## The HIP Alignment Method

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## Outline

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## Introduction

- Develop an algorithm that can be used for detector alignment esp. planar sensors (MVD, LMD,...)
- Find the correct position of the detector (or its individual component) by calculating the 6 parameters (3 for the translation and 3 the rotation) of the transformation from the GCS to LCS.
- This can be achieved by using the "Hits and Impact Points (HIP) Alignment Method".


## Basic Formalism of the HIP Alignment Method

It is consisting on finding the correct transformation by minimizing of the hit residuals $q_{m}\left(u_{m}, v_{m}, 0\right)$ of the measured point and $q_{x}\left(u_{x}, v_{x}, 0\right)$ of the track impact point in LCS ( $\hat{W}$, the $3^{\text {rd }}$ component unit vector, is perpendicular to the detector plane).

$$
\begin{aligned}
& \chi^{2}=\sum_{j}^{\text {Ntrack }} \varepsilon_{j}^{T} V_{j}^{-1} \varepsilon_{j} \\
& \text { where: } \quad \varepsilon=\binom{\varepsilon_{u}}{\varepsilon_{v}}=\binom{u_{x}-u_{m}}{v_{x}-v_{m}}
\end{aligned}
$$

$V_{j}$ is the sum of measured and impact point positions covariance matrices of track $j$.

## The Alignment Parameters

The correct total transformation from the GCS to the LCS:

$$
\begin{aligned}
& q=\Delta R R\left(r-r_{0}\right)-\Delta q \\
& \text { where: } \boldsymbol{q}(\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w}) \text { : point coordinates in LCS, } \\
& r(x, y, z) \text { : point coordinates in GCS, } \\
& r_{0}\left(x_{0}, y_{0}, z_{0}\right) \text { : origin coordinates of LCS in GCS, } \\
& R=R_{\alpha} R_{\beta} R_{\gamma} \quad \text { : (ideal) rotation matrix, } \\
& \Delta R=R_{\Delta \alpha} R_{\Delta \beta} R_{\Delta \gamma} \text { :corrective rotation matrix, } \\
& \Delta \boldsymbol{q}=\Delta R R \Delta r=(\Delta u, \Delta v, \Delta w): \text { corrective translation } .
\end{aligned}
$$

$p=(\Delta u, \Delta v, \Delta w, \Delta \boldsymbol{\alpha}, \Delta \boldsymbol{\beta}, \Delta \boldsymbol{\gamma})$ is the alignment parameter.

The alignment corrections are expected to be small, the fitted trajectories can be approximated with a straight line in a vicinity of the detector plane. The line equation is:

$$
r_{s}=r_{x}+h \hat{s}
$$

As a consequence, $q_{s}(h)=\Delta R R\left(r_{x}+h \hat{s}-r_{0}\right)-\Delta q$

$$
\begin{aligned}
& r_{x} \text { is the trajectory impact point on the detector, } \\
& \hat{s} \text { is the unit vector parallel to the line, } \\
& h \text { is a parameter which satisfies } q_{s}\left(h_{x}\right) \cdot \hat{w}=0
\end{aligned}
$$

Finally:

$$
q_{x}^{c}=\Delta R q_{x}+\left(\Delta w-\left[\Delta R q_{x}\right]_{3}\right) \cdot \frac{\Delta R \hat{t}}{[\Delta R \hat{t}]_{3}}-\Delta q
$$

" C " indicates the corrected position after the alignment,
" 3 " indicates the $3^{\text {rd }}$ component,
$\hat{t}=R \hat{s} \quad$ is the uncorrected trajectory direction in LCS

## Residuals Expressions

The matrix rotation following the $z x z$ convention van be written as:

$$
\Delta R=\left(\begin{array}{ccc}
1 & \Delta \gamma+\Delta \alpha & 0 \\
-\Delta \gamma-\Delta \alpha & 1 & \Delta \beta \\
0 & -\Delta \beta & 1
\end{array}\right)
$$

Therefore:

$$
\begin{gathered}
\varepsilon_{u}=u_{x}-\Delta u+(\Delta \gamma+\Delta \alpha) v_{x}+\left(\Delta w+\Delta \beta v_{x}\right) \tan \psi-u_{m} \\
\varepsilon_{v}=v_{x}-\Delta v-(\Delta \gamma+\Delta \alpha) u_{x}+\left(\Delta w+\Delta \beta v_{x}\right) \tan \theta-v_{m} \\
\text { where }: \tan \theta=\hat{t}_{l} \mid \hat{t}_{3} \quad(\theta \text { is the angle between uw-plane and the track }) \\
\tan \psi=\hat{t}_{1} \mid \hat{t}_{3} \quad(\psi \text { is the angle between } v w-\text { plane and the track })
\end{gathered}
$$

## Simulation Studies



Initial geometry: 4 layers of Silicon Strip Sensors parallel to $x y-p l a n e$ and have axis the $z$-axis
geometry modified : at the $4^{\text {th }}$ plane
$\rightarrow$ x-shift: -0.0284 cm
$\rightarrow y$-shift: 0.0279 cm
$\rightarrow$ z-shift: 0.0 cm
$\rightarrow \phi=50 \mathrm{mrad}$
$\rightarrow \theta=20 \mathrm{mrad}$
$\rightarrow \psi=25 \mathrm{mrad}$
100000 proton tracks at $1000 \mathrm{GeV} / \mathrm{c}$ (to minimize the effect of scattering) are generated. So,

$$
\begin{aligned}
& V_{j}=V=\left(\begin{array}{cc}
\sigma_{x}^{2}+\sigma_{m}^{2} & 0 \\
0 & \sigma_{x}^{2}+\sigma_{m}^{2}
\end{array}\right)=\left(\begin{array}{cc}
\sigma_{m}^{2} & 0 \\
0 & \sigma_{m}^{2}
\end{array}\right) \\
& \sigma_{m}=\text { pitch } / \sqrt{12} \quad \text { : error from the digitization }
\end{aligned}
$$

Result

|  | Displacement |
| :--- | :--- |
| $\Delta x(\mathrm{~cm})$ | -0.0284 |
| $\Delta y(\mathrm{~cm})$ | 0.0279 |
| $\phi(\mathrm{mrad})$ | 50 |
| $\theta(\mathrm{mrad})$ | 20 |
| $\psi(\mathrm{mrad})$ | 25 |


| Params | Values |
| :--- | :--- |
| $\Delta \mathrm{u}(\mathrm{cm})$ | $0.0277(6.87 \mathrm{e}-06)$ |
| $\Delta \mathrm{v}(\mathrm{cm})$ | $-0.0274(6.80 \mathrm{e}-06)$ |
| $\Delta \alpha(\mathrm{mrad})$ | $-49.99(3.63 \mathrm{e}-04)$ |
| $\Delta \beta(\mathrm{mrad})$ | $-19.32(1.37 \mathrm{e}-03)$ |
| $\Delta \mathrm{y}(\mathrm{mrad})$ | $-22.62(1.54 \mathrm{e}-04)$ |

## Residuals

black: before alignment blue: after alignment


## Outlook

- Use the algorithm for experimental data (Bonn TS beam test at COSY) : test algorithm for more than one plane.
- Calcul of the covariante matrix for relative lower beam momenta

