





Study of the ppbar->e⁺e⁻π⁰ process in the unphysical region with PANDARoot

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23.06.2020

Outline

- Theoretical description of the signal differential cross section within the one nucleon exchange model
- Determination of statistical errors on the proton form factors in the unphysical region assuming **100% signal efficiency and acceptance**
- Results within PANDARoot

Electromagnetic Form Factors of the Proton



- Electric G_E and magnetic G_M proton FFs are analytical functions of the momentum transfer squared q^2
- Dispersion relations connect space-like and time-like form factors
- No experimental data in the unphysical region

Feynman diagrams for the process pbar+p -> e⁺e⁻π⁰



Give access to the proton form factors in the unphysical region Dominates at backward pion angles |u|, |t| <<s

- M. P. Rekalo, Sov. J. Nucl. Phys. 1 (1965) 760
- C. Adamuscin et al., Phys. Rev. C 75, 045205 (2007)
- A.Z. Dubnickova , S. Dubnicka , M.P. Rekalo, Z. Phys. C 70, 473–481 (1996)
- G. I. Gakh et al. PHYSICAL REVIEW C 83, 025202 (2011)
- Feasibility studies by J. Boucher; PhD thesis (BaBar Framework)
- J. Guttmann, M. Vanderhaeghen, PLB B 719 (2013) 136–142



- Expected to play a role at moderate values of √s, when the pion is emitted around 90° in the center of mass.
- Forward and backward region: suppressed by the phase volume factor |t|/s or |u|/s.

E. A. Kuraev et al., J.Exp.Theor.Phys. 115 (2012) 93-104



Pion-nucleon (Pseudoscalar, pseudovector) coupling:

$$\Gamma_{\pi NN}(q_{\pi}) = g_{\pi NN} \left[\gamma_5, \gamma_5 p_{\alpha}^{\pi} \gamma^{\alpha} \right]$$

C. Adamuscin, E.A. Kuraev, E. Tomasi-Gustafsson, F.E. Maas, Phys. Rev. C 75 (2007) 045205. 5

J. Van de Wiele (PhD thesis of J. Boucher, Paris-Sud and JGU mainz, Orsay, 2011)

$$\overline{p}p \rightarrow \pi^0 \gamma$$



0.5

J. Van de Wiele (PhD thesis of J. Boucher, Paris-Sud and JGU mainz, Orsay, 2011)

$$\overline{p}p \rightarrow \pi^0 e^+ e^-$$

Modified nucleon propagator:

$$t - channel: \frac{1}{t - M^2} \rightarrow \frac{1}{t - M^2}.F$$
$$u - channel: \frac{1}{u - M^2} \rightarrow \frac{1}{u - M^2}.F$$
$$F = \left[\frac{\lambda^2 - M^2}{\lambda^2 - t}\right] \left[\frac{\lambda^2 - M^2}{\lambda^2 - u}\right]$$





- 3 body final state: differential cross section is described by 5 independent kinematic variables (one choice: s, q^2 , $\theta_{\pi 0}$, θ_e^* , ϕ_e^*)
 - $\theta_{\pi 0}$ in the laboratory frame
 - θ_e^*, φ_e^* in the virtual photon rest frame (*)

 $L_{\mu\nu}$: leptonic tensors describing the γ^* ->e+e- process

 $H_{\mu\nu}$: hadronic tensors describes pion-nucleon coupling, nucleon propagator (corrected to fit the $\pi^0\gamma$ data from Fermilab) and contains the information on G_E and G_M

J. Van de Wiele (PhD thesis of J. Boucher, Paris-Sud and JGU mainz, Orsay, 2011) ₈



In the γ^* rest frame (unpolarized experiment):

$$\frac{d\sigma}{dq^2 d\cos\theta_{\pi^0} d\Omega_e^*} = 4\pi e^2 q^2 (H_{11} + H_{22} + H_{33}) - 8e^2 p_e^{*2} \left(\frac{H_{11} + H_{22}}{2} + \frac{H_{11} - H_{22}}{2} \sin^2\theta_e^* \cos^2\theta_e^* + 2H_{13} \sin\theta_e^* \cos\theta_e^* \cos^2\theta_e^* + \frac{1}{2}(2H_{33} - H_{11} - H_{22})\cos^2\theta_e^* + \frac{1}{2}(2H_{3$$

• The polar and azimuthal angular distributions of e⁺/e⁻ (θ_e^* , ϕ_e^*) gives access to 4 H_{µv} (H₁₁, H₂₂, H₃₃, H₁₃)

$$H_{\mu\nu} = |G_M|^2 [\alpha_{\mu\nu} R^2 + \beta_{\mu\nu} + \gamma_{\mu\nu} R\cos(\phi_E - \phi_M)], R = |G_E| / |G_M|$$

• $\alpha_{\mu\nu}$, $\beta_{\mu\nu}$, $\gamma_{\mu\nu}$ depend on s, q^2 and $\theta_{\pi 0}$

J. Van de Wiele (PhD thesis of J. Boucher, Paris-Sud and JGU mainz, Orsay, 2011)

 θ_{e}^{*}

- For one value of s, and fixing the q^2 and $\theta_{\pi 0}$ intervals:
 - Extract the proton form factors directly from the 2D distribution:

$$\frac{d^2\sigma}{d\Omega_e^*} = \int_{\Delta q^2} \int_{\Delta \theta_{\pi^0}} \frac{d\sigma}{dq^2 d\cos\theta_{\pi^0}} d\Omega_e^* dq^2 d\cos\theta_{\pi^0}$$

Integration over one variable (to avoid low statistics bins):

$$\frac{dN_1}{d\cos\theta_e^*} = L \int_{\Delta q^2} \int_{\Delta \theta_{a^0}} \int_0^{2\pi} \frac{d\sigma}{dq^2 d\cos\theta_{\pi^0} d\Omega_e^*} dq^2 d\cos\theta_{\pi^0} d\varphi_e^* = A(1 + B\cos^2\theta_e^*)$$

$$\frac{dN_2}{d\varphi_e^*} = L \int_{\Delta q^2} \int_{\Delta \theta_{a^0}} \int_{-1}^1 \frac{d\sigma}{dq^2 d\cos\theta_{\pi^0} d\Omega_e^*} dq^2 d\cos\theta_{\pi^0} d\cos\theta_e^* = C(1 + D\cos 2\varphi_e^*)$$

$$\frac{dN_3}{d\varphi_e^*} = L \int_{\Delta q^2} \int_{\Delta \theta_{a^0}} \int_0^1 \frac{d\sigma}{dq^2 d\cos\theta_{\pi^0} d\Omega_e^*} dq^2 d\cos\theta_{\pi^0} d\cos\theta_e^* = E(1 + F\cos 2\varphi_e^* + G\cos\varphi_e^*)$$

J. Van de Wiele (PhD thesis of J. Boucher, Paris-Sud and JGU mainz, Orsay, 2011)

Time-like proton form factors (VMD model)



Determination of the proton form factors For each interval of q^2 and $\theta_{\pi 0}$:

- 1. Calculate the theoretical number of counts $N^{th[}[i,j]$ in each bin of the 2Ddistribution (θ^*_{e}, ϕ^*_{e})
- 2. Calculate the number of reconstructed events N^{rec[}[i,j] taking into account signal efficiency
- 3. Calculate the number of observed events N^{obs[}[i,j] taking into account statistical fluctuations by generating a random number in each bin of the 2D-distribution (Poisson distribution with mean N^{rec[}[i,j])
- 4. Correct the observed events by the signal efficiency: N^{cor[}[i,j]
- 5. Projections to obtain the three 1D-distrbutions:

$$N_{1} = \sum_{j=0}^{j < Nj} N^{cor}[i, j], N_{2} = \sum_{i=0}^{i < Ni} N^{cor}[i, j], N_{3} = \sum_{i=0}^{i < Ni/2} N^{cor}[i, j]$$

- 6. Extract the proton form factors by fitting the three 1D-distrubtions
- Error estimation of the proton FFs by generating ~100 histograms using N^{obs[}[i,j] as input for random generation

A, B, C, D, E, F, G are functions of the proton form factors

	R	$\cos(\phi_{\rm E}-\phi_{\rm M})$	
Theoretical values	1.066	0.998	
Fit results (one simulation)	1.067 ± 0.006	1.040 ± 0.085	

	R	$\cos(\phi_{\rm E} - \phi_{\rm M})$
Theoretical values	1.066	0.998
Fit results (500 simulations)	1.063 ± 0.007	1.034 ± 0.084

• $q^2=0.605 \pm 0.015 (GeV/c^2)^2$, 100% signal efficiency

R	$10^{\circ} < \theta_{\pi 0} < 30^{\circ}$	$30^{\circ} < \theta_{\pi 0} < 50^{\circ}$	$80^{\circ} < \theta_{\pi 0} < 100^{\circ}$	$120^{\circ} < \theta_{\pi 0} < 140^{\circ}$	$140^{\circ} < \theta_{\pi 0} < 160^{\circ}$
Events N th	2.9 . 10 ⁶	2.5 . 10 ⁶	$1.4 . 10^{6}$	840234	517112
Theoretical values	1.066	1.066	1.066	1.066	1.066
Fit results (this work)	1.063±0.007 (0.7%)	1.065±0.004 (0.4%)	1.052±0.007 (0.6%)	1.024±0.022 (2.1%)	1.068±0.070 (6.4%)

$\cos(\phi_{\rm E} - \phi_{\rm M})$	$10^{\circ} < \theta_{\pi 0} < 30^{\circ}$	$30^{\circ} < \theta_{\pi 0} < 50^{\circ}$	$80^{\circ} < \theta_{\pi 0} < 100^{\circ}$	$120^{\circ} < \theta_{\pi 0} < 140^{\circ}$	$140^{\circ} < \theta_{\pi 0} < 160^{\circ}$
Theoretical values	0.998	0.998	0.998	0.998	0.998
Fit results (this work)	1.034±0.084 (8%)	1.086±0.063 (6%)	0.956±0.058 (6%)	0.665±0.129 (19%)	Х

• $q^2 = 2 \pm 0.125 (GeV/c^2)^2$, 100% signal efficiency

R	$10^{\circ} < \theta_{\pi 0} < 30^{\circ}$	$30^{\circ} < \theta_{\pi 0} < 50^{\circ}$	$80^{\circ} < \theta_{\pi 0} < 100^{\circ}$	$120^{\circ} < \theta_{\pi 0} < 140^{\circ}$	$140^{\circ} < \theta_{\pi 0} < 160^{\circ}$
Events N th	18441	17379	9362	4989	2954
Theoretical values	0.802	0.802	0.802	0.802	0.802
Fit results (this work)	0.802±0.026 (3%)	0.809±0.017 (2%)	0.785±0.028 (3.5%)	0.761±0.075 (10%)	Х

$\cos(\phi_{\rm E} - \phi_{\rm M})$	$10^{\circ} < \theta_{\pi 0} < 30^{\circ}$	$30^{\circ} < \theta_{\pi 0} < 50^{\circ}$	$80^{\circ} < \theta_{\pi 0} < 100^{\circ}$	$120^{\circ} < \theta_{\pi 0} < 140^{\circ}$	$140^{\circ} < \theta_{\pi 0} < 160^{\circ}$
Theoretical values	0.999	0.999	0.999	0.999	0.999
Fit results (this work)	1.006±0.082 (8%)	0.905±0.076 (8.5%)	0.904±0.090 (10%)	0.929±0.244 (28%)	Х

Feasibility studies with PANDARoot

Description of the Monte Carlo simulations

Monte Carlo Simulation Studies:

Background suppression versus signal efficiency

Background $\overline{p}p \rightarrow \pi^+\pi^-\pi^0$ Signal $\overline{p}p \rightarrow e^+e^-\pi^0$

- PANDARoot version oct19, FairSoft jun19p1, FairRoot v18.2
- Simulation macros from master directory
- **Event** generation ٠

 - Antiproton momentum (lab) $p_{pbar}=1.7 \text{ GeV/c}$ $q^2 = 0.605 \pm 0.015 (\text{GeV/c}^2)^2$: $M_{inv}=[0.77-0.79] \text{ GeV/c}$
 - $q^2 = 2.0 \pm 0.375 (GeV/c^2)^2$: $M_{inv} = [1.27 1.54] GeV/c$
 - PHSP angular distributions, PHOTOS switched on
 - 5.10⁷ events for the signal in each q2 interval
 - 10⁸ events for the background in each q2 interval
 - **Reconstruction and particle identification**
 - Kalman Filter with muon hypothesis ٠
 - PID Correlator with pion hypothesis

Signal and background cross sections

- P=1.7 GeV/c
- $q^2=0.6 \text{ (GeV}^2/c^2)$, $M_{inv}=0.78 \text{ GeV}/c$

$$\frac{\sigma(\overline{p}p \to \pi^+ \pi^- \pi^0)}{\sigma(\overline{p}p \to e^+ e^- \pi^0)} \sim 10^3 - 10^4$$

- Background rejection ~ 10⁻⁶-10⁻⁷ is needed for a signal pollution < 1%</p>
- $q^2=2$ (GeV²/c²), M_{inv}=1.4 GeV/c

$$\frac{\sigma(\overline{p}p \to \pi^+ \pi^- \pi^0)}{\sigma(\overline{p}p \to e^+ e^- \pi^0)} \sim 10^6 - 10^7$$

Background rejection < 10⁻⁸ is needed for a signal pollution at few percent level

Event Selection $\overline{p}p \rightarrow e^+e^-\pi^0$

I. Charged track selection (related to the background suppression)

- a) Events with only two charged tracks are selected
- b) PID probability for the detected particle to be identified as e+/elarger than 99% or 99.8% (EMC+STT+MVD+DRC)
- c) EMC deposit energy over the tracking momentum $E_{EMC}/p>0.8$
- d) Energy loss in the STT dE/dx(STT) > 5.8

II. Pion reconstruction (RN-QCD-2018-002)

- At least two photons/event with EMC raw energy > 15 MeV
- Pion mass cut: $M_{\pi 0} 0.05 < M_{\gamma \gamma} < M_{\pi 0} 0.05 (GeV/c^2)$
- Mass constraint fit to the nominal π0 mass : Prob. > 10⁻³. In case of more than one reconstructed pion/event, the pair (γγ) of higher fit probability is selected.

III. Events selection

• 4C fit is applied to the reconstructed three particles ($\chi 2 < 50$ or $\chi 2 < 30$)

Signal and background Efficiencies

$q^2=0.605 \pm 0.015 (GeV/c^2)^2$			$q^2=2.0 \pm 0.375 \ (GeV/c^2)^2$		
Sequential eff.	Signal	background		Signal	Background
Reconstruction	65%	63 %	Reconstruction	69 %	68%
PID (Prob.>99%, E _{EMC} /p, dE/dx)	22%	4.10-5	PID (Prob.>99.8%, E _{EMC} /p, dE/dx)	30%	2.10-5
pi0 Rec.	16%	7.10-6	pi0 Rec.	19%	9.10-6
4C-fit (χ2_4C<50)	13%	2.10-7	4C-fit (χ2_4C<30)	15%	<10-8

$$\bar{p}p \rightarrow e^+ e^- \pi^0$$

Invariant mass squared of the selected e⁺e⁻

• $q^2=0.605 \pm 0.015 (GeV/c^2)^2$

"Before Bremsstrahlung correction, without 4C kinematic fit"

"After Bremsstrahlung correction without 4C kinematic fit" (Methode described in: E. ATOMSSA TN-STT-2015-001)

"After Bremsstrahlung correction with 4C kinematic fit"

Measurement of the proton FFs in small intervals of q^2 (in the unphysical region) is possible at PANDA

• $q^2=0.605 \pm 0.015 (GeV/c^2)^2$, $\theta_{\pi 0}=[10^{\circ}-30^{\circ}]$,

Determination of the proton form factors (Results)

• $q^2=0.605 \pm 0.015 (GeV/c^2)^2$

R	$10^{\circ} < \theta_{\pi 0} < 30^{\circ}$	$30^{\circ} < \theta_{\pi 0} < 50^{\circ}$	$80^{\circ} < \theta_{\pi 0} < 100^{\circ}$	$120^{\circ} < \theta_{\pi 0} < 140^{\circ}$	$140^{\circ} < \theta_{\pi 0} < 160^{\circ}$
Theoretical values	1.066	1.066	1.066	1.066	1.066
Fit results (100% signal efficiency)	1.063±0.007	1.065±0.004	1.052±0.007	1.024±0.022	1.068±0.070
	(0.7%)	(0.4%)	(0.6%)	(2.1%)	(6.4%)
Fit results	1.061±0.029	1.059±0.009	1.072±0.018	1.171±0.672	X
(PANDARoot)	(2.7%)	(0.8%)	(1.6%)	(57%)	

$\cos(\phi_{\rm E} - \phi_{\rm M})$	$10^{\circ} < \theta_{\pi 0} < 30^{\circ}$	$30^{\circ} < \theta_{\pi 0} < 50^{\circ}$	$80^{\circ} < \theta_{\pi 0} < 100^{\circ}$	$120^{\circ} < \theta_{\pi 0} < 140^{\circ}$	$140^{\circ} < \theta_{\pi 0} < 160^{\circ}$
Theoretical values	0.998	0.998	0.998	0.998	0.998
Fit results (100% signal efficiency)	1.034±0.084 (8%)	1.086±0.063 (6%)	0.956±0.058 (6%)	0.665±0.129 (19%)	X
Fit results (PANDARoot)	1.302±0.334 (26%)	1.156±0.131 (11%)	0.785±0.138 (18%)	X	X

Determination of the proton form factors (Results)

• $q^2 = 2 \pm 0.125 (GeV/c^2)^2$

R	$10^{\circ} < \theta_{\pi 0} < 30^{\circ}$	$30^{\circ} < \theta_{\pi 0} < 50^{\circ}$	$80^{\circ} < \theta_{\pi 0} < 100^{\circ}$	$120^{\circ} < \theta_{\pi 0} < 140^{\circ}$	$140^{\circ} < \theta_{\pi 0} < 160^{\circ}$
Theoretical values	0.802	0.802	0.802	0.802	0.802
Fit results (100% signal efficiency)	0.802±0.026 (3%)	0.809±0.017 (2%)	0.785±0.028 (3.5%)	0.761±0.075 (10%)	X
Fit results (PANDARoot)	0.638±0.099 (16%)	0.745±0.041 (5%)	0.628±0.068 (11%)	0.656±0.165 (25%)	X

$\cos(\phi_{\rm E} - \phi_{\rm M})$	$10^{\circ} < \theta_{\pi 0} < 30^{\circ}$	$30^{\circ} < \theta_{\pi 0} < 50^{\circ}$	$80^{\circ} < \theta_{\pi 0} < 100^{\circ}$	$120^{\circ} < \theta_{\pi 0} < 140^{\circ}$	$140^{\circ} < \theta_{\pi 0} < 160^{\circ}$
Theoretical values	0.999	0.999	0.999	0.999	0.999
Fit results (100% signal efficiency)	1.006±0.082 (8%)	0.905±0.076 (8.5%)	0.904±0.090 (10%)	0.929±0.244 (28%)	X
Fit results (PANDARoot)	1.179±0.343 (29%)	1.576±0.210 (13%)	0.556±0.302 (54%)	X	X

Summary

- The process **ppbar->e+e** π^0 is considered within the one nucleon exchange model in t-and u-channels
 - to access the proton form factors in the unphysical region
- Monte Carlo simulations for signal and background processes are performed with PANDARoot. The results show that
 - Background can be sufficiently suppressed
 - The proton form factor ratio and the relative phase term $\cos(\phi_E \phi_M)$ can be determined in different intervals of q2 and thetepi0 with good precisions
- Next step: investigation of the **ppbar->e+e** π^0 process beyond the one nucleon exchange model:
 - Theoretical analysis of the ppbar->e⁺e⁻π⁰ process within a Regge framework (J. Guttmann, M. Vanderhaeghen, PLB 719 (2013) 136–142)-> exchange of dominant baryon Regge trajectories

Backup slides

Differential cross section within Regge framework

J. Guttmann, M. Vanderhaeghen / PLB 719 (2013) 136–142

Regge trajectory for the nucleon

• In the γ^* rest frame (unpolarized experiment):

$$L^{\mu\nu}H_{\mu\nu} = 2e^2q^2(H_{11} + H_{22} + H_{33})$$

-8e^2p_e^{*2}(H_{11}\sin^2\theta_e^*\cos^2\varphi_e^* + 2H_{13}\sin\theta_e^*\cos\theta_e^*\cos\varphi_e^*
+H_{22}\sin^2\theta_e^*\sin^2\varphi_e^* + H_{33}\cos^2\theta_e^*)

- Non zero hadronic tensors H₁₁, H₂₂, H₃₃, H₁₃
- $H_{0\nu}=0$ by gauge invariance (in this frame $q_{\mu}H^{\mu\nu}=q_{0}H^{0\nu}+q_{i}H^{i\nu}=0$)
- H₁₂, H₂₃ numerically zero

J. Van de Wiele (PhD thesis of J. Boucher, Paris-Sud and JGU mainz, Orsay, 2011)

Electromagnetic Form Factors of the Proton

- Electric G_E and magnetic G_M proton FFs are functions of q^2
- Unphysical region can be accessed by:

Phys. C 70, 473-481 (1996) Phys. Rev. C 75, 045205 (2007)

Electromagnetic form factors of the proton (Time-Like region)

Charged Track Reconstruction

Charged Track Selection (PID Probabilities)

q²=0.605 ± 0.015 (GeV/c²)²

PID probability for the negative track to be identified as electron:

PID probability for the detected particle to be identified as e+/e- larger than 99% (EMC+STT+MVD+DRC)

Charged Track Selection (E_{EMC}/P and dE/dx STT)

$q^2=0.605 \pm 0.015 (GeV/c^2)^2$,

Charged Track Reconstruction (PID)

 $q^2=0.605 \pm 0.015 (GeV/c^2)^2$

- PID probability for the detected particle to be identified as e+/e-larger than 99% (EMC+STT+MVD+DRC)
- $E_{EMC}/p>0.8$ and dE/dx(STT) > 5.8

Signal eff. after PID prob.: 42%

Signal efficiency after PID prob., E_{EMC}/p and dE/dx(STT) cut: 22%

Charged Track Reconstruction and PID

 $q^2=0.605 \pm 0.015 (GeV/c^2)^2$

Rec. efficiency 61%

Efficiency after PID (>0.99) 19%

Neutral pion reconstruction (photon candidates)

 $q^2=0.605 \pm 0.015 (GeV/c^2)^2$

- At least two photons/event with EMC raw energy > 15 MeV
- Pion mass cut: $M_{\pi 0} 0.05 < M_{\gamma \gamma} < M_{\pi 0} 0.05 (GeV/c^2)$
- Mass constraint fit to the nominal π0 mass : Prob. > 10⁻³. In case of more than one reconstructed pion/event, the pair (γγ) of higher fit probability is selected.

Signal eff. after pi0 selection 16%

Event selection (4C kinematic fit)

Charged tracks and neutral pion selection conditions are applied

• $q^2=0.605 \pm 0.015 (GeV/c^2)^2$, **100% signal efficiency**

R	$10^{\circ} < \theta_{\pi 0} < 30^{\circ}$	$30^{\circ} < \theta_{\pi 0} < 50^{\circ}$	$80^{\circ} < \theta_{\pi 0} < 100^{\circ}$	$120^{\circ} < \theta_{\pi 0} < 140^{\circ}$	$140^{\circ} < \theta_{\pi 0} < 160^{\circ}$
Events N th	2.9 . 10 ⁶	2.5 . 10 ⁶	$1.4 . 10^{6}$	840234	517112
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Fit results (this work)	1.063±0.007 (0.7%)	1.065±0.004 (0.4%)	1.052±0.007 (0.6%)	1.024±0.022 (2.1%)	1.068±0.070 (6.4%)
Fit results (J. Boucher PhD)	1.066±0.008		1.063±0.007		1.067±0.074

$\cos(\phi_{\rm E} - \phi_{\rm M})$	$10^{\circ} < \theta_{\pi 0} < 30^{\circ}$	$30^{\circ} < \theta_{\pi 0} < 50^{\circ}$	$80^{\circ} < \theta_{\pi 0} < 100^{\circ}$	$120^{\circ} < \theta_{\pi 0} < 140^{\circ}$	$140^{\circ} < \theta_{\pi 0} < 160^{\circ}$
Theoretical values	0.998	0.998	0.998	0.998	0.998
Fit results (this work)	1.034±0.084 (8%)	1.086±0.063 (6%)	0.956±0.058 (6%)	0.665±0.129 (19%)	Х
Fit results (J. Boucher PhD)	1.049±0.103		0.997±0.063		Х

• $q^2 = 2 \pm 0.125 (GeV/c^2)^2$, **100% signal efficiency**

R	$10^{\circ} < \theta_{\pi 0} < 30^{\circ}$	$30^{\circ} < \theta_{\pi 0} < 50^{\circ}$	$80^{\circ} < \theta_{\pi 0} < 100^{\circ}$	$120^{\circ} < \theta_{\pi 0} < 140^{\circ}$	$140^{\circ} < \theta_{\pi 0} < 160^{\circ}$
Events N th	18441	17379	9362	4989	2954
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Fit results (this work)	0.802±0.026 (3%)	0.809±0.017 (2%)	0.785±0.028 (3.5%)	0.761±0.075 (10%)	Х
Fit results (J. Boucher PhD)	0.8±0.028		0.802±0.031		Х

$\cos(\phi_{\rm E} - \phi_{\rm M})$	$10^{\circ} < \theta_{\pi 0} < 30^{\circ}$	$30^{\circ} < \theta_{\pi 0} < 50^{\circ}$	$80^{\circ} < \theta_{\pi 0} < 100^{\circ}$	$120^{\circ} < \theta_{\pi 0} < 140^{\circ}$	$140^{\circ} < \theta_{\pi 0} < 160^{\circ}$
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Fit results (J. Boucher PhD)	1.001±0.076		0.999±0.084		Х

Time-like proton form factors (VMD model)

Vector Meson Dominance (VMD) F. Iachello, PRC 69, 055204 (2004)

 $q^2=0.605\pm0.005 (GeV/c^2)^2$ R=1.066, $\cos(\phi_E-\phi_M)=0.998$ (4°)

 $q^2=2\pm0.125 (GeV/c^2)^2$ R=0.802, $\cos(\phi_E-\phi_M)=0.999$ (3°)

2 fb⁻¹

s=5.4 GeV ²	q ² =0.605±0.005 (GeV/c ²) ²	$q^2 = 2 \pm 0.125 \ (GeV/c^2)^2$
$10^{\circ} < \theta_{\pi 0} < 30^{\circ}$	2.91271 . 10 ⁶	18441
$30^{\circ} < \theta_{\pi 0} < 50^{\circ}$	2.47392 . 10 ⁶	17379
$90^{\circ} < \theta_{\pi 0} < 100^{\circ}$	$1.40351.10^{6}$	9362
$120^{\circ} < \theta_{\pi 0} < 140^{\circ}$	840234	4989
$140^{\circ} < \theta_{\pi 0} < 160^{\circ}$	517112	2954

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