Calculations of Λnn, ${}^{3}_{\Lambda}\mathrm{H}^{*}$, and ${}^{5}_{\Lambda}\mathrm{He}$ at LO $\not\pi$EFT

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Why *s*-shell hypernuclei ?

What we have ?

- experimentaly observed more than 30 Λ-hypernuclei and three well-established ΛΛ-hypernuclei (emulsion experiments)
  $\rightarrow$ available experimental B_Λ separation energies
  $\rightarrow$rather precise spectroscopic Λ-hypernucear data
  (for p-shell hypernuclei extremely precise)

on the other hand ...

- scarce ΛN scattering data
  $\rightarrow$ large theoretical model dependencies
- femtoscopy

s-shell hypernuclei

- precise B_Λ separation energies $B_\Lambda({}^{A}_{\Lambda}\mathrm{X}) = B({}^{A}_{\Lambda}\mathrm{X})$-$B({}^{A-1}\mathrm{X})$
- few-body character of these systems makes easier to track effects of underlying hyperon-nucleon(s) interactions
- ${}^{5}_{\Lambda}\mathrm{He}$ overbinding problem, $\Lambda N - \Sigma N$ mixing, charge symmetry breaking
- discussed $\Lambda\mathrm{nn}$, ${}^{3}_{\Lambda}\mathrm{H}^{*}$, $\Lambda\Lambda\mathrm{n}$, $\Lambda\Lambda\mathrm{nn}$
- question of bound ${}^{5}_{\Lambda\Lambda}\mathrm{He}$ (J-PARC P75 proposal) and ${}^{4}_{\Lambda\Lambda}\mathrm{H}(1^{+})$

YN scattering data

- cross-section datapoints for $p_{\mathrm{lab}} \gtrsim 100$ MeV
  - 12 d.p. for $\Lambda + p \to \Lambda + p$
  - 22 d.p. for $\Sigma^- + p \to \Lambda + n$, $\Sigma^+ + p \to \Sigma^+ + p$, $\Sigma^- + p \to \Sigma^- + p$, and $\Sigma^- + p \to \Sigma^0 + n$
- no information regarding spin-depence

- **Alexander et al.** (PR173, 1452, 1968)
  $a_{\Lambda N}(^1S_0)$=-1.8 fm
  $a_{\Lambda N}(^3S_1)$=-1.6 fm
- **Sechi-Zorn et al.** (PR175, 1735, 1968)
  $0 > a_{\Lambda N}(^1S_0) > -9.0$ fm
  $-0.8 > a_{\Lambda N}(^3S_1) > -3.2$ fm

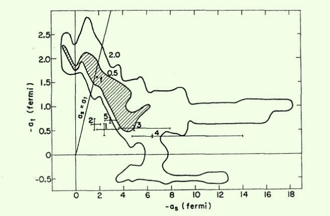


Fig. 9. Mapping of the likelihood function L in the a_s-a_t plane for the four-parameter fit. The shaded area includes all points with likelihood values above L_{max}/exp0.5, where L_{max} is the value of the best fit (point f). The external smooth curve encloses likelihood values lying above L_{max}/exp2.0. Points 1–5 represent scattering lengths derived from early hypernuclei calculations.

Hypernucler trios ${}^3_\Lambda\mathrm{H}$, ${}^3_\Lambda\mathrm{H}^*$, $\Lambda\mathrm{nn}$ - physical motivation

${}^3_\Lambda\mathrm{H}$

- lightest bound hypernucleus with $1/2^+$ spin-parity g.s.

  $\rightarrow$ established from hypertriton weak-decay measurements

$R_3 = \frac{\Gamma({}^3_\Lambda\mathrm{H} \rightarrow \pi^- + {}^3\mathrm{He})}{\Gamma_{\pi^-}({}^3_\Lambda\mathrm{H})} = 0.35 \pm 0.04$

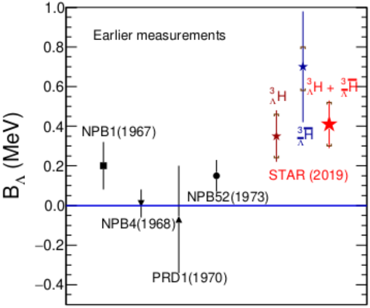
(G. Keyes et al., NPB67, 269, 1973)

$J^\pi = 1/2^+$ requires R_3 about 0.4
$J^\pi = 3/2^+$ requires R_3 about 0.1
(Bertrand et al., NPB16, 77, 1970)

Experimental status of $B_\Lambda({}^3_\Lambda\mathrm{H})$

Bubble chamber experiments :

- $B_\Lambda({}^3_\Lambda\mathrm{H})$ extracted from 4 different sets of bubble chamber data
- widely accepted value
  $B_\Lambda^{\mathrm{BC}}({}^3_\Lambda\mathrm{H}) = 0.13 \pm 0.05$ MeV
  (M. Jurić et al., NPB52, 1, 1973)



STAR Collaboration measurement :

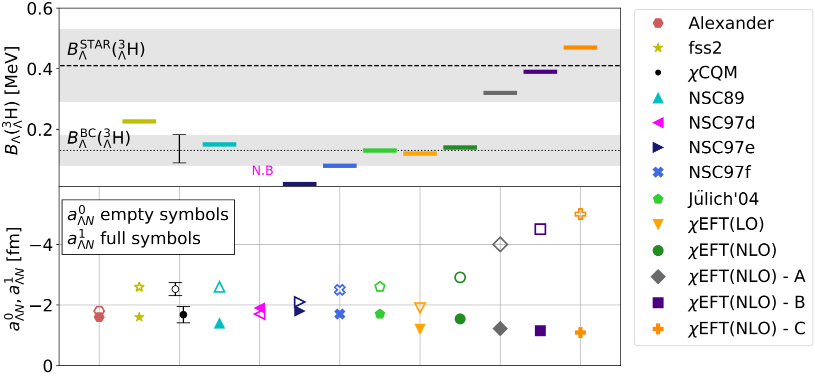
- HI collisions (Au+Au @$\sqrt{s_{NN}} = 200$ GeV)
- $B_\Lambda^{\mathrm{STAR}}({}^3_\Lambda\mathrm{H}) = 0.41 \pm 0.12(\mathrm{stat.}) \pm 0.11(\mathrm{syst.})$ MeV
  (STAR Collaboration, Nat. Phys. 16, 409, 2020)

$+$ **Hypertriton lifetime constraints :**

- $B_\Lambda({}^3_\Lambda\mathrm{H}) \lesssim 0.1$ MeV (ALICE); $B_\Lambda({}^3_\Lambda\mathrm{H}) \gtrsim 0.2$ MeV (STAR)
  (A. Pérez-Obiol et al., PLB811, 135916, 2020)

$B_\Lambda({}^3_\Lambda\mathrm{H})$ as a constraint in *YN* interaction models

Hypertriton serves as a highly important constraint in YN models !!



(Alexander et al., PR173, 1452, 1968; Y. Fujiwara et al., PRC77, 027001, 2008; H. Garcilazo et al. PRC75, 034002, 2007; A. Nogga, NPA914, 140, 2013; H. Le et al., PLB801, 135189, 2020)

Hypernucler trios ${}^3_\Lambda$H, ${}^3_\Lambda$H*, Λnn - physical motivation

${}^3_\Lambda$H($1/2^+$)

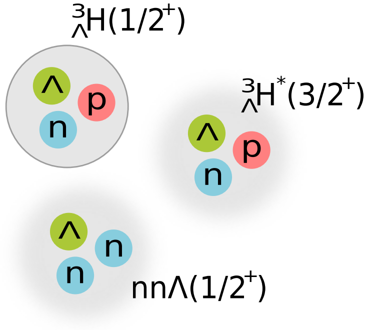
- lightest bound hypernucleus
- currently no experimental consensus on its B_Λ
- constraint in ΛN interaction models

${}^3_\Lambda$H*($3/2^+$)

- no experimental evidence
- strict constraint on ΛN $S = 1$ interaction
- JLab C12-19-002 proposal

Λnn($1/2^+$)

- experiment (HypHI)
- JLab E12-17-003 experiment
- valuable source of Λn interaction
- structure of neutron-rich Λ-hypernuclei



Outline of this talk

Nature of the $\Lambda\mathrm{nn}$ and ${}^{3}_{\Lambda}\mathrm{H}^{*}$ states:

- (hyper)nuclear LO $\not\pi$EFT

  (L. Contessi, N. Barnea, and A. Gal, PRL121, 102502, 2018)

- few-body methods
- results for $\Lambda\mathrm{nn}$ and ${}^{3}_{\Lambda}\mathrm{H}^{*}$

  (M. Schäfer, B. Bazak, N. Barnea, J. Mareš, PLB808, 135614, 2020)

  (M. Schäfer, B. Bazak, N. Barnea, J. Mareš, arXiv:2007.10264 [nucl-th], 2020)

Implications of increased $B_{\Lambda}({}^{3}_{\Lambda}\mathrm{H})$:

- ${}^{5}_{\Lambda}\mathrm{He}$, $\Lambda\mathrm{nn}$, ${}^{3}_{\Lambda}\mathrm{H}^{*}$

  (manuscript in preparation)

Nature of the Λnn and ${}^{3}_{\Lambda}H^*$ states

Λnn and ${}^3_\Lambda H^*$ - early work

- **R. H. Dalitz, B. W. Downs** (PR110, 958, 1958; PR111, 967, 1958; PR114, 593, 1959)
  $\rightarrow$ first calculation, variational approach, unbound Λnn

- **H. Garcilazo** (J. Phys. G: Nucl. Phys. 13, 63, 1987)
  $\rightarrow$ Faddeev equations, separable potentials, unbound Λnn

- **K. Miyagawa et al.** (PRC51, 2905, 1995)
  $\rightarrow$ Faddeev equations, realistic Nijmegen interaction, unbound Λnn and ${}^3_\Lambda H^*$

- **H. Garcilazo et al.** (PRC75, 034002, 2007; PRC76, 034001, 2007)
  $\rightarrow$ Faddeev equations, Chiral Quark Model ($N\Lambda - N\Sigma$ coupling, tensor force)
  $\rightarrow$ unbound Λnn
  $\rightarrow$ constraints on $a_{\Lambda N}^{S=0}$, $a_{\Lambda N}^{S=1}$ from ${}^3_\Lambda H$, unbound ${}^3_\Lambda H^*$, and Λp data

- **V. B. Belyaev et al.** (NPA803, 210, 2008)
  $\rightarrow$ first resonance calculation, 3-body Jost function, phenomenological potential
  $\rightarrow$ Λnn pole just above/below the threshold, large widths

$\Lambda\mathrm{nn}$ and ${}^3_\Lambda\mathrm{H}^*$ - current status

HypHI Collaboration (PRC88, 041001(R), 2013)

→ suggestion of bound $\Lambda\mathrm{nn}$, ${}^6\mathrm{Li} + {}^{12}\mathrm{C}$ @ 2A GeV
→ invariant mass distribution $d + \pi^-(\Lambda nn \to t^* + \pi^- \to d + n + \pi^-)$
→ invariant mass distribution $t + \pi^-(\Lambda nn \to t + \pi^-)$

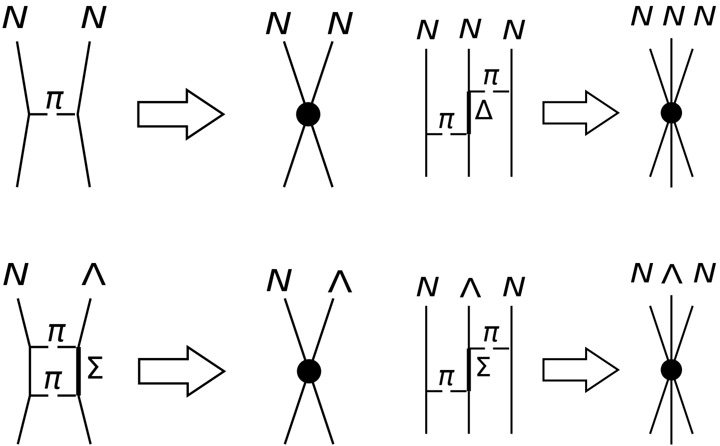
- **E. Hiyama et al.** (PRC89, 061302(R), 2014)
  → YN model equivalent to NSC97f; changing ${}^3V^T_{N\Lambda-N\Sigma}$, ${}^0V_{NN}$ to bind $\Lambda\mathrm{nn}$
  → nonexistence of bound $\Lambda\mathrm{nn}$ (${}^3_\Lambda\mathrm{H}$, ${}^3_\Lambda\mathrm{H}^*$, ${}^4_\Lambda\mathrm{H}$, ${}^3\mathrm{H}$)

- **A. Gal, H. Garcilazo** (PLB736, 93, 2014)
  → Faddeev equations, separable potentials
  → nonexistence of bound $\Lambda\mathrm{nn}$ ($\sigma_{\Lambda p}$, ${}^3_\Lambda\mathrm{H}$, and ${}^4_\Lambda\mathrm{H}$ exc. energy)

- **I. R. Afnan, B. F. Gibson** (PRC92, 054608, 2015)
  → Faddeev equations, $\Lambda\mathrm{nn}$ resonance calculations, separable potentials
  → subthreshold (non-physical) $\Lambda\mathrm{nn}$ resonance

Single-Λ LO $\not{\pi}$EFT - basic idea

(L. Contessi, N. Barnea, and A. Gal, Phys. Rev. Lett. 121, 102502, 2018)



Single-Λ LO $\not\pi$EFT

(L. Contessi, N. Barnea, and A. Gal, Phys. Rev. Lett. 121, 102502, 2018)

Regularization/Renormalization

$$C\ \delta(\mathrm{r}_{ij}) \rightarrow C(\lambda)\left(\frac{\lambda}{2\sqrt{\pi}}\right)^{3} \mathrm{e}^{\frac{-\lambda^{2} r_{ij}^{2}}{4}}$$

$$D\ \delta(\mathrm{r}_{ij})\delta(\mathrm{r}_{jk}) \rightarrow D(\lambda)\left(\frac{\lambda}{2\sqrt{\pi}}\right)^{6} \mathrm{e}^{\frac{-\lambda^{2}(r_{ij}^{2}+r_{jk}^{2})}{4}}$$

- $C(\Lambda), D(\lambda)$ are low energy constants (LECs) tuned to reproduce two-body resp. three-body observables for each λ
- required (RG invariance for $\lambda >> M$)
  $\rightarrow$ all observable will become λ independent when $\lambda \rightarrow \infty$

  $O_\lambda = O_\infty + \frac{\alpha}{\lambda} + \frac{\beta}{\lambda^2} + \frac{\gamma}{\lambda^3} + \ldots$

Single-Λ LO $\not\pi$EFT - fitting low energy constants

- 8 LEC constants (3 nuclear and 5 hypernuclear) for each value of cut-off λ which we fit using available experimental data

Nuclear LECs :

C_1	NN	$S=0\ I=1$	$a_{NN}(^1S_0)$	$=$ -18.63 fm
C_2	NN	$S=1\ I=0$	$B(^2\mathrm{H})$	$=$ 2.22452 MeV
D_1	NNN	$S=\frac{1}{2}\ I=\frac{1}{2}$	$B(^3\mathrm{H})$	$=$ 8.482 MeV

Hypernuclear LECs :

C_3	ΛN	$S=0\ I=\frac{1}{2}$	$a_{\Lambda N}(^1S_0)$	
C_4	ΛN	$S=1\ I=\frac{1}{2}$	$a_{\Lambda N}(^3S_1)$	
D_2	ΛNN	$S=\frac{1}{2}\ I=0$	$B_\Lambda(^3_\Lambda\mathrm{H})$	$=$ 0.13(5) MeV
D_3	ΛNN	$S=\frac{3}{2}\ I=0$	$B_\Lambda(^4_\Lambda\mathrm{H}, 0^+)$	$=$ 2.16(8) MeV
D_4	ΛNN	$S=\frac{1}{2}\ I=1$	$E_{ex}(^4_\Lambda\mathrm{H}, 1^+)$	$=$ 1.09(2) MeV

Single-Λ LO $\not\pi$EFT - ΛN scattering data

ΛN **interaction models** (Rev. Mod. Phys.88, 035004, 2016)

Model	$a_{\Lambda N}(^1S_0)$	$r^{eff}_{\Lambda N}(^1S_0)$	$a_{\Lambda N}(^3S_1)$	$r^{eff}_{\Lambda N}(^3S_1)$
NSC89	-2.79	2.89	-1.36	3.18
NSC97e	-2.17	3.22	-1.84	3.17
NSC97f	-2.60	3.05	-1.71	3.33
ESC08c	-2.54	3.15	-1.72	3.52
Jülich '04	-2.56	2.75	-1.66	2.93
χEFT(LO)	-1.91	1.40	-1.23	2.20
χEFT(NLO)	-2.91	2.78	-1.54	2.27

Experimental data

- Alexander et al. (PR173, 1452, 1968)
  $a_{\Lambda N}(^1S_0)$=-1.8 fm
  $a_{\Lambda N}(^3S_1)$=-1.6 fm
  (Alexander)

Stochastic Variational Method

(K. Varga et al., NPA571, 447, 1994)
(K. Varga, Y. Suzuki, PRC52, 2885, 1995)

optimizes variational basis in a **random trial and error procedure**

Variational basis states

- antisymmetrized correlated Gaussians (assuming L=0)

$$\psi_{SM_S TM_T}(\mathbf{x}, A) = \mathcal{A}\{G_A(\mathbf{x})\chi_{SM_S}\eta_{TM_T}\}, \quad G_A(\mathbf{x}) = e^{-\frac{1}{2}\mathbf{x}A\mathbf{x}}$$

- Jacobi coordinates **x**, symmetric positive definite matrix of variational parameters A, spin χ_{SM_S} and isospin η_{TM_T} parts
- $\frac{N(N-1)}{2}$ real parameters for one basis state
- explicit antisymmetrization → computational complexity grows with N!

$$\mathcal{A} = \sum_{i=1}^{N!} p_i \mathcal{P}_i$$

Complex Scaling Method (CSM)

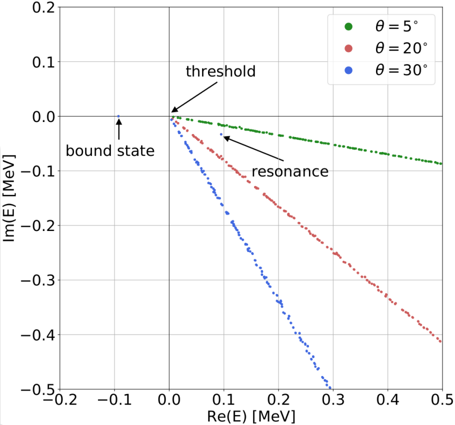
CSM transformation :

$$U(\theta)\ \mathrm{r}\ U^{-1}(\theta) = \mathrm{r}\ e^{\mathrm{i}\theta}$$

$$H^{\theta}\psi^{\theta} = E^{\theta}\psi^{\theta}$$

ABC theorem :

1. Energies of bound states remain unchanged.
2. Continuum spectra rotate clokwise by angle 2θ from the real axis with its centre at respective threshold.
3. For $\theta > \theta_r = \arctan\left(\frac{\gamma_r}{\kappa_r}\right)$ corresponding to the resonance pole $k_r = \kappa_r - \mathrm{i}\gamma_r$, the resonance is described by square-integrable functions and it is not changed by the complex scaling anymore.



→ **direct method :**
$E_r = \mathrm{Re}(E^{\theta})$ and $\Gamma = -2\mathrm{Im}(E^{\theta})$

→ for Gaussian potential $\theta < 45^{\circ}$

E. Balslev and J.M. Combes
(Commun. Math. Phys. 22, 280, 1971)

Inverse analytic continuation in the coupling constant (IACCC)

(V. I. Kukulin et al., *Theory of resonances*, Springer, 1988)
(J. Horáček and L. Pichl, Commun. Comput. Phys. 21, 1154, 2017)

Auxiliary short range interaction :

$$H \to H^{phys} + V^{IACCC} = H^{phys} + \delta U$$

$\to$ for certain δ_0 the system starts to be bound

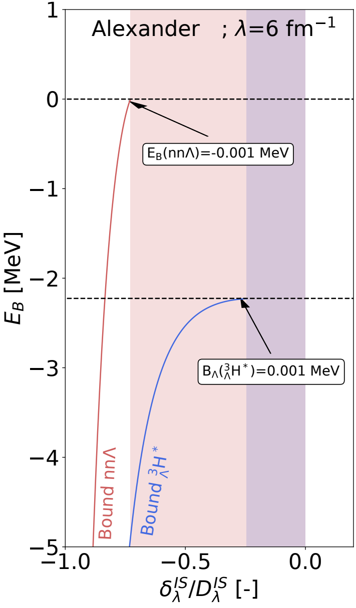
Padé approximation :
$\to$ calculation of bound state energies E_i for several values of δ_i
$\to$ construction of Padé approximant using $\delta_i(\kappa_i)$

$$\delta(\kappa) \approx \frac{P_M}{Q_N} = \frac{\sum_{i=0}^{M} c_i \kappa^i}{1 + \sum_{j=0}^{N} d_i \kappa^i}; \qquad \kappa = -\mathrm{i}k = -\mathrm{i}\sqrt{E}$$

Physical solution :
$\to$ search for roots of a polynomial

$$\delta(\kappa) = 0 \quad \Longrightarrow \quad \sum_{i=0}^{M} c_i \kappa^i = 0$$

Evolution of $\Lambda\mathrm{nn}$ and ${}^3_\Lambda\mathrm{H}^*$ pole with 3-body force



Hamiltonian of $\Lambda\mathrm{nn}$ and ${}^3_\Lambda\mathrm{H}^*$ systems:

$$H = \overbrace{T_k + V_2 + V_3}^{H^{phys}} + V_3^{IACCC}$$

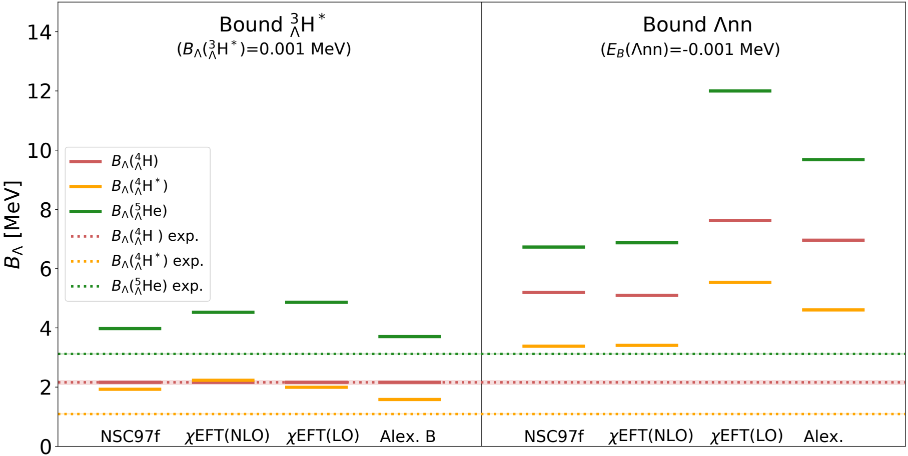
$$V_3 = D^{IS}_\lambda \sum_{i<j<k} \mathcal{Q}^{IS}_{ijk} \sum_{\mathrm{cyc}} e^{-\frac{\lambda^2}{4}(\mathrm{r}_{ij}^2+\mathrm{r}_{jk}^2)}$$

$$V_3^{IACCC} = \delta^{IS}_\lambda \sum_{i<j<k} \mathcal{Q}^{IS}_{ijk} \sum_{\mathrm{cyc}} e^{-\frac{\lambda^2}{4}(\mathrm{r}_{ij}^2+\mathrm{r}_{jk}^2)}$$

Three important points $(\lambda >> 2\mathrm{m}_\pi)$**:**

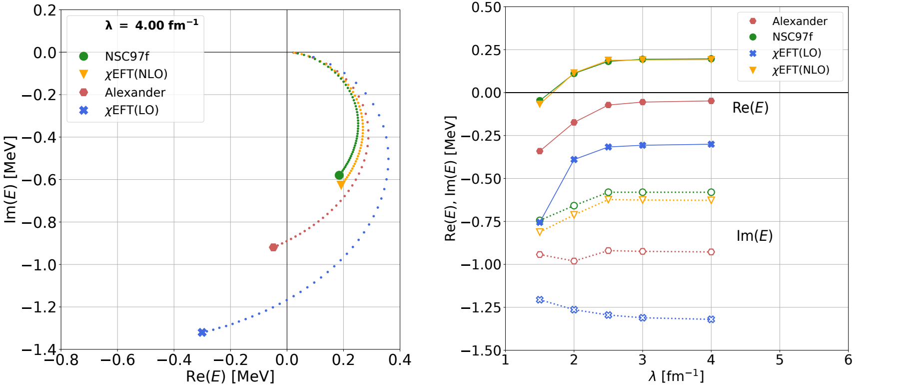
1. **No three-body force** $\quad \delta^{IS}_\lambda / D^{IS}_\lambda = -1$
   →Thomas collapse
2. **Just bound $\Lambda\mathrm{nn}$ or ${}^3_\Lambda\mathrm{H}^*$** $\quad \delta^{IS}_\lambda / D^{IS}_\lambda = ?$
   →implications to 4 and 5-body s-shell hypernuclei ?
3. **Physical Hamiltonian** $\quad \delta^{IS}_\lambda / D^{IS}_\lambda = 0$
   → zero V_3^{IACCC} force
   → $\Lambda\mathrm{nn}$, ${}^3_\Lambda\mathrm{H}^*$ resonances, virtual states ?

Implications of just bound Λnn and ${}^3_\Lambda\mathrm{H}^*$ ($\lambda = 6\ \mathrm{fm}^{-1}$)



- $B_\Lambda({}^3_\Lambda\mathrm{H})$ is used to fix three-body force in $I, S = 0, 1/2$ channel and remains unaffected

Resonance in Λnn system



- Λnn resonance pole moves with increasing cut-off towards physical Riemann sheet

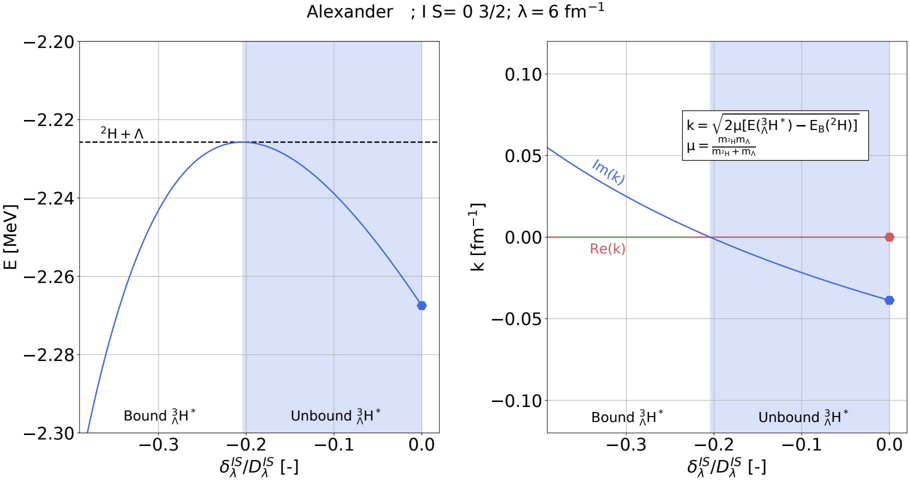
Where does ${}^{3}_{\Lambda}\mathrm{H}^*$ pole go ?

CSM

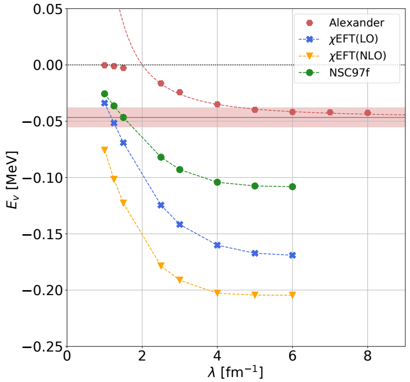
→ no sign of resonance

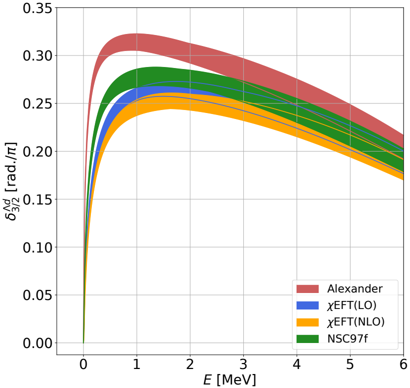
IACCC

→ $\delta(\kappa)$, $\kappa = -\mathrm{i}k = -\mathrm{i}\sqrt{E}$



Excited state of hypertriton ${}^3_\Lambda\mathrm{H}^*$ as a virtual state





- ${}^3_\Lambda\mathrm{H}^*$ virtual state solution for all considered cut-offs and scattering lengths
- convergence of ${}^3_\Lambda\mathrm{H}^*$ virtual state pole with increasing cut-off
- at LO $\not\pi$EFT there is a virtual state lying from 0.02 up to 0.25 MeV near the ${}^2\mathrm{H}+\Lambda$ threshold

Implications of increased $B_\Lambda(^3_\Lambda\mathrm{H})$

Implications of increased $B_\Lambda({}^3_\Lambda\mathrm{H})$

What we want to know ?

- consistency of increased $B_\Lambda({}^3_\Lambda\mathrm{H})$ with respect to experimentally measured properties of 4, 5, and higher-body hypernuclei
- so far not experimentally observed $\Lambda\mathrm{nn}$ or ${}^3_\Lambda\mathrm{H}^*(\frac{3}{2}^+)$ systems

H. Le et al. (PLB801, 135189, 2020)

$\rightarrow$ 3 version of χEFT(NLO) interaction (A,B,C) each constrained by $B_\Lambda^{\mathrm{STAR}}({}^3_\Lambda\mathrm{H})$
$\rightarrow$ study of ${}^4_\Lambda\mathrm{He}$ and ${}^7_\Lambda\mathrm{Li}$
$\rightarrow$ larger $a^0_{\Lambda N}$, smaller $a^1_{\Lambda N}$
$\rightarrow$ **no principle reason against larger $B_\Lambda({}^3_\Lambda\mathrm{H})$!**

In this work (preliminary) :

$\rightarrow$ LO (hyper)nuclear $\not\pi$EFT
$\rightarrow$ large range of ΛN scattering lengths; $B_\Lambda^{\mathrm{STAR}}({}^3_\Lambda\mathrm{H})$
$\rightarrow$ implications to ${}^5_\Lambda\mathrm{He}$, $\Lambda\mathrm{nn}$, and ${}^3_\Lambda\mathrm{H}^*$

Single-Λ LO $\not\pi$EFT - low energy constants

- 8 LEC constants (3 nuclear and 5 hypernuclear) for each value of cut-off λ which we fit using available experimental data

Nuclear LECs :

C_1	NN	$S=0$ $I=1$	$a_{NN}(^1S_0)$	= -18.63 fm
C_2	NN	$S=1$ $I=0$	$B(^2\mathrm{H})$	= 2.22452 MeV
D_1	NNN	$S=\frac{1}{2}$ $I=\frac{1}{2}$	$B(^3\mathrm{H})$	= 8.482 MeV

Hypernuclear LECs :

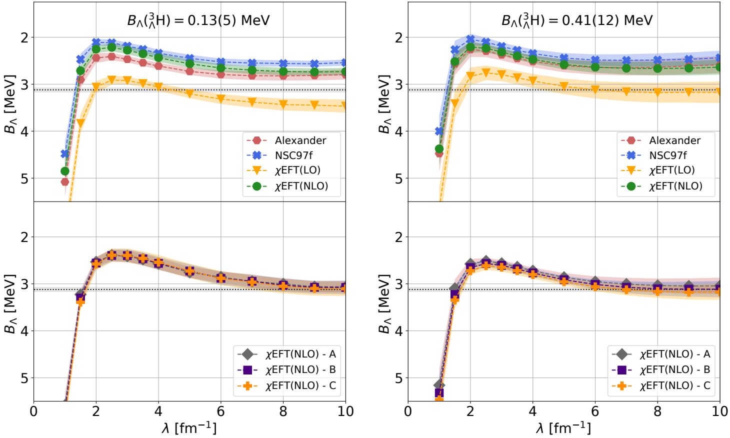
C_3	ΛN	$S=0$ $I=\frac{1}{2}$	$a_{\Lambda N}(^1S_0)$	$\left(\begin{array}{l}\text{Alexander, NSC97f, }\chi\text{EFT(LO),}\\ \chi\text{EFT(NLO), }\chi\text{EFT(NLO)}-\text{A, B, C}\end{array}\right)$
C_4	ΛN	$S=1$ $I=\frac{1}{2}$	$a_{\Lambda N}(^3S_1)$	
D_2	ΛNN	$S=\frac{1}{2}$ $I=0$	$B_\Lambda(^3_\Lambda\mathrm{H})$	= 0.13(5) MeV or 0.41(12) MeV
D_3	ΛNN	$S=\frac{3}{2}$ $I=0$	$B_\Lambda(^4_\Lambda\mathrm{H},0^+)$	= 2.16(8) MeV
D_4	ΛNN	$S=\frac{1}{2}$ $I=1$	$E_{ex}(^4_\Lambda\mathrm{H},1^+)$	= 1.09(2) MeV

Predictions : $\Lambda\mathrm{nn}$ $(J^\pi=\frac{1}{2}^+; I=1)$, $^3_\Lambda\mathrm{H}^*$ $(J^\pi=\frac{3}{2}^+; I=0)$, $^5_\Lambda\mathrm{He}$ $(J^\pi=\frac{1}{2}^+; I=0)$

${}^{5}_{\Lambda}\mathrm{He}$

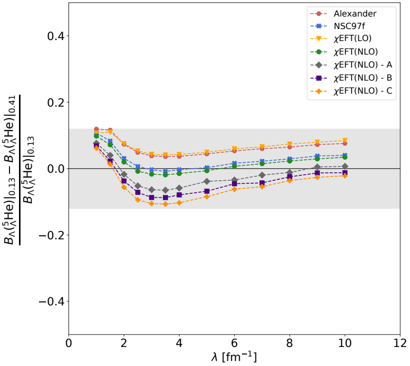
shaded areas - propagated exp. errors in $B_\Lambda({}^{3}_{\Lambda}\mathrm{H})$, $B_\Lambda({}^{4}_{\Lambda}\mathrm{H}; 0^+)$, and $E_{ex}({}^{4}_{\Lambda}\mathrm{H}; 1^+)$ constraints

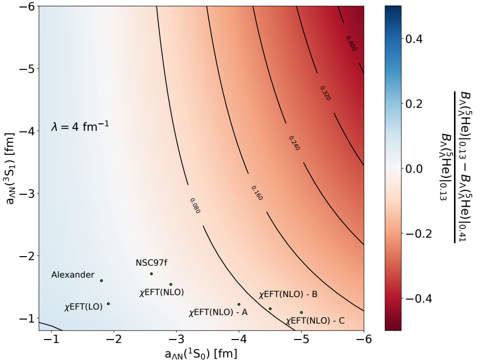
(Preliminary results - not fully converged $B_\Lambda({}^{5}_{\Lambda}\mathrm{He})$ for $\lambda \geq 7\ \mathrm{fm}^{-1}$)



→ both $B_\Lambda^{\mathrm{STAR}}({}^{3}_{\Lambda}\mathrm{H})$ and $B_\Lambda^{\mathrm{BC}}({}^{3}_{\Lambda}\mathrm{H})$ are acceptable with LO $\not{\pi}$EFT accuracy

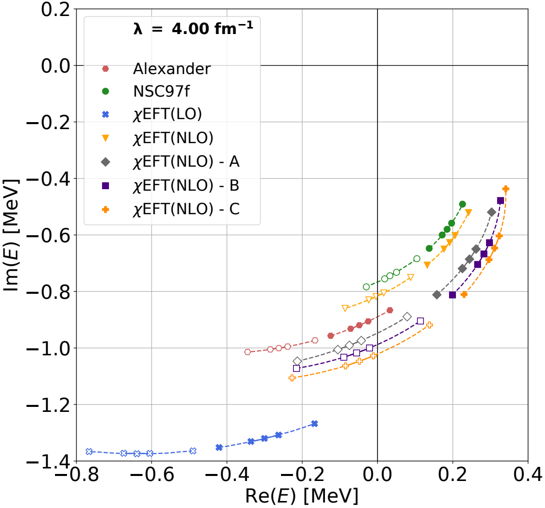
${}^{5}_{\Lambda}\mathrm{He}$





→ at LO $\not\pi$EFT the relative difference $\left|\frac{B_\Lambda({}^{5}_{\Lambda}\mathrm{He})|_{0.13} - B_\Lambda({}^{5}_{\Lambda}\mathrm{He})|_{0.41}}{B_\Lambda({}^{5}_{\Lambda}\mathrm{He})|_{0.13}}\right| \leq 12\%$ for all considered sets of ΛN scattering lengths and values of λ

$\Lambda\mathrm{nn}$ ($J^\pi = 1/2^+; I = 1$) resonance



- full symbols
  $B_\Lambda(^3_\Lambda\mathrm{H}) = 0.13(5)$ MeV
- empty symbols
  $B_\Lambda(^3_\Lambda\mathrm{H}) = 0.41(12)$ MeV

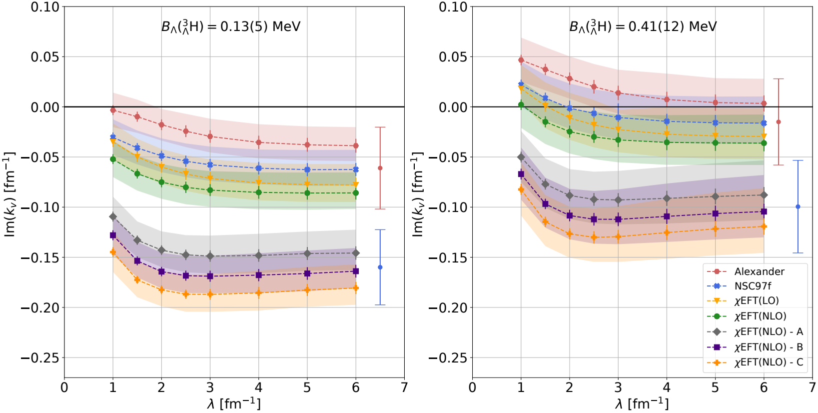
$\rightarrow$ increasing $B_\Lambda(^3_\Lambda\mathrm{H})$ shifts $\Lambda\mathrm{nn}$ resonance pole towards the third quadrant

$\rightarrow$ $B_\Lambda(^3_\Lambda\mathrm{H})$ experimental error yields considerable uncertainty in $E_{\Lambda\mathrm{nn}}$ prediction

$\rightarrow$
$\Gamma_{\Lambda\mathrm{nn}} = -2\mathrm{Im}(E_{\Lambda\mathrm{nn}}) \geq 0.8$ MeV

Excited state of the hypertriton ${}^3_\Lambda\mathrm{H}^*$ ($J^\pi = 3/2^+; I = 0$)

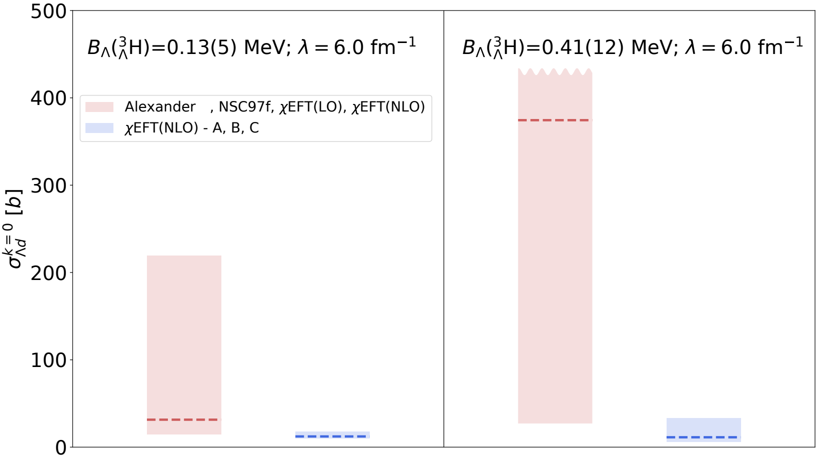
error bars - propagated exp. errors in $B_\Lambda({}^4_\Lambda\mathrm{H}; 0^+)$ and $B_\Lambda({}^4_\Lambda\mathrm{H}; 1^+)$ constraints
shaded areas - same as above plus exp. error in $B_\Lambda({}^3_\Lambda\mathrm{H})$ constraint



→ increasing $B_\Lambda({}^3_\Lambda\mathrm{H})$ moves ${}^3_\Lambda\mathrm{H}^*$ virtual state pole closer to the Λd threshold or into the bound state region

Λd scattering

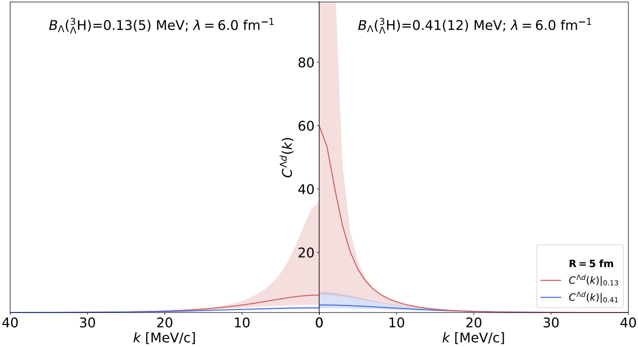
$$\sigma_{\Lambda d}^{k=0} = 4\pi\left[\frac{1}{3}A_{\Lambda d}^2(1/2^+) + \frac{2}{3}A_{\Lambda d}^2(3/2^+)\right] \simeq 4\pi\left[\frac{1}{3}\frac{1}{2\mu_{\Lambda d}B_\Lambda({}^3_\Lambda\mathrm{H};1/2^+)} + \frac{2}{3}\frac{1}{k_v^2(3/2^+)}\right]$$



Two-body Λd momentum correlation function $C^{\Lambda d}(k)$

- measured in HI or high energy *pp* collisions might provide information on hadron-hadron forces at low energies ($\Lambda\Lambda$, $p\Omega$, $\Omega\Omega$, $K^- d$)
- recently, pointed out that $C^{\Lambda d}(k)$ should be considered to study Λd and underlying ΛN interaction (J. Haidenbauer, PRC102, 034001, 2020)

→ LL model; $C(k) \simeq C(A, r, R; k)$ (R. Lednicky, V. L Lyuboshitz, SJNP770, 35, 1982) predicted $A_{\Lambda d}(1/2^+)$, $A_{\Lambda d}(3/2^+) + r_{\Lambda d}(1/2^+) = 3$ fm and $r_{\Lambda d}(3/2^+) = 4$ fm



Conclusions

- successful extension of SVM to unbound region using CSM and IACCC
- comprehensive study of the Λnn and ${}^{3}_{\Lambda}\mathrm{H}^*$ system in LO $\not\pi$EFT
  $\rightarrow$ various ΛN scattering lengths, three-body forces, connection to ${}^{4}_{\Lambda}\mathrm{H}$, ${}^{4}_{\Lambda}\mathrm{H}^*$, ${}^{5}_{\Lambda}\mathrm{He}$

$\Lambda\mathrm{nn}(\frac{1}{2}^+)$ - resonant state

- question of experimentally observable Λnn resonance (physical Riemann sheet)
- if Λnn just bound $\rightarrow$ serious disagreement with B_Λ^{exp} of 4- and 5-body hypernuclei

${}^{3}_{\Lambda}\mathrm{H}^*(\frac{3}{2}^+)$ - virtual state

- virtual state from 0.02 up to 0.25 MeV below the ${}^{2}\mathrm{H} + \Lambda$ threshold
- if ${}^{3}_{\Lambda}\mathrm{H}^*$ just bound $\rightarrow$ B_Λ of 4- and 5-body hypernuclei do not change dramatically

Conclusions

- comprehensive study of the ${}^{5}_{\Lambda}\mathrm{He}$, $\Lambda\mathrm{nn}$, and ${}^{3}_{\Lambda}\mathrm{H}^*$ systems within LO $\not\pi$EFT

  $\rightarrow$ various $a_{\Lambda N}$ sets, $B_{\Lambda}^{\mathrm{BC}}({}^{3}_{\Lambda}\mathrm{H}) = 0.13(5)$ MeV and $B_{\Lambda}^{\mathrm{STAR}}({}^{3}_{\Lambda}\mathrm{H}) = 0.41(12)$ MeV

At LO $\not\pi$EFT both $B_{\Lambda}^{\mathrm{BC}}({}^{3}_{\Lambda}\mathrm{H})$ and $B_{\Lambda}^{\mathrm{STAR}}({}^{3}_{\Lambda}\mathrm{H})$ are consistent with $B_{\Lambda}^{\mathrm{exp}}({}^{5}_{\Lambda}\mathrm{He}) = 3.12(2)$ MeV.

$\boldsymbol{\Lambda\mathrm{nn}(\frac{1}{2}^+)}$

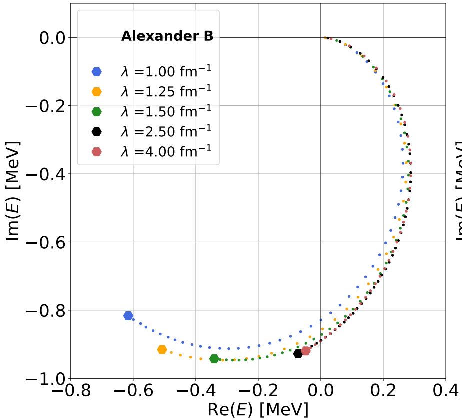
- increasing $B_{\Lambda}({}^{3}_{\Lambda}\mathrm{H})$ shifts $\Lambda\mathrm{nn}$ resonance pole towards the third quadrant of the complex energy plane $(\mathrm{Re}(E) < 0,\ \mathrm{Im}(E) < 0)$; $\Gamma_{\Lambda\mathrm{nn}} \geq 0.8$ MeV

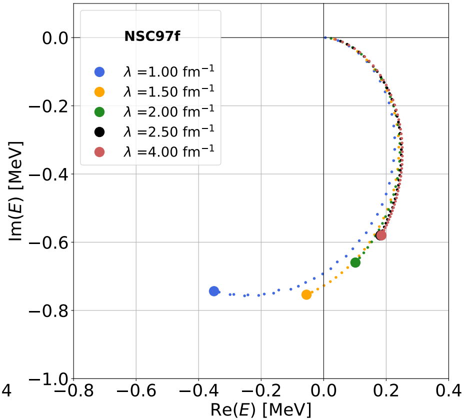
$\boldsymbol{{}^{3}_{\Lambda}\mathrm{H}^*(\frac{3}{2}^+)}$

- increasing $B_{\Lambda}({}^{3}_{\Lambda}\mathrm{H})$ pushes ${}^{3}_{\Lambda}\mathrm{H}^*$ virtual state pole closer to the Λd threshold or into the bound state region
- pole position sensitive to increase of the ΛN spin-singlet strength at the expense of the ΛN spin-triplet channel (measurement of $C^{\Lambda d}(k)$ is desirable)

Backup slides

Resonance in Λnn system - cut-off dependence





- pole of Λnn resonance moves with increasing cut-off towards physical Riemann sheet
- both physical positions and trajectories $E_{\Lambda nn}(\delta_\lambda^{I=1,S=1/2}, \lambda)$ converge with increasing cut-off

Phenomenological approach

I. R. Afnan and B. F. Gibson (PRC92, 054608, 2015)

- separable nonlocal potential fitted to reproduce ΛN spin-singlet/triplet scattering lengths and effective ranges of NSC97f, XEFTNLO, Julich'04, and Nijmegen D models
- $V(k,k') = g(k)\ C\ g(k')$, $g(k) = \frac{1}{k^2+\beta^2}$
- prediction for ${}^{3}_{\Lambda}\mathrm{H}$ and $\Lambda\mathrm{nn}$ resonance pole

→ **Let's try to follow the same path :**

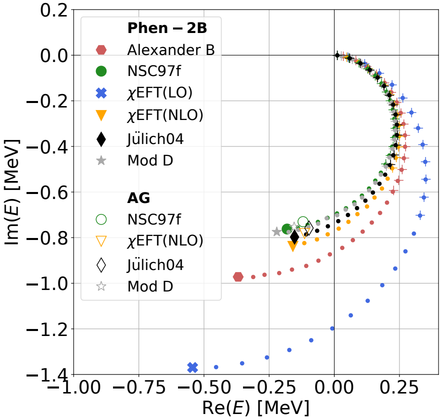
- single Gaussian fitted to reproduce NN and ΛN spin-singlet/triplet scattering lengths and effective ranges; prediction for $\Lambda\mathrm{nn}$ pole and B_Λ for ${}^{3}_{\Lambda}\mathrm{H}$, ${}^{3}_{\Lambda}\mathrm{H}^*$, ${}^{4}_{\Lambda}\mathrm{H}$, ${}^{4}_{\Lambda}\mathrm{H}^*$, and ${}^{5}_{\Lambda}\mathrm{He}$
- $V^{IS}(r) = C^{IS}\exp(-\frac{\lambda^{IS}}{4}r^2)$

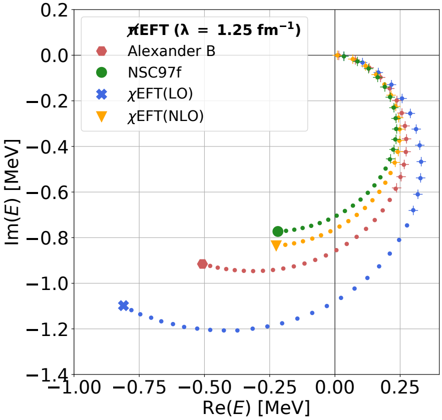
	$a^{S=0}$	$r^{S=0}$	$a^{S=1}$	$r^{S=1}$
Mod. D	-2.03	3.66	-1.84	3.32
Julich'04	-2.56	2.74	-1.67	2.93
XEFTNLO	-2.91	2.78	-1.54	2.75
NSC97f	-2.60	3.05	-1.72	3.32

	$a^{S=0}$	$r^{S=0}$	$a^{S=1}$	$r^{S=1}$
Alex. B	-1.80	2.80	-1.60	3.30
XEFTLO	-1.91	1.40	-1.23	2.20
NN	-18.90	2.75	5.42	1.75

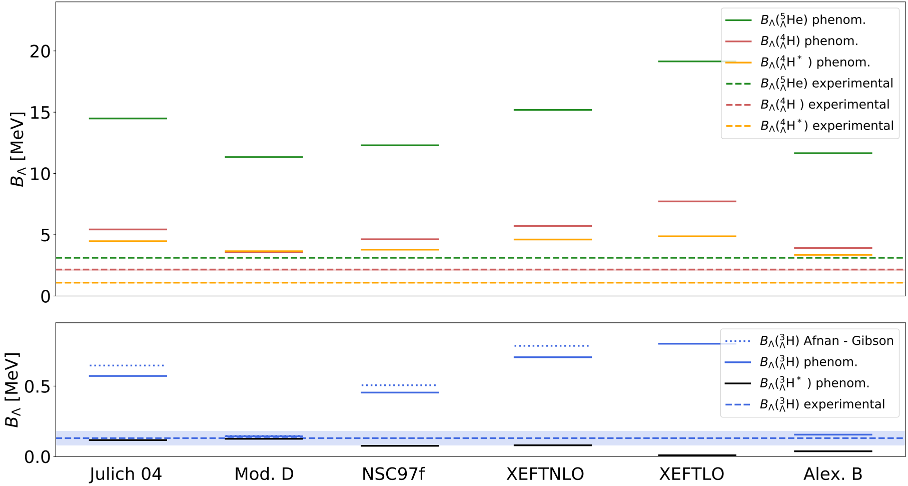
*all a^S, r^S in fm

Phenomenological approach - Λnn resonance





Phenomenological approach - s-shell hypernuclei



- reasonable reproduction of $B_\Lambda({}^3_\Lambda H)$ of Afnan and Gibson
- bound ${}^3_\Lambda H^*$ for all considered sets of scattering lengths and effective ranges
- 4 and 5-body hypernuclear systems severely overbound

LO truncation error

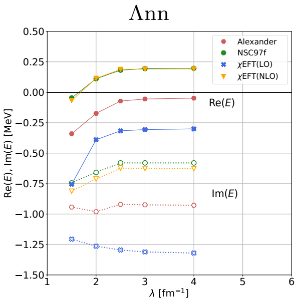
- small relevant energies of both $\Lambda\mathrm{nn}$ and ${}^3_\Lambda\mathrm{H}^*$
  $E \lesssim 1$ MeV

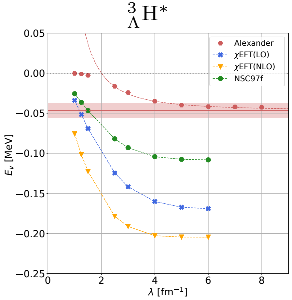
${}^3_\Lambda\mathbf{H}^*$

- Λ typical momentum $p_\Lambda \sim \sqrt{2\mu E} \approx 37$ MeV
  (μ is Λ-deuteron reduced mass)
- leading effective range correction $O(QR)$
  $\rightarrow$ for $R \sim r_s^{\Lambda N} \approx 2.5$ fm error $\approx 47\%$
- (p_Λ of $\Lambda\mathrm{nn}$ should be roughly the same)

Residual cutoff dependence:

- corrected by higher order terms
- λ dependence from $\lambda = 1.25\ \mathrm{fm}^{-1}$ to $\lambda \rightarrow \infty$





Few-body techniques

Bound states :

- Stochastic Variational Method with correlated Gaussian basis (SVM)

Continuum :

- resonances, virtual states
- two independent methods
  → Inverse Analytic Continuation in the Coupling Constant (IACCC)
  → Complex Scaling Method (CSM)

→ for more details on hyper(nuclear) $\not\pi$EFT and application of aforementioned few-body techniques see :

(L. Contessi, N. Barnea, A. Gal, PRL121, 102502, 2018)

(M. Schäfer, B. Bazak, N. Barnea, J. Mareš, PLB808, 135614, 2020)

(M. Schäfer, B. Bazak, N. Barnea, J. Mareš, arXiv:2007.10264 [nucl-th], 2020)

Auxiliary interaction in $\not\pi$EFT

→ **three-body force**

$$V_3^{IACCC} = \delta_\lambda^{IS} \sum_{i<j<k} \mathcal{Q}_{ijk}^{IS} \sum_{\text{cyc}} e^{-\frac{\lambda^2}{4}(r_{ij}^2+r_{jk}^2)}$$

→ cut-off λ consistent with H^{phys}

→ just one three-body force for each 3-body system
$^3_\Lambda\mathrm{H}$ $(I=0\ S=1/2)$
$\Lambda\mathrm{nn}$ $(I=1\ S=1/2)$
$^3_\Lambda\mathrm{H}^*$ $(I=0\ S=3/2)$

Advantages with respect to 2-body auxiliary interaction :

→ position of 2+1 threshold does not change with δ_λ^{IS} ($^2\mathrm{H}+\Lambda$ in $^3_\Lambda\mathrm{H}^*$)

→ for each value of δ_λ^{IS} respective two-body part of the Hamiltonian remains the same (two-body observables are δ_λ^{IS} independent)

→ we might expect RG invariance of the $\Lambda\mathrm{nn}/^3_\Lambda\mathrm{H}^*$ pole trajectory

$$\lim_{\lambda\to\infty} E(\delta_\lambda^{IS};\lambda) = E(\delta^{IS})$$

How to optimize the SVM basis ?

IACCC

→ required numerical precision of bound state energies $E_i(\delta_i)$ up to 4-5 digits

→ **increasing numerical accuracy of E_i :**

$$H_i = H^{phys} + \delta_i U$$

1. selecting several δ_i such that $E_{max} < E_i < E_{min}$
2. for each δ_i we select SVM basis minimizing respective total binding energy E_i
3. we merge basis states stochastically selected for each δ_i into final **SVM-IACCC** basis
4. for any δ_i such that $E_{max} < E_i < E_{min}$ we obtain E_i diagonalizing Hamiltonian H_i in the **SVM-IACCC** basis

Complex Scaling Method

→ **for resonances only stationary principle**

→ not possible to select basis states variationally minimizing the total binding energy of respective system

→ **bypassing the issue using HO trap :**

$$H_b = H^{phys} + V_b^{HO}$$

1. we select several HO trap lengths b
2. for each b we generate SVM basis minimizing energies of excited states in the HO trap
3. we merge basis states stochastically selected for each b into final **SVM-CSM** basis

Benchmark - resonance in 3-body bosonic system (1+1+1)

(N. Tanaka, Y. Suzuki and K. Varga, Phys. Rev. C56 (1997) 562)

Properties:

$\rightarrow \hbar^2/m = 41.47$

$$H = T_k + V + V^{IACCC}$$

Auxiliary 2-body potential :

$$V = -120e^{-r^2}, \quad V_{2b}^{IACCC} = 3\lambda e^{-\frac{r^2}{9}}$$

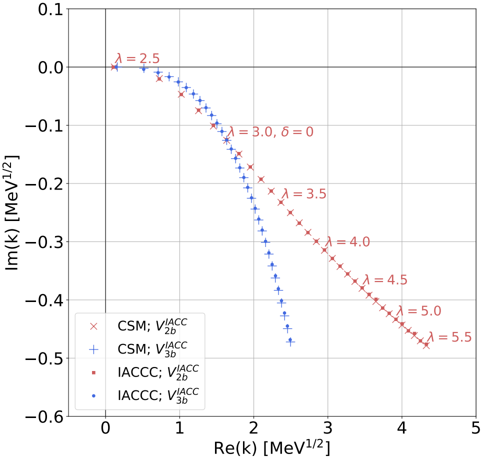
Auxiliary 3-body potential :

$$V = -120e^{-r^2} + 9e^{-\frac{r^2}{9}}$$

$$V_{3b}^{IACCC} = \delta \sum_{i<j<k} \sum_{cyc} e^{-0.25(r_{ij}+r_{jk})}$$

Results ($\lambda = 3$ and $\delta = 0$):

k(CSM)	1.6347 - i0.1259 $MeV^{1/2}$
k(IACCC; 2b)	1.6350 - i0.1247 $MeV^{1/2}$
k(IACCC; 3b)	1.6347 - i0.1253 $MeV^{1/2}$



* note that $k = \sqrt{E}$

Benchmark - resonance in 3-body bosonic system (2+1)

Properties:

→ m=938 MeV

$$V = -55e^{-0.2r^2} + 1.5e^{-0.01(r-5)^2}$$

→ 2-body and 3-body bound state

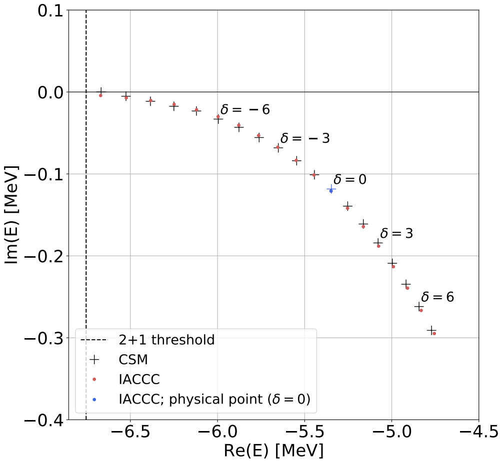
Auxiliary 3-body potential :

$$H = T_k + V + V^{IACCC}$$

$$V^{IACCC} = \delta \sum_{i<j<k} \sum_{cyc} e^{-0.25(r_{ij}+r_{jk})}$$

Results :

E(CSM)	-5.346 - i0.118 MeV
E(IACCC)	-5.348 - i0.121 MeV



Other available results :

E	-5.310 - i0.117 MeV	(Phys. Rev. A75 (2007) 042508)
E	-5.96 - i0.4 MeV	(Few-Body Syst. 33 (2003) 153)
E	-5.32(1) (width not given)	(Phys. Rev. C98 (2018) 034004)