













## Chirally motivated $K^- N$ approaches

effective separable potentials constructed to match the chiral amplitudes

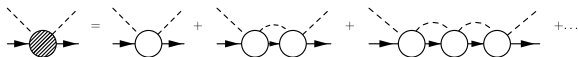
$$V_{ij}(k, k'; \sqrt{s}) = g_i(k^2) v_{ij}(\sqrt{s}) g_j(k'^2)$$

$$v_{ij}(\sqrt{s}) = -\frac{C_{ij}(\sqrt{s})}{4\pi f_i f_j} \sqrt{\frac{M_i M_j}{s}}$$

- energy dependent couplings  $C_{ij}$  determined by the chiral SU(3) symmetry
- the formfactors  $g_j(k) = 1/[1 + (k/\alpha_j)^2]$  account naturally for the off-shell effects with the inverse ranges  $\alpha_j$  fitted to the low energy  $\bar{K}N$  data

Lippmann-Schwinger equation used to solve exactly the loop series

$$T = V + V G T$$



## Chirally motivated $K^-N$ approaches

$$F_{ij}(k, k'; \sqrt{s}) = g_i(k^2) f_{ij}(\sqrt{s}) g_j(k'^2)$$

$$f_{ij}(\sqrt{s}) = \left[ (1 - v \cdot G(\sqrt{s}))^{-1} \cdot v \right]_{ij}$$

with a loop Green function (in a free space)

$$G_n(\sqrt{s}) = -4\pi \int \frac{d^3 p}{(2\pi)^3} \frac{g_n^2(p^2)}{k_n^2 - p^2 + i0} = \frac{(\alpha_n + i k_n)^2}{2\alpha_n} [g_n(k_n)]^2$$

**nuclear medium treatment**

$$G_n(\sqrt{s}) = -4\pi \int_{\Omega_i(\rho)} \frac{d^3 p}{(2\pi)^3} \frac{g_n^2(p^2)}{k_n^2 - p^2 - \Pi_i(\omega_i, E_i, \vec{p}; \rho) + i0}$$

- integration domain  $\Omega_i(\rho)$  is limited by the Pauli principle in the  $\bar{K}N$  channels
- $\Pi_i$  represents a sum of meson and baryon self-energies in channel  $i$
- $\Pi_{\bar{K}} \sim F_{\bar{K}N} \rho \Rightarrow$  selfconsistent treatment required



## Chirally motivated $K^-N$ approaches

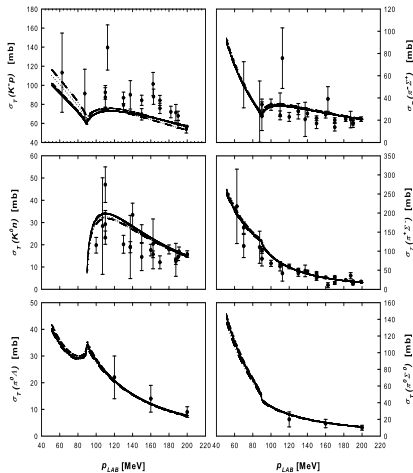
- **Kyoto-Munich (KM)**  
*Y. Ikeda, T. Hyodo, W. Weise, Nucl. Phys. A 881 (2012) 98*
- **Murcia ( $M_I, M_{II}$ )**  
*Z. H. Guo, J. A. Oller, Phys. Rev. C 87 (2013) 035202*
- **Bonn ( $B_2, B_4$ )**  
*M. Mai, U.-G. Meißner - Eur. Phys. J. A 51 (2015) 30*
- **Prague (P)**  
*A. C., J. Smejkal, Nucl. Phys. A 881 (2012) 115*
- **Barcelona (BCN)**  
*A. Feijoo, V. Magas, À. Ramos, Phys. Rev. C 99 (2019) 035211*

Model parameters (couplings, inverse interaction ranges or subtraction constants) fixed in **fits to low energy  $K^-p$  data** (and more in some cases).

Comparative analysis of the first four approaches presented in  
*A. C., M. Mai, U.-G. Meißner, J. Smejkal - Nucl. Phys. A 954 (2016) 17*

# $K^-p$ data (at and above threshold)

low energy cross sections:



threshold branching ratios:

$$\gamma = \frac{\Gamma(K^-p \rightarrow \pi^+\Sigma^-)}{\Gamma(K^-p \rightarrow \pi^-\Sigma^+)} = 2.36 \pm 0.04$$

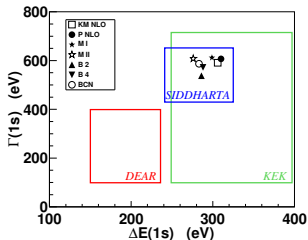
$$R_c = \frac{\Gamma(K^-p \rightarrow \text{charged})}{\Gamma(K^-p \rightarrow \text{all})} = 0.664 \pm 0.011$$

$$R_n = \frac{\Gamma(K^-p \rightarrow \pi^0\Lambda)}{\Gamma(K^-p \rightarrow \text{neutral})} = 0.189 \pm 0.015$$

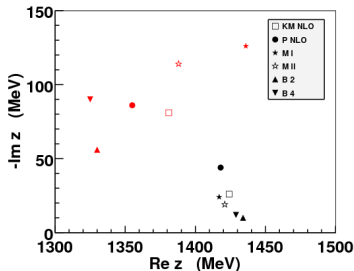
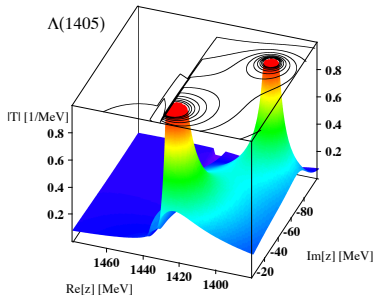
kaonic hydrogen:

$$\Delta E_N(1s) = 283 \pm 36(\text{stat.}) \pm 6(\text{syst.}) \text{ eV}$$

$$\Gamma(1s) = 541 \pm 89(\text{stat.}) \pm 22(\text{syst.}) \text{ eV}$$



# Model predictions - $\Lambda(1405)$ resonance



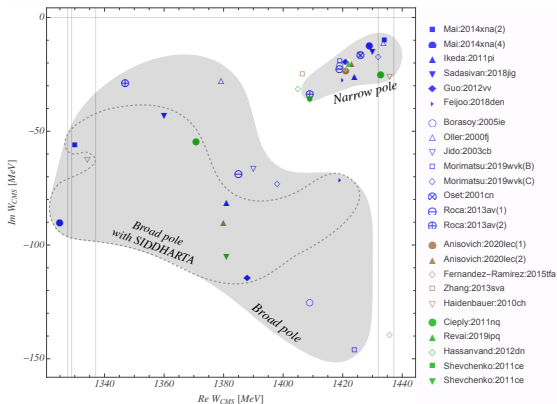
Hyodo, Jido - Prog. Part. Nucl. Phys. 67 (2012) 55

A.C., Mai, Meißner, Smejkal - NPA 954 (2016) 17

- the *higher* pole around 1425 MeV couples more strongly to  $\bar{K}N$ , the *lower* pole is much further from the real axis and has larger coupling to  $\pi\Sigma$
- all models tend to agree on the position of the  $\bar{K}N$  related pole
- the data are not very sensitive to the position of the  $\pi\Sigma$  related pole
- illustrative picture:  $\bar{K}N$  bound state submerged in  $\pi\Sigma$  continuum

# Model predictions - $\Lambda(1405)$ resonance

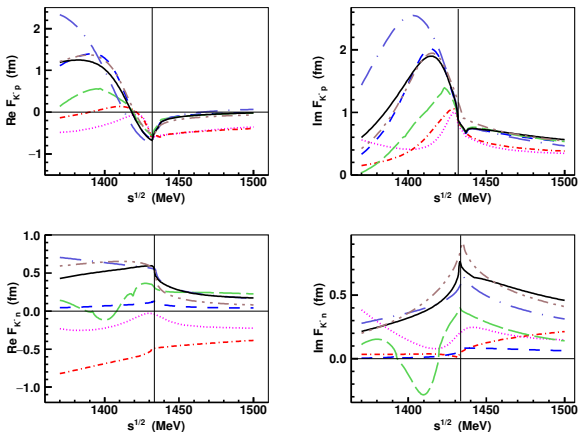
all recent (year  $\geq 2000$ )  $\Lambda(1405)$  poles predictions



M. Mai - arXiv:2010.00056 [nucl-th]

# Model predictions - $K^- N$ amplitudes (free space)

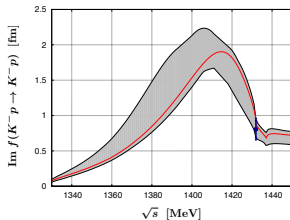
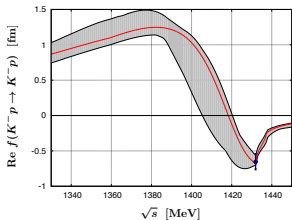
$K^- p$  and  $K^- n$  elastic amplitudes



$B_2$  (dotted, purple),  $B_4$  (dot-dashed, red),  $M_I$  (dashed, blue),  $M_{II}$  (long-dashed, green),  $P$  (dot-long-dashed, violet),  $BCN$  (dot-dot-dashed, brown),  $KM$  (continuous, black)

# Model predictions - $K^- N$ amplitudes (free space)

For a comparison, uncertainty bands provided in  
Y. Ikeda, T. Hyodo, W. Weise, Nucl. Phys. A 881 (2012) 98



bands calculated by varying the  $KM_{NLO}$  model parameters to get the  $K^- p$  scattering length within SIDDHARTA experimental error bars

the theoretical ambiguities below the  $\bar{K}N$  threshold are much larger than indicated by such uncertainty bounds provided for a specific model and a given  $\chi^2$  local minimum !!!

# Model predictions - $K^- N$ amplitudes (free space)

the  $I = 1$  sector is not restricted by the fitted experimental data leading to varied predictions for the  $K^- n$  scattering amplitude

## New data needed !!!

- new precise data for the isovector  $K^- p \rightarrow \pi^0 \Lambda$  and isoscalar  $K^- p \rightarrow \pi^0 \Sigma^0$  reactions at as low energies as possible are highly desired
- $\pi \Sigma$  mass spectra at subthreshold energies should help provided we understand the process dynamics
- the  $K_L^0 p \rightarrow K^+ \Xi^0$  reaction ( $I=1$ ) to be measured at KLF@JLAB
- decays of heavy baryons into channels including MB( $S=-1$ ), e.g.  $\Lambda_b \rightarrow J/\psi \Lambda(1405)$ , measured at LHC
- kaonic deuterium measurement (AMADEUS, Frascati) will also add to the picture

## Dynamically generated resonances/poles

Where do the poles come from? (demonstration for the Prague approach)

The amplitude has poles for complex energies  $z$  (equal to  $\sqrt{s}$  on the real axis) if a determinant of the inverse matrix is equal to zero,

$$\det|f^{-1}(z)| = \det|v^{-1}(z) - G(z)| = 0$$

The origin of the poles can be traced to the

**zero coupling limit:**  $C_{ij} = 0$  for  $i \neq j$  (interchannel couplings switched off)

for  $C_{i,j \neq i} = 0$  the condition for a pole of the amplitude becomes

$$\prod_n [1/v_{nn}(z) - G_n(z)] = 0$$

There will be a pole in channel  $n$  at a Riemann sheet  $[+/-]$  (phys./unphys.) if the following condition is satisfied for any complex energy  $z$ :

$$\frac{4\pi f_n^2}{C_{nn}(z)} \frac{z}{M_n} + \frac{(\alpha_n + ik_n)^2}{2\alpha_n} [g_n(k_n)]^2 = 0$$

Only states with nonzero diagonal couplings  $C_{i,j=i}$  can generate the poles!



# Dynamically generated resonances/poles

What channels have nonzero diagonal couplings?

For simplicity, we first look only at the leading order WT term couplings

Sample results for the  $P_{WT}$  model:

sector	channel	ZCL state	resonance
$I = 0$	$\pi\Sigma$	resonance	$\Lambda(1405)$
	$\bar{K}N$	bound	$\Lambda(1405)$
	$K\Xi$	bound	$\Lambda(1670)$
$I = 1$	$\pi\Sigma$	resonance	—
	$\bar{K}N$	virtual	$K^-n$ amplitude $\pi\Sigma$ photoproduction (CLAS data)
	$K\Xi$	virtual	$\Sigma(1750)$

In general, the exact situation (existence of a ZCL pole in a given channel) depends on the model parameters (subtraction constants, NLO contributions that generate sufficiently large couplings  $C_{nn}$ )

## Dynamically generated resonances/poles

**Notation:** Each Riemann sheet is labeled by the signs of the imaginary parts of the CMS momenta in the meson-baryon channels ordered according to their thresholds. The poles may move from one Riemann sheet to another one by crossing the real axis through the branch cuts.

For  $I = 0$  the RS  $[++++]$  is the physical sheet, for  $I = 1$  it is the  $[++++]$  RS. The Riemann sheets accessed by crossing the real axis in between the  $\pi\Sigma$  and  $\bar{K}N$  thresholds are  $[-+++]$  and  $[- -+++]$ , respectively.

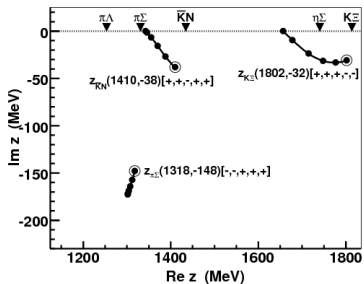
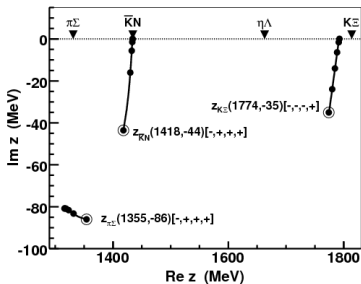
R.J. Eden, J.R. Taylor (1964):

- Each pole observed in the ZCL will develop into **multiplets of shadow poles** observed at Riemann sheets with the same original sign (+/-) in the respective channel.
- **Physical observables are affected by the pole that is closest to the physical region** in the limit of physical (fully switched on) interchannel couplings.
- This **dominant pole and any other shadow pole may switch roles** when the model parameters are varied, the couplings of various channels to a specific pole evolve as well.

# Dynamically generated resonances/poles

Pole movements upon scaling the nondiagonal interchannel couplings

$$C_{i,j \neq i} \text{ replaced by } x \cdot C_{i,j \neq i}$$



$P_{NLO}$  model, left panel: isoscalar states, right panel: isovector states  
 The pole positions in the physical limit are emphasized with large empty circles.  
 The triangles at the top of the real axis indicate the channel thresholds.

# Dynamically generated resonances/poles

## Pole content of the considered models

resonance	models / ZCL channels					
	$P_{NLO}$	$KM_{NLO}$	$M_I$	$M_{II}$	$B_2$	$B_4$
$\Lambda_1(1405)$	$\pi\Sigma$	$\pi\Sigma$	$\pi\Sigma$	$\pi\Sigma$	$\pi\Sigma$	$\pi\Sigma$
$\Lambda_2(1405)$	$\bar{K}N$	$\bar{K}N$	$\bar{K}N$	$\eta\Lambda$	$\eta\Lambda$	$\bar{K}N$
$\Lambda(1670)$	$K\Xi$	—	$K\Xi$	$K\Xi$	—	$K\Xi$
$\bar{K}N(I = 1)$	$\bar{K}N$	$\eta\Sigma$	$\bar{K}N$	$\bar{K}N$	—	—
$\Sigma(1750)$	$K\Xi$	—	—	$K\Xi$	—	$K\Xi$

$M_{II}$  and  $B_2$  models generate the  $\Lambda_2(1405)$  pole from the  $\eta\Lambda$  ZCL bound state.

earlier reports on the isovector  $\bar{K}N$  related pole:

- J. Oller, U.-G. Meißner - Phys. Lett. B 500 (2001) 263
- D. Jido, J.A. Oller, E. Oset, A. Ramos, U.-G. Meißner - NPA 725 (2003) 181
- A.C., E. Friedman, A. Gal, D. Gazda, J. Mareš - PRC 84 (2011) 045206
- A.C., J. Smejkal - Few Body Syst. 54 (2013) 1183

## One or two $\Lambda(1405)$ poles?

The multichannel chiral approaches generate two  $\Lambda(1405)$  poles !!!  
questioned recently in J. Révai - *Few Body Syst.* 59 (2018) 49

$$T(p', p; \sqrt{s}) = V(p', p; \sqrt{s}) + \int d^3 q V(p', q; \sqrt{s}) \frac{2\mu}{k^2 - q^2 + i\epsilon} T(q, p; \sqrt{s})$$

**on-shell factorisation:** the loop momentum  $q$  in the argument of  $T$  and  $V$  replaced by its on-shell value  $k$ , then  $T$  and  $V$  pulled out of the integral  
**equivalent to introducing tad-pole contributions to the interaction kernel  $V$**   
that can be absorbed into a renormalisation of the meson-baryon vertex

J. Révai put this procedure under question demonstrating that the  $\pi\Sigma$  related pole disappears if the off-shell part of the loop function integral is not dropped

- very specific form of the potential  $V$  adopted
- **non-relativistic treatment of the  $MB$  energies and momenta**

## One or two $\Lambda(1405)$ poles?

only Weinberg-Tomozawa term considered, taken in a form

$$\langle q_i | V_{ij} | q_j \rangle = u_i(q_i) \langle q_i | v_{ij} | q_j \rangle u_j(q_j) = u_i(q_i) \lambda_{ij} \left( m_i + \frac{q_i^2}{2\mu_i} + m_j + \frac{q_j^2}{2\mu_j} \right) u_j(q_j)$$

with the central piece equivalent to the on-shell form  $(2\sqrt{s} - M_i - M_j)$   
and a dipole form-factor  $u_i(q) = [1/(1 + q^2/\beta_i^2)]^2$

parameters  $\beta_j$  fitted to the  $\bar{K}N$  data for both choices

result: **one pole** vs **two poles**

However, ...

## One or two $\Lambda(1405)$ poles?

P. Bruns, A.C. - Nucl. Phys. A 996 (2020) 121702

We were able to rewrite the Révai's algebraic T-matrix solution as

$$T_{\text{on}}(k) = u(k) \left[ \tilde{W}_{\text{JR}}^{-1} - G_{\text{on}} \right]^{-1} u(k),$$

where the effective potential  $\tilde{W}_{\text{JR}}$  is again a real coupled-channel matrix depending only on the on-shell momenta  $k^2$ ,

$$\tilde{W}_{\text{JR}} = [1 + \lambda l_0]^{-1} (\bar{\gamma} \lambda + \lambda \bar{\gamma} - \lambda l_1 \lambda) [1 + l_0 \lambda]^{-1},$$

where  $\gamma(q) = \frac{q^2}{2\mu} + m$ ,  $\bar{\gamma} = \gamma(k)$  and the tad-pole integrals are

$$l_n := \frac{4\pi}{(2\mu)^n} \int_0^\infty dq q^2 (u(q))^2 (q^2 - k^2)^n$$

In this form, the on-shell approximation is quite transparent, equivalent to neglecting the integrals  $l_n$ .

## One or two $\Lambda(1405)$ poles?

The JR amplitude gives scattering lengths that do not vanish in the SU(3) chiral limit! Demonstration: one channel case, at threshold ( $k = 0$  and  $\bar{\gamma} = m$ )

$$a_{0+}^{\text{JR}} = -4\pi^2 \mu \left[ \frac{(1 + \lambda l_0)^2}{2\lambda m - \lambda l_1 \lambda} + 4\pi^2 \mu \frac{5\beta}{16} \right]^{-1}$$

on-shell approximation:  $l_0, l_1 \rightarrow 0$ ,  $a_{0+} \sim \mathcal{O}(m)$

off-shell effects in:  $l_{0,1} \neq 0$ ,  $a_{0+} \sim \mathcal{O}(m^0)$

**The JR approach violates the chiral symmetry!**

The origin of this violation can be traced to the use of non-relativistic kinematics.

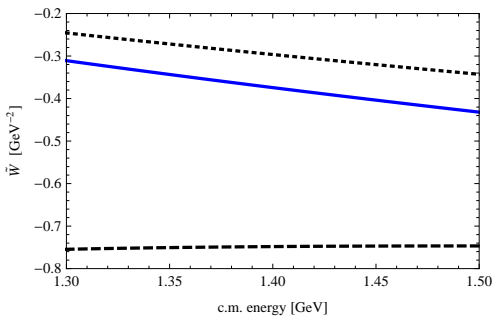


## One or two $\Lambda(1405)$ poles?

A proper relativistic treatment is possible, though the respective formulas are much more complicated.

The effective relativistic potential, a counterpart of  $\tilde{W}_{JR}$ , can be written as

$$\tilde{W}_{BC}(\sqrt{s}) = \sqrt{\frac{E_B + M}{\mu}} \frac{\mathcal{W}(\sqrt{s})}{4(2\pi)^3\sqrt{s}} \left[ 1 + (I_B(\beta) - I_M(\beta)) \frac{\mathcal{W}(\sqrt{s})}{2\sqrt{s}} \right]^{-1} \sqrt{\frac{E_B + M}{\mu}}$$



$\bar{K}N(I = 0)$  effective kernels:  $W_{WT}$  (dotted),  $W_{JR}$  (dashed),  $W_{BC}$  (blue)

## One or two $\Lambda(1405)$ poles?

the corresponding amplitudes satisfy the chiral symmetry strictures,

$$a_{0+,BC}^{\bar{K}N,I=0} \sim \mathcal{O}(m)$$

Model parameters, inverse ranges  $\beta_j$ , determined in fits to kaonic hydrogen data,  $K^-p$  threshold branching ratios and cross sections. The ratio of the meson decay constants fixed as  $F_K/F_\pi = 1.193^{n-1}$  providing two models  $BC_1$  and  $BC_2$ .

Pole positions (in MeV) on the  $[-,+]$  and  $[-,-,+]$  Riemann sheets for the  $l = 0$  and  $l = 1$  sectors, respectively.

model	$z_1 (l = 0)$	$z_2 (l = 0)$	$z_3 (l = 1)$
CS	(1432.8, -24.9)	(1370.8, -54.2)	(1408.9, -199.7)
JR	(1422.9, -25.7)	—	(1106.5, -71.6)
$BC_1$	(1439.9, -23.3)	(1316.0, -6.76)	(1361.1, -166.9)
$BC_2$	(1437.8, -20.9)	(1251.1, 0.0)	(1337.4, -117.3)

Note: There are two poles but their positions are not where we would like to have them.

## One or two $\Lambda(1405)$ poles?

A.V. Anisovich et al. - Eur. Phys. J. A 56 (2020) 5, 139

partial wave analysis of low-energy data on  $K^-p$  and  $\pi\Sigma$  interactions:

- bubble chamber data on  $K^-p \rightarrow \pi\pi\pi\Sigma$
- $K^-p \rightarrow \pi^0\pi^0\Lambda$ ,  $\pi^0\pi^0\Sigma^0$  from Crystal Ball at BNL
- $\gamma p \rightarrow K^+\pi\Sigma$  from CLAS at JLab
- $K^-p$  total cross sections
- kaonic hydrogen from SIDDHARTA at Frascati

Two equivalent solutions found:

- one pole  $z = (1421 \pm 3) - i(23 \pm 3)$  MeV, SU(3) singlet; compatible with the quark-model predictions
- two poles with  $z_1 = (1423 \pm 3) - i(20 \pm 3)$  MeV, SU(3) octet, and a second pole fixed at  $z_1 = 1380 - i90$  MeV, SU(3) singlet; compatible with an earlier analysis by D. Jido et al. - Nucl. Phys. A 725 (2003) 181

## Yet another $\bar{K}N$ model (preliminary!)

a new **Prague model** developed together with P. Bruns

- based on effective chiral Lagrangian (manifestly Lorentz invariant)
- improved treatment of the Born terms
- $\eta\Lambda$  and  $\eta\Sigma^0$  cross sections included to cover the  $\Lambda(1670)$  region
- 12 parameters fitted to the data:  $b_0$ ,  $b_F$ , 4  $d$ 's, 6 inverse ranges defining the Yamaguchi form factors that regularize the loop function integrals

fit quality:  $\chi^2/\text{dof} \approx 1.3$

**Pole positions (isoscalar sector):**

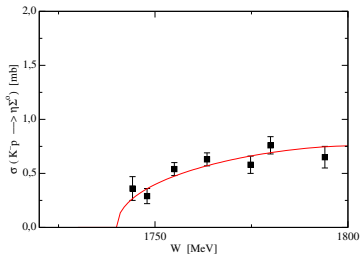
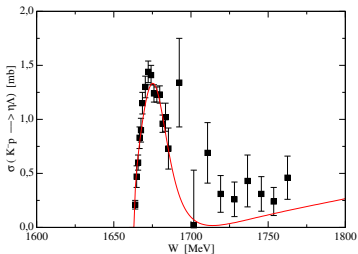
$$\Lambda(1405) \ z_{\pi\Sigma}[-, +, +, +] = (1353, -43) \text{ MeV}$$

$$\ z_{\bar{K}N}[-, +, +, +] = (1428, -24) \text{ MeV}$$

$$\Lambda(1670) \ z_{K\Xi}[-, -, -, +] = (1677, -14) \text{ MeV}$$

# Yet another $\bar{K}N$ model (preliminary!)

$\eta\Lambda$  and  $\eta\Sigma^0$  data reproduction



**Our aim:** study the SU(3) flavor structure of the poles assigned to  $\Lambda(1405)$  and  $\Lambda(1670)$  in some detail

## Summary

- The up-to-date (NLO) chirally motivated  $\bar{K}N$  models provide **very different predictions for the  $K^-N$  amplitudes at subthreshold energies.**
- **Two  $\Lambda(1405)$  poles in the chiral models.** Pole movements on the complex energy manifold give us additional insights on the the dynamically generated meson-baryon resonances. The **origin of the poles can be tracked to the nonzero diagonal inter-channel couplings.**
- Available experimental data can be reproduced about equally well by models with just one pole in the  $\Lambda(1405)$  region.
- Despite our progress over the last 60 years, the character of  $\Lambda(1405)$  resonance is still not reliably established.

**Have a peaceful holidays!**