Marriage & divorce of "Feedbacks" with "Landau damping" – transverse

Nicolas Mounet, CERN/BE-ABP-HSC

With lots of inputs from S. Antipov, S. Arsenyev, X. Buffat, A. Burov, A. Chao, S. Furuseth, G. Iadarola, N. Klinkenberg, J. Komppula, K. Li, A. Maillard, E. Métral, A. Oeftiger, G. Rumolo, M. Schenk, V. Vaccaro, D. Valuch.

Feedbacks – in a nutshell

- In presence of a coherent instability leading to a complex frequency shift $\Delta\omega_{coh}$

$$\langle x\rangle(t) \propto e^{j\Delta\omega_{coh}t} = e^{-\Im(\Delta\omega_{coh})t}e^{j\Re(\Delta\omega_{coh})t}$$

… it is natural to try to damp the exponentially growing term ($\Im(\Delta\omega_{coh}) < 0$) with a counteracting damping exponential:

$$\langle x\rangle(t) \propto e^{-g_{damp}t}$$

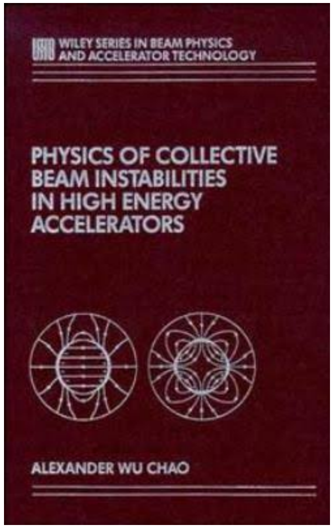
- In other words the growth rate $-\Im(\Delta\omega_{coh}) > 0$ gets cancelled out by the damper gain $-g_{damp} < 0$.

- To do so, one "only" need to measure the beam position $\langle x\rangle(t)$ and kick proportionally to it with a phase of $\frac{\pi}{2}$.

$\Longrightarrow$ "ideal" (bunch-by-bunch) damper, which acts only on the bunch centroid.

Landau damping – in a nutshell

Alex W. Chao: "[…] there are a large number of collective instability mechanisms acting on a high intensity beam in an accelerator […]. Yet the beam as a whole seems basically stable, as evidenced by the existence of a wide variety of working accelerators[…]. One of the reasons for this fortunate outcome is Landau damping, which provides a natural stabilizing mechanism against collective instabilities if particles in the beam have a small spread in their natural […] frequencies."



Mathematically, the coherent frequency of the motion Ω must self-consistently obey the dispersion relation involving the frequency spread $\rho(\omega)$:

$$1 = -\Delta\omega_{coh} \int d\omega \frac{\rho(\omega)}{\Omega - \omega} = -\Delta\omega_{coh} \left[P.V \int_{-\infty}^{+\infty} d\omega \frac{\rho(\omega)}{\Omega - \omega} - j\pi\rho(\Omega) \right]$$

with $\Delta\omega_{coh}$ the complex frequency shift of the instability in the absence of frequency spread.

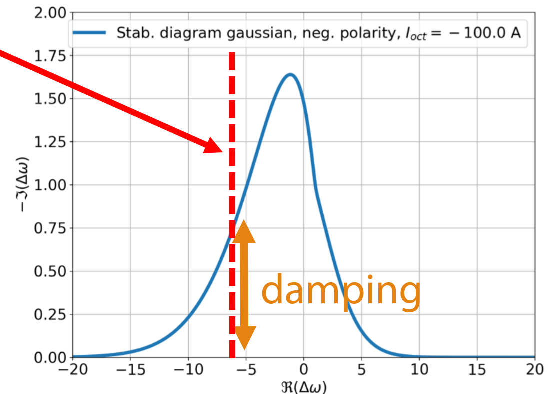
Landau damping – transverse

- Dispersion relation:

$$1 = -\Delta\omega_{coh}\left[P.V\int_{-\infty}^{+\infty} d\omega \frac{\rho(\omega)}{\Omega-\omega} - j\pi\rho(\Omega)\right]$$

- What does this mean?
  - Even with Ω real, there are both a real and imaginary part between the square brackets.
  - This means the equation can hold even when $\Delta\omega_{coh}$ is complex and the final coherent frequency Ω is real (i.e. stable).
  - At a given real freq. shift $\Re(\Delta\omega_{coh})$, everything is as if the instability gets a damping term equal to the stability diagram imaginary part computed at $\Re(\Delta\omega_{coh})$

To some extent it looks like the action of a feedback…

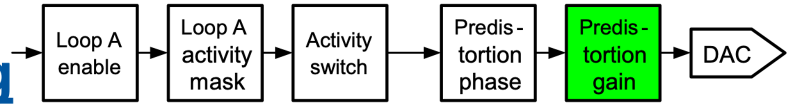


Stability diagrams ⇒ modes with a freq. shift inside the diagram are stable

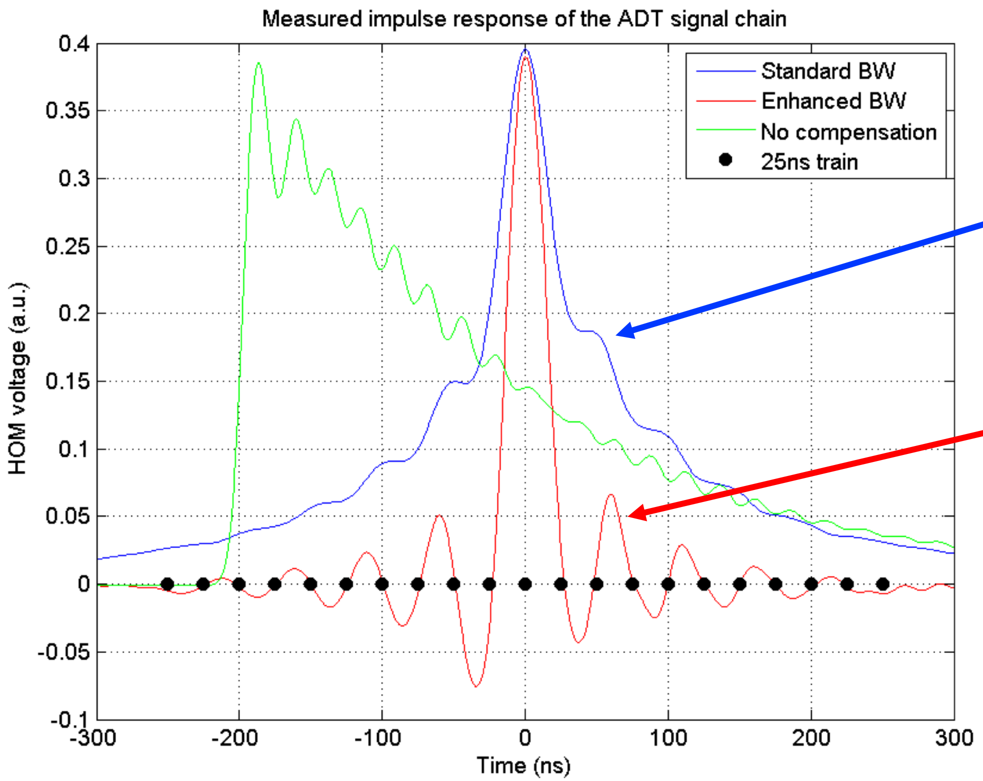
Stab. diagram theory from ***A. Ruggiero and V. Vaccaro,*** *CERN-ISR-TH/68-33*

Feedbacks – in reality (LHC)

ADT Digital Signal Processing



- Impulse response comparison of "standard bandwidth" and "enhanced bandwidth" operation



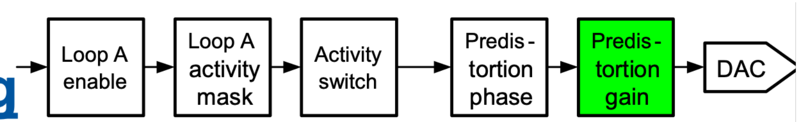
Bunches "talk to each other" through the feedback

Damper closer to ideal

From ***D. Valuch,*** LBOC 24/05/016

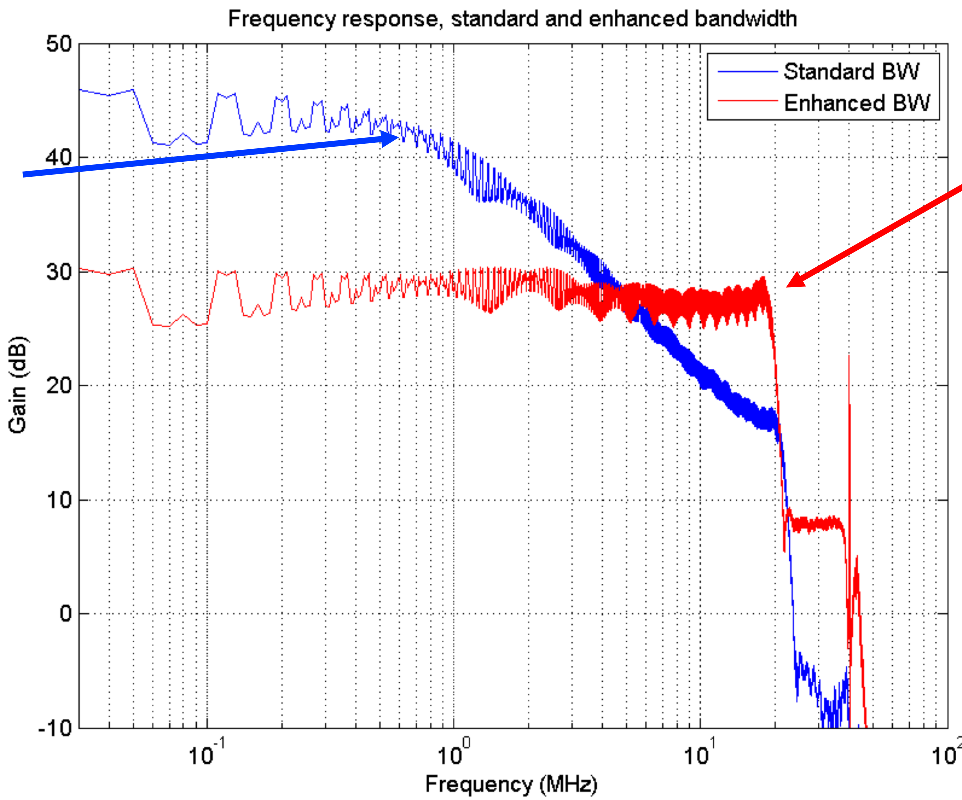
Feedbacks – in reality (LHC)

ADT Digital Signal Processing



- Frequency response comparison of “standard bandwidth” and “enhanced bandwidth” operation

Non ideal feedback has higher gain for modes at lower frequency



…while in ideal feedback the gain is the same for all modes.

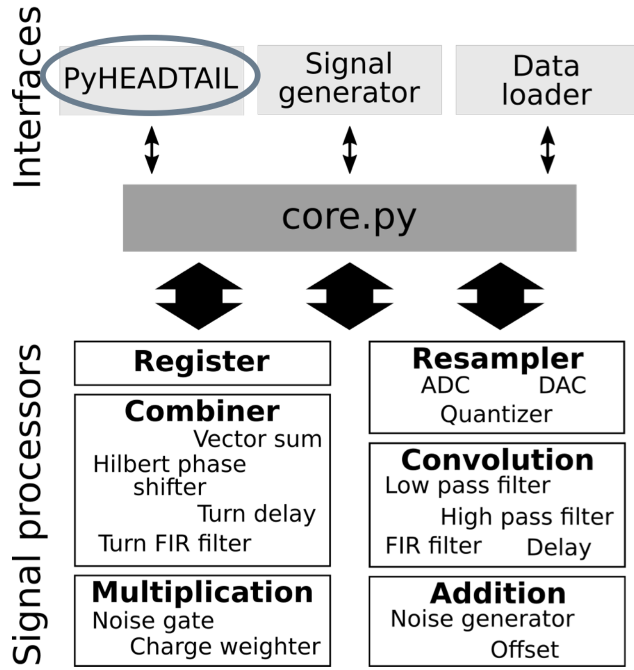
From ***D. Valuch,*** LBOC 24/05/016

Real frequency response is smooth, please disregard the ripple visible in the plot

Example: the PyHEADTAIL ADT model

- Detailed model can be build with a list of signal processors

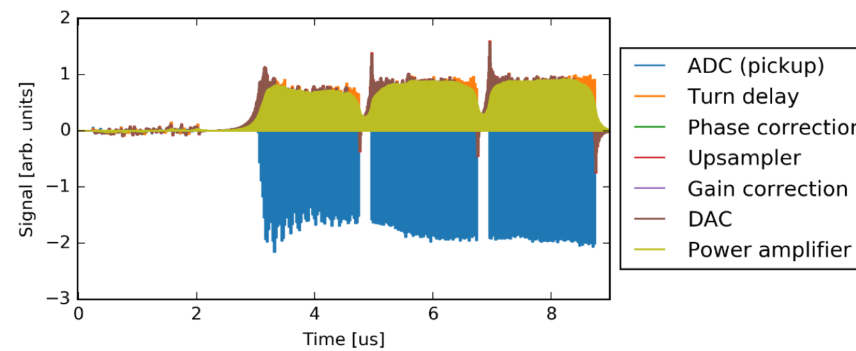
```
processors_x = [
        ChargeWeighter
#         NoiseGenerat
        HarmonicADC(1*
                    n_
        TurnFIRFilter(
        FIRFilter(FIR_
        Upsampler(3, [
        FIRFilter(FIR_
        DAC(ADC_bits,
        Lowpass(fc, f_
        BackToOriginal
]
```



J. Komppula

Example visualization: 3-batch injection

- Direct data from signal processors in debug mode



From ***K. Li & J. Komppula***, HSC meeting, 18/03/2019

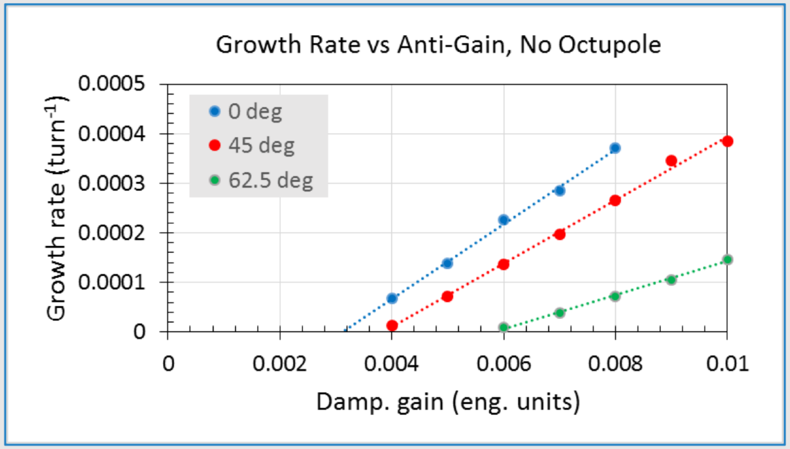
18.03.2019 HSC Meeting - Kevin Li 18

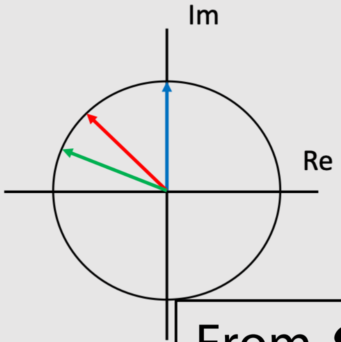
Feedbacks can be used in various ways…

- Instead of being resistive, they can also be used as reactive feedback, i.e. fighting the real part of the frequency shift
  → used to stabilise TMCI, with limited success (not more than 5-10 % increase of LEP TMCI threshold - several models developed [Danilov-Perevedentsev 1993, Sabbi 1996, Brandt et al 1995].

- More generally, one can play with the phase and even use the damper as an instability exciter (was done as a machine study in the LHC):

ADT fully qualified to act as a controlled source of impedance





From ***S. Antipov***, HSC meeting, 04/11/2018

Can measure Landau damping from nonlinearities at injection

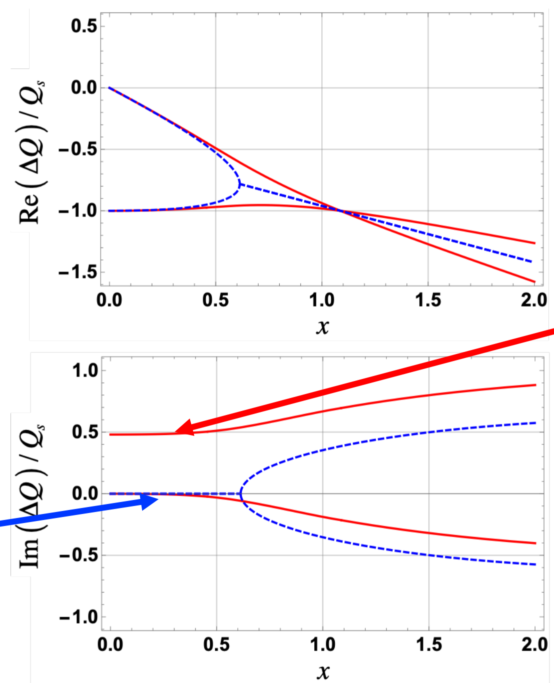
- Next step – compare with the measurements (E. McLean *et al.*)

Feedbacks can also create instabilities…

9th International Particle Accelerator Conference IPAC2018, Vancouver, BC, Canada JACoW Publishing
ISBN: 978-3-95450-184-7 doi:10.18429/JACoW-IPAC2018-THPAF048

DESTABILISING EFFECT OF THE LHC TRANSVERSE DAMPER

E. Métral†, D. Amorim, S. Antipov, N. Biancacci, X. Buffat and K. Li, Geneva, Switzerland



The damper creates an instability.

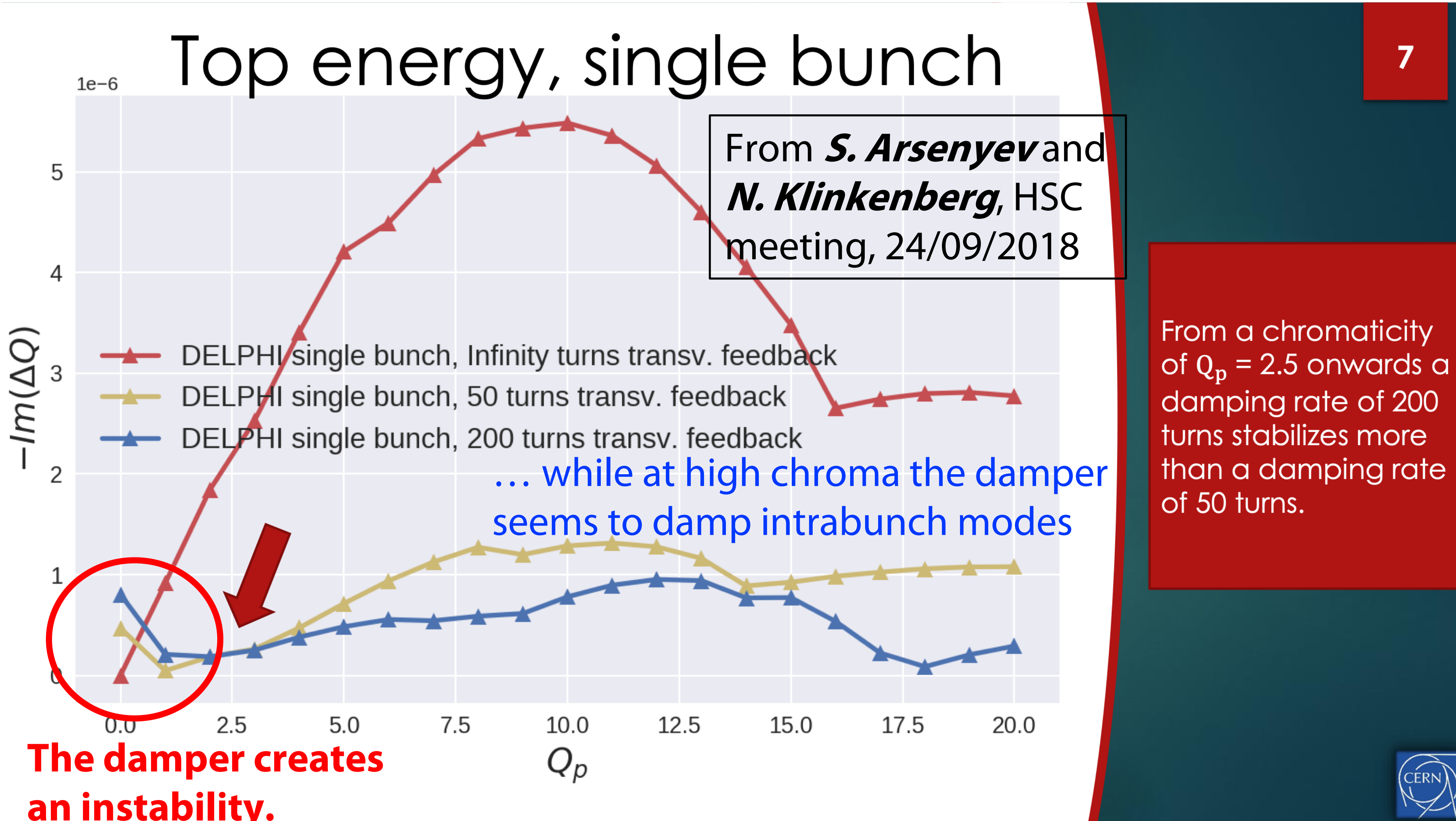
No damper ⟶ no instability at zero chroma and low intensity.

Figure 3: Solutions of the diagonalisation of the 2×2 matrix of Eq. (7): without (blue) and with (red) the damper.

From ***E. Métral*** et al, IPAC'18

… but also unexpectedly damp others

➢ Case of FCC (it is very similar in the LHC):



Landau damping – a more complex reality

- Sometimes tunespread also has a detrimental effect:

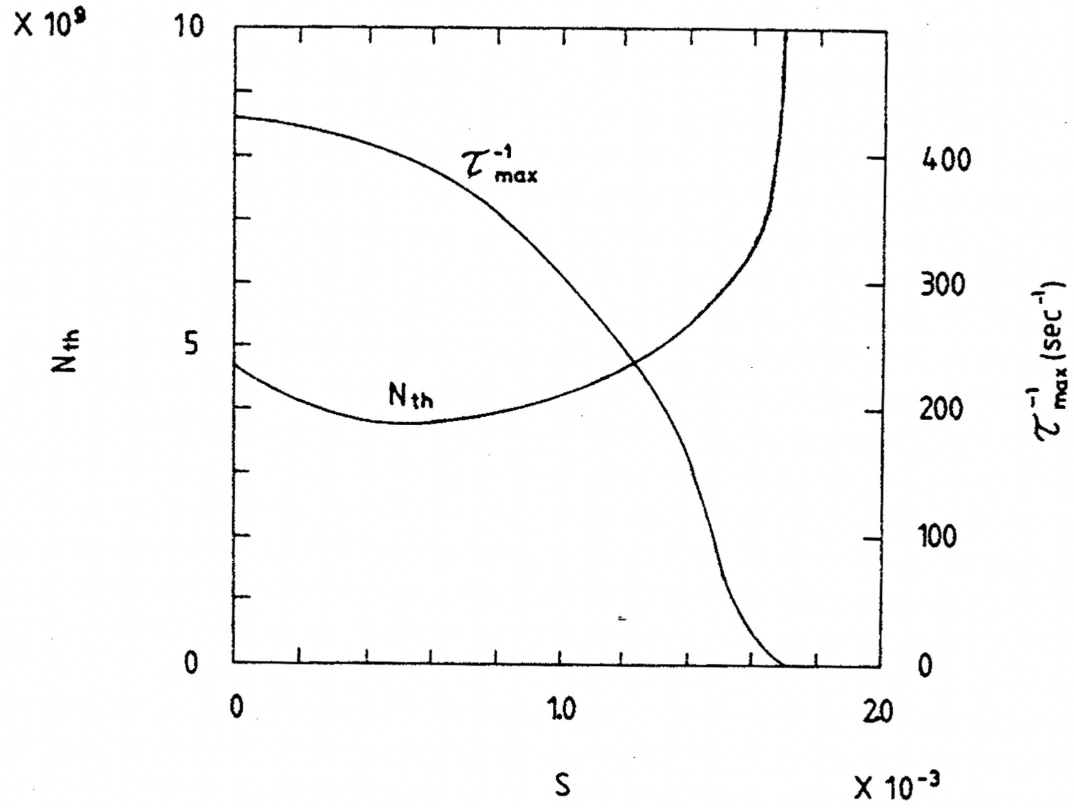


Fig. 5 The threshold intensity and the maximum growth rate as a function of the tune spread.

From ***Y. Chin***, *CERN/SPS/85-09* (1985).

Landau damping – a more complex reality

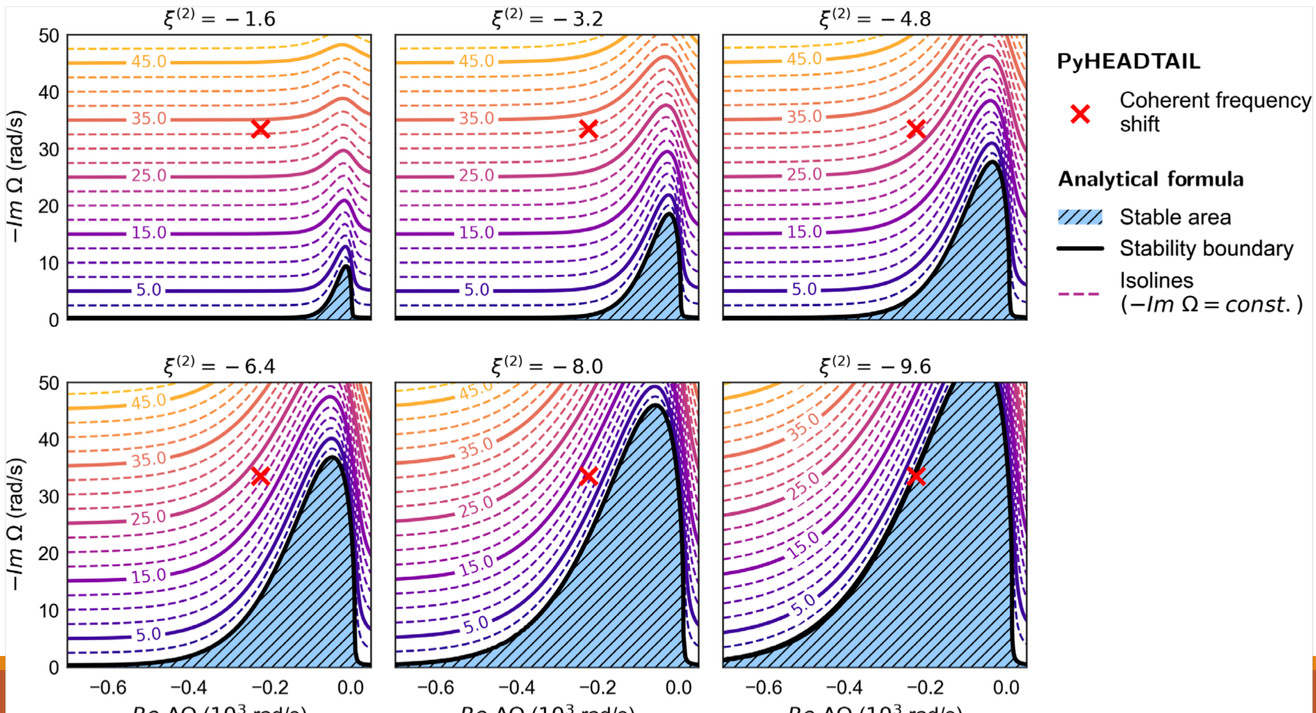
PHYSICAL REVIEW ACCELERATORS AND BEAMS **21**, 084402 (2018)

Editors' Suggestion

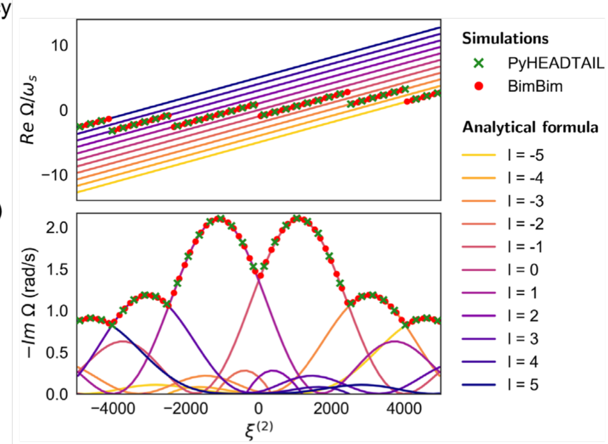
Vlasov description of the effects of nonlinear chromaticity on transverse coherent beam instabilities

M. Schenk,[1,2,*] X. Buffat,[1] K. Li,[1] and A. Maillard[3]

Stability diagrams, on one side…

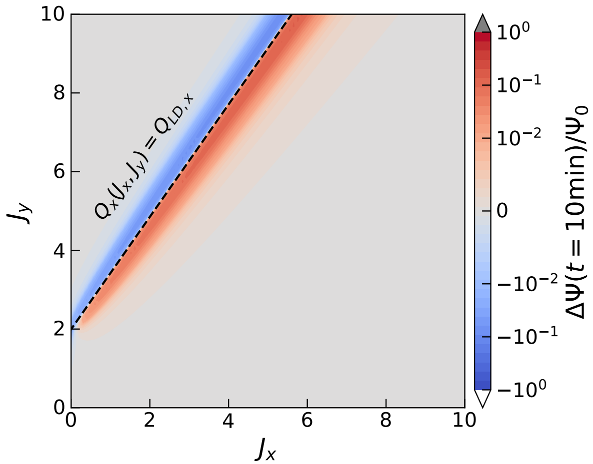


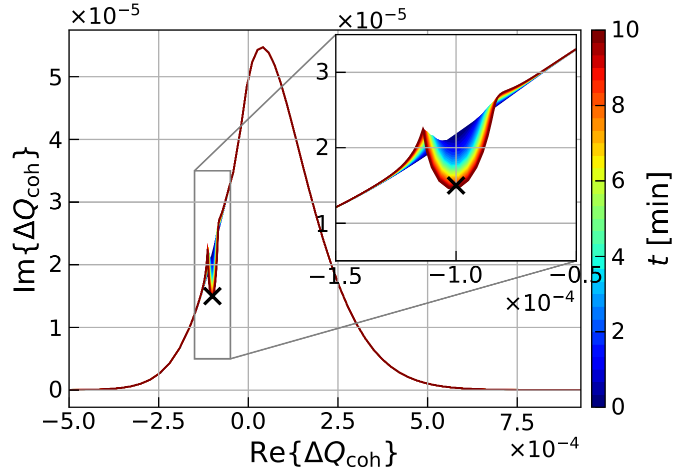
… but on the other hand modes get also modified by tunespread.



Landau damping – a more complex reality

➢ Other effects (e.g. noise) can modify the stability diagram:





From ***S. Furuseth & X. Buffat***, MCBI Workshop, 26/09/2019

⟹ Diffusion effects seem to be able to drill “holes” in the stability diagram.

Feedbacks and Landau damping

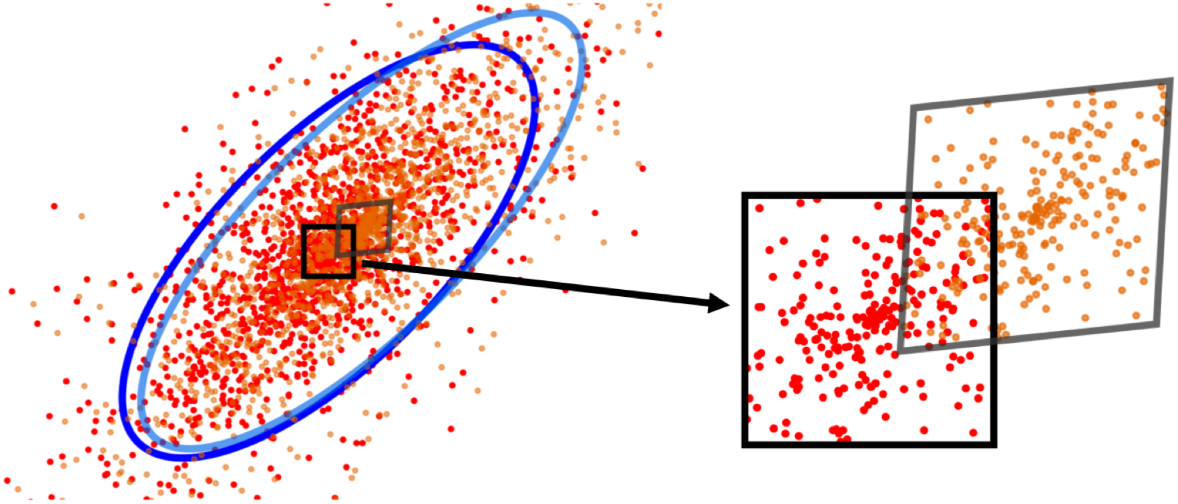
- Simplistic view: they both introduce a damping rate.
- Reality is much more complex:
  - complex feedback system,
  - non-ideal feedbacks,
  - complex modification of the stability diagram.
- … and some effects are really contradicting the simple picture:
  - destabilizing effect of damper close to zero chromaticity,
  - destabilizing effect of tunespread on TMCI,
  - modification of coherent modes due to the source of tunespread.

$\Longrightarrow$ to get all these effects one needs a more realistic modelling,

$\Longrightarrow$ macroparticle simulations or Vlasov solvers.

Vlasov equation [**A. A. Vlasov**, *J. Phys. USSR 9, 25 (1945)*]

- Vlasov equation is based on Liouville theorem (or equivalently, on the collisionless Boltzmann transport equation), which expresses that the local phase space density does not change when one follows the flow (i.e. the trajectory) of particles.
- In other words: local phase space area is conserved in time: $\frac{d\psi}{dt} = 0$



Red particles at time *t* become the orange ones at time *t* + *dt*, and the black square becomes the grey parallelogram which contains the same number of particles.

From Vlasov equation to Sacherer equation

- Vlasov equation is a priori a partial differential equation of 7 variables:

$$\frac{d\psi}{dt} = \frac{\partial\psi}{\partial t} + \frac{\partial\psi}{\partial x}\frac{dx}{dt} + \frac{\partial\psi}{\partial p_x}\frac{dp_x}{dt} + \frac{\partial\psi}{\partial y}\frac{dy}{dt} + \frac{\partial\psi}{\partial p_y}\frac{dp_y}{dt} + \frac{\partial\psi}{\partial z}\frac{dz}{dt} + \frac{\partial\psi}{\partial p_z}\frac{dp_z}{dt} = 0$$

- In the context of transverse beam instabilities from impedance, after a number of assumptions and decompositions, and looking for a single coherent mode, it can be recast into the simpler Sacherer equation [F. J. Sacherer, CERN/SI-BR/72-5 (1972)]:

$$\left(\Omega - Q_{y0}\omega_0 - l\omega_s\right)R_l(r) = \frac{jN\omega_0 e^2}{4\pi\gamma m_0 v Q_{y0}} g_0(r) \sum_{l'=-\infty}^{+\infty} j^{l'-l}$$
$$\times \sum_{k=-\infty}^{+\infty} \int_0^{+\infty} \tilde{r}d\tilde{r} R_{l'}(\tilde{r}) J_{l'}\left[\left(Q_{y0} + k - \frac{Q'_y}{\eta}\right)\frac{\tilde{r}}{R}\right]$$
$$\times Z_y\left(\left(Q_{y0}+k\right)\omega_0\right) J_l\left[\left(Q_{y0} + k - \frac{Q'_y}{\eta}\right)\frac{r}{R}\right]$$

These are the unknown (frequency and radial distribution of the mode)

This is the impedance

N: number of particles of charge e,
η: slippage factor,
R: machine radius,
Q_{y0}: unperturbed transverse tune,
ω_0: angular revolution frequency,
$\omega_s = \omega_0 Q_s$: synchrotron frequency,
v: beam velocity,
γ: relativistic mass factor
m_0: particle rest mass
Q'_y: chromaticity,
$g_0(r)$: longitudinal distribution,
J_l: Bessel function

Extension of Sacherer equation with feedback

- A non-ideal (but still linear) feedback can be modelled as in impedance $\longrightarrow$ already in the formalism
- An ideal feedback is a delta function impedance (or constant wake) $\longrightarrow$ needs an extension of Sacherer equation to include it [NM, *CERN Yellow Reports: Conf. Proc., 1 (2018) p. 77*]:

$$\left(\Omega - Q_{y0}\omega_0 - l\omega_s\right)R_l(r) = \frac{jN\omega_0 e^2}{4\pi\gamma m_0 v Q_{y0}} g_0(r) \sum_{l'=-\infty}^{+\infty} j^{l'-l} \times \sum_{k=-\infty}^{+\infty} \int_0^{+\infty} \tilde{r}d\tilde{r}R_{l'}(\tilde{r}) \left\{ \mu J_{l'}\left[\left(-\frac{Q'_y}{\eta}\right)\frac{\tilde{r}}{R}\right] J_l\left[\left(-\frac{Q'_y}{\eta}\right)\frac{r}{R}\right] + \sum_{k=-\infty}^{+\infty} Z_y\left(\left(Q_{y0}+k\right)\omega_0\right) J_{l'}\left[\left(Q_{y0}+k-\frac{Q'_y}{\eta}\right)\frac{\tilde{r}}{R}\right] J_l\left[\left(Q_{y0}+k-\frac{Q'_y}{\eta}\right)\frac{r}{R}\right]\right\}$$

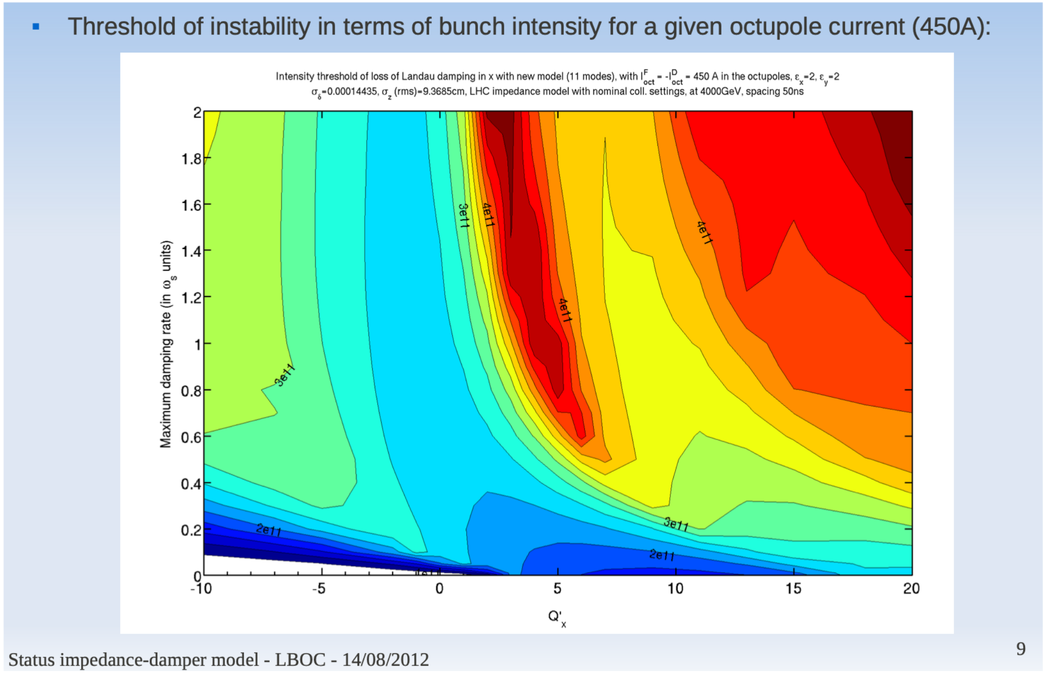
Damper part (μ is proportional to damping gain)

Equation can be recast into a standard eigenvalue problem.

Sacherer equation with feedback

Including the feedback in the dynamical system (rather than a posteriori as a simple damping rate) unveiled a number of effects:

- destabilising effect of the damper close to zero chromaticity (see before),
- damping of high order headtail modes at high Q' with bunch-by-bunch damper:



A. Burov, NM et al, ABA model, talk at LBOC, 14/08/2012

⟹ This supported the idea of running at high Q' with high damper gain in the LHC.

The equation is implemented in several Vlasov solvers (**NHTVS** [A. Burov, PR ST-AB, 17 (2014), **DELPHI** [NM, *CERN Yellow Reports: Conf. Proc., 1 (2018) p. 77],* **GALACTIC** [E. Métral et al, *IPAC'18 (2018) pp. 3076–3079*].

Extension of Sacherer equation with tunespread

- It is also possible to extend Sacherer equation including linear amplitude detuning (and going beyond the stability diagram theory).
- Defining the dispersion integral as

Transverse distribution

$$I(\Omega - l\omega_s) = \int_0^{+\infty} \frac{dJ_y \cdot J_y \cdot \frac{df_0}{dJ_y}}{\Omega - \omega_0 \left(Q_{y0} + a_{yy} J_y\right) - l\omega_s}$$

Amplitude detuning

we can get:

$$\frac{\rho_l(r)}{I(\Omega - l\omega_s)} = \frac{-j\omega_s \omega_0 e^2}{2\gamma m_0 \eta v^2 Q_{y0}} g_0(r) \sum_{l'=-\infty}^{+\infty} j^{l'-l}$$

$$\times \sum_{k=-\infty}^{+\infty} Z_y\left[\left(Q_{y0} + k\right)\omega_0\right] J_l\left[\left(Q_{y0} + k - \frac{Q'_y}{\eta}\right)\frac{r}{R}\right]$$

$$\times \int_0^{+\infty} \tilde{r} d\tilde{r} \rho_{l'}(\tilde{r}) J_{l'}\left[\left(Q_{y0} + k - \frac{Q'_y}{\eta}\right)\frac{\tilde{r}}{R}\right]$$

Equation obtained first by **Y. Chin**, *CERN/SPS/85-09* (1985).

The determinant equation

- After some expansion on orthogonal polynomials, one gets an equation of the form

$$0 = \sum_{l'=-\infty}^{+\infty} \sum_{n'=0}^{+\infty} \left(\frac{\delta_{ll'}\delta_{nn'}}{I(\Omega - l'\omega_s)} + \mathcal{M}_{ln,l'n'} \right) c_{l'}^{n'}$$

where each coefficient of the matrix $\mathcal{M}$ can be computed analytically.

- Non trivial solutions are found if and only if Ω is the root of

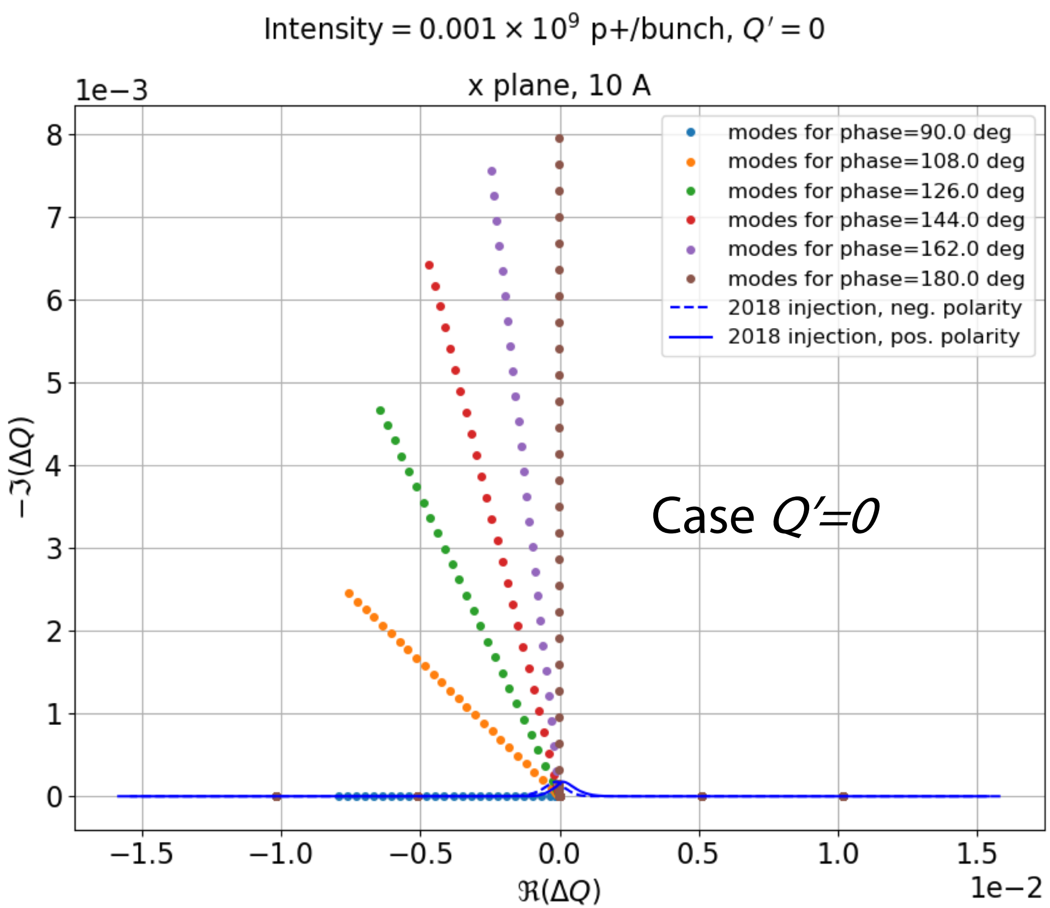
$$\det\left(\left[\frac{\delta_{ll'}\delta_{nn'}}{I(\Omega - l'\omega_s)} + \mathcal{M}_{ln,l'n'}\right]\right) = 0$$

- Issue: there is a priori no general strategy to find all the roots.

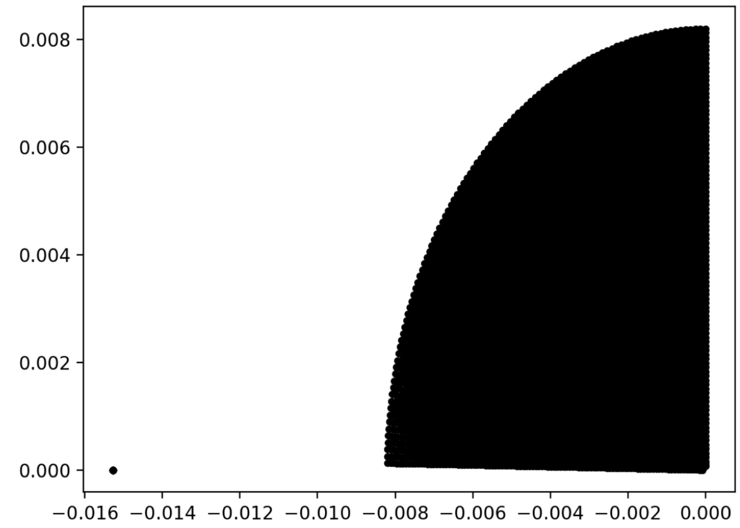
$\Longrightarrow$ this is a very difficult equation to solve.

Is there a stability diagram still in the general case?

Without trying to solve the full problem, we try to map the complex plane of tuneshifts, thanks a broad scan of the phases and gains of a damper (inspired by the LHC study by **S. Antipov** et al, *CERN-ACC-NOTE-2019-0034*).

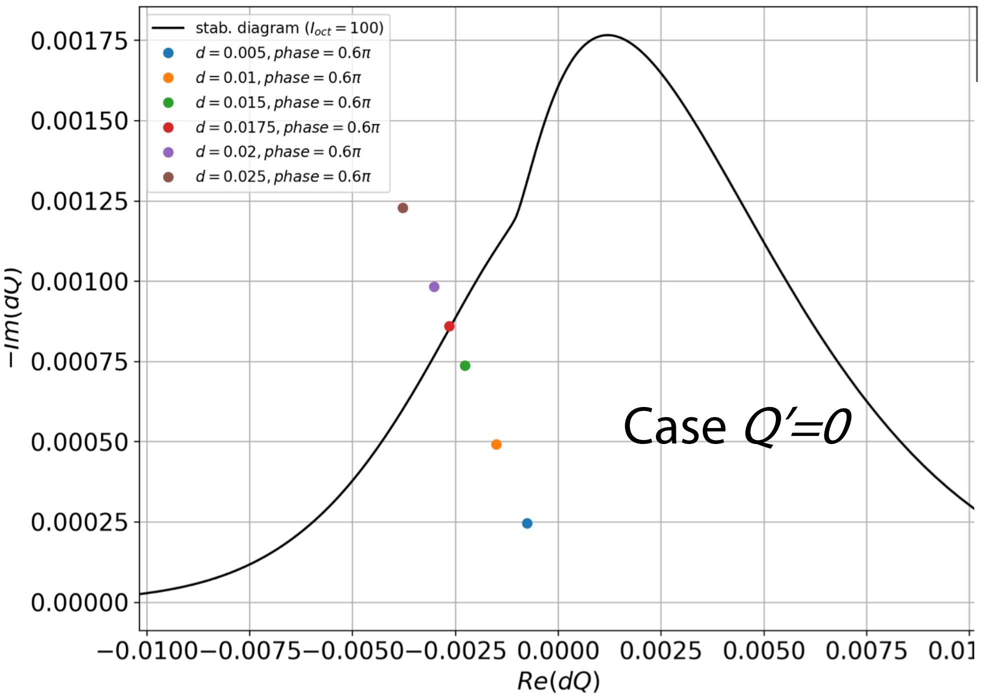


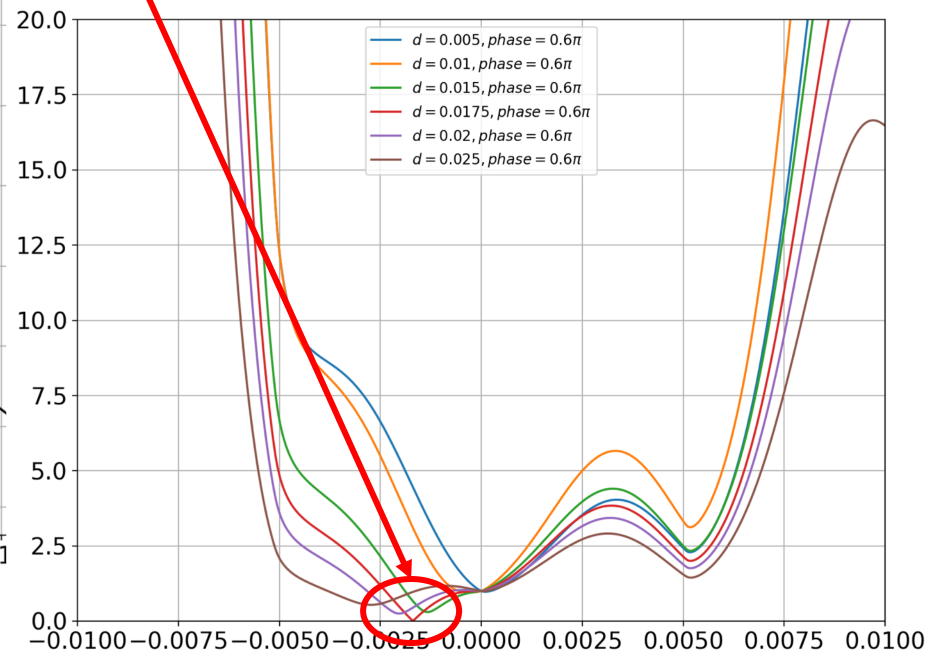
Making a fine mesh of phases and gains, we can cover a large area in the complex plane:



Can we recover the stability diagram theory?

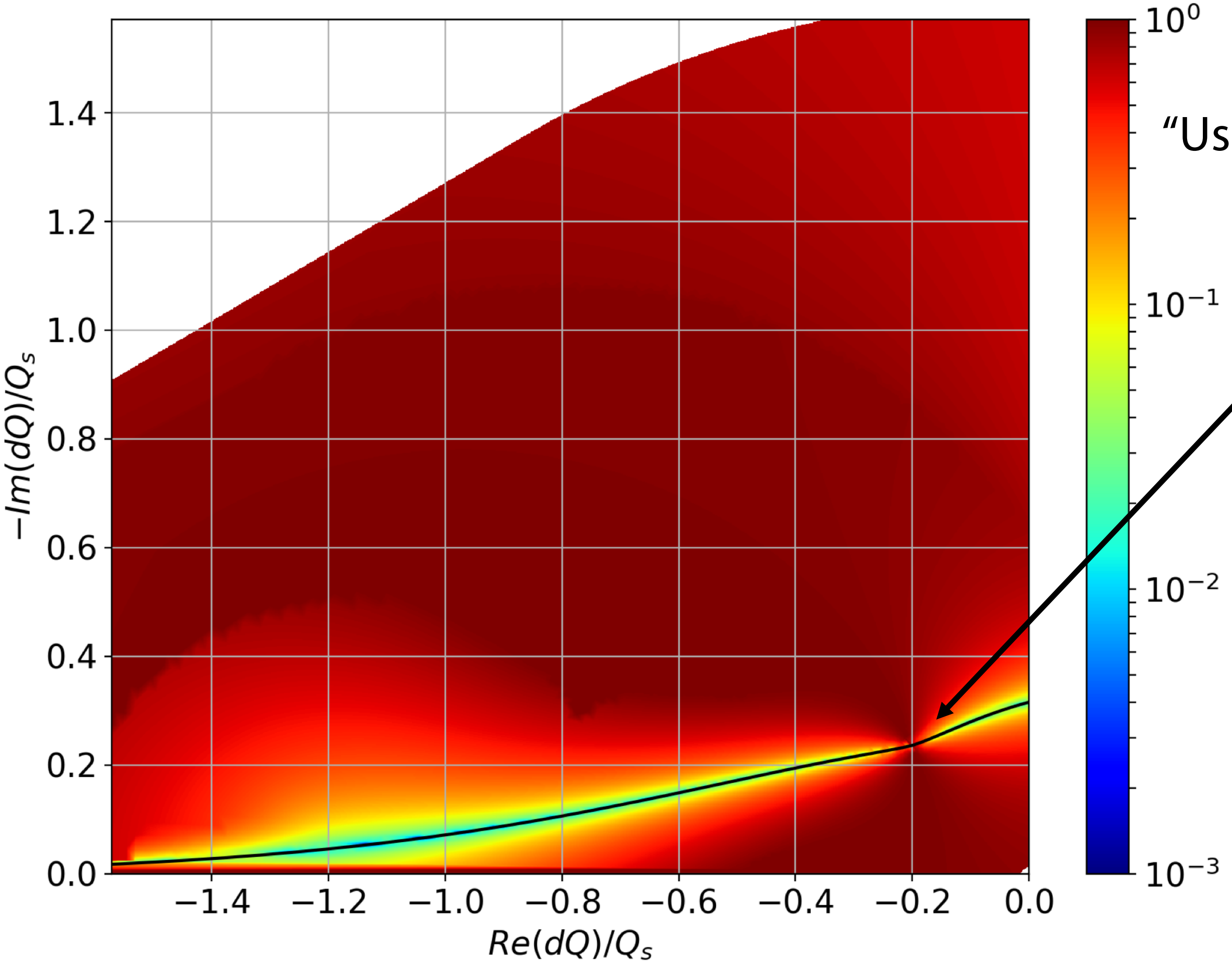
- Strategy: for each gain/phase of the damper, we compute the determinant along the real tune shifts→ when it touches the stability diagram, the minimum of this 1D curve should go to zero.





Generalized stability diagrams

The color represents the minimum of the previous 1D curves:

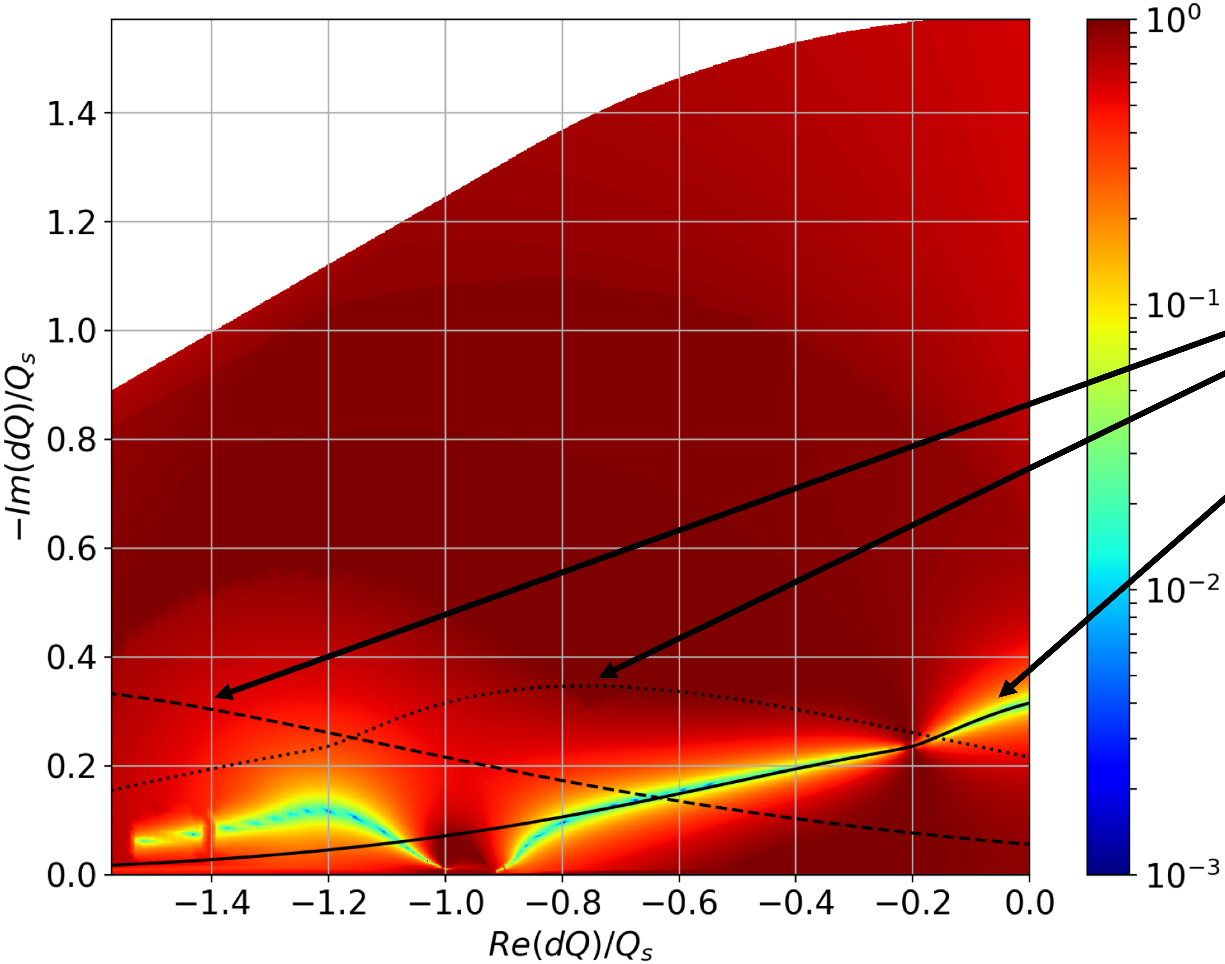


"Usual" stability diagram

Case *Q′=0*

Effect of chromaticity

The color represents again the minimum of the 1D curves:

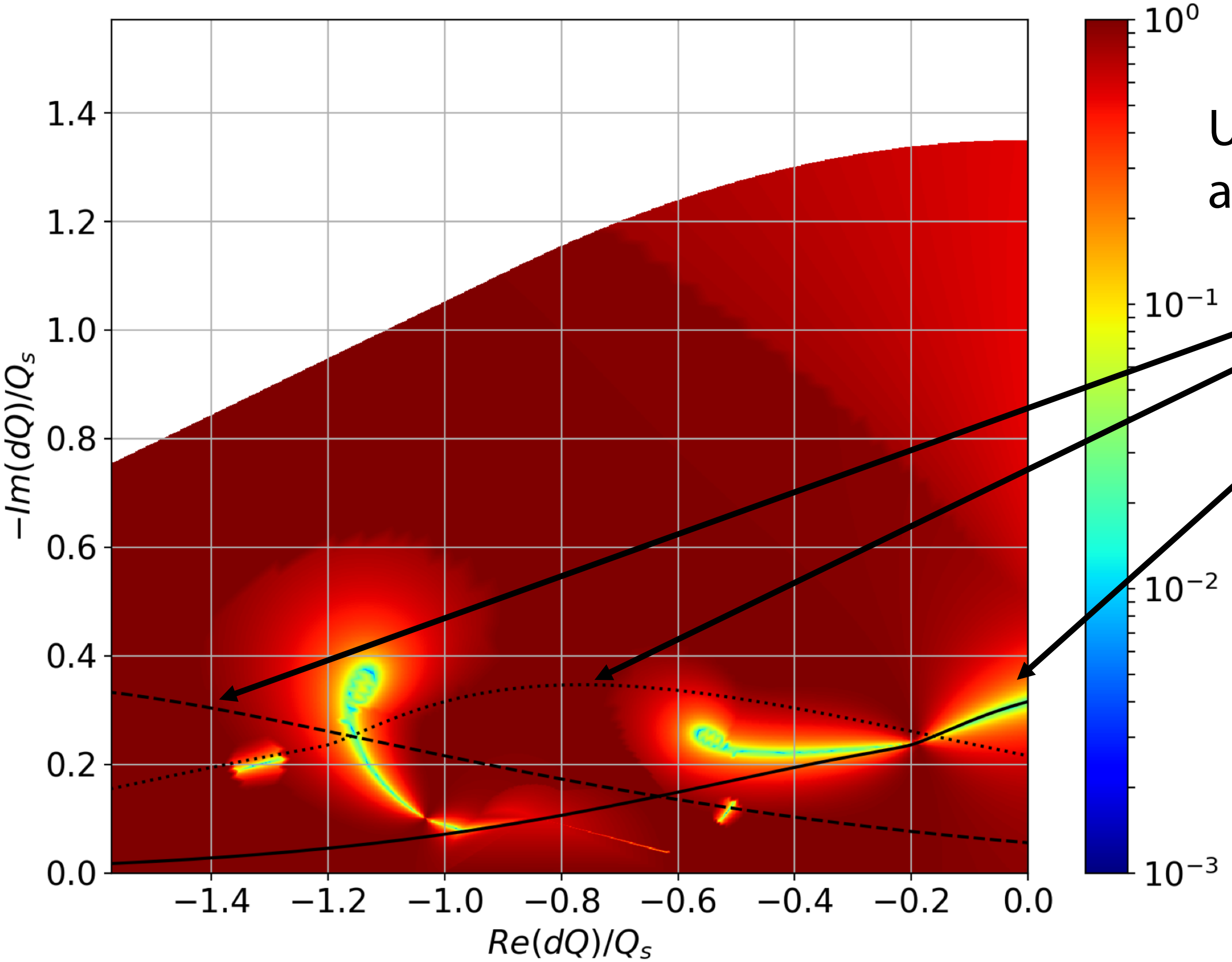


Usual stability diagrams around 0, $-Q_s$ and $-2Q_s$

Case *Q'=5*

Effect of chromaticity

The color represents again the minimum of the 1D curves:



Usual stability diagrams around 0, $-Q_s$ and $-2Q_s$

Case *Q′=15*

→ Generalized stability diagrams, different from the "usual" ones and chromaticity dependent.

Feedbacks and Landau damping

- Both can be understood heuristically as providing a damping rate for certain modes.

- However, such a simple picture does not withhold a number of observations, e.g. destabilizing effects.

- In a Vlasov treatment, a more realistic picture can be obtained by:

  - extending Sacherer equation with an ideal, bunch-by-bunch damper,

  $\Longrightarrow$ unveils the ability of such a damper to stabilize high order headtail modes and to destabilize the beam in some conditions,

  - generalizing the same equation with linear amplitude detuning,

  $\Longrightarrow$ could potentially extend the stability diagram theory, but requires to overcome the difficulties to solve the obtained determinant equation.