

Machine Learning Based Track Reconstruction

Joint GlueX-PANDA-EIC ML virtual workshop

22.09.2020 | Waleed Esmail, Tobias Stockmanns and James Ritman

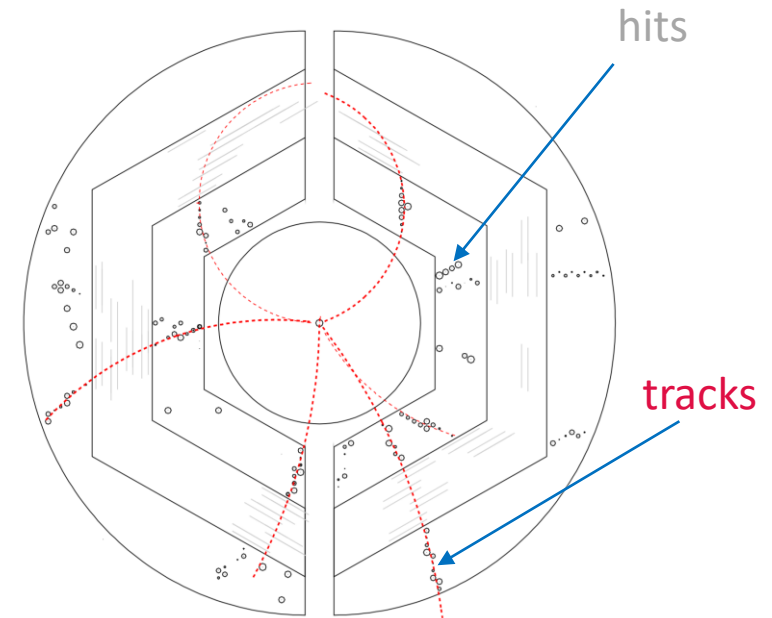
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Outlines:

- Introduction
- TrackML challenge
- PANDA FTS
- ANN and RNN application to FTS
- GNN application to FTS
- Track fitting
- Hands-on tutorials

Introduction:

- Track reconstruction is a pattern recognition task
- Two main steps: **Track Finding** and **Track Fitting** (usually done in iterative procedure).
- **Track Finding**: assign position measurements (**hits**) to track candidates (**particle paths**)
- **Track Fitting**: determine track parameters and covariance matrix for each track
- Track finding is usually the most time-consuming part in the reconstruction process
- There are two generic approaches for track finding:
 1. **Local approach**: find track candidates consecutively
 2. **Global approach**: find all track candidates at once
- Good tracking algorithm should be **high efficiency, high purity, low fake rate, and fast algorithm**

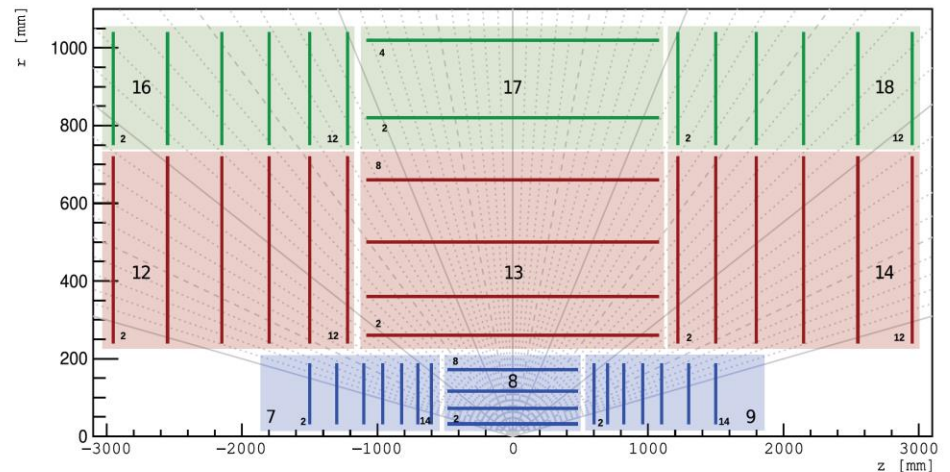
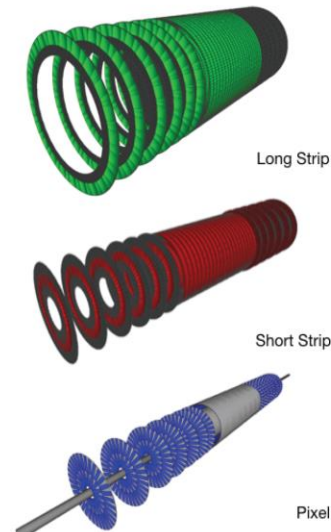


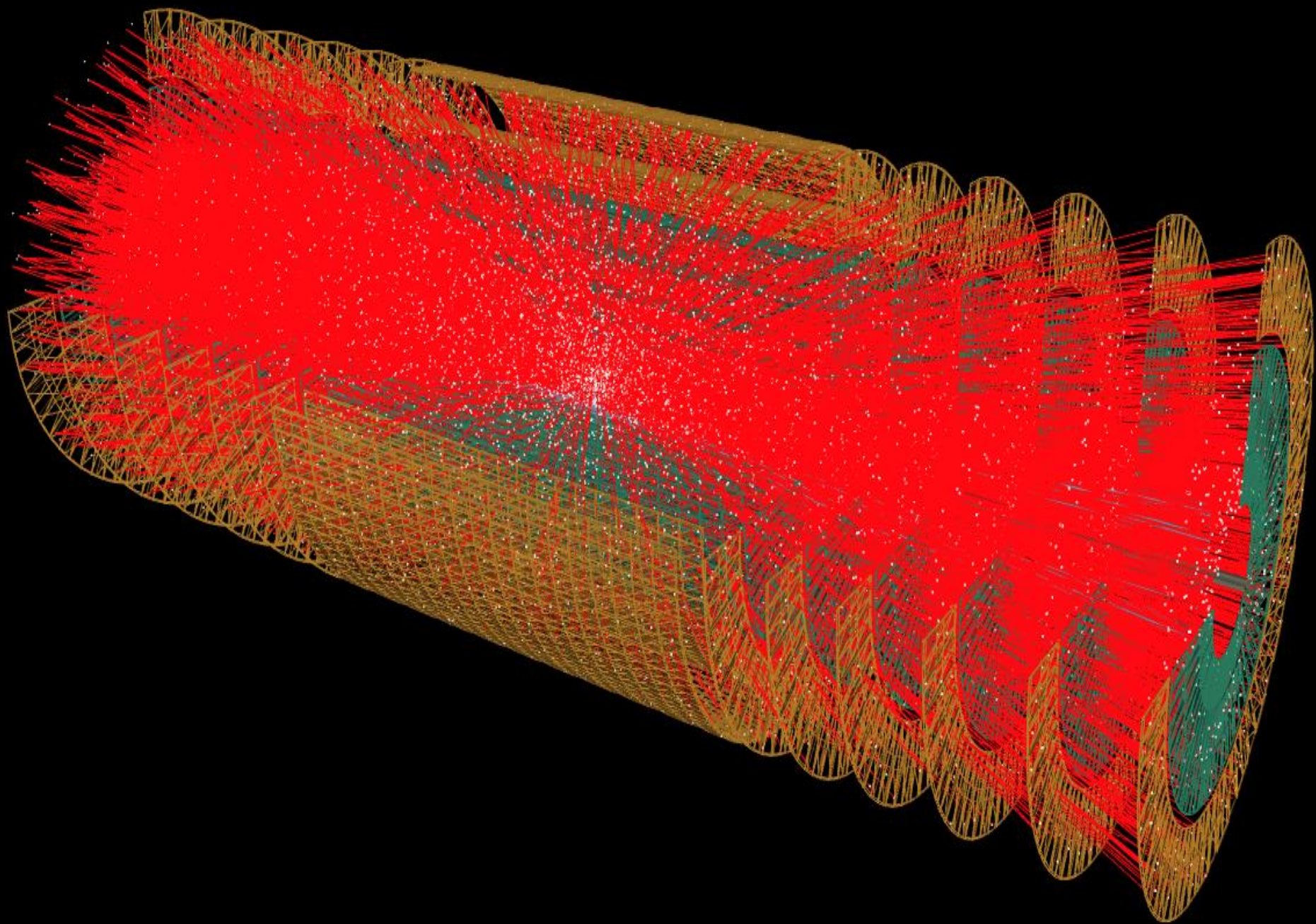
TrackML challenge I:

- A **competition** hosted by [Kaggle](#) ([Accuracy](#)) and [Codalab](#) ([Accuracy & Speed](#))
- A participant is challenged to build an algorithm that quickly reconstructs particle tracks from 3D points (hits)
- Can ML tracking compete traditional approaches (for HL-LHC)?!
- Realistic detector model to simulate measured particle hits (ACTS simulation)
- A hard QCD interaction overlaid with soft QCD interactions (pileup)

~ 10000 tracks/event

~ 10 hits/track





TrackML challenge II:

➤ What is provided

1. 3D space points in global coordinate system (hits)
2. Cells: Each hit originates from one or more active detector cells
3. Ground Truth information (for supervised models)

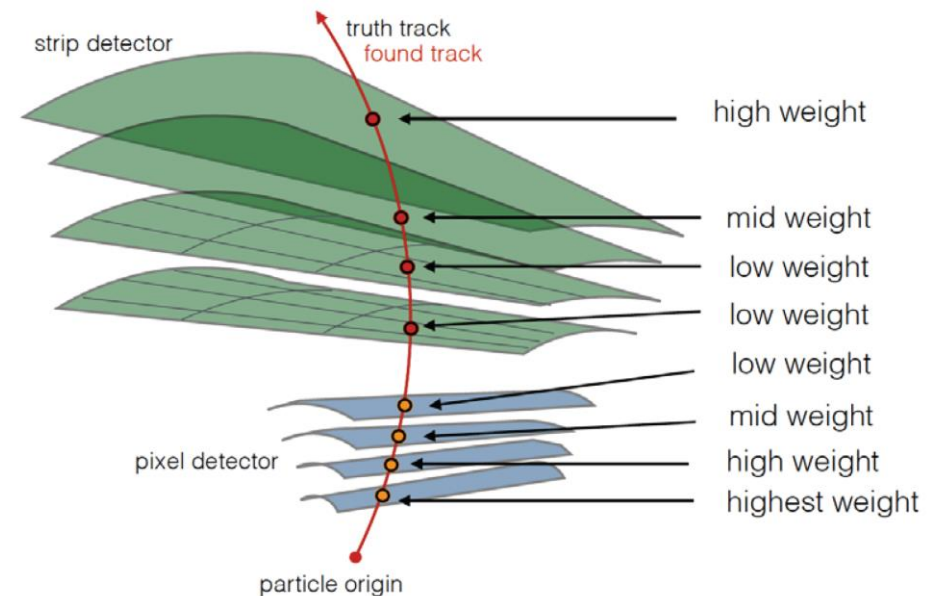
➤ What is expected

1. High score

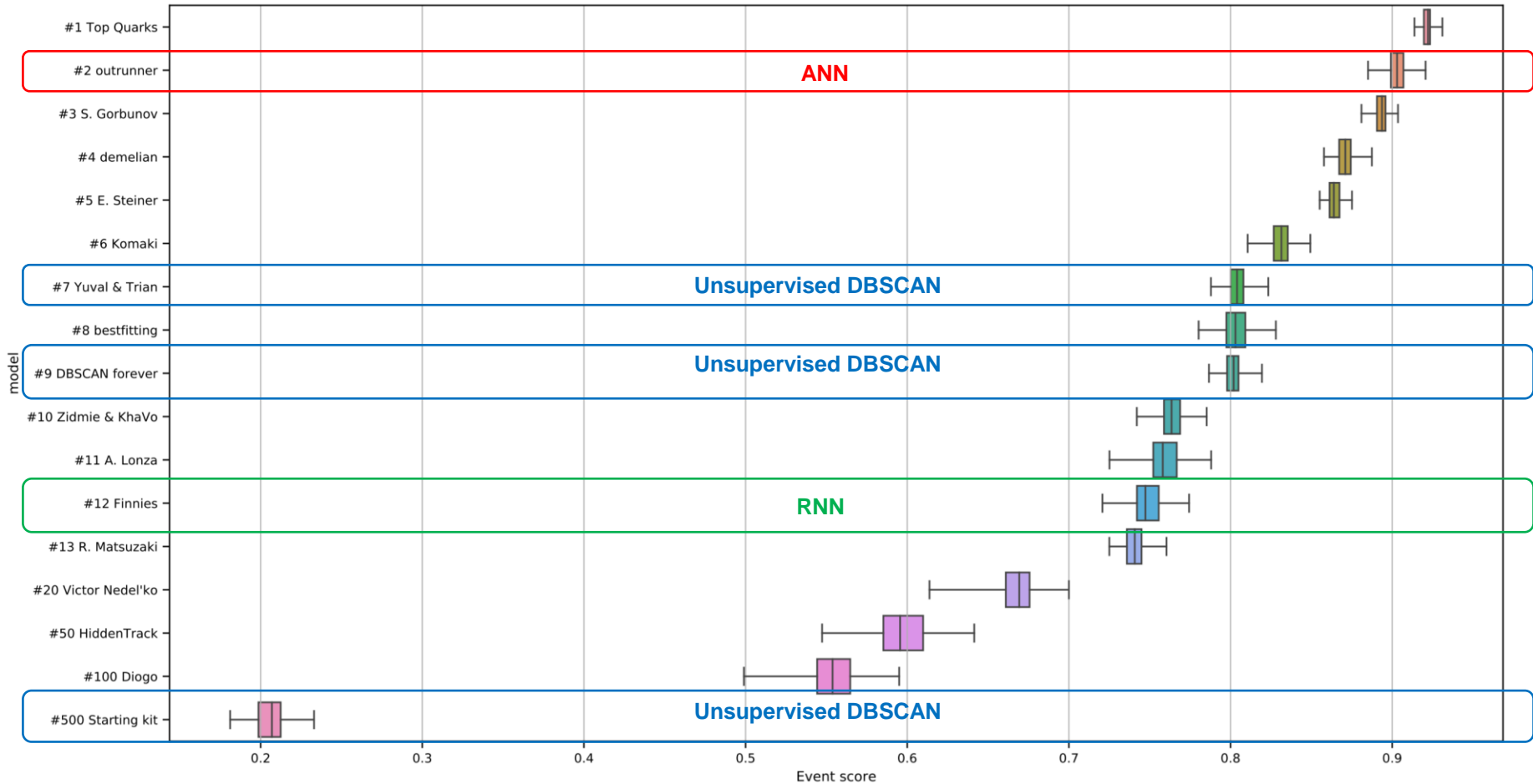
$$S \sim \sum_{\{\text{events}\}} \sum_{\{\text{tracks}\}} \begin{cases} 0 & \# \text{good hits} < 50\%, \# \text{hits} < 3 \\ \sum_{\{\text{good hits}\}} w_i & \text{else} \end{cases}$$

$$S_{\text{perfect}} = 1$$

$$w_i = w_i(\text{hit order, particle } p_{\perp}) .$$



TrackML Solutions

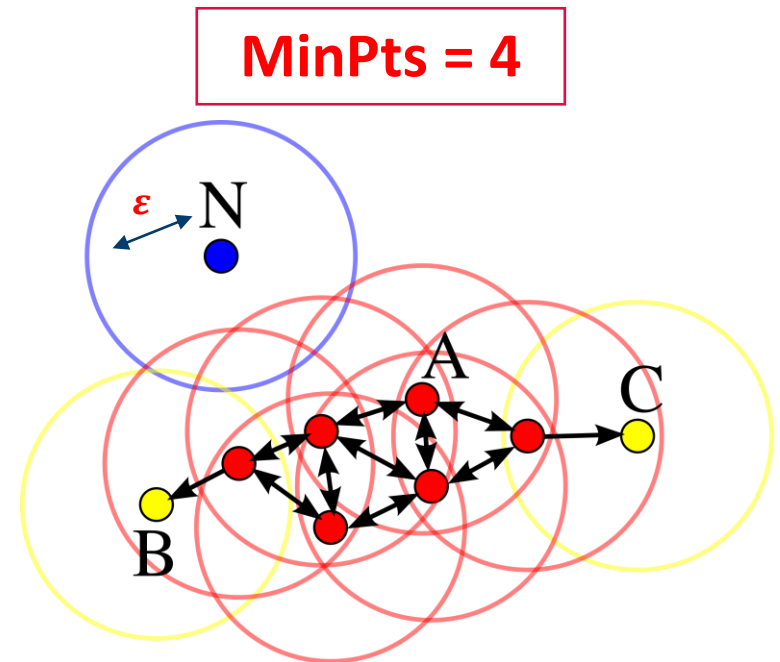


TrackML Solutions: (DBSCAN I)

- Track finding is a clustering process, so why not to use **unsupervised** methods!
- **Clustering**: cluster data points (**hits**) that are more similar to each other
- **DBSCAN** **D**ensity **B**ased **S**patial **C**lustering of **A**pplications with **N**oise
- **Density** is parametrized by a hyperparameter ϵ .

- Label is assigned to each data point
 1. **core point** (\geq min # of points **MinPts** within ϵ)
 2. **boarder point** ($<$ min # of points **MinPts** within ϵ)
 3. all other points are **noise points**.

- A point q is **directly reachable** from p if point q is within distance ϵ from core point p , or if there is a path of points.



Wikipedia/DBSCAN

TrackML Solutions: (DBSCAN II)

➤ One of the baseline solution with accuracy ~ 0.25

➤ Idea is to do **hit transformation**

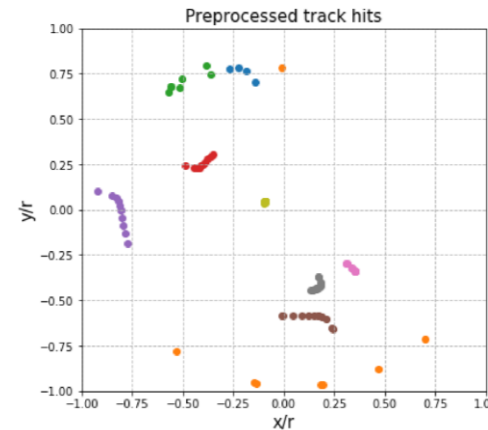
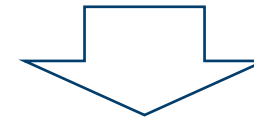
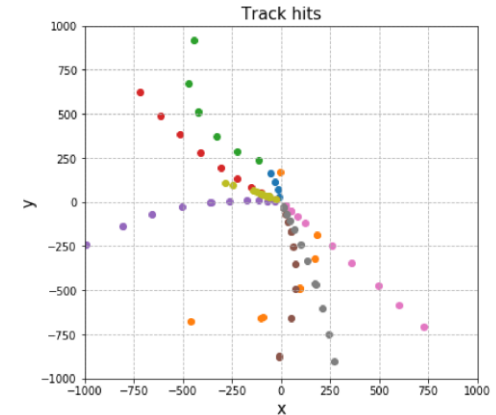
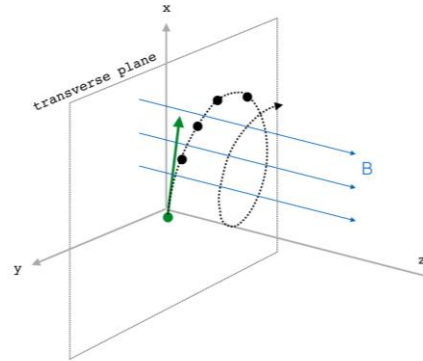
$$r_1 = \sqrt{x^2 + y^2 + z^2}$$

$$x_2 = x/r_1$$

$$y_2 = y/r_1$$

$$r_2 = \sqrt{x^2 + y^2}$$

$$z_2 = z/r_2$$



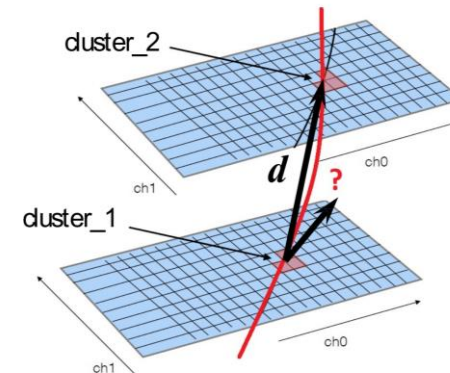
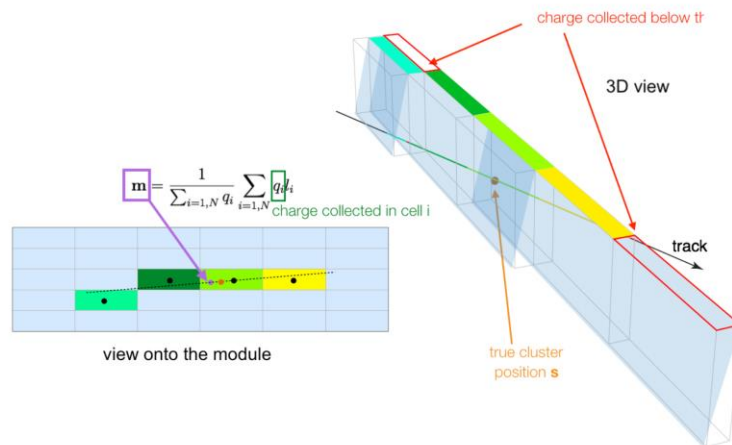
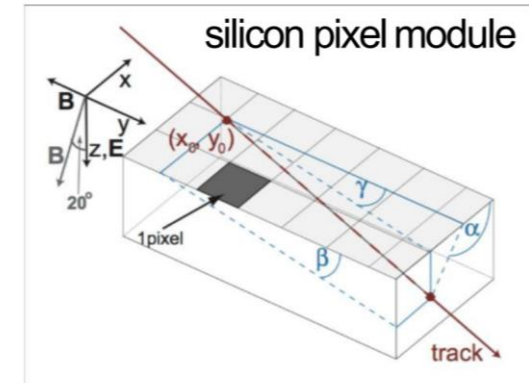
➤ Many other solutions based on DBSCAN are heavily dependent on **preprocessing (feature engineering)**

➤ Core idea is to **unroll the helix**

TrackML Solutions: (DL solution I)

➤ The solution that ranked **second in the challenge** is using an artificial neural network.

- Input [two hits] -> DNN -> output [pair quality]
- Input features (x, y, z, direction from cells, ...)
- Output (pair probability) -> (Adjacency Matrix)



TrackML Solutions: (DL solution II)

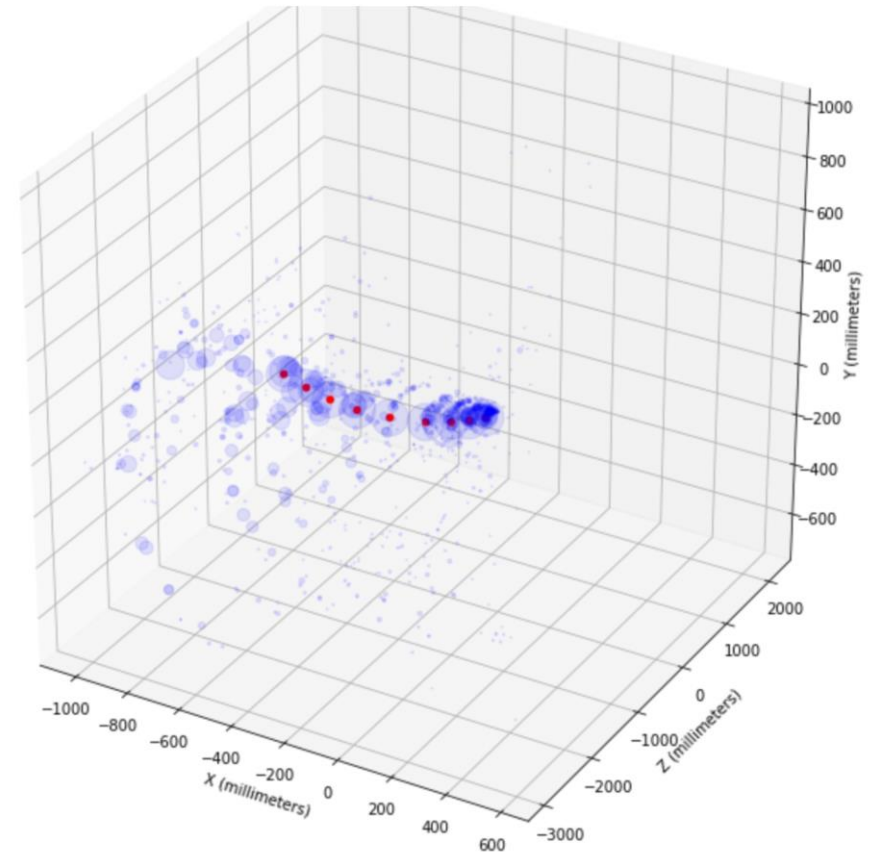
- Build tracks by maximizing the sum of probabilities

	h1	h2	h3	h4	h5
h1	-	0.8	0.2	0.9	0.4
h2	0.8	-	0.5	0.7	0.7
h3	0.2	0.5	-	0.3	0.4
h4	0.9	0.7	0.3	-	0.4
h5	0.4	0.7	0.4	0.4	-

- If threshold = 0.65

	h1	h2	h3	h4	h5
h1	-	0.8	-	0.9	-
h2	0.8	-	-	0.7	0.7
h3	-	-	-	-	-
h4	0.9	0.7	-	-	-
h5	-	0.7	-	-	-

- $p(\text{h1}, \text{h4}) = 0.9 > 0.65$



TrackML Solutions: (DL solution II)

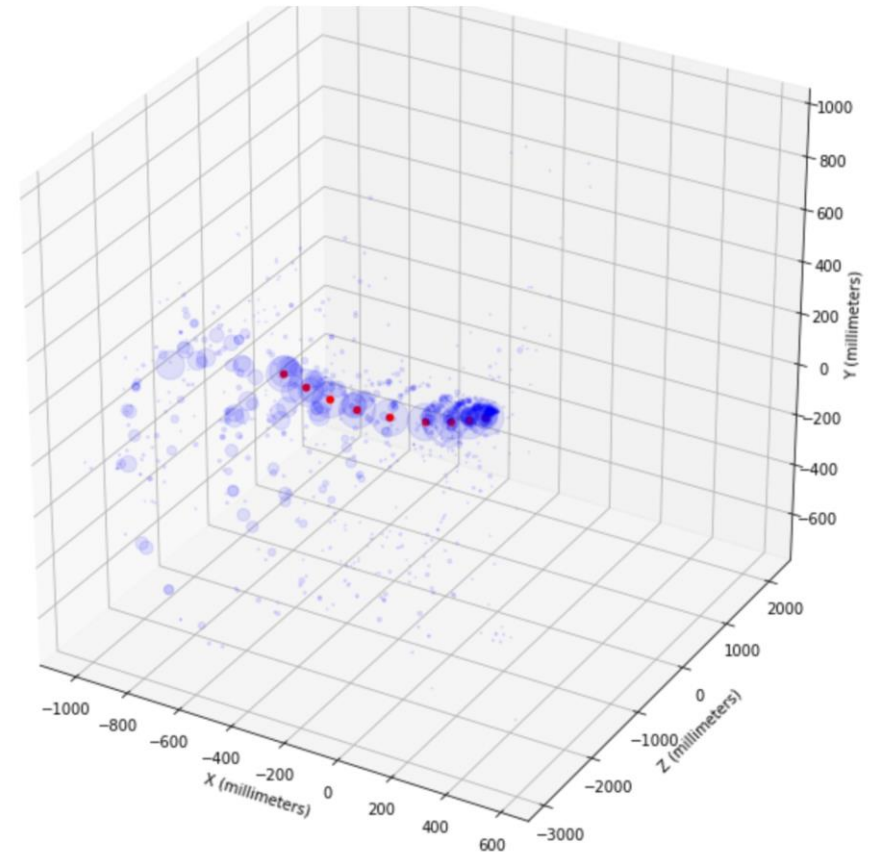
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h4	0.9	0.7	0.3	-	0.4
h5	0.4	0.7	0.4	0.4	-

- If threshold = 0.65

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h4	0.9	0.7	-	-	-
h5	-	0.7	-	-	-

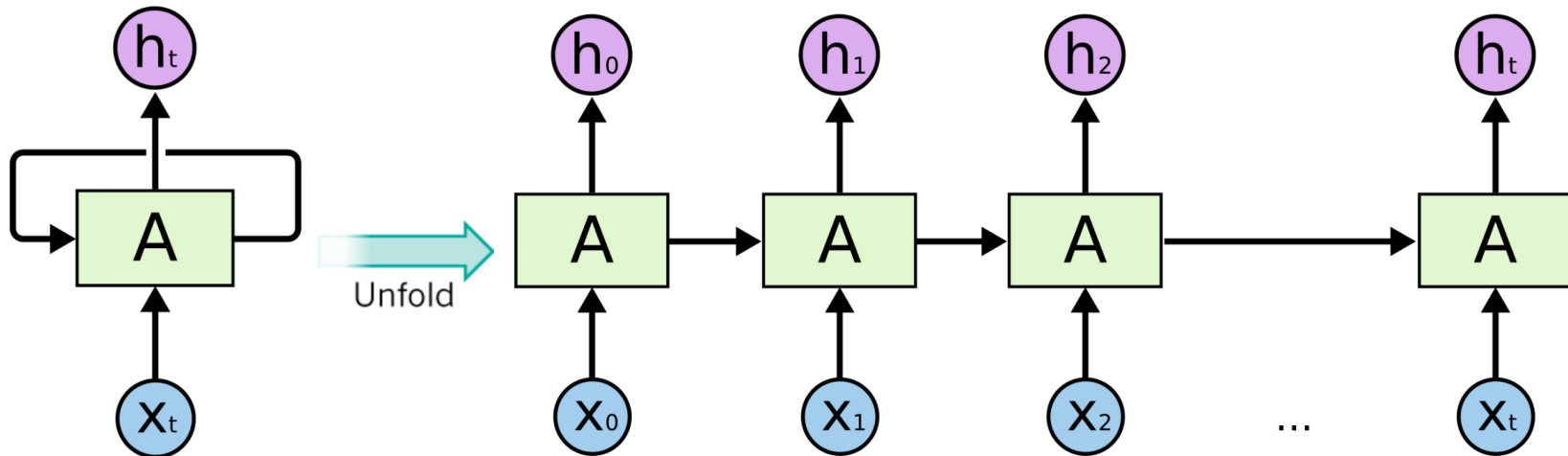
- $p(h1, h2, h4) = 1.5 > 0.65 \rightarrow h1, h2, h4$ same track



TrackML Solutions: (RNN solution I)

Recurrent Neural Networks

- The solution that ranked **12th in the challenge** is using recurrent neural networks
- **Artificial Neural Networks ANN** is also known as **feed forward network**, because each input shown to them is processed independently
- **Recurrent Neural Network RNN** processes sequences by **iterating** through the **sequence** elements and maintaining a state containing information relative to what it has seen so far

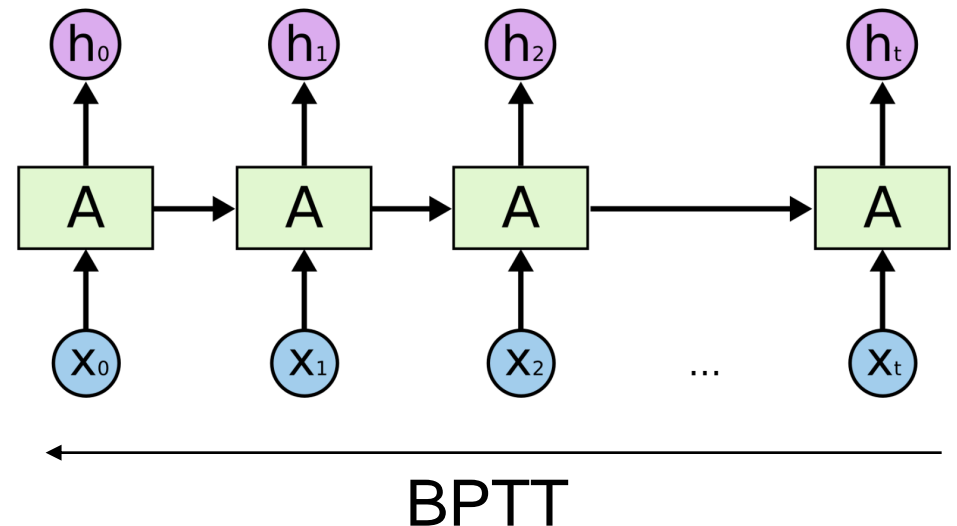
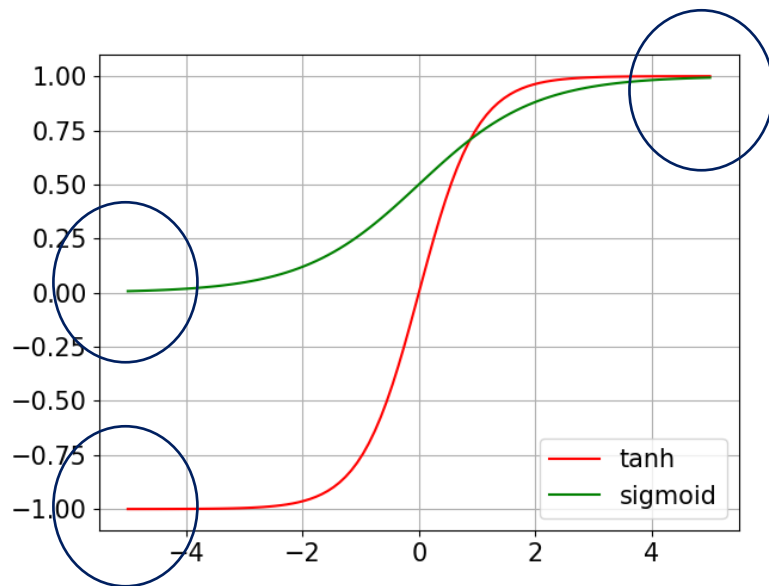


$$y_{(t)} = \phi(\mathbf{W}_x \mathbf{x}_{(t)} + \mathbf{W}_y \mathbf{y}_{(t-1)} + \mathbf{b})$$

TrackML Solutions: (RNN solution II)

Recurrent Neural Networks

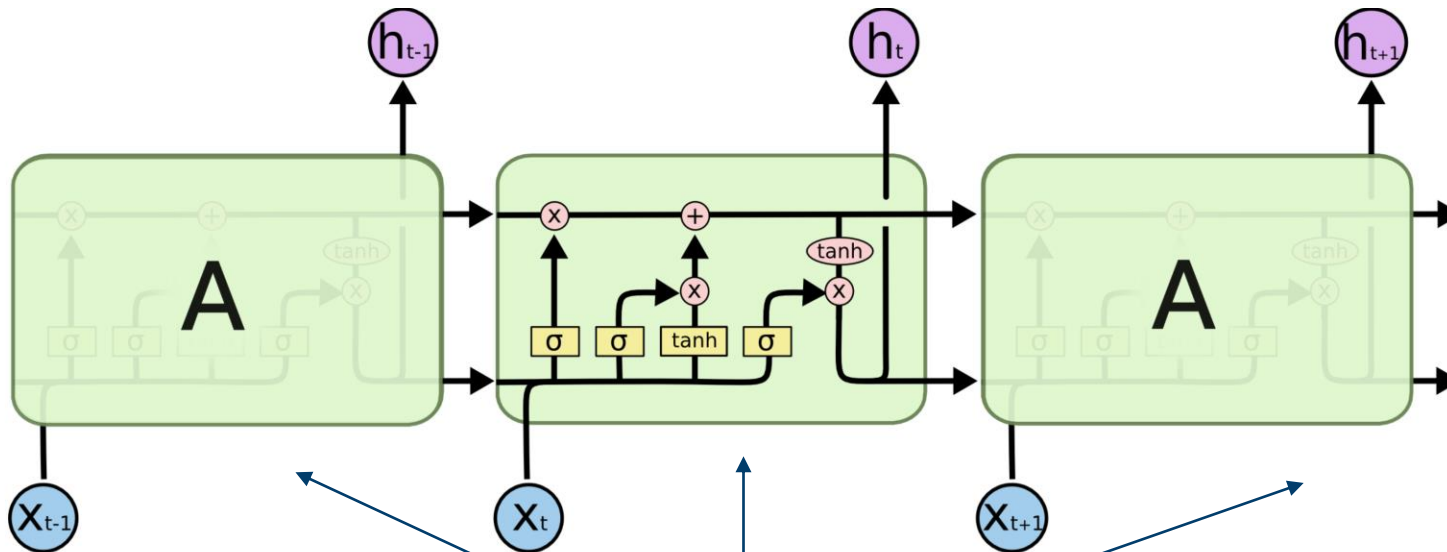
- RNNs are trained using the **backpropagation through time (BPTT)**
- Processing RNN for **long sequences** leads to **vanishing/exploding gradient** problem.
- **lower layers do not learn anything.**



TrackML Solutions: (RNN solution III)

Long Short-Term Memory LSTM

- LSTM is a variant of RNN that overcomes the vanishing/exploding gradient.
- LSTM has **memory cells** and can process very long sequences.
- **Gates** regulate the information flow

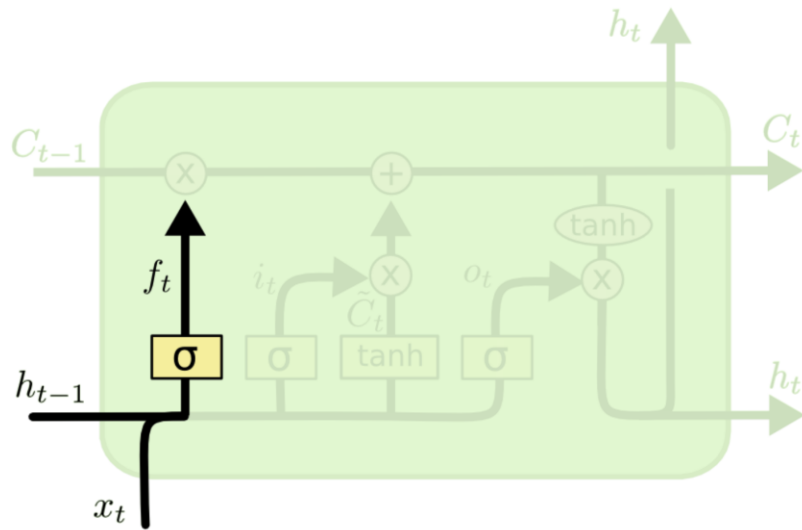


Memory cell unfolded

TrackML Solutions: (RNN solution IV)

Long Short-Term Memory LSTM

➤ Forget gate

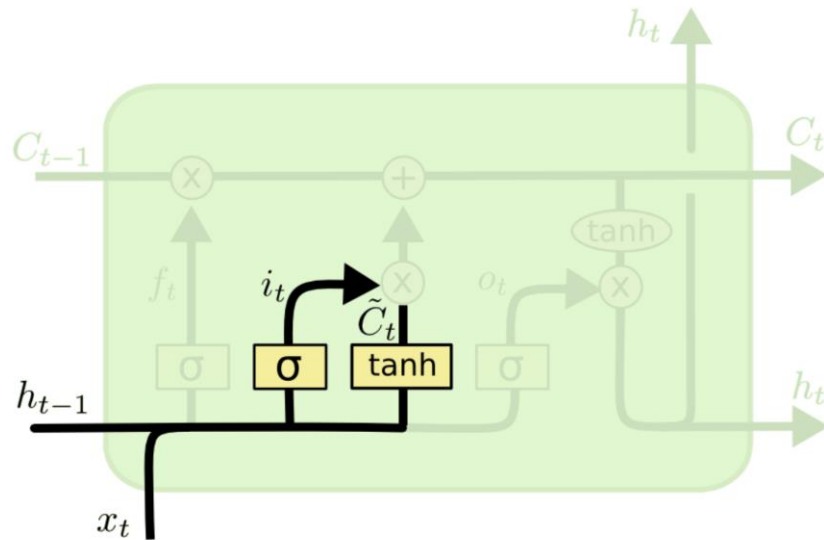


$$f_t = \sigma (W_f \cdot [h_{t-1}, x_t] + b_f)$$

TrackML Solutions: (RNN solution IV)

Long Short-Term Memory LSTM

➤ Input gate

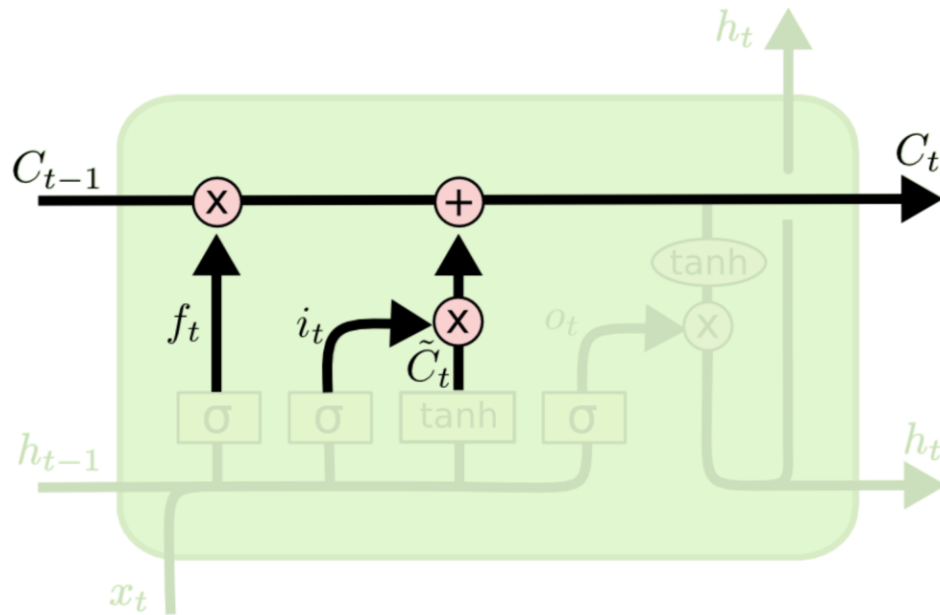


$$i_t = \sigma(W_i \cdot [h_{t-1}, x_t] + b_i)$$
$$\tilde{C}_t = \tanh(W_C \cdot [h_{t-1}, x_t] + b_C)$$

TrackML Solutions: (RNN solution IV)

Long Short-Term Memory LSTM

➤ Input gate (cell state)

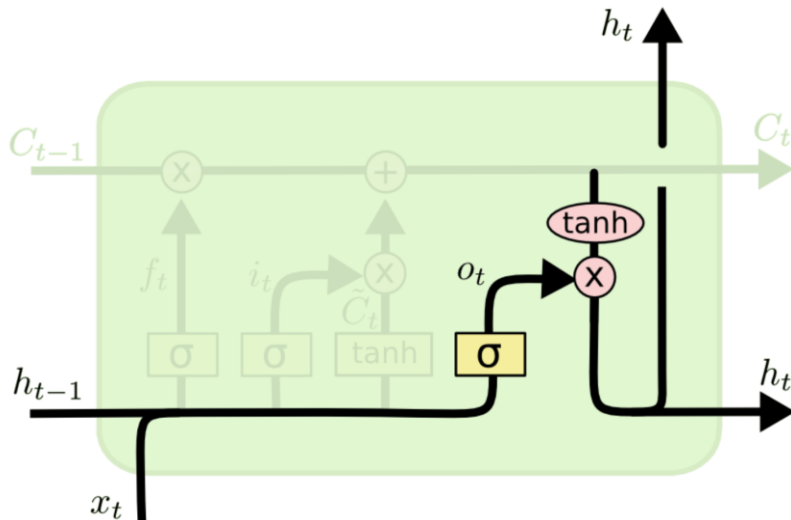


$$C_t = f_t * C_{t-1} + i_t * \tilde{C}_t$$

TrackML Solutions: (RNN solution IV)

Long Short-Term Memory LSTM

➤ Output gate



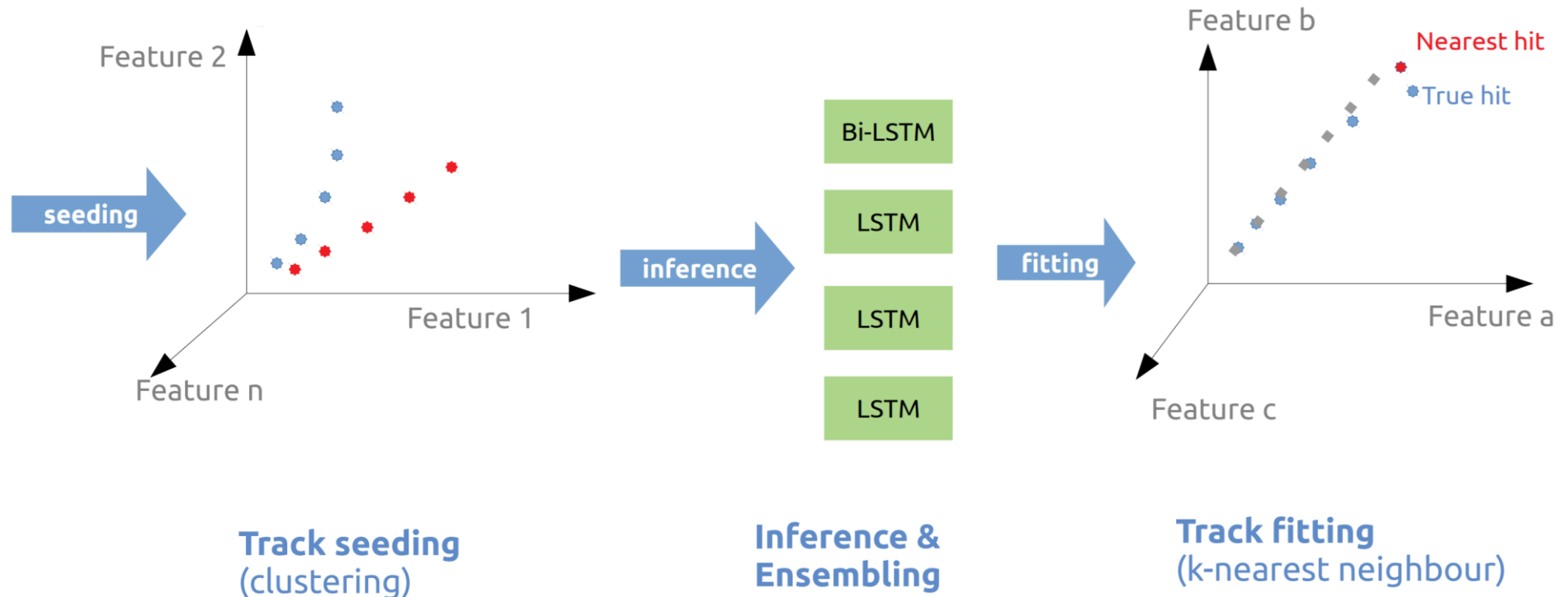
$$o_t = \sigma (W_o [h_{t-1}, x_t] + b_o)$$

$$h_t = o_t * \tanh (C_t)$$

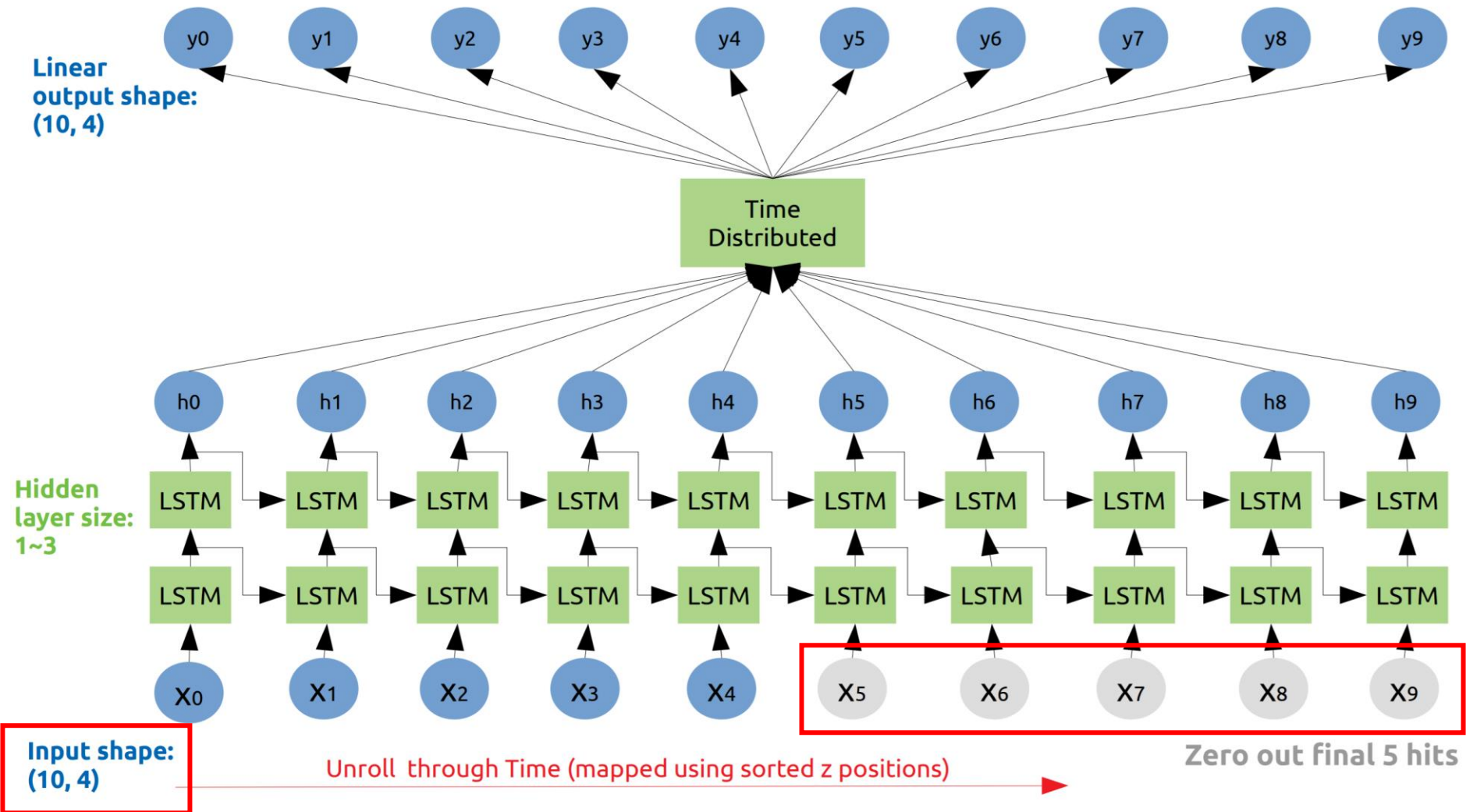
TrackML Solutions: (RNN solution V)

➤ Solution Pipeline:

1. Seed finding (DBSCAN)
2. LSTM for Track Following
3. k-D tree search for hit association



TrackML Solutions: (RNN solution VI)



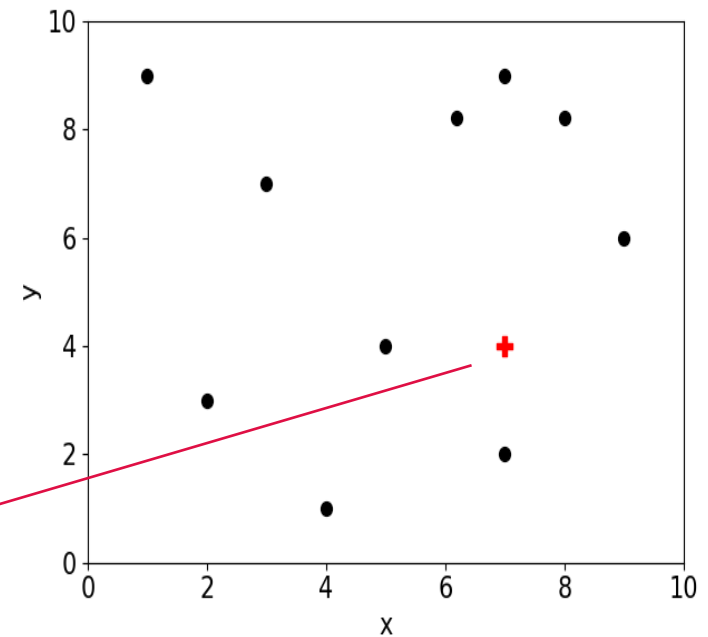
TrackML Solutions: (RNN solution VI)

- Multiple architectures LSTMs are trained
- Ensembled with averaging to provide the final prediction
- Build a binary tree to search for the **nearest neighbor (4-D Tree)**

A simple k-D Tree

(1,9), (2,3), (4,1), (3,7), (5,4), (6,8), (7,2), (8,8), (7,9), (9,6)

Query point

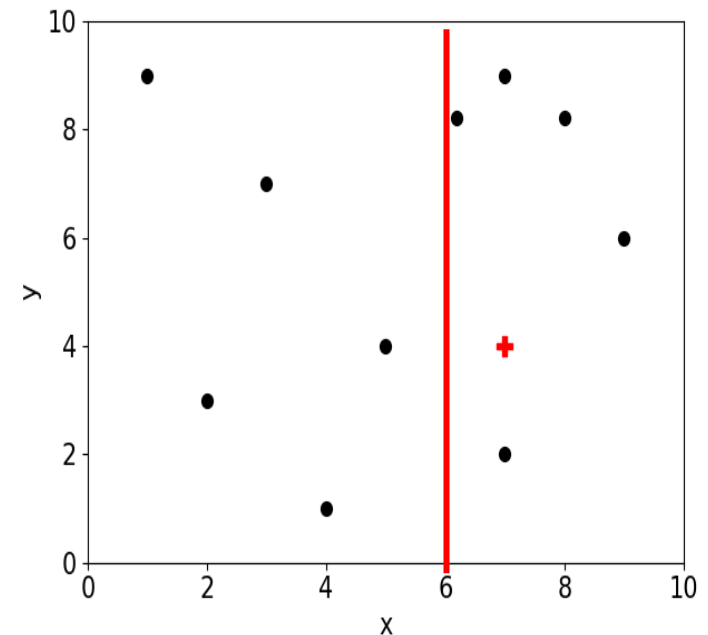
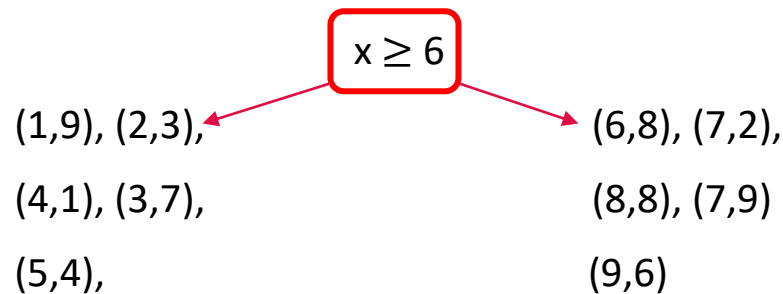


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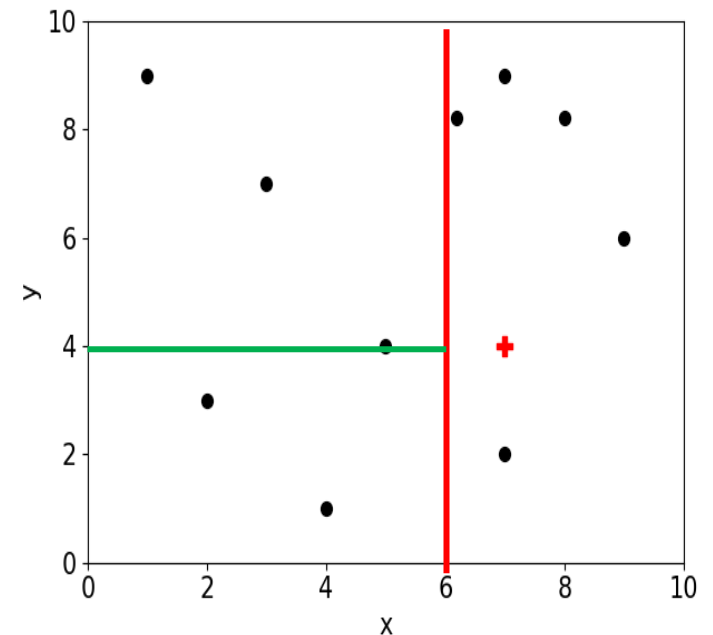
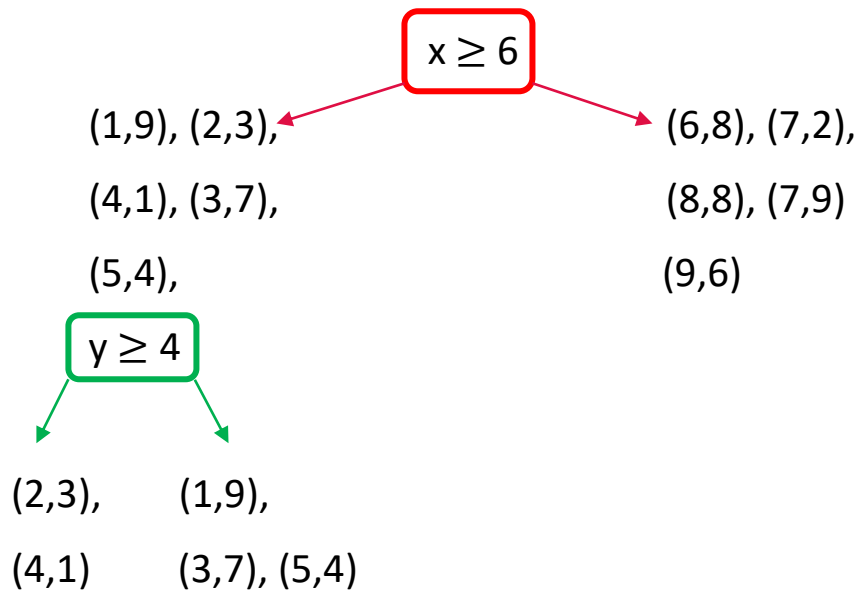


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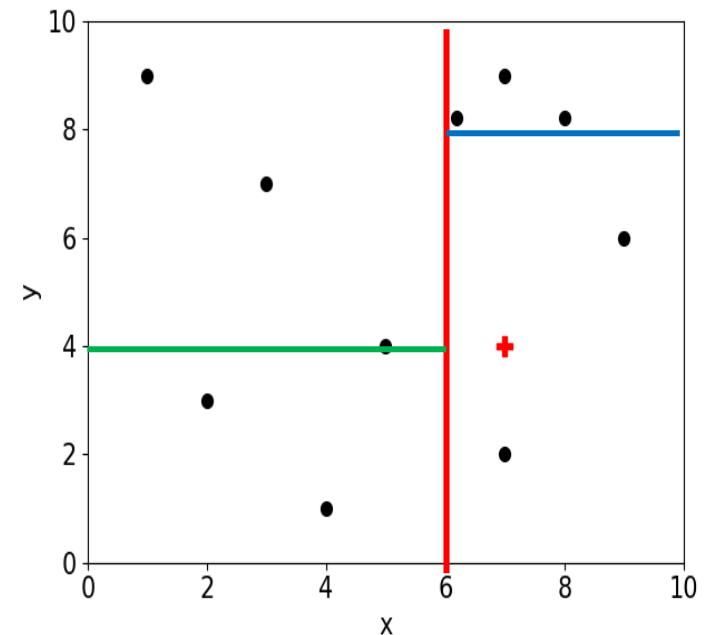
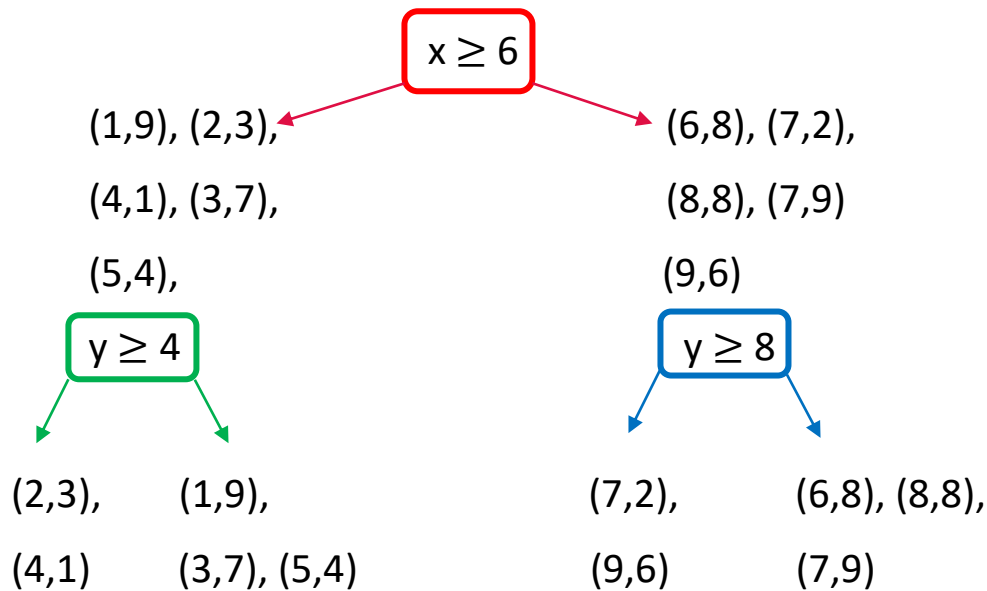


TrackML Solutions: (RNN solution VI)

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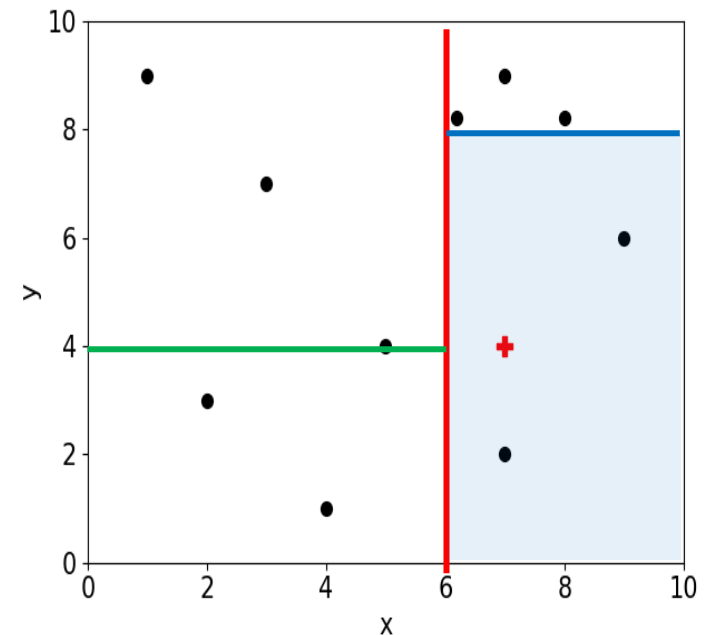
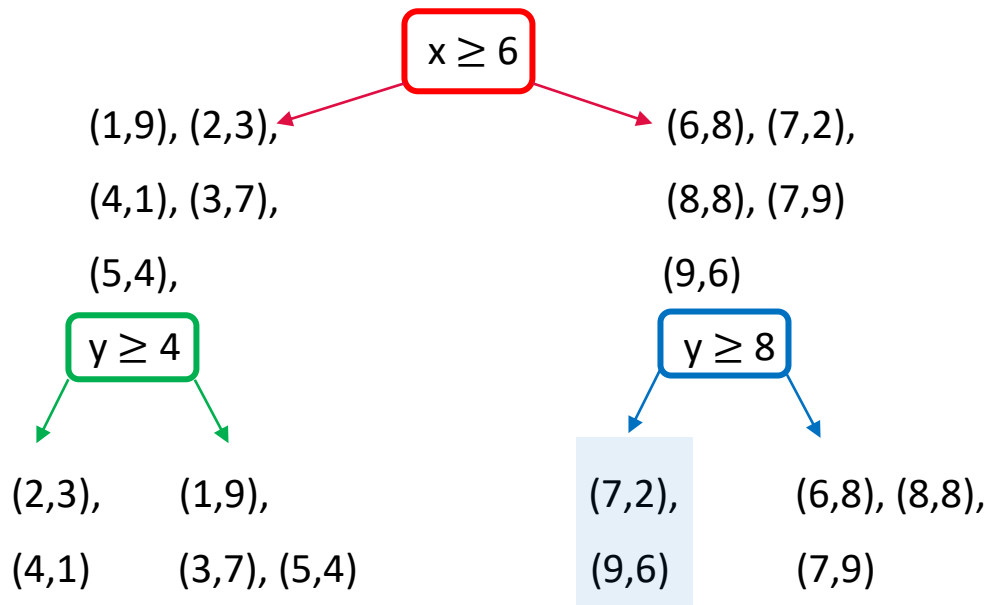


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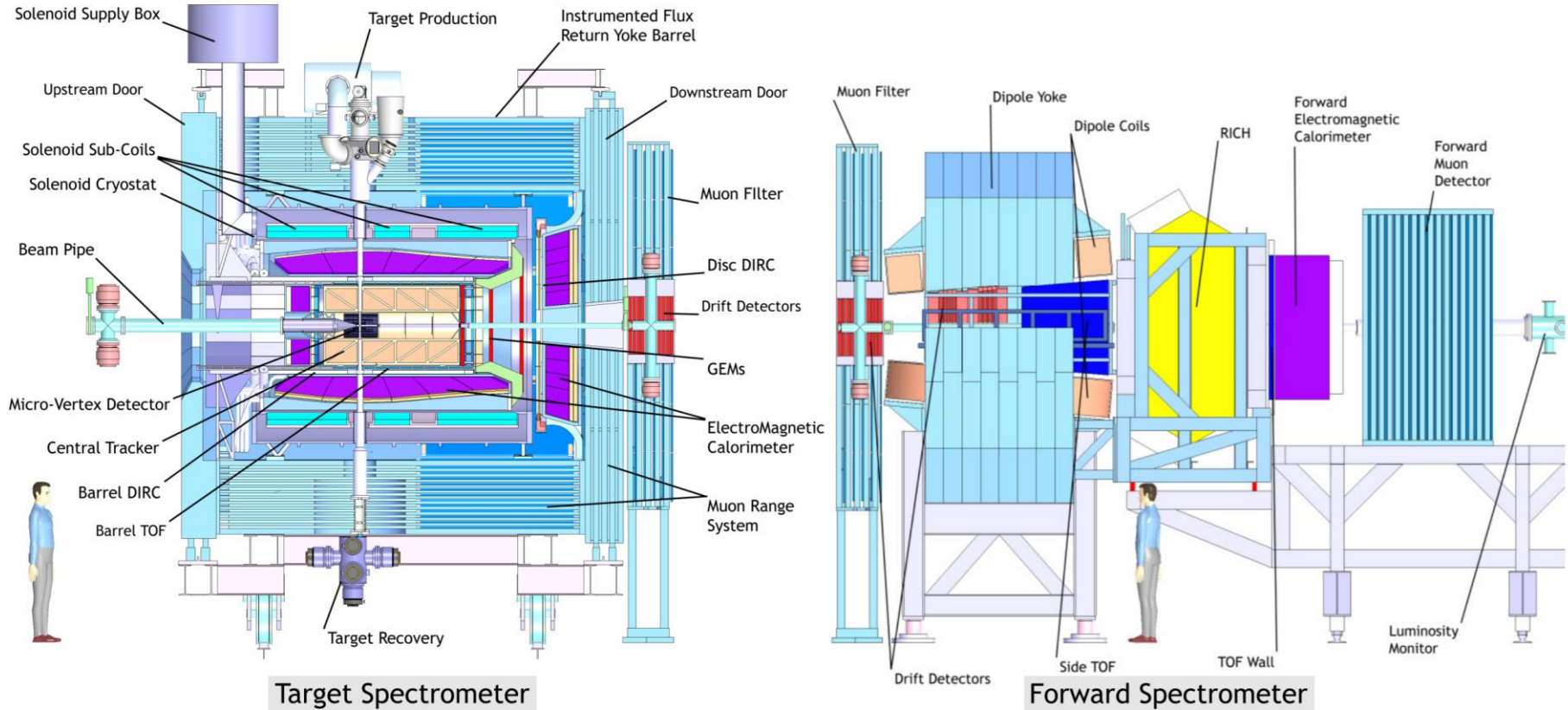
(1,9), (2,3), (4,1), (3,7), (5,4), (6,8), (7,2), (8,8), (7,9), (9,6)



ML Based Track Finding at PANDA FTS

PANDA Detector:

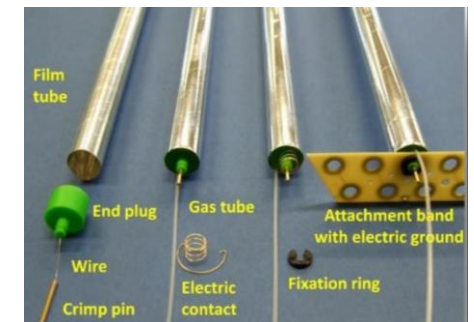
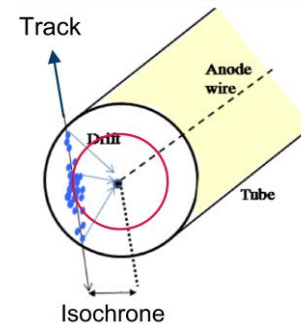
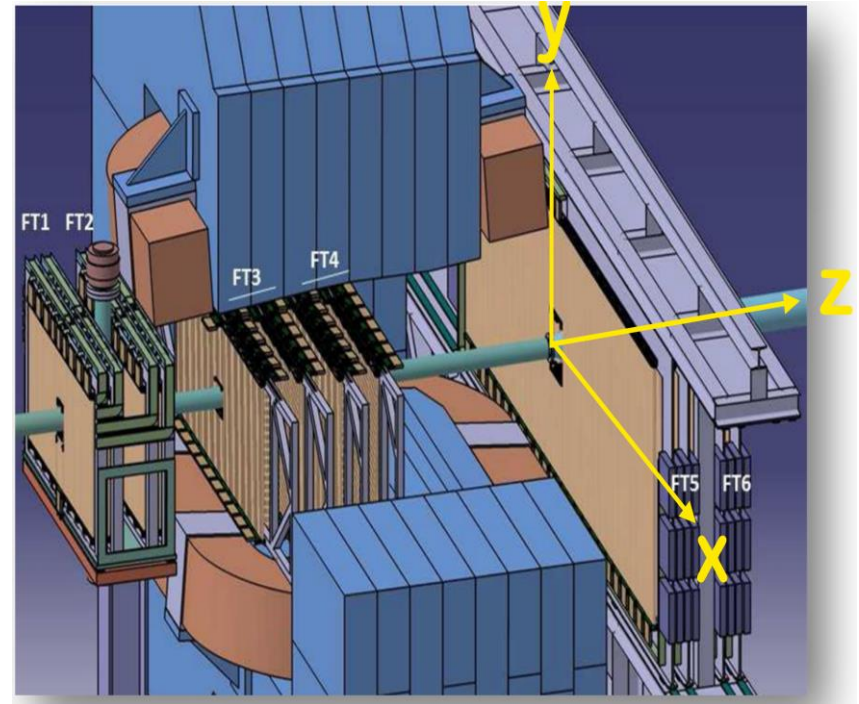
antiProton ANnihillation at DArmstadt



PANDA FTS:

Forward Tracking Stations:

- Straw tubes, same as in the barrel, vertically arranged in double layers
- 3 stations with 2 chambers each
 - FTS1&2 : No magnetic field
 - FTS3&4 : Inside the field (2Tm)
 - FTS5&6 : No magnetic field
- 8 double layers per chamber.
- Orientations $0^\circ/+5^\circ/-5^\circ/0^\circ$ per chamber
- Tracks are defined by distance of closest approach to the wire (Isochrones)
- **Inputs: Wire position (hits), Isochrones,**
...



PANDA FTS Algorithm I:

Local approach:

I. Create track segments (tracklets) using Artificial Neural Network

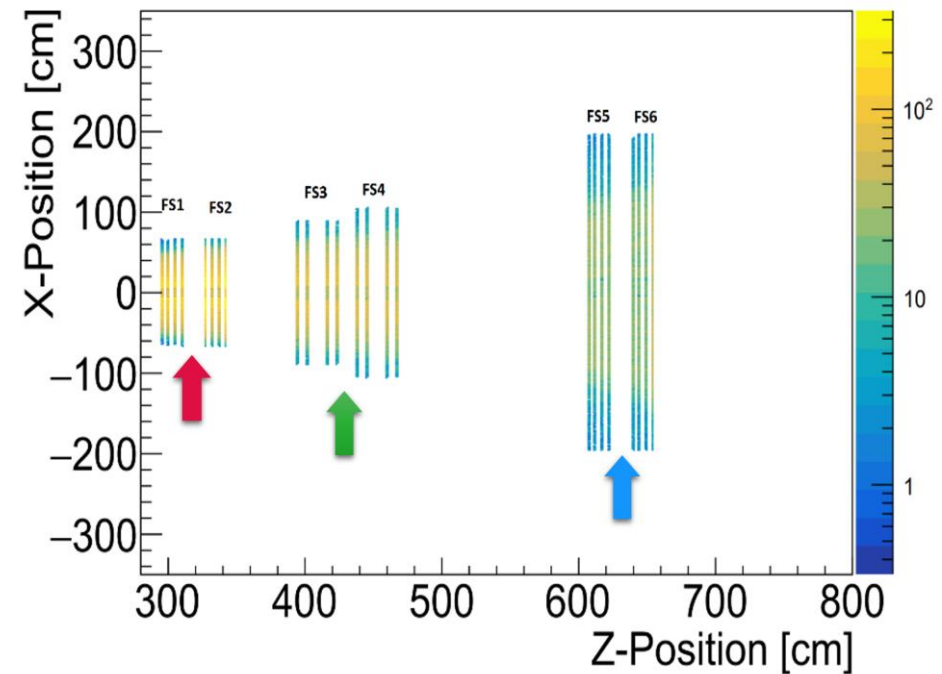
FTS1 & FTS2

FTS3 & FTS4

FTS5 & FTS6

II. Connect the segments using LSTM

Make all possible combinations of tracklets

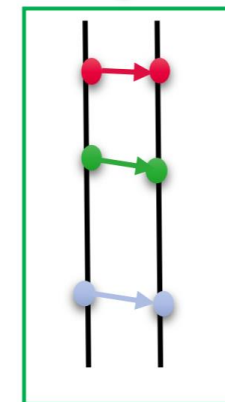
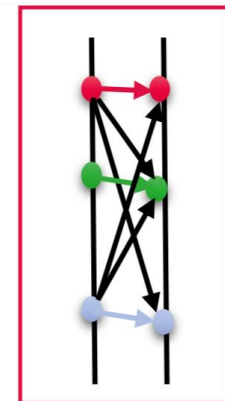


PANDA FTS Algorithm I: Step I

- All possible combinations of hit pairs **ONLY adjacent layers**
- **ONLY vertical layers**
- Network predict the quality of the pair
- **Input Hit Pair**(x, z, r) -> **DNN** -> **Probability**
- Training data -> 5 tracks/event (particle gun)

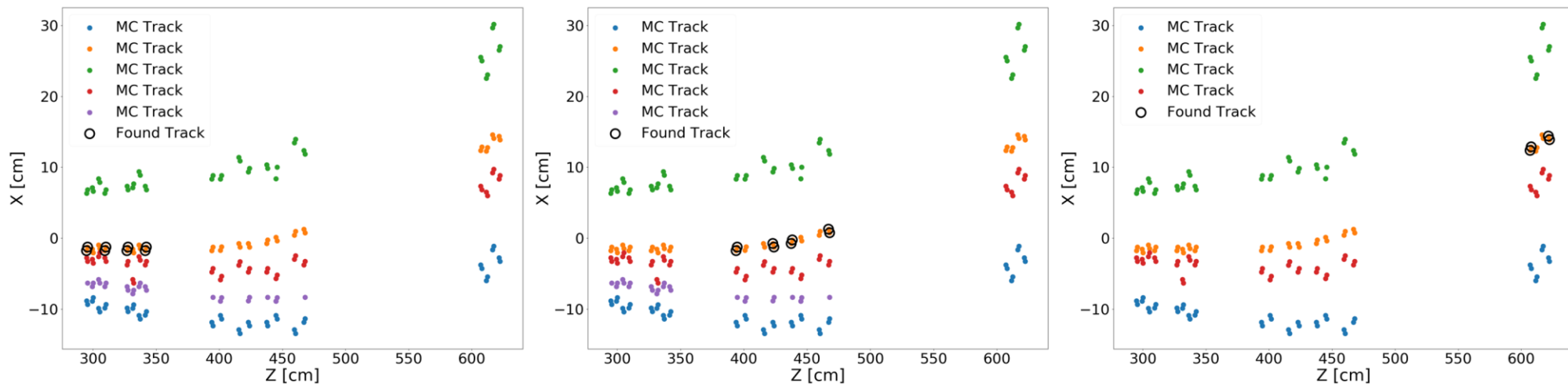
- Clustering using the probability output (**threshold**)
 1. if $\text{Prob}(h_1, h_2) > \text{threshold}$ and,
 2. $\text{Prob}(h_2, h_3) > \text{threshold}$

(h_1, h_2, h_3) on the **same track**



Accuracy ~ 96%

PANDA FTS Algorithm I: Step I



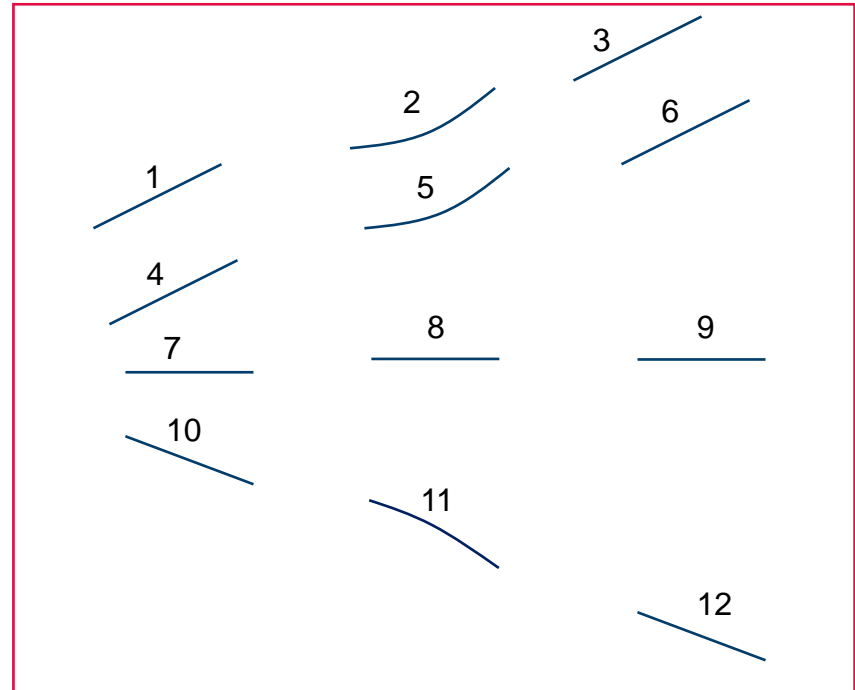
PANDA FTS Algorithm I: Step II

➤ All possible combinations of tracklets

[1,2,3], [1,2,6], [1,5,3], ...

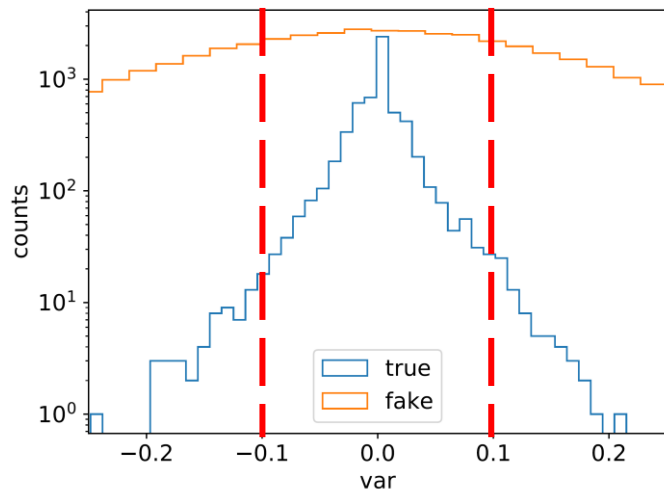
➤ LSTM is used as a classification model ~ 98%

➤ Network predict the quality of the track candidate



PANDA FTS Algorithm I: Step II

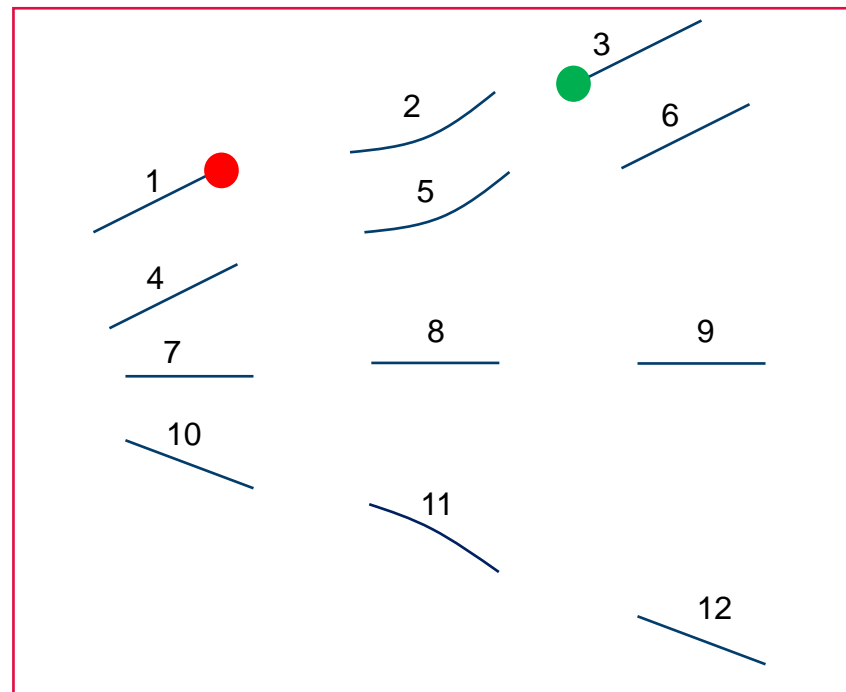
- All possible combinations of tracklets
[1,2,3], [1,2,6], [1,5,3], ...
- Network predict the quality of the track candidate
- **Preprocess -> Balance the training set**



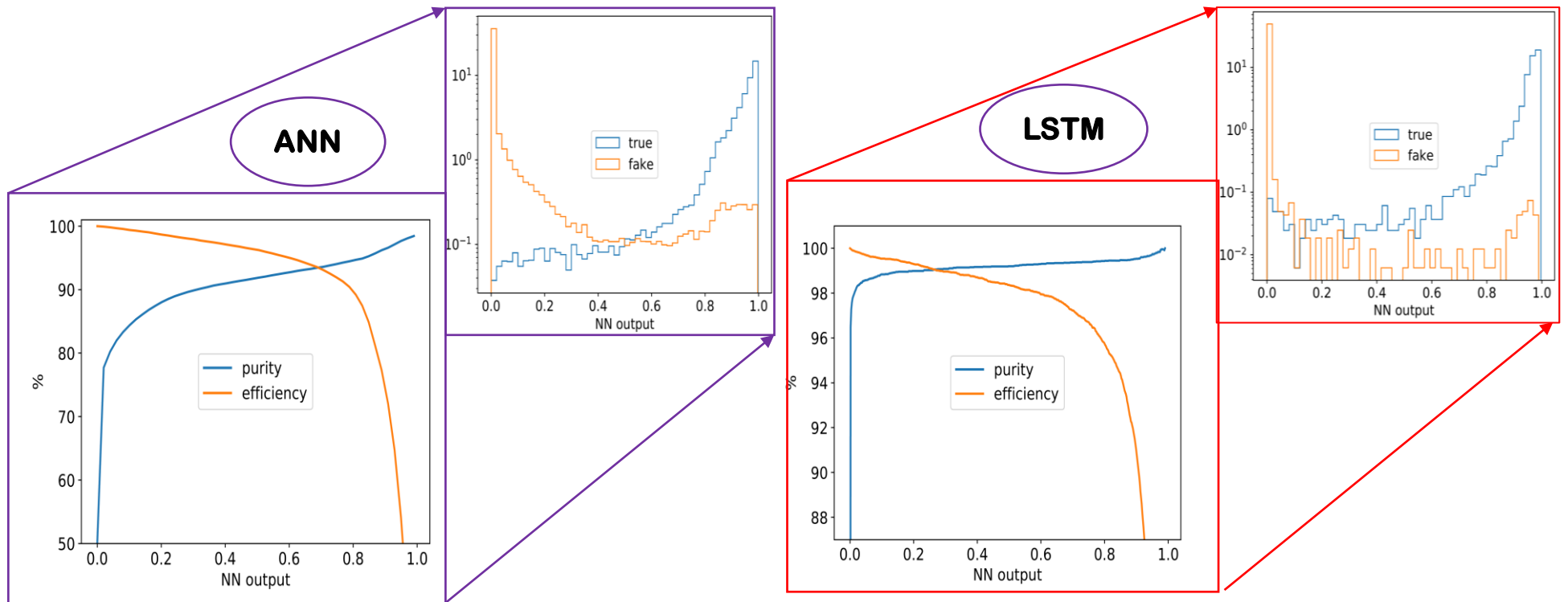
➤ $a = z/x$

➤ $\text{var} = a_2 - a_1$

➤ $R(\text{fake}/\text{true}) \sim 8 \rightarrow 4$



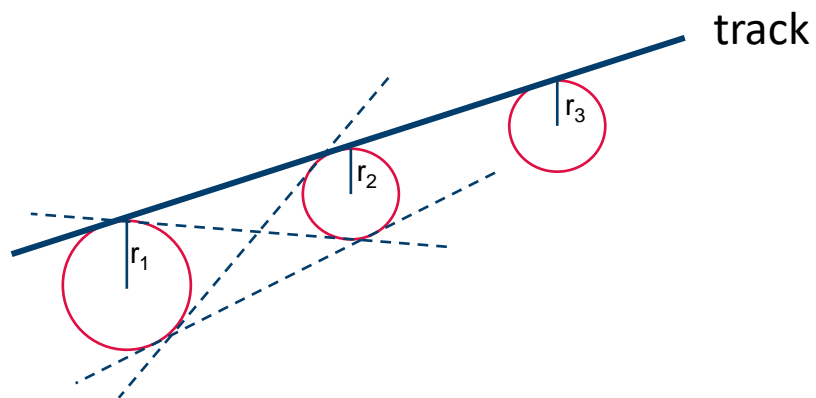
PANDA FTS Algorithm I: Optimizing Probability Cuts:



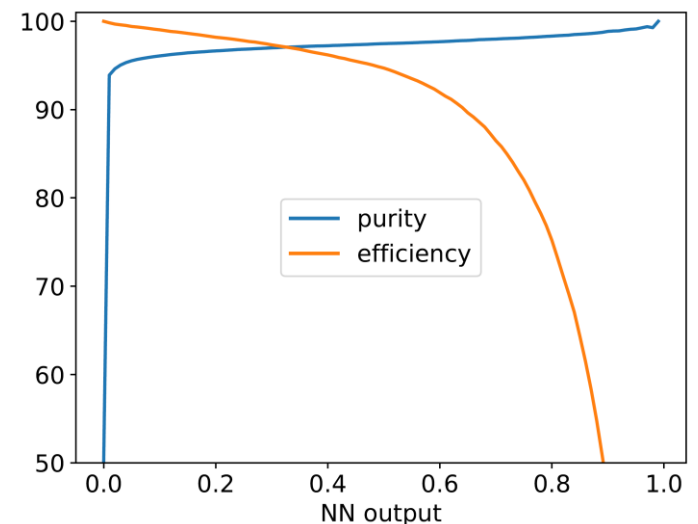
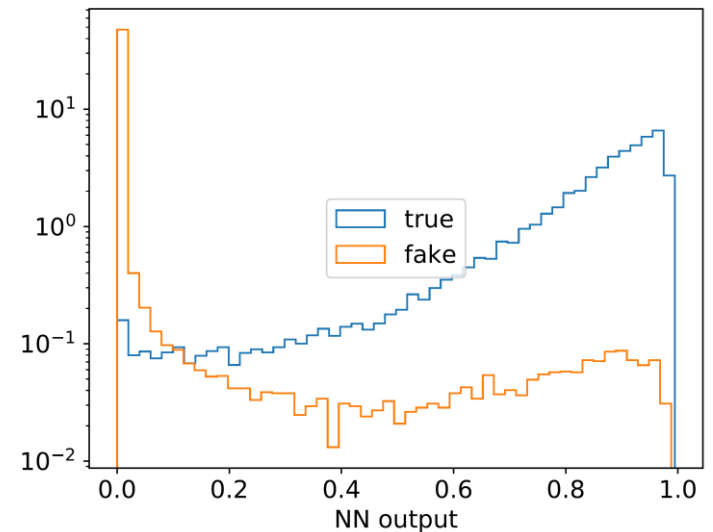
- Purity = true that pass the cut / all that pass the cut
- efficiency = true that pass the cut / all true
- Overall **tracking efficiency** ranging from ~ 80 ~ 100 %

PANDA FTS Algorithm I: Resolving Ambiguity:

- All possible combinations of triplets **ONLY adjacent layers**
- **ONLY vertical layers**
- Network predict the quality of the triplet
- Input Hit Triplet(x, z, r) -> DNN -> Probability



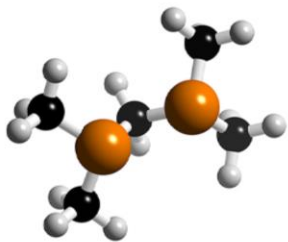
- Overall **tracking efficiency** comparable to hit-pairs.



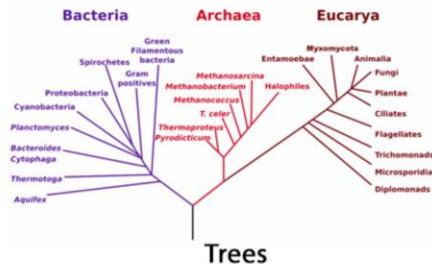
Tracking Using Graph Neural Networks

Geometric Deep Learning GDL

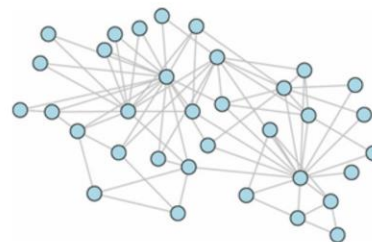
- Images, text, audio, and many others are called **euclidean data**
- **Non-euclidean data** can represent more complex items and concepts with more accuracy than 1D or 2D representation
- **GDL** is about building neural networks that can learn from **non-euclidean data**
- **Non-euclidean data** can be resented as a **Graph**



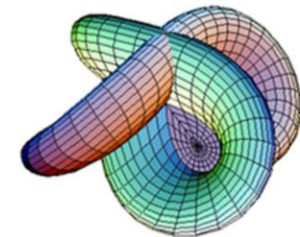
Molecules



Trees



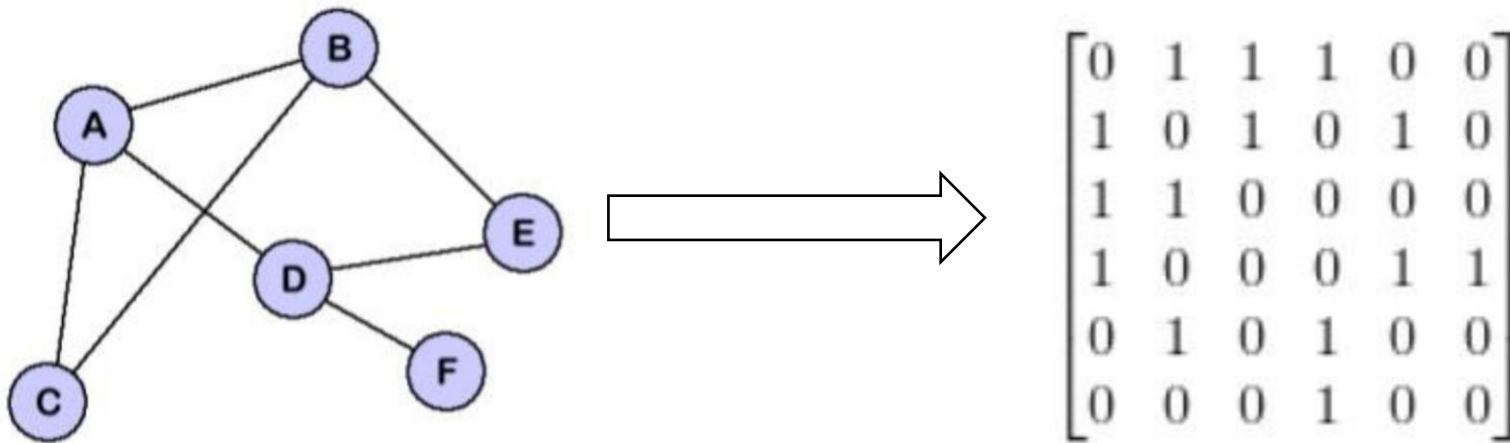
Networks



Manifolds

Graph Concept

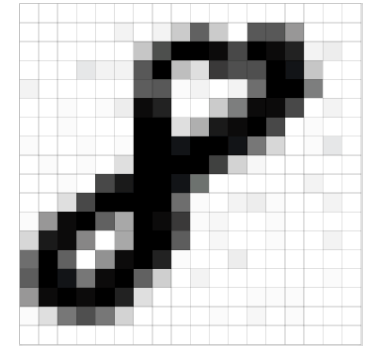
- A **graph** is a data structure comprising of **nodes (vertices)** and **edges** connecting nodes
- **Graph = $G(X,E)$** can be resented by a matrix (e.g. **Adjacency Matrix**)



- Graph can be directed or undirected
- The neural network itself can be viewed as a graph, where **nodes are neurons** and **edges are weights**

Convolution Operation

- An image can be represented as a matrix of pixel values
- The purpose of Convolution is to **extract features** from the input image



1	1	1	0	0
0	1	1	1	0
0	0	1	1	1
0	0	1	1	0
0	1	1	0	0

*

1	0	1
0	1	0
1	0	1

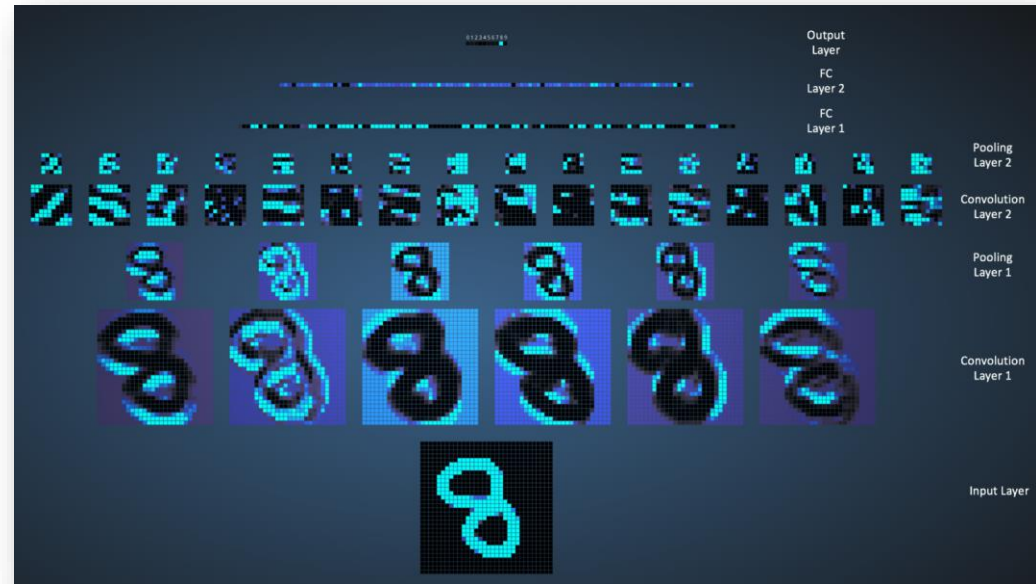
Matrix Multiplication

1 _{x1}	1 _{x0}	1 _{x1}	0	0
0 _{x0}	1 _{x1}	1 _{x0}	1	0
0 _{x1}	0 _{x0}	1 _{x1}	1	1
0	0	1	1	0
0	1	1	0	0

4		

Image

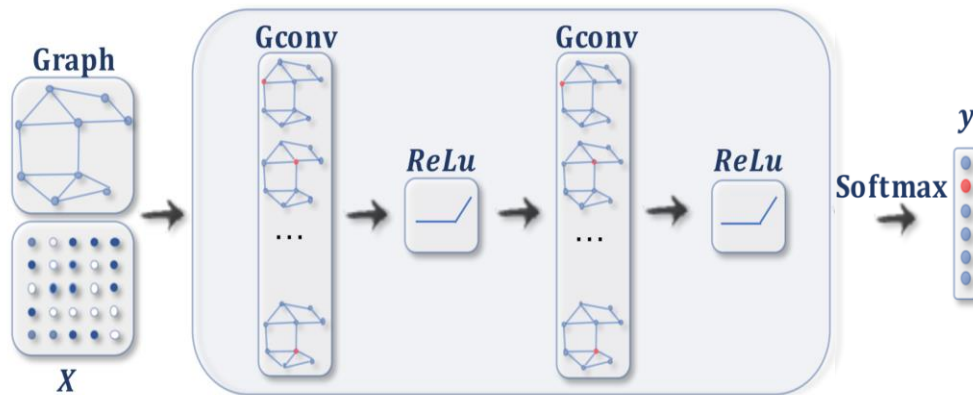
Convolved
Feature



1. [An Intuitive Explanation of Convolutional Neural Networks 2016](#)

Graph Neural Networks GNN

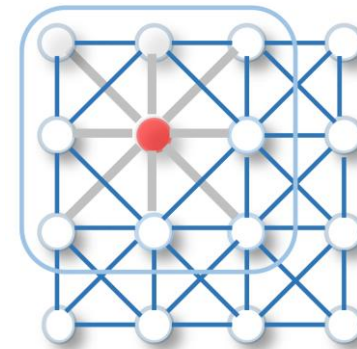
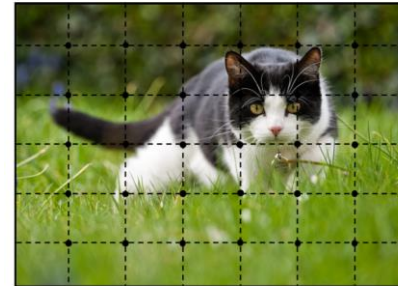
- Motivated by CNN and graph embeddings
- **RecGNNs, ConvGNNs, GAEs, ...**



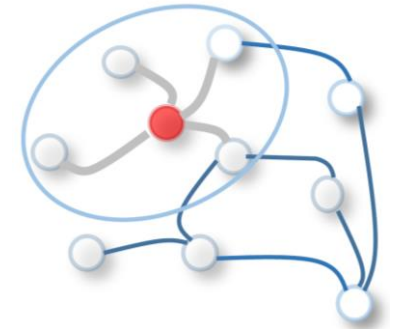
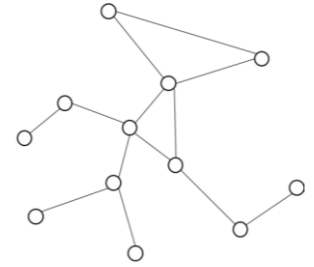
The target of GNN is to learn a state embedding (neighborhood relations)

$$H^{t+1} = F(X, H^t)$$

Euclidean



Non-Euclidean



Tasks: Node-level, **Edge-level**, Graph-level.

1. Graph Neural Networks: A Review of Methods and Applications Jie Zhou 2019
2. A Comprehensive Survey on Graph Neural Networks Zonghan Wu 2019

GNN applied to FTS:

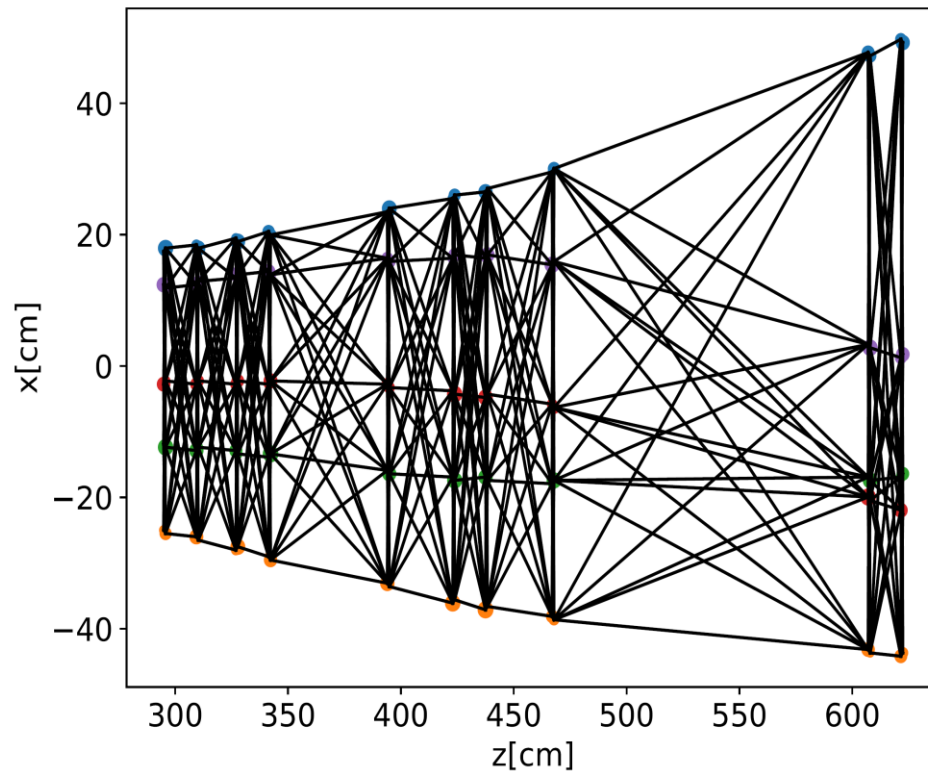
- Global approach
- GNN is used as a binary classifier (**hit-pairs classification or edge classification**)
- Input is a graph (FTS hits of one event).
- Two main components: **edge network** and **node network**
- **Edge network** uses the node features to compute **edge weights**
- **Node network** aggregates node features with the edge weights and **updates node features**
- With each graph iteration, the model propagates information through the graph, strengthens important connections, and weakens useless ones.

node features = [x, z, isochrone]

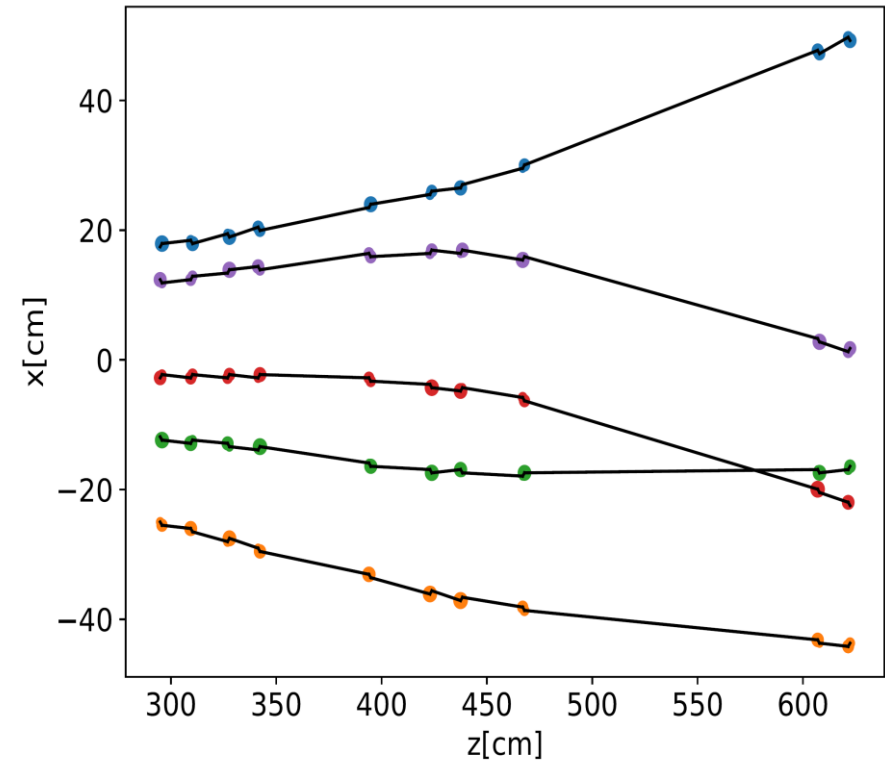
graph iterations = 3

GNN applied to FTS:

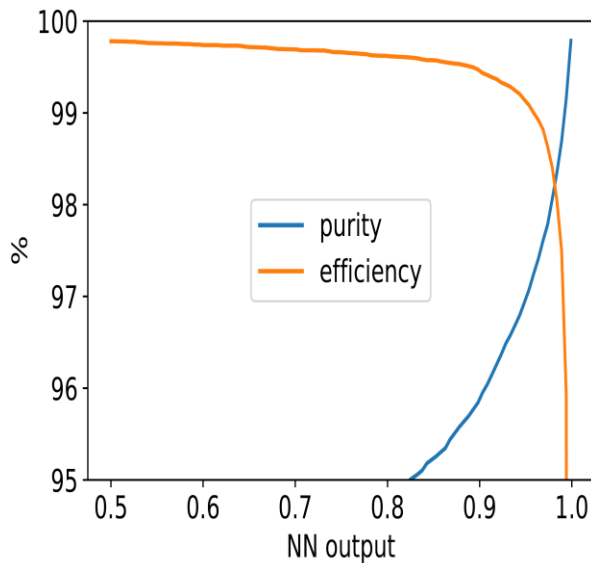
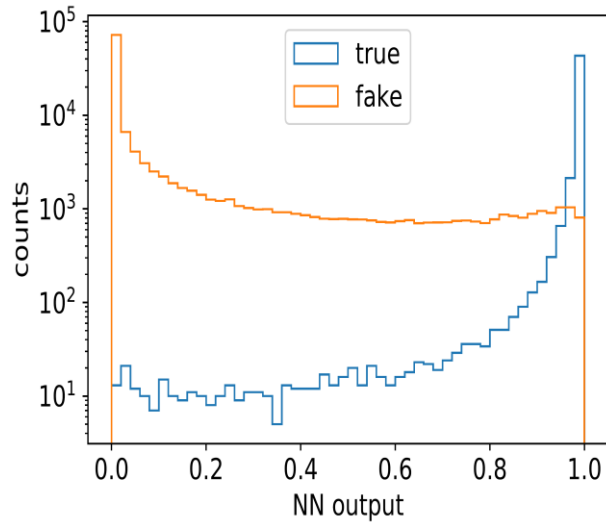
Input Graph



Ideal Graph

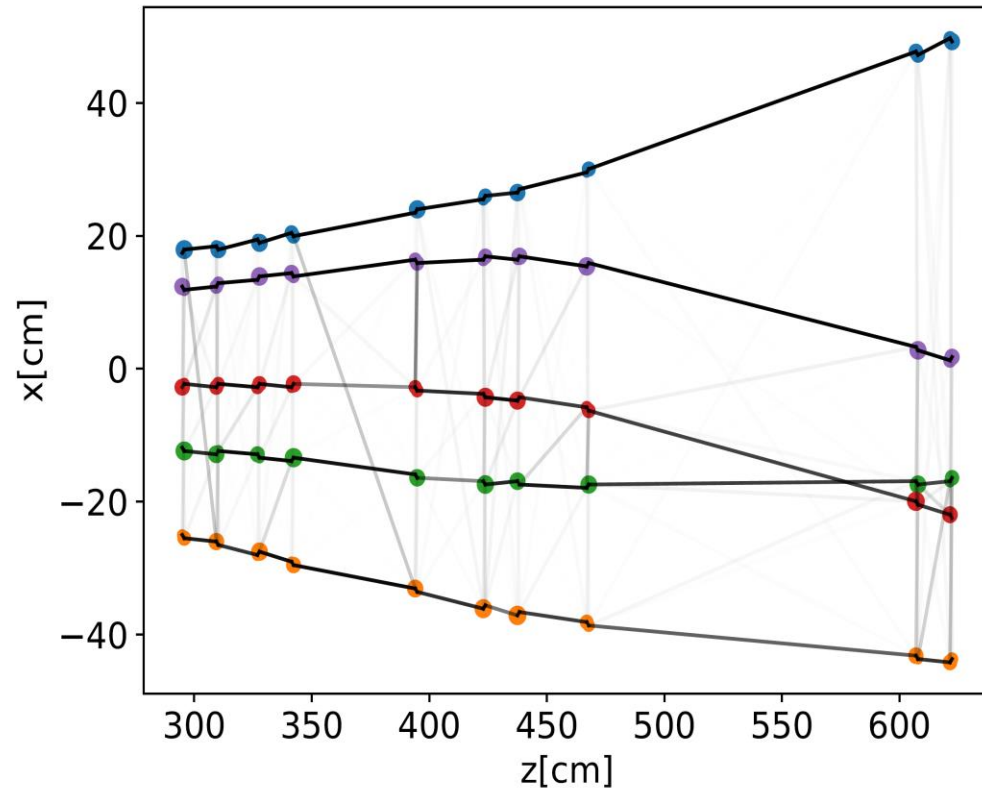


GNN applied to FTS:

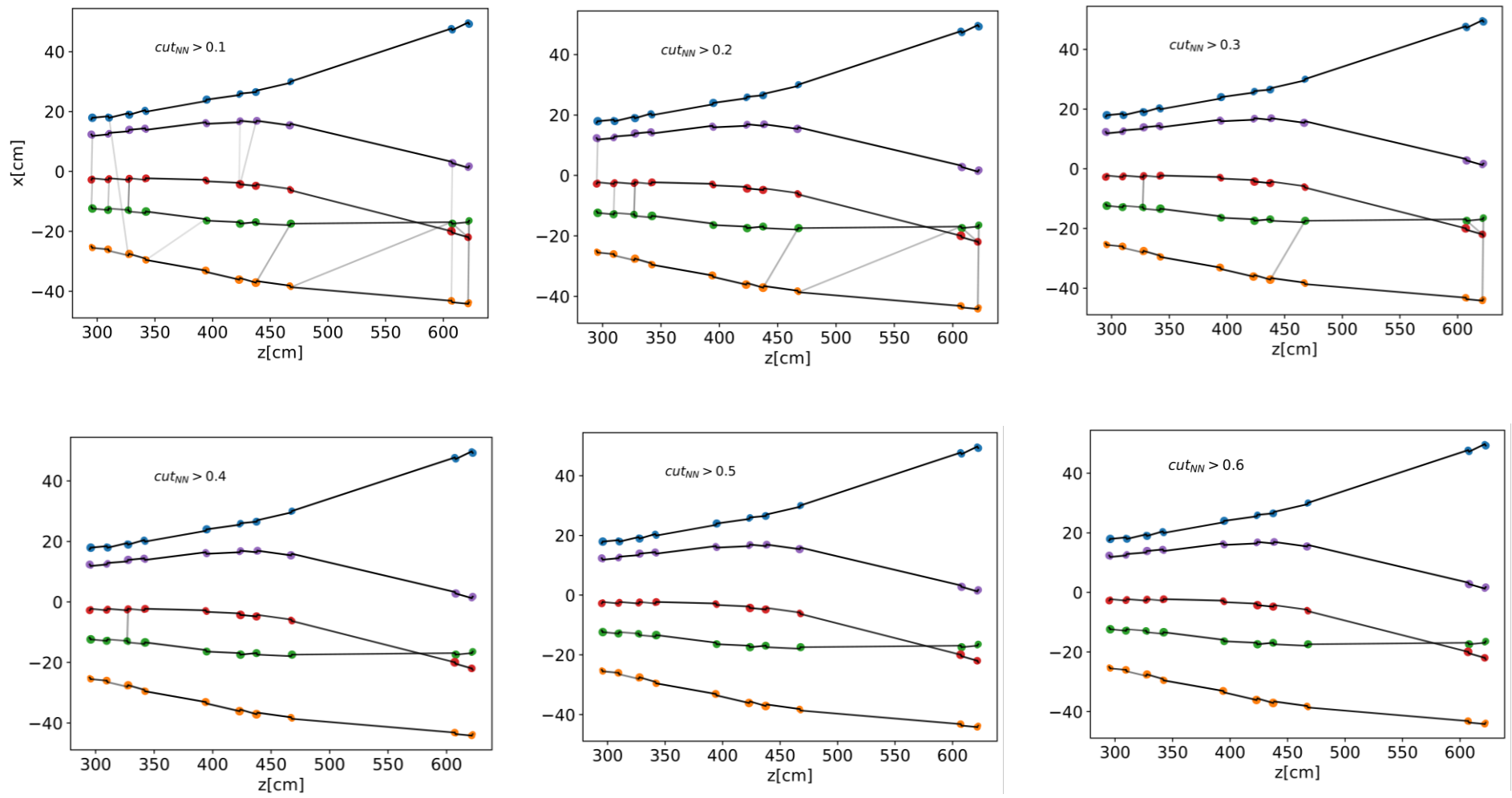


➤ Accuracy ~ 99%

An output graph



GNN applied to FTS:



GNN applied to FTS:

- Finding tracks is finding graph connected components (subgraphs)
- A traversal algorithm, starting at vertex v_i then visit all vertices.

```
Mark all vertices as not visited
```

```
For every vertex v:
```

```
    if v is not visited call DFS()
```

```
DFS()
```

```
    Mark v as visited
```

```
    store v in a list
```

```
        For every edge (adjacent vertices v and u):
```

```
            if u is not visited, then recursively call DFS()
```

Tracking QA:

1. Track efficiency

- How many MC tracks have been found by track finderfinder

2. Purity

- Belong all hits of one found track belong to one MC track.

3. Ghosts

- How many hits not belonging to an MC track have been found

4. Partially found

- Not all hits belonging to one track have been found but all hits belong to one MC track

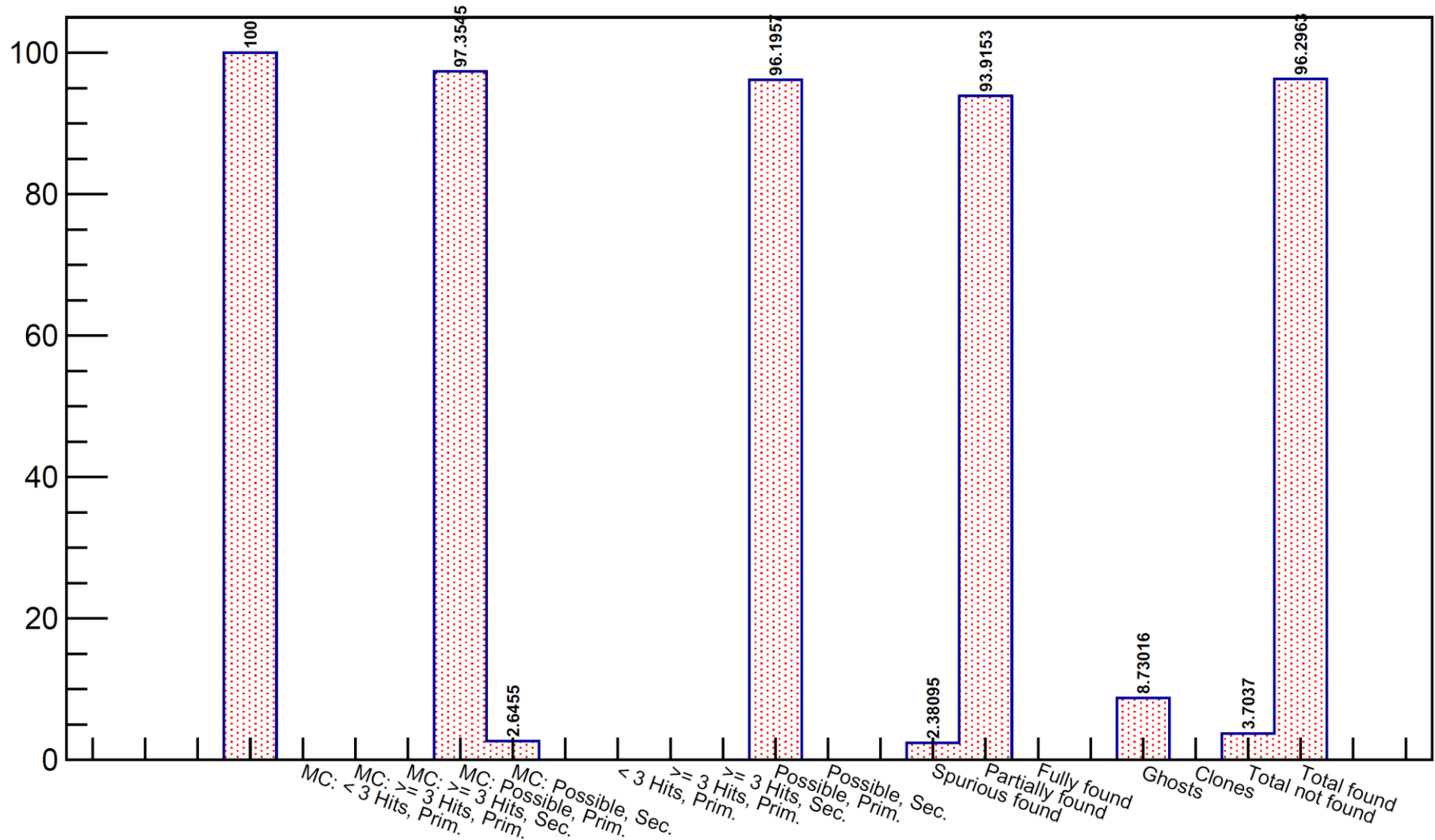
5. Spurious found

- > 70% found hits belong to one MC track

6. Fully found

- 100 % of MC hits have been found and no other hits are part of the track

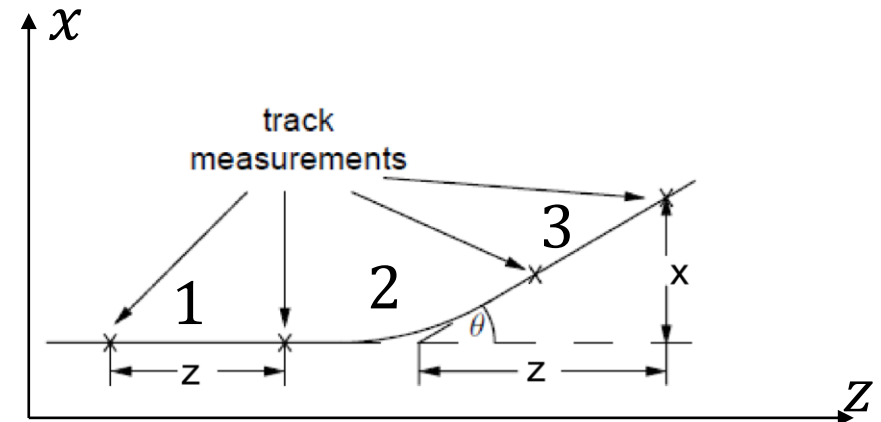
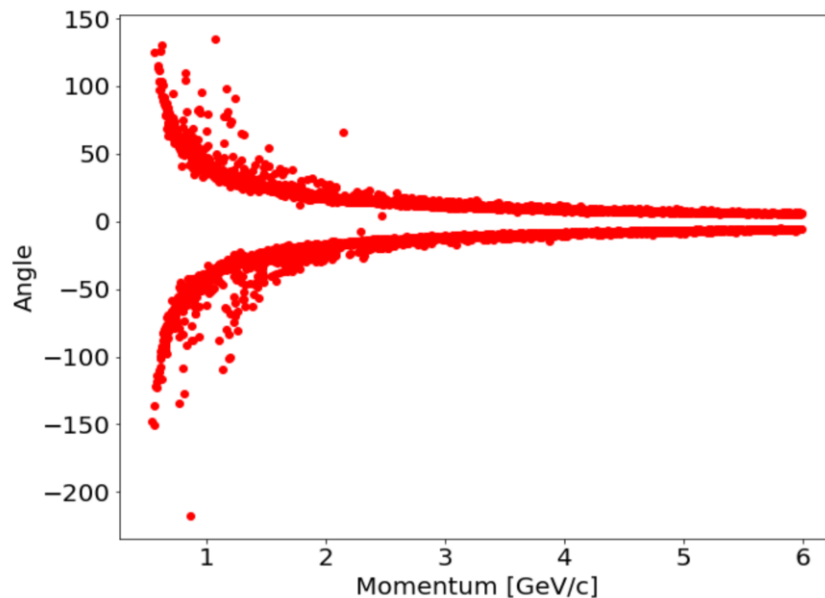
Tracking QA:



Track Fitting:

- Track Reconstruction = Track Finding + **Track Fitting**
- Standard approach in many experiments is the **Kalman Filter**
- Kalman Filter needs starting values (seed)
- Track Fitting delivers parameters needed for physics analysis (e.g. Momentum)

- Momentum is estimated from track curvature



$$\rho \text{ [m]} = \frac{p \text{ [GeV/c]}}{0.3 B \text{ [T]}}$$

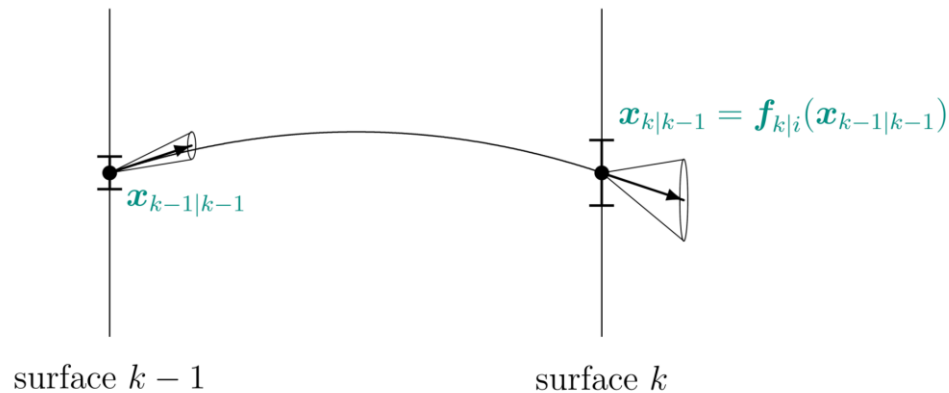
$$\theta = \frac{L}{\rho} = \frac{L}{p} eB$$

Track Fitting:

- **Track Model** is a parametrization of the track (**state vector**)

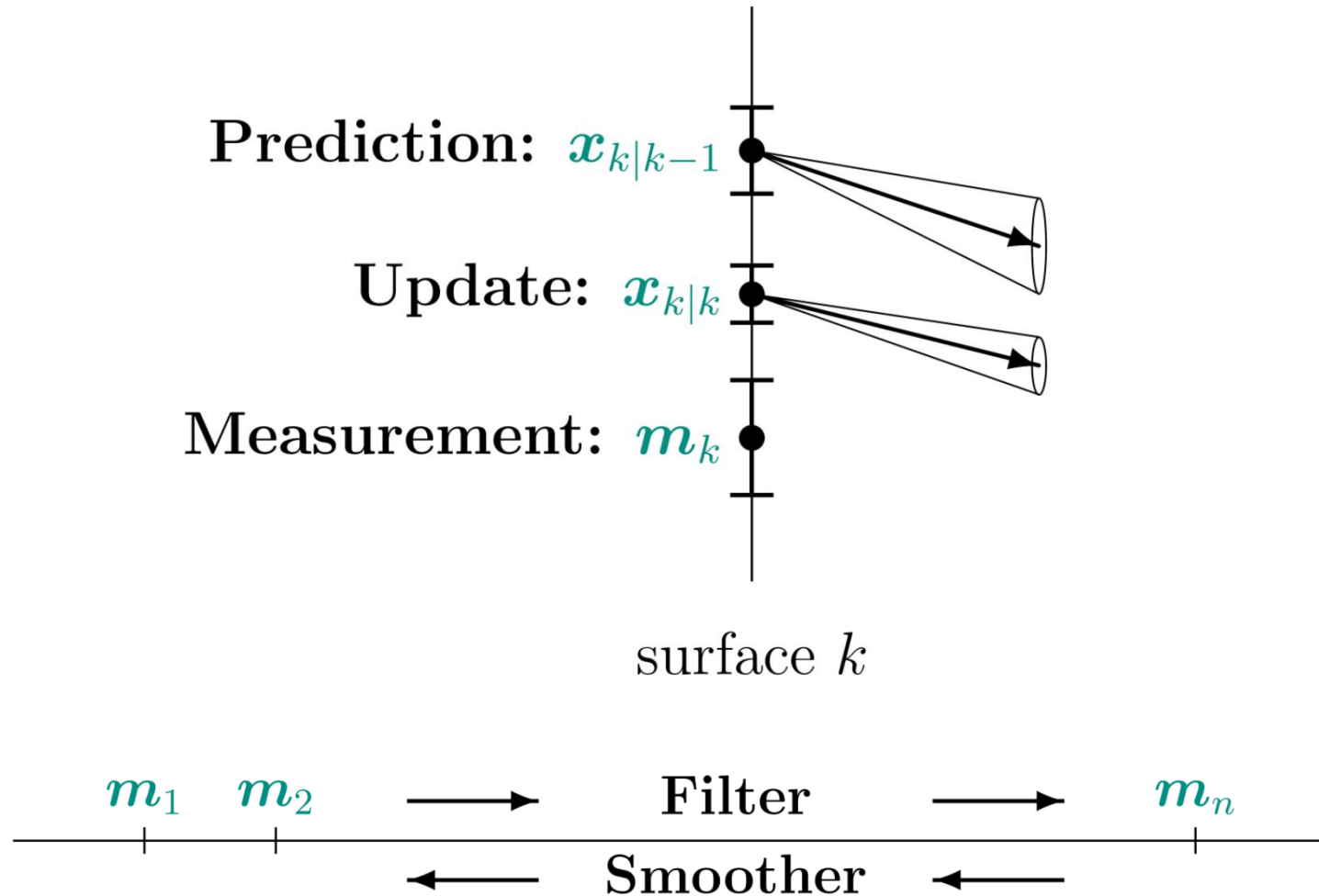
$$\begin{pmatrix} x \\ y \\ t_x \\ t_y \\ q/p \end{pmatrix} \quad \text{with } t_x = \frac{\partial x}{\partial z} \text{ and } t_y = \frac{\partial y}{\partial z}$$

- Kalman filter has two steps that are repeated **prediction** and **update**



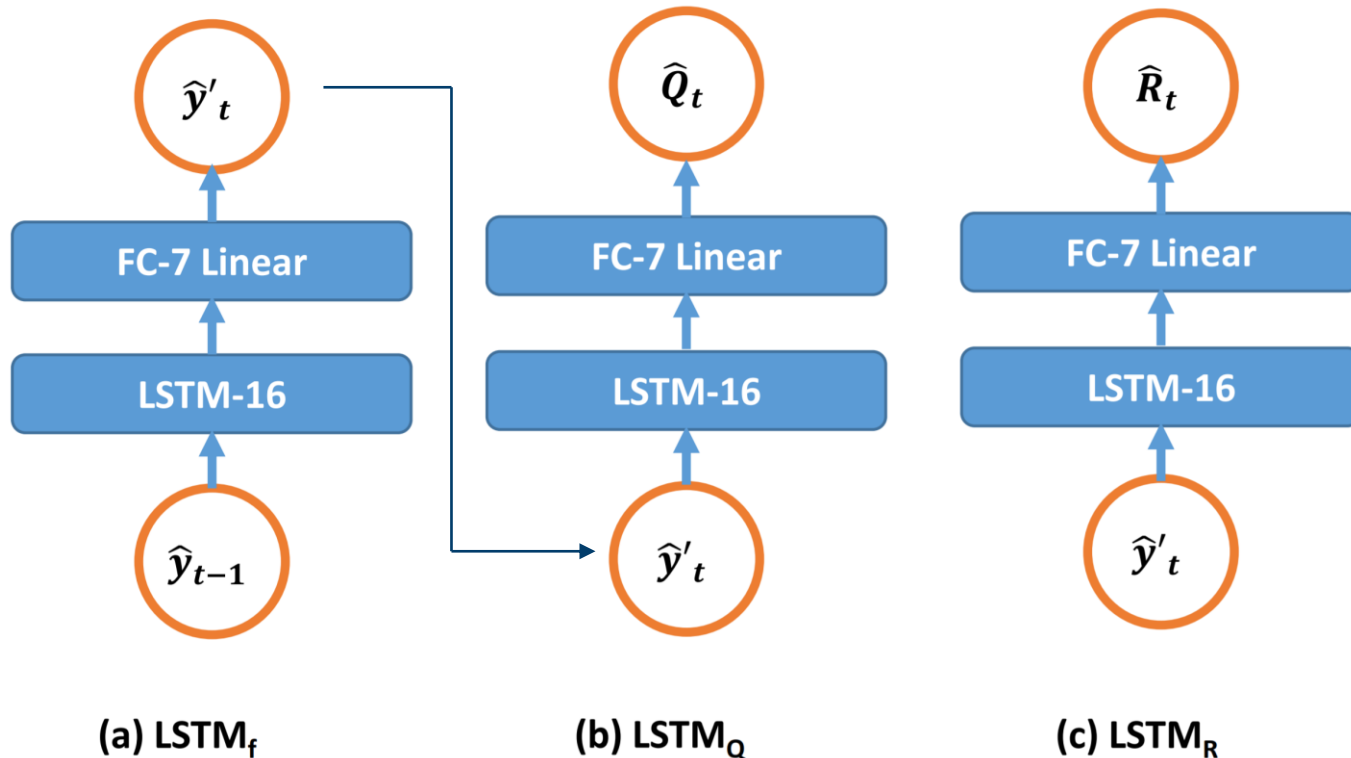
Prediction step of the Kalman filter

Track Fitting:



Deep Learning Track Fitting:

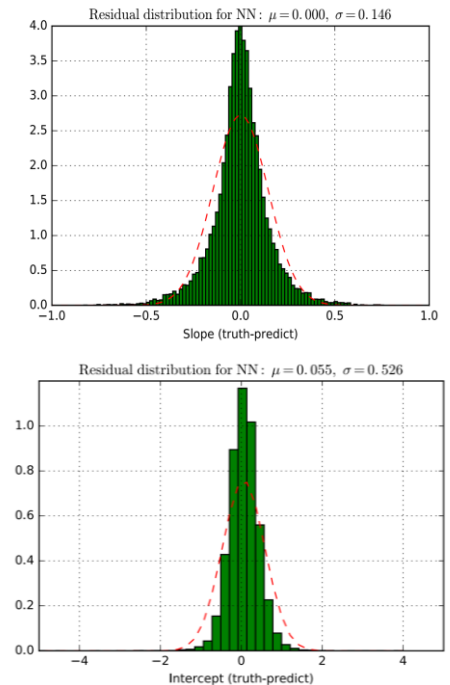
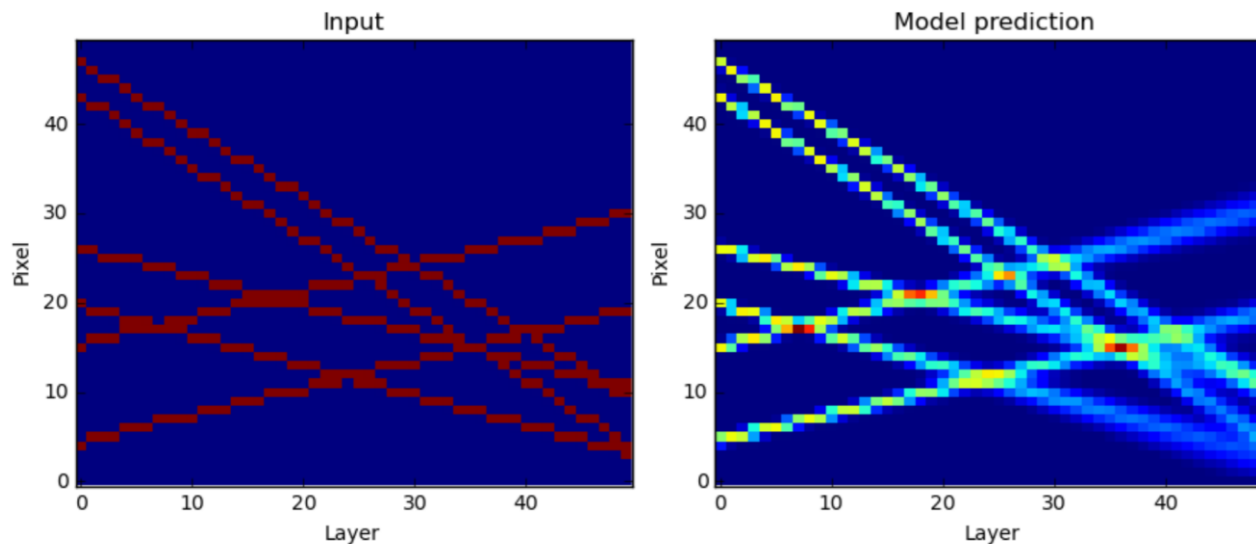
- **Inputs:** seed state vector --> **Model** --> best estimate of state vector
- **LSTM model Kalman update**



Deep Learning Track Fitting:

- The **HEP.TrkX** project is exploring the applicability of advanced machine learning algorithms to HL-LHC track reconstruction
- **CNN + LSTM** model
- Toy dataset with **custom loss function** (see David's talk Thursday)

$$L(x, y) = \log |\Sigma| + (y - f(x))^T \Sigma^{-1} (y - f(x))$$



The HEP.TrkX Project: deep neural networks for HL-LHC online and offline tracking, Steven Farrell 2017

Hands-on Tutorials:

➤ <https://github.com/wesmail/ML-GlueX-EIC-PANDA>

```
git clone https://github.com/wesmail/ML-GlueX-EIC-PANDA.git
```

➤ Tutorials in perfect order

1. `data_exploration.ipynb`
2. `DBSCAN.ipynb`
3. `hit_pairs.ipynb`
4. `RNN.ipynb`
5. `GNN.ipynb`

Thank You