Gauge/Gravity Duality and Strongly Coupled Matter

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Motivation

Matter under Extreme Conditions $\Leftrightarrow$ Strongly Coupled Matter

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$\Rightarrow$ Gauge/Gravity Duality

- maps strongly coupled gauge theories to weakly coupled gravity theories
- originates from string theory

Gauge/Gravity Duality

- Gauge/gravity duality: Generalizations of AdS/CFT correspondence

  AdS: Anti-de Sitter space, CFT: Conformal field theory

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  - creating new links between strongly correlated quantum liquids

    quark-gluon plasma

    ultracold atoms

    neutron matter

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- Top-down approach: Solve equations of motion in 10d gravity

Outline

1. Basics of Gauge/Gravity Duality

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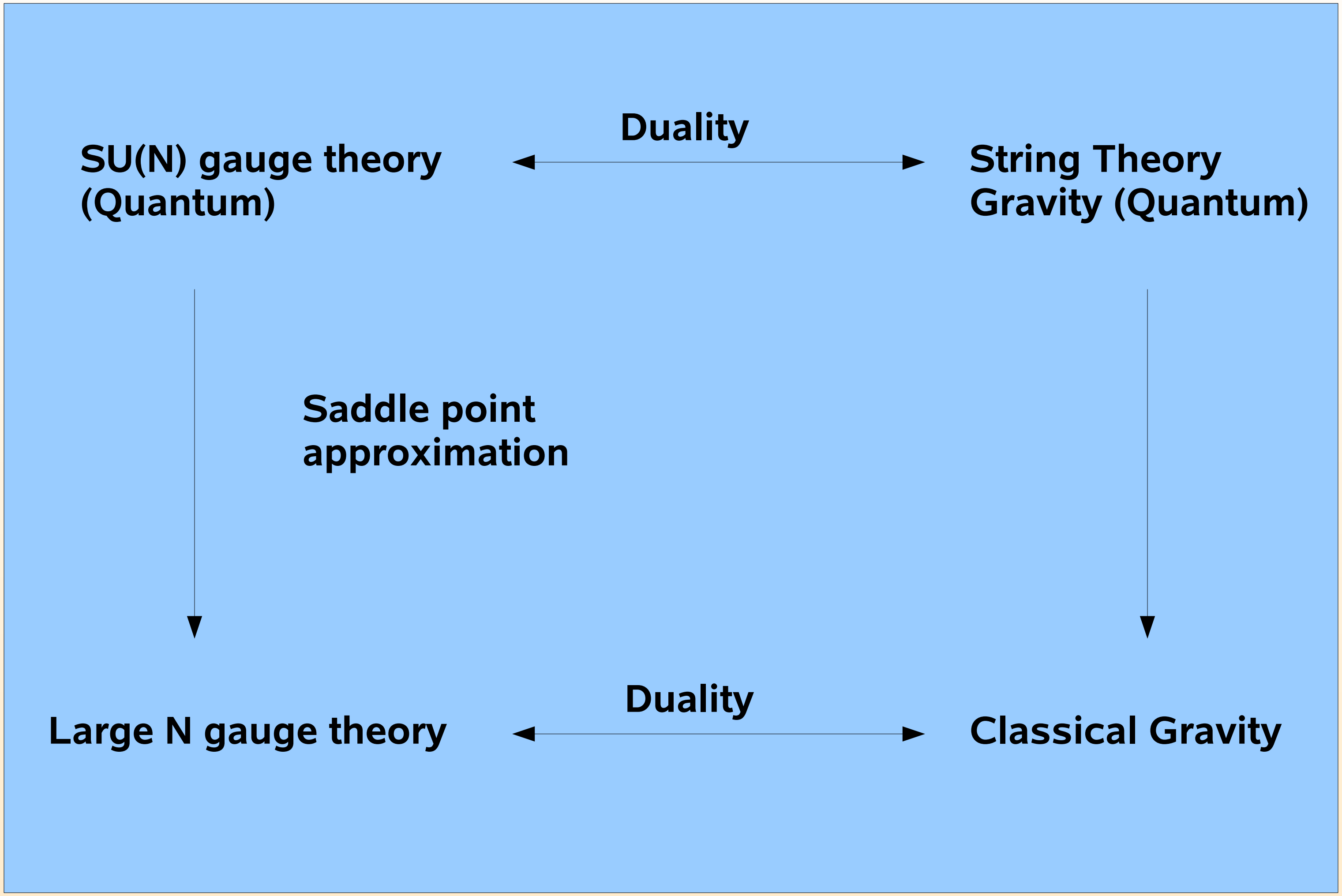
2. An Example for Applications:

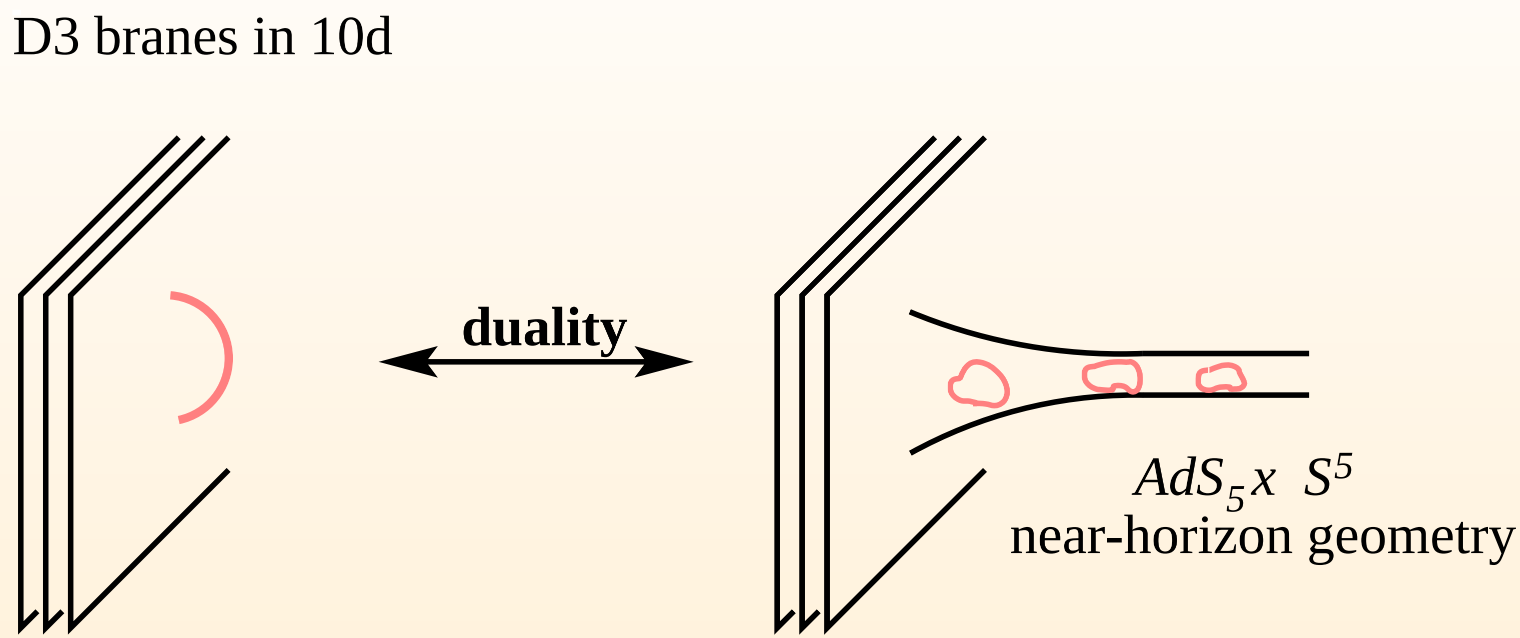
   - Superconductivity/Superfluidity at finite isospin density

1. Basics of Gauge/Gravity Duality

AdS/CFT correspondence (Maldacena 1997)



String theory origin of AdS/CFT correspondence



$\Downarrow$ Low-energy limit

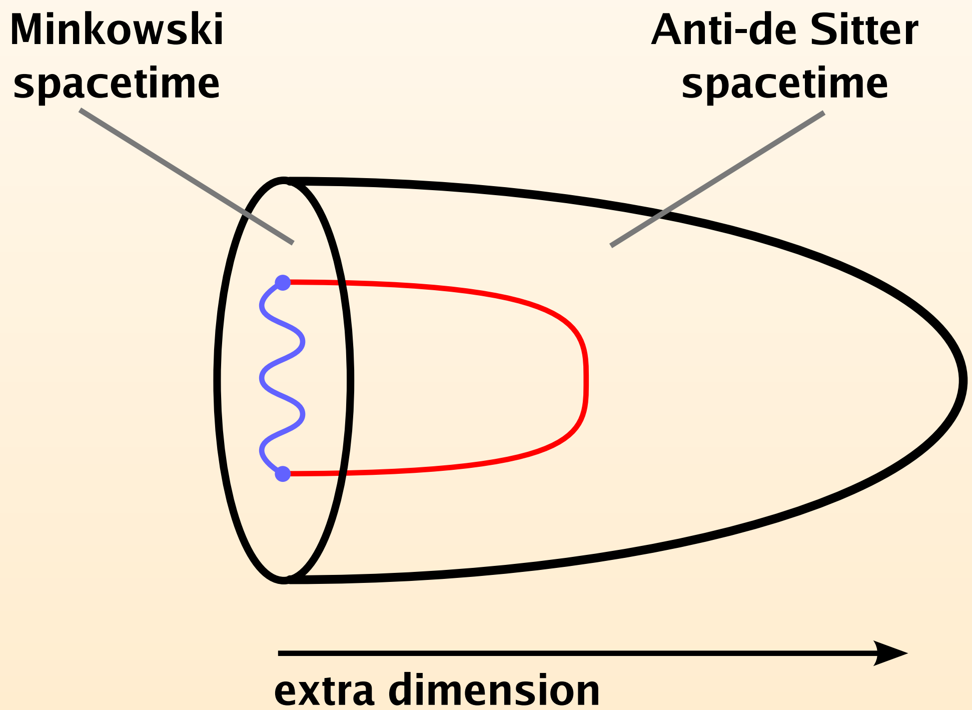
$\mathcal{N} = 4$ $SU(N)$ theory in four dimensions ($N \to \infty$)

Supergravity on $AdS_5 \times S^5$

AdS/CFT correspondence (Maldacena 1997)

AdS: Anti-de Sitter space in $d = 5$ $\Leftrightarrow$ CFT: conformal field theory in $d = 4$

Symmetry properties coincide



'Dictionary' Gauge invariant operators in field theory $\Leftrightarrow$ Fields in gravity theory

Generalizations of the AdS/CFT Correspondence

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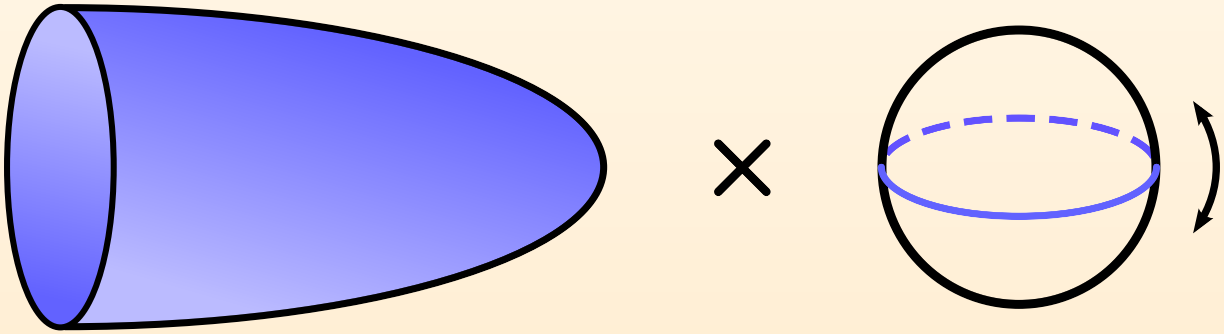
3. Non-relativistic theories

Quarks (fundamental fields) within the AdS/CFT correspondence

Adding D7 brane probe: Babington, J.E., Evans, Guralnik, Kirsch PRD 2004

	0	1	2	3	4	5	6	7	8	9
D3	X	X	X	X						
D7	X	X	X	X	X	X	X	X		

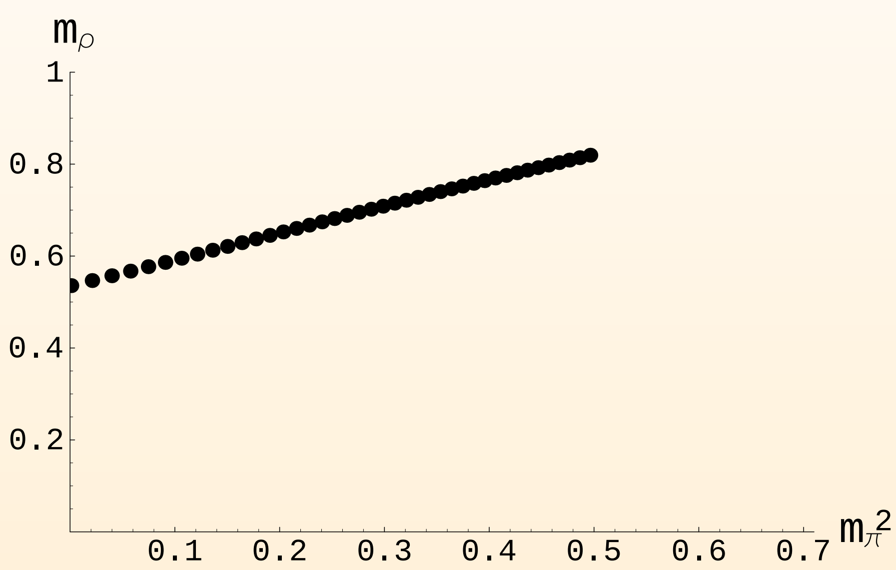


π pseudoscalar meson mass: From fluctuations of hypersurface

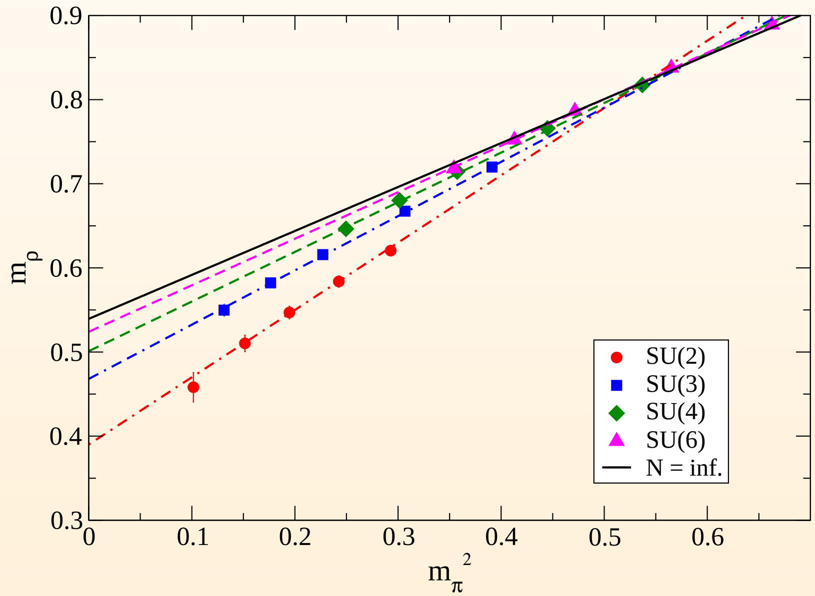
ρ vector meson mass: From fluctuations of gauge field on hypersurface

Comparison to lattice gauge theory

Mass of ρ meson as function of π meson mass^2 (for $N \to \infty$)



J.E., Evans, Kirsch, Threlfall '07, review EPJA



Lattice: Lucini, Del Debbio, Patella, Pica '07

AdS/CFT result:

$$\frac{m_\rho(m_\pi)}{m_\rho(0)} = 1 + 0.307 \left(\frac{m_\pi}{m_\rho(0)}\right)^2$$

Lattice result (from Bali, Bursa '08): slope 0.341 ± 0.023

Part II:

Application Example

Superconductivity/Superfluidity

Superfluidity in quark-gluon plasma:

Instability at finite isospin density

ρ meson condensate, flavour degrees of freedom can move without friction

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Global $U(1)$ toy model for electromagnetism

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Moreover:

Global $U(1)$ toy model for electromagnetism

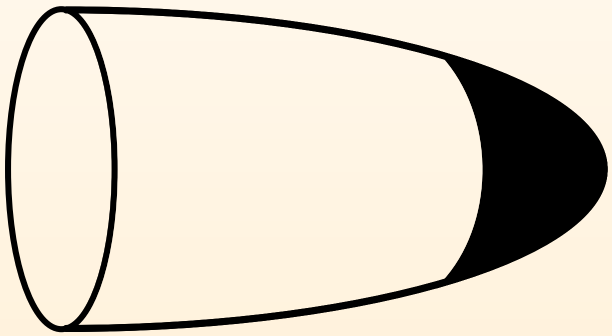
Fermionic excitations in superfluid

Excitations near zeroes of the energy gap (order parameter)

Gauge/Gravity Duality at Finite Temperature

$\mathcal{N} = 4$ Super Yang-Mills theory at finite temperature is dual to AdS black hole

Witten 1998



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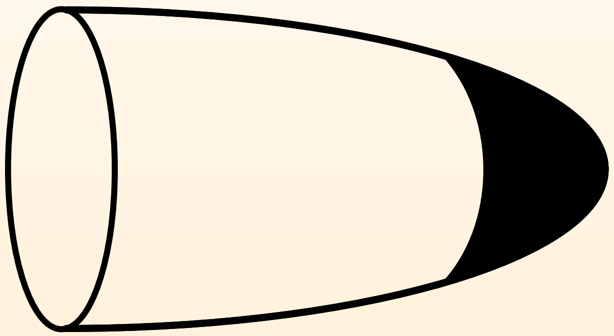
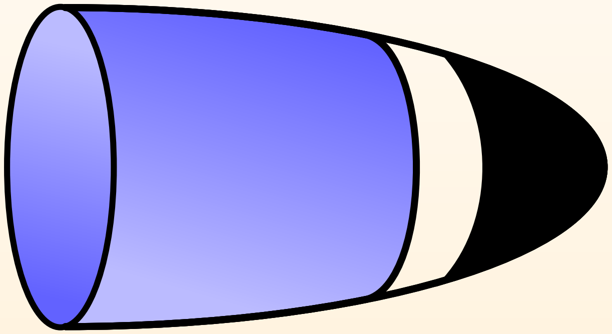
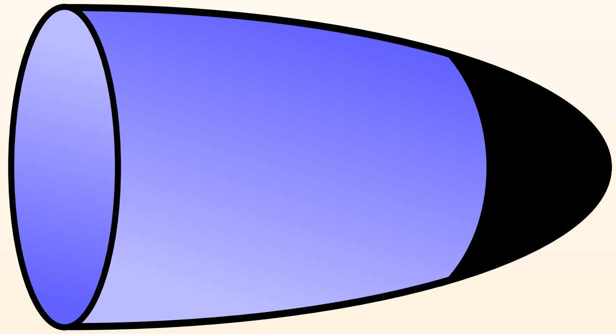


Toy model for quark-gluon plasma

D7 brane embedding in black hole background





Minkowski phase

Black hole phase

First order phase transition

Babington, J.E., Evans, Guralnik, Kirsch
Mateos, Myers, Thomson

Masses and decay widths of mesons

Standard procedure in D3/D7: Mateos, Myers et al 2003

Meson masses calculated from linearized fluctuations of D7 embedding

Fluctuations: $\delta w(x,\rho) = f(\rho) e^{i(\vec{k}\cdot\vec{x}-\omega t)}$, $M^2 = -k^2$

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For black hole embeddings, ω develops negative imaginary part

$\Rightarrow$ damping $\Rightarrow$ decay width

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Quasinormal modes of D7 brane fluctuations determine meson mass and width

Landsteiner, Hoyos, Montero 2006

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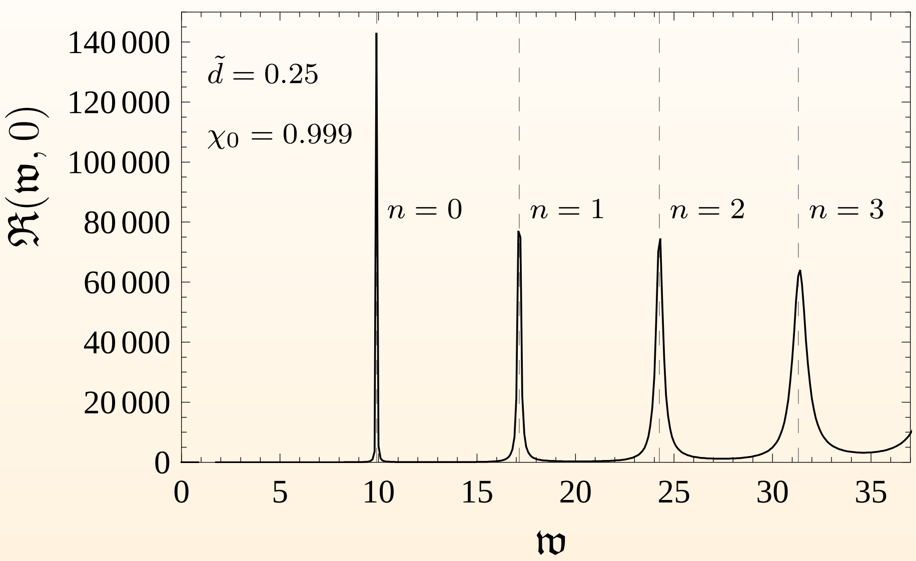
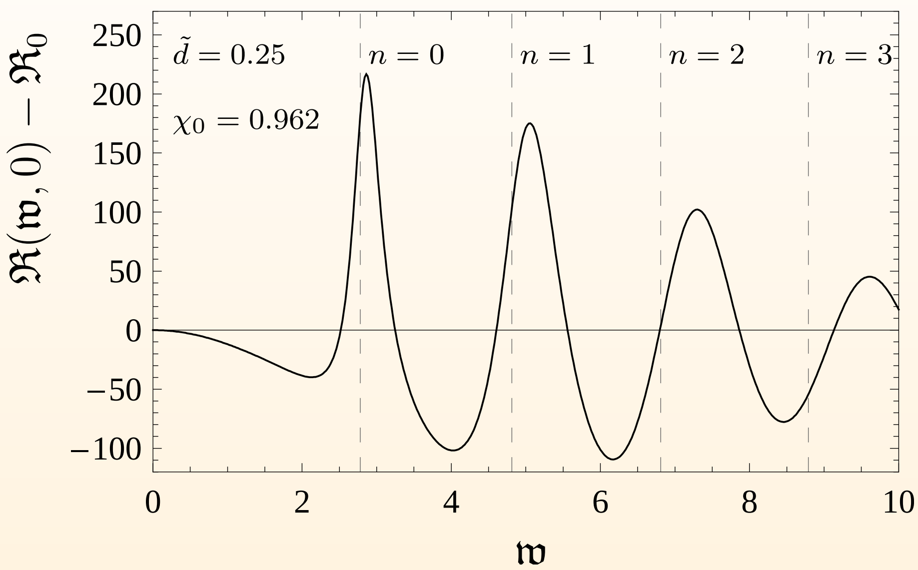
Excitations in strongly coupled system

Quasinormal Modes and Spectral Functions





Minkowski phase

Mesons stable

Normal modes real

Black hole phase

Mesons decay

Quasinormal modes complex

Finite $U(1)$ baryon density

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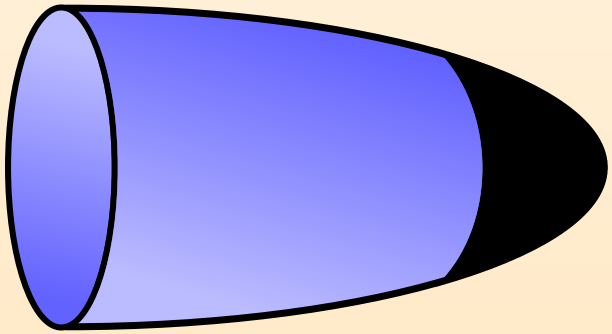
Mateos, Myers, Matsuura et al

Baryon density n_B and $U(1)$ chemical potential μ
from non-trivial profile for gauge field time component:

$$\bar{A}_0(\rho) \sim \mu + \frac{\tilde{d}}{\rho^2}\,, \qquad \tilde{d} = \frac{2^{5/2}}{N_f\sqrt{\lambda}T^3}\,n_B$$

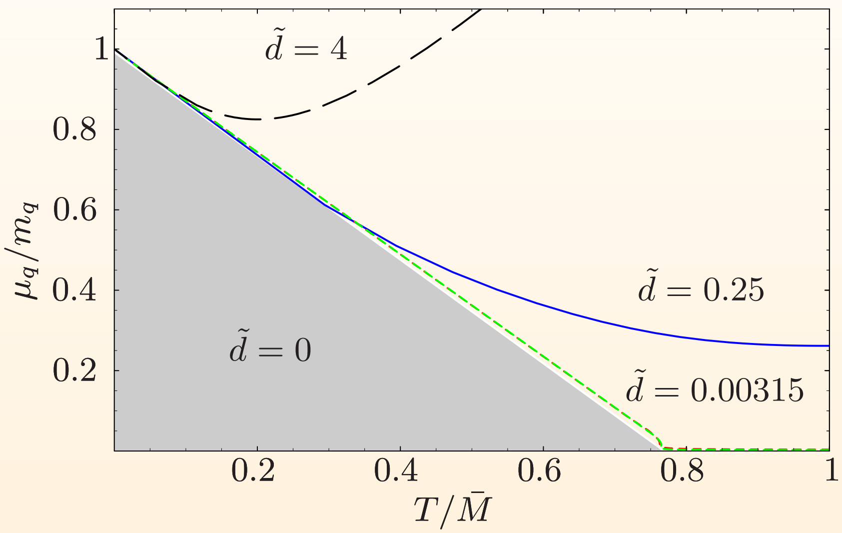
At finite baryon density, all embeddings are black hole embeddings



Phase diagram with finite $U(1)$ baryon density

Phase diagram:

grey region: $n_B = 0$
white region: $n_B \neq 0$



Isospin chemical potential and density

- Embed two coincident D7-branes into AdS-Schwarzschild
  gauge fields $A_\mu = A_\mu^a \sigma^a \in u(2) = u(1)_B \oplus su(2)_I$
- Finite isospin density: $A_0^3 \neq 0 \Rightarrow$ Explicit breaking to $u(1)_3$

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Field theory described:

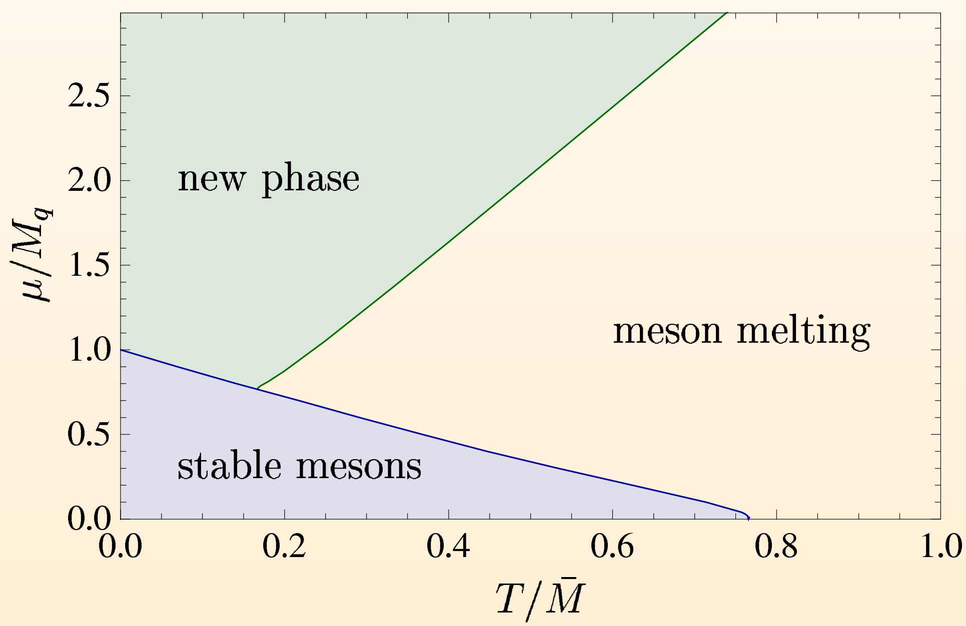
$\mathcal{N} = 4$ Super Yang-Mills plus two flavors of fundamental matter

at finite temperature and finite isospin density

ρ meson condensation

J.E., Kaminski, Kerner, Rust 0807.2663

Above a critical isospin density, a new phase forms



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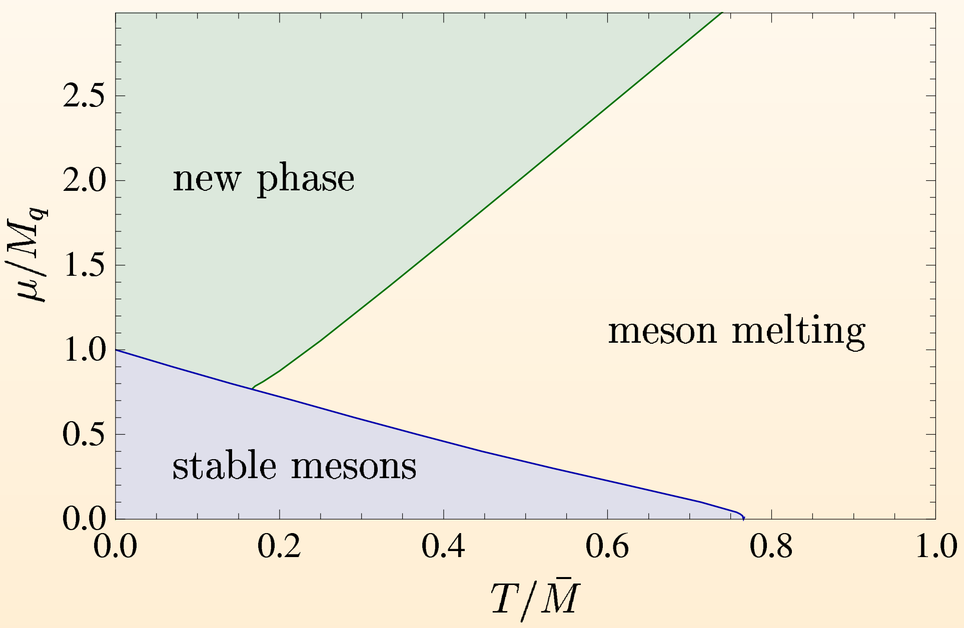
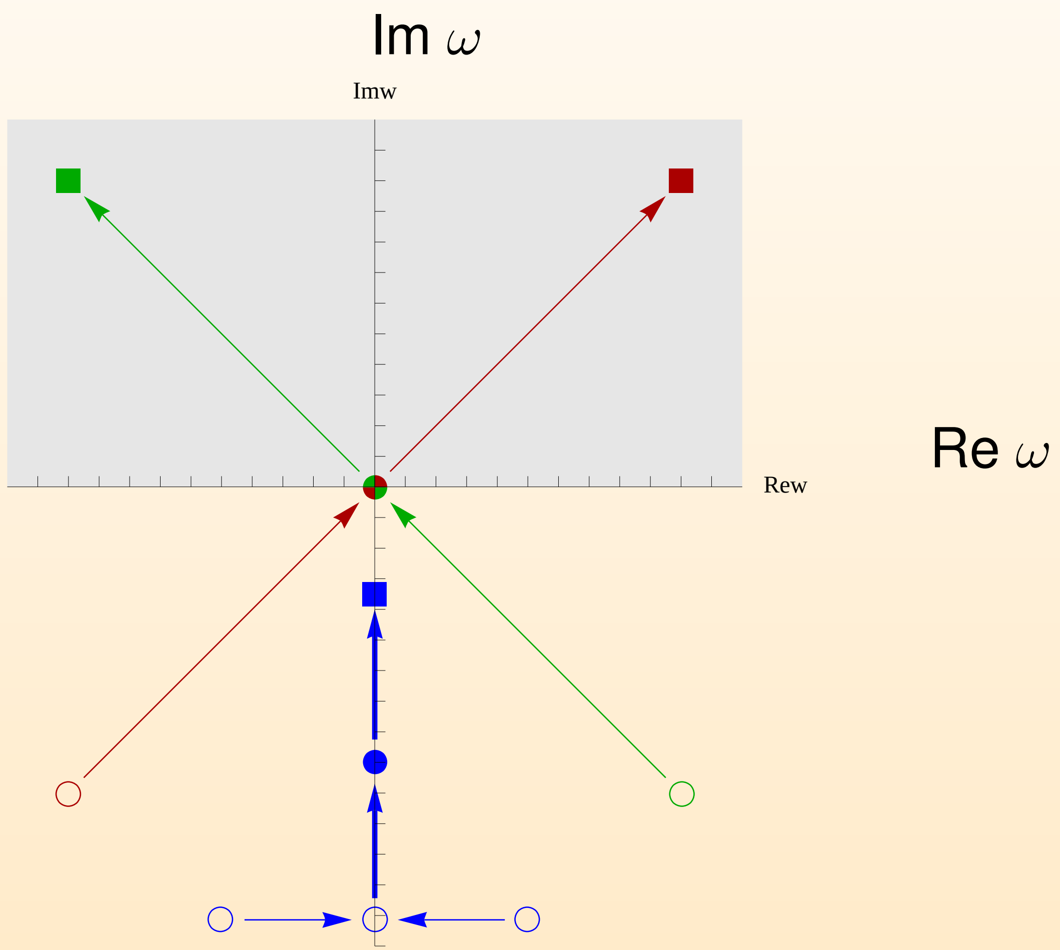


New phase is unstable

Instability:



A new ground state forms

There is a new solution to the equations of motion

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$$A_0^3 = \mu - \frac{\tilde{d}_0^3}{2\pi\alpha'}\frac{\rho_H}{\rho^2} + \ldots\,, \qquad A_3^1 = -\frac{\tilde{d}_1^3}{2\pi\alpha'}\frac{\rho_H}{\rho^2} + \ldots$$

A new ground state forms

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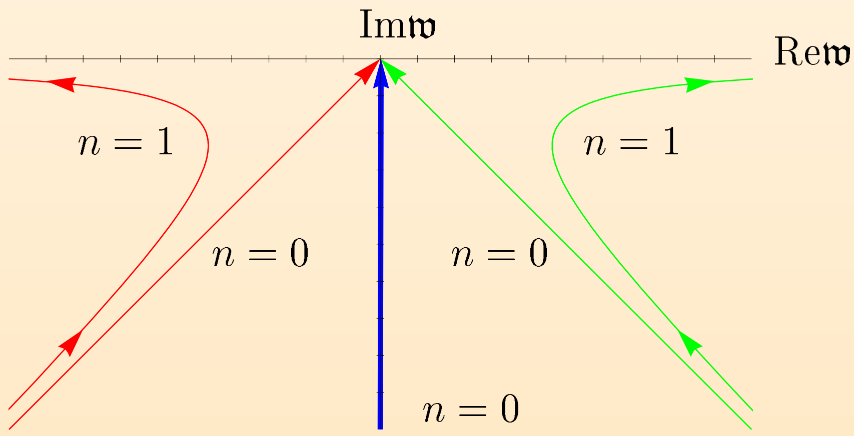
Pole structure:



Superconductivity

Ammon, J.E., Kaminski, Kerner 0810.2316, 0903.1864

The new ground state has properties known from superconductors:

- infinite DC conductivity, gap in the AC conductivity
- second order phase transition, critical exponent of 1/2 (mean field)
- a remnant of the Meissner–Ochsenfeld effect

Superfluidity and Superconductivity

Order parameter $\tilde{d}_3^1 \propto \langle \bar{\psi}_u \gamma_3 \psi_d + \bar{\psi}_d \gamma_3 \psi_u + \, bosons \, \rangle \neq 0$

ρ meson condensate (p-wave symmetry)

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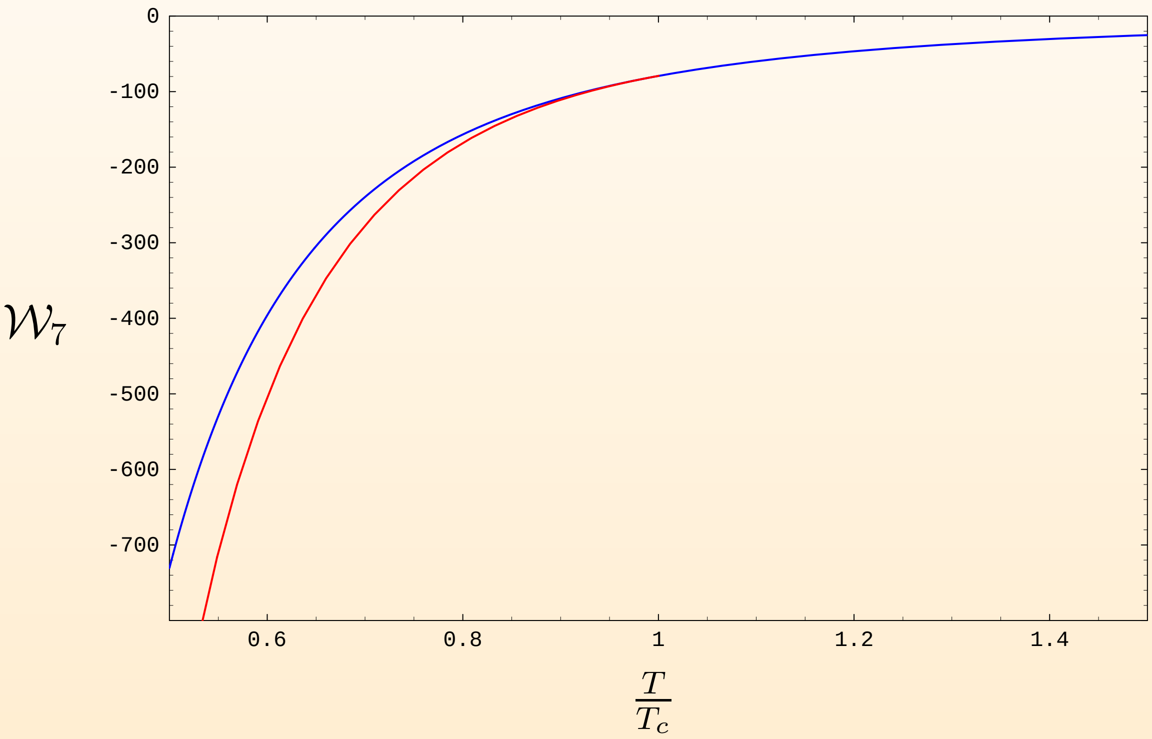
Flavour superfluid

Within QCD: Superconductivity in presence of magnetic field Chernodub 2010

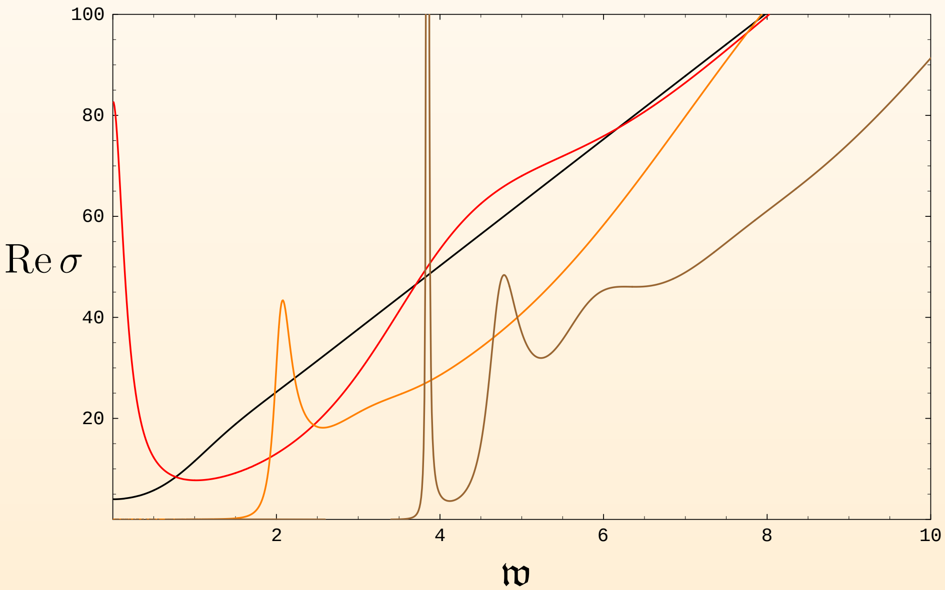
Flavour contribution to Grand potential vs. temperature



Conductivity

Frequency-dependent conductivity $\sigma(\omega) = \frac{i}{\omega} G^R(\omega)$

G^R retarded Green function for fluctuation a_2^3



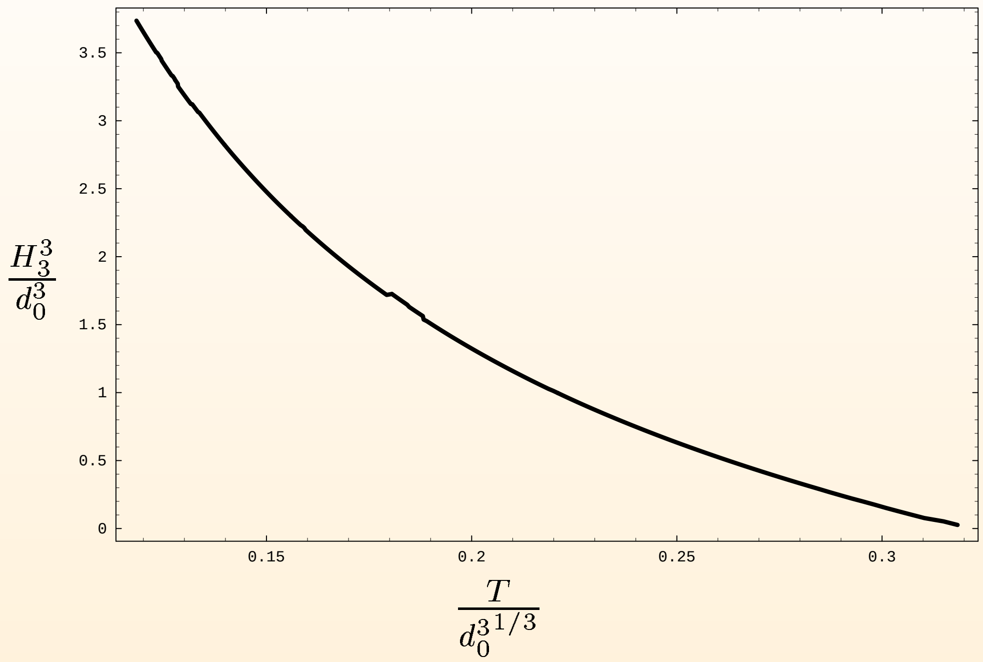
$\mathfrak{w} = \omega/(2\pi T)$

T/T_c: Black: ∞, Red: 1, Orange: 0.5, Brown: 0.28.

(Vanishing quark mass)

Interpretation: Frictionless motion of mesons through plasma

Meissner effect



Lower phase: magnetic field and condensate coexist

Upper phase: condensate vanishes

Quantum phase transition

Turn on both isospin and baryon chemical potential

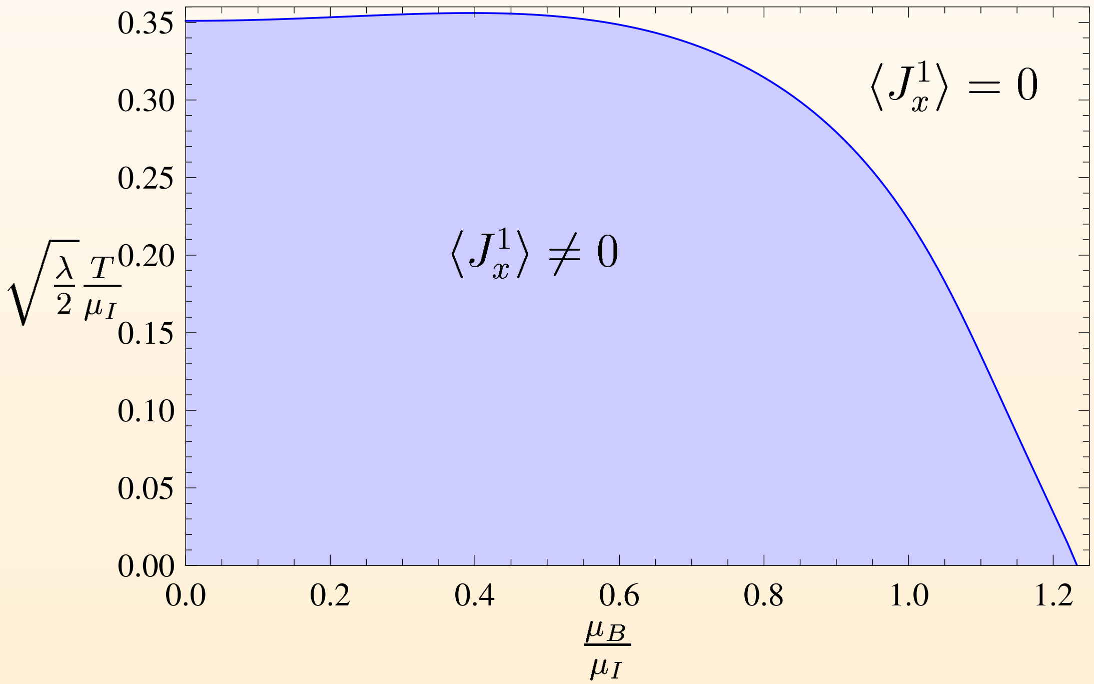
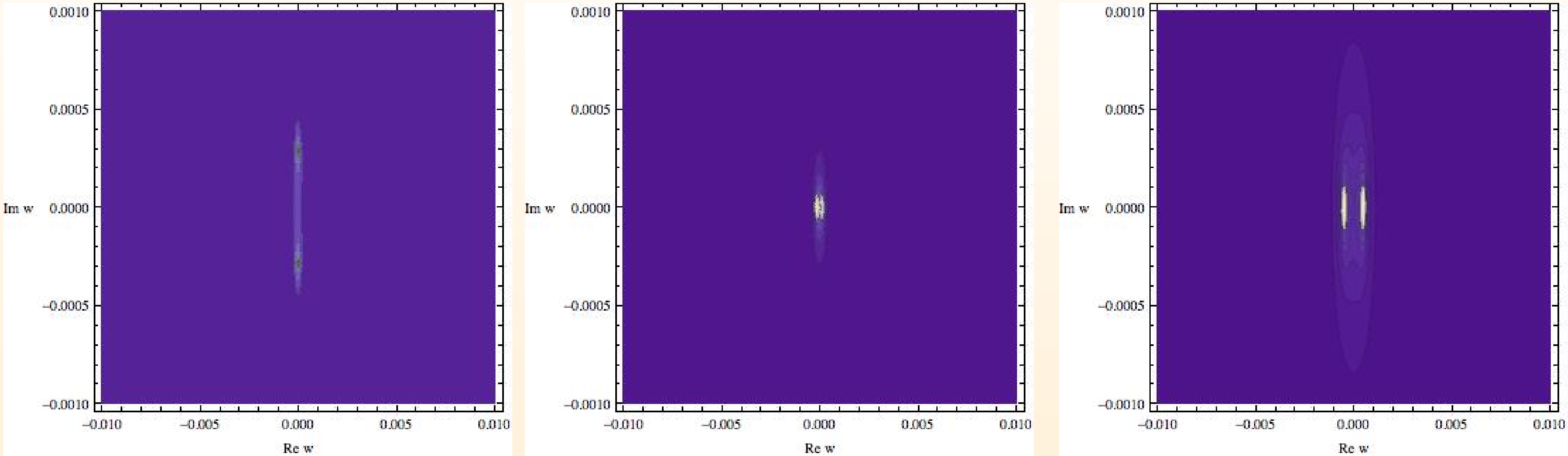


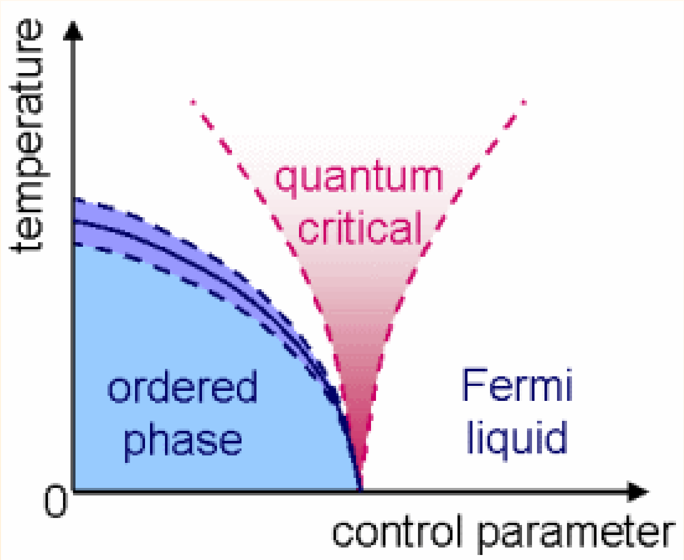
Figure by Patrick Kerner

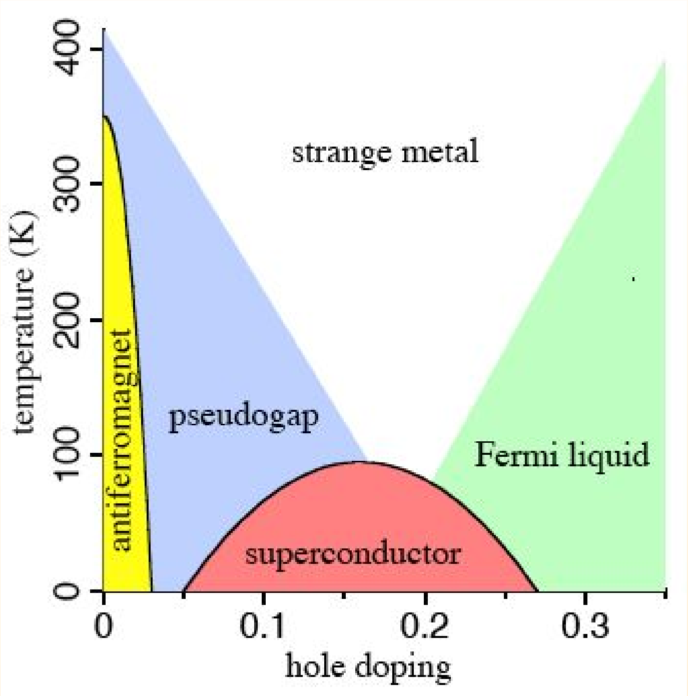
Example for Quantum Phase Transition

Movement of quasinormal modes



Phase diagrams



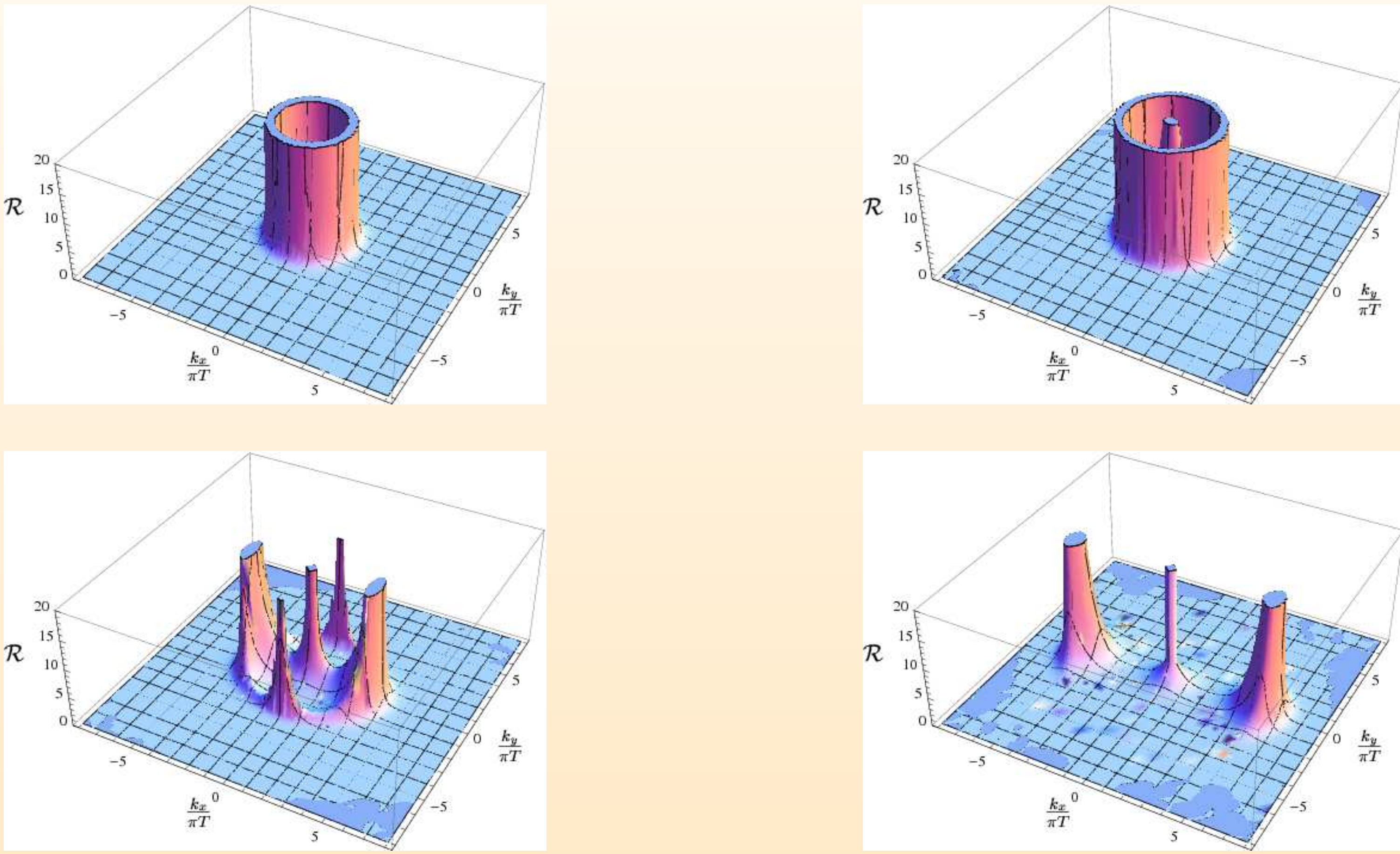


Quantum Phase Transition

Superconductor

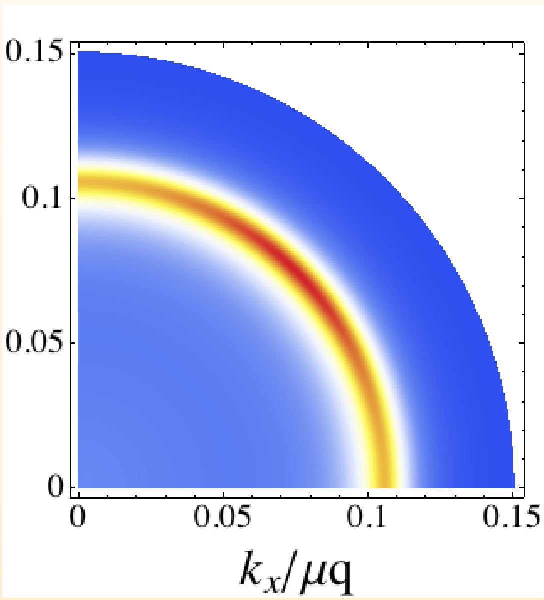
Fermionic excitations in holographic p-wave superfluids

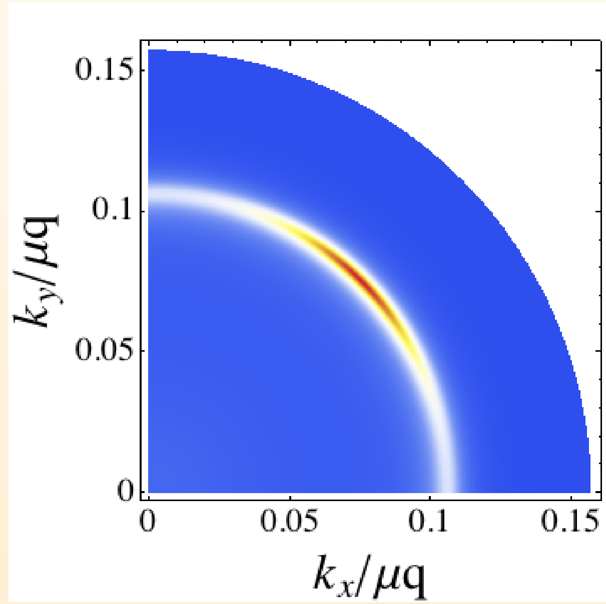
Ammon, J.E., Kaminski, O'Bannon 1003.1134



Benini, Herzog, Yarom 2010

Fermi arcs





$T = 0.59T_c$ $T = 0.49T_c$

Density plot of Fermion spectral function

Summary

- An example of gauge/gravity duality applied to

  phase transitions in strongly coupled theories

  at finite temperature and density
- Meson melting
- At finite isospin density: ρ meson condensation
- Superfluidity: Frictionless motion of mesons
- Fermionic excitations
- Quantum phase transitions

(Selection of) Current activities and new results in gauge/gravity duality

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1. Time-dependent processes, thermalization — Romatschke; Chesler+Yaffe, . . .

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2. Axial anomaly and hydrodynamics at finite baryon density
   J.E., Haack, Kaminski, Yarom; Loganayagam et al

   Chiral phase separation in rotating relativistic fluids — Son, Surowka

   Chiral magnetic effect in gauge/gravity duality
   Landsteiner, Rebhan et al, Yee et al

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5. Quantum phase transitions — Horowitz et al, ...

Thanks to the Gauge/Gravity Duality group at MPI for Physics, Munich

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Hai Thanh Ngo
Patrick Kerner
Constantin Greubel
Steffen Müller
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Matthias Kaminski (now Princeton)
Martin Ammon (now UCLA)

Diplom/Master students:

Saeid Aminian

Jegors Korovins

Hans-Jörg Zeller