Optics correction schemes of conventional and advanced accelerators

ARIES-APEC workshop "Mitigation Approaches for Storage Rings and Synchrotrons"

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Office of Science

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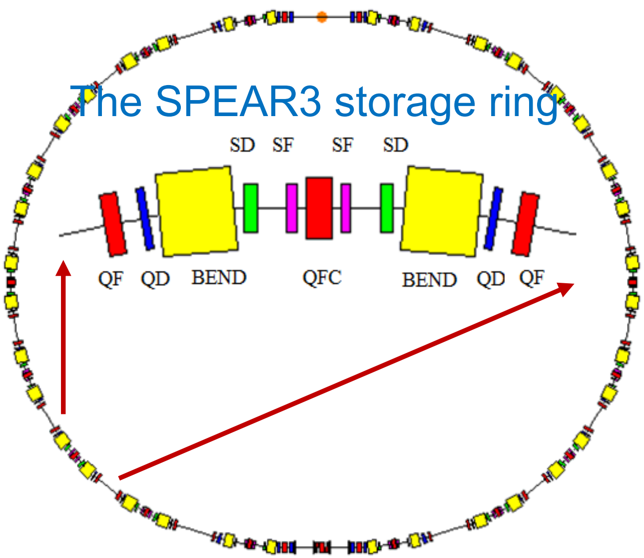
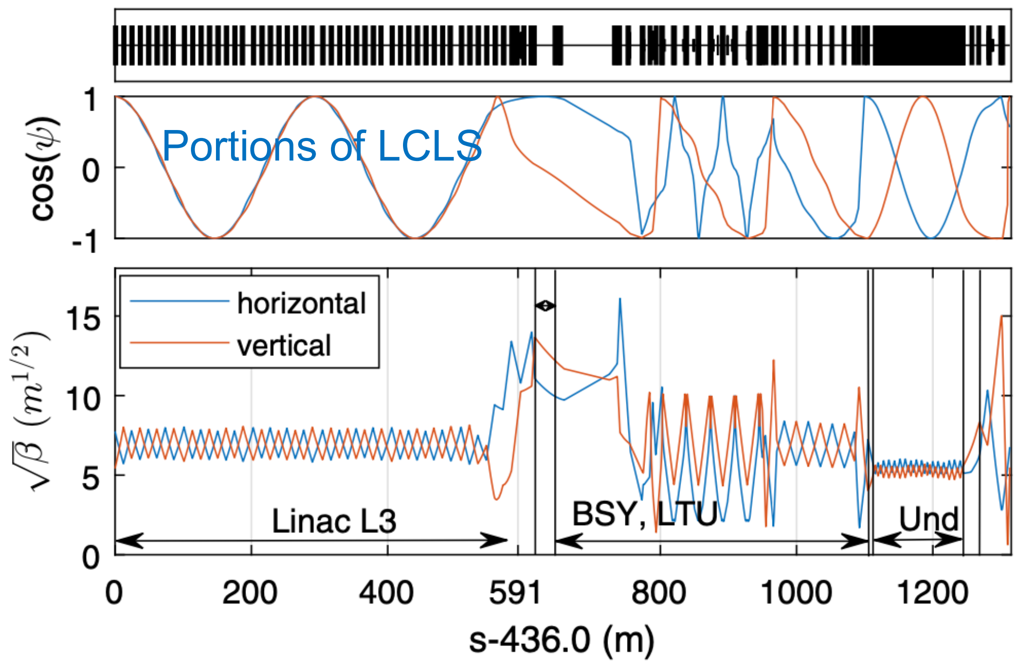
Outline

- Optics errors in accelerators and impact
- Characterization of optics errors
- Global optics correction schemes
- Fitting with constraints
- Coupling correction
- Summary

Accelerator lattices

- (Periodic) placement of focusing elements in the accelerator lattice determines its optics.





The optics has big impact to the performance of the machine: orbit control, beam loss, beam distribution, dynamic aperture, …

Characterization of optics

- Transfer matrix

  Between two points $\mathbf{X}_2 = \mathbf{M}_{21}\mathbf{X}_1$ $\mathbf{X} = (\mathrm{x}, \mathrm{x}', \mathrm{y}, \mathrm{y}')^{\mathrm{T}}$

- Beta functions and phase advances

  For uncoupled, periodic lattices, Courant-Snyder parametrization links beta functions and phase advances with transfer matrices

$$\mathbf{M} = \begin{pmatrix} \cos\Phi + \alpha\sin\Phi & \beta\sin\Phi \\ -\gamma\sin\Phi & \cos\Phi - \alpha\sin\Phi \end{pmatrix}$$

$$\psi_{21} = \tan^{-1}\frac{M_{12}}{M_{11}\beta_1 - M_{12}\alpha_1}$$

- Moments of transverse beam distribution

  If beam is matched to the optics (e.g., in the equilibrium distribution in a ring)

$$\begin{pmatrix} \sigma_y^2 & \sigma_{yy'} \\ \sigma_{yy'} & \sigma_{y'}^2 \end{pmatrix} = \epsilon_{\mathrm{rms}} \begin{pmatrix} \bar{\beta} & -\bar{\alpha} \\ -\bar{\alpha} & \bar{\gamma} \end{pmatrix}$$

- Indirect characterization

  For example, closed-orbit response matrix in rings.

$$y_{\mathrm{co}}(s) = \frac{\theta\sqrt{\beta(s)\beta_0}}{2\sin\pi\nu}\cos(|\psi(s) - \psi_0| - \pi\nu)$$

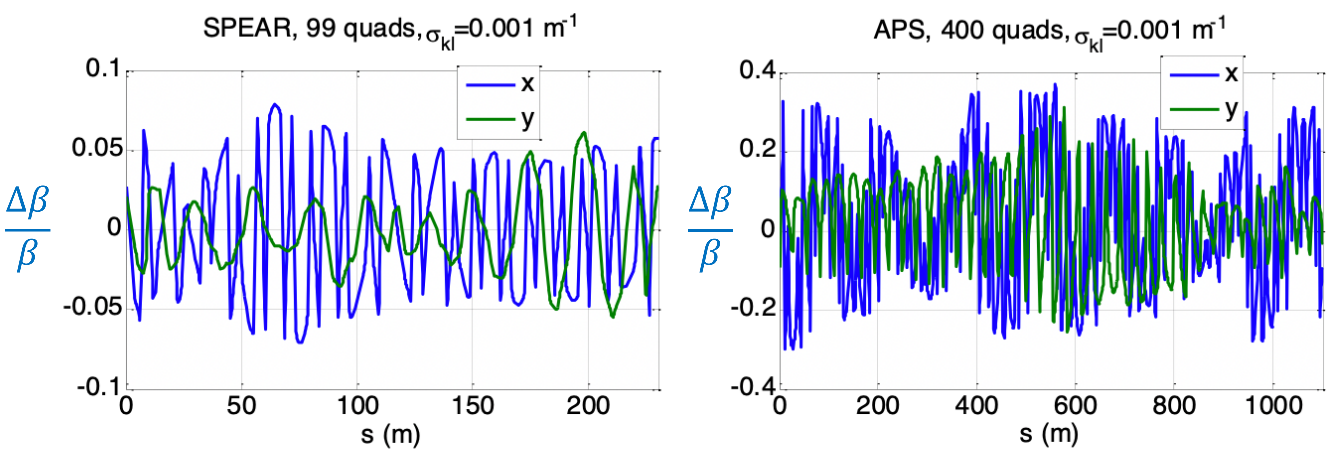
Optics errors

- Optics in an actual machine deviates from the ideal design because of
  - Errors in quadrupole magnet strengths
  - Feed-down quadrupole components from nonlinear magnets
  - Other focusing effects not accounted for in the lattice model (e.g., insertion devices, fringe fields, impedance …)

- More sources, more errors

Rms beta beat from N uncorrelated quadrupole error sources

$$< \frac{\Delta\beta}{\beta} >_{rms} = \frac{\sqrt{N}\bar{\beta} < \Delta K l >_{rms}}{2\sqrt{2}\sin 2\pi\nu}$$



Phase advance errors for the leading harmonic

$$< \Delta\psi >_{rms} = \frac{1}{2} < \frac{\Delta\beta}{\beta} >_{rms}$$

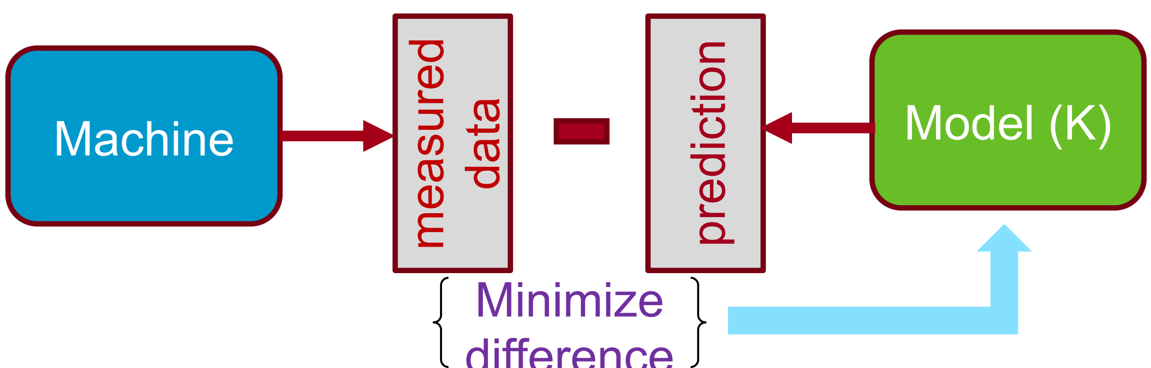
Correction of optics errors

- Optics errors can be corrected by adjusting the quadrupole strengths in the lattice
- For correction, we need
  - Measurements of optics errors (direct or indirect)
  - Quadrupole strength knobs
  - A method to determine the required knob changes from the measured errors
- Local correction – at one or a few locations
  - at injection point, interaction point, FEL undulator entrance, …
- Global correction – throughout a transport line or a ring

We'll focus on global optics correction in the following.

Global optics correction schemes

- Orbit response matrix
  - Linear optics from closed-orbit (LOCO)
- Turn-by-turn BPM orbit data
  - Determination of phase advances and beta functions
  - Or, resonance driving terms (RDT), f_{2000} and f_{0020}
  - Or, fit orbit data directly to lattice model
- Beta function measured by quadrupole modulation
  - Measure tune change due to quadrupole variation



Most schemes fit the measured optics data to the lattice model with least-square fitting.

Fitting orbit response matrix (LOCO)

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- Orbit response matrix measurement

$$\mathbf{R}_x = \frac{\mathbf{x}_+ - \mathbf{x}_-}{2\Delta\theta}, \quad \mathbf{R}_y = \frac{\mathbf{y}_+ - \mathbf{y}_-}{2\Delta\theta}$$

- Least-square fitting

$$\chi^2 = \sum_{ij} \frac{1}{\sigma_i^2}(R_{ij}^{\text{meas}} - R_{ij}^{\text{model}})^2 + \left(\frac{\alpha_c f_{\text{rf}}}{\Delta f}\right)^2 \sum_i \frac{1}{\sigma_i^2}(D_i^{\text{meas}} - D_i^{\text{model}})^2,$$

  - Usually dispersion functions are also included as fitting data.
  - BPM and corrector gains are fitted
  - Include the effects of energy shifts due to horizontal corrector kicks
  - Linear coupling are often fitted together (including cross-plane orbit response, fit skew quads and BPM and corrector rolls)

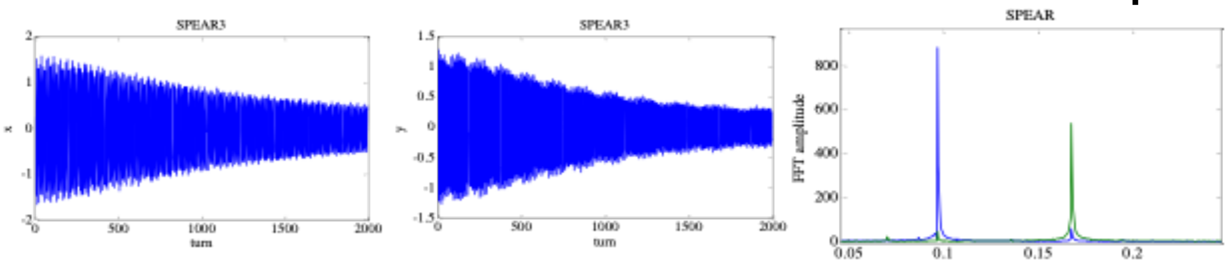
BPM gain and coupling

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} g_x & c_x \\ c_y & g_y \end{pmatrix} \begin{pmatrix} \tilde{x} \\ \tilde{y} \end{pmatrix}$$

J. Safranek, NIMA, 388(1):27 – 36, 1997
J. Safranek, G. Portmann, and A. Terebilo. Matlab-based LOCO. EPAC'02, 1184–1186, 2002.

Turn-by-turn (TbT) orbit data

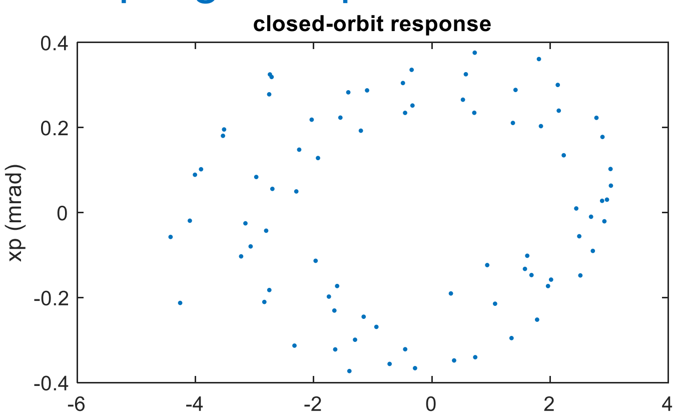
- Coherent oscillation of an 'excited' beam samples the optics

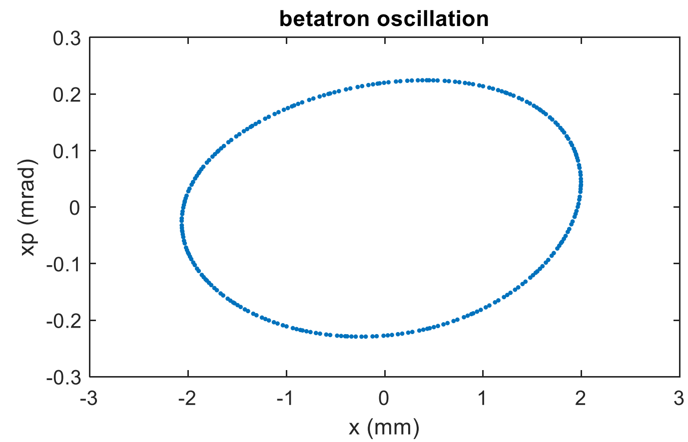


The oscillation amplitude and phase are related to the beta function and phase advance

$$x_m(t) = \sqrt{2J(t)\beta_m}\cos(\phi(t)+\psi_m)$$

Closed-orbit response and turn-by-turn BPM data are very much equivalent in sampling the optics:





Determination of phase advances from TbT orbit data

- Harmonic analysis

P. Castro, et al, PAC'93

$C = \sum_n x(n) \cos 2\pi n\nu$, $S = \sum_n x(n) \sin 2\pi n\nu$,

Then amplitude and phase are

$A = \frac{2\sqrt{C^2+S^2}}{N}$, and $\psi = -\cot^{-1}\frac{S}{C}$,

where N is the number of turns.

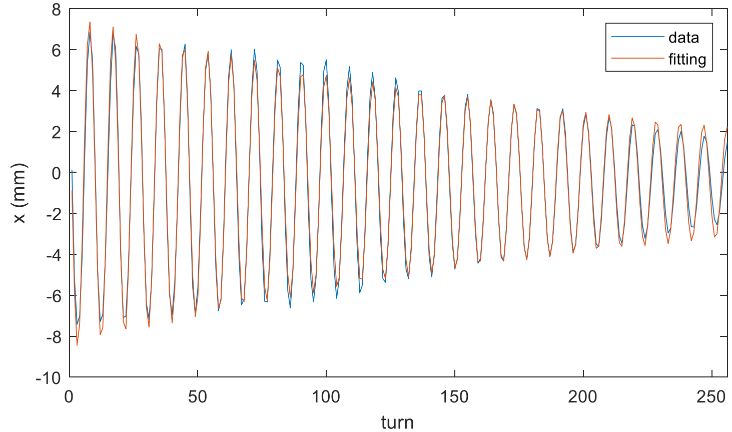
Tunes can be determined accurately with NAFF or interpolated FFT

NAFF, J. Laskar, et al,1990, Physica D, 67, 257, (1993);
Interpolated FFT, R. Bartolini, et al, EPAC 96

- Fitting in time domain

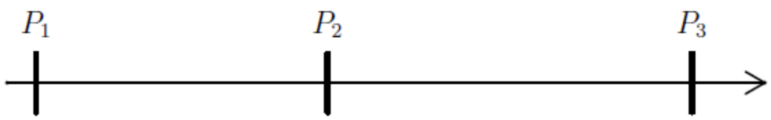
$$x(n) = x_0 + Ae^{-\alpha n} \sin(2\pi\nu n + \psi)$$

with 5 free variables, x_0, A, α, ν, ψ



The three-BPM method

- Courant-Snyder parameters can be determined from the measured phase advances and the lattice model.



Denote the transfer matrix from P_1 to P_2, $\mathbf{A} = \mathbf{M}(P_2|P_1)$, from P_1 to P_3, $\mathbf{B} = \mathbf{M}(P_3|P_1)$. We have

$$\frac{A_{11}}{A_{12}} = \frac{\cot\psi_{21} + \alpha_1}{\beta_1}, \quad \frac{B_{11}}{A_{12}} = \frac{\cot\psi_{31} + \alpha_1}{\beta_1},$$

where ψ_{21} and ψ_{31} are phase advances from P_1 to P_2 and P_3, respectively, and α_1 and β_1 are C-S parameters at P_1.

P. Castro, et al, PAC'93

Hence
$$\beta_1|_{\text{meas}} = \beta_1|_{\text{model}} \frac{\cot\psi_{31} - \cot\psi_{21}|_{\text{meas}}}{\cot\psi_{31} - \cot\psi_{21}|_{\text{model}}}$$

and similarly for the α parameter.

This method has been extended to use additional neighboring BPMs (the N-BPM method). A. Langer, R. Tomás, PRSTAB 18, 031002 (2015)

Model independent analysis (MIA) and independent component analysis (ICA)

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- The TbT data at different BPMs are observations of the same physical processes (source signals, e.g., betatron oscillations).

$$x_i(t) = \sum_j a_{ij} s_j(t) + n_j(t)$$ for the i'th BPM

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t)$$ $\mathbf{A}$ is the mixing matrix

- Collecting TbT data from all BPMs in a matrix, principal component analysis (PCA, used for MIA) and ICA can be used to separate the source signals

$$\mathbf{x} = \begin{pmatrix} x_1(1) & x_1(2) & \cdots & x_1(T) \\ x_2(1) & x_2(2) & \cdots & x_2(T) \\ \vdots & \vdots & \ddots & \vdots \\ x_m(1) & x_m(2) & \cdots & x_m(T) \end{pmatrix}$$

MIA uses SVD for signal separation:
J. Irwin, et al, PRL 82, 1684, (1999);
Chun-xi Wang, et al. PR-STAB 6, 104001 (2003).

ICA uses simultaneous diagonalization of time-shifted covariance matrices:
X. Huang, et al, PRSTAB, 8, 064001, 2005

PCA is based on the uncorrelatedness of the source signals, while ICA takes advantages of other features of the independent source signals (e.g., non-overlapping of the frequency spectrum, or non-Gaussianity)

Phase advance and beta function from MIA/ICA modes

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- Phase advances and beta functions are obtained from the spatial vectors.

Uncoupled betatron motion is decomposed into two orthogonal modes:

$$\mathbf{x} = \mathbf{U}\mathbf{S}\mathbf{V}^{\mathrm{T}} = s_+ \mathbf{u}_+ \mathbf{v}_+^{\mathrm{T}} + s_- \mathbf{u}_- \mathbf{v}_-^{\mathrm{T}}$$

Chun-xi Wang, et al. PR-STAB 6, 104001 (2003).

u: spatial vector

$$u_{+,m} = \frac{1}{s_+}\sqrt{<J>\beta_m}\cos(\phi_0 + \psi_m),$$

$$u_{-,m} = \frac{1}{s_-}\sqrt{<J>\beta_m}\sin(\phi_0 + \psi_m)$$

v: temporal vector

$$v_+(t) = \sqrt{\frac{2J(t)}{T<J>}}\cos(\phi(t) - \phi_0),$$

$$v_-(t) = -\sqrt{\frac{2J(t)}{T<J>}}\sin(\phi(t) - \phi_0)$$

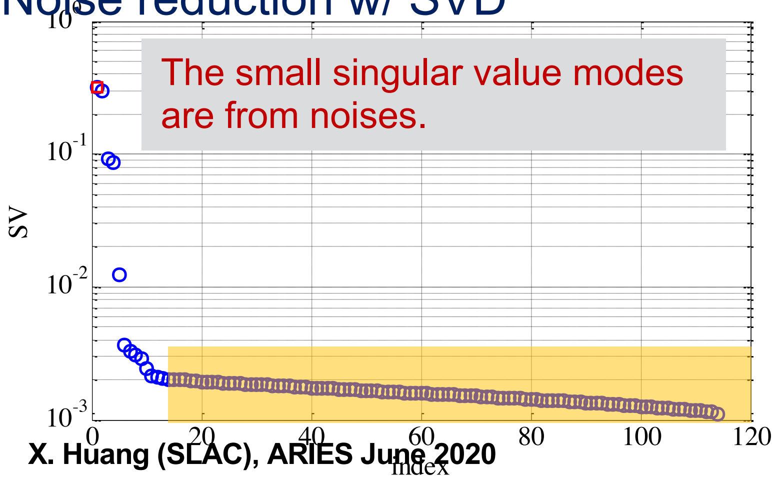
Betatron phase advance (at BPM m)

$$\psi_m = \tan^{-1}\left(\frac{s_- u_{-,m}}{s_+ u_{+,m}}\right)$$

Beta function (at BPM m)

$$\beta_m = \frac{1}{<J>}[(s_+ u_{+,m})^2 + (s_- u_{-,m})^2]$$

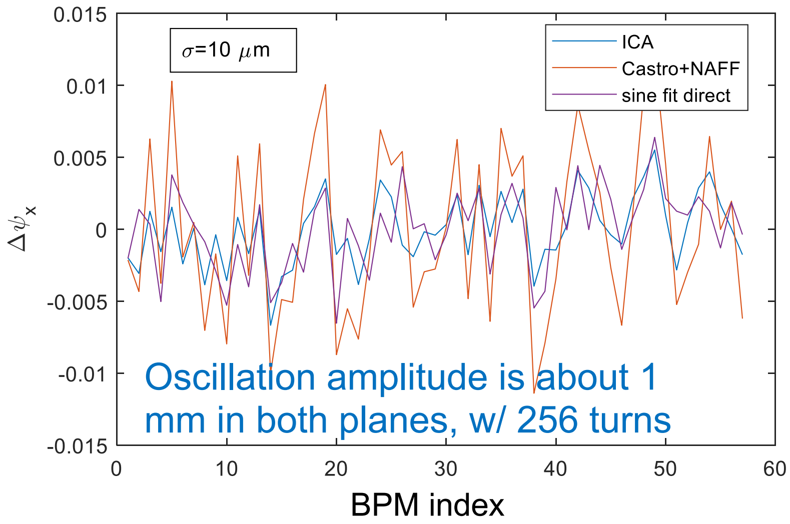
Noise reduction w/ SVD

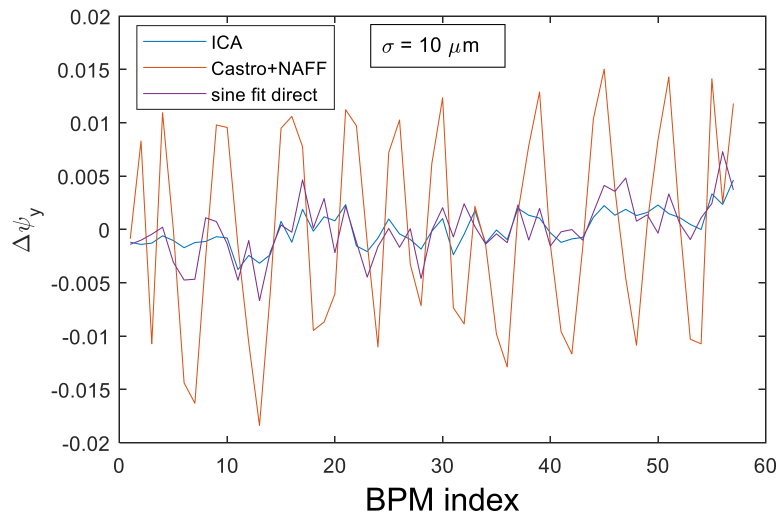


- PCA fails to decouple the source signals in case of generate modes, due to equal variance in betatron/synchrotron modes or bad/contaminated BPMs
- ICA can separate betatron, synchrotron modes despite of bad BPMs and can help identify bad BPMs

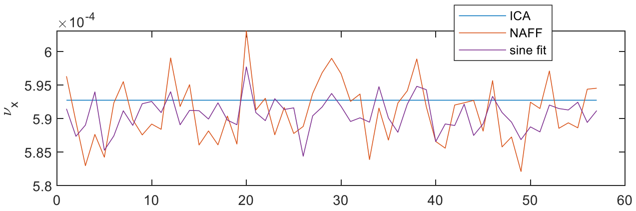
Comparison of accuracy in phase advance determination

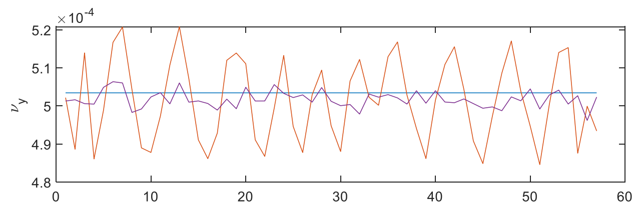
- Simulation shows that ICA and time-domain fit give better accuracy than harmonic analysis in phase determination





- The finite number of turns has an impact to the phase advance determination for the harmonic analysis.
- Treating TbT data in all BPMs as common modes as in MIA/ICA is beneficial.



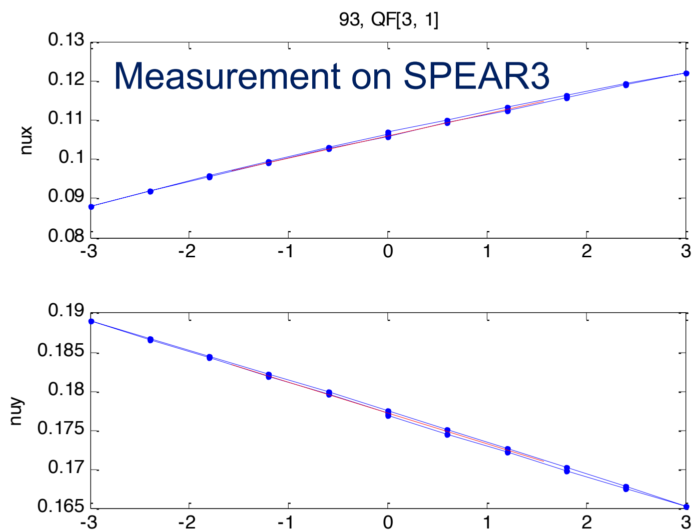


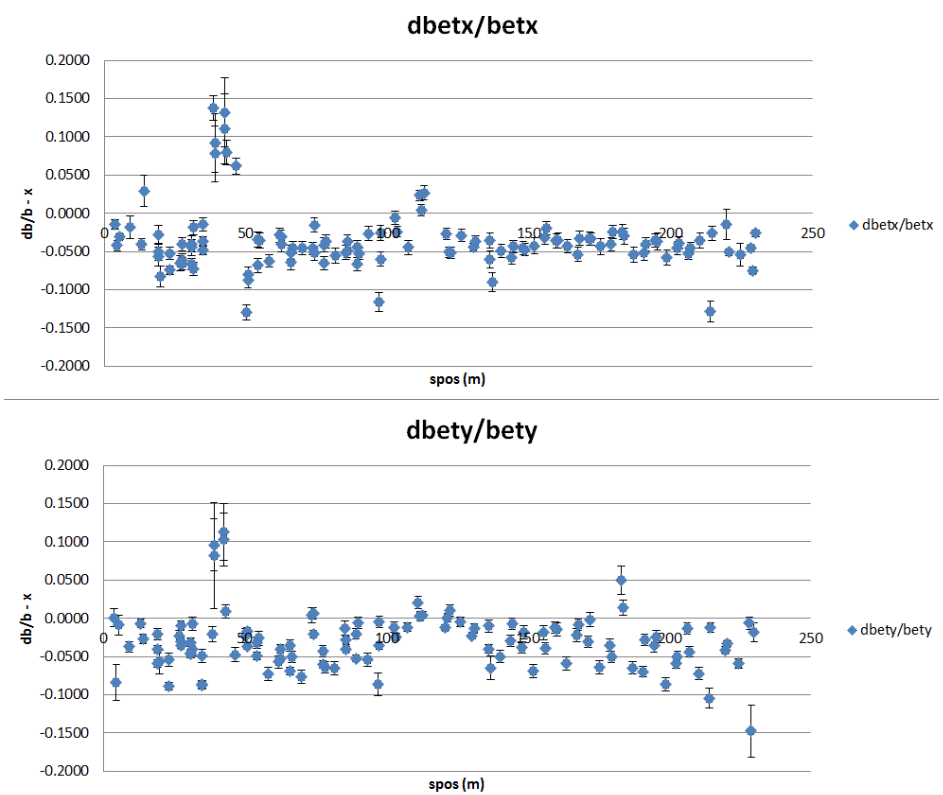
X. Huang, Beam-based correction and optimization for accelerators, CRC Press, (2019)

Beta function measurement w/ quadrupole modulation

- Tune change due to quadrupole modulation can be used to measure beta function at the location of the quadrupole

$$\Delta\nu = \frac{1}{4\pi}[\Delta KL]\beta \quad \rightarrow \quad \beta = \frac{4\pi\Delta\nu}{[\Delta KL]}$$





Measured average beta was found to be lower than design values at most locations, possibly due to systematic errors.
(negative offsets also observed by Aiba et al)

M. Aiba, et al, PRSTAB, 16, 012803 (2013)

Fitting lattice with measured optics functions

- The measured optics functions can be used for fitting

$$f(\mathbf{q}) = \chi^2 = \frac{1}{2}\sum_{i=1}^{240} r_i^2, \qquad r_i = \frac{y_i(\mathbf{q}) - y_i^d}{\sigma_i},$$

$$\mathbf{y} = (w_1\boldsymbol{\beta}_x, w_2\Delta\psi_x, w_3\boldsymbol{\beta}_z, w_4\boldsymbol{\Delta\psi}_z, w_5\mathbf{D}_x),$$

X. Huang, et al, PRSTAB, 8, 064001 (2005)

Many choose not to include beta functions in lattice fitting to be unaffected by BPM calibration errors.

This is later extended to include linear coupling and demonstrated on NSLS-II. BPM gains and rolls can be fitted along with quadrupoles.

X. Yang, X. Huang, NIMA, 828, 97, (2016)

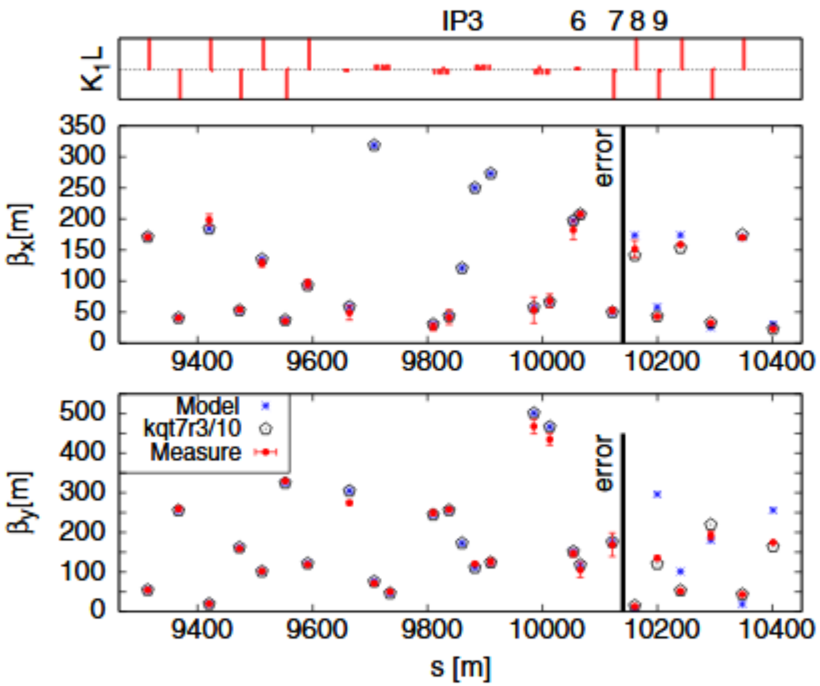
Fitted lattice parameters by ICA and LOCO before and after corrections.

Parameters	Before		After	
	ICA	LOCO	ICA	LOCO
rms $\Delta\beta_x/\beta_x$	0.0678	0.0780	0.0051	0.0194
rms $\Delta\beta_y/\beta_y$	0.0937	0.0991	0.0038	0.0110
rms ΔD_x	0.0167	0.0227	0.0056	0.0045
rms ΔD_y	0.0080	0.0085	0.0030	0.0046
Mean ϵ_y/ϵ_x	0.0147	0.0129	0.0027	0.0031

LOCO data taken after correction in this table were contaminated by a closed-orbit jump during measurement.

Segment-by-segment technique (SBST) for optics correction

- For LHC, the global correction schemes based on response matrix (orbit or optics functions) were considered not practical because of the size of the system, thus the SBST was proposed.
- The alpha- and beta- functions at one location are propagated within the segment and compared to the model.



Initial optics measurement and correction for LHC.

Example: IR3 region

M. Aiba, et al, PRSTAB, 12,081002 (2009);

This method has been applied to electron storage rings (ALBA, ESRF).

Resonance driving terms and optics correction

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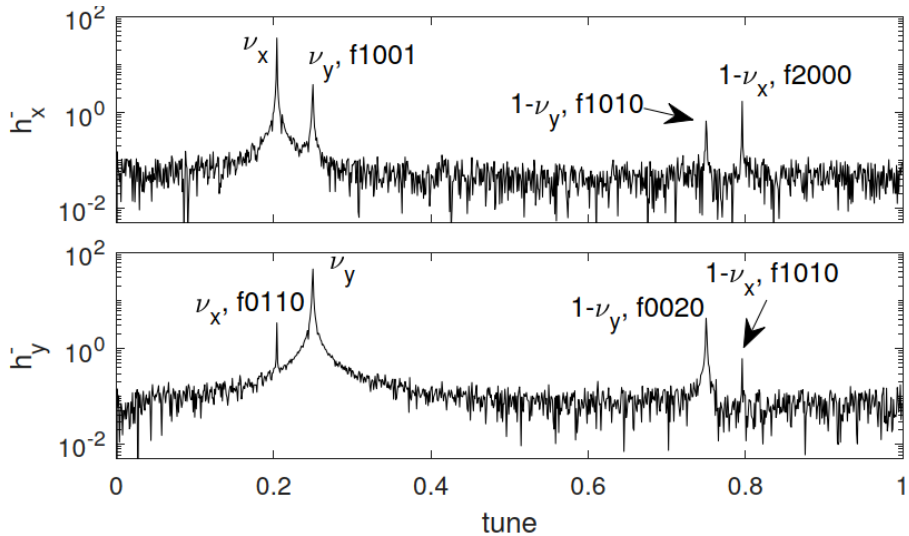
- Normal quadrupole errors are associated with specific RDTs

$$f_{2000} = \frac{\sum_k \Delta b_{1,k} L_k \beta_{x,k} e^{i2\psi_{x,k}}}{8(1-e^{i2\pi\nu_x})}, \quad f_{0020} = -\frac{\sum_k \Delta b_{1,k} L_k \beta_{y,k} e^{i2\psi_{y,k}}}{8(1-e^{i2\pi\nu_y})},$$



R. Bartolini, F. Schmidt, Part. Accel., 59:93–106, 1997

$$h_x^{\pm} = \bar{x} \pm i\bar{p}_x = \sqrt{2J_x} e^{\mp i\phi_x}$$

Figure taken from X. Huang, BBC&O, CRC Press, (2019)

- Optics correction can be done by minimizing the RDTs

$$\begin{pmatrix} \vec{f}_{2000} \\ \vec{f}_{0020} \end{pmatrix}_{\text{meas}} = -\mathbf{N}\vec{K}_c,$$

K_c vector are integrated quadrupole strength. $\mathbf{N}$ is the RDT response matrix w.r.t. the quadrupoles

A. Franchi, et al, PRSTAB, 14,034002 (2011)

Similar work was also done at NSLS-II. Y. Hidaka, et al, NAPAC 2016

Fitting TbT data directly to tracking data

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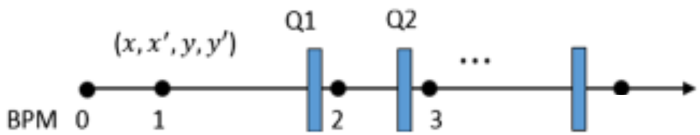
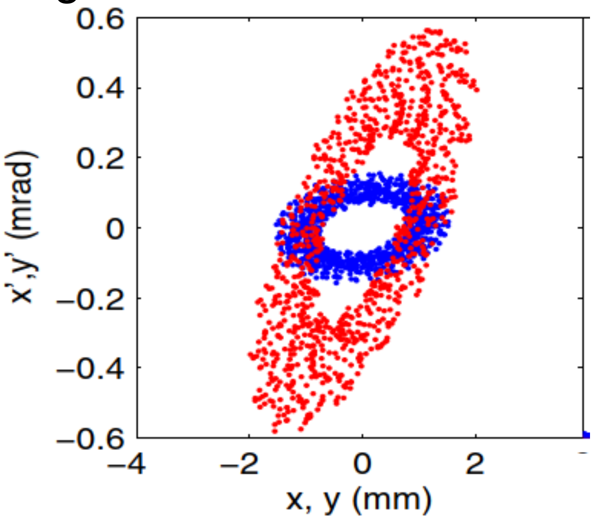
- Calculating the angle coordinates from position coordinates on two adjacent BPMs, one can predict the orbit by tracking.
  - Two BPMs separated by a drift is ideal for this purpose.
- The lattice can be fitted with the TbT orbit data directly.
  - Can fit BPM gains and rolls, and linear coupling



$$\chi^2 = \sum_{n=1}^{N} \sum_{i=2}^{M+1} \left[\left(\frac{x_i(n) - \tilde{x}_i[\mathbf{p}; \mathbf{X}_1(\mathbf{n})]}{\sigma_{xi}} \right)^2 + \left(\frac{y_i(n) - \tilde{y}_i[\mathbf{p}; \mathbf{X}_1(\mathbf{n})]}{\sigma_{yi}} \right)^2 \right],$$

X. Huang, et al, PRSTAB 13, 114002 (2010)

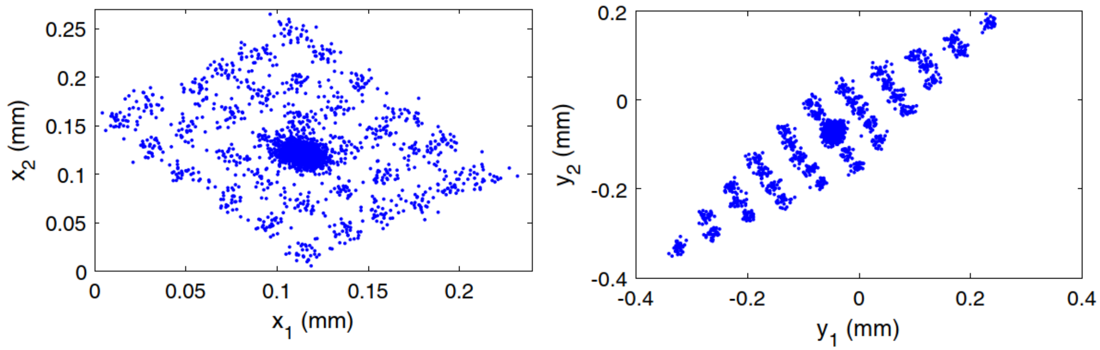
Demonstrated on a section of SPEAR3 and verified with NSLS-II data (see X. Huang, BBC&O).



- It is possible to fit the x' and y' by tracking multiple turns.

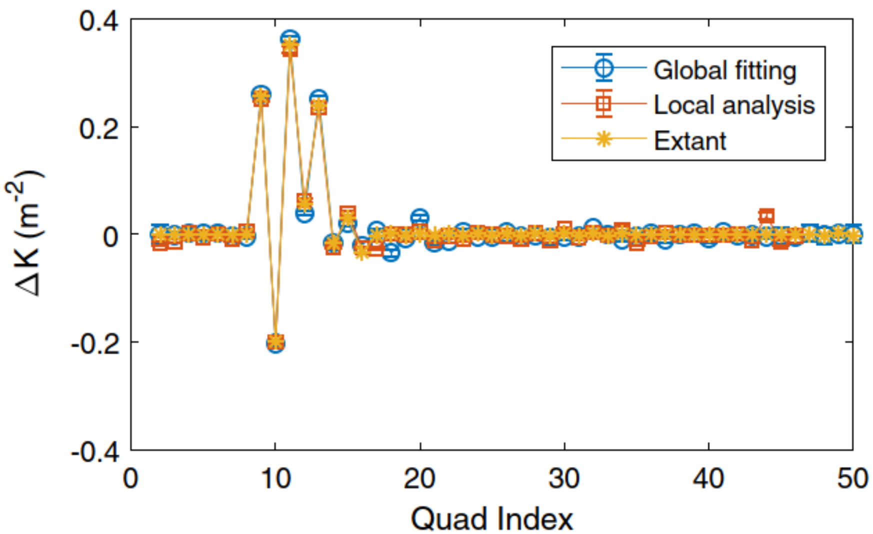
Optics correction for one-pass systems

- The direct fitting method is applicable to one-pass systems (e.g. linac)

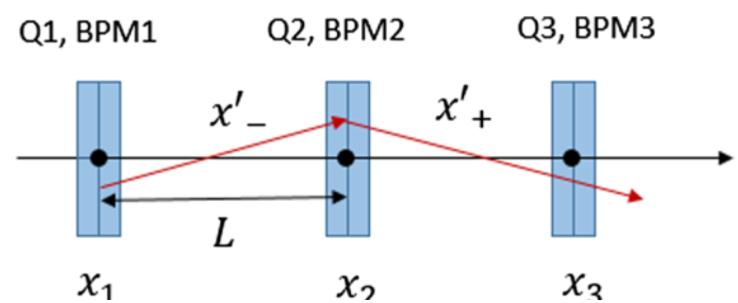


LCLS trajectory grid scan

Lattice fitting for LCLS Linac section L3.



A special case for quadrupole error determination: one BPM for one nearby quad



$$\Delta x_2' \equiv x_+' - x_-' = [KL_q]_2 x_2$$

T. Zhang, et al, PRAB, 21, 092801 (2018)

Solving the least-square fitting problem

- Optics correction is typically done by first fitting the lattice
- It is a least-square problem: multiple knobs, multiple targets

The linearized problem

$$\mathbf{b}(\mathbf{p}) \approx \mathbf{b}_0 + \mathbf{J}\Delta\mathbf{p} = \mathbf{b}_\mathbf{t}$$

is solved, with target vector $\mathbf{b}_t$, present observation $\mathbf{b}_0$, desired knob change$\boldsymbol{\Delta}\mathbf{p} = \mathbf{p} - \mathbf{p}_\mathbf{0}$, and $\mathbf{J}$ is the Jacobian matrix (or response matrix).

Equivalent least-square problem

$$f(\mathbf{p}) = \chi^2 = \mathbf{r}^T\mathbf{r}$$

with residual vector $\mathbf{r} = \mathbf{b}(\mathbf{p})$-$\mathbf{b}_\mathbf{t}$. Current residual vector $\mathbf{r}_0 = \mathbf{b}_0 - \mathbf{b}_\mathbf{t}$.

Solution can be found iteratively, with (the Gauss-Newton method):

$$\Delta\mathbf{p} = -(\mathbf{J}^T\mathbf{J})^{-1}\mathbf{J}^T\mathbf{r}_0$$

Problem is solved if matrix $\mathbf{J}^T\mathbf{J}$ can be inverted

$$\Delta\mathbf{p} = -\Sigma_i \frac{1}{s_i}\mathbf{v}_i(\mathbf{u}_i^T\mathbf{r}_0), \text{ with } \mathbf{J} = \mathbf{USV}^T = \Sigma_i s_i \mathbf{u}_i \mathbf{v}_i^T.$$

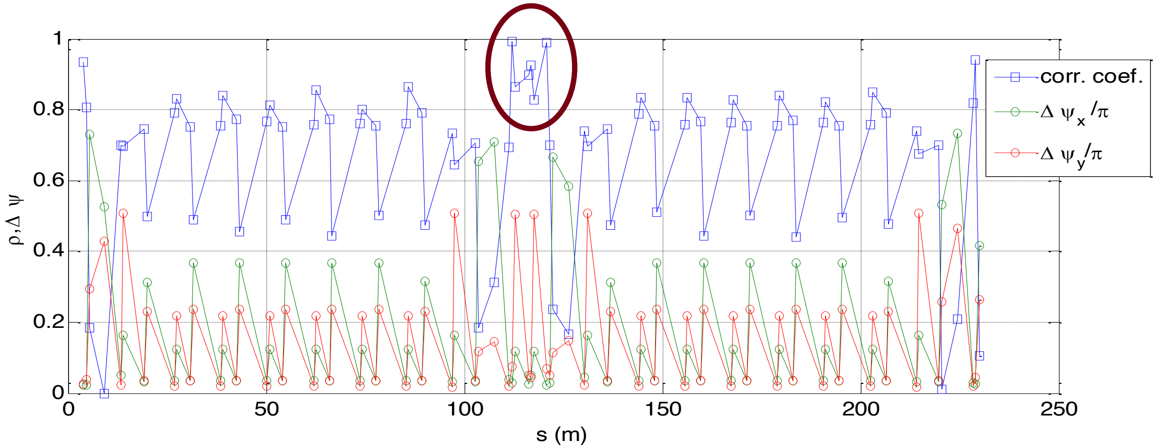
Overfitting due to near degeneracy

- The fitting problem is near degenerate when two variables have similar effect to the measurements

Columns in the Jacobian matrix are similar ↔ Jacobian matrix has small singular values

Adjacent quadrupoles w/ small phase advances in between tend to cause degeneracy.

- When there are small SVs, errors in the residual vector and the Jacobian matrix cause large spurious excursion on $\Delta \mathbf{p}$



Cutting off SVs or eliminating fitting variables are often the remedies. But they are hard measures that reduce accuracy in optics control.

X. Huang, et al, ICFA Newsletter, 44 (2007)

Fitting with constraints

- Fitting with constraints helps alleviate overfitting

$$\chi_c^2 = \sum_{ij} \frac{1}{\sigma_{ij}^2}(R_{ij}^{\rm meas} - R_{ij}^{\rm model})^2 + \frac{1}{\sigma_K^2}\sum_{i=1}^{N_q} w_i^2 \Delta K_i^2$$

Constraints for quadrupole variables ΔK

$$\Delta \mathbf{p} = -(\mathbf{J}^T\mathbf{J} + \boldsymbol{W}^T\boldsymbol{W})^{-1}\mathbf{J}^T\mathbf{r}_0 \qquad W_{ii} = \frac{w_i}{\sigma_K}$$

- Constraints can be implemented with Levenberg-Marquardt method

$$\Delta \mathbf{p} = -(\mathbf{J^TJ} + \lambda \mathrm{diag}(\mathbf{J^TJ}))^{-1}\mathbf{J^T r_0}$$

scaled L-M method with a finite λ

- Constraints can be added to discourage certain patterns in $\Delta \mathbf{p}$
  - SVD patterns with small singular patterns are natural choices

$$\chi_c^2 = \chi^2 + \sum^P \lambda_i^2 (\mathbf{v}_i^T \Delta \mathbf{p})^2$$

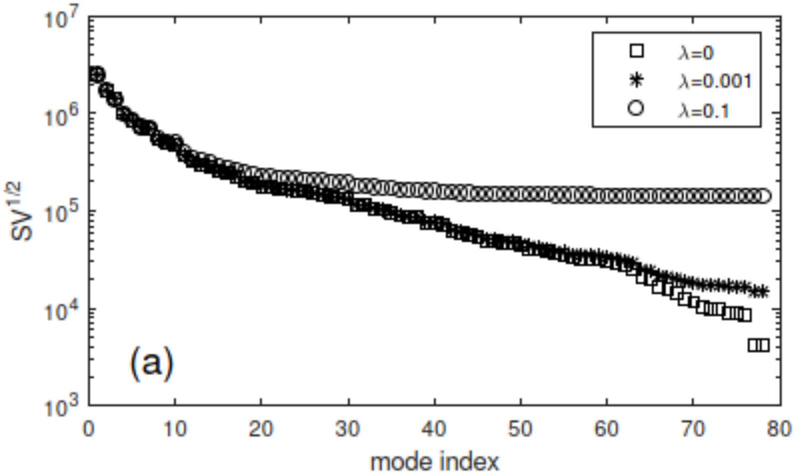
corresponding to $\mathbf{W} = \boldsymbol{\Lambda V}$ with $\Lambda_{ii} = \lambda_i$

$$\Delta \mathbf{p} = -\mathbf{V}(\mathbf{S}^2 + \boldsymbol{\Lambda}^2)^{-1}\mathbf{S}\mathbf{U}^T\mathbf{r}_0 = -\sum_{i=1}^{P} \frac{s_i}{s_i^2 + \lambda_i^2}\mathbf{v}_i(\mathbf{u}_i^T \mathbf{r}_0)$$

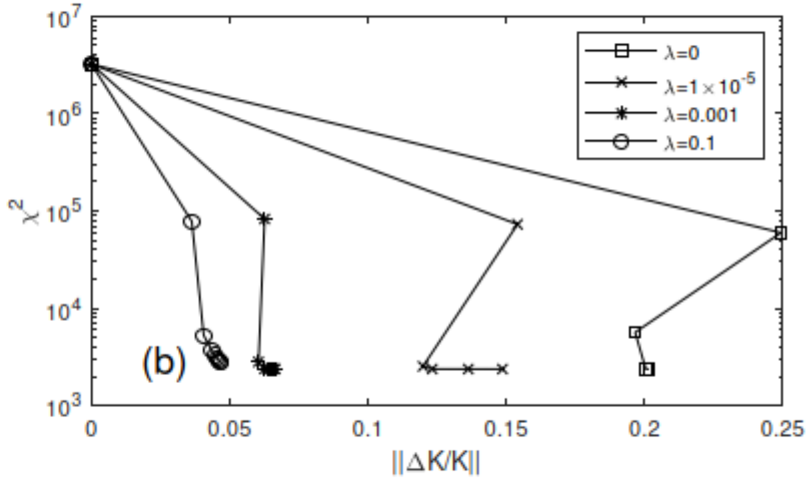
X. Huang, et al, PAC 05 (2005); X. Huang, et al, ICFA Newsletter, 44 (2007); X. Huang, BBC&O, (2019)

Benefits of constrained fitting

- Constrained fitting makes optics correction possible or results in better optics accuracy



Modification of SV spectrum by constraints
(scaled L-M method)



Same level of χ^2, but much less quadrupole variation, when constraints are introduced.

X. Huang, BBC&O, CRC Press (2019)

Comparison of optics correction methods

- The NSLS-II study

V. Smaluk, et al, IPAC 2016

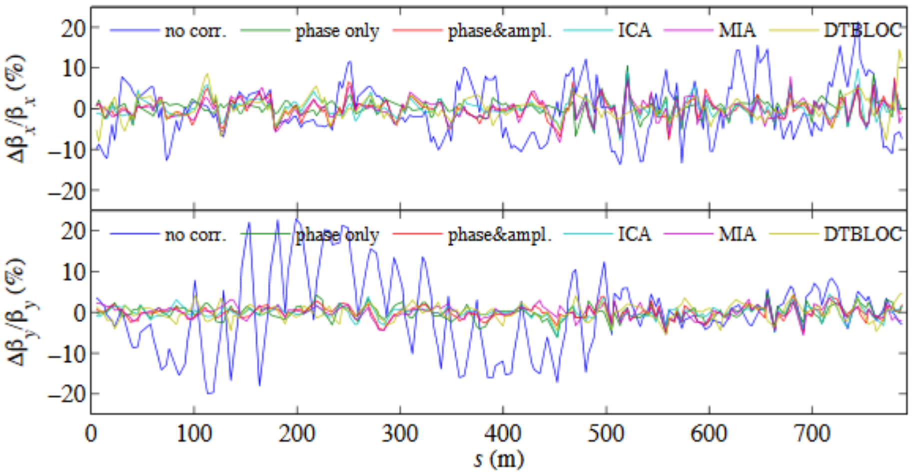


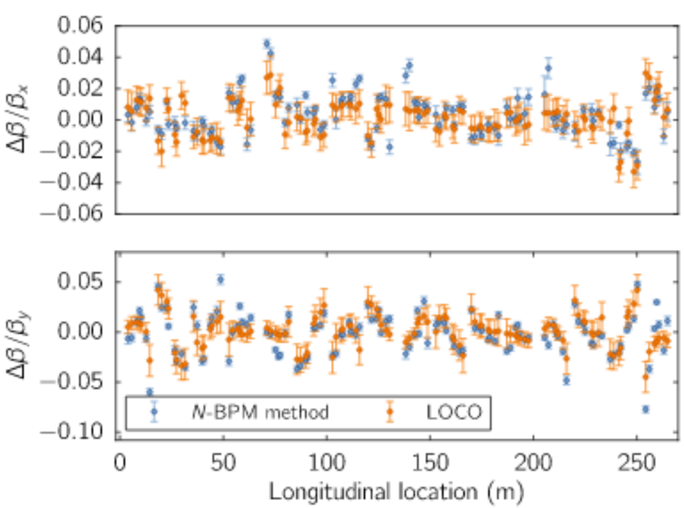
Table 1: Residual Errors

Algorithm	$\Delta\beta_x/\beta_x$ %	$\Delta\beta_y/\beta_y$ %	$\Delta\psi_x$ °	$\Delta\psi_y$ °	$\Delta\eta_x$ mm	η_y mm
no corr.	8	10	4.5	3.5	18	8
LOCO	2.1	1.4	0.5	0.2	2.6	4.4
phase only[1]	2.3	1.8	0.6	0.5	39	9.9
phase&amp.[1]	2.8	1.7	0.7	0.9	11	7.8
ICA	2.6	1.6	0.5	0.4	5.0	2.3
MIA	2.8	1.7	0.7	1.0	5.4	6.8
DTBLOC	3.0	1.9	0.4	0.8	2.3	4.5

[1] no dispersion corrected

- The ALBA study

A. Langner et al, PRAB 19, 092803 (2016)



Method vs. Nominal model	rms β-beating (%) Horizontal	Vertical
N-BPM (phase)	1.4	2.0
From amplitude	2.0	2.7
LOCO	1.1	1.6
Method 1 vs. Method 2		
N-BPM (phase) vs. LOCO	1.0	1.3
N-BPM (phase) vs. amplitude	1.7	1.9
From amplitude vs. LOCO	1.4	1.7
***N*-BPM using LOCO model**		
N-BPM (phase) vs. LOCO	0.8	1.1

Coupling correction

- LOCO correct coupling by fitting skew quads for the off-diagonal orbit responses

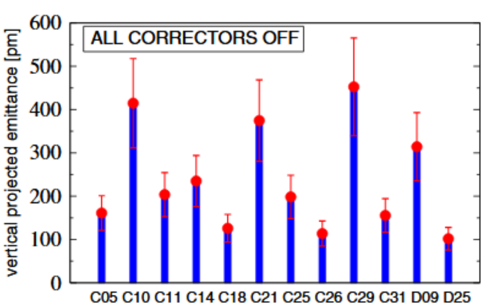
J. Safranek, NIMA, 388(1):27 – 36, 1997
J. Safranek, G. Portmann, and A. Terebilo. Matlab-based LOCO. EPAC'02, 1184–1186, 2002.

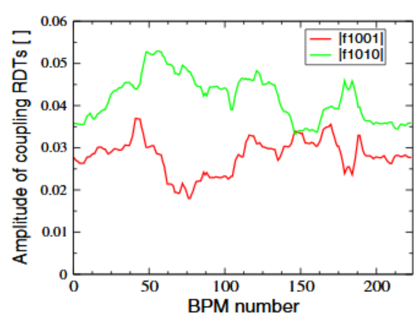
- RDTs f_{1001} and f_{1010} from TbT data

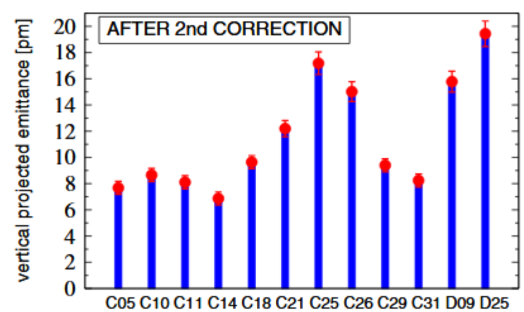
A. Franchi, et al, PRSTAB 14, 034002 (2011)

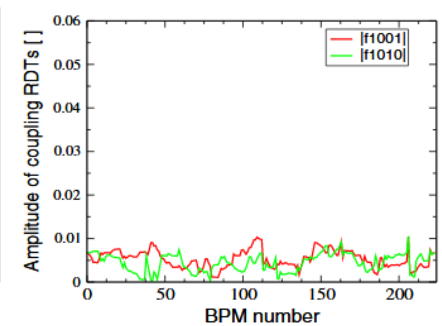
$$f(s)_{\substack{1001\\1010}} = -\frac{1}{4(1-e^{2\pi i(Q_x \mp Q_y)})}\sum_l k_l \sqrt{\beta_x^l \beta_y^l}\, e^{i(\Delta\phi_x^{sl} \mp \Delta\phi_y^{sl})}$$

$$\begin{pmatrix} a_1 \vec{f}_{1001} \\ a_1 \vec{f}_{1010} \\ a_2 \vec{D}_y \end{pmatrix}_{\text{meas}} = -\mathbf{M}\vec{J}_c,$$









Coupling correction - continued

- Amplitude and phase of the ICA modes

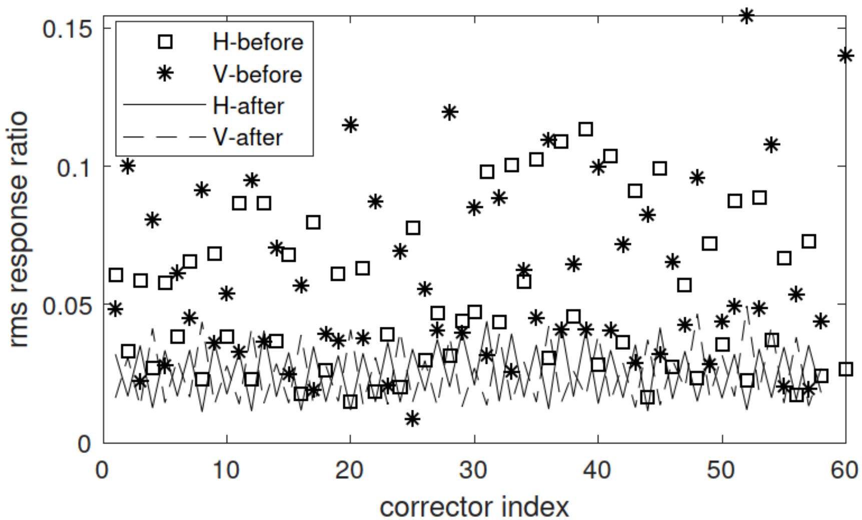
Model prediction

$$\mathbf{X}(n) = \mathbf{P}\mathbf{\Theta}(n), \quad \text{with } \mathbf{\Theta}(n) = \begin{pmatrix} \sqrt{2J_1}\cos\Phi_1(n) \\ -\sqrt{2J_1}\sin\Phi_1(n) \\ \sqrt{2J_2}\cos\Phi_2(n) \\ -\sqrt{2J_2}\sin\Phi_2(n) \end{pmatrix}$$

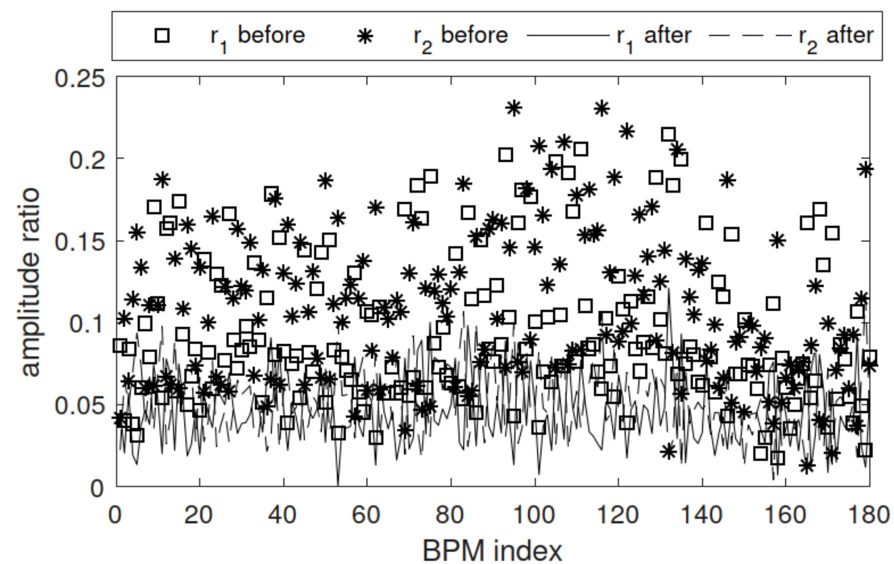
ICA decomposition of measured TbT data

$$x_n = A\cos\Psi_{1n} - B\sin\Psi_{1n} + c\cos\Psi_{2n} - d\sin\Psi_{2n},$$
$$y_n = a\cos\Psi_{1n} - b\sin\Psi_{1n} + C\cos\Psi_{2n} - D\sin\Psi_{2n},$$

X. Yang, X. Huang, NIMA, 828, 97, (2016)



X. Huang, BBC&O, CRC Press (2019)



$$r_1 = \frac{\sqrt{a^2 + b^2}}{\sqrt{A^2 + B^2}} = \frac{\sqrt{p_{31}^2 + p_{32}^2}}{p_{11}},$$

$$r_2 = \frac{\sqrt{c^2 + d^2}}{\sqrt{C^2 + D^2}} = \frac{\sqrt{p_{13}^2 + p_{14}^2}}{p_{33}}$$

- Fitting TbT orbit data directly to model.
  - Applicable to linacs

Summary

- BPM data can effectively sample linear optics errors
  - Closed orbit response
  - Turn-by-turn orbit data with coherent betatron oscillations
- Fitting data to the lattice model can find the quadrupole errors for optics correction
  - Fitting orbit response matrix
  - Fitting beta functions and phase advances derived from TbT BPM data
  - Fitting turn-by-turn data directly
  - Fitting RDTs
- Near degeneracy is a common issue. The constrained fitting approach helps find solutions applicable for correction and improve optics correction accuracy.