S-shell $\Lambda\Lambda$ Hypernuclei Based on Chiral Interactions

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Theia-Strong2020 Web Seminar

Outline

- Introduction
  - Baryon-Baryon interactions in chiral effective field theory (χEFT)
  - Few-body techniques for S=-2
- Numerical approach:
  - Similarity Renormalization Group (SRG)
  - Jacobi no-core shell model (J-NCSM) for S=-2
- Results for A=4-6 $\Lambda\Lambda$ hypernuclei
- Conclusions & outlook

Introduction

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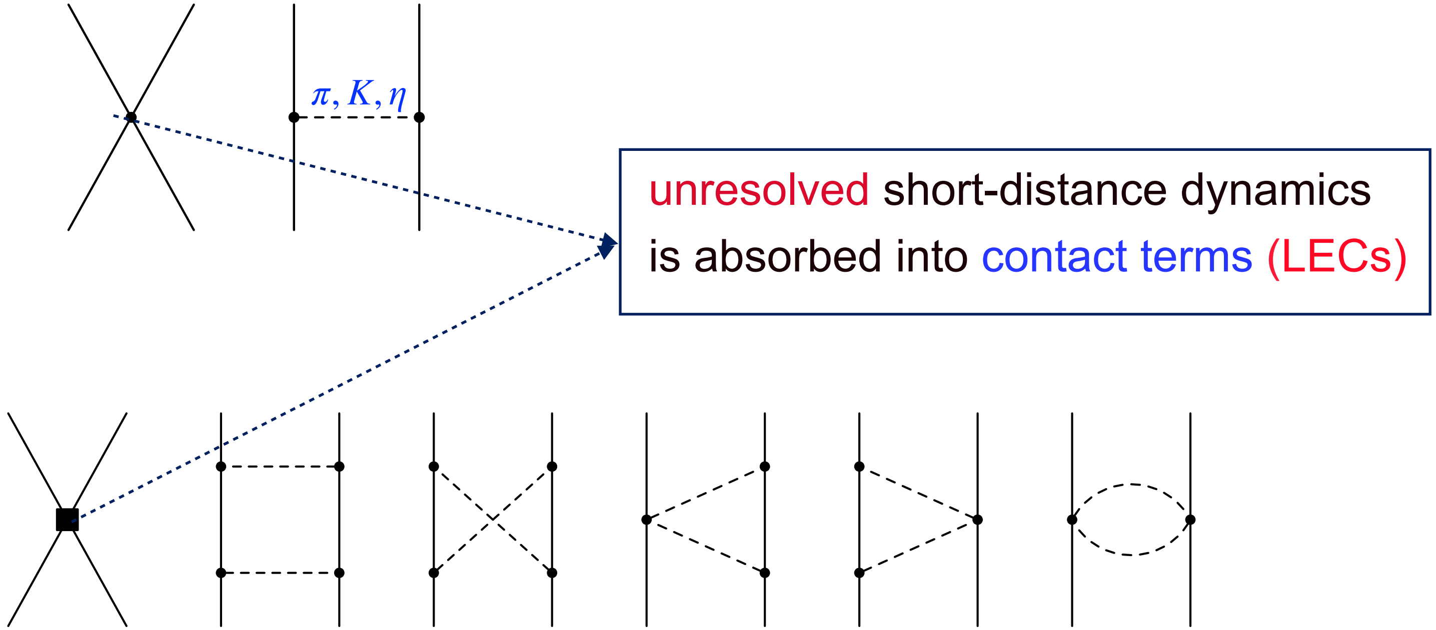
Baryon-Baryon interactions in χEFT

LO: H. Polinder et al., NPA 779 (2006). NLO: J. Haidenbauer et al., NPA 915 (2013)

- degrees of freedom: octet baryons $(N, \Lambda, \Sigma, \Xi)$, pseudoscalar mesons (π, K, η)
- based on Weinberg power counting as in the NN case

LO:

NLO:



- exploit $SU(3)_f$ to fix BBM couplings and relate various LECs, allow $SU(3)_f$ breaking where it seems appropriate

→ number of LECs:

NN:	2 (LO)	7 (NLO)
YN:	+3 (LO)	+11 (NLO)
YY:	+1 (LO)	+4 (NLO)

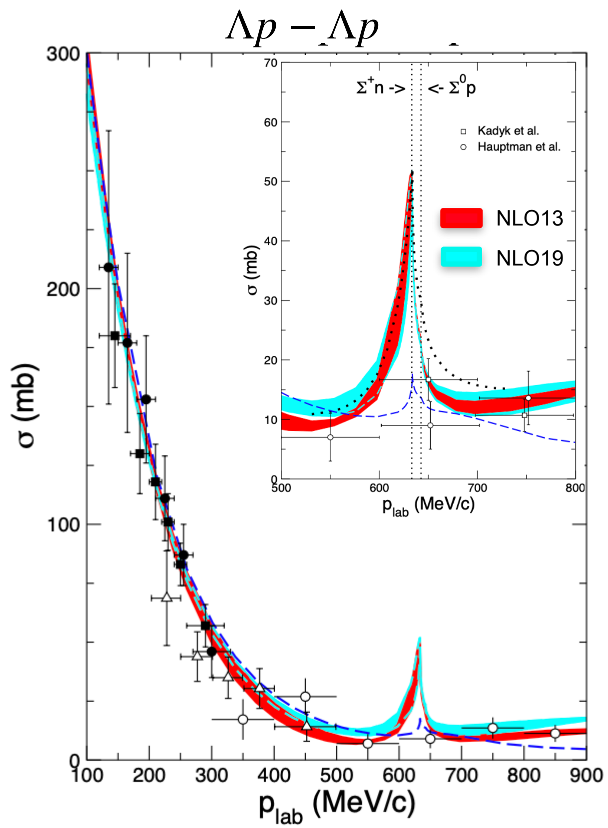
YN interactions at NLO

NLO13: J. Haidenbauer et al., NPA 915 (2013), NLO19: EPJ A 56 (2019) 91

- most of YN LECs are fitted to 36 YN data points $(\Lambda p \to \Lambda p,\ \Sigma N \to \Sigma N,\ \Sigma N \to \Lambda N)$
- use $B_\Lambda({}^3_\Lambda\text{H})$ to fix relative strength of ΛN singlet/triplet interaction

- two realisations at NLO: NLO13 and NLO19
  - almost phase equivalent
  - NLO13 leads to a larger transition potential $V_{\Lambda N \leftrightarrow \Sigma N}$ (not an observable)

→ NLO13 and NLO19 as a tool to estimate effects from three-body forces (Haidenbauer et al. EPJA 56 (2019))



(Haidenbauer 2019)

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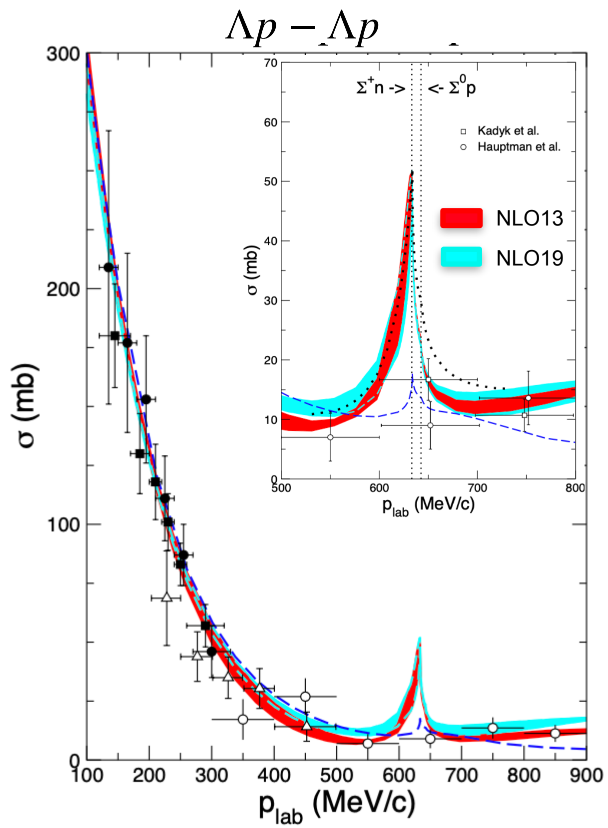

(Haidenbauer 2019)

- chiral YN interactions give reasonable predictions for B_Λ and energy level splitting up to p-shell hypernuclei
  - R. Wirth et al., PRL (2014, 2016) PRC (2019) up to ${}^{13}_{\Lambda}C$ using LO
  - H. Le et al., PLB 801 (2020), EPJ A 56 (2020) up to ${}^{7}_{\Lambda}Li$ using NLO

Constraints from S=-2 sector

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- information about S=-2 sector is very limited:
  - some data/ limits for ΞN (in)elastic cross sections $(200 < P_\Xi < 800$ MeV/c)
    J.K. Ahn et al, PLB 633 (2006) 214
  - Nagara event: $\Delta B_{\Lambda\Lambda} = B_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}\text{He}) - 2B_\Lambda({}^{5}_{\Lambda}\text{He}) = 1.01 \pm 0.2$ MeV
    $= 0.67 \pm 0.17$ MeV

    H. Takahashi et al., PRL 87 (2001) 212502
    K. Nakazawa et al., NPA 835 (2010) 207

    $\Rightarrow$ $-1.32 < a_{\Lambda\Lambda} < -0.73$ fm (based on the 2001 value)
  - some events support Ξ-bound states

    $\Xi^- - {}^{14}N$ K. Nakazawa et al., PTEP (2015) 033D02 (KISO), arXiv : 22010.14317 [nucl-ex] (IBUKI)

    $\Xi^- - {}^{11}B$ T. Nagae et al., PoS (INPC2016) 038, AIP Conf. Proc 2130 (2019), talk Theia-strong2020

    $\Rightarrow$ a weakly attractive $U_\Xi \approx -14$ MeV
- exploit strict $SU(3)_f$ to relate LECs in S=-2 sector to LECs in S=0,-1 sectors
  $\Rightarrow$ 5 unknown LECs (2 in s-wave, 3 in p-wave) at NLO

YY interactions up to NLO

LO: H. Polinder et al., PLB 653 (2007) 29.

NLO: J. Haidenbauer et al., NPA 954 (2016) 273, EPJA 55 (2019) 23

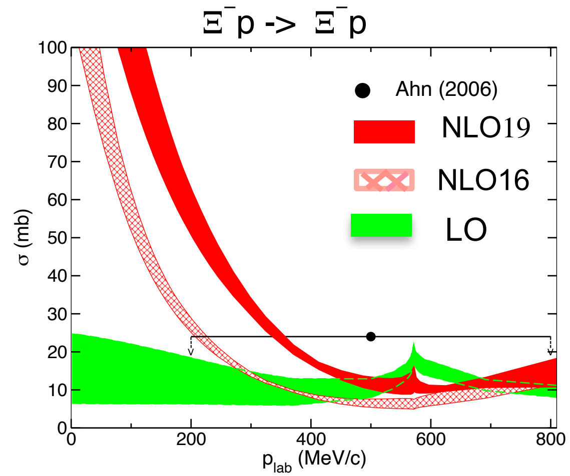
- two unknown s-wave LECs are determined via a fit to the scarce YY data

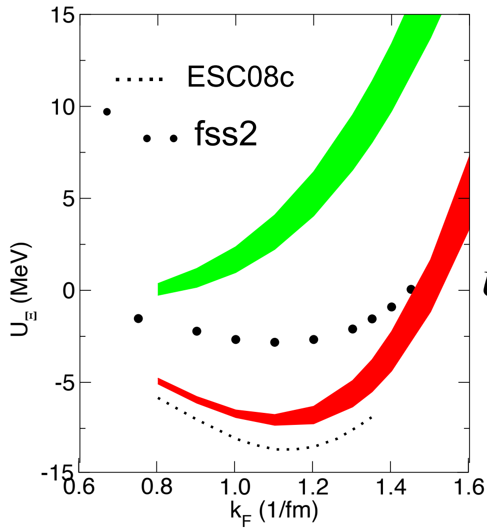
→ only qualitative determinations of LECs are feasible

YY	$a_{\Lambda\Lambda}(^1S_0)$ [fm]	$\Delta B_{\Lambda\Lambda}(^{6}_{\Lambda\Lambda}\text{He})$ [MeV]
LO(600)	-1.52	?
NLO19(600)	-0.66	?
ESC08c	-5.36	0.97,..,1.37
fss2	0.32	1.27,…, 1.41
Experiment	?	1.01; 0.67

ESC08c: M. M. Nagels et al., arXiv:1504:02634

fss2: Y. Fujiwara et al., Prog. PNP 58 (2007)





Haidenbauer(2019)

$U_\Xi(\text{NLO16}) \approx$ $22.4 \cdots 27.7$ MeV

$U_\Xi(\text{NLO19}) \approx$ $-5.5 \cdots -3.8$ MeV

- additional constraints on YY interactions are expected from few-body calculations or lattice simulations

Few-body Techniques for S=-2

Stochastic variational method $A \leq 6$

H. Nemura et al., PTP 103 (1999), PRL 94 (2005)

- ΛN interactions based on soft-core Nijmegen, adjusted to reproduce $A = 3-5\ B_\Lambda$
- central YY potentials, all particle conversions in S=-1, -2 are considered

→ ${}^{6}_{\Lambda\Lambda}$He, ${}^{5}_{\Lambda\Lambda}$H/He are strongly bound, $B_{\Lambda\Lambda}({}^{4}_{\Lambda\Lambda}\mathrm{H}) \approx 2$ keV

L. Contessi et al., PLB 797 (2019)

- use pionless EFT interactions at LO

→ $|a_{\Lambda\Lambda}| > 1.5$ fm in order to obtain bound ${}^{4}_{\Lambda\Lambda}$H

→ the existence of ${}^{4}_{\Lambda\Lambda}$H is incompatible with the Nagara result for ${}^{6}_{\Lambda\Lambda}$He

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Cluster models: $\alpha's + \Lambda's + N's$

Jacobian-coordinate Gaussian expansion method $A \leq 11$

E. Hiyama et al., PTP 97 (1997), PRC 66 (2002), Ann. RNP Sci. (2018)

- use simulated G-matrix potentials derived from OBE interactions
- $\Lambda N - \Sigma N$, $YY - \Xi N$ conversions are not treated explicitly

→ may affect the predictions for s-shell $\Lambda\Lambda$ hypernuclei, but not p-shell

(E. Hiyama talk LEAP (2013))

Few-body Techniques for S=-2

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Faddeev-Yakubovsky calculations $A \leq 10$

I. Filikhin A. Gal PRL 89 (2002), NP A 707 (2002)

- employ two-range Gaussian $V_{\alpha\Lambda}, V_{\alpha\alpha}$, and the simulated V_{YY} but restrict to s-wave

→ predictions for ${}^{4}_{\Lambda\Lambda}$H are model sensitive:

  - no bound state using $\Lambda + \Lambda + n + p$ model
  - particle-stable within $\Lambda + \Lambda + d$ model for $-a_{\Lambda\Lambda} \geq 0.5$ fm

Few-body Techniques for S=-2

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Our aim:

- develop Jacobi NCSM for S=-2 hypernuclei
  - based on realistic chiral NN, YN and YY interactions
  - $\Lambda N - \Sigma N$, $YY - \Xi N$ conversions are explicitly taken into account

→ study predictions of LO and NLO YY potentials for A=4-6 $\Lambda\Lambda$ hypernuclei

→ provide useful constraints to improve YY interactions

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Similarity Renormalization Group (SRG)

Similarity Renormalization Group (SRG)

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- BB interactions contain short-range and tensor correlations that couple low- and high-momentum states ➡ NCSM calculations converge slowly
- pre-diagonalize the Hamiltonian via SRG

  F.J. Wegner NPB 90 (2000). S.K. Bogner et al., PRC 75 (2007)

$$\frac{dV_s}{ds} = \Big[\Big[\sum \frac{p^2}{2\mu}, V_s\Big], H_s\Big], \qquad H_s = T_{rel} + V_s = T_{rel} + V_s^{NN} + V_s^{YN} + V_s^{YY} \qquad \text{Eq.(1)}$$

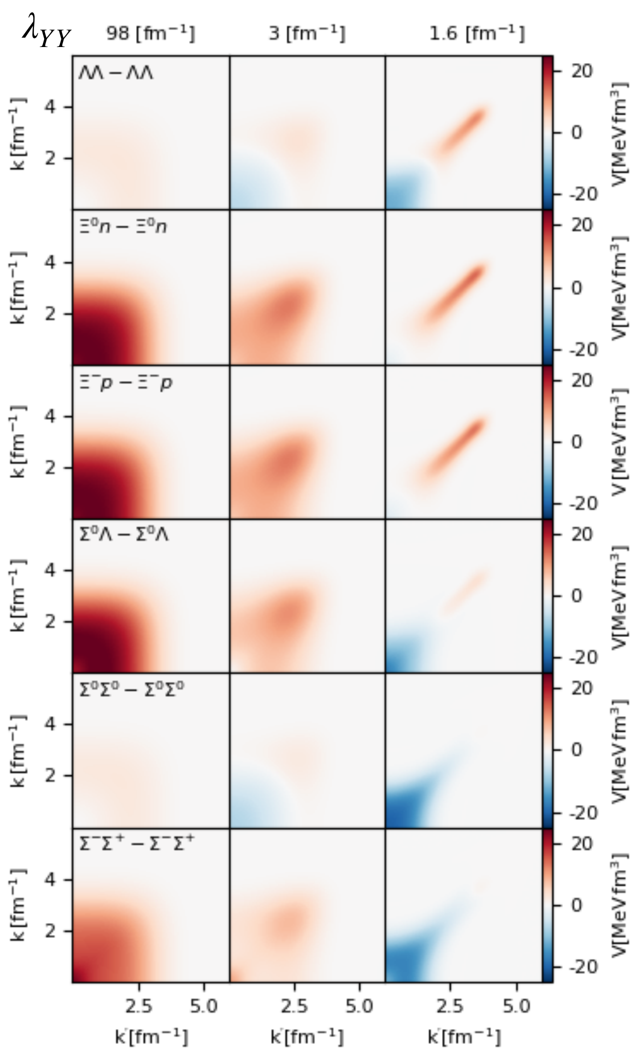
- restrict to 2-body space $\Rightarrow$ $V_s^{NN}, V_s^{YN}, V_s^{YY}$ can be evolved separately
- project Eq.(1) on partial-wave momentum basis: $|p\alpha\rangle \equiv |p(ls)J; t_1\, m_{t_1} S_1\, t_2\, m_{t_2} S_2\rangle$

$$\Rightarrow \quad \frac{dV_s^{\alpha\alpha'}(pp')}{ds} = \underbrace{\Big\{T_{rel}^{\alpha}(p)\frac{p'^2}{2\mu^{\alpha'}} + T_{rel}^{\alpha'}(p')\frac{p^2}{2\mu^{\alpha}} - T_{rel}^{\alpha}(p)\frac{p^2}{2\mu^{\alpha}} - T_{rel}^{\alpha'}(p')\frac{p'^2}{2\mu^{\alpha'}}\Big\} V_s^{\alpha\alpha'}(pp')}_{\text{drives V towards the diagonal}}$$

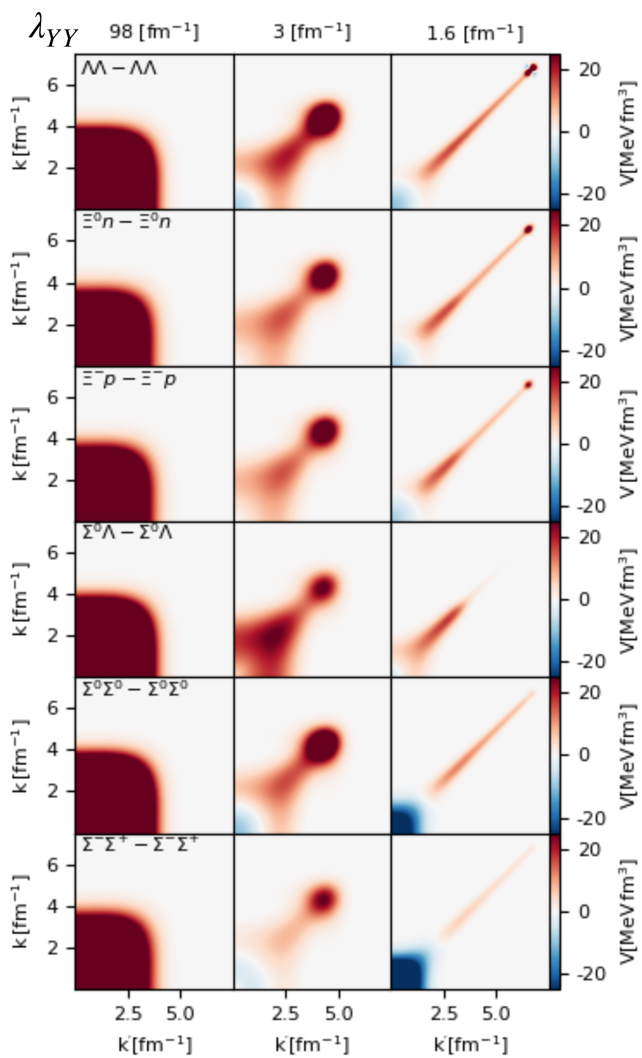
$$+ \sum_{\tilde\alpha} \underbrace{\int_0^\infty dk k^2 \Big\{\frac{p^2}{2\mu^{\alpha}} + \frac{p'^2}{2\mu^{\alpha'}} - \frac{k^2}{\mu^{\tilde\alpha}}\Big\} V_s^{\alpha\tilde\alpha}(pk) V_s^{\tilde\alpha\alpha'}(kp')}_{\text{preserves unitarity}}$$

- $\lambda = (4\mu^2/s)^{1/4}$: a measure of the width of V in p-space (S.K. Bogner et al. PRC 75 (2007))
- omit SRG-induced 3B, 4B… forces $\Rightarrow E_b = E_b(\lambda)$

SRG-evolved YY potentials



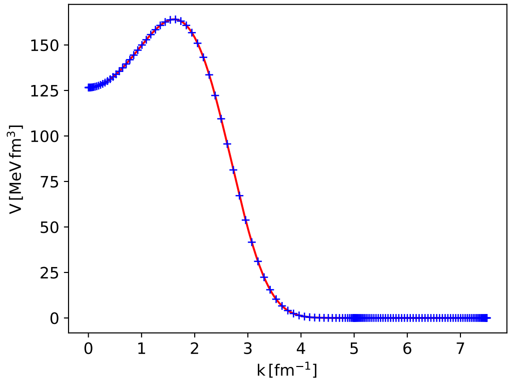
YY-LO, $\Lambda_{YY} = 600$ MeV



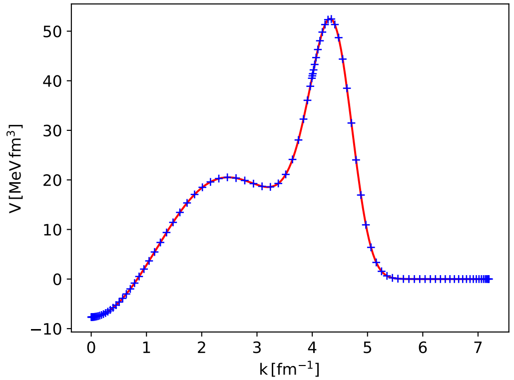
YY-NLO, $\Lambda_{YY} = 600$ MeV

$^1S_0$ partial wave, charge Q=0

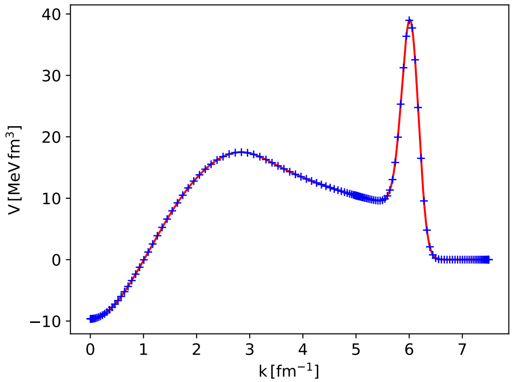
SRG-evolved YY potentials



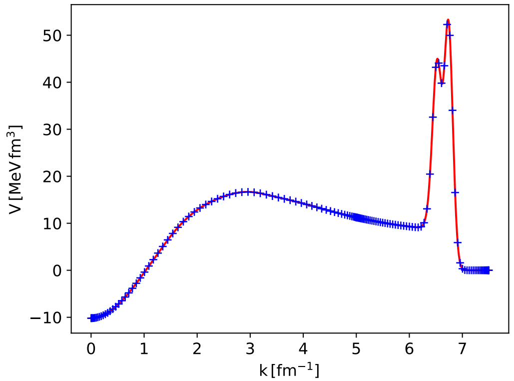
(a) λ_{YY} = 98 fm^{-1}



(b) λ_{YY} = 3 fm^{-1}



(c) λ_{YY} = 2.0 fm^{-1}



(d) λ_{YY} = 1.6 fm^{-1}

$^1S_0$ partial wave, charge Q=0

Diagonal matrix elements of $V_{\Lambda\Lambda-\Lambda\Lambda}$ potential.

Initial potential: YY-NLO, $\Lambda_{YY} = 600$ MeV.

Jacobi No-Core Shell Model (J-NCSM)

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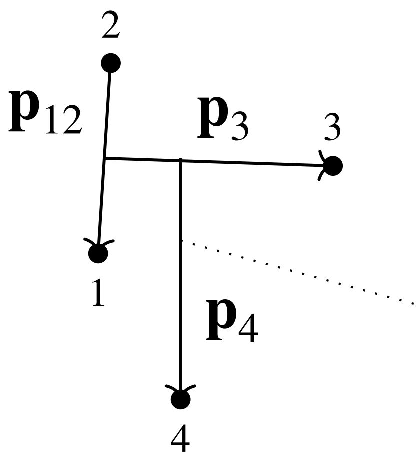
- an expansion of the wavefunction in a many-body HO basis depending on Jacobi coordinates

$$\mathbf{p}_{12} = \frac{m_2}{m_1+m_2}\mathbf{k}_1 - \frac{m_1}{m_1+m_2}\mathbf{k}_2,$$

$$\mathbf{p}_3 = \frac{m_1+m_2}{m_1+m_2+m_3}\mathbf{k}_3 - \frac{m_3}{m_1+m_2+m_3}(\mathbf{k}_1+\mathbf{k}_2)$$

$$\ldots$$



  - explicit removal of c.m. motion $\Rightarrow$ significantly reduce dimensionality as compared to other realisations (e.g., m-scheme NCSM)
  - inclusion of higher-body forces is straightforward
  - antisymmetrization of basis states is demanding $(A \leq 9)$
- all particles are active (no inert core) $\Rightarrow$ employ microscopic BB interactions
- converge slowly $\Rightarrow$ require soft interactions (use techniques e.g., Vlow_k, SRG)

Jacobi Basis States for S=-2

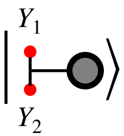
- diagonalize the hypernuclear Hamiltonian,

$$H = T_{rel} + V_{NN}^{S=0} + V_{YN}^{S=-1} + V_{YY}^{S=-2} + \Delta M + \cdots$$

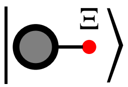
in a finite A-particle harmonic oscillator (HO) basis

- allow all possible particle conversions:
  - $\Lambda N \leftrightarrow \Sigma N$ in $S = -1$ sector
  - $Y_1 Y_2 \leftrightarrow \Xi N$ and $Y_1 Y_2 \leftrightarrow Y_1 Y_2$; $Y_1, Y_2 = \Lambda, \Sigma$ in $S = -2$ sector

⇒ split basis functions into two orthogonal sets



$$= |\mathcal{N} J T, \alpha_{A-2} \underbrace{\alpha_{Y_1 Y_2}}_{|\Lambda\Lambda\rangle, |\Lambda\Sigma\rangle, |\Sigma\Sigma\rangle} n_\lambda \lambda; ((l_{Y_1 Y_2} S_{Y_1 Y_2}) J_{Y_1 Y_2} (\lambda J_{A-2}) I_\lambda) J, ((t_{Y_1} t_{Y_2}) T_{Y_1 Y_2} T_{A-2}) T\rangle \equiv |\alpha^{*(Y_1 Y_2)}\rangle$$



$$= |\mathcal{N} J T, \alpha_{A-1} n_\Xi l_\Xi t_\Xi; (J_{A-1} (l_\Xi s) I_\Xi) J, (T_{A-1} t_\Xi) T\rangle \equiv |\alpha^{*(\Xi)}\rangle$$

⇒ hypernuclear wavefunction

$$|\Psi(\pi, J, T)\rangle = \sum_{\alpha^{*(Y_1 Y_2)}} C_{\alpha^{*(Y_1 Y_2)}} |\alpha^{*(Y_1 Y_2)}(\mathcal{N} J T)\rangle + \sum_{\alpha^{*(\Xi)}} C_{\alpha^{*(\Xi)}} |\alpha^{*(\Xi)}(\mathcal{N} J T)\rangle$$

S=-2 Hamiltonian in Jacobi Coordinates

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$$\langle \Psi(\pi JT) \,|\, H \,|\, \Psi(\pi JT) \rangle = \sum_{\substack{\alpha^{*(Y_1Y_2)} \\ \alpha'^{*(Y_1Y_2)}}} C_{\alpha^{*(Y_1Y_2)}} C_{\alpha'^{*(Y_1Y_2)}} \langle \alpha^{*(Y_1Y_2)} \,|\, H \,|\, \alpha'^{*(Y_1Y_2)} \rangle$$

$$+ \sum_{\substack{\alpha^{*(\Xi))} \\ \alpha'^{*(\Xi)}}} C_{\alpha^{*(\Xi)}} C_{\alpha'^{*(\Xi)}} \langle \alpha^{(\Xi)} \rangle \,|\, H \,|\, \alpha'^{*(\Xi)} \rangle$$

$$+ \sum_{\substack{\alpha^{*(Y_1Y_2)} \\ \alpha'^{*(\Xi)}}} 2\, C_{\alpha^{*(Y_1Y_2)}} C_{\alpha'^{*(\Xi)}} \langle \alpha^{*(Y_1Y_2)} \,|\, H \,|\, \alpha'^{*(\Xi)} \rangle$$

⇒ we distinguish three parts of the Hamiltonian:

$$H_{Y_1Y_2} = \sum_{i<j=1}^{A-2} \left(\frac{2p_{ij}^2}{M(t_{Y_1}, t_{Y_2})} + V_{ij}^{s=0} \right)$$

$$+ \sum_{i=1}^{A-2} \left(\frac{m_N + m(t_{Y_1})}{M(t_{Y_1}, t_{Y_2})} \frac{p_{iY_1}^2}{2\mu_{iY_1}} + V_{iY_1}^{s=-1} + \frac{m_N + m(t_{Y_2})}{M(t_{Y_1}, t_{Y_2})} \frac{p_{iY_2}^2}{2\mu_{iY_2}} + V_{iY_2}^{s=-1} \right)$$

$$+ \frac{m_{t_{Y_1}} + m_{t_{Y_2}}}{M(t_{Y_1}, t_{Y2})} \frac{p_{Y_1Y_2}^2}{2\mu_{Y_1Y_2}} + V_{Y_1Y_2}^{s=-2} + \left(m(t_{Y_1}) + m(t_{Y_2}) - 2m_\Lambda \right)$$

} system of (A-2)N + 2Y

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S=-2 Hamiltonian in Jacobi Coordinates

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$$+ \frac{m_{t_{Y_1}} + m_{t_{Y_2}}}{M(t_{Y_1}, t_{Y2})} \frac{p_{Y_1Y_2}^2}{2\mu_{Y_1Y_2}} + V_{Y_1Y_2}^{s=-2} + \left(m(t_{Y_1}) + m(t_{Y_2}) - 2m_\Lambda \right)$$

system of (A-2)N + 2Y

$$H_\Xi = \sum_{i<j=1}^{A-1} \left(\frac{2p_{ij}^2}{M(\Xi)} + V_{ij}^{s=0} \right)$$
$$+ \sum_{i=1}^{A-1} \left(\frac{m_N + m_\Xi}{M(\Xi)} \frac{p_{\Xi i}^2}{2\mu_{\Xi i}} + V_{\Xi i}^{s=-2} \right) + \left(m_\Xi + m_N - 2m_\Lambda \right)$$

system of (A-1)N + Ξ

$$H_{Y_1Y_2-\Xi N}^{S=-2} = \sum_{i=1}^{A-1} V_{Y_1Y_2-\Xi i}^{s=-2}$$

transition Hamiltonian

Evaluating Hamiltonian Matrix Elements

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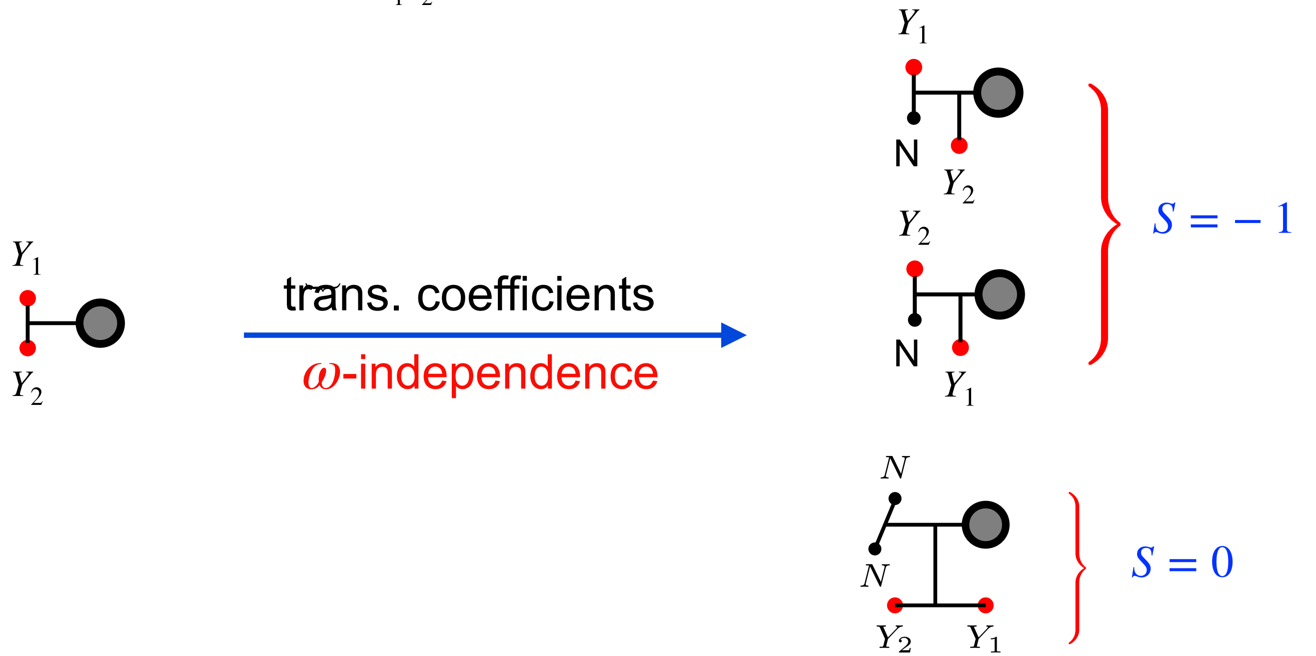
- for evaluating, e.g. $\langle \alpha^{*(Y_1Y_2)} | H_{Y_1Y_2} | \alpha'^{*(Y_1Y_2)} \rangle$ we need to transform to other bases



- basis truncation:

$$\mathcal{N} = \mathcal{N}_{A-2} + N_{Y_1Y_2} + 2n_\lambda + \lambda \leq \mathcal{N}_{max} \Rightarrow E_b = E_b(\omega, \mathcal{N}_{max})$$

→ require $\mathcal{N}_{max} \to \infty$ extrapolation

- all bases states + trans. coefficients are independent of interactions and HO-ω

⇒ enable a series of calculations with different interactions, a range of HO-ω and $\mathcal{N}_{max}$

different interactions: study chiral forces

a range of HO-ω and $\mathcal{N}_{max}$: extract converged E_b

Results for ${}^{6}_{\Lambda\Lambda}$He, ${}^{5}_{\Lambda\Lambda}$He, ${}^{4}_{\Lambda\Lambda}$H

(H. Le, J. Haidenbauer, U.-G. Meißner, A. Nogga in preparation)

${}^{6}_{\Lambda\Lambda}$He$(0^+,0)$

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- for all calculations use:
  - NN : N^{4}LO$^+$$(450)$, $\lambda_{NN} = 1.6\,\text{fm}^{-1}$ (P. Reinert et al. EPJA 54 (2018))
  - YN : NLO19(650), $\lambda_{YN} = 0.87\,\text{fm}^{-1}$

→ reproduce separation energies of ${}^{4}_{\Lambda}$He(1^+), ${}^{5}_{\Lambda}$He, ${}^{7}_{\Lambda}$Li, but slightly underbind ${}^{4}_{\Lambda}$He(0^+)

H. Le et al., EPJA 56 (2020)

  - YY : LO(600), NLO19(600) $1.4 \le \lambda_{YY} \le 3.0$ fm^{-1} → study effects of SRG-YY evolution

- separation-energy difference: $\Delta B_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}\text{He}) = B_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}\text{He}) - 2B_{\Lambda}({}^{5}_{\Lambda}\text{He})$

$$= 2E({}^{5}_{\Lambda}\text{He}) - E({}^{4}\text{He}) - E_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}\text{He})$$

→ contains information about $\Lambda\Lambda$ interaction strength, spin-dependent part of Λ-core interactions, core distortions

M. Danysz et al., NP 49 (1963). E. Hiyama et al., PRC 66 (2002)

${}^{\ 6}_{\Lambda\Lambda}He(0^+,0)$

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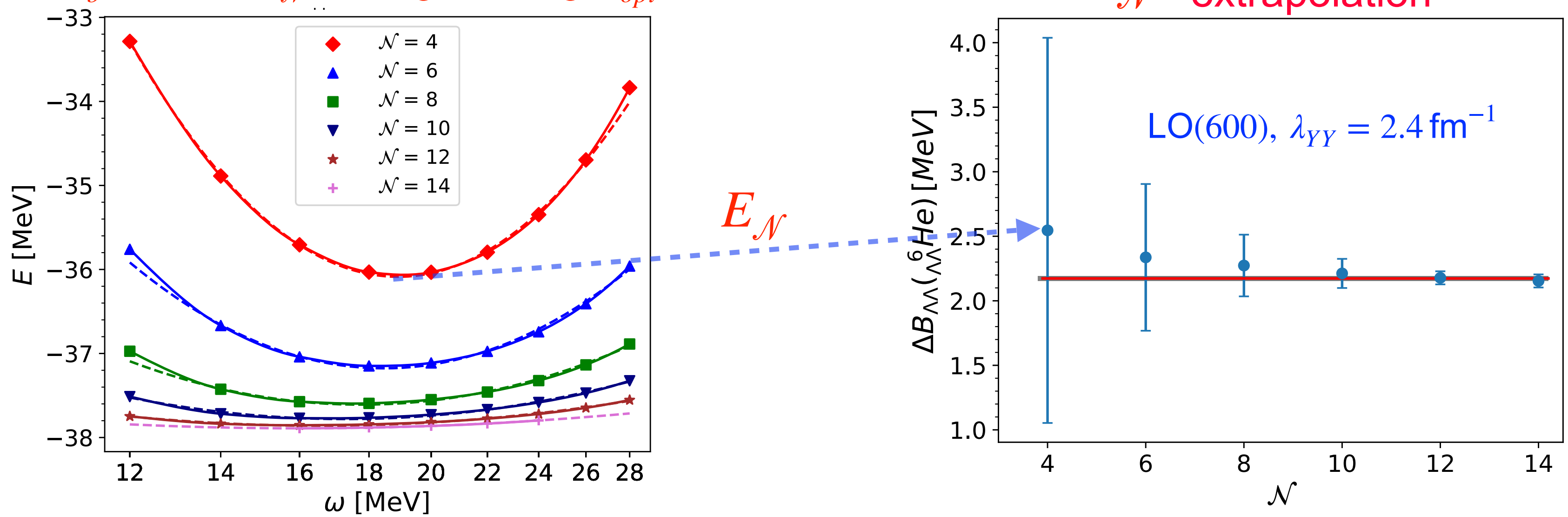
- separation-energy difference: $\Delta B_{\Lambda\Lambda}({}^{\ 6}_{\Lambda\Lambda}\mathrm{He}) = 2E({}^{5}_{\Lambda}\mathrm{He}) - E({}^{4}\mathrm{He}) - E_{\Lambda\Lambda}({}^{\ 6}_{\Lambda\Lambda}\mathrm{He})$

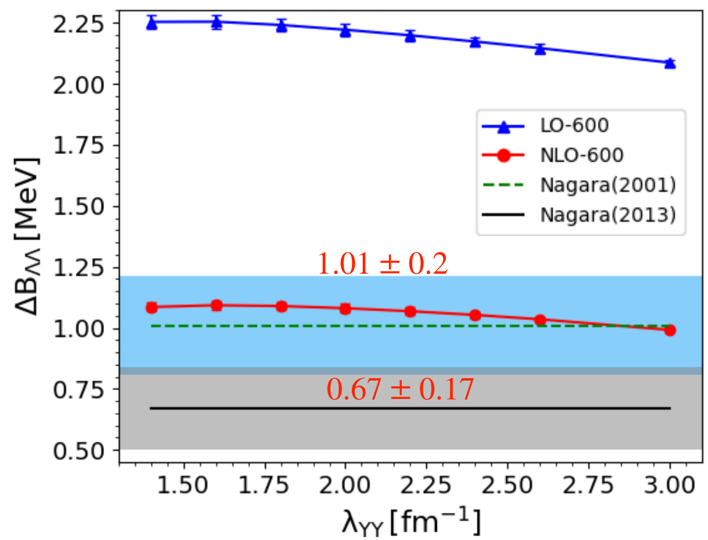
$E_b(\omega, \mathcal{N}) = E_{\mathcal{N}} + \kappa(\log(\omega) - \log(\omega_{opt}))^2$

$\mathcal{N}$ – extrapolation



$P_{\Sigma}({}^{5}_{\Lambda}\mathrm{He}$, YN-NLO19$) = 0.07\ \%$

λ_{YY} fm^{-1}	NLO(600)			LO(600)		
	$P_{\Lambda\Sigma}$	$P_{\Sigma\Sigma}$	P_{Ξ}	$P_{\Lambda\Sigma}$	$P_{\Sigma\Sigma}$	P_{Ξ}
1.4	0.13	0.11	0.02	0.17	0.04	0.5
2.0	0.13	0.11	0.07	0.17	0.05	0.84
3.0	0.12	0.13	0.12	0.18	0.08	1.08



- Effect of SRG-induced YYN forces is negligible
- NLO results are comparable to the Nagara
- NLO lead to smaller $\Delta B_{\Lambda\Lambda}$ and P_{Ξ}
- (no simple one-to-one connection between $\Delta B_{\Lambda\Lambda}$ and P_{Ξ})

${}^{5}_{\Lambda\Lambda}\text{He}(\frac{1^+}{2},\frac{1}{2})$

${}^{3}\text{He}(1/2) + \Lambda(1/2)$ → ↑↑ ${}^{4}_{\Lambda}\text{He}(1^+)$

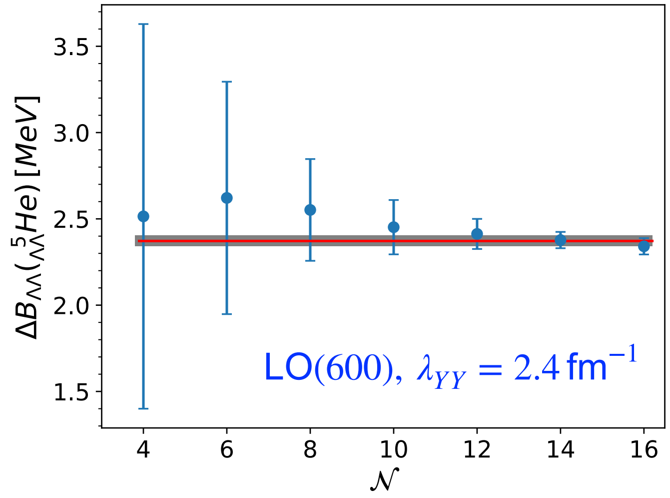
${}^{3}\text{He}(1/2) + \Lambda(1/2)$ → ↑↓ ${}^{4}_{\Lambda}\text{He}(0^+)$

→ use spin-averaged $\overline{B}_\Lambda$ (Danysz, Hiyama)

$$\overline{B}_\Lambda({}^{4}_{\Lambda}\text{He}) = \frac{1}{4} B_\Lambda({}^{4}_{\Lambda}\text{He}, 0^+) + \frac{3}{4} B_\Lambda({}^{4}_{\Lambda}\text{He}, 1^+)$$

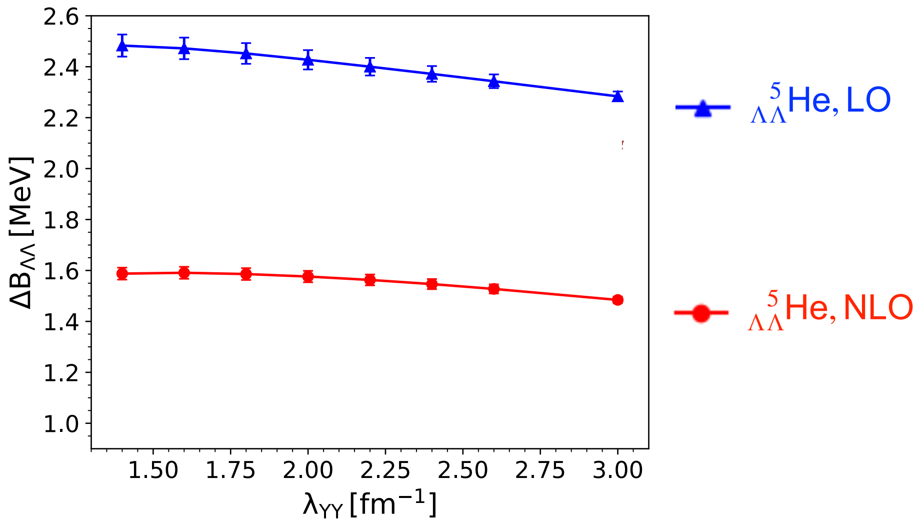
- effect of SRG-induced YYN forces on $\Delta B_{\Lambda\Lambda}$ is minor

$$\Delta B_{\Lambda\Lambda}({}^{5}_{\Lambda\Lambda}\text{He}) = B_{\Lambda\Lambda}({}^{5}_{\Lambda\Lambda}\text{He}) - 2\overline{B}_\Lambda({}^{4}_{\Lambda}\text{He})$$



$\mathcal{N}_{max}({}^{5}_{\Lambda\Lambda}\text{He}) = 16$

$\mathcal{N}_{max}({}^{4}_{\Lambda}\text{He}) = 22$



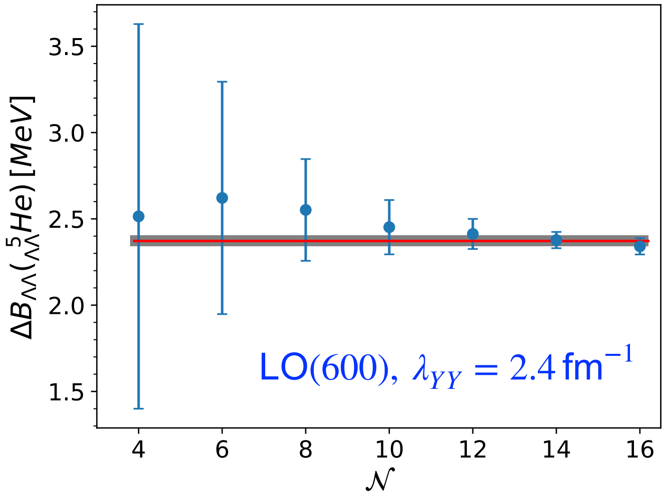
${}^{5}_{\Lambda\Lambda}\mathrm{He}(\frac{1}{2}^{+},\frac{1}{2})$

${}^{3}\mathrm{He}(1/2) + \Lambda(1/2)$ → $\uparrow\uparrow$ ${}^{4}_{\Lambda}\mathrm{He}(1^{+})$
${}^{3}\mathrm{He}(1/2) + \Lambda(1/2)$ → $\uparrow\downarrow$ ${}^{4}_{\Lambda}\mathrm{He}(0^{+})$

→ use spin-averaged $\overline{B}_{\Lambda}$ (Danysz, Hiyama)

$$\overline{B}_{\Lambda}({}^{4}_{\Lambda}\mathrm{He}) = \frac{1}{4}B_{\Lambda}({}^{4}_{\Lambda}\mathrm{He},0^{+}) + \frac{3}{4}B_{\Lambda}({}^{4}_{\Lambda}\mathrm{He},1^{+})$$

$$\Delta B_{\Lambda\Lambda}({}^{5}_{\Lambda\Lambda}\mathrm{He}) = B_{\Lambda\Lambda}({}^{5}_{\Lambda\Lambda}\mathrm{He}) - 2\overline{B}_{\Lambda}({}^{4}_{\Lambda}\mathrm{He})$$

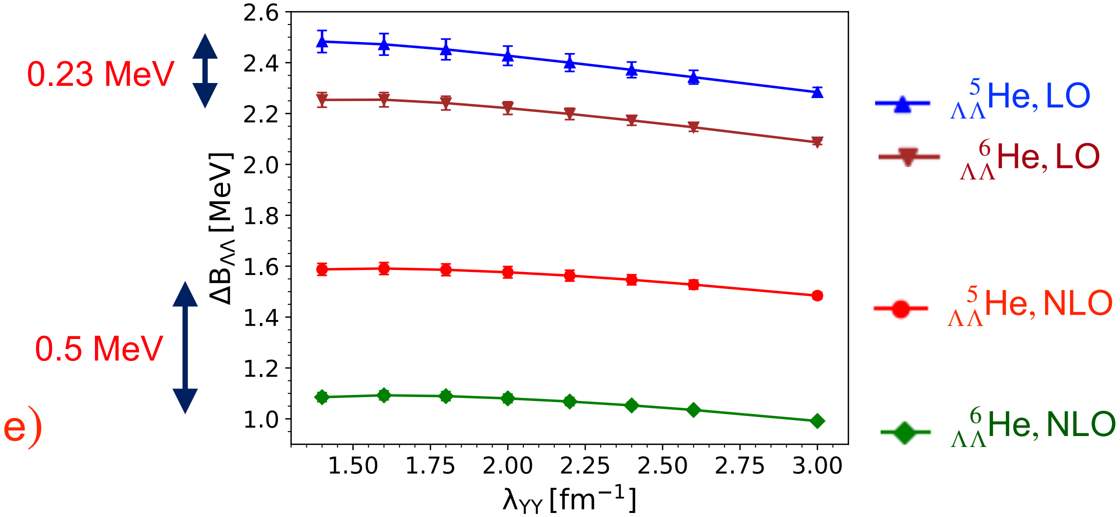


$\mathcal{N}_{max}({}^{5}_{\Lambda\Lambda}\mathrm{He}) = 16$

$\mathcal{N}_{max}({}^{4}_{\Lambda}\mathrm{He}) = 22$

- effect of SRG-induced YYN forces on $\Delta B_{\Lambda\Lambda}$ is minor
- large $\Lambda\Lambda$-separation energy difference:
  $\Delta B_{\Lambda\Lambda}({}^{5}_{\Lambda\Lambda}\mathrm{He}) > \Delta B_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}\mathrm{He})$
- FY calculations: $\Delta B_{\Lambda\Lambda}({}^{5}_{\Lambda\Lambda}\mathrm{He}) < \Delta B_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}\mathrm{He})$

  (I. Filikhin, A. Gal NPA 707 (2002)



${}^{5}_{\Lambda\Lambda}\mathrm{He}(\frac{1}{2}^{+},\frac{1}{2})$

- $P_{\Lambda\Sigma}, P_{\Sigma\Sigma}, P_{\Xi}$ are not very sensitive to SRG YY
- LO predicts significantly larger P_{Ξ}

λ_{YY}	YY-NLO(600)			YY-LO(600)		
fm^{-1}	$P_{\Lambda\Sigma}$	$P_{\Sigma\Sigma}$	P_{Ξ}	$P_{\Lambda\Sigma}$	$P_{\Sigma\Sigma}$	P_{Ξ}
1.4	0.61	0.07	0.4	0.53	0.02	1.25
2.0	0.6	0.08	0.38	0.51	0.03	1.36
3.0	0.57	0.08	0.23	0.51	0.05	1.35

$P_{\Sigma}({}^{4}_{\Lambda}\mathrm{He}, \mathrm{YN\text{-}NLO19}) = 0.43\,\%$

probabilities (%) of finding $\Sigma, \Sigma\Sigma, \Xi$ in ${}^{5}_{\Lambda\Lambda}\mathrm{He}$

	${}^{5}_{\Lambda\Lambda}\mathrm{He}$			${}^{6}_{\Lambda\Lambda}\mathrm{He}$		
	$P_{\Lambda\Sigma}$	P_{Ξ}	$B_{\Lambda\Lambda}$	$P_{\Lambda\Sigma}$	P_{Ξ}	$B_{\Lambda\Lambda}$
NLO($\lambda_{YY} = 2.0$)	0.6	0.38	3.67 ± 0.03	0.13	0.07	7.62 ± 0.02
LO($\lambda_{YY} = 2.0$)	0.51	1.36	4.53 ± 0.01	0.17	0.84	8.40 ± 0.02
mNDs*			3.66	1.17	0.28	7.54

$P_{\Sigma}({}^{5}_{\Lambda}\mathrm{He}, \mathrm{YN\text{-}NLO19}) = 0.07\,\%$

probabilities (%) of finding Σ, Ξ and $B_{\Lambda\Lambda}$ (MeV) in ${}^{5}_{\Lambda\Lambda}\mathrm{He}$, ${}^{6}_{\Lambda\Lambda}\mathrm{He}$

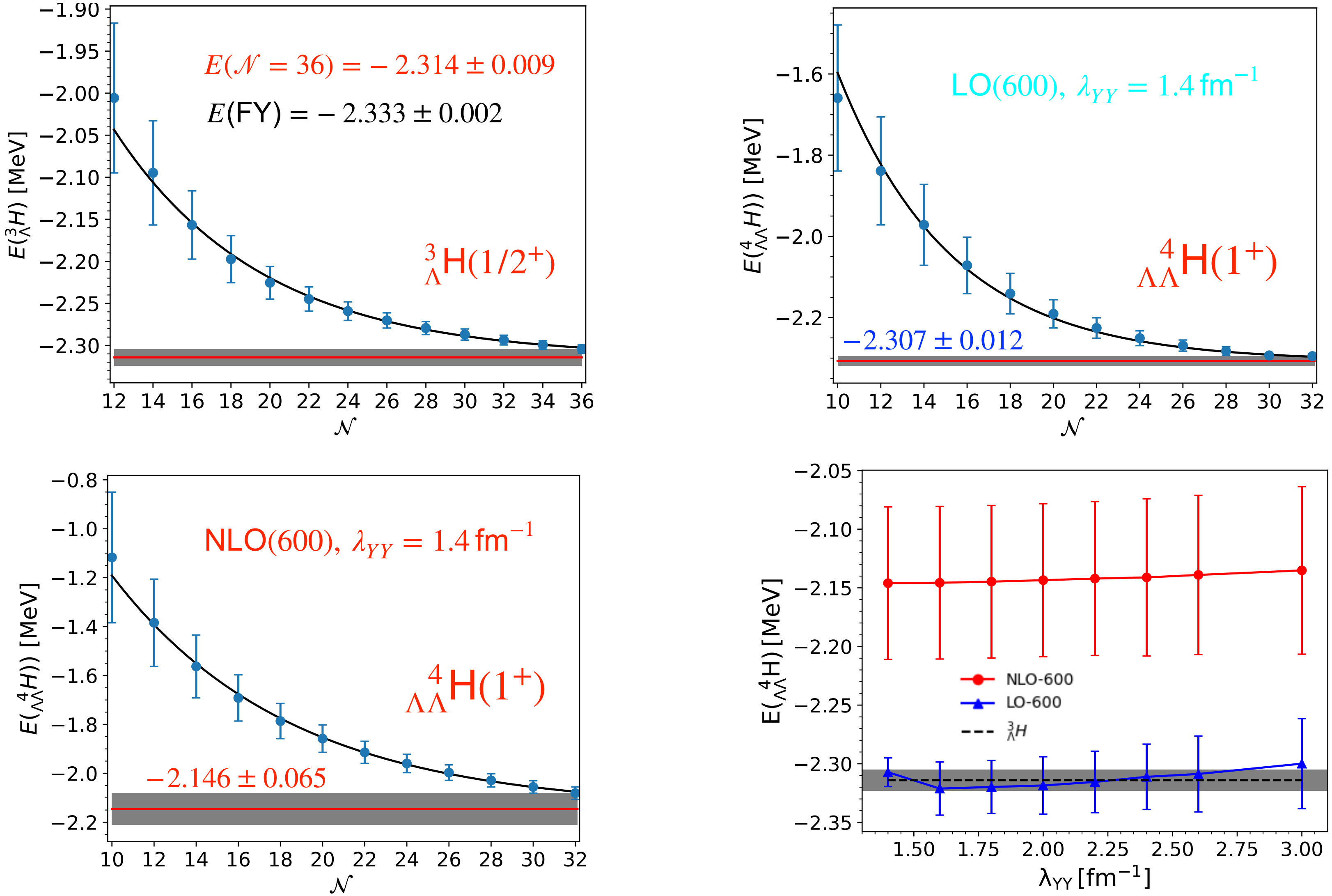
* H. Nemura et al., PRL 94 (2005)

- $P_{\Xi}({}^{6}_{\Lambda\Lambda}\mathrm{He}) < P_{\Xi}({}^{5}_{\Lambda\Lambda}\mathrm{He}) \Rightarrow \Lambda\Lambda - \Xi N$ transition is suppressed in ${}^{6}_{\Lambda\Lambda}\mathrm{He}$

B. F. Gibson PTPS 117, 339 (1994), E. Hiyama talk LEAP (2013)

$^{4}_{\Lambda\Lambda}$H$(1^{+},0)$

- Is $^{4}_{\Lambda\Lambda}$H stable against the breakup to $^{3}_{\Lambda}$H + Λ?



→ NLO leads to a particle unstable $^{4}_{\Lambda\Lambda}$H. LO results do not allow for a definite conclusion

Conclusions & outlook

JÜLICH FORSCHUNGSZENTRUM

- developed ab-initio Jacobi NCSM for $\Lambda\Lambda$ hypernuclei up to p-shell
- studied the predictions of χ YY LO and NLO for $^{6}_{\Lambda\Lambda}$He, $^{5}_{\Lambda\Lambda}$He, $^{4}_{\Lambda\Lambda}$H
  - SRG YY evolution has minor effects on $\Delta B_{\Lambda\Lambda}$ and $P_{\Lambda\Sigma}$, $P_{\Sigma\Sigma}$
  - LO strongly overbinds $^{6}_{\Lambda\Lambda}$He; NLO results are comparable to experiment
  - both interactions result in $B_{\Lambda\Lambda}(^{6}_{\Lambda\Lambda}\text{He}) < B_{\Lambda\Lambda}(^{5}_{\Lambda\Lambda}\text{He})$
  - NLO predicts a particle unstable $^{4}_{\Lambda\Lambda}$H, LO results do not allow for a clear conclusion yet

- need to investigate impacts of χ NN and YN interactions on $B_{\Lambda\Lambda}$
- inclusion of χ and SRG-induced 3N forces, SRG-induced YNN forces is in progress

  → provide meaningful constraints to improve YY interactions

Thank You!

Λp Phase shift