Application of Equation of Motion Phonon Method for Calculations of Hypernuclei

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Motivation

Work on development of **many-body model**(s) suitable for description of **nuclear and hypernuclear structure** which aim to describe wide range of hypernuclei including medium-size & heavy systems.

This model can be used in calculations of **hypernuclear production** – especially the hypernuclei whose production is planned by experimentalists in close future (**$^{40}_{\Lambda}$K, $^{48}_{\Lambda}$K, $^{208}_{\Lambda}$Tl**).

Outline of the seminar:

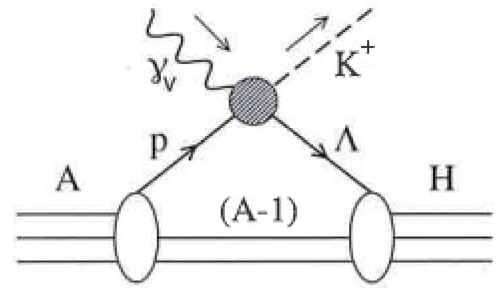
- **Electroproduction** of Hypernuclei
- **Equation of Motion Phonon Method** (EMPM) in the Calculations of **Nuclei**
- Extension of **EMPM** for the Calculations of **Hypernuclei**
- **Results** – EMPM Demonstrated on Calculations of **$^{5}_{\Lambda}$He, $^{17}_{\Lambda}$O, $^{16}_{\Lambda}$O, $^{16}_{\Lambda}$N**
- **Summary** and Future Plans

Electroproduction of Hypernuclei

Hypernuclear production:

$$\gamma_v(P_\gamma) + \mathrm{A}(P_\mathrm{A}) \longrightarrow \mathrm{H}(P_\mathrm{H}) + \mathrm{K}^+(P_\mathrm{K})$$

- Elementary process of electroproduction: **p (e,e' K$^+$) Λ**
- **Kinematics** of the reaction
- Information about (many-body) **nuclear & hypernuclear structure**
- Information about the **ΛN(N) interaction**s



All “ingredients“ important for description of hypernuclear production.

$$\frac{d^3\sigma}{dE'_e d\Omega'_e d\Omega_K} = \Gamma\left[\frac{d\sigma_U}{d\Omega_K} + \varepsilon_L \frac{d\sigma_L}{d\Omega_K} + \varepsilon \frac{d\sigma_P}{d\Omega_K}\cos 2\Phi_K + \sqrt{2\varepsilon_L(1+\varepsilon)}\,\frac{d\sigma_I}{d\Omega_K}\cos\Phi_K\right]$$

$$\frac{d\sigma_U}{d\Omega_K} = \frac{\beta}{2(2J_A+1)}\sum_{jm}\frac{1}{2j+1}\{|A^{+1}_{jm}|^2 + |A^{-1}_{jm}|^2\}\,,$$

$$\frac{d\sigma_P}{d\Omega_K} = -\frac{\beta}{2J_A+1}\sum_{jm}\frac{1}{2j+1}\mathrm{Re}\{A^{+1}_{jm}A^{-1*}_{jm}\}\,,$$

$$\frac{d\sigma_L}{d\Omega_K} = \frac{\beta}{2J_A+1}\sum_{jm}\frac{1}{2j+1}|A^{0}_{jm}|^2\,,$$

$$\frac{d\sigma_I}{d\Omega_K} = \frac{\beta}{2J_A+1}\sum_{jm}\frac{1}{2j+1}\mathrm{Re}\{A^{0*}_{jm}[A^{+1}_{jm} - A^{-1}_{jm}]\}$$

Transition amplitude

$$T^{(1)}_\lambda = \frac{Z}{[J_H]}\sum_{S\eta}\mathcal{F}^S_{\lambda\eta}\sum_{LM}\sum_{Jm}C^{Jm}_{LMS\eta}\,C^{J_H M_H}_{J_A M_A Jm}\,(\,J_H||\,F_{LM}[Y_L\otimes\sigma^S]^J\,||\,J_A\,)$$

Operator can be expressed in the second quantized form.

We evaluate $(\,\Phi_H\,||\,[b^+_{\alpha'}\otimes a_\alpha]^J\,||\,\Phi_A\,)$

hypernucleus

creation Λ

annihilation **p**

nucleus

Electroproduction of Hypernuclei

Hypernuclear production:

$$\frac{d^3\sigma}{dE'_e d\Omega'_e d\Omega_K} = \Gamma\left[\frac{d\sigma_U}{d\Omega_K} + \varepsilon_L \frac{d\sigma_L}{d\Omega_K} + \varepsilon \frac{d\sigma_P}{d\Omega_K}\cos 2\Phi_K + \sqrt{2\varepsilon_L(1+\varepsilon)}\frac{d\sigma_I}{d\Omega_K}\cos\Phi_K\right]$$

$$\mathrm{M}_\mu = \langle \Psi_\mathrm{H} | \langle \chi_\mathrm{K} | \sum_{j=1}^{Z} \hat{\mathrm{J}}_\mu(\mathrm{j}) \, | \chi_\gamma \rangle | \Psi_\mathrm{A} \rangle \qquad (\, \Phi_H \, || \, [b^+_{\alpha'} \otimes a_\alpha]^J \, || \, \Phi_A \,)$$

$$\gamma_\mathrm{v}(P_\gamma) + \mathrm{A}(P_\mathrm{A}) \longrightarrow \mathrm{H}(P_\mathrm{H}) + \mathrm{K}^+(P_\mathrm{K})$$

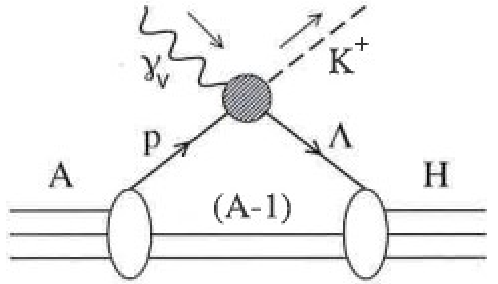
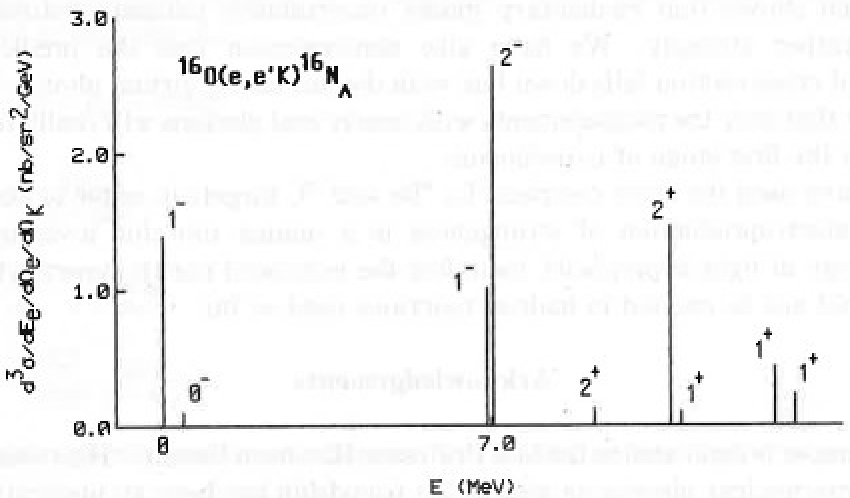


Fig. from M. **Sotona**, S. **Frullani**,
Prog. Theor. Phys. Suppl. 117, 151 (1994)

Study of the **effects** of the **ΛN(N) interactions** & the used **many-body model**...



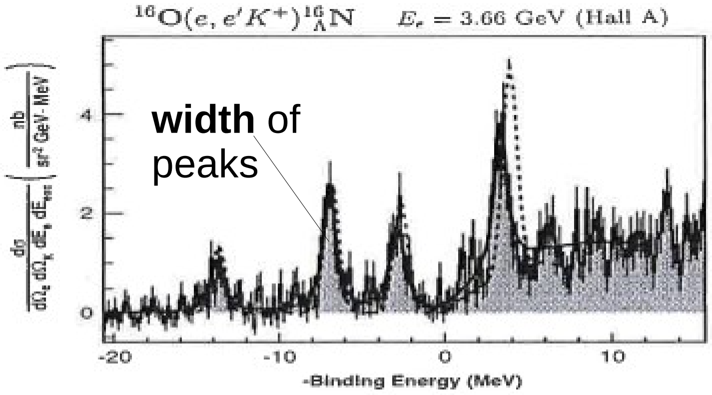


Fig. 17. Experimental spectrum of the $^{16}\mathrm{O}(e, e'K^+)^{16}_{\Lambda}\mathrm{N}$ reaction obtained at JLab Hall A. Taken from Ref. 7).

7) F. Cusanno et al., Phys. Rev. Lett. **103** (2009), 202501.

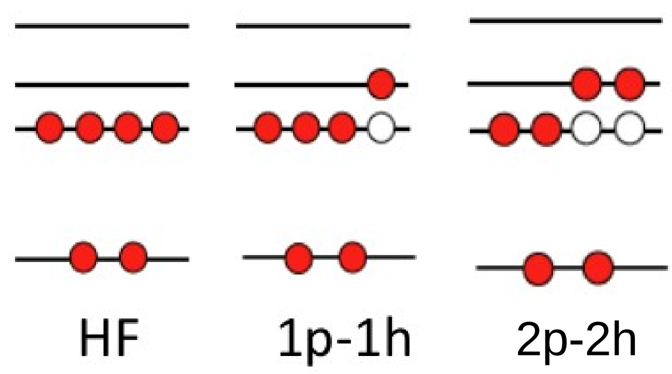
Equation of Motion Phonon Method

Equation of Motion Phonon Method (EMPM) was developed to study the structure of nuclei.

Hilbert space – divided into subspaces

$$\mathcal{H} = \mathcal{H}_0 \oplus \mathcal{H}_1 \oplus \mathcal{H}_2 \oplus ... \oplus \mathcal{H}_n$$

HF – Hartree-Fock state (nucleons occupy lowest single-particle levels)
1p-1h = 1**particle** – 1**hole** excitation of HF
2p-2h = 2**particle** – 2**hole** excitation of HF
.
.
np-nh = n**particle** – n**hole** excitation of HF



Instead of multiple **particle-hole** excitations we can excite multiple **TDA phonons**

$$\begin{aligned}
\mathcal{H}_0 &= \{|HF>\} \\
\mathcal{H}_1 &= \{O^\dagger_{\nu_1}|HF>\} \\
\mathcal{H}_2 &= \{O^\dagger_{\nu_1}O^\dagger_{\nu_2}|HF>\} \\
&\;\;. \\
&\;\;. \\
\mathcal{H}_n &= \{O^\dagger_{\nu_1}O^\dagger_{\nu_2}...O^\dagger_{\nu_n}|HF>\}
\end{aligned}$$

These basis sets are in general **overcomplete**!

We need to solve our Eigenvalue problem(s) in the **linearly independent subsets** of basis states.

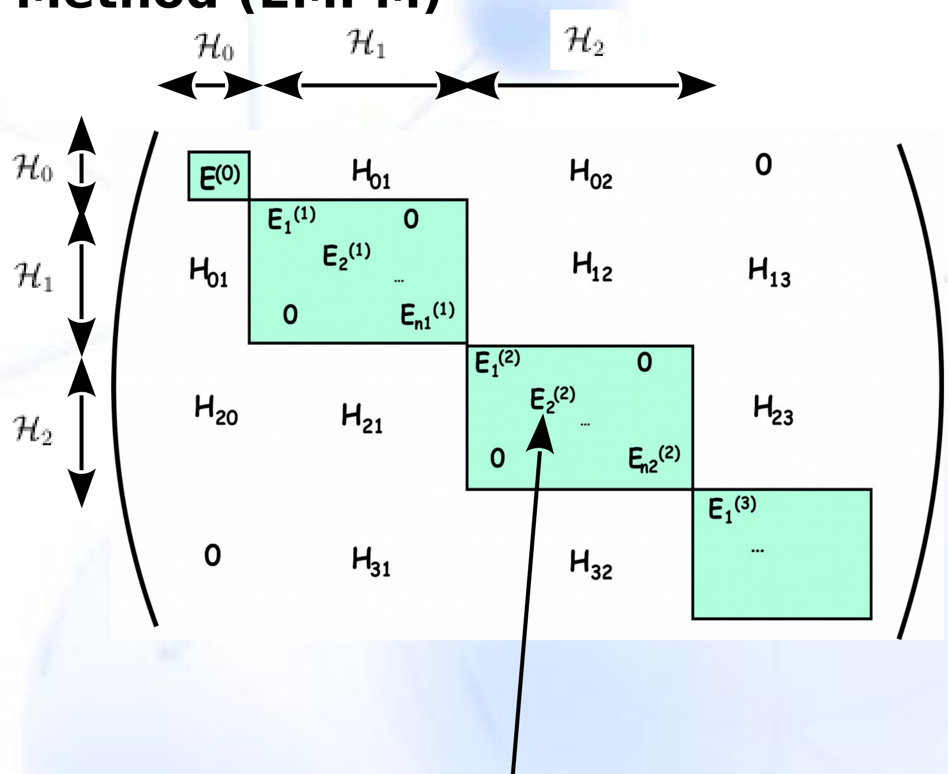
Linear Algebra methods – e.g. **Choleski method**

HF → TDA → EMPM

Equation of Motion Phonon Method

Equation of Motion Phonon Method (EMPM)

$$
\begin{aligned}
\mathcal{H}_0 &= \{|HF>\} \\
\mathcal{H}_1 &= \{O^\dagger_{\nu_1}|HF>\} \\
\mathcal{H}_2 &= \{O^\dagger_{\nu_1}O^\dagger_{\nu_2}|HF>\} \\
&\;\;. \\
&\;\;. \\
\mathcal{H}_n &= \{O^\dagger_{\nu_1}O^\dagger_{\nu_2}...O^\dagger_{\nu_n}|HF>\}
\end{aligned}
$$



the total **Hamiltonian** mixes configurations from different **Hilbert subspaces**

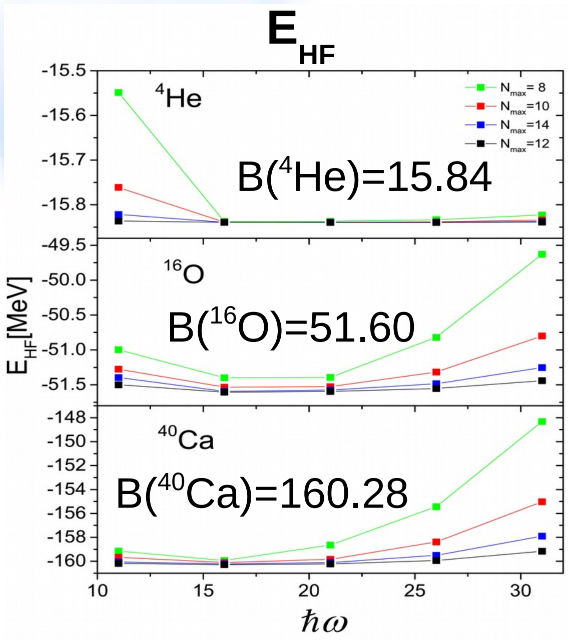
Equation of Motion (EoM) – recursive eq. to solve **eigen-energies** on each **i**-phonon subspace while knowing the (**i-1**)-phonon solution

$$< i,\beta_i|[\hat{H},O^\dagger_\nu]|i-1,\alpha_{i-1} >= (E^i_{\beta_i} - E^{i-1}_{\alpha_{i-1}}) < i,\beta_i|O^\dagger_\nu|i-1,\alpha_{i-1} >$$

non-diagonal blocks of **Hamiltonian** calculated from amplitudes $< i,\beta_i|O^\dagger_\nu|i-1,\alpha_{i-1} >$

we diagonalize the total **Hamiltonian**

Nuclear ground state properties (^{4}He, ^{16}O, ^{40}Ca)



Chiral **NNLO$_{\mathbf{opt}}$** potential:

N$_{\mathbf{max}}$ – maximal osc. shell (defines how big basis is)

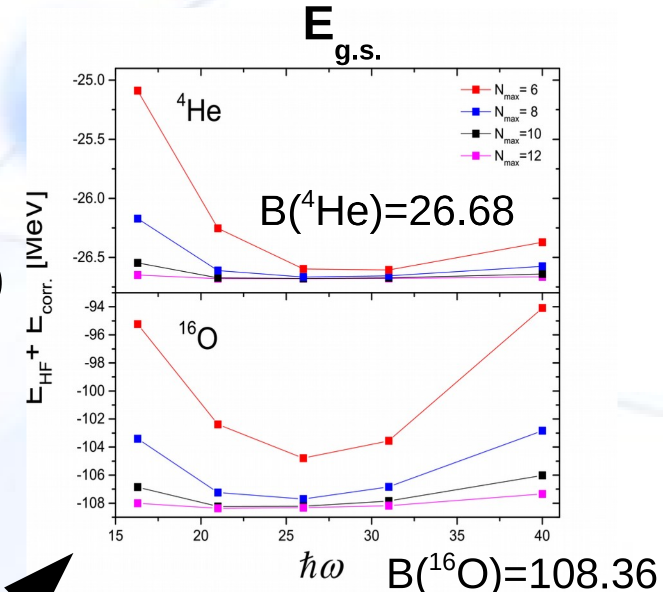
hω – parameter of basis

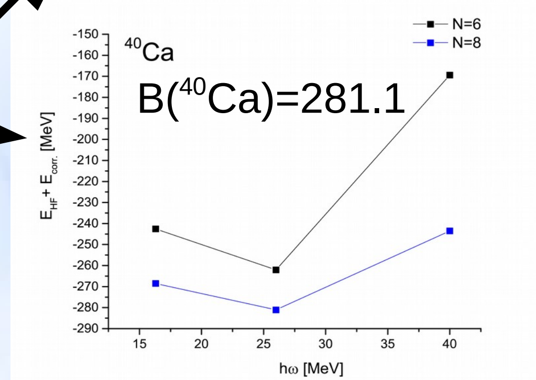
Final energy must be converged with respect to **N$_{\mathbf{max}}$** and for **N$_{\mathbf{max}}$** big enough independent on **hω** ...

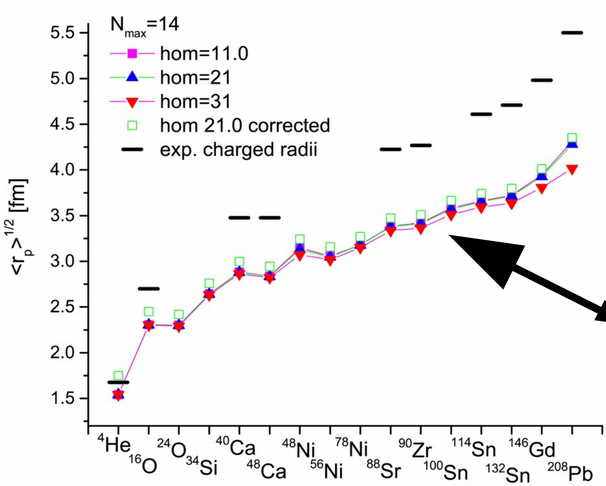
We obtain important contribution into $E_{g.s.}$ from beyond mean-field correlations!!

The effect on radii – much smaller!!

Phys Rev C 95, 024306, (2017)







Experimental values:

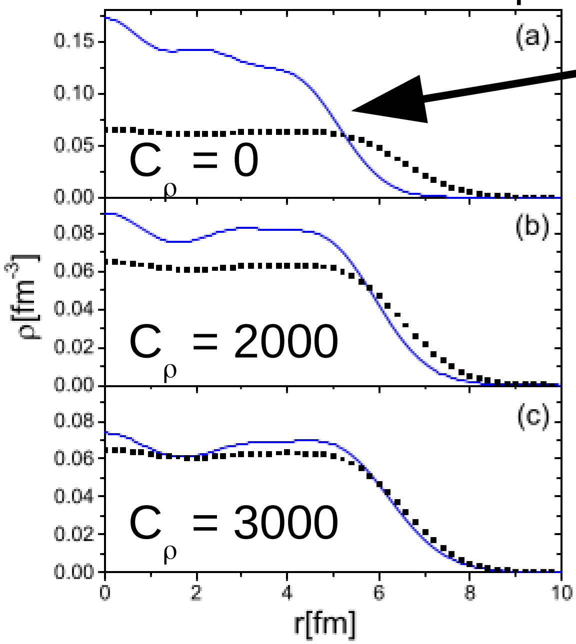
B(^{4}He)=28.256 MeV

B(^{16}O)=127.619 MeV

B(^{40}Ca)=342.052 MeV

Nuclear energy spectra (^{208}Pb)

2-phonon calculation of ^{208}Pb – see in **Phys. Rev. C 92**, 054315 (2015), Chiral **NNLO**$_{\textbf{opt}}$ + phenomenological **density dependent force**



nuclear density distribution

study of the dipole photoabsorption spectrum

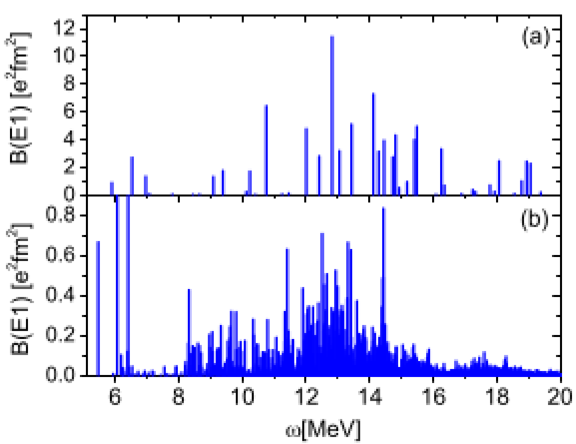
$B(E1, 0^+_{g.s.} \rightarrow 1^-_{exc.})$

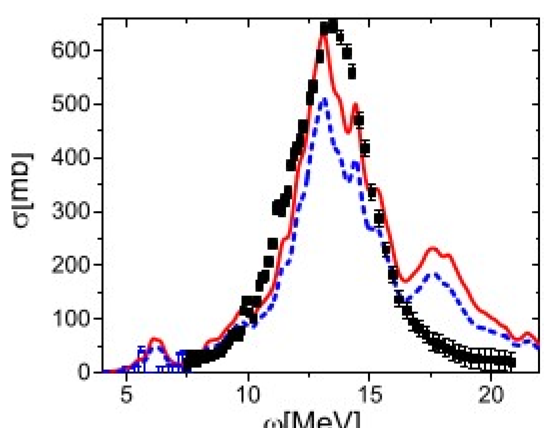
2-phonon configurations very important to describe richness of spectrum → **multifragmentation** of dipole resonance...
we describe width of res.

most of 1^- states have configurations beyond 1ph

TABLE I. Phonon composition of the lowest twenty 1^- states.

J^π_ν	ω_ν (MeV)	$\lvert C_1^{(\nu)}\rvert^2$	$\lvert C_2^{(\nu)}\rvert^2$
1^-_1	4.42780	0.00017	0.99983
1^-_2	4.67271	0.00083	0.99917
1^-_3	4.96609	0.00014	0.99986
1^-_4	5.46012	0.95558	0.04442
1^-_5	5.93408	0.03132	0.96868
1^-_6	6.05979	0.90712	0.09288
1^-_7	6.18594	0.05422	0.94578
1^-_8	6.25179	0.04936	0.95064
1^-_9	6.26285	0.05409	0.94591
1^-_{10}	6.27701	0.00310	0.99690
1^-_{11}	6.38869	0.15931	0.84069
1^-_{12}	6.40474	0.69907	0.30093
1^-_{13}	6.42531	0.03371	0.96629
1^-_{14}	6.43502	0.03215	0.96785
1^-_{15}	6.48971	0.86985	0.13015
1^-_{16}	6.53002	0.00956	0.99044
1^-_{17}	6.55127	0.00485	0.99515
1^-_{18}	6.64103	0.00346	0.99654
1^-_{19}	6.71925	0.01301	0.98699
1^-_{20}	6.73778	0.00058	0.99942

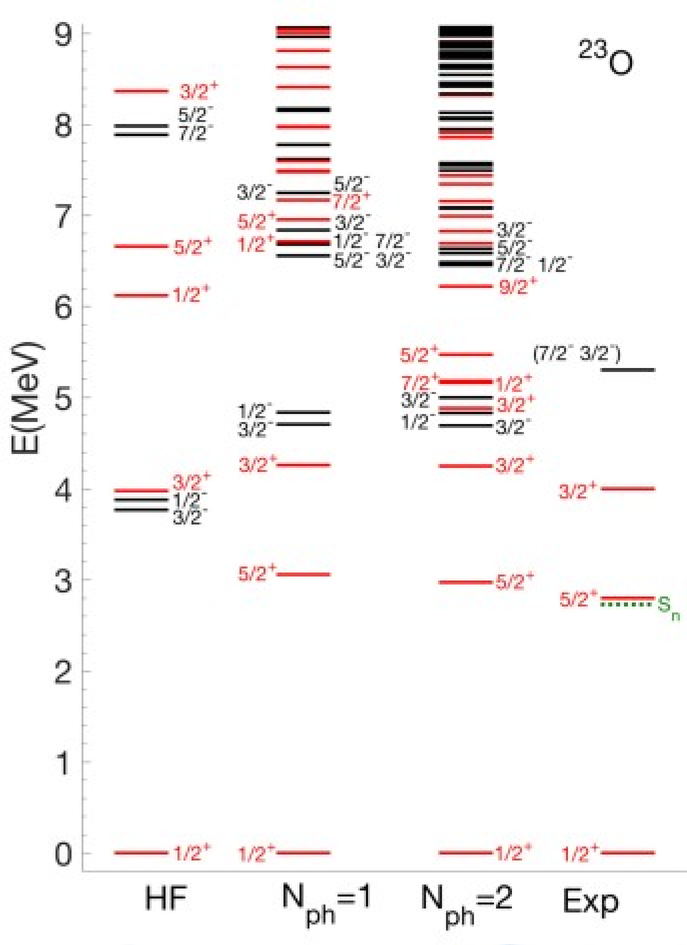




EMPM for odd-even nuclei

NN interaction - χ **NNLO**$_{\mathbf{opt}}$

Phys. Rev. C 97, 034311 (2018)



extension of **EMPM** – formalism in Phys Rev C95, 034327 (2017)

Valence nucleon is coupled to $\mathbf{N_{ph}}$**–phonon** excitations of the core

TABLE IV. Phonon composition of the low-lying states in ^{23}O.

J^π	E^ν	$\lvert C_0\rvert^2$	$\lvert C_1\rvert^2$	$\lvert C_2\rvert^2$
$1/2_1^+$	0.000	0.9404	0.0594	0.0002
$5/2_1^+$	2.973	0.0003	0.9948	0.0049
$3/2_1^+$	4.244	0.9507	0.0478	0.0015
$3/2_1^-$	4.688	0.9049	0.0905	0.0046
$1/2_1^-$	4.827	0.9686	0.0307	0.0007
$3/2_2^+$	4.880	0.0000	0.0000	1.0000
$3/2_2^-$	4.996	0.0498	0.8645	0.0852
$7/2_1^+$	5.159	0.0000	0.0000	1.0000
$1/2_2^+$	5.181	0.0000	0.0000	1.0000
$5/2_2^+$	5.468	0.0000	0.0000	1.0000
$9/2_1^+$	6.218	0.0000	0.0000	1.0000
$7/2_1^-$	6.452	0.0095	0.9374	0.0531
$5/2_1^-$	6.482	0.0004	0.9924	0.0072
$1/2_2^-$	6.583	0.0011	0.9930	0.0059
$3/2_3^-$	6.633	0.0284	0.9656	0.0060

Phonon composition of each state of ^{23}O

EMPM for Hypernuclei

EMPM extended on single-Λ hypernuclei

I)

hypernuclei with Λ in **even-even nuclear cores**

$$\mathcal{H} = \mathcal{H}_0 \oplus \mathcal{H}_1 \oplus \mathcal{H}_2 \oplus ... \oplus \mathcal{H}_n$$

We **couple** creation particle of Λ with the |HF> and **(multi)phonon** excitations of |HF>

$$\begin{aligned} \mathcal{H}_0 &= \{c_p^\dagger|\mathrm{HF}>\} \\ \mathcal{H}_1 &= \{c_p^\dagger O_{\mu_1}^\dagger|\mathrm{HF}>\} \\ \mathcal{H}_2 &= \{c_p^\dagger O_{\mu_1}^\dagger O_{\nu_1}^\dagger|\mathrm{HF}>\} \end{aligned}$$

We can apply this method to calculate structure of **$^{5}_{\Lambda}$He, $^{17}_{\Lambda}$O, $^{41}_{\Lambda}$Ca, $^{49}_{\Lambda}$Ca** etc.

II)

hypernuclei with Λ in **even-odd nuclear cores**

$$\mathcal{H} = \mathcal{H}_0 \oplus \mathcal{H}_1 \oplus \mathcal{H}_2 \oplus ... \oplus \mathcal{H}_n$$

We **couple ΛN TDA** states with |HF> and **(multi)phonon** excitations of |HF>

$$\begin{aligned} \mathcal{H}_0 &= \{R_\nu^\dagger|\mathrm{HF}>\} \\ \mathcal{H}_1 &= \{R_\nu^\dagger O_{\mu_1}^\dagger|\mathrm{HF}>\} \\ \mathcal{H}_2 &= \{R_\nu^\dagger O_{\mu_1}^\dagger O_{\nu_1}^\dagger|\mathrm{HF}>\} \end{aligned}$$

We can apply this method to calculate structure of **$^{4}_{\Lambda}$H, $^{4}_{\Lambda}$He, $^{16}_{\Lambda}$N, $^{16}_{\Lambda}$O, $^{40}_{\Lambda}$K, $^{40}_{\Lambda}$Ca, $^{48}_{\Lambda}$K, $^{48}_{\Lambda}$Ca** etc.

Nuclei:	**HF**	→	**TDA**	→	**EMPM**
Hypernuclei:	**p-n-Λ HF**	→	**NΛ TDA**	→	**ext. EMPM**

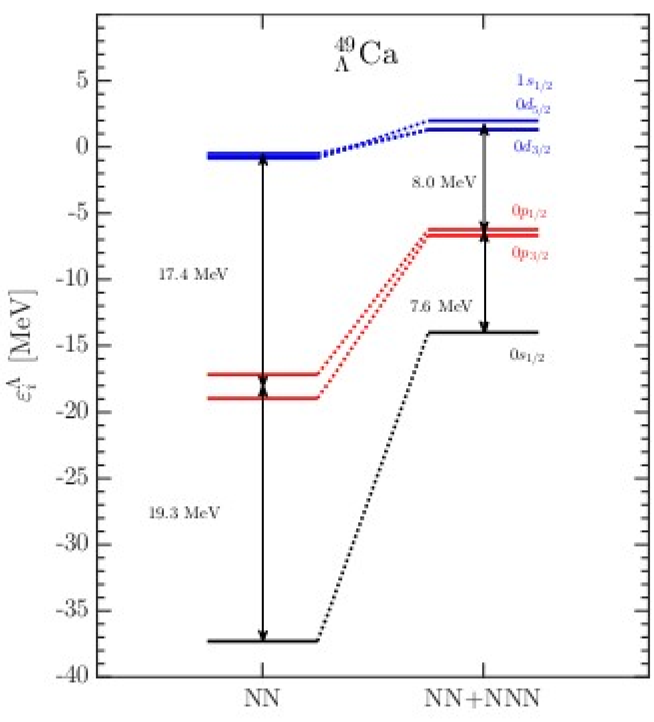
p-n-Λ Hartree-Fock Method

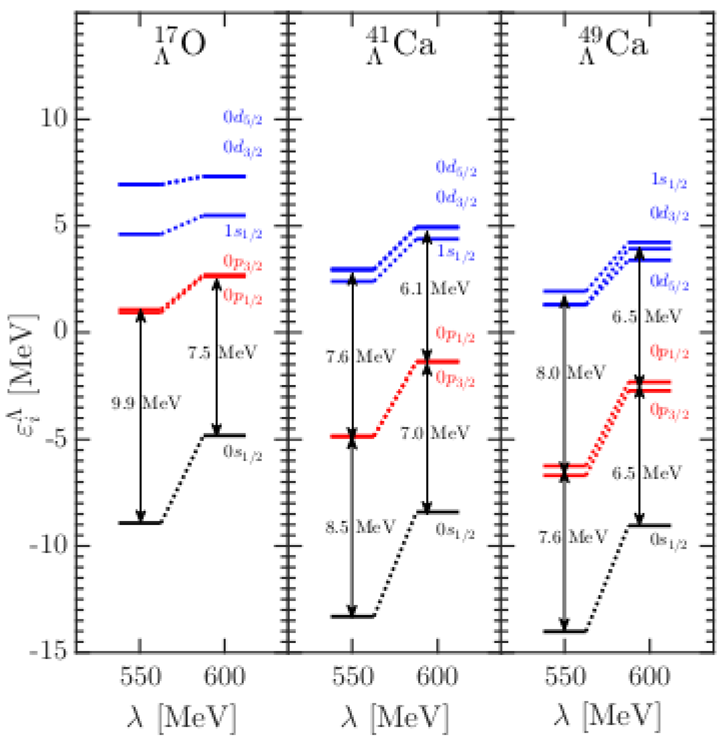
p-n-Λ HF = Hartree-Fock method in the proton-neutron-Λ formalism

- diploma thesis of **J. Pokorný** “*Three-body Interactions in Mean-Field Model of Nuclei and Hypernuclei*“, **Czech Technical University**, (2018)
- **Phys. Scr. 94, 014006, (2019)**; **Acta Phys. Pol. B Proc. Suppl. 12, 657, (2019)**

We obtain: - **single-particle levels** of protons, neutrons and Λ

Single-particle Λ energies:





realistic chiral **NN+NNN** potential **NNLO$_{sat}$**

realistic chiral LO **YN** potential (ΛN-ΛN channel) with different **regulator cut-off λ**

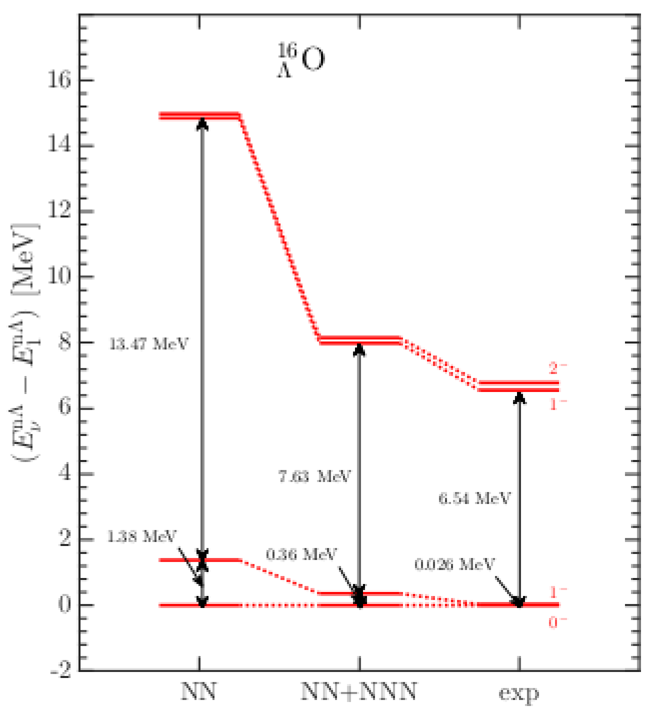
NΛ Tamm-Dancoff

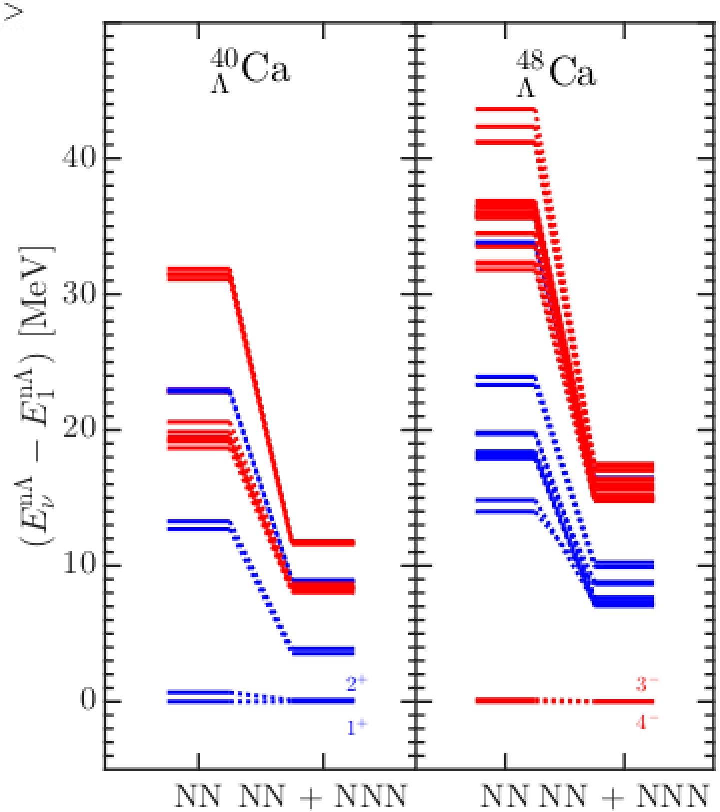
NΛ TDA = Nucleon-Λ Tamm-Dancoff Approximation

- diploma thesis of **J. Pokorný** "*Three-body Interactions in Mean-Field Model of Nuclei and Hypernuclei*", **Czech Technical University**, (2018)
- **Phys. Scr. 94, 014006, (2019)**; **Acta Phys. Pol. B Proc. Suppl. 12, 657, (2019)**

Suitable for **hypernuclei** with **Λ** in **even-odd nuclear cores**

NΛ TDA Phonon $R^{\dagger}_{\nu}|\mathrm{HF}> = \sum_{\mathrm{ph}} r^{\nu}_{\mathrm{ph}} c^{\dagger}_{\mathrm{p}} a_{\mathrm{h}} |\mathrm{HF}>$





realistic chiral **NN+NNN** potential **NNLO$_{\mathbf{sat}}$**

realistic chiral LO **YN** potential (ΛN-ΛN channel) with different **regulator cut-off λ**

EMPM for Hypernuclei

EMPM extended on single-Λ hypernuclei

I) **hypernuclei** with Λ in **even-even nuclear cores**

$$\hat{H} = \hat{T}^{N} + \hat{T}^{\Lambda} + \hat{V}^{NN} + \hat{V}^{NNN} + \hat{V}^{\Lambda N} + \hat{V}^{\Lambda NN} - \hat{T}_{CM}.$$

$$\mathcal{H} = \mathcal{H}_0 \oplus \mathcal{H}_1 \oplus \mathcal{H}_2 \oplus \ldots \oplus \mathcal{H}_n$$

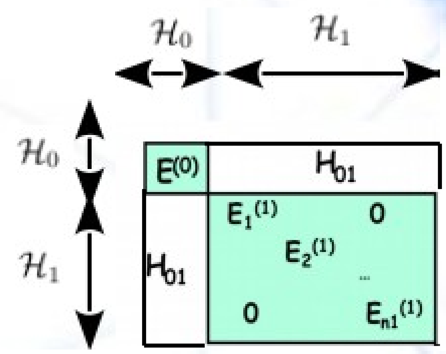
Within this formalism we can study **$^{5}_{\Lambda}$He, $^{17}_{\Lambda}$O, $^{41}_{\Lambda}$Ca, $^{49}_{\Lambda}$Ca, $^{209}_{\Lambda}$Pb ...**

$$\mathcal{H}_0 = \{c_p^\dagger |\mathrm{HF}>\}$$
$$\mathcal{H}_1 = \{c_p^\dagger O_{\mu_1}^\dagger |\mathrm{HF}>\}$$
$$\mathcal{H}_2 = \{c_p^\dagger O_{\mu_1}^\dagger O_{\nu_1}^\dagger |\mathrm{HF}>\}$$

Our theoretical **formalism**:

$$|\nu>= c_\nu^\dagger |\mathrm{HF}>$$

$$|\beta>= \sum_{\nu\mu} X^{\beta}_{\nu\mu} c_\nu^\dagger Q_\mu^\dagger |\mathrm{HF}>$$



We construct the **Hamiltonian** matrix:

1) The diagonal block $\mathcal{H}_0 \times \mathcal{H}_0$ = s.p. Λ energies
2) The diagonal block $\mathcal{H}_1 \times \mathcal{H}_1$ = coupling of Λ to phonon exc. of core
3) The nondiagonal block $\mathcal{H}_0 \times \mathcal{H}_1$ => not difficult to calculate

Diagonalization of whole matrix leads to **correlated states** → they already cannot be interpreted as purely **"single-particle"** states of Λ.

Also energy spectrum can get much richer by inclusion configurations from **larger Hilbert space**.

EMPM for Hypernuclei

EMPM extended on single-Λ hypernuclei

II) hypernuclei with Λ in **even-odd nuclear cores**

$$\hat{H} = \hat{T}^N + \hat{T}^\Lambda + \hat{V}^{NN} + \hat{V}^{NNN} + \hat{V}^{\Lambda N} + \hat{V}^{\Lambda NN} - \hat{T}_{CM}$$

$$\mathcal{H} = \mathcal{H}_0 \oplus \mathcal{H}_1 \oplus \mathcal{H}_2 \oplus \ldots \oplus \mathcal{H}_n$$

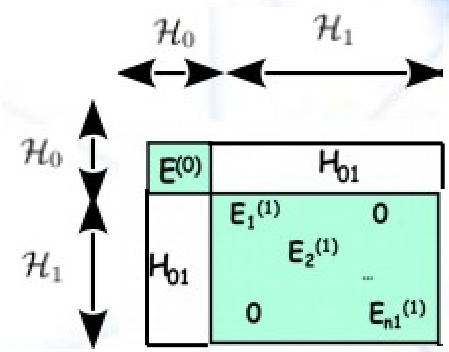
$$\begin{aligned} \mathcal{H}_0 &= \{R^\dagger_\nu|\mathrm{HF}>\} \\ \mathcal{H}_1 &= \{R^\dagger_\nu O^\dagger_{\mu_1}|\mathrm{HF}>\} \\ \mathcal{H}_2 &= \{R^\dagger_\nu O^\dagger_{\mu_1} O^\dagger_{\nu_1}|\mathrm{HF}>\} \end{aligned}$$

It is more important to study such hypernuclei from the point of view of experiment (production of hypernuclei **$^{4}_{\Lambda}$H, $^{16}_{\Lambda}$O, $^{16}_{\Lambda}$N, $^{40}_{\Lambda}$K, $^{48}_{\Lambda}$K,...**)

Our theoretical **formalism**:

$$|\nu >= R^\dagger_\nu|\mathrm{HF}>$$

$$|\beta >= \sum_{\nu\mu} X^\beta_{\nu\mu} R^\dagger_\nu Q^\dagger_\mu|\mathrm{HF}>$$



$$-B_\Lambda = E_i + \varepsilon_F^N$$

We construct the **Hamiltonian** matrix:

1) The diagonal block $\mathcal{H}_0 \times \mathcal{H}_0$ = **NΛ TDA** energies
2) The diagonal block $\mathcal{H}_1 \times \mathcal{H}_1$ = **Equation of Motion**
3) The nondiagonal block $\mathcal{H}_0 \times \mathcal{H}_1$ => not difficult to calculate

Equation of Motion:

$$AX = EX$$

A-matrix $A =< \beta|[\hat{H}, R^\dagger_\nu]|\mu > + E_\mu < \beta|R^\dagger_\nu|\mu >$

Eigen-value problem in an **overcomplete non-orthogonal basis**…

$$\overline{ADC} = E\overline{DC}$$

Eigen-value problem in the **reduced space** (linearly independent subset of states)

D-matrix = overlap matrix of the basis states
(A.D) – must be hermitian

Description of Hypernuclei

Hypernuclei with **single-Λ** particle:
Hamiltonian $\widehat{H}=\widehat{T}^{N}+\widehat{T}^{\Lambda}+\widehat{V}^{NN}+\widehat{V}^{NNN}+\widehat{V}^{\Lambda N}+\widehat{V}^{\Lambda NN}-\widehat{T}_{CM}.$

Example: NN(+NNN) potential

- phenomenological **NN** potential
  Brink-Boeker
  Nucl. Phys. **A91**, 1, (1967)

- realistic chiral **NN** potential
  NNLO$_{opt}$
  Phys. Rev. Lett. **110**, 192502, (2013)

- realistic chiral **NN+NNN** potential
  NNLO$_{sat}$
  Phys. Rev. **C91**, 051301(R), (2015)

ΛN potential

- phenomenological Gaussian **ΛN** potential
  Prog. Theor. Phys. **70**, 189, (1983)

$$v_{\Lambda N}(r)=v^0_{\Lambda N}e^{-(r/\beta_{\Lambda N})^2}(1+\eta\sigma_\Lambda\cdot\sigma_N),$$

$$v_{\Lambda N}=-38.19\text{ MeV},\quad \beta_{\Lambda N}=1.034\text{ fm},\quad \eta=-0.1.$$

- **G-matrix** effective **ΛN** potential derived from **Juelich-A YN**
  Prog. Theor. Phys. Suppl. **117**, 361 (1994)

$$V_{\Lambda N}(r)=\sum_{i=1}^{3}(a_i+b_ik_F+c_ik_F^2)\exp[-r^2/\beta_i^2]$$

	β_i(fm)	1.25	0.70	0.45
	a	−25.82	−389.4	859.0
^1E	b	−12.51	401.2	−303.2
	c	2.437	−136.0	188.8
	a	−45.01	−296.6	1094.
^3E	b	4.620	218.3	−504.6
	c	.7500	−92.50	230.0
	a	−14.54	144.7	734.6
^1O	b	3.615	27.50	76.37
	c	−.8750	−5.000	3.125
	a	−25.91	248.1	615.3
^3O	b	5.410	210.9	−1260.
	c	.5000	−123.1	734.8

Description of Hypernuclei

EMPM extended on single-Λ hypernuclei

I) **hypernuclei** with Λ in **even-even nuclear cores**

$$\hat{H} = \hat{T}^{N} + \hat{T}^{\Lambda} + \hat{V}^{NN} + \hat{V}^{NNN} + \hat{V}^{\Lambda N} + \hat{V}^{\Lambda NN} - \hat{T}_{CM}.$$

$$\mathcal{H} = \mathcal{H}_0 \oplus \mathcal{H}_1 \oplus \mathcal{H}_2 \oplus \ldots \oplus \mathcal{H}_n$$

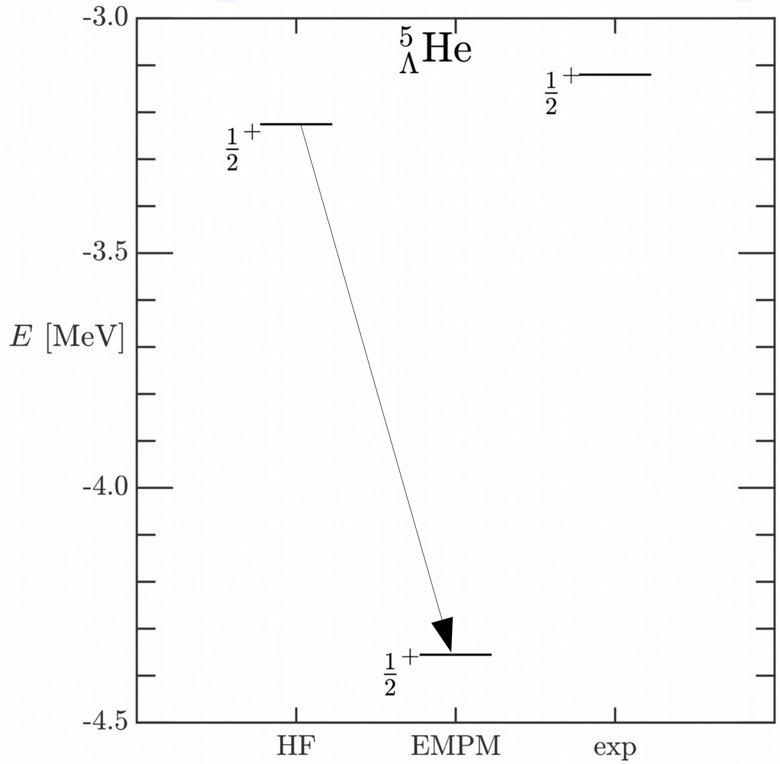
$$\begin{aligned} \mathcal{H}_0 &= \{c_p^\dagger |\mathrm{HF}>\} \\ \mathcal{H}_1 &= \{c_p^\dagger O_{\mu_1}^\dagger |\mathrm{HF}>\} \\ \mathcal{H}_2 &= \{c_p^\dagger O_{\mu_1}^\dagger O_{\nu_1}^\dagger |\mathrm{HF}>\} \end{aligned}$$

realistic chiral **NN+NNN** potential **NNLO$_{\mathrm{sat}}$**

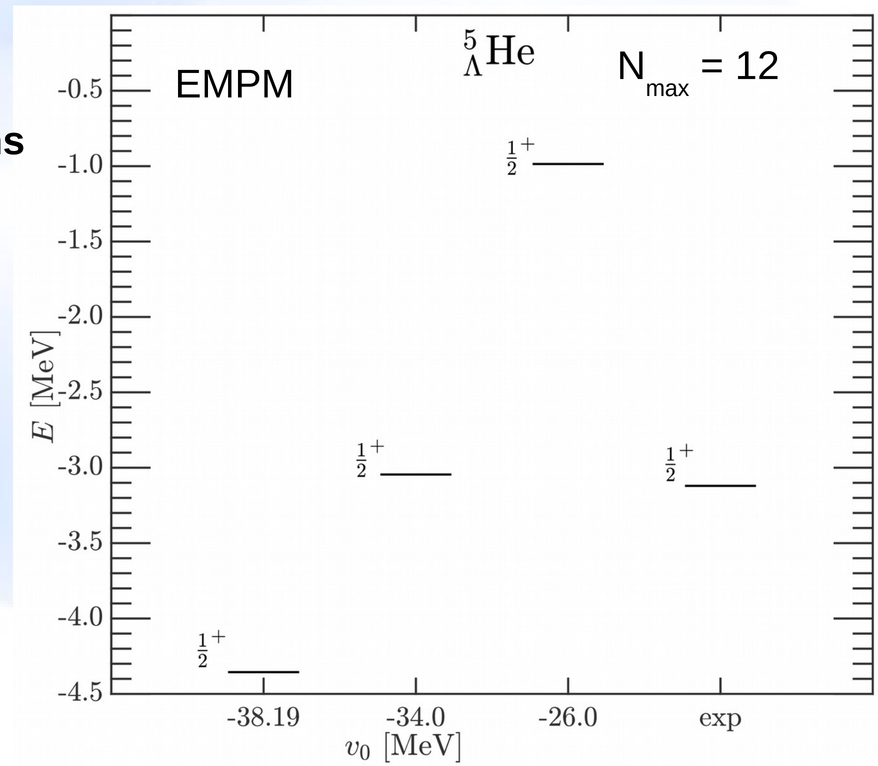
$$v_{\Lambda N}(r) = v_{\Lambda N}^{0} e^{-(r/\beta_{\Lambda N})^2}(1+\eta \boldsymbol{\sigma}_\Lambda \cdot \boldsymbol{\sigma}_N),$$

$$v_{\Lambda N} = -38.19 \text{ MeV}, \quad \beta_{\Lambda N} = 1.034 \text{ fm}, \quad \eta = -0.1.$$

we tune the strength of **ΛN**



Effect of **correlations → down shift** in energy



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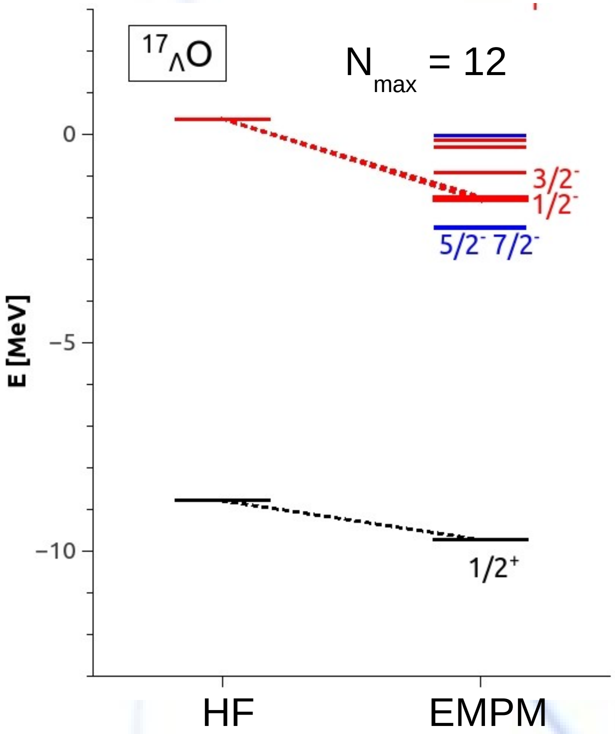
we tune the strength of **ΛN**
$V_{\Lambda N}$ = -34.0 MeV



- Down shift in energy for the whole spectrum due to the coupling of Λ with phonon excitations of the nuclear core

- More complex energy spectrum in EMPM (there is multiplet of negative energy states instead of $1/2^-$ and $3/2^-$ only)

- No direct “experimental“ binding B_Λ of $^{17}{}_\Lambda$O. Usualy exper. value taken from experiment on $^{16}{}_\Lambda$O.

Description of Hypernuclei

EMPM extended on single-Λ hypernuclei

II) hypernuclei with Λ in **even-odd nuclear cores**

$$\hat{H} = \hat{T}^{N} + \hat{T}^{\Lambda} + \hat{V}^{NN} + \hat{V}^{NNN} + \hat{V}^{\Lambda N} + \hat{V}^{\Lambda NN} - \hat{T}_{CM}$$

$$\mathcal{H} = \mathcal{H}_0 \oplus \mathcal{H}_1 \oplus \mathcal{H}_2 \oplus \ldots \oplus \mathcal{H}_n$$

$$\begin{aligned} \mathcal{H}_0 &= \{R^{\dagger}_{\nu}|\mathrm{HF}>\} \\ \mathcal{H}_1 &= \{R^{\dagger}_{\nu}O^{\dagger}_{\mu_1}|\mathrm{HF}>\} \\ \mathcal{H}_2 &= \{R^{\dagger}_{\nu}O^{\dagger}_{\mu_1}O^{\dagger}_{\nu_1}|\mathrm{HF}>\} \end{aligned}$$

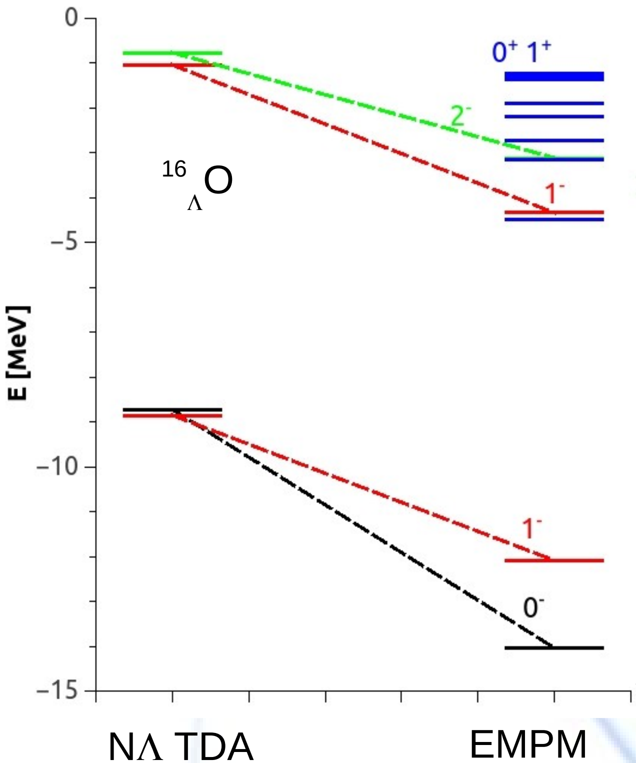
realistic chiral **NN+NNN** potential **NNLO$_{sat}$**

$N_{max} = 8$

$$v_{\Lambda N}(r) = v^{0}_{\Lambda N}\, e^{-(r/\beta_{\Lambda N})^2}(1 + \eta\, \boldsymbol{\sigma}_{\Lambda} \cdot \boldsymbol{\sigma}_N),$$

$$v_{\Lambda N} = -38.19 \text{ MeV}, \quad \beta_{\Lambda N} = 1.034 \text{ fm}, \quad \eta = -0.1\,.$$

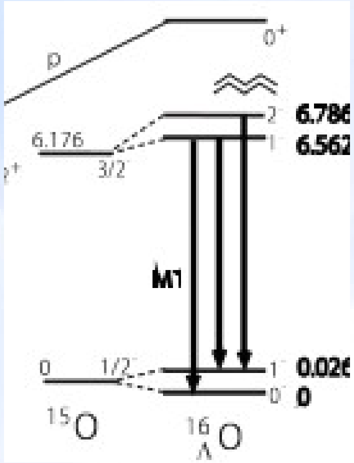
we tune the strength of Λ**N**
$V_{\Lambda N}$ = -34.0 MeV



there will be rich multiplet of positive parity states 0^+ 1^+ 2^+ 3^+

exper. B_{Λ} = -13 MeV
explanation for too big splitting (?)

experimental energies of $^{16}_{\Lambda}O$



Description of Hypernuclei

- **G-matrix** effective **ΛN** potential derived from **Juelich-A YN**
  Prog. Theor. Phys. Suppl. **117**, 361 (1994)
  Gaussian-like form – easy to implement, interaction is effective (we can take just ΛN-ΛN part) but dependent on a parameter $\mathbf{k_F}$

$$V_{\Lambda N}(r)=\sum_{i=1}^{3}(a_i+b_ik_F+c_ik_F^2)\exp[-r^2/\beta_i^2]$$

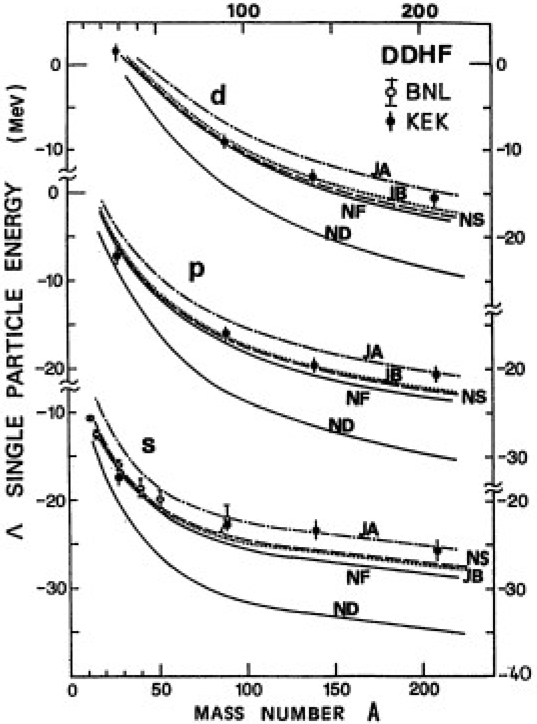


Fig. 2. The Λ single-particle energies $\epsilon_\Lambda(nl;A)$, nl $=0s$, $0p$ and $0d$, calculated in DDHF with five effective interactions. The experimental data

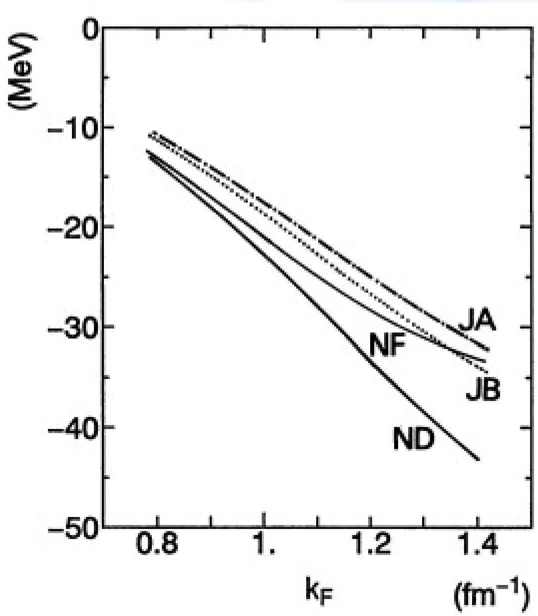


Fig. 1. The Λ single-particle potentials calculated as a function of k_F. The different interaction

Dependence of the **Λ single-particle** energies on $\mathbf{k_F}$

$\mathbf{k_F}$ as a parameter to tune the proper effective ΛN interaction. But tuning should be done at the level of the beyond mean-field calculation.

Prog. Theor. Phys. Suppl. **117**, 361 (1994)

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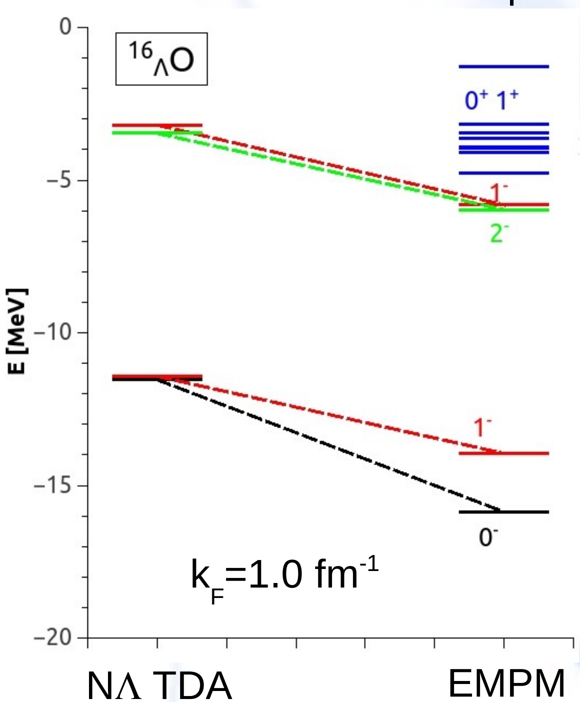
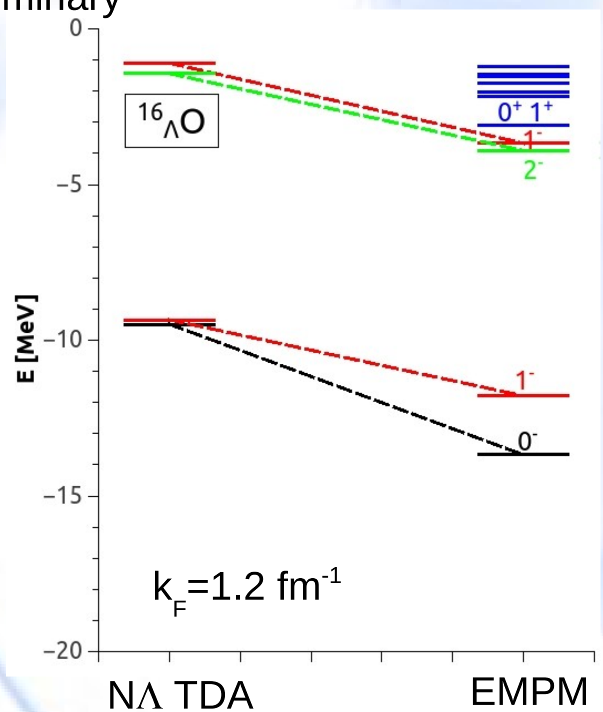
realistic
chiral
NN+NNN
potential
$\mathrm{NNLO}_{\mathrm{sat}}$

$$V_{\Lambda N}(r) = \sum_{i=1}^{3}(a_i + b_i k_F + c_i k_F^2)\exp[-r^2/\beta_i^2]$$

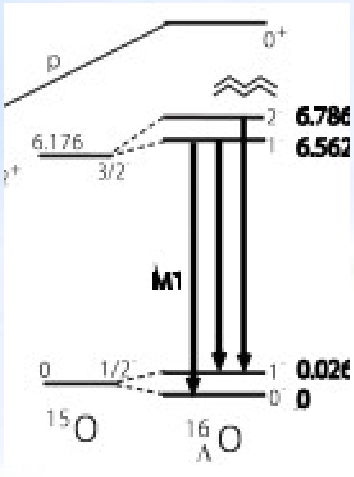
YNG Juelich-A

preliminary





experimental energies of $^{16}{}_{\Lambda}$O



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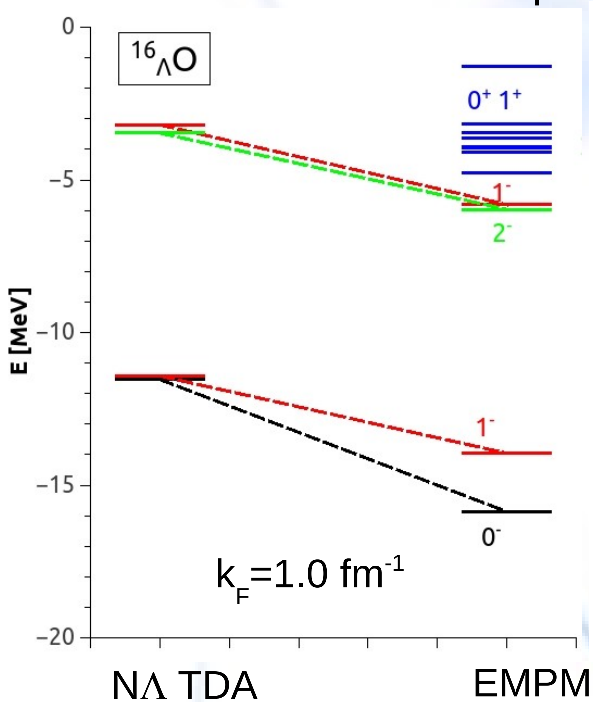
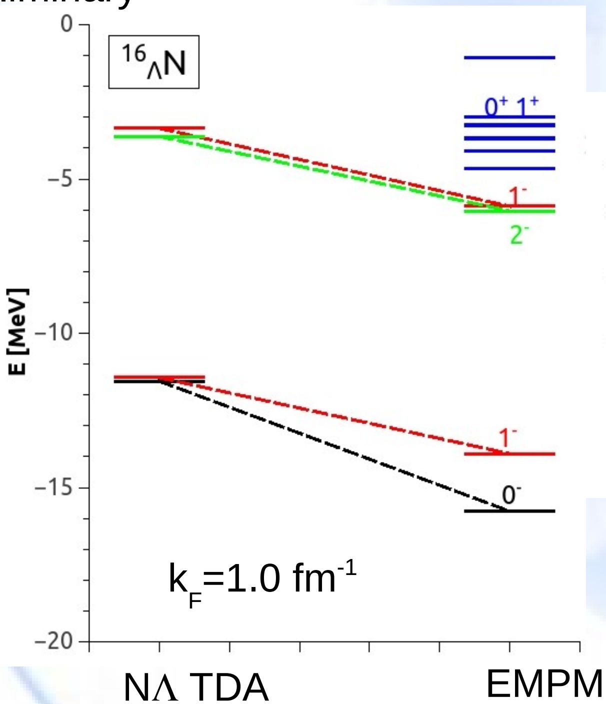
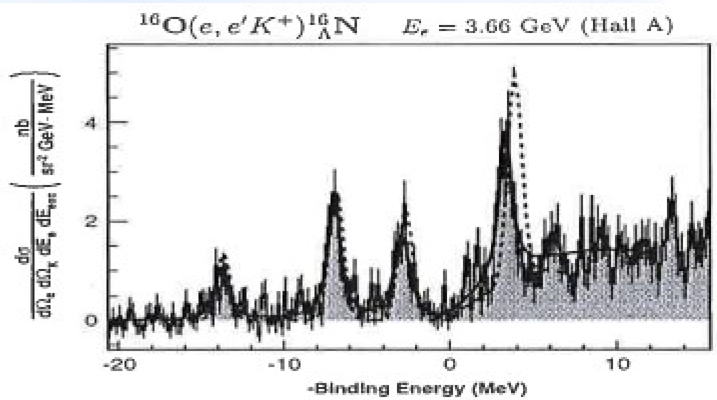
YNG Juelich-A

preliminary





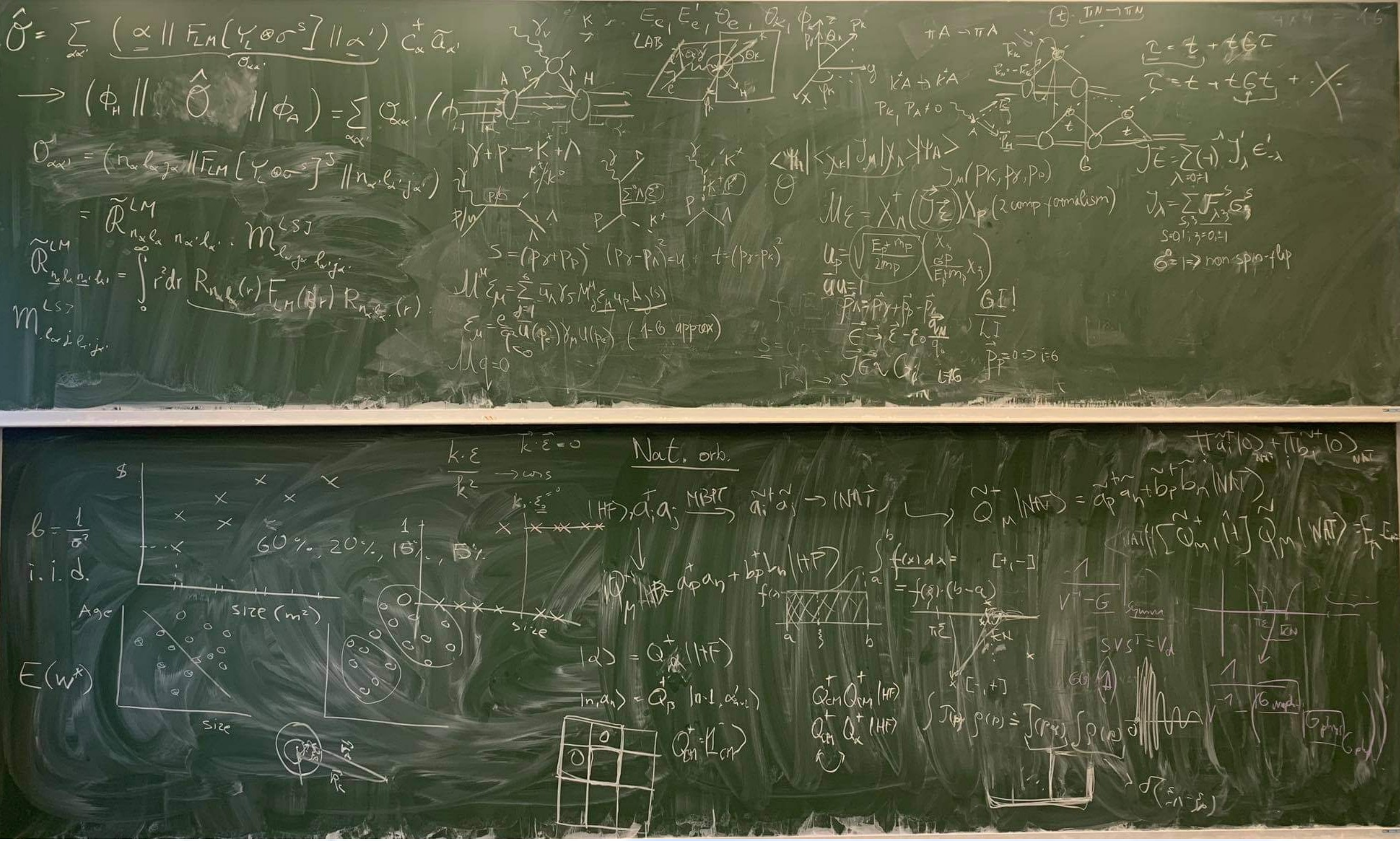


7) F. Cusanno et al., Phys. Rev. Lett. **103** (2009), 202501.

Summary

- **EMPM** was originally introduced for the calculation of energy spectra of medium and heavy nuclei
- **Extensions** of EMPM to calculate single-Λ **hypernuclei** (with even-even & odd-even cores)
- Both nuclear & hypernuclear calcs. useful to study **production of hypernuclei**
- **Proof of principles** calculations of **$^{5}_{\Lambda}$He, $^{17}_{\Lambda}$O, $^{16}_{\Lambda}$O, $^{16}_{\Lambda}$N** with EMPM – phenom. **ΛN** interaction
- **Tasks to be addressed**:
  - various effective **ΛN** potentials can be used (important to tune them in the beyond mean-field level)
  - study of **$^{40}_{\Lambda}$K, $^{48}_{\Lambda}$K, $^{208}_{\Lambda}$Tl** (their exper. measurement is planned in close future)
  - formalism to study directly **cross section of electroproduction**
  - formalism to study isospin dependence of **ΛNN** interaction (**$^{40}_{\Lambda}$K & $^{48}_{\Lambda}$K**)
- More **long-term tasks**:
  - further **development of EMPM** itself (coupling to 2-phonon states)
  - formulation of whole method in **deformed HF** basis

Many thanks to all my collaborators!!
P. Bydžovský, G. De Gregorio, D. Denisova, F. Knapp, N. Lo Iudice, D. Petrellis, J. Pokorný, D. Skoupil



Thank you for attention!