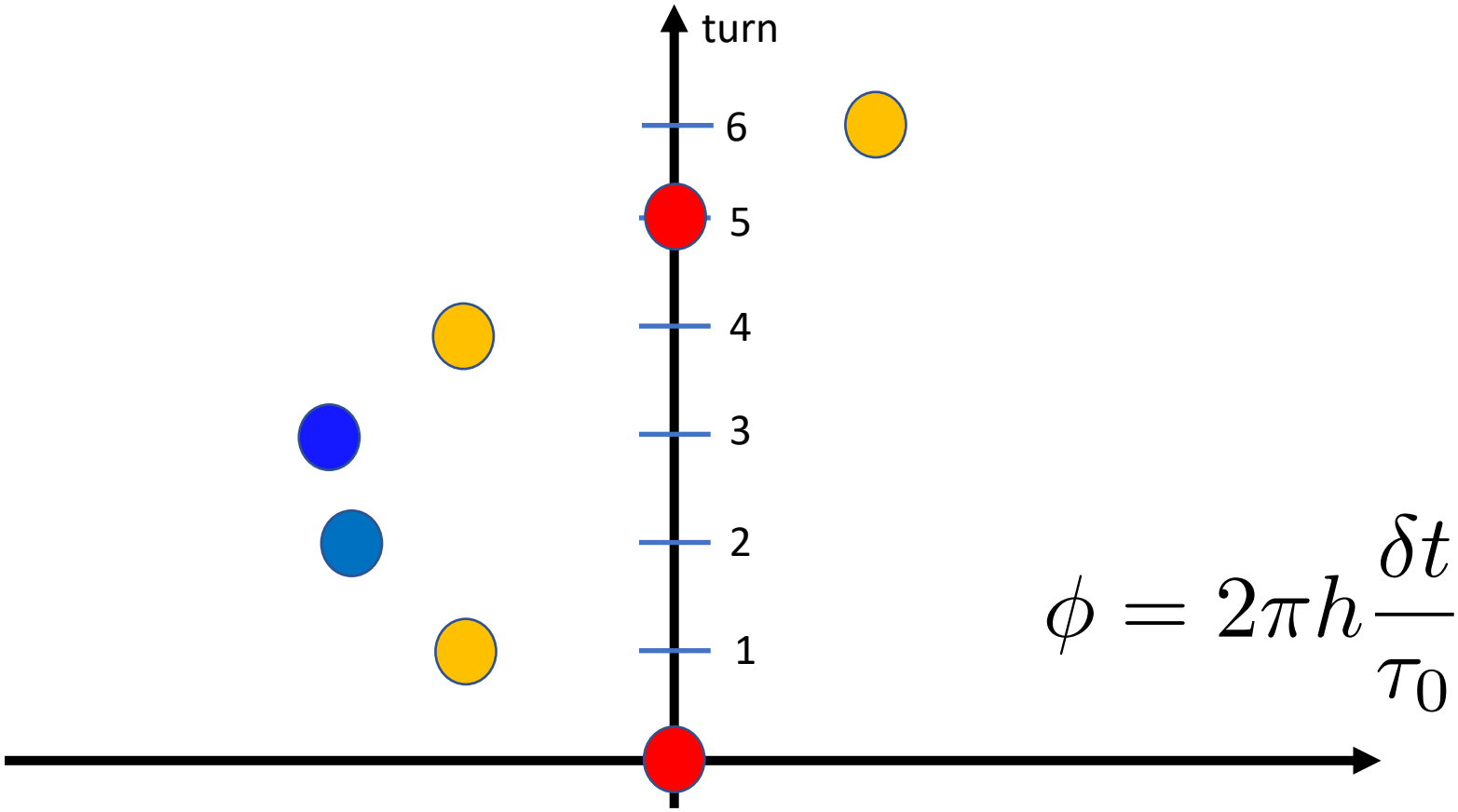
Particle position measured with time



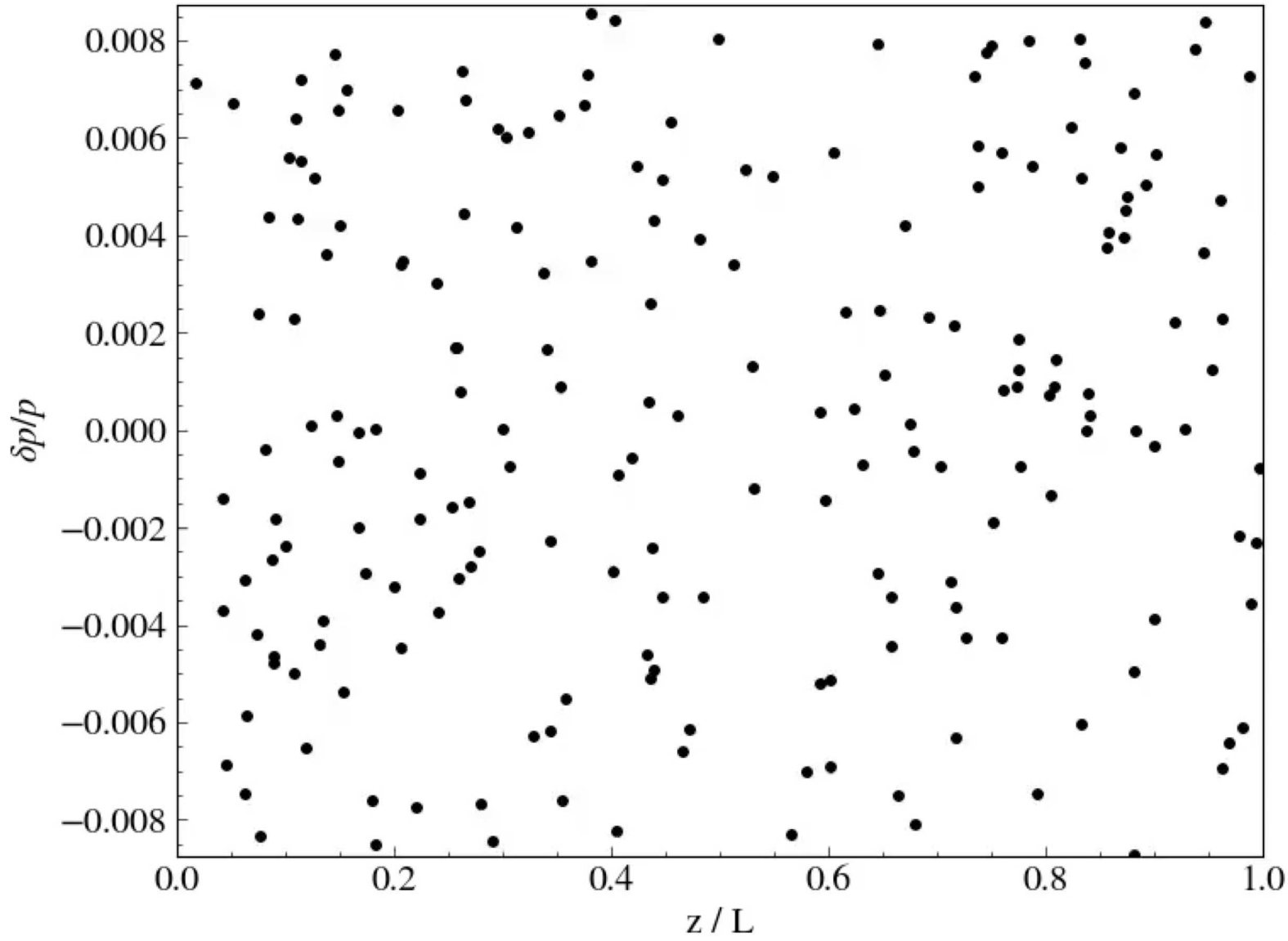
Evolution

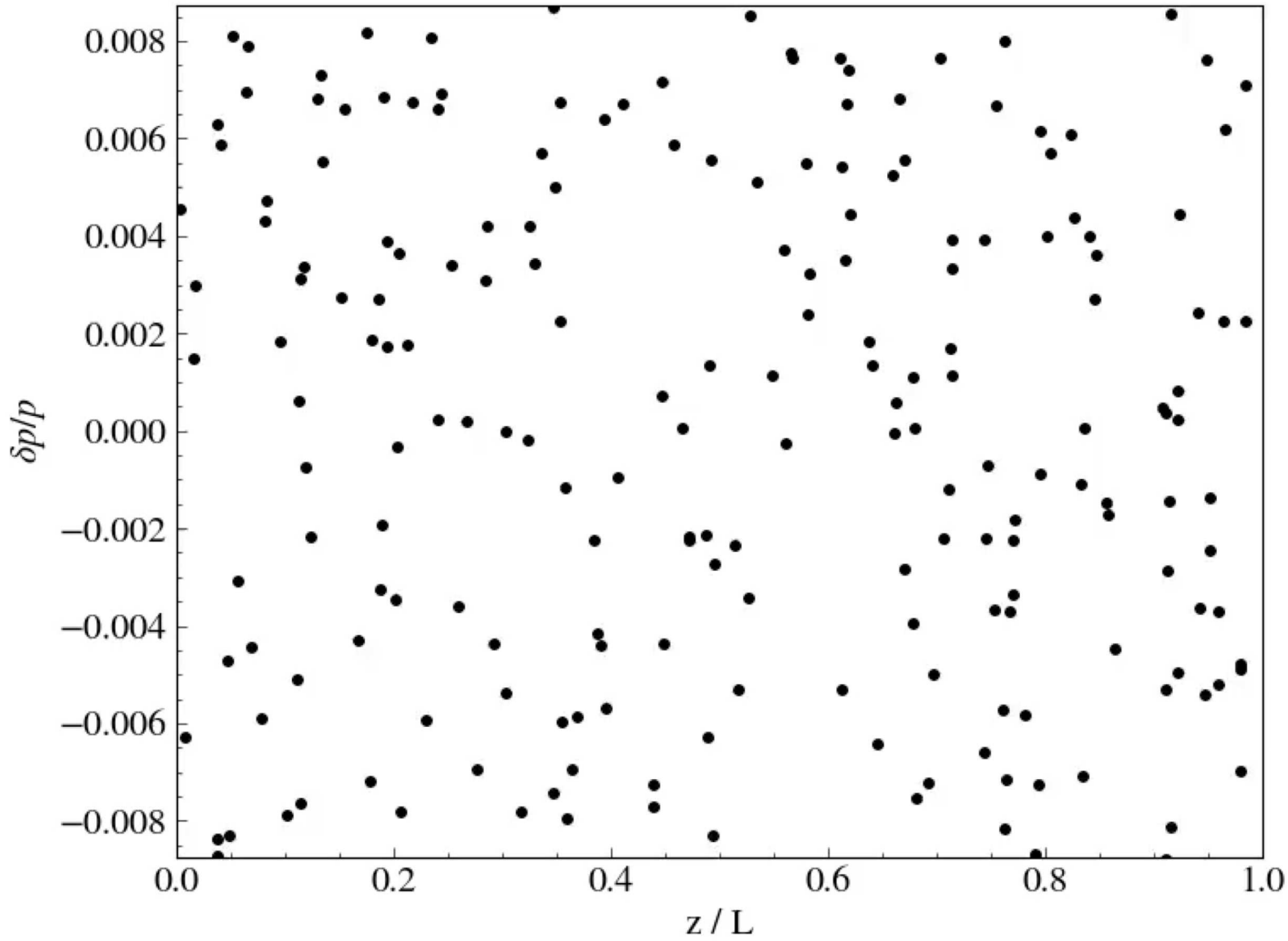


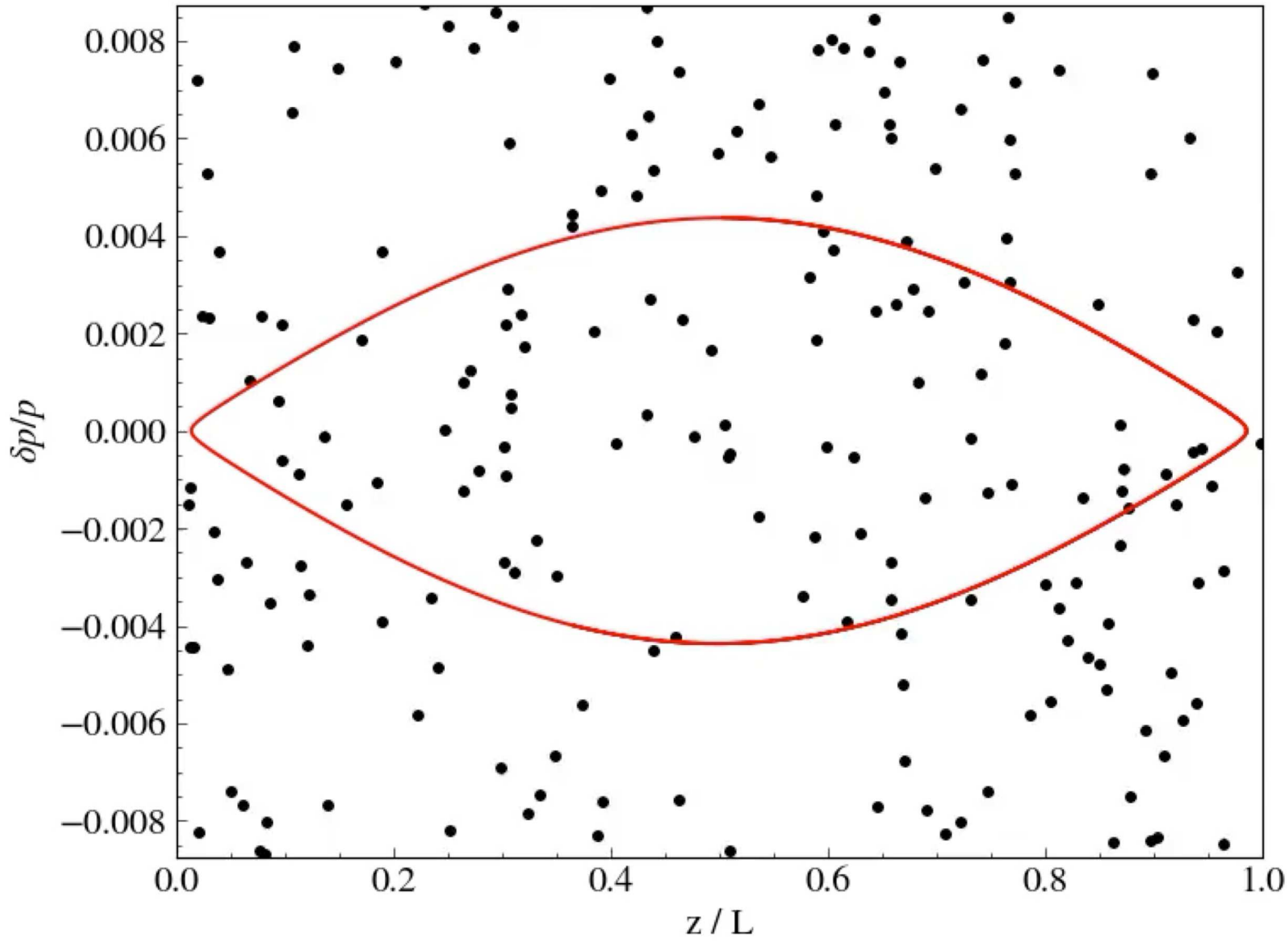
Instead of δt, one can use the cavity phase



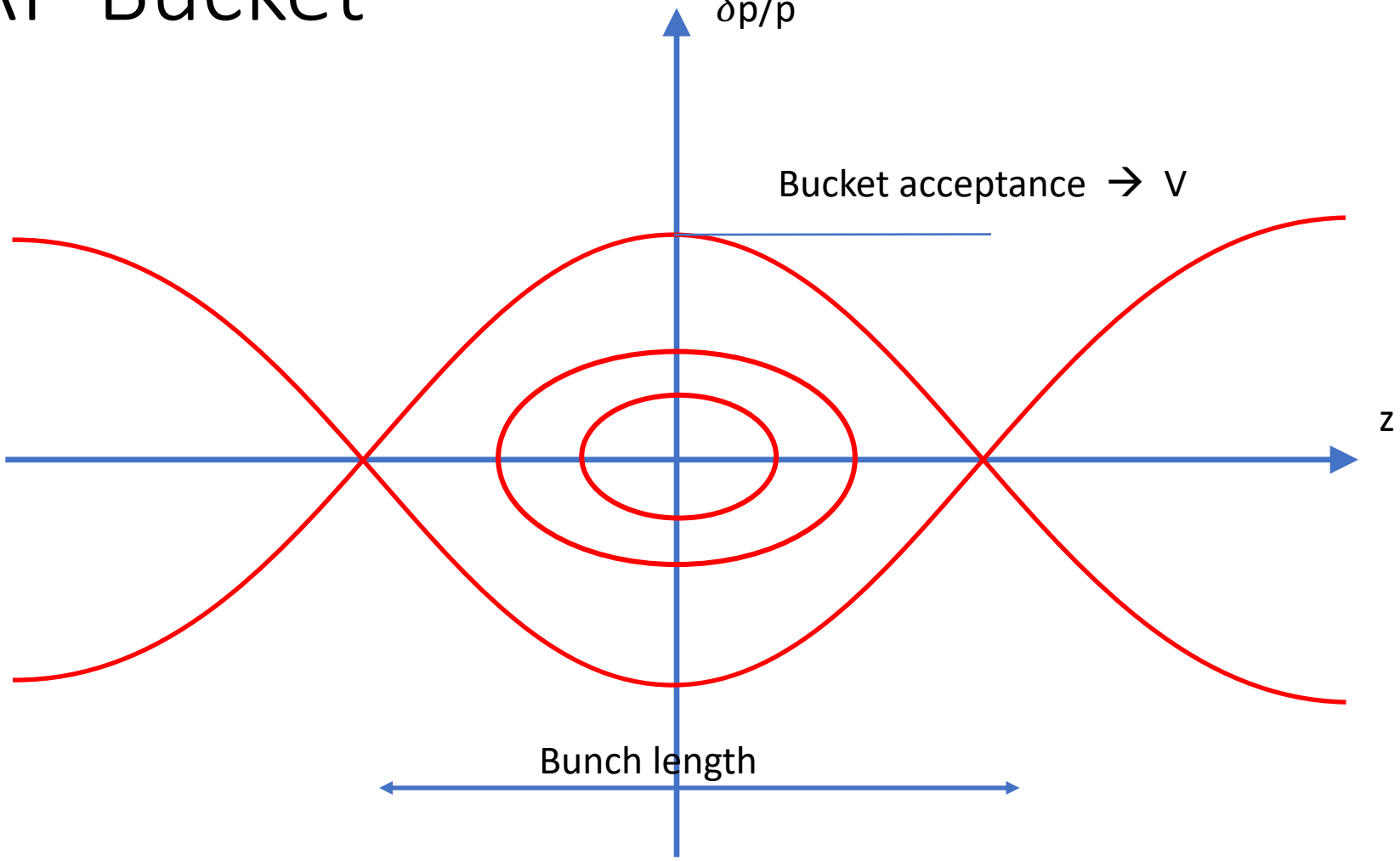
$$\phi = 2\pi h \frac{\delta t}{\tau_0}$$


0.008
0.006
0.004
0.002
0.000
−0.002
−0.004
−0.006
−0.008
δp/p
0.0
0.2
0.4
0.6
0.8
1.0
z / L

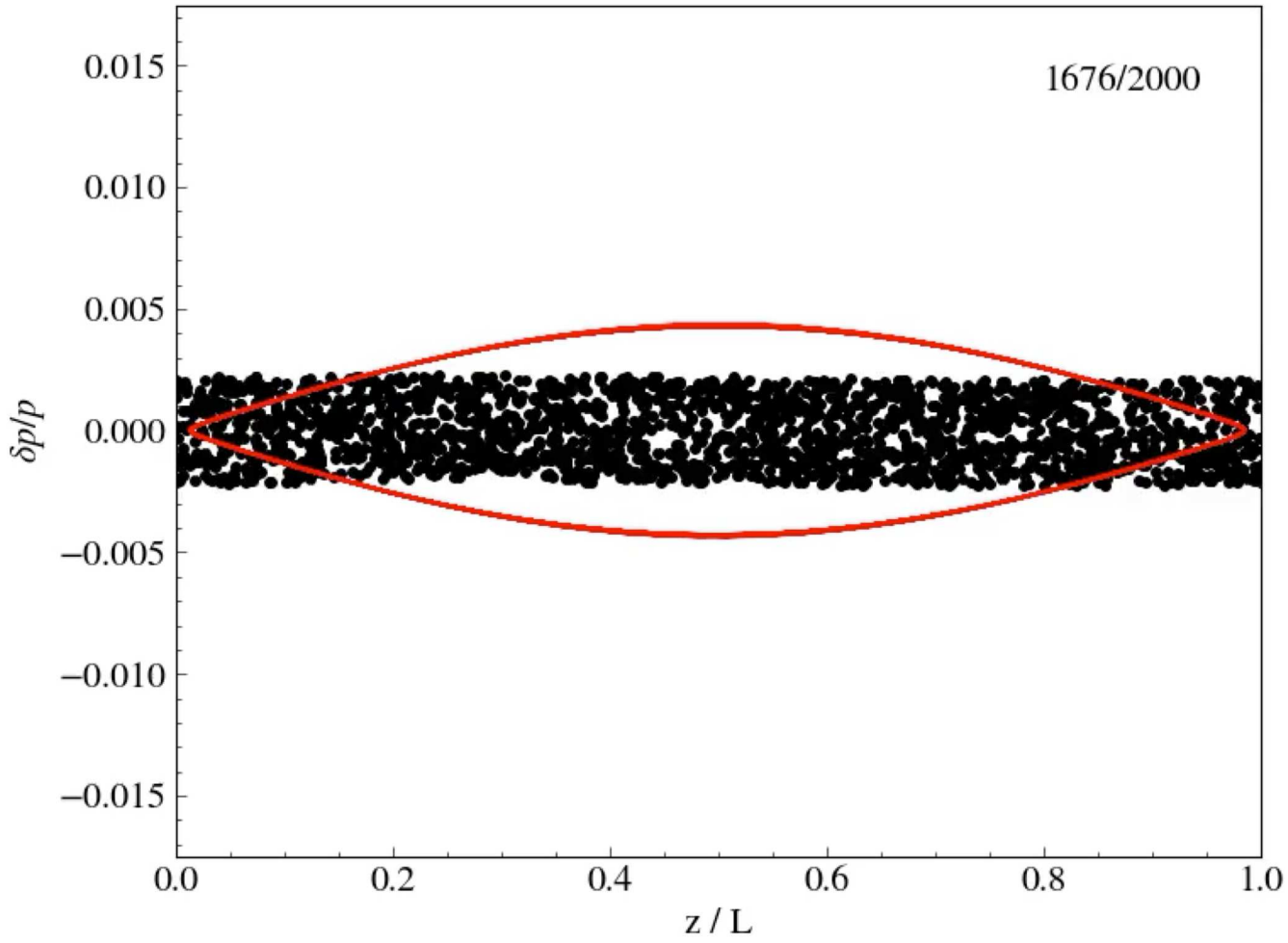

0.008
0.006
0.004
0.002
0.000
−0.002
−0.004
−0.006
−0.008
δp/p
0.0
0.2
0.4
0.6
0.8
1.0
z / L

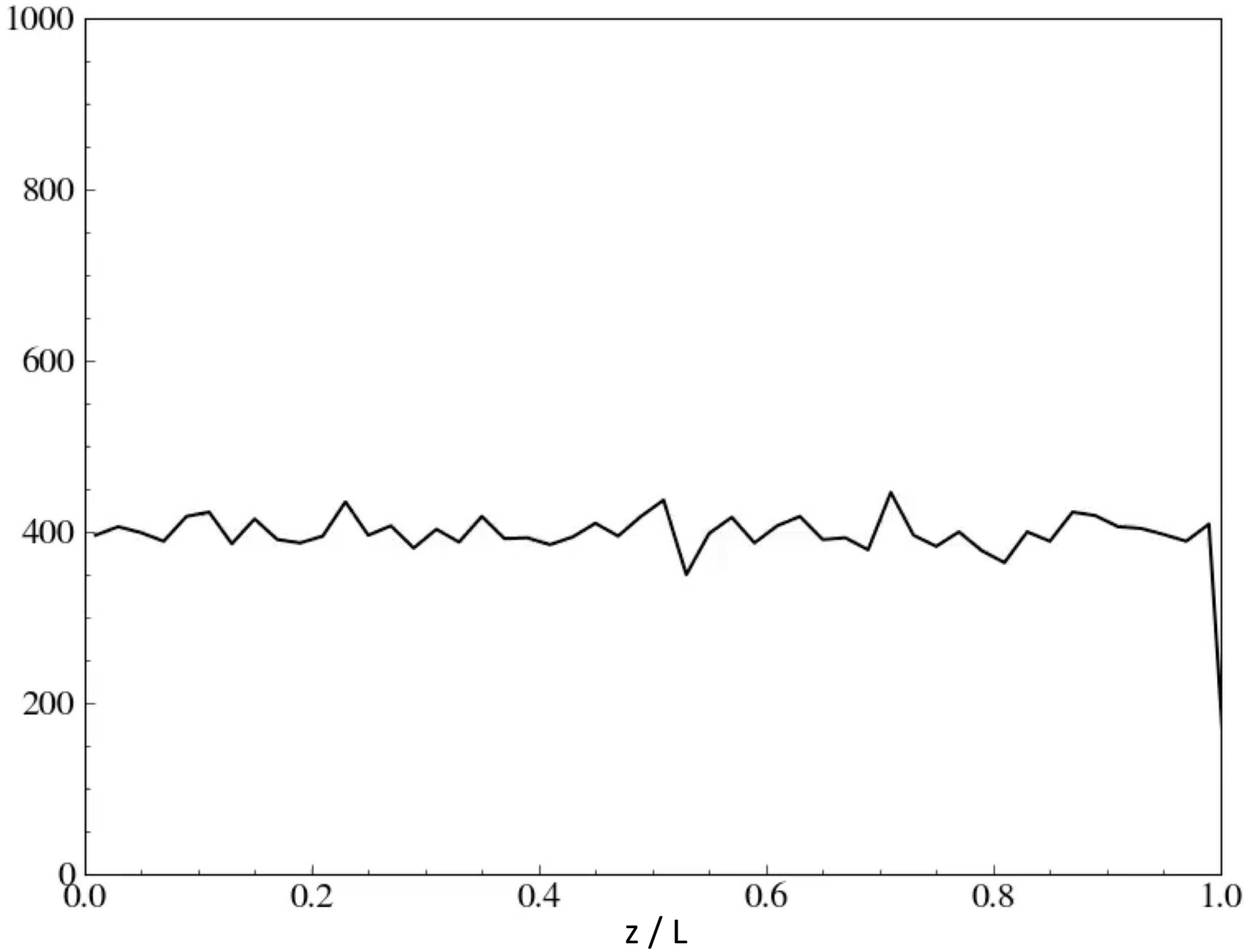

0.008
0.006
0.004
0.002
0.000
−0.002
−0.004
−0.006
−0.008
δp/p
0.0
0.2
0.4
0.6
0.8
1.0
z / L

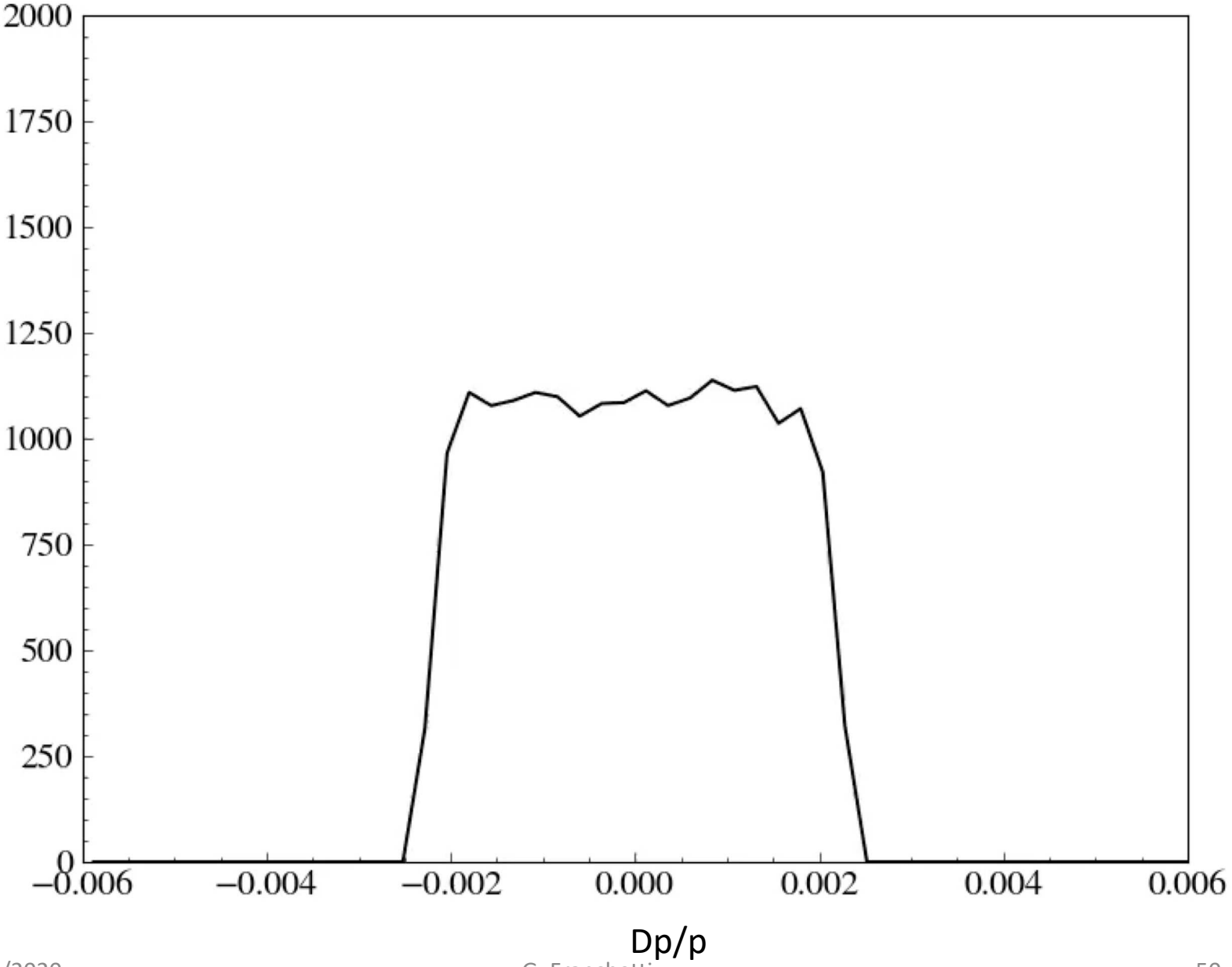
RF-Bucket

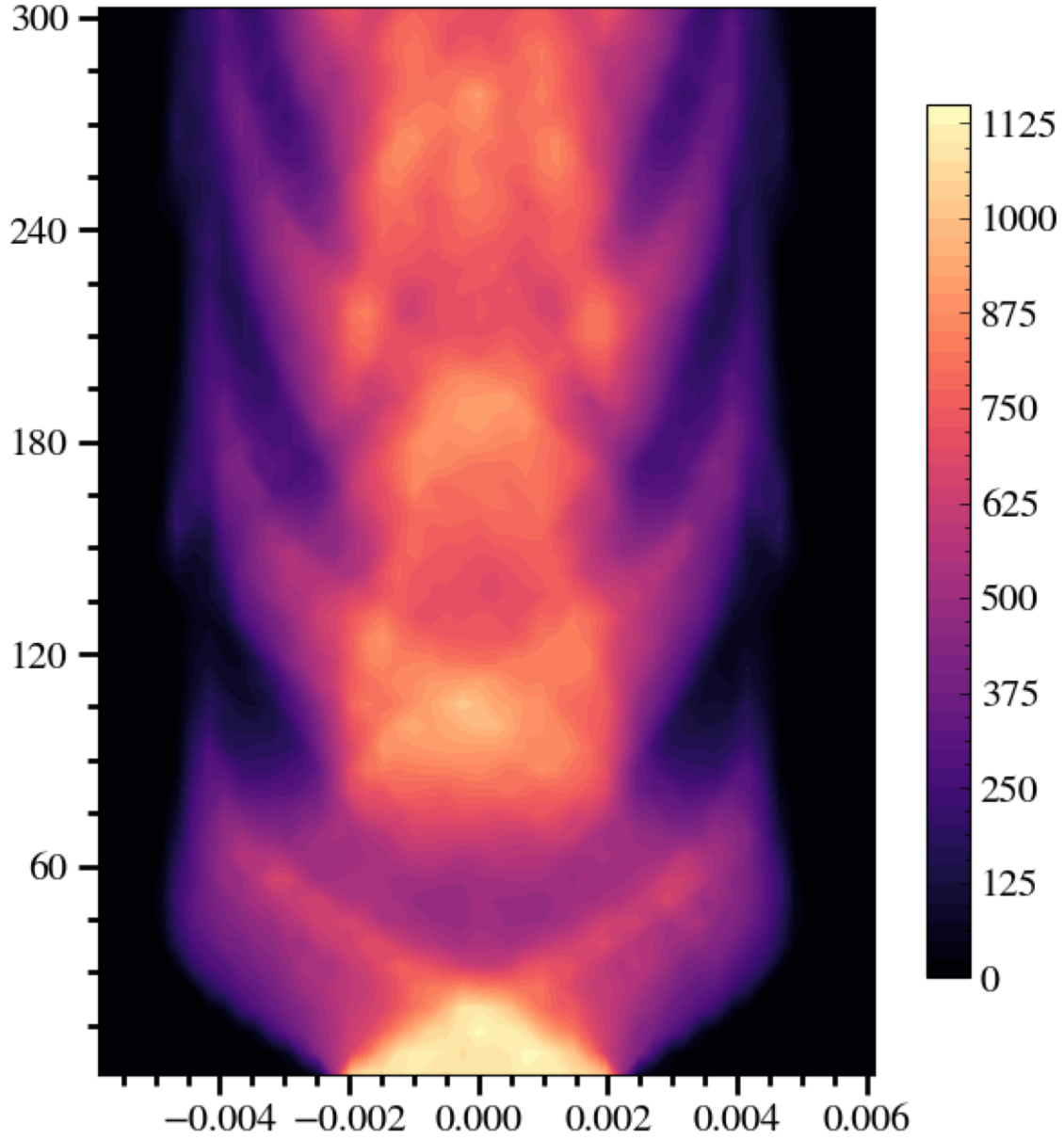


Bunching

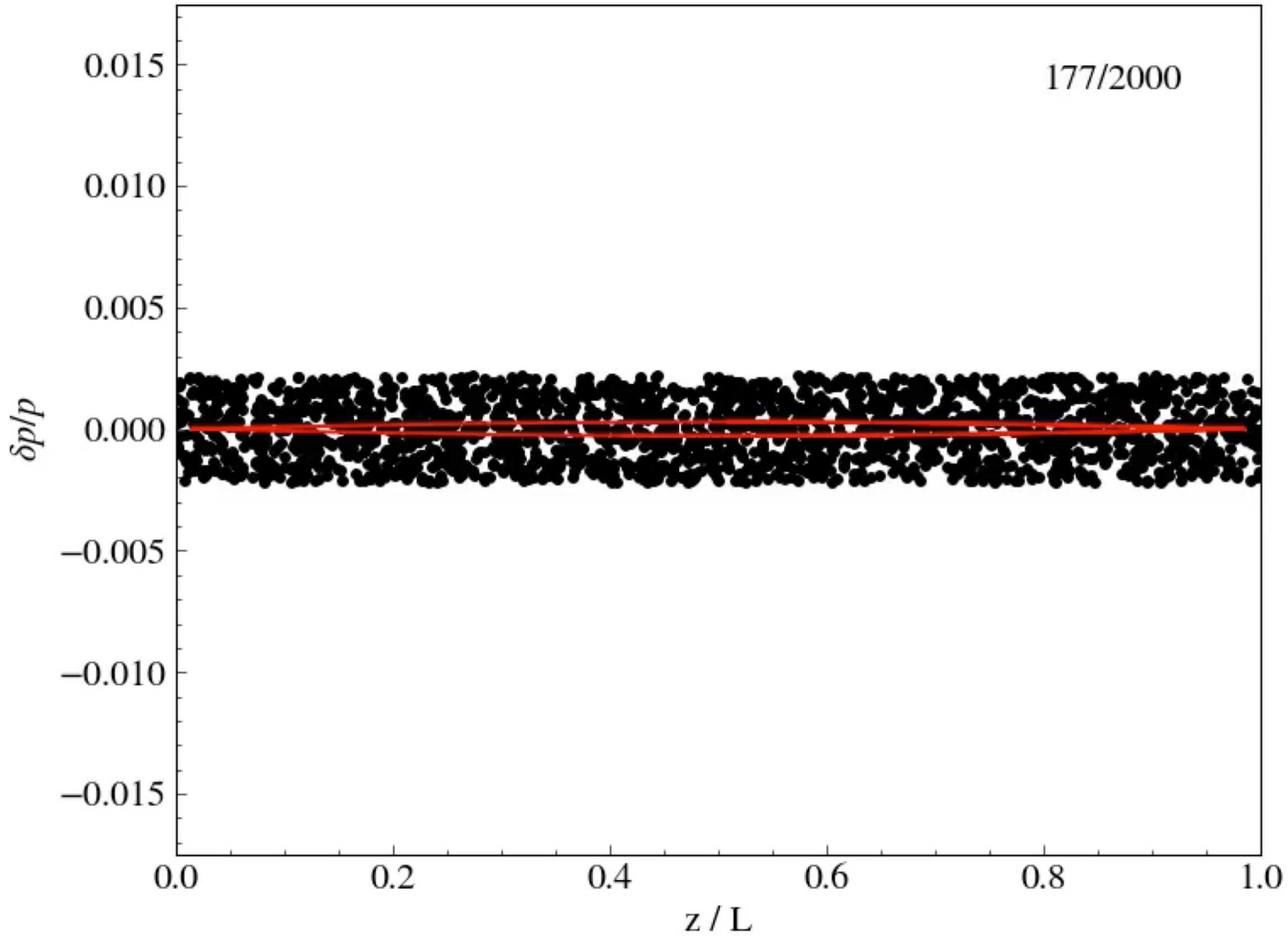

1676/2000
0.015
0.010
0.005
0.000
−0.005
−0.010
−0.015
δp/p
0.0
0.2
0.4
0.6
0.8
1.0
z / L


1000
800
600
400
200
0
0.0
0.2
0.4
0.6
0.8
1.0
z / L


2000
1750
1500
1250
1000
750
500
250
0
−0.006
−0.004
−0.002
0.000
0.002
0.004
0.006
Dp/p


300
240
180
120
60
−0.004
−0.002
0.000
0.002
0.004
0.006
Dp/p
1125
1000
875
750
625
500
375
250
125
0

Adiabatic bunching



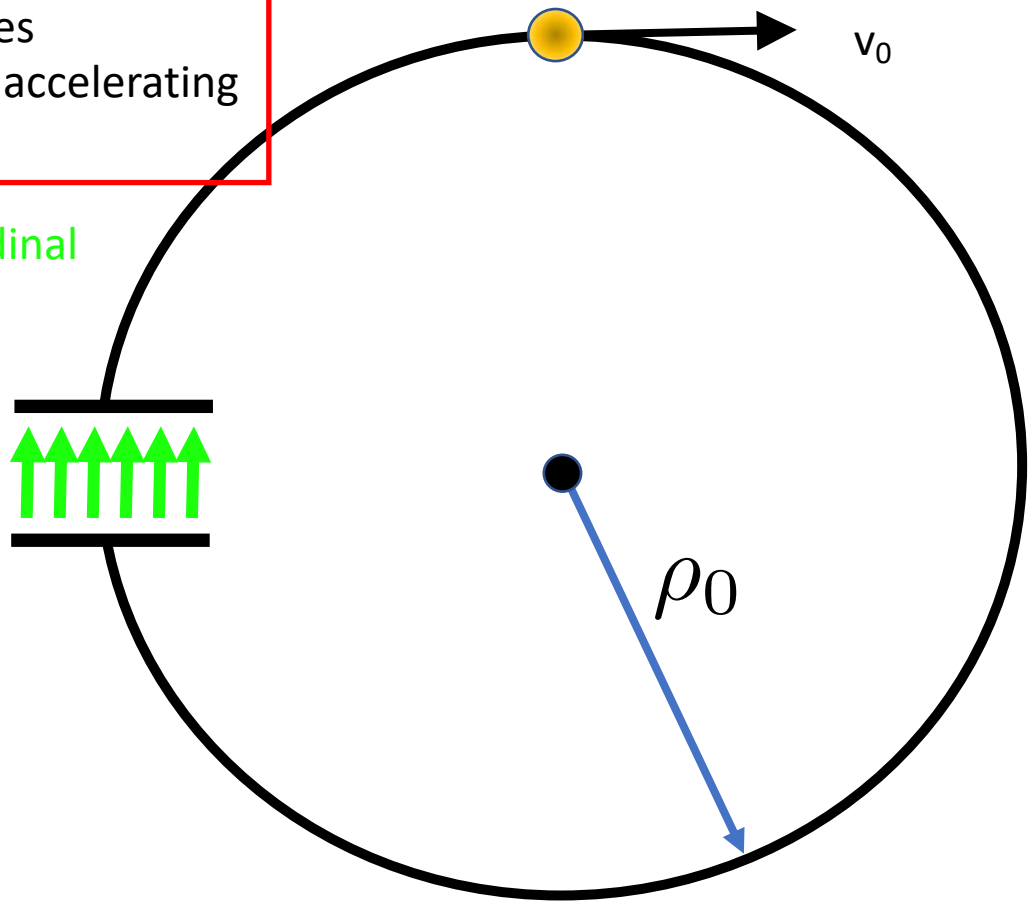
Acceleration

Every time the reference particle goes Through the cavity, it sees the same accelerating electric field

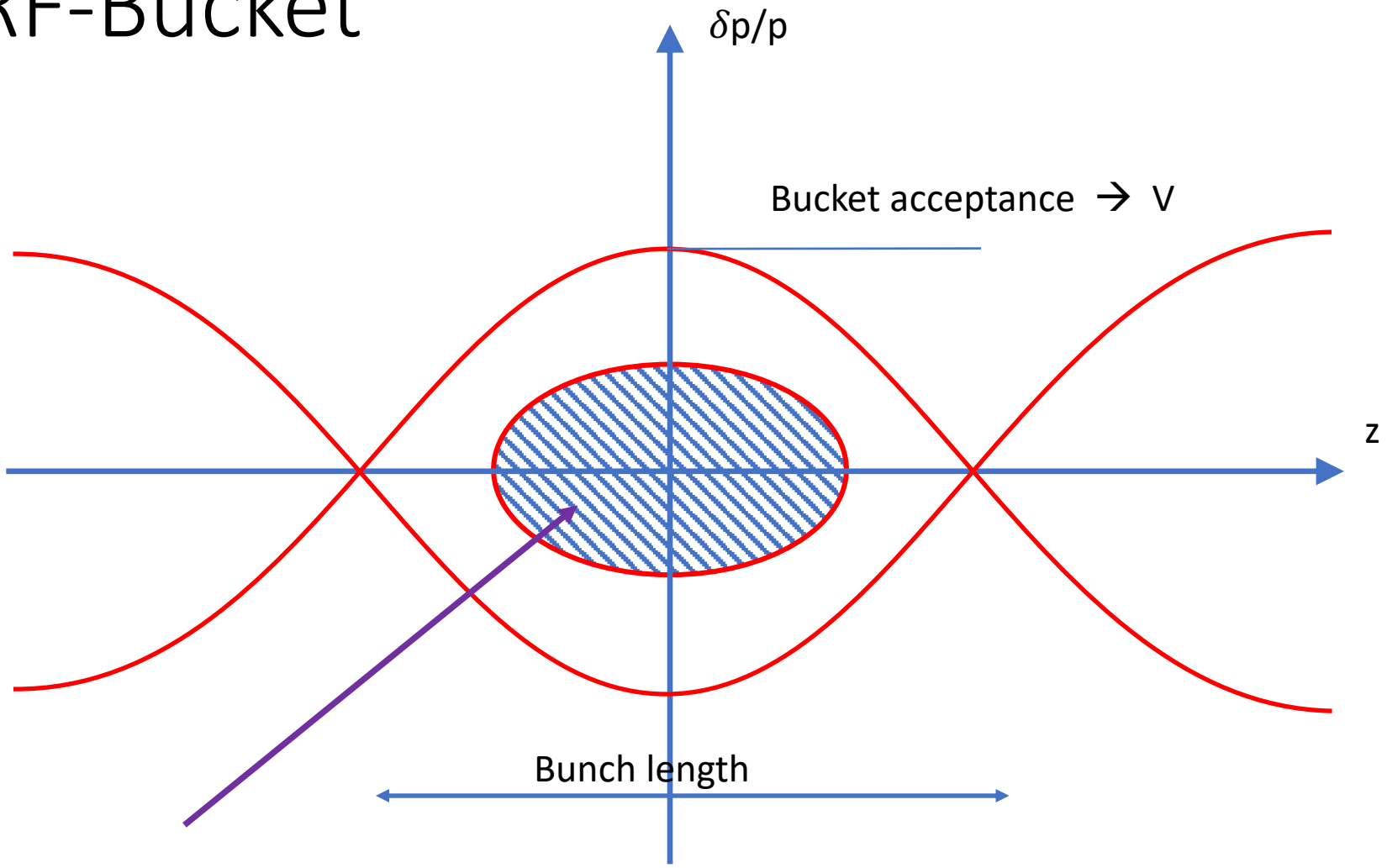
Longitudinal Electric field

$$E_z = V \sin(2\pi f t)$$

The field oscillates with frequency f

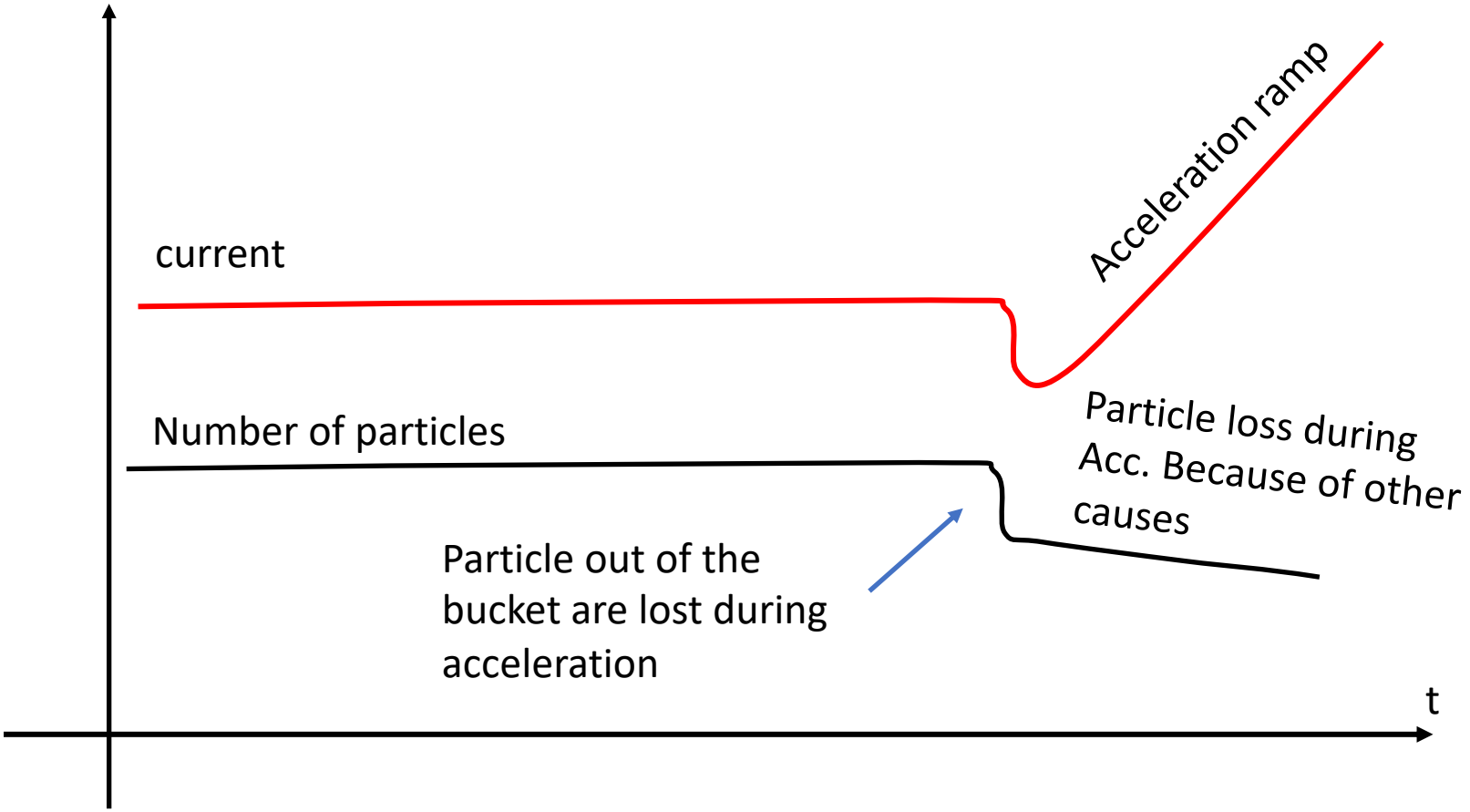


RF-Bucket



Only this area is accelerated: particle out of the bucket are lost during acceleration

Effect of RF-capture / acceleration



More topics…

RF-gymnastic

Bunch compression

Effect of high intensity

Negative mass region

Isochronous modes

Dual RF systems

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