Hypernuclear Physics from Nuclear Lattice EFT

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supported by DFG, SFB/TR-110 by CAS, PIFI by VolkswagenStftung by EU

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- Nuclear lattice EFT - what and why?
- Chiral EFT on a lattice
- Assorted recent results
- Strangeness nuclear physics
  - YN/YY scattering in chiral EFT
  - Impurity lattice Monte Carlo algorithm
  - Applications in condensed matter physics
  - s-shell hypernuclei
- Summary & outlook

Nuclear lattice EFT: what and why ?

THE NUCLEAR LANDSCAPE: AIMS & METHODS

- Theoretical methods:

– Lattice QCD: *A = 0,1,2, . . .*

– NCSM, Faddeev-Yakubowsky, GFMC, ... :
A = 3 - 16

– SM, coupled cluster, . . .: *A = 16 -100*

– density functional theory, . . .: $A \geq 10(0)$

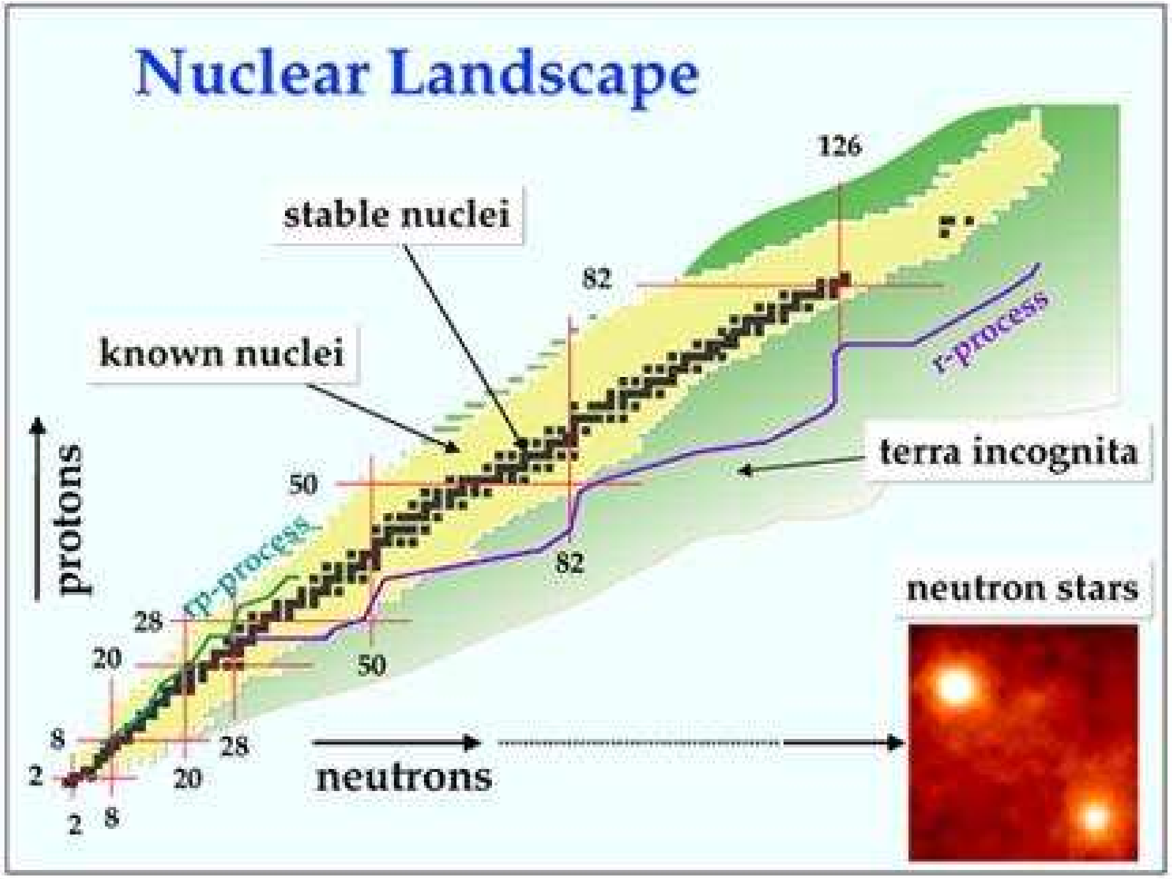


Image source: www.orau.org/RIA

- Chiral EFT:

– provides **accurate 2N, 3N and 4N forces**

– successfully applied in light nuclei
with *A = 2, 3, 4*

Epelbaum, Hammer, UGM, Rev. Mod. Phys. **81** (2009) 1773

– combine with simulations to get to larger A

$\Rightarrow$ Nuclear Lattice Effective Field Theory

AB INITIO NUCLEAR STRUCTURE and SCATTERING

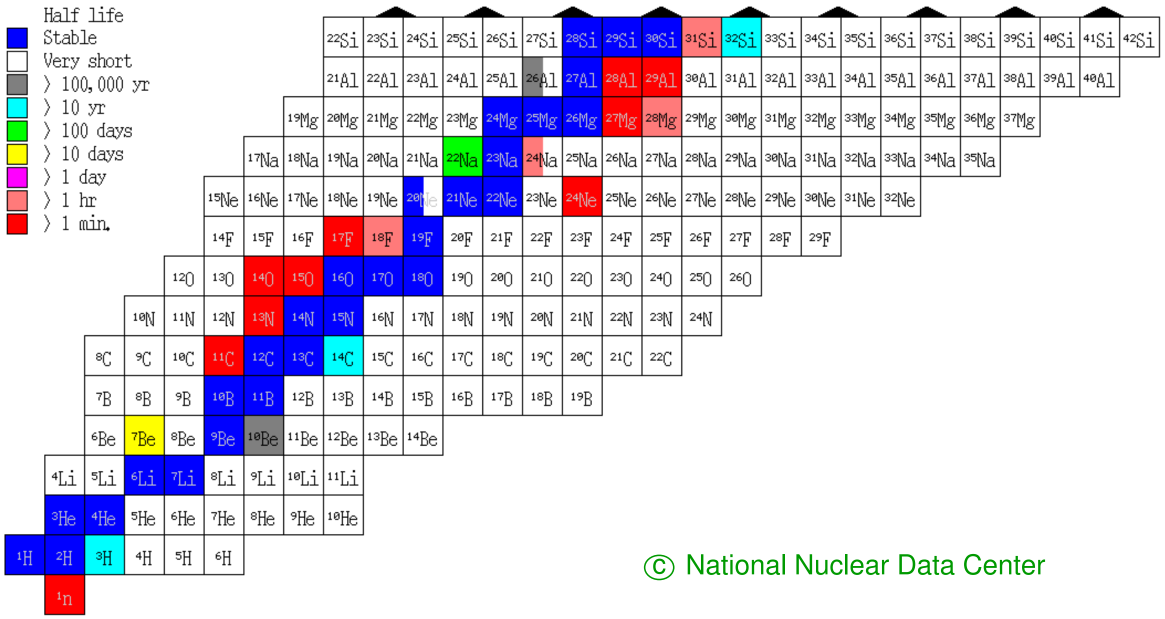
- Nuclear structure:

  ⋆ 3-nucleon forces

  ⋆ limits of stability

  ⋆ alpha-clustering

  ⋮



- Nuclear scattering: processes relevant for nuclear astrophysics

  ⋆ alpha-particle scattering: $^4\text{He} + {}^4\text{He} \to {}^4\text{He} + {}^4\text{He}$

  ⋆ triple-alpha reaction: $^4\text{He} + {}^4\text{He} + {}^4\text{He} \to {}^{12}\text{C} + \gamma$

  ⋆ alpha-capture on carbon: $^4\text{He} + {}^{12}\text{C} \to {}^{16}\text{O} + \gamma$

  ⋮

MANY–BODY APPROACHES

- nuclear physics = notoriously difficult problem: strongly interacting fermions

- define *ab initio*: combine the precise and well-founded forces from *chiral EFT* with a many-body approach

- two different approaches followed in the literature:

  $\star$ combine chiral NN(N) forces with standard many-body techniques

  Dean, Duguet, Hagen, Navratil, Nogga, Papenbrock, Schwenk, Soma . . .

  $\rightarrow$ successful, but problems with cluster states (SM, NCSM,...)

  $\star$ combine chiral forces and lattice simulations methods

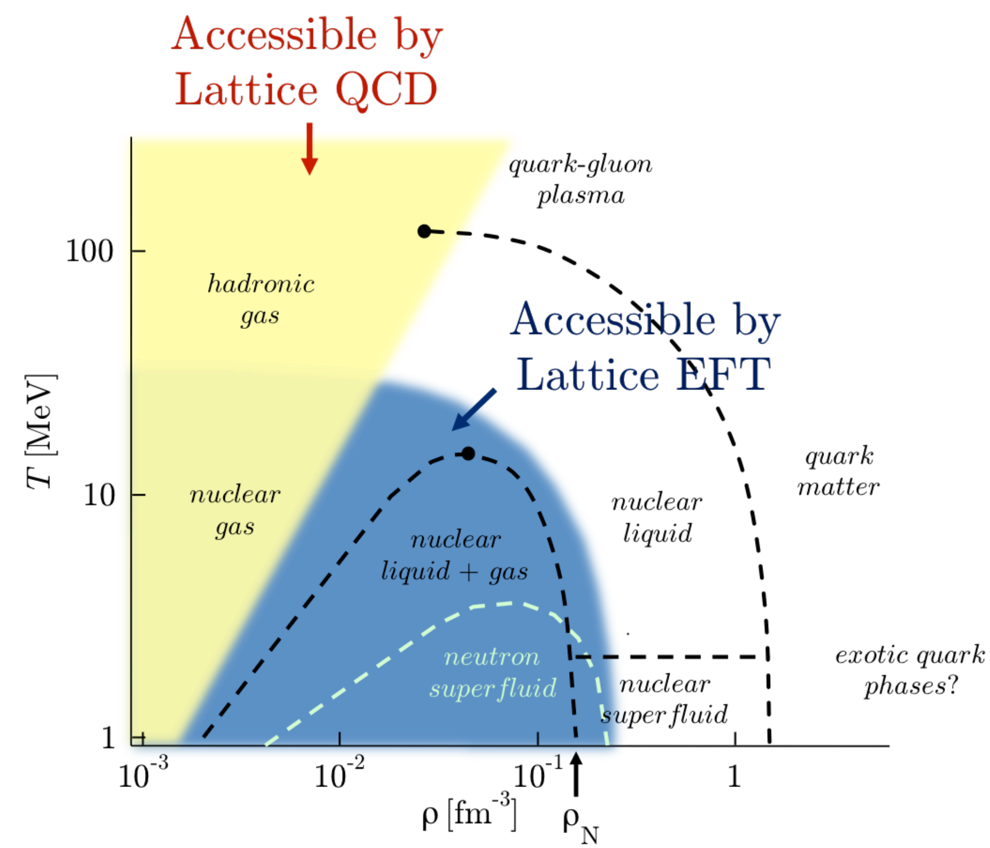
  $\rightarrow$ this new method is called *Nuclear Lattice Effective Field Theory* (NLEFT)

  Borasoy, Elhatisari, Epelbaum, Krebs, Lee, Lähde, UGM, Rupak, . . .

  $\rightarrow$ rest of the talk

COMPARISON to LATTICE QCD

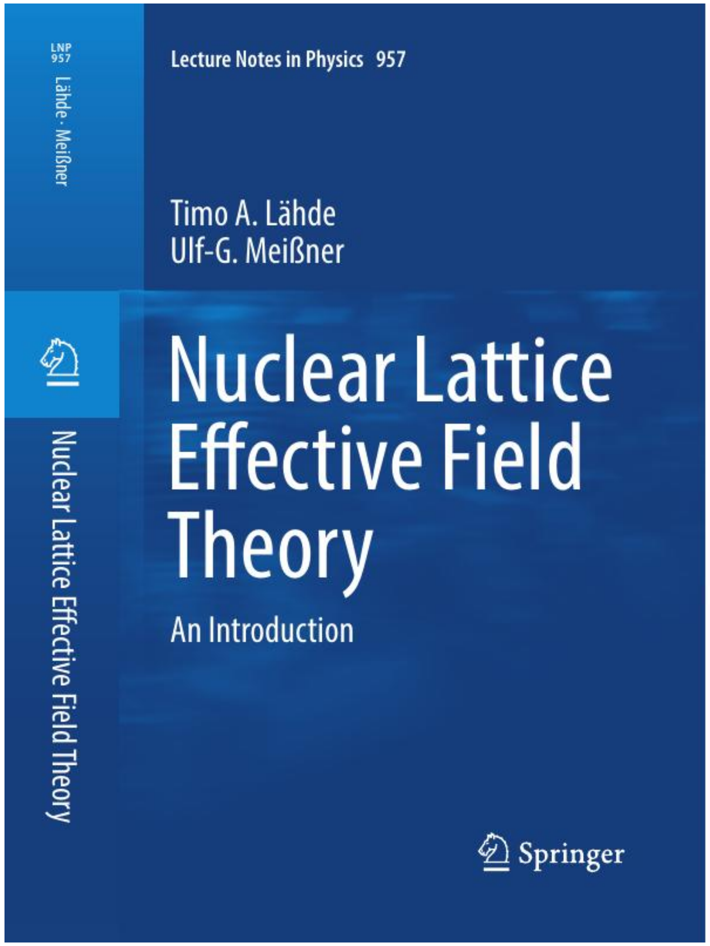
LQCD (quarks & gluons)	NLEFT (nucleons & pions)
relativistic fermions	non-relativistic fermions
renormalizable th'y	EFT
continuum limit	no continuum limit
(un)physical masses	physical masses
Coulomb - difficult	Coulomb - easy
high T/small ρ	small T/nuclear densities
sign problem severe	sign problem moderate



- similar methods:
  hybrid MC, parallel computing, . . .
  ↪ not treated here

- what I want to discuss within the time limitations:
  - ↪ how to put the chiral EFT on a lattice
  - ↪ show some recent results
  - ↪ chiral EFT for baryon-baryon interactions
  - ↪ Impurity lattice MC (ILMC)
  - ↪ Applications in condensed matter and hypernuclear physics

Chiral EFT on a lattice



T. Lähde & UGM

Nuclear Lattice Effective Field Theory - An Introduction

Springer Lecture Notes in Physics **957** (2019) 1 - 396

NUCLEAR LATTICE EFFECTIVE FIELD THEORY

Frank, Brockmann (1992), Koonin, Müller, Seki, van Kolck (2000) , Lee, Borasoy, Schäfer (2004), . . .
Borasoy, Krebs, Lee, UGM, Nucl. Phys. **A768** (2006) 179; Borasoy, Epelbaum, Krebs, Lee, UGM, Eur. Phys. J. **A31** (2007) 105

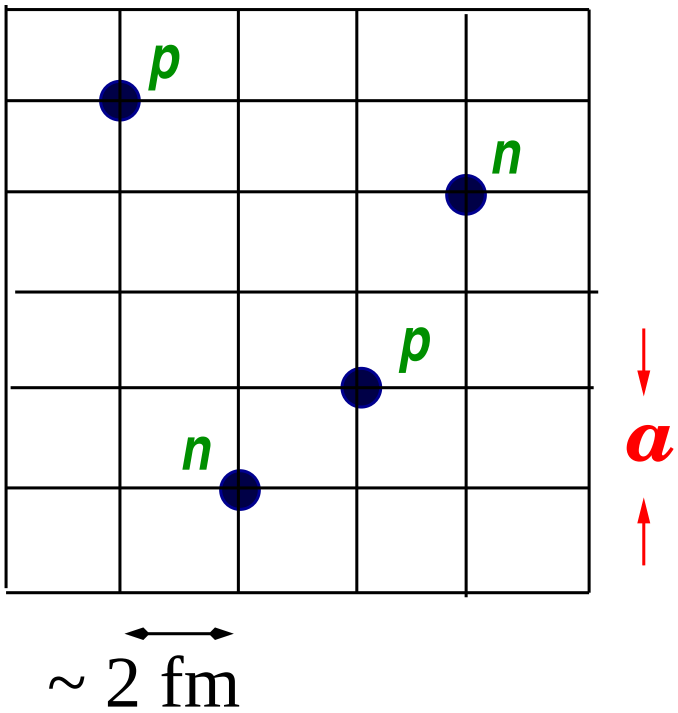
- *new method* to tackle the nuclear many-body problem

- discretize space-time $V = L_s \times L_s \times L_s \times L_t$:
  nucleons are point-like particles on the sites

- discretized chiral potential w/ pion exchanges
  and contact interactions + Coulomb

  $\rightarrow$ see Epelbaum, Hammer, UGM, Rev. Mod. Phys. **81** (2009) 1773

- typical lattice parameters



$$p_{\max} = \frac{\pi}{a} \simeq 314 \text{ MeV [UV cutoff]}$$

- strong suppression of sign oscillations due to approximate Wigner SU(4) symmetry

  E. Wigner, Phys. Rev. **51** (1937) 106; T. Mehen et al., Phys. Rev. Lett. **83** (1999) 931; J. W. Chen et al., Phys. Rev. Lett. **93** (2004) 242302

- physics independent of the lattice spacing for $a = 1 \ldots 2$ fm [$\gtrsim$ N2LO]

  Alarcon, Du, Klein, Lähde, Lee, Li, Lu, Luu, UGM, EPJA **53** (2017) 83; Klein, Elhatisari, Lähde, Lee, UGM, EPJA **54** (2018) 121

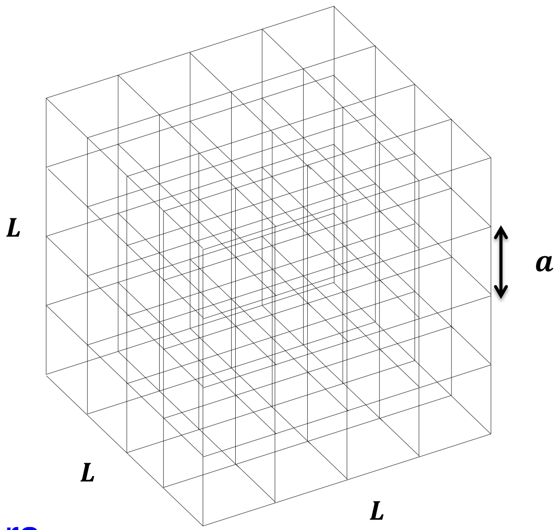
LATTICE NOTATION

- nucleon annihilation ops: $a_{0,0} \equiv a_{\uparrow,p},\; a_{1,0} \equiv a_{\downarrow,p},\; a_{0,1} \equiv a_{\uparrow,n},\; a_{1,1} \equiv a_{\downarrow,n}$

  $\rightarrow$ labeling **spin** and **isospin**

- spatial & temporal lattice spacing: $a,\, a_t \rightarrow \alpha_t \equiv a_t/a$

- lattice size: $L \equiv Na,\, L_t \equiv N_t a_t$



- lattice momenta: $\vec{k} = (k_1, k_2, k_3) \equiv \left(\frac{2\pi}{N}\hat{k}_1, \frac{2\pi}{N}\hat{k}_2, \frac{2\pi}{N}\hat{k}_3\right)$,

  $\rightarrow$ in the first Brillouin zone: $|k_i| < \pi$ and $0 \leq |\hat{k}_i| < N/2$

- any derivative operator requires *improvement*, as the simplest representation in terms of two neighboring points is afflicted by the largest discretization errors

$$k_l \equiv \sum_{j=1}^{\nu+1} (-1)^{j+1} \theta_{\nu,j} \sin(jk_l) \; + \mathcal{O}(a^{2\nu+2})$$

$$\frac{k_l^2}{2} \equiv \sum_{j=0}^{\nu+1} (-1)^{j} \omega_{\nu,j} \cos(jk_l) \; + \mathcal{O}(a^{2\nu+2})$$

$\hookrightarrow$ no improvement ($\nu = 0$): $\theta_{0,1} = 1,\;\; \omega_{0,0} = 1,\;\; \omega_{0,1} = 1$

LATTICE NOTATION continued

- Order a^2 improvement ($\nu = 1$): $\theta_{1,1} = \frac{4}{3}$, $\theta_{1,2} = \frac{1}{6}$, $\omega_{1,0} = \frac{5}{4}$, $\omega_{1,1} = \frac{4}{3}$, $\omega_{1,2} = \frac{1}{12}$

- Order a^4 improvement ($\nu = 2$): $\theta_{2,1} = \frac{3}{2}$, $\theta_{2,2} = \frac{3}{10}$, $\theta_{2,3} = \frac{1}{30}$

  $\omega_{2,0} = \frac{49}{36}$, $\omega_{2,1} = \frac{3}{2}$, $\omega_{2,2} = \frac{3}{20}$, $\omega_{2,3} = \frac{1}{90}$

$\hookrightarrow$ definition of the first order spatial derivative:

$$\nabla_{l,(\nu)} f(\vec{n}) \equiv \frac{1}{2} \sum_{j=1}^{\nu+1} (-1)^{j+1} \theta_{\nu,j} \left[f(\vec{n} + j\hat{e}_l) - f(\vec{n} - j\hat{e}_l) \right]$$

$\hookrightarrow$ second order spatial derivative:

$$\tilde{\nabla}^2_{l,(\nu)} f(\vec{n}) \equiv - \sum_{j=0}^{\nu+1} (-1)^{j} \omega_{\nu,j} \left[f(\vec{n} + j\hat{e}_l) + f(\vec{n} - j\hat{e}_l) \right]$$

has two zeros in per Brillouin zone $\longrightarrow$ beneficial feature for tuning NLO coefficients

$\hookrightarrow$ improved lattice dispersion relation: $\omega^{(\nu)}(\vec{p}) \equiv \frac{1}{\tilde{m}_N} \sum_{j=0}^{\nu+1} \sum_{l=1}^{3} (-1)^j \omega_{\nu,j} \cos(jp_l)$

$$\boxed{\tilde{m}_N \equiv m_N a}$$

TRANSFER MATRIX METHOD

- Correlation–function for A nucleons: $Z_A(\tau) = \langle \Psi_A | \exp(-\tau H) | \Psi_A \rangle$

  with Ψ_A a Slater determinant for A free nucleons

  [or a more sophisticated (correlated) initial/final state]

Euclidean time

- Transient energy

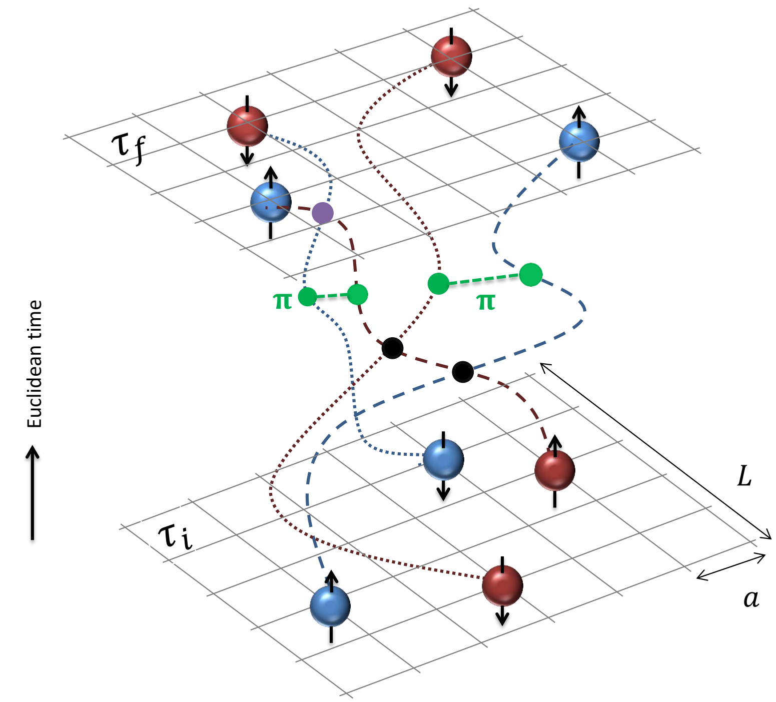
$$E_A(\tau) = -\frac{d}{d\tau} \ln Z_A(\tau)$$

$\rightarrow$ ground state: $E_A^0 = \lim_{\tau \to \infty} E_A(\tau)$

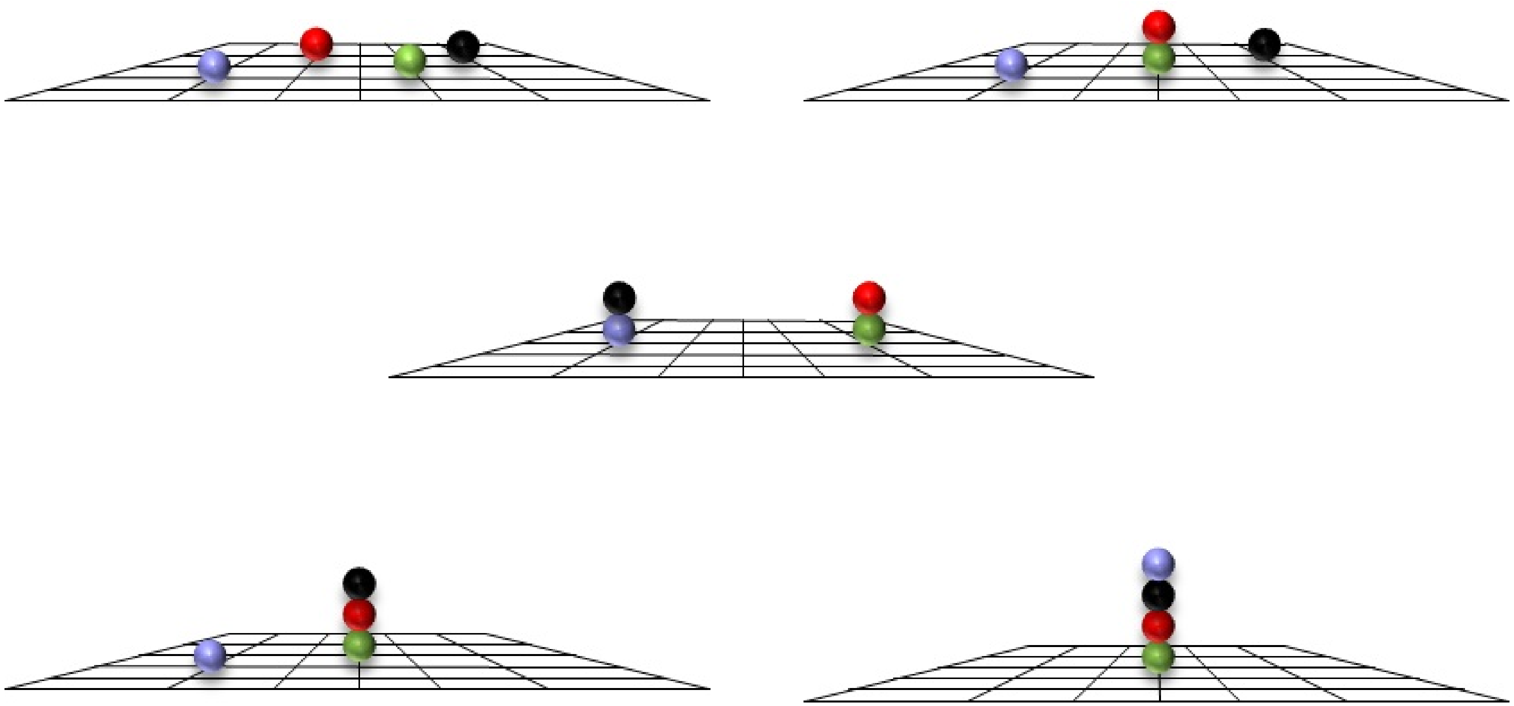
- Exp. value of any normal–ordered operator $\mathcal{O}$

$$Z_A^{\mathcal{O}} = \langle \Psi_A | \exp(-\tau H/2)\, \mathcal{O}\, \exp(-\tau H/2)\, | \Psi_A \rangle$$

$$\lim_{\tau \to \infty} \frac{Z_A^{\mathcal{O}}(\tau)}{Z_A(\tau)} = \langle \Psi_A | \mathcal{O} \, | \Psi_A \rangle$$



CONFIGURATIONS



⇒ all *possible* configurations are sampled

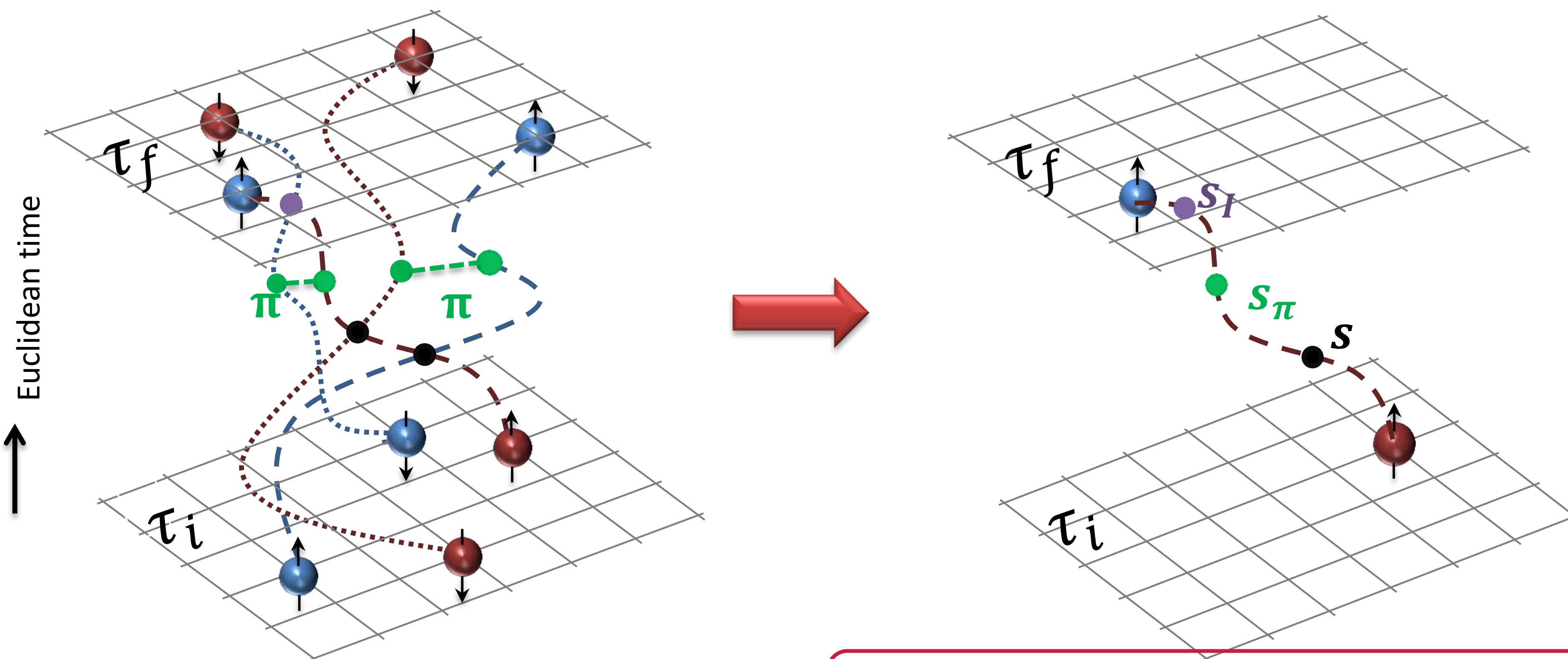
⇒ preparation of *all possible* initial/final states

⇒ *clustering* emerges *naturally*

AUXILIARY FIELD METHOD

- Represent interactions by auxiliary fields:

$$\exp\left[-\frac{C}{2}\left(N^\dagger N\right)^2\right] = \sqrt{\frac{1}{2\pi}}\int ds \exp\left[-\frac{s^2}{2} + \sqrt{C}\, s\left(N^\dagger N\right)\right]$$



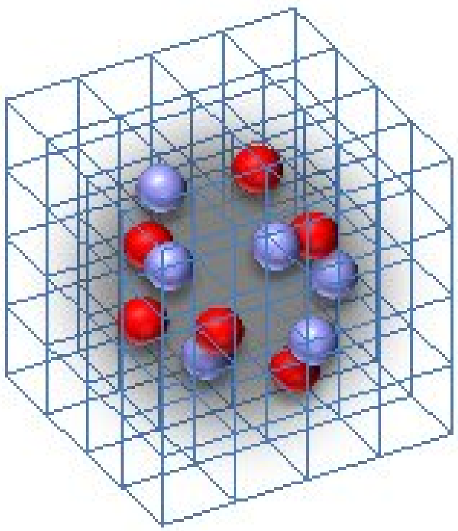
optimally suited for parallel computing!

COMPUTATIONAL EQUIPMENT

- Past = JUQUEEN (BlueGene/Q)
- Present = JUWELS (modular system) + SUMMIT + ...

12 Pflops

Assorted recent results



NLEFT

ENERGIES for SELECTED NUCLEI

Lu, Li, Elhatisari, Epelbaum, Lee, UGM. Phys. Lett. B **797** (2019) 134863 [arXiv:1812.10928]

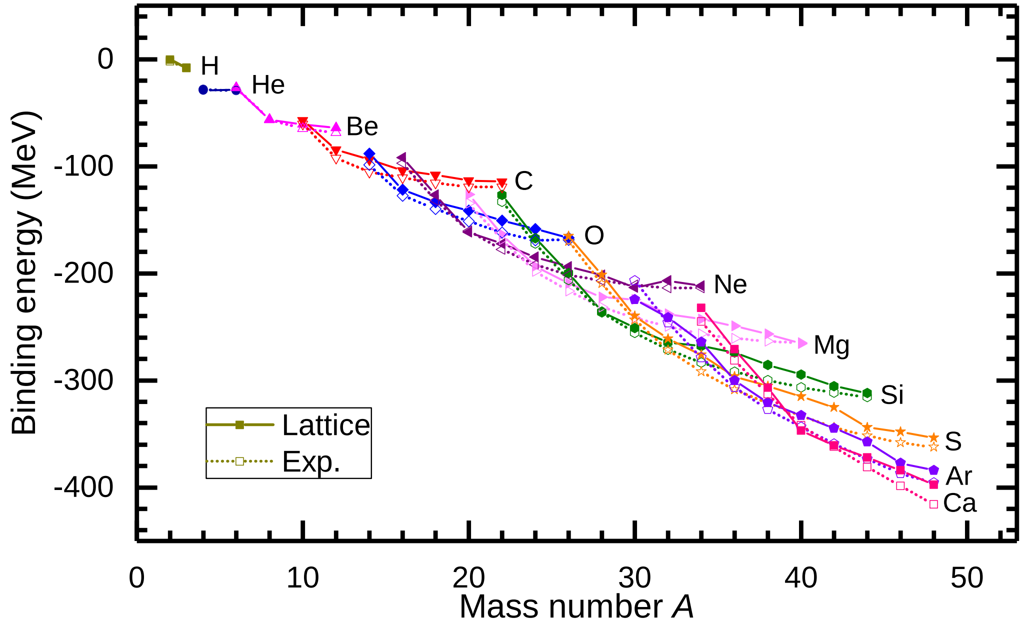
- Highly improved SU(4) symmetric LO action with 2- and 3-nucleon forces

- Calculated binding energies for 3N & alpha-type nuclei:

	B [MeV]	Coul. [MeV]	B/Exp.
^{3}H	8.48(2)*	0.0	1.00
^{3}He	7.75(2)	0.73(1)	1.00
^{4}He	28.89(1)	0.80(1)	1.02
^{16}O	121.9(3)	13.9(1)	0.96
^{20}Ne	161.6(1)	20.2(1)	1.01
^{24}Mg	193.5(17)	28.0(2)	0.98
^{28}Si	235.8(17)	37.1(4)	1.00
^{40}Ca	346.8(8)	71.7(6)	1.01

[* = input]

- Binding energies for 86 even-even nuclei



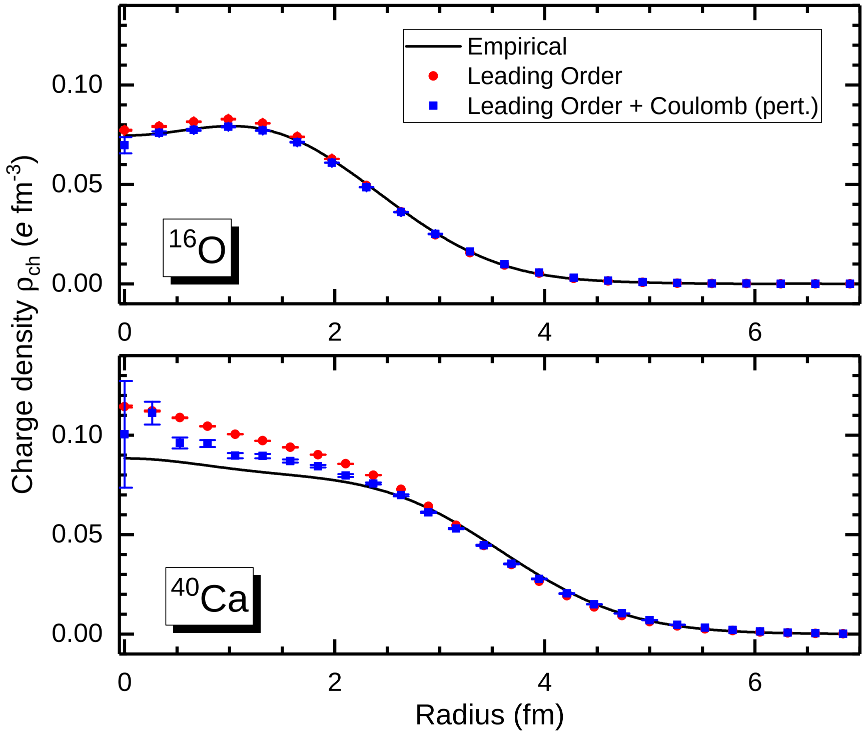
- Selected nuclei: amazingly precise, largest deviation about 9% in ^{12}C
- Even-even isotopic chains come out amazingly precise, general trends reproduced
- But remember: this is only leading order! work on N3LO corrections under way...

RADII for SELECTED NUCLEI

- Calculated charge radii for 3N & alpha-type nuclei:

	R_{ch}	Exp.	R_{ch}/Exp.
^{3}H	1.90(1)	1.76	1.08
^{3}He	1.99(1)	1.97	1.01
^{4}He	1.72(3)	1.68	1.02
^{16}O	2.74(1)	2.70	1.01
^{20}Ne	2.95(1)	3.01	0.98
^{24}Mg	3.13(2)	3.06	1.02
^{28}Si	3.26(1)	3.12	1.04
^{40}Ca	3.42(3)	3.48	0.98

- Charge distributions:



- Radii quite well described

$\hookrightarrow$ overcomes earlier problems (see PRL 109 (2012) 252501, 112 (2014) 102501)

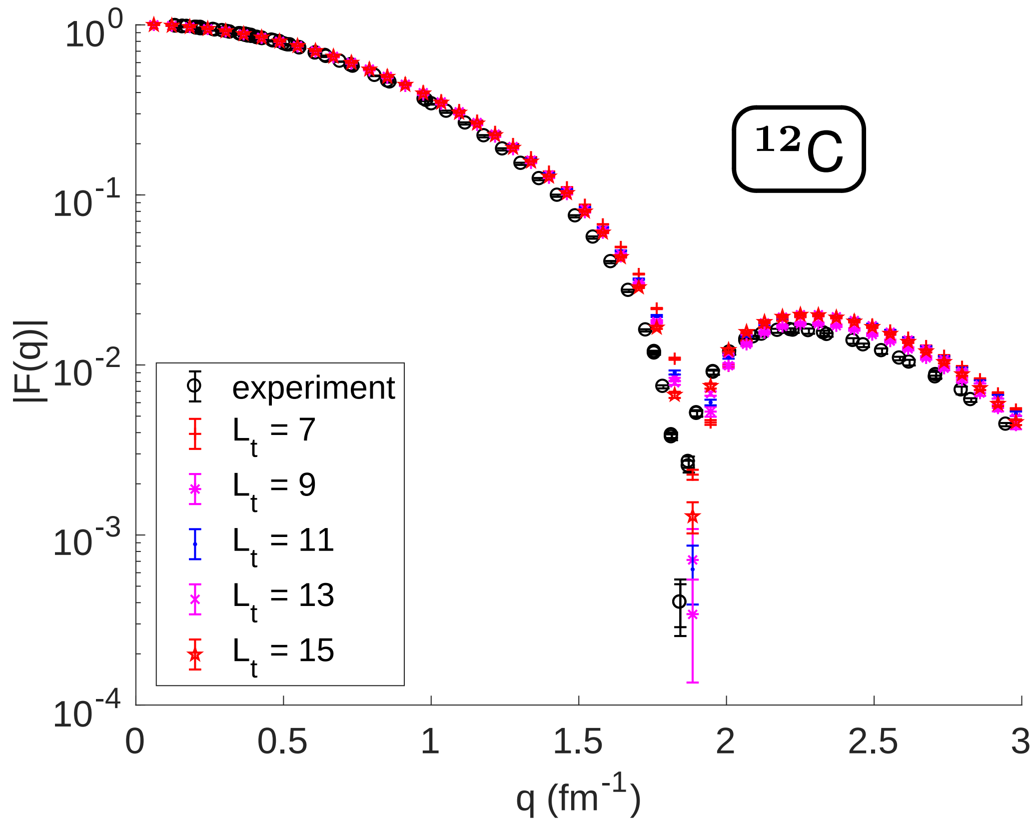
- Also a fair description of the charge distributions at LO!

- Form factors can also be calculated (pinhole algorithm)

FORM FACTORS

- Fit charge distributions by a Wood-Saxon shape
  $\hookrightarrow$ get the form factor from the Fourier-transform (FT)
  $\hookrightarrow$ uncertainties from a direct FT of the lattice data

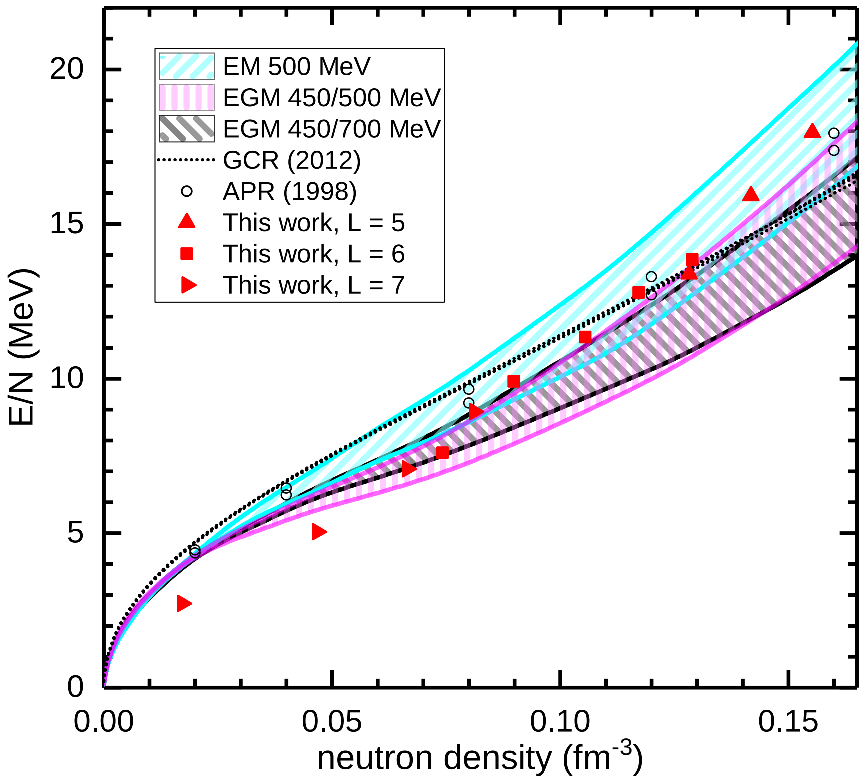
Elhatisari et al., Phys. Rev. Lett. 119 (2017) 222505

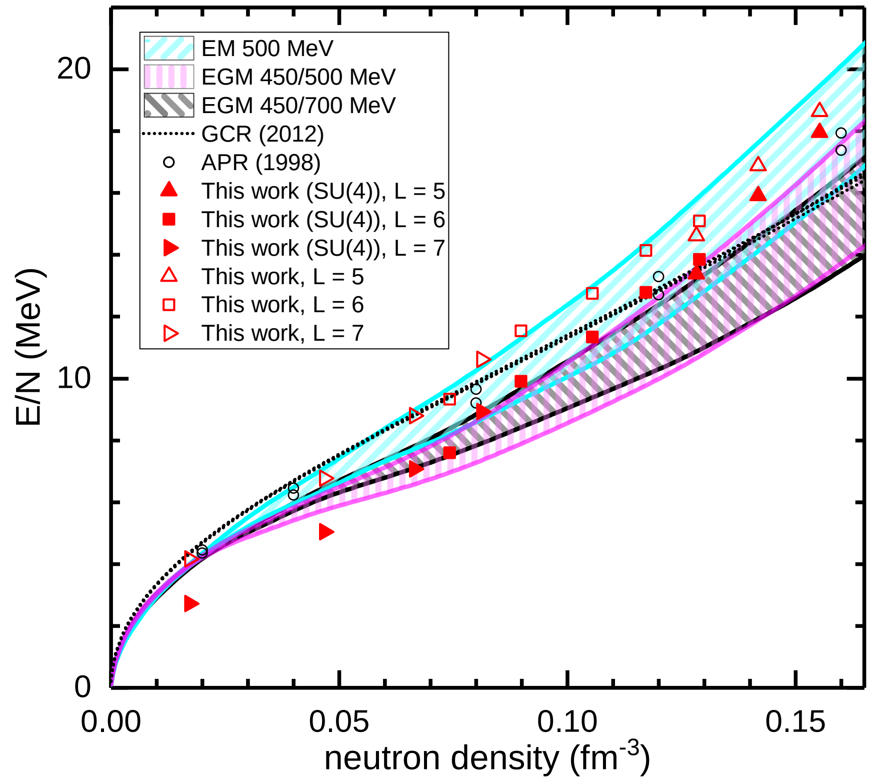


$\Rightarrow$ detailed structure studies become possible

NEUTRON MATTER

- 14 to 66 neutrons in $L = 5, 6, 7 \rightarrow \rho = 0.02 - 0.15\,\text{fm}^{-3}$





- exact SU(4)

$\hookrightarrow$ deviations at low densities

- SU(4) breaking term $\rightarrow a_{nn}$ ✓

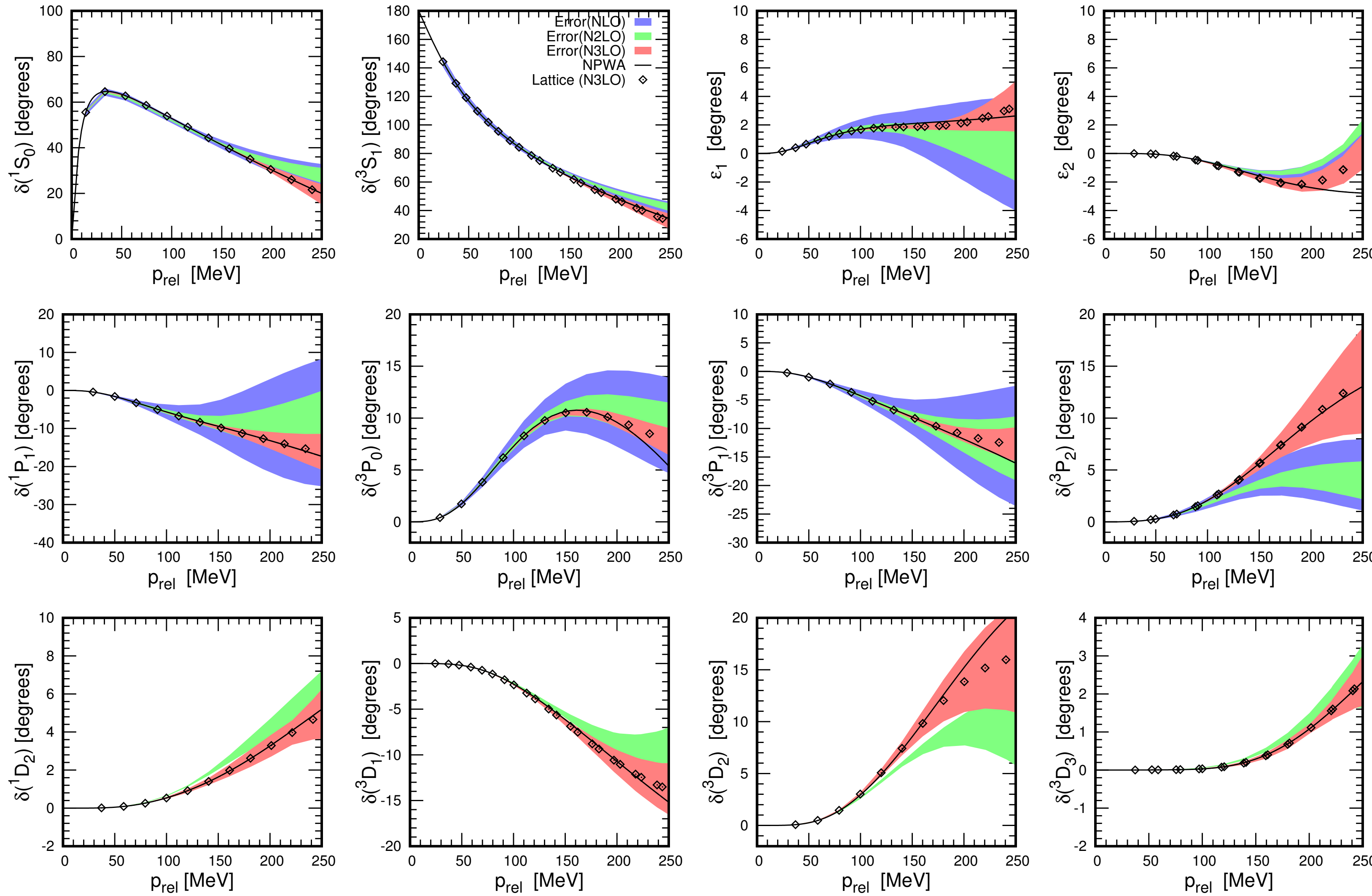
$\hookrightarrow$ good overall description

APR = Akmal, Pandharipande, Ravenhall, Phys. Rev. C **58** (1998) 1804; GCR = Gandolfi, Carlson, Reddy, Phys. Rev. C **85** (2012) 032801; all others in: Tews et al., Phys. Rev. Lett. **110** (2013) 032504.

NN INTERACTION at N3LO

- np phase shifts including uncertainties for $a = 1.64$ fm (cf. Nijmegen PWA)

NLO
N2LO
N3LO

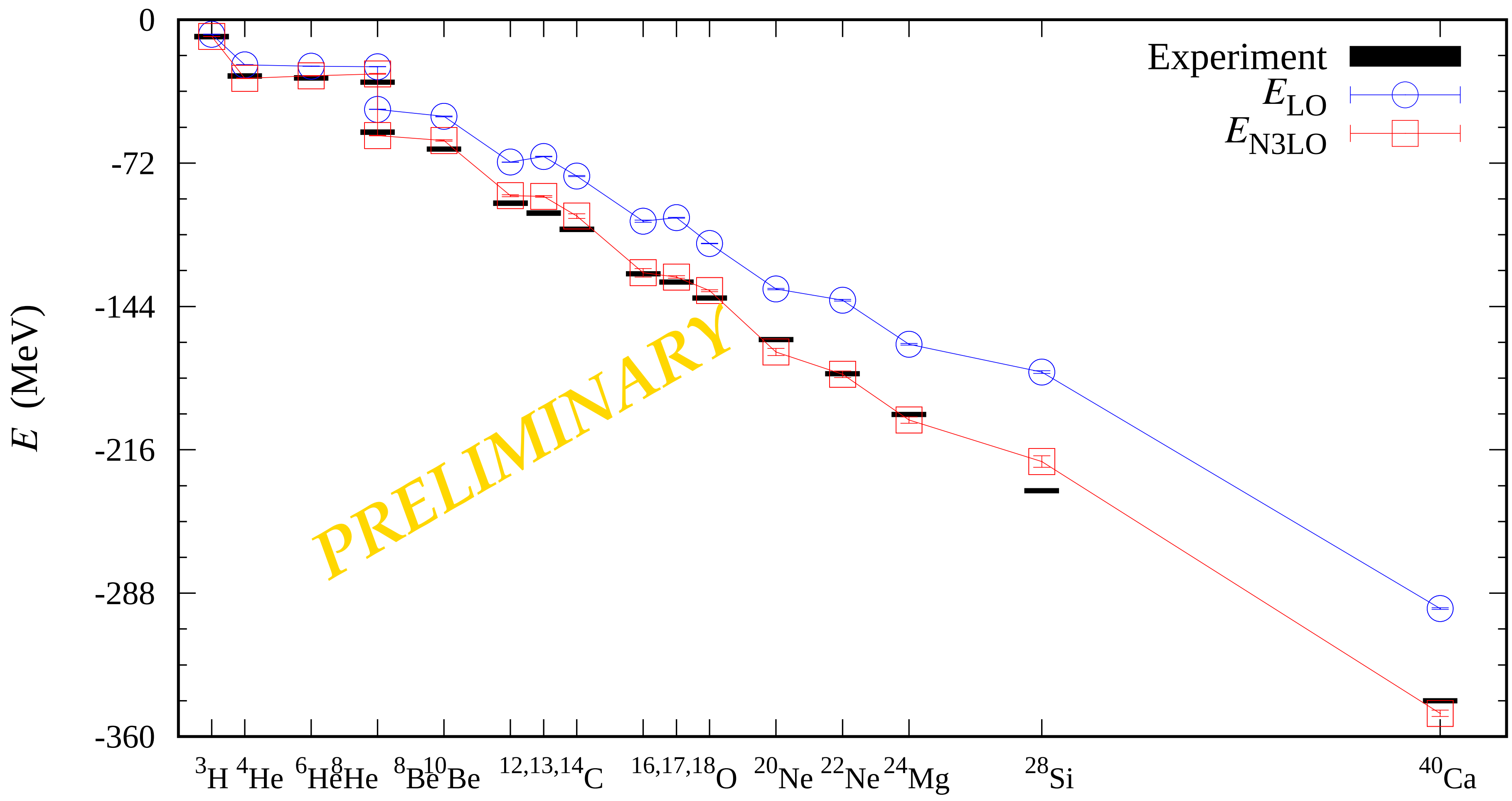


uncertainty estimates á la Epelbaum, Krebs, UGM, Eur. Phys. J. A **51** (2015) 53 [same for other a]

Li et al., Phys. Rev. C **98** (2018) 044002; Phys. Rev. C **99** (2019) 064001

NUCLEI at N3LO

- Binding energies of nuclei for $a = 1.64$ fm [w/o 3NFs]



$\rightarrow$ excellent starting point for precision studies

Hypernuclear physics

STRANGENESS NUCLEAR PHYSICS

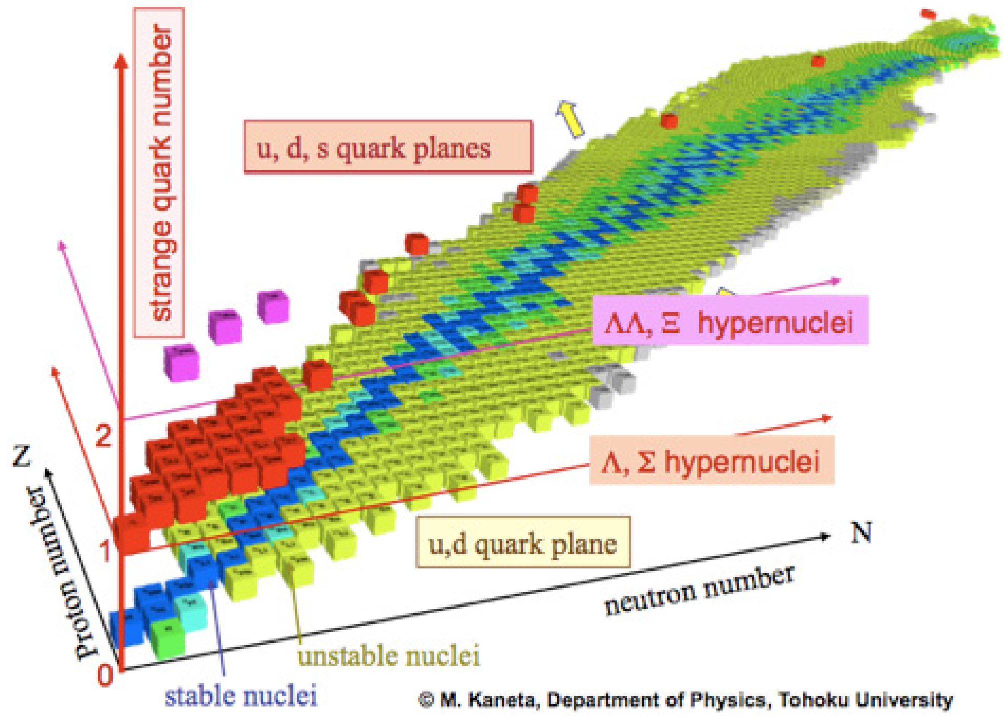
- Substitute one (or two) nucleon(s) by a hyperon (Λ, Σ)

- A few known **hypernuclei**

- Also: very few hyperon-nucleon scattering data

$\Rightarrow$ important role of hypernuclear spectra

$\Rightarrow$ lattice can make an impact!



- Step 1: Crash course on YN/YY scattering in chiral EFT

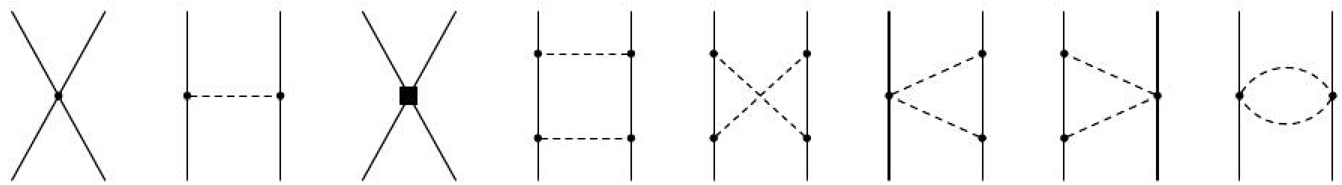
- Step 2: Impurity Lattice Monte Carlo (ILMC) algorithm

- Step 3: Applications in condensed matter and nuclear physics

BARYON-BARYON INTERACTIONS in CHIRAL EFT

LO: Polinder, Haidenbauer, UGM, Nucl. Phys. A **779** (2006) 244

NLO: Haidenbauer, Petschauer, Kaiser, UGM, Nogga, Weise, Nucl. Phys. A **915** (2013) 24



- Goldstone boson octet interacts with the the ground-state baryon octet and via contact interactions (just like NN)

$$M = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta}{\sqrt{6}} \end{pmatrix} \qquad B = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ -\Xi^- & \Xi^0 & -\frac{2\Lambda}{\sqrt{6}} \end{pmatrix}$$

- Use SU(3) symmetry to relate MBB couplings and the various contact term LECs

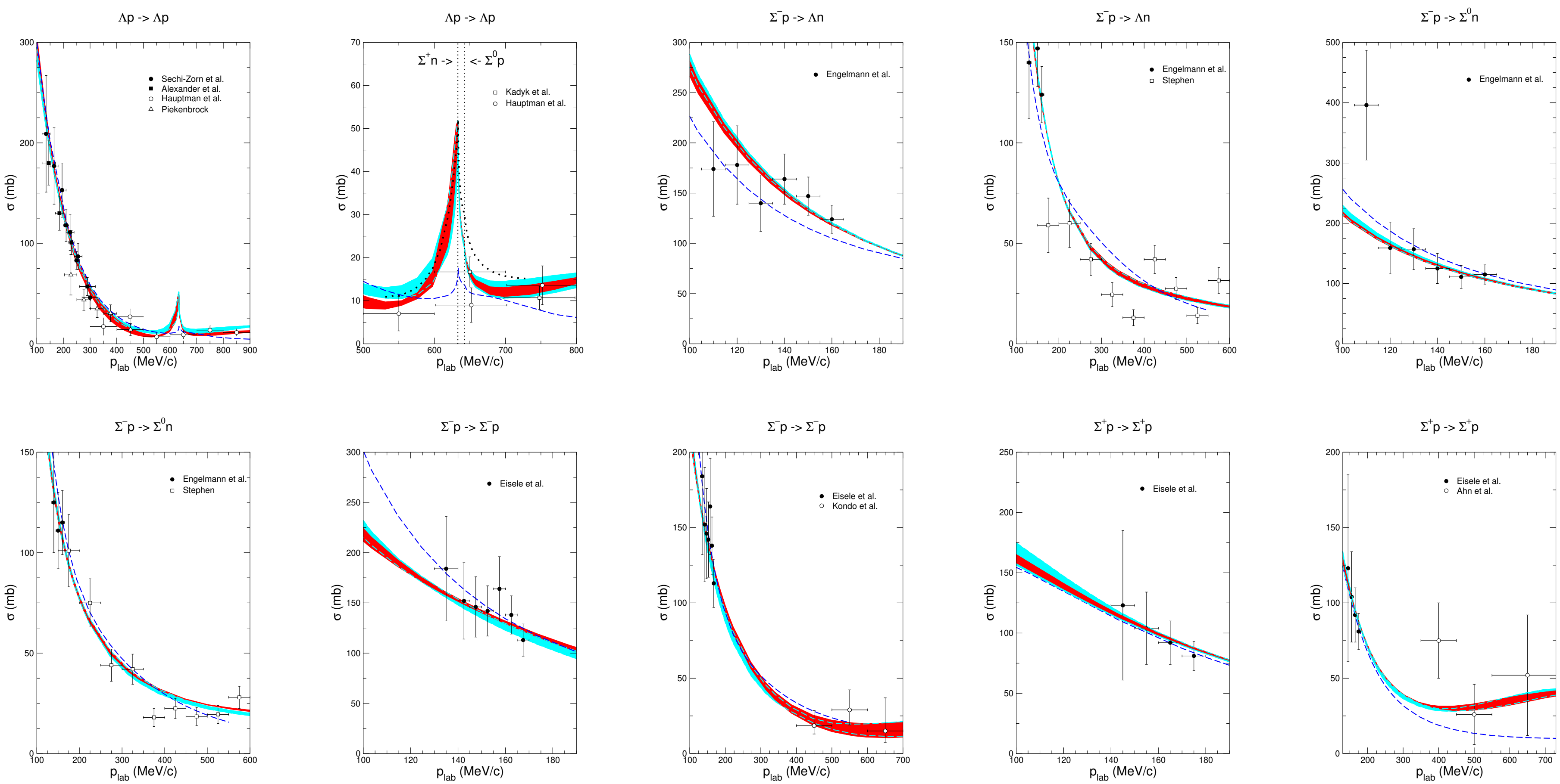
- Need SU(3) breaking for a combined description of NN and YN interactions

- 5 (6) LECs contribute to YN (YY) scattering at LO

BARYON-BARYON INTERACTIONS in CHIRAL EFT

- Total XS results (fit to 36 low-energy data points, only cut-off variations)

[better uncertainty estimate available for NLO19]



NLO13

NLO19

J'04

closed symbols: fit
open symbols: prediction

Jülich '04 potential: Haidenbauer and UGM, Phys. Rev. C **72** (2005) 044005

HYPERON-NUCLEON INTERACTIONS in LIGHT NUCLEI

- Separation energies in light hyper-nuclei (all in MeV)

YN interaction	$E_\Lambda(^3_\Lambda\mathrm{H})$	$E_\Lambda(^4_\Lambda\mathrm{He}(0^+))$	$E_\Lambda(^4_\Lambda\mathrm{He}(1^+))$
NLO13(500)	0.135	1.705	0.790
NLO13(550)	0.097	1.503	0.586
NLO13(600)	0.090	1.477	0.580
NLO13(650)	0.087	1.490	0.615
NLO19(500)	0.100	1.643	1.226
NLO19(550)	0.094	1.542	1.239
NLO19(600)	0.091	1.462	1.055
NLO19(650)	0.095	1.530	0.916
Jülich'04	0.046	1.704	2.312
Expt.	0.13(5)	2.39(3)	0.98(3)

- NLO13 as described before

- NLO19: make use of explicit SU(3) breaking contact terms at NLO
  - $\hookrightarrow$ remedy friction between the NN and YN S-waves
  - $\hookrightarrow$ perform improved uncertainty analysis

Haidenbauer, UGM, Nogga, Eur.Phys.J.A **56** (2020) 91

LATTICE FORMULATION

- Simpler physics as there are no unnaturally large scattering lengths
  (as far as they are known)

- Formulation as for the NN is possible, spin-flavor matrices:

$$\begin{pmatrix} a_{0,0} \\ a_{1,0} \\ a_{0,1} \\ a_{1,1} \\ a_{0,2} \\ a_{1,2} \\ a_{0,3} \\ a_{1,3} \\ a_{0,4} \\ a_{1,4} \\ a_{0,5} \\ a_{1,5} \end{pmatrix} = \begin{pmatrix} a_{\uparrow,p} \\ a_{\downarrow,p} \\ a_{\uparrow,n} \\ a_{\downarrow,n} \\ a_{\uparrow,\Lambda} \\ a_{\downarrow,\Lambda} \\ a_{\uparrow,\Sigma^+} \\ a_{\downarrow,\Sigma^+} \\ a_{\uparrow,\Sigma^0} \\ a_{\downarrow,\Sigma^0} \\ a_{\uparrow,\Sigma^-} \\ a_{\downarrow,\Sigma^-} \end{pmatrix}$$

- LO simulations for the contact interactions
  $\Rightarrow$ feasible, LECs fitted to threshold ratios
  $\Rightarrow$ volume dependence of the scattering lengths
  consistent with the Lüscher formula

  S. Bour, diploma thesis, Bonn, 2009

- however, no follow-up due to missing SU(4) Wigner symmetry

  $\hookrightarrow$ too little control on the sign oscillations
  (expectation, not a calculation)

- is there another method to deal with hyperons in nuclei?

IMPURITY MONTE CARLO

Elhatisari, Lee, Phys. Rev. C **90** (2014) 046001

- Basic idea: Consider the hyperon(s) as **impurity(ies)** in a sea of nucleons
- Benchmark calculation: a ↓-particle in a sea of ↑-particles ($m_\uparrow = m_\downarrow = m$)
- Lattice Hamiltonian ($H_0 + V$):

$$H_0 = H_0^\uparrow + H_0^\downarrow$$

$$H_0^s = \frac{1}{2m}\sum_{l=1}^{3}\sum_{\vec n}\left[2a_s^\dagger(\vec n)a_s(\vec n) - a_s^\dagger(\vec n)a_s(\vec n+\hat l) - a_s^\dagger(\vec n)a_s(\vec n-\hat l)\right]$$

$$V = C_0\sum_{\vec n}\rho_\uparrow(\vec n)\,\rho_\downarrow(\vec n) \qquad (s=\uparrow,\downarrow)$$

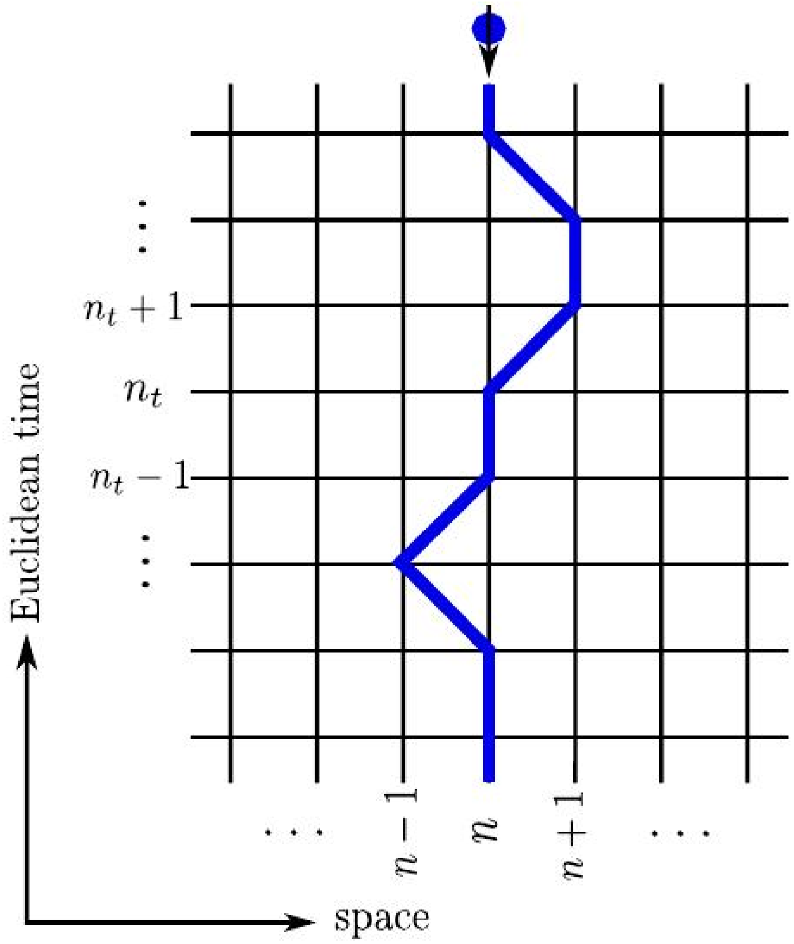
- Work in occupation number basis:

$$\left|\chi_{n_t}^\uparrow, \chi_{n_t}^\downarrow\right\rangle = \prod_{\vec n}\left\{\left[a_\uparrow^\dagger(\vec n)\right]^{\chi_{n_t}^\uparrow(\vec n)}\left[a_\downarrow^\dagger(\vec n)\right]^{\chi_{n_t}^\downarrow(\vec n)}\right\}, \quad \chi_{n_t}^s(\vec n) = 0 \text{ or } 1$$

- allows to calculate the transfer matrix: $\langle\chi_{n_t+1}^\uparrow, \chi_{n_t+1}^\downarrow|\, M\, |\chi_{n_t}^\uparrow, \chi_{n_t}^\downarrow\rangle$

IMPURITY MONTE CARLO continued

- Worldline configuration and the reduced transfer matrix (integrate out the impurity)



- impurity makes one spatial hop:

$$M_{\vec{n}''\pm\hat{l},\vec{n}''} = \left(\frac{\alpha_t}{2m}\right) : \exp\left[-\alpha_t H_0^{\uparrow}\right] :$$

- impurity worldline remains stationary:

$$M_{\vec{n}'',\vec{n}''} = \left(1 - \frac{3\alpha_t}{m}\right)$$
$$\times : \exp\left[-\alpha_t H_0^{\uparrow} - \frac{\alpha_t C_0}{1-3\alpha_t/m}\rho_{\uparrow}(\vec{n}'')\right] :$$

- Induced three-body forces $\sim a$

- can also be extended to the Adiabatic Projection Method $\to$ reactions

Elhatisari et al., Nature **528** (2015) 111

IMPURITY MONTE CARLO: BENCHAMRK CALCULATION

Bour, Lee, Hammer, UGM, Phys. Rev. Lett. **115** (2015) 185301

- $\mathbf{9\,|\!\uparrow\rangle + 1\,|\!\downarrow\rangle\,,\, L = 10^3}$, zero range interaction

- caluclate the ground-state energy:

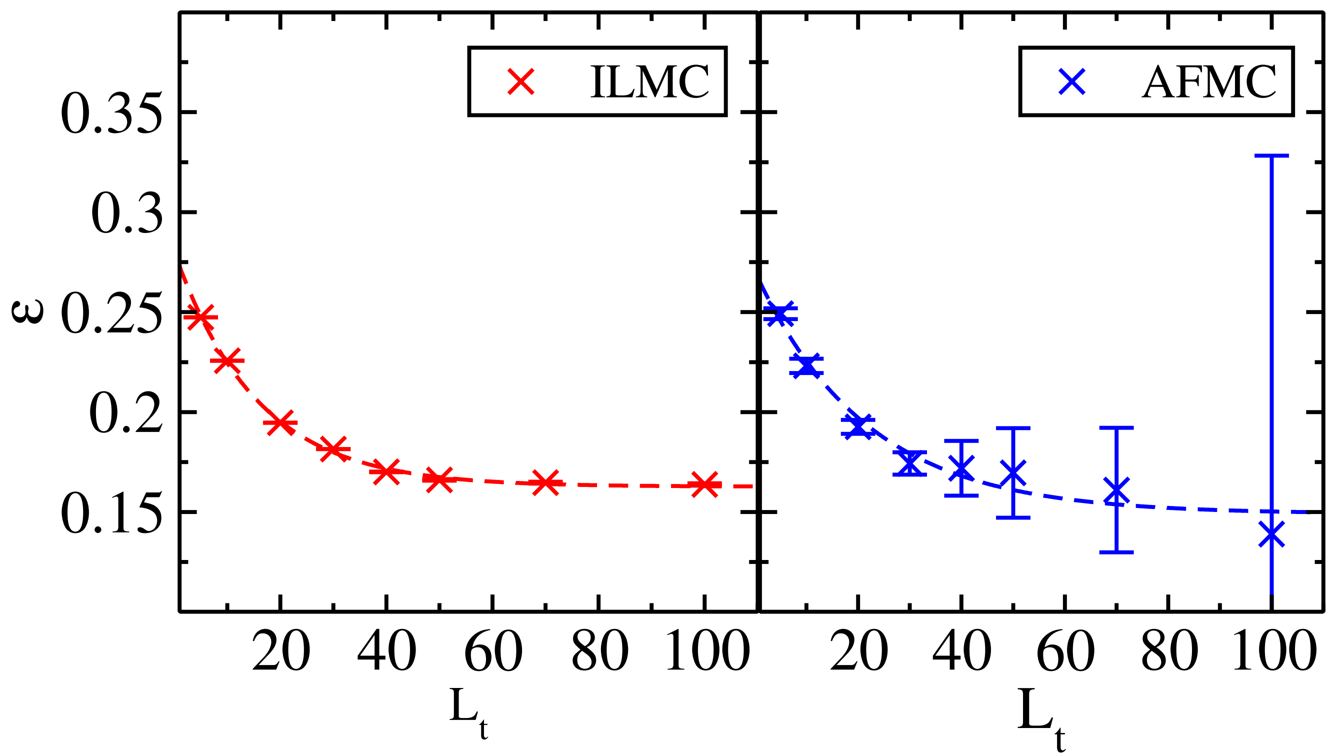
$$\epsilon = \frac{1}{a_t} \lim_{L_t \to \infty} \ln \frac{Z(L_t - 1)}{Z(L_t)}$$

- ILMC outperforms AFMC

$\hookrightarrow$ computationally simpler and faster

$\hookrightarrow$ far smaller sign oscillations

- apply the method to the polaron in two and three dimensions



WHAT is a POLARON?

- Polaron = quasiparticle to understand the electron-atom interactions

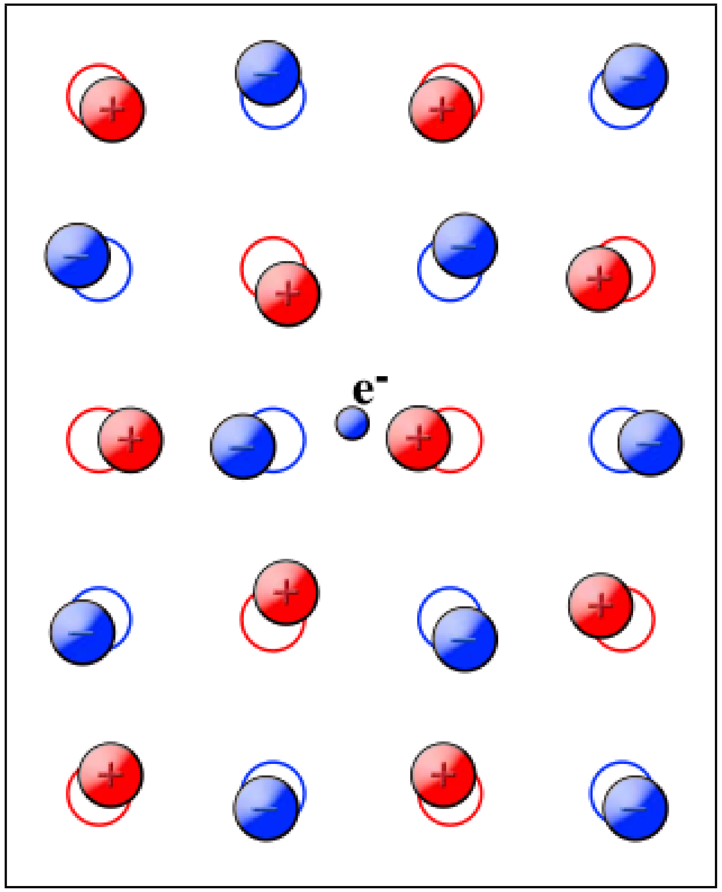
Landau, Phys. Z. Sowjetunion **3** (1933) 644

- consider an electron moving in a dielectric crystal

⇒ the atoms move from their equilibrium positions
to effectively screen the charge of an electron
(phonon cloud)

⇒ this lowers the electron mobility
and increases the electron's effective mass

- for details, see:
J. T. Devreese
Encyclopedia of Applied Physics **14** (1996) 383



@Wikipedia

IMPURITY MONTE CARLO: POLARON RESULTS

Bour, Lee, Hammer, UGM, Phys. Rev. Lett. **115** (2015) 185301

- Energy of the 3D polaron (in units of the Fermi energy) in the unitary limit

- linear fit in $\mathbf{1/N}$ (particle no.):

$$\boldsymbol{\epsilon_P/\epsilon_F = -0.622(9)}$$

- Diagrammatic MC:

$$\boldsymbol{\epsilon_P/\epsilon_F = -0.618}$$

Prokofev, Svistunov, Phys. Rev. B **77** (2008) 020408

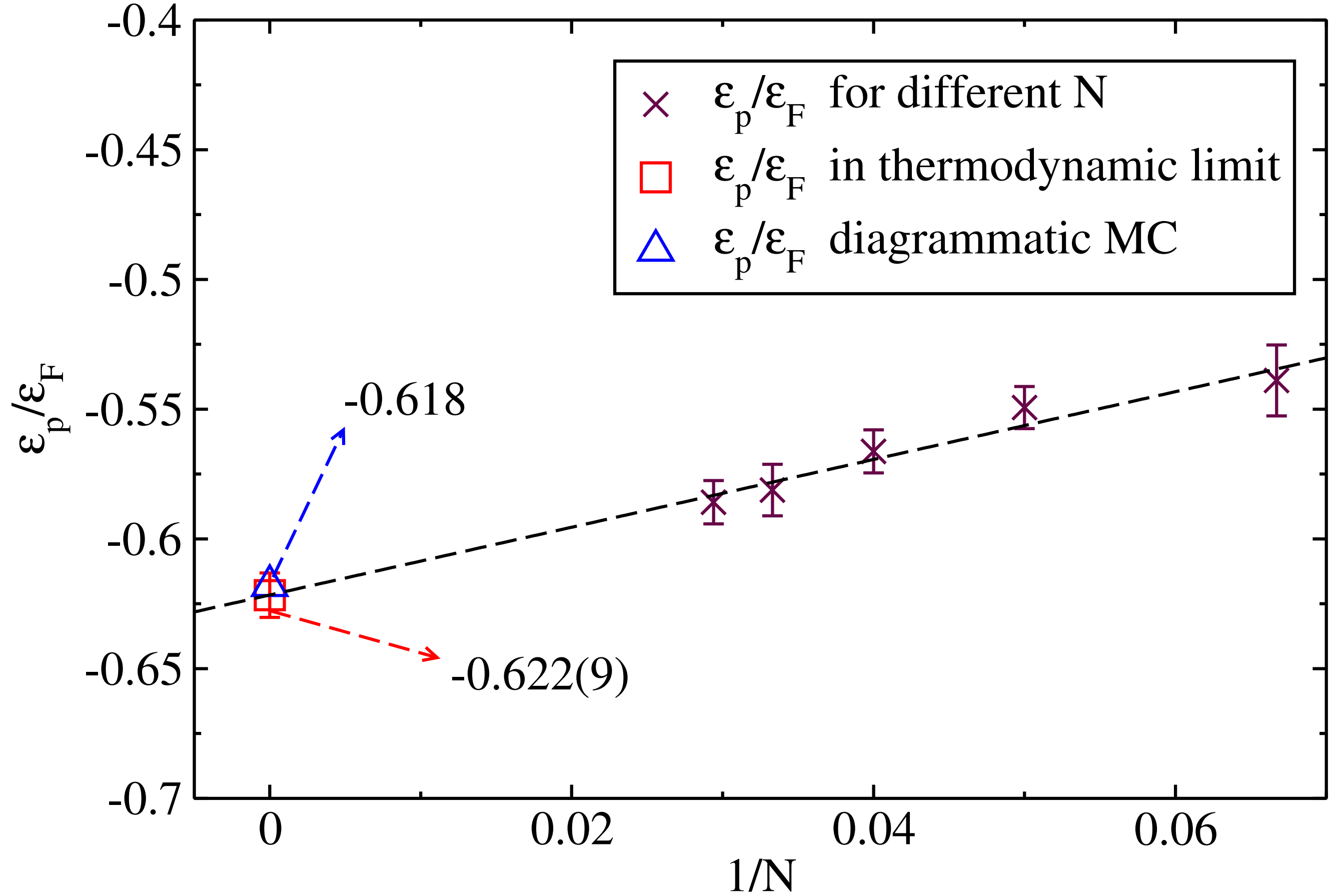
- Experiment:

$$\boldsymbol{\epsilon_P/\epsilon_F = -0.58(5)}$$

$$\boldsymbol{\epsilon_P/\epsilon_F = -0.64(7)}$$

Shin, Phys. Rev. A **77** (2008) 041603

Schirotzek et al., Phys. Rev. Lett. **102** (2009) 023402



IMPURITY MONTE CARLO: POLARON RESULTS

Bour, Lee, Hammer, UGM, Phys. Rev. Lett. **115** (2015) 185301

- Attractive polarons in 2D
  (two-body bound state develops)

- First calculation that covers
  the whole range in η

  $$\eta = \frac{1}{2}\ln(2\epsilon_F/|\epsilon_B|)$$

- good agreement with earlier calculations
  and experiment (where available)

- smooth crossover form the polaron
  to the molecular state (new!) by looking
  at density-density correlations

- ILMC is a **powerful** method

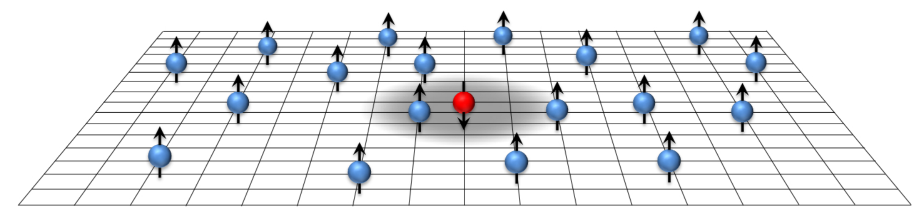
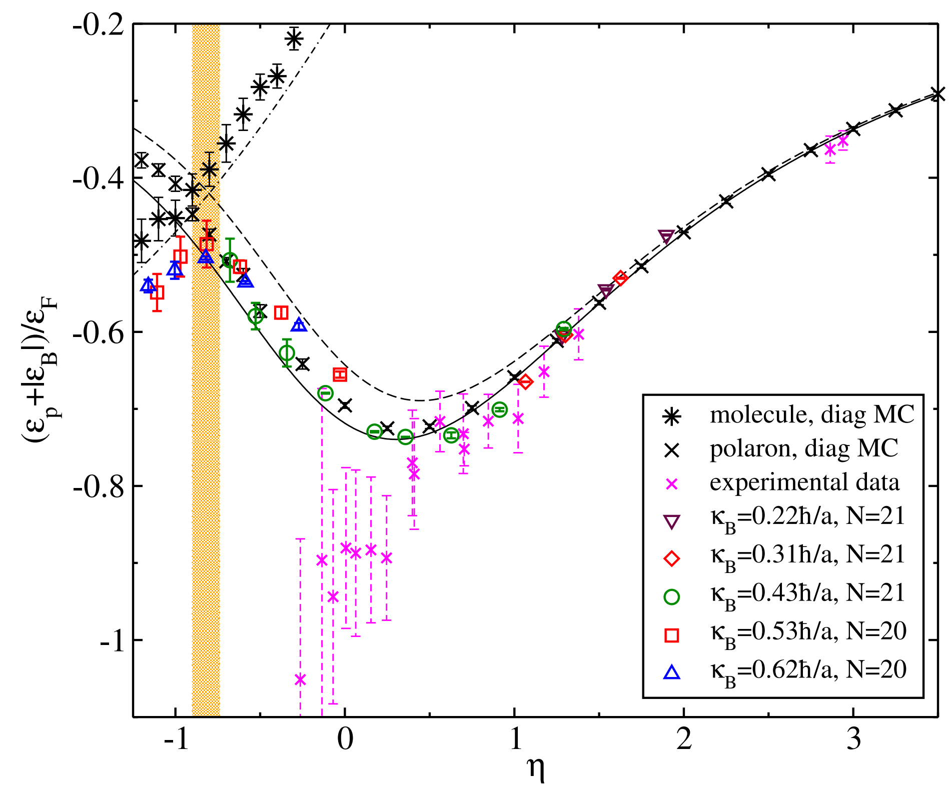


Figure courtesy Dean Lee



S-SHELL HYPERNUCLEI

Frame, Lähde, Lee, UGM, EPJA **56**:248 (2020) [arXiv:2007.06335 [nucl-th]]

- Exploratory study:

  $\hookrightarrow$ LO SU(4) symmetric NN+NNN interaction

  $\hookrightarrow$ simple YN interaction

$$V_{\mathbf{YN}} = C_{\mathbf{YN}} \int d^3r\, \hat{\rho}_Y(\vec{r})\hat{\rho}_N(\vec{r})$$

- average over Wigner SU(4) & Λ spin components

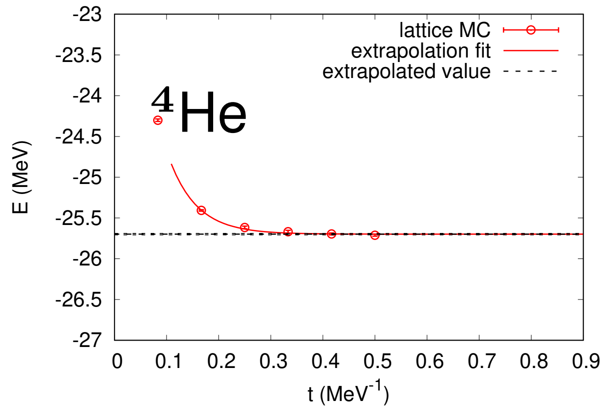
- determine $C_{\mathbf{YN}}$ from an overall fit to the Λ separation energies: $C_{\mathbf{YN}} = 1.6 \cdot 10^{-5}$ MeV^{-2}

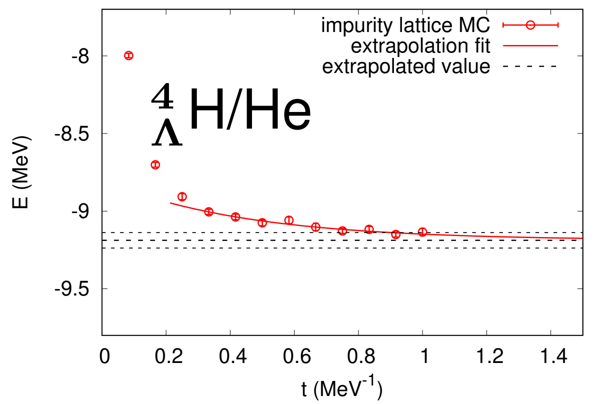
- ${}^3_\Lambda$H calculated exactly (Lanzcos) and with ILMC

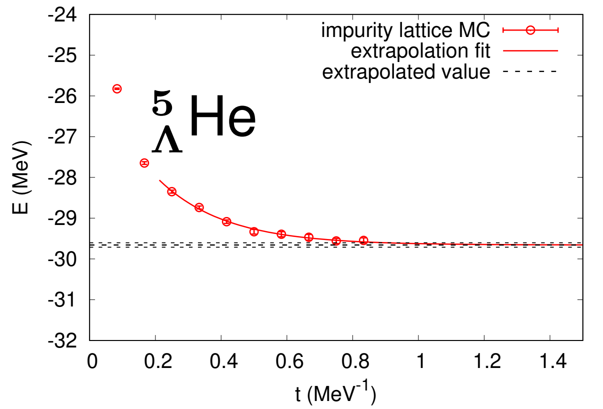
  $\hookrightarrow$ perfect agreement

- Other nuclei/hypernuclei with MC/ILMC

$$E(t) = E_0 + c\exp(-\Delta E\, t)$$







S-SHELL HYPERNUCLEI continued

Frame, Lähde, Lee, UGM, EPJA **56**:248 (2020) [arXiv:2007.06335 [nucl-th]]

- Results for the separation energies (error only from large-t extrapolation):

	B_Λ [MeV]	$B_\Lambda^{\rm exp}$ [MeV]
${}^{3}_{\Lambda}$H	0.22	0.13(5)
${}^{4}_{\Lambda}$H/He	0.46(5)	1.39(4)
${}^{5}_{\Lambda}$He	3.96(6)	3.12(2)

$\Rightarrow$ Good starting point for systematic studies of hypernuclei

$\Rightarrow$ Favorable scaling: $\boxed{t_{\rm CPU} \sim A}$

$\Rightarrow$ Next steps:

– use the (N)LO (realistic) YN interaction

– include the $\Lambda N \leftrightarrow \Sigma N$ conversion

– extend ILMC to double hypernuclei

SUMMARY & OUTLOOK

- Nuclear lattice simulations: a new quantum many-body approach

  $\rightarrow$ based on the successful continuum nuclear chiral EFT

  $\rightarrow$ a number of highly visible results already obtained

- Hypernuclear physics: first steps

  $\rightarrow$ impurity lattice Monte Carlo algorithm

  $\rightarrow$ interesting results in condensed matter systems

  $\rightarrow$ s-shell hypernuclei based on a simple YN contact interaction

  $\rightarrow$ promising proof of principle

- Outlook

  $\rightarrow$ nuclear spectra and reactions to N3LO (high precision)

  $\rightarrow$ hypernuclear physics based on (N)LO YN interactions

  $\rightarrow$ extension of the ILMC algorithm to double hypernuclei

SPARES

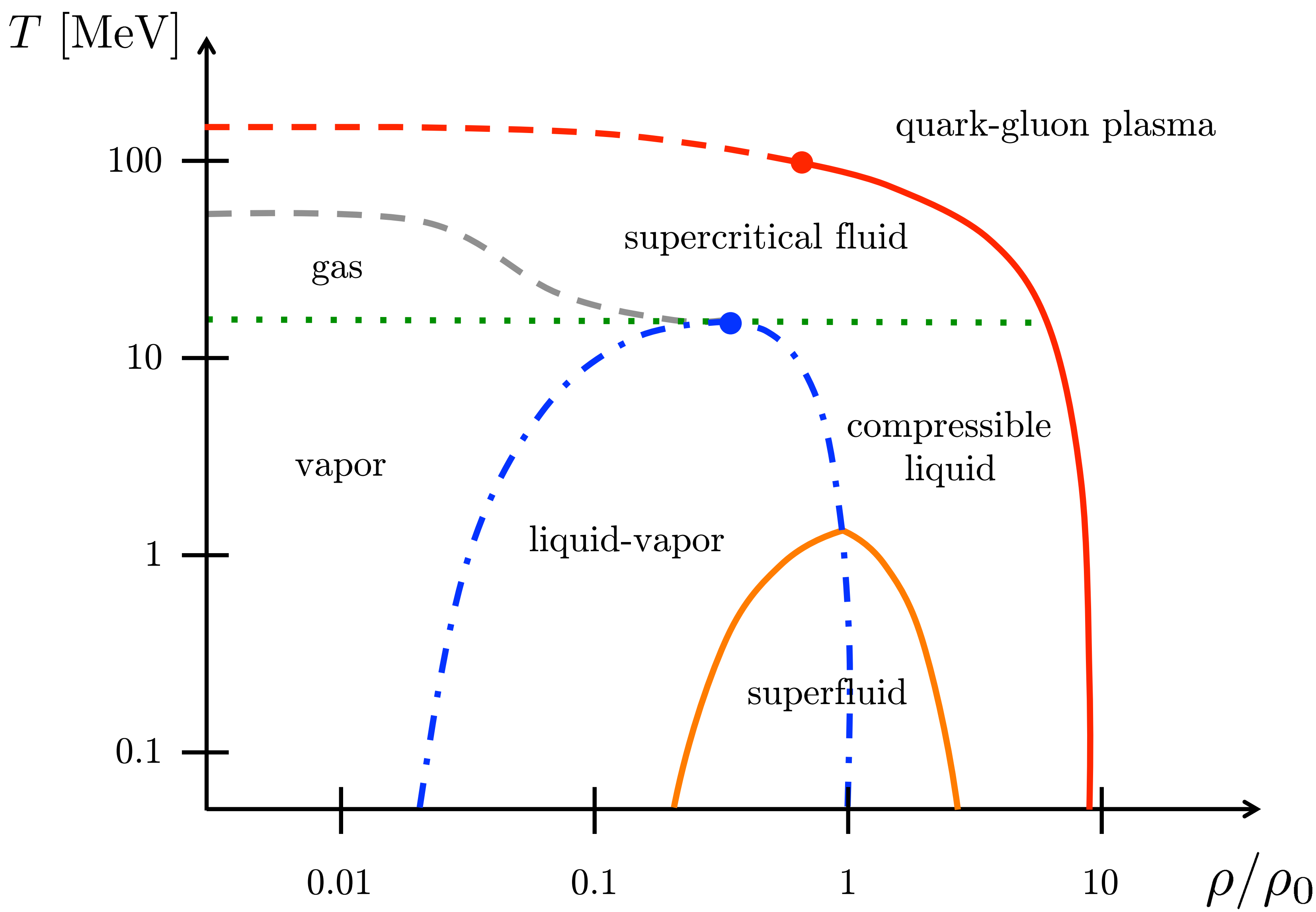
Ab Initio Nuclear Thermodynamics

B. N. Lu, N. Li, S. Elhatisari, D. Lee, J. Drut, T. Lähde, E. Epelbaum, UGM, *Phys. Rev. Lett.* (2020) in press [arXiv:1912.05105]

PHASE DIAGRAM

- Phase diagram of strongly interacting matter

Fig. courtesy B.-N. Lu



NEW PARADIGM for NUCLEAR THERMODYNAMICS

- The PTA allows for simulations with fixed neutron & proton numbers at non-zero T

$\hookrightarrow$ thousand times faster than existing codes using the grand-canonical ensemble:

$$t_{\rm CPU}^{\rm PTA} \sim A^2 V L_t \text{ vs. } t_{\rm CPU}^{\rm BBS} \sim A V^2 L_t$$

Blankenbecler, Scalapino, Sugar, Phys. Rev. D **24** (1981) 2278

- Only a mild sign problem $\rightarrow$ pinholes are dynamically driven to form pairs

- Typical simulation parameters:

up to $N = 144$ nucleons in volumes $L^3 = 4^3, 5^3, 6^3$

$\hookrightarrow$ densities from 0.008 fm^{-3} ... 0.20 fm^{-3}

$a = 1.32$ fm $\rightarrow \Lambda = \pi/a = 470$ MeV , $a_t \simeq 0.1$ fm

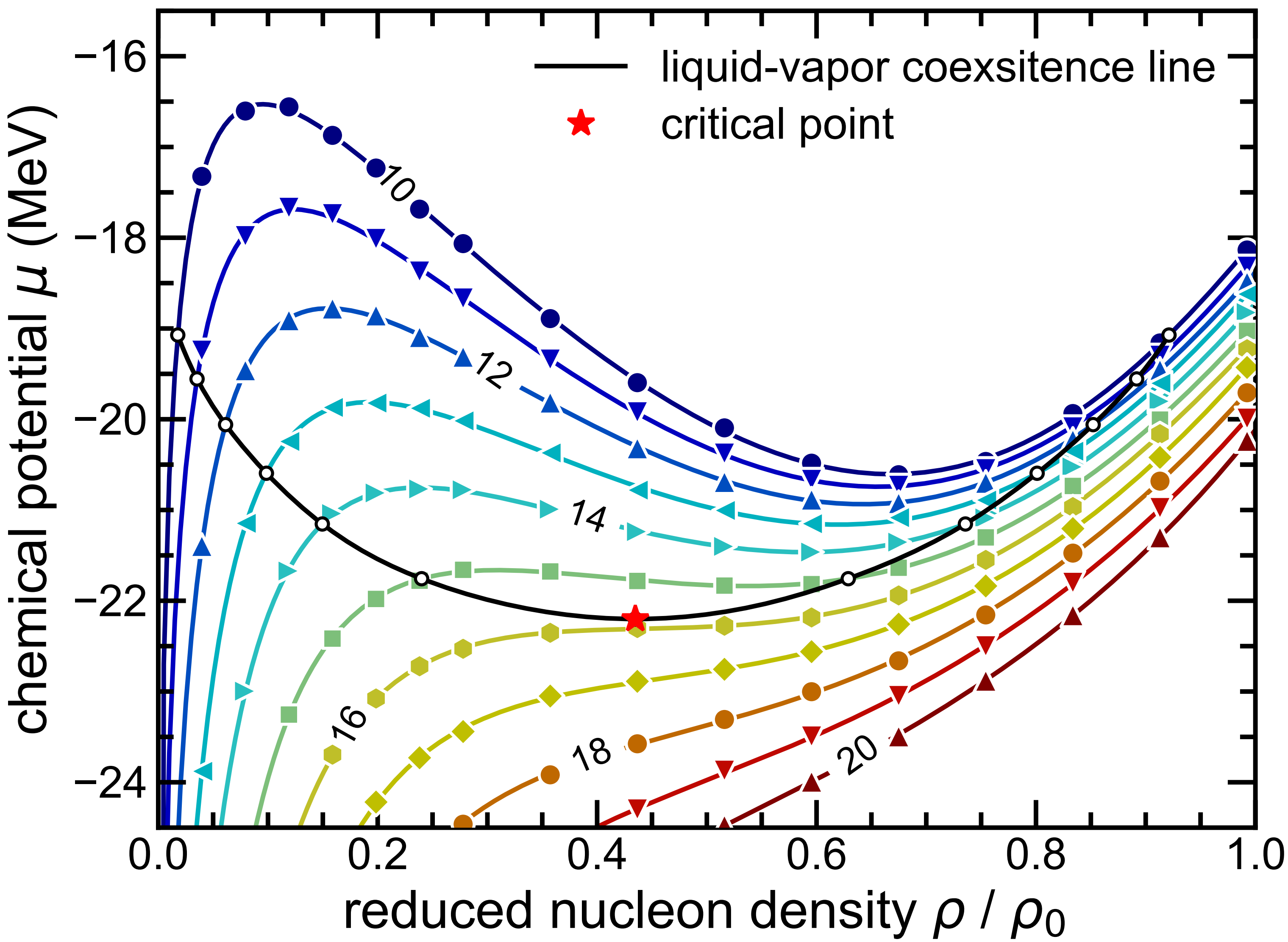
consider $T = 10 \ldots 20$ MeV

- use twisted bc's, average over twist angles $\rightarrow$ acceleration to the td limit

CHEMICAL POTENTIAL

- Calculated from the free energy: $\mu = (F(N+1) - F(N-1))/2$
- Quantum Widom insertion method

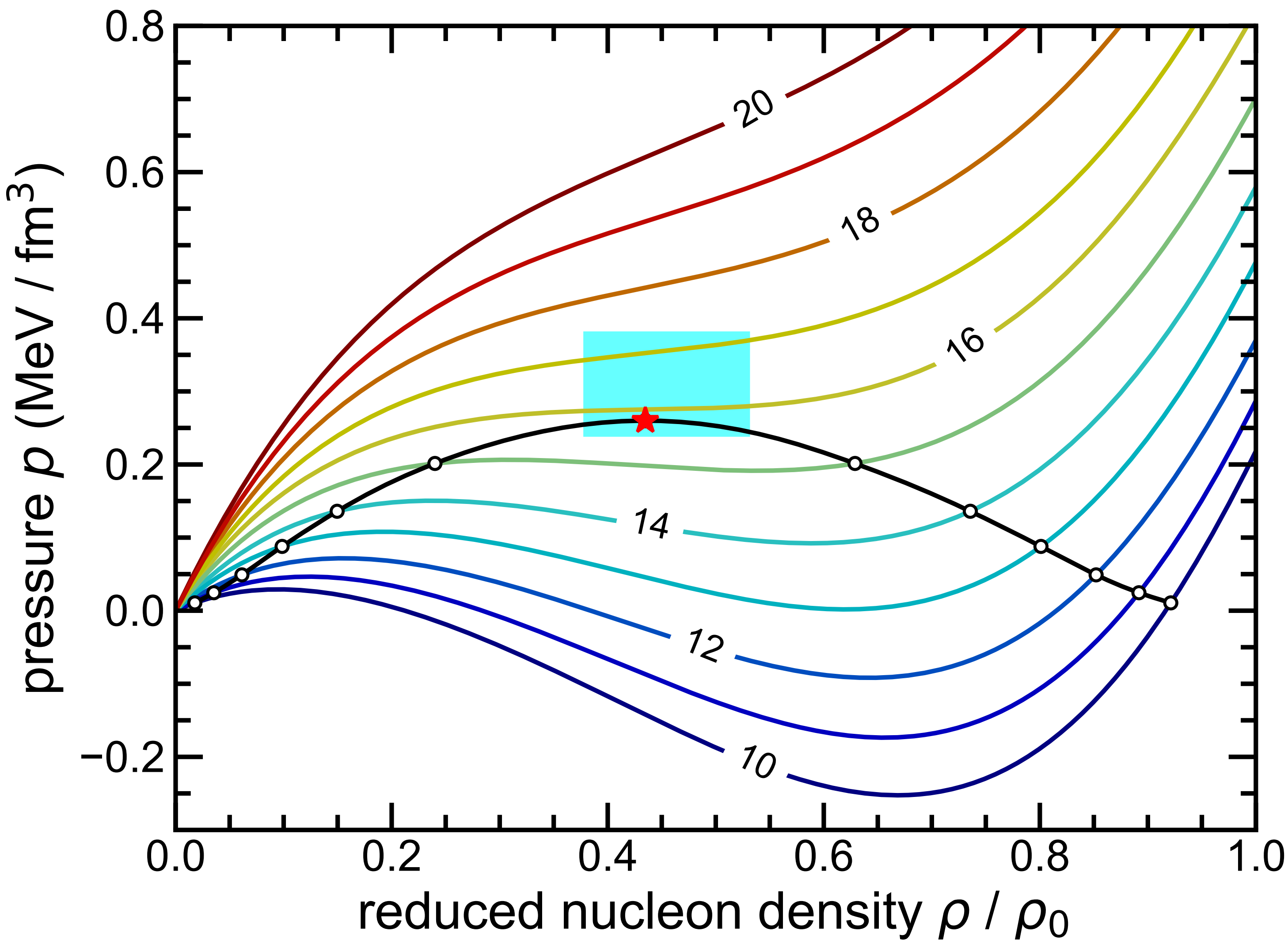
Widom, J. Chem. Phys. **39** (1963) 2808



at very low densities like the ideal gas $\mu \propto \log(\rho)$

EQUATION of STATE

- Calculated by integrating Gibbs-Duhem: $dP = \rho\, d\mu$ w/ $p = \rho = 0$
- Crtitical point: $T_c = 15.8(1.6)$ MeV, $P_c = 0.26(3)$ MeV/fm^3, $\rho_c = 0.089(18)$ fm^{-3}



Experiment: $T_c = 15.0(3)$ MeV, $P_c = 0.31(7)$ MeV/fm^3, $\rho_c = 0.06(2)$ fm^{-3}

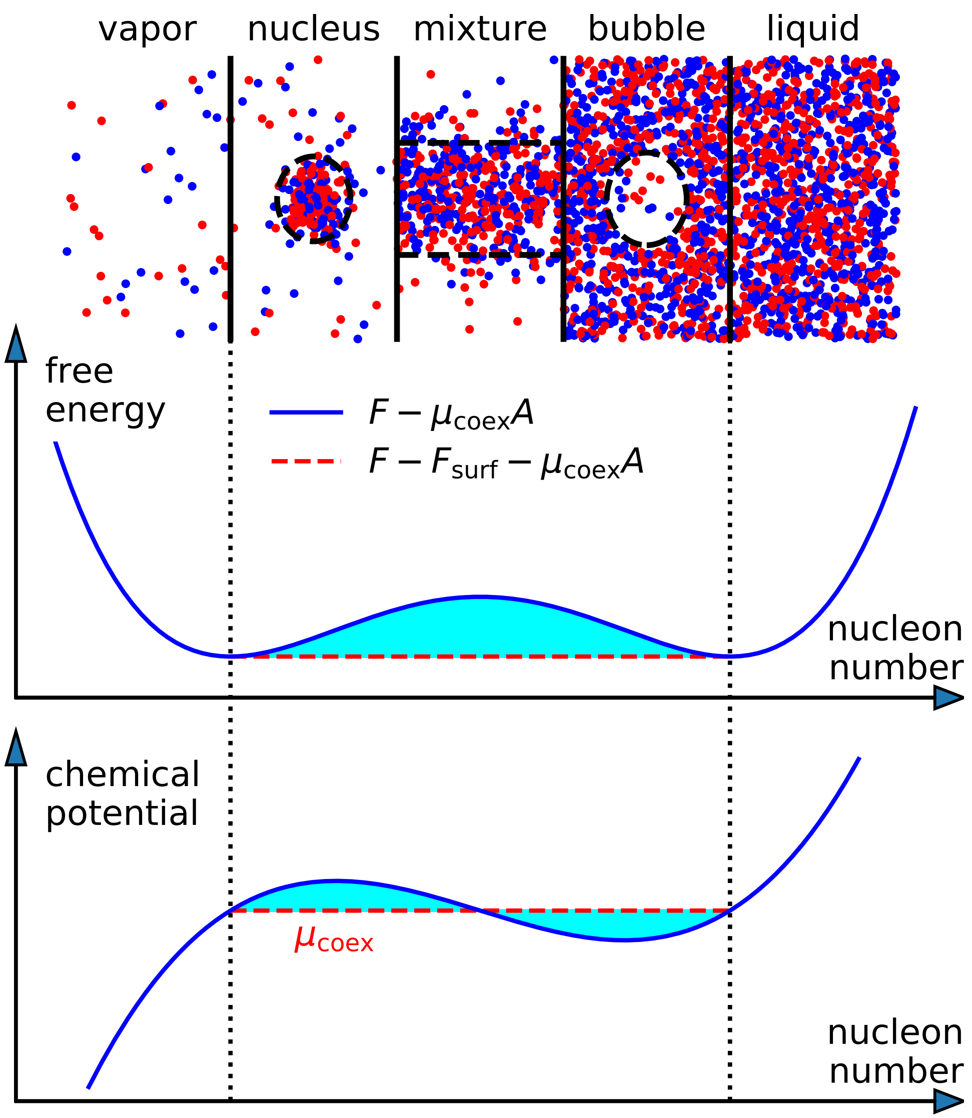
VAPOR-LIQUID PHASE TRANSITION

- Vapor-liquid phase transition
  in a finite volume V & $T < T_c$

- the most probable configuration
  for different nucleon number A

- the free energy

- chemical potential $\mu = \partial F / \partial A$



STRANGENESS NUCLEAR PHYSICS

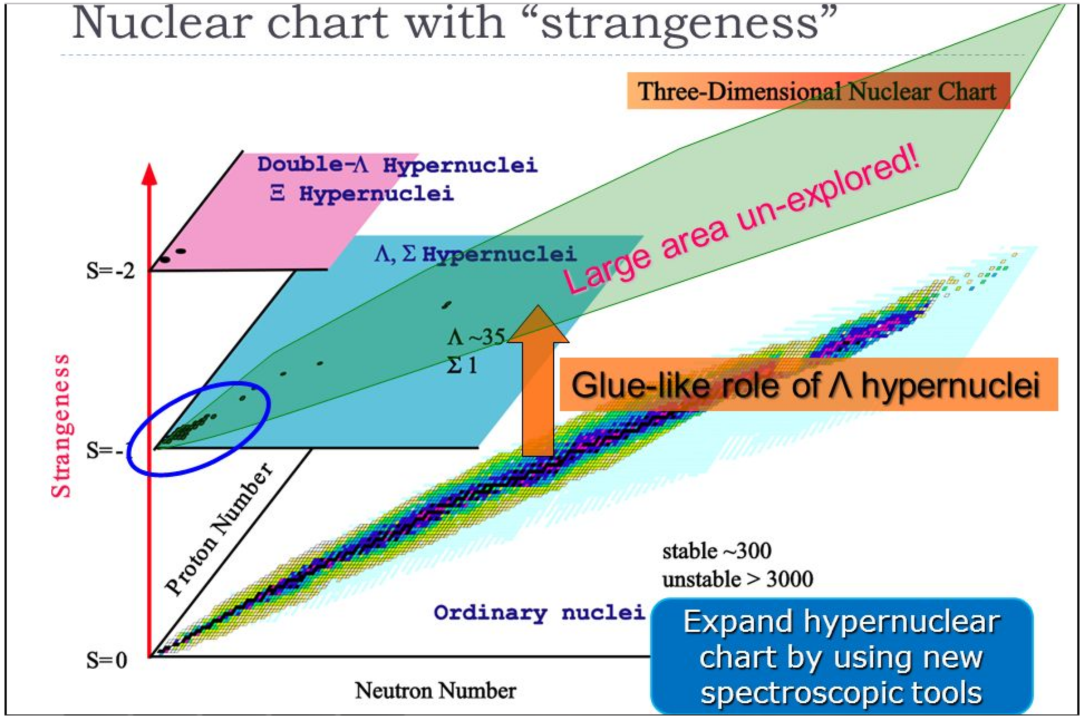
- Substitute one (or two) nucleon(s) by a hyperon (Λ, Σ)

- A few known **hypernuclei**

- Also: very few hyperon-nucleon scattering data

$\Rightarrow$ important role of hypernuclear spectra

$\Rightarrow$ lattice can make an impact!



- New tool : Impurity Lattice Monte Carlo (ILMC) algorithm

  Elhatisari, Lee, Phys. Rev. C **90** (2014) 046001

  Bour, Lee, Hammer, UGM, Phys. Rev. Lett. **115** (2015) 185301

  Frame, Lähde, Lee, UGM, [arXiv:2007.06335 [nucl-th]]