Theory of Relativistic Heavy-Ion Collisions: Achievements and Challenges

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GSI, November 2010

Heavy-Ion Collisions

Heavy-Ion Colliders: The Bevalac (1971-1993)

Heavy-Ion Colliders: Relativistic Heavy-Ion Collider (RHIC) (2000-)

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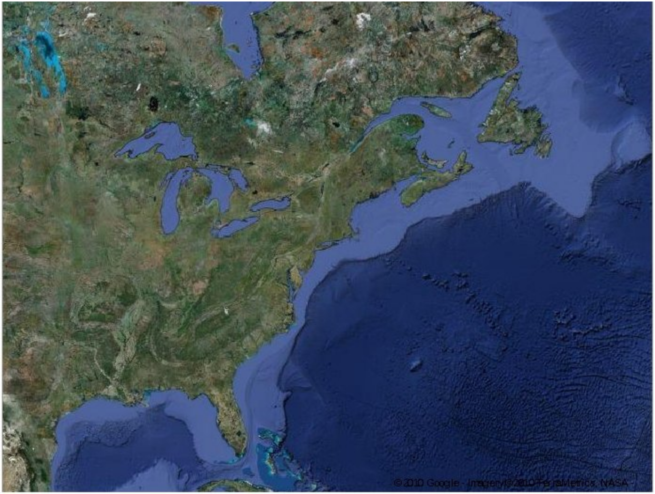

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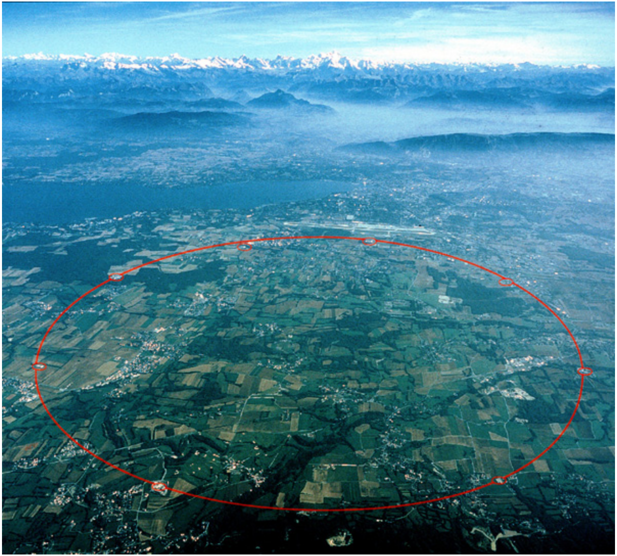
Heavy-Ion Colliders: Large Hadron Collider (LHC) (2009-???)



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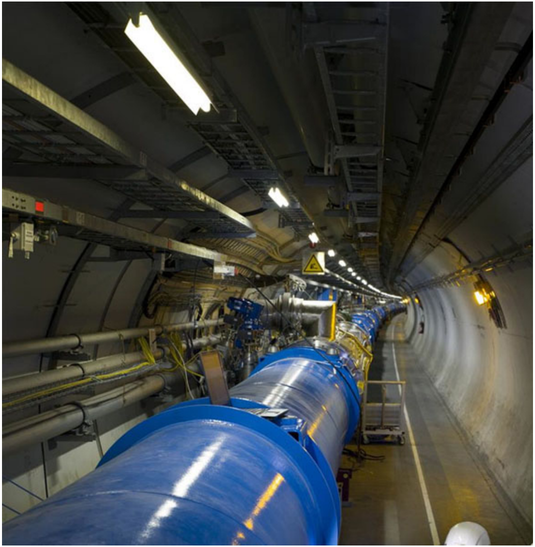
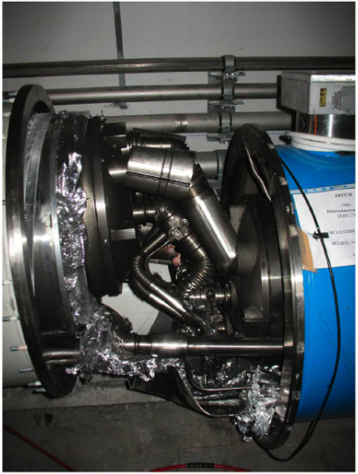


Heavy-Ion Colliders: LHC “incident” (2008)



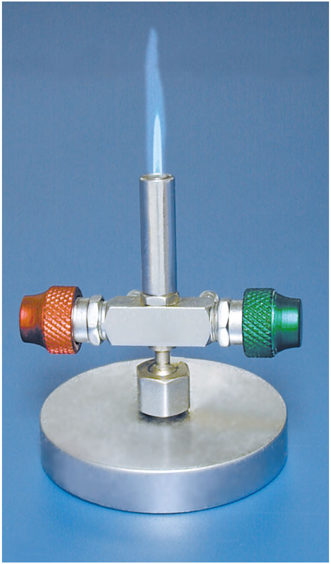
Heavy-Ions

Definition: Ions heavier than carbon

Heavy-Ions in CH: Lead (LHC)

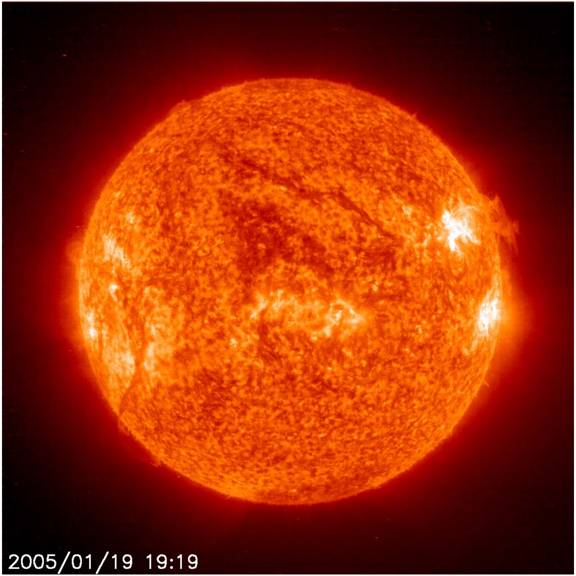
Heavy-Ions in the USA: Gold (RHIC)

Matter in extreme conditions: A flame



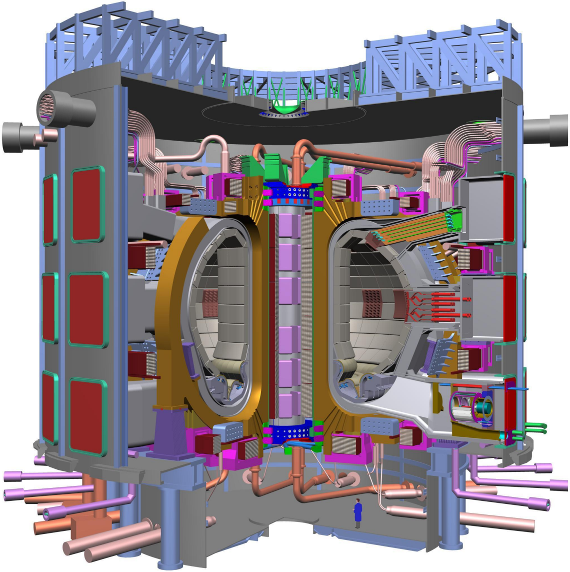
Bunsen Burner: $T \sim 1700°\mathrm{C}$

Matter in extreme conditions: Sun's Core



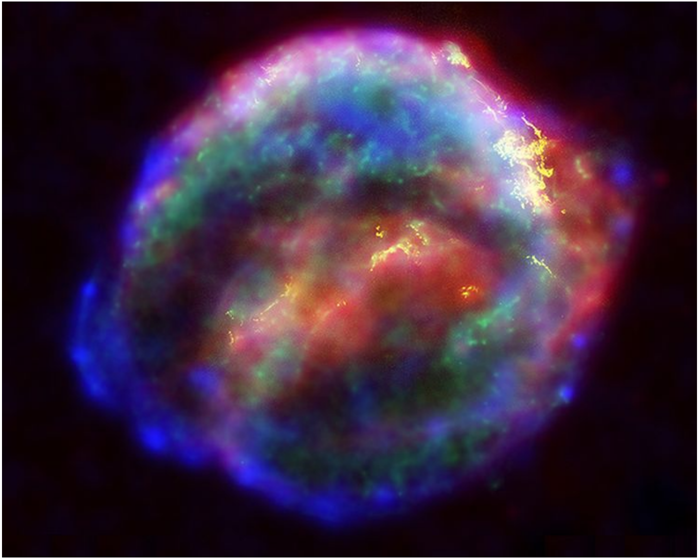
Sun's Core: $T \sim 10^7$ K

Matter in extreme conditions: Fusion on Earth



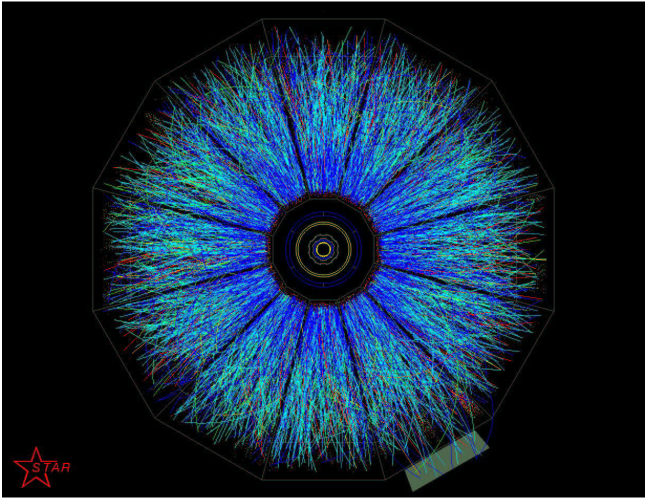
Nuclear fusion at ITER: $T \sim 10^8$ K

Matter in extreme conditions: Supernova



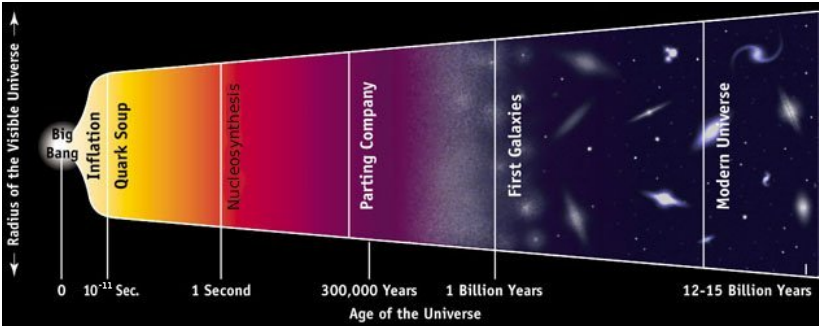
Supernova: $T \sim 10^{11}$ K

Matter in extreme conditions: Heavy-ion collisions



Heavy-Ion Collisions RHIC : $T \sim 10^{12}$ K

History of the Universe (Sketch)

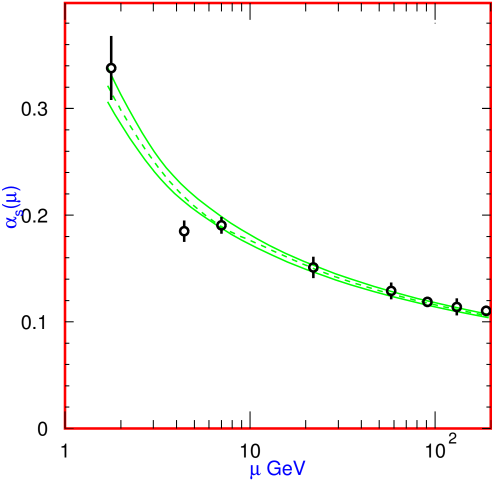


Physics of Heavy-Ion Collisions

- Heavy-Ions $\longrightarrow$ Nuclear Physics
- Nuclear Physics $\longrightarrow$ Strong Interactions
- Theory of Strong Interactions$\longrightarrow$ Quantum Chromo Dynamics (QCD)

Asymptotic Freedom in QCD

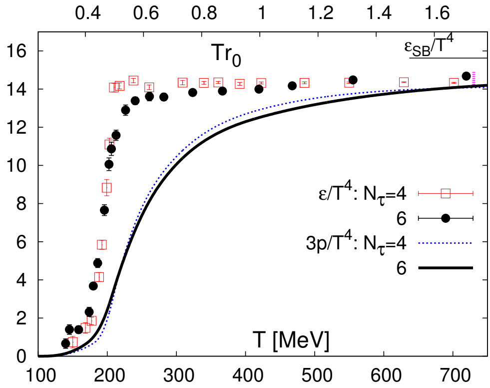
Strength of the Interaction: α_s



Nobel prize 2004: Gross, Politzer, Wilczek

The QCD Phase Transition

Simulating QCD in Equilibrium:



Rapid rise of energy density ϵ and pressure p

[M. Cheng *et al.* 08]

Heavy-Ion Colliders: Relativistic Heavy-Ion Collider

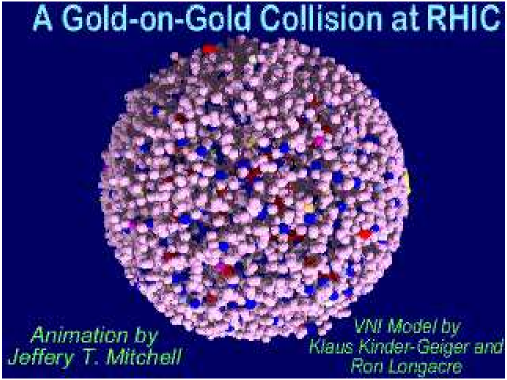
Numbers

- QCD Phase Transition: $T \sim 200$ MeV $\rightarrow$ $\epsilon \sim 12 \times (0.2\ \mathrm{GeV})^4 \sim 2\mathrm{GeV}/\mathrm{fm}^3$
- RHIC:

$$\epsilon \sim \frac{2Am_N}{(6\ \mathrm{fm})^3}\gamma \sim \frac{2A\sqrt{s}}{(6\ \mathrm{fm})^3} \sim 200\mathrm{GeV}/\mathrm{fm}^3$$

- But not all of ϵ_{RHIC} is thermalized!!!

Au+Au Collisions at RHIC



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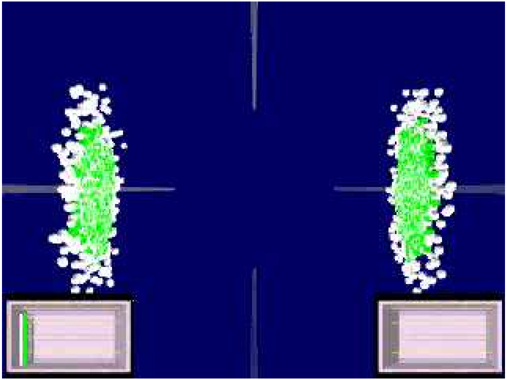

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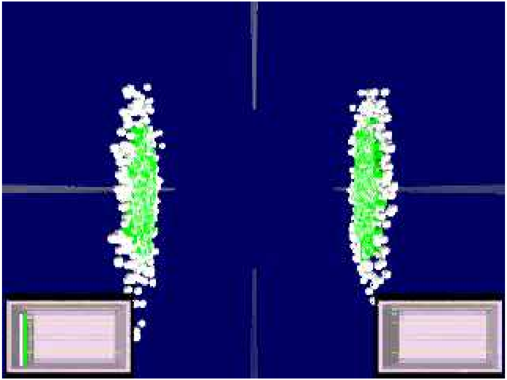

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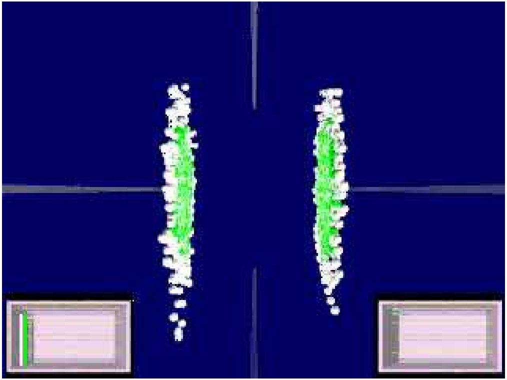

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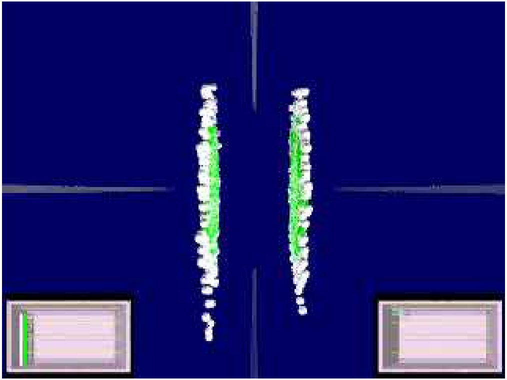

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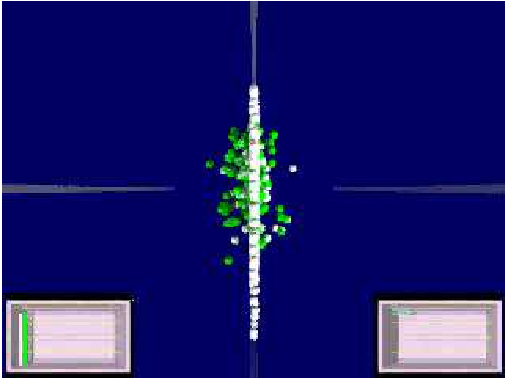

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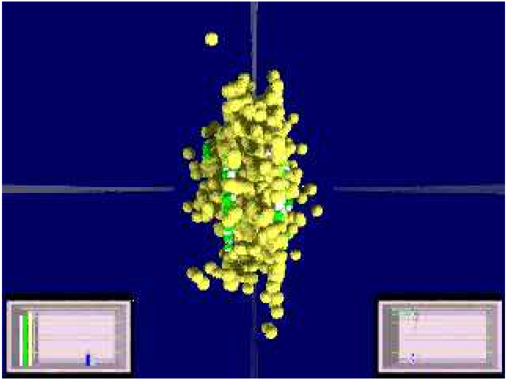

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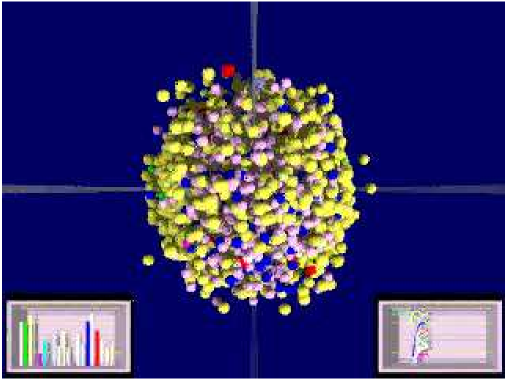
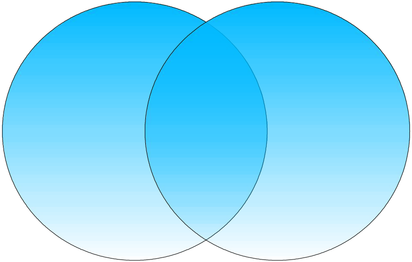


Experimental Observables



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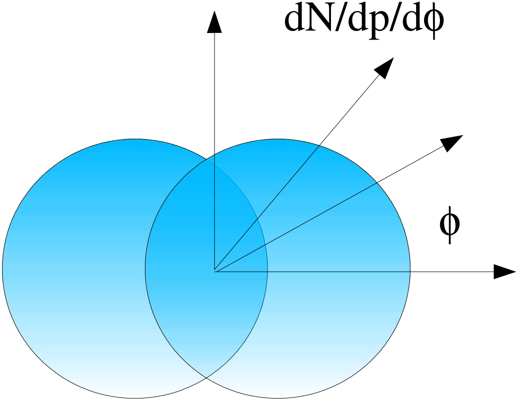


Experimental Observables

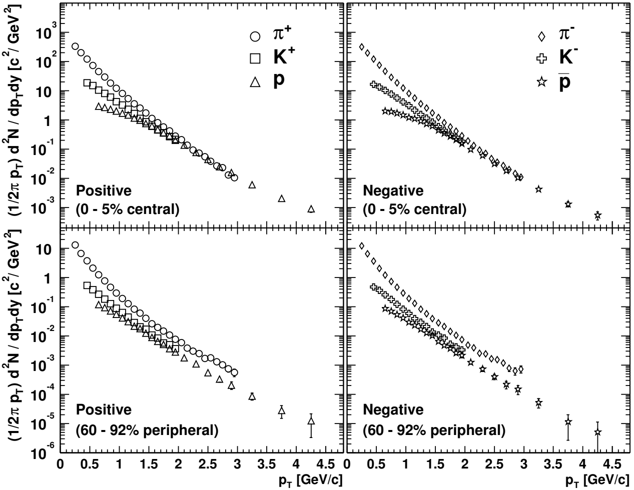
- For ultrarelativistic heavy-ion collisions,

$$\frac{dN}{dp_\perp d\phi dy} = \langle \frac{dN}{dp_\perp d\phi dy} \rangle_\phi (1 + 2v_2(p_\perp)\cos(2\phi) + \ldots)$$

- Radial flow: $\langle \frac{dN}{dp_\perp dy} \rangle_\phi$
- Elliptic flow: $v_2(p_\perp)$

Experimental Data

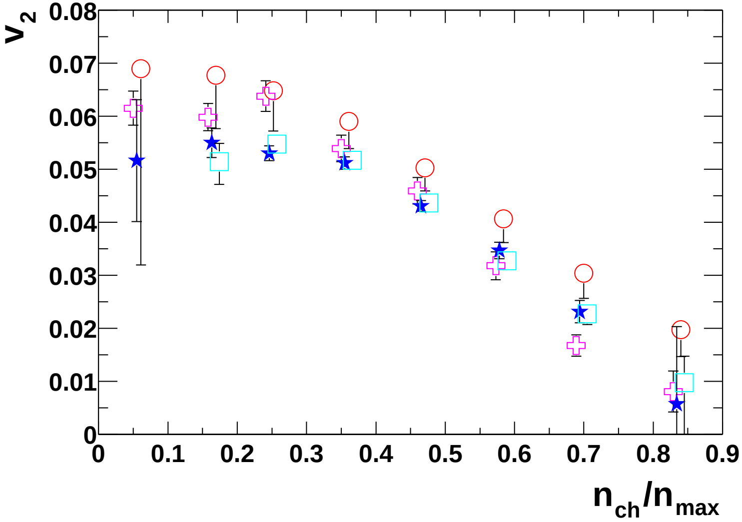
Hadron Spectra at RHIC $\sqrt{s} = 200$ GeV



[PHENIX, 2004]

Experimental Data

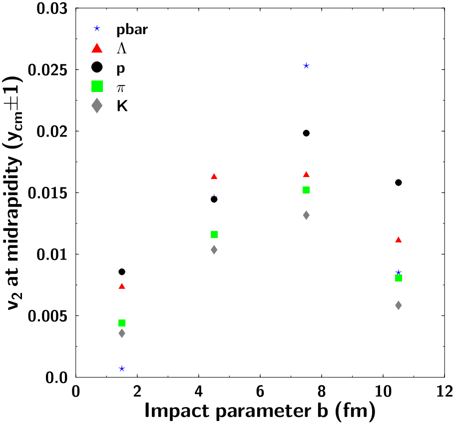
Elliptic Flow at RHIC $\sqrt{s} = 130$ GeV



[STAR, 2002]

What kind of physics dominates at RHIC?

Is it Kinetic Theory?



[Bleicher & Stöcker, 2002]

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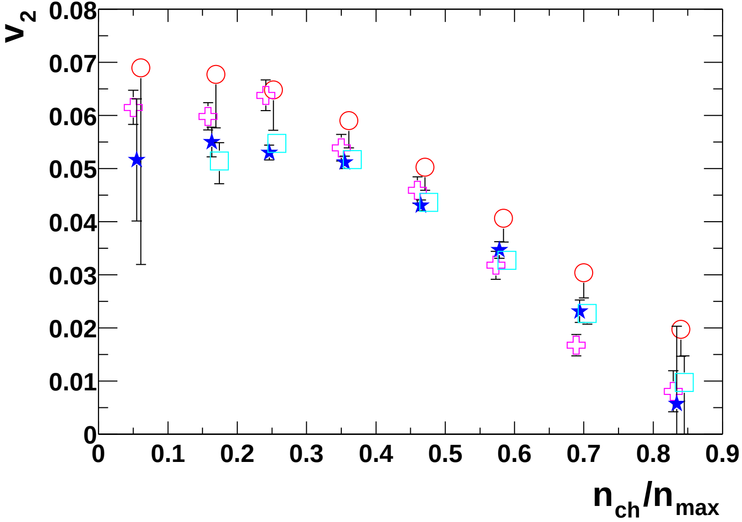
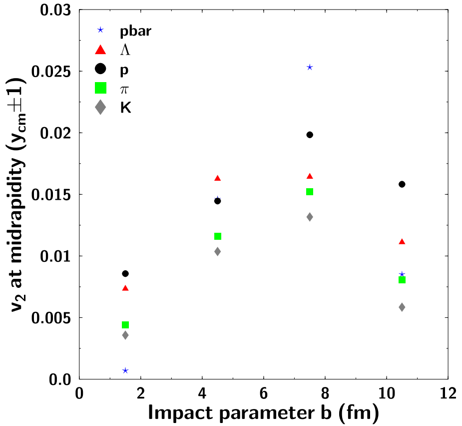


[STAR, 2002]

What kind of physics dominates at RHIC?

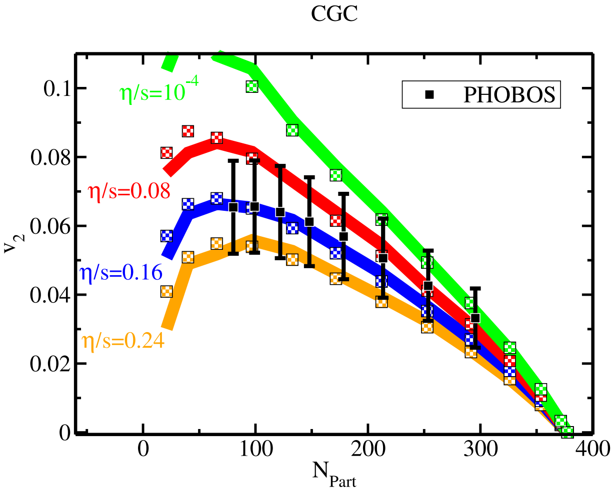
Is it Kinetic Theory? No!



[Bleicher & Stöcker, 2002]

What kind of physics dominates at RHIC?

Is it Fluid Dynamics? **Yes!**



[Luzum & Romatschke, 2008]

Why Fluid Dynamics?

The Quark-Gluon Plasma:

- Temperature $\sim 4 \times 10^{12}$ °C
- Lifetime $\sim 10^{-23}$sec
- Size $\sim 10^{-14}$m

Why can we describe the QGP with fluid dynamics?

Why Fluid Dynamics?

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Why can we describe the QGP with fluid dynamics?

Viscous Fluid Dynamics

Fluid Dynamics

=

Conservation of Energy+Momentum for long wavelength modes[1]

[1] long wavelength modes =
looking at the system for a very long time from very far away

$$T_{id}^{\mu\nu} = \epsilon u^\mu u^\nu - p(g^{\mu\nu} - u^\mu u^\nu)$$ (Fluid EMT, no gradients)

+

$$\partial_\mu T^{\mu\nu} = 0$$ (“EMT Conservation”)

=

Ideal Fluid Dynamics

Non-relativistic Ideal Fluid Dynamics

$$\partial_t v^i + v^m \partial_m v^i = -\frac{1}{\rho} \partial_j \delta^{ij} p$$

[L. Euler, 1755]

- Non-linear
- Non-dissipative: "Ideal Fluid Dynamics"

Relativistic Ideal Fluid Dynamics

$$T^{\mu\nu} = T^{\mu\nu}_{id}$$ (Fluid EMT, no gradients)

$$+$$

$$\partial_\mu T^{\mu\nu} = 0$$ ("EMT Conservation")

$$=$$

Ideal Fluid Dynamics

Relativistic Viscous Fluid Dynamics

$$T^{\mu\nu} = T^{\mu\nu}_{\rm id} + \Pi^{\mu\nu}$$ (Fluid EMT, 1^{st} o. gradients)

+

$$\partial_\mu T^{\mu\nu} = 0$$ ("EMT Conservation")

=

Relativistic Navier-Stokes Equation

Relativistic Viscous Fluid Dynamics

- L. Euler, 1755:

$$\partial_t v^i + v^m \partial_m v^i = -\frac{1}{\rho} \partial_j \delta^{ij} p$$

- C. Navier, 1822; G. Stokes 1845:

$$\partial_t v^i + v^m \partial_m v^i = -\frac{1}{\rho} \partial_j \left[\delta^{ij} p + \Pi^{ij} \right] \, ,$$

$$\Pi^{ij} = -\eta \left(\frac{\partial v^i}{\partial x^j} + \frac{\partial v^j}{\partial x^i} - \frac{2}{3} \delta^{ij} \frac{\partial v^l}{\partial x^l} \right) - \zeta \delta^{ij} \frac{\partial v^l}{\partial x^l} \, ,$$

- $\eta, \zeta \ldots$ transport coefficients (“viscosities”)

Fluid Dynamics

=

Effective Theory of Small Gradients

Relativistic Navier-Stokes Equation

- Good enough for non-relativistic systems
- NOT good enough for relativistic systems

Navier-Stokes: Problems with Causality

Consider small perturbations around equilibrium

- Transverse velocity perturbations obey

$$\partial_t \delta u^y - \frac{\eta}{\epsilon + p} \partial_x^2 \delta u^y = 0$$

- Diffusion speed of wavemode k:

$$v_T(k) = 2k \frac{\eta}{\epsilon + p} \to \infty \; (k \gg 1)$$

- Know how to regulate: “second-order” theories:

$$\tau_\pi \partial_t^2 \delta u^y + \partial_t \delta u^y - \frac{\eta}{\epsilon + p} \partial_x^2 \delta u^y = 0$$

[Maxwell (1867), Cattaneo (1948)]

Second Order Fluid Dynamics

- Limiting speed is finite

$$\lim_{k\to\infty} v_L(k) = \sqrt{c_s^2 + \frac{4\eta}{3\tau_\pi(\epsilon + p)} + \frac{\zeta}{\tau_\Pi(\epsilon + p)}}$$

[Romatschke, 2009]

- $\tau_\pi, \tau_\Pi \ldots \ldots$: "2$^{\rm nd}$ order" regulators for "1$^{\rm st}$ order" fluid dynamics
- Regulators acts in UV, low momentum (fluid dynamics) regime is still Navier-Stokes

Second Order Fluid Dynamics

$$T^{\mu\nu} = T^{\mu\nu}_{\mathrm{id}} + \Pi^{\mu\nu}$$ (Fluid EMT, 2^{nd} o. gradients)

+

$$\partial_\mu T^{\mu\nu} = 0$$ ("EMT Conservation")

=

"Causal" Relativistic Viscous Fluid Dynamics

First complete 2^{nd} theory only in 2007 !

[Baier, Romatschke, Son, Starinets 2007]

Second Order Fluid Dynamics

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“Causal” Relativistic Viscous Fluid Dynamics

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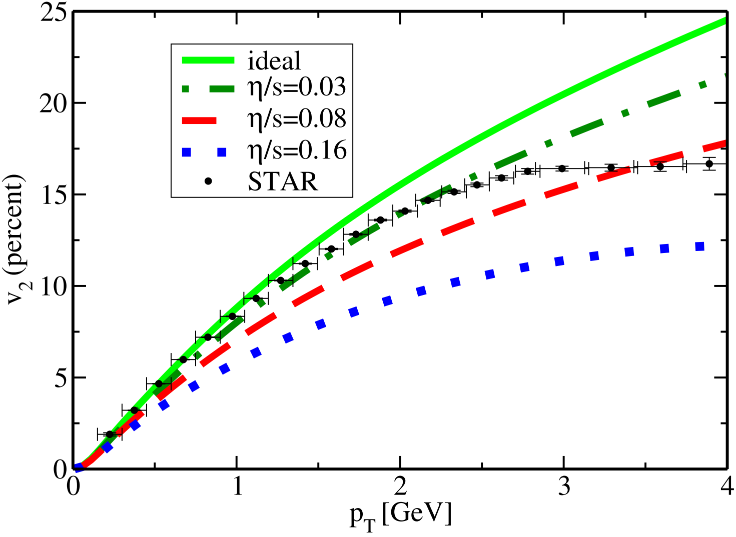
First Relativistic Viscous Fluid Dynamics Simulation



[Paul and Ulrike Romatschke 2007]

Lessons Learned

- Fluid dynamics is effective theory of long wavelength modes
- Fluid dynamic equations are universal$\rightarrow$ apply at many different scales
- Coefficients (speed of sound, viscosity) depend on specific system
- Effect theory breaks down if gradient expansion fails

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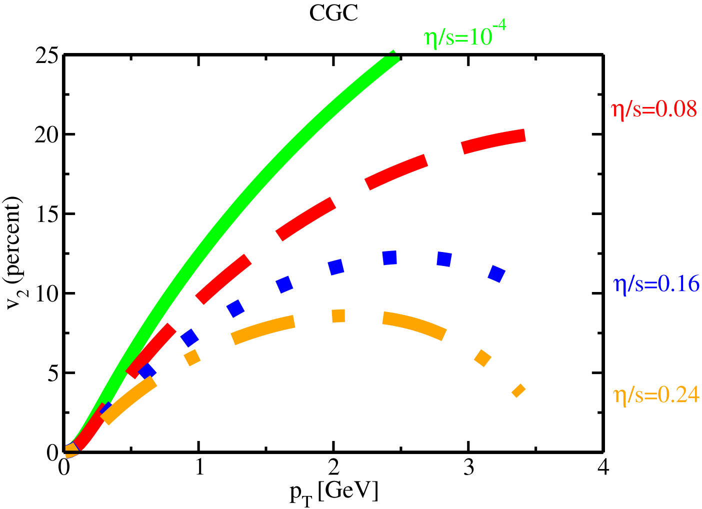
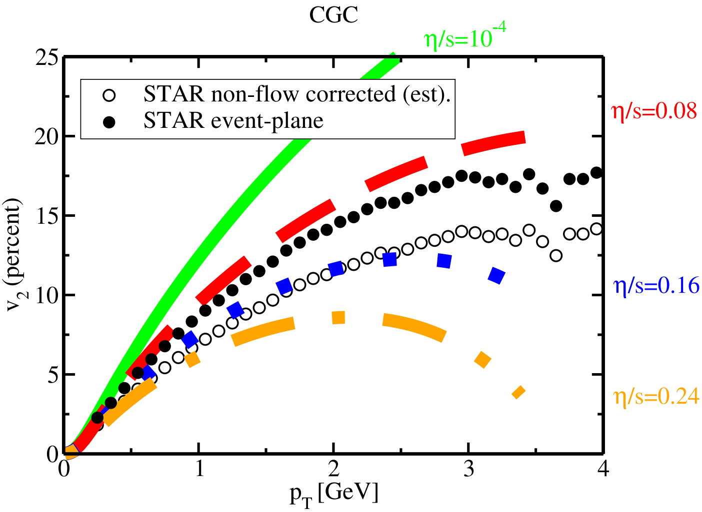
Quark Gluon Plasma: Fluid Dynamics versus data



[Luzum and Romatschke, 2008]

Quark Gluon Plasma: Fluid Dynamics versus data



[Luzum and Romatschke, 2008]

Constraints on η/s: Implications

- Find $\eta/s < 0.5$ at RHIC
- Weak coupling QCD calculations:

$$\frac{\eta}{s} \sim \frac{1}{g^4 \ln g^{-1}} \to \sim 1 \text{ for RHIC}$$

[Hosoya and Kajantie, 1985]

- Strong coupling calculations via gauge/gravity duality (not QCD!!)

$$\frac{\eta}{s} \sim \frac{1}{4\pi} \sim 0.08$$

[Policastro, Son, Starinets 2001]

Theory Achievements

Theory Achievements (based on RHIC data)

- The bulk of the matter produced in heavy-ion collisions behaves fluid-like
- The Quark Gluon Plasma is less viscous than superfluid helium ($\eta/s \sim 1$)!
- Value of η/s suggests the QGP is non-perturbative
- Gauge/gravity duality prediction for η/s is close to experiment

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Challenges

- Presently, initial equilibration of system is assumed
- Equilibration in heavy-ion collisions is hard problem: non-perturbative, real-time (no lattice QCD calculations)
- So far not even a credible model exists!
- Maybe gauge/gravity duality can help?

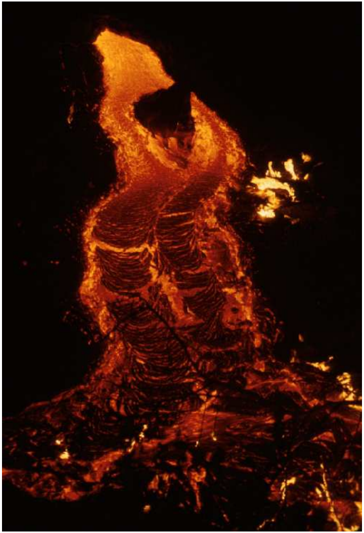
Conclusions

- Heavy-Ion Collisions probe Matter under extreme conditions
- Available experimental data is well described by fluid dynamics, while most other approaches fail
- Heavy-Ion Collision Data together with Fluid Dynamics constrain properties of nuclear matter at $T \sim 200$ MeV

Bonus Material

Non-linear & Non-dissipative: Turbulence

Non-linear & Dissipative: Laminar



Non-linear & Dissipative: Laminar

Viscosity dampens turbulent instability!