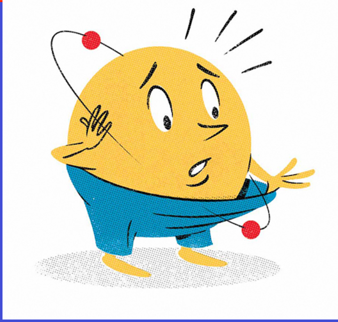


Proton Charge Radius from electron scattering… Is this meaningful?

Egle Tomasi-Gustafsson

CEA, IRFU, DPhN et Université Paris-Saclay, France

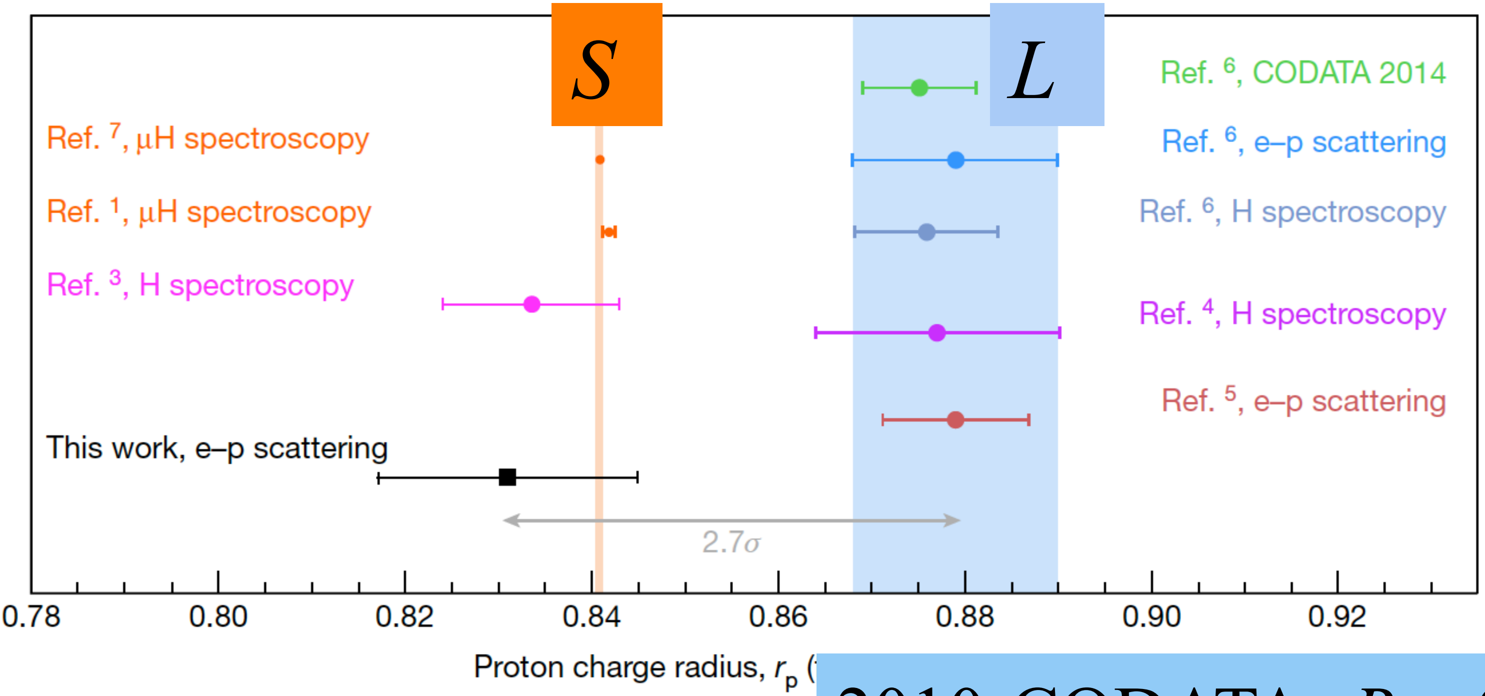
in collaboration with

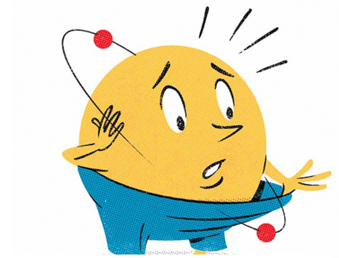
Simone Pacetti

INFN e Università di Perugia,Italia

PANDA Coll. Meeting, EM Session, March 10, 2019

The SIZE of the proton





The New York Times

2010-CODATA: Rp=0.8775(51) fm

Rp=0.84087(39) fm (muonic H)

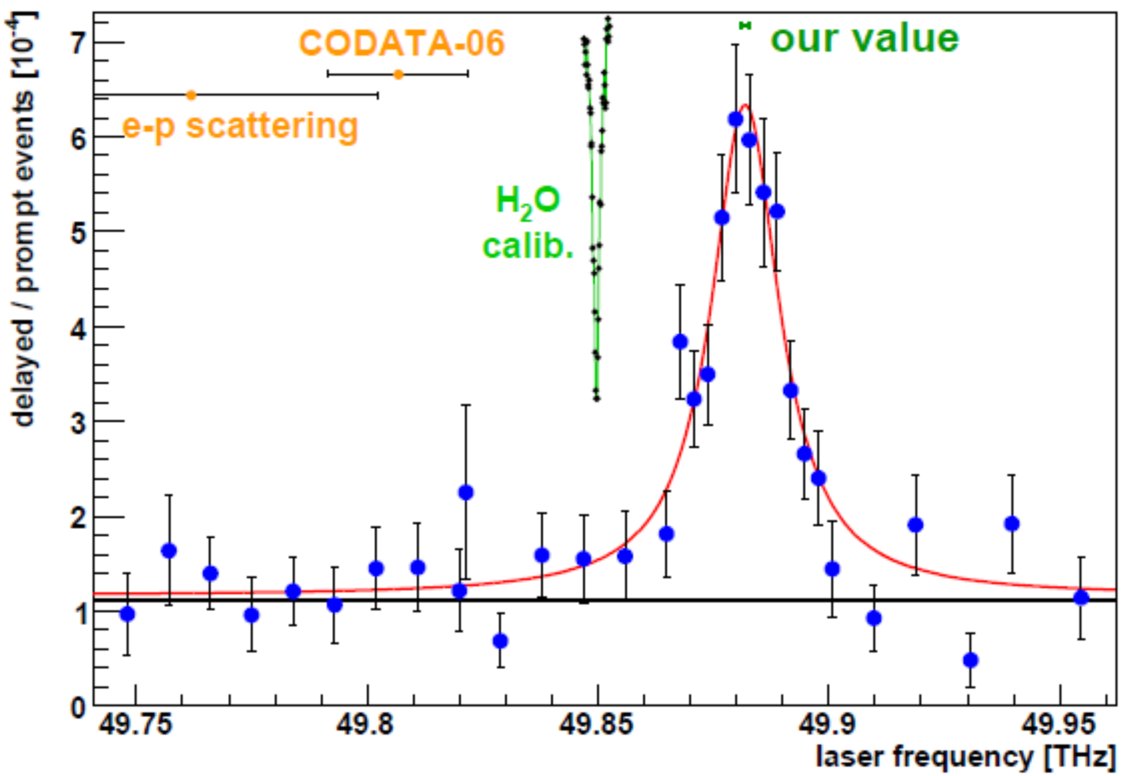
Rp=0.8335(95) fm (new H)

ATOMIC PHYSICS

International Nuclear Physics Conference 2010 (INPC2010) IOP Publishing
Journal of Physics: Conference Series **312** (2011) 032002 doi:10.1088/1742-6596/312/3/032002

The proton radius puzzle

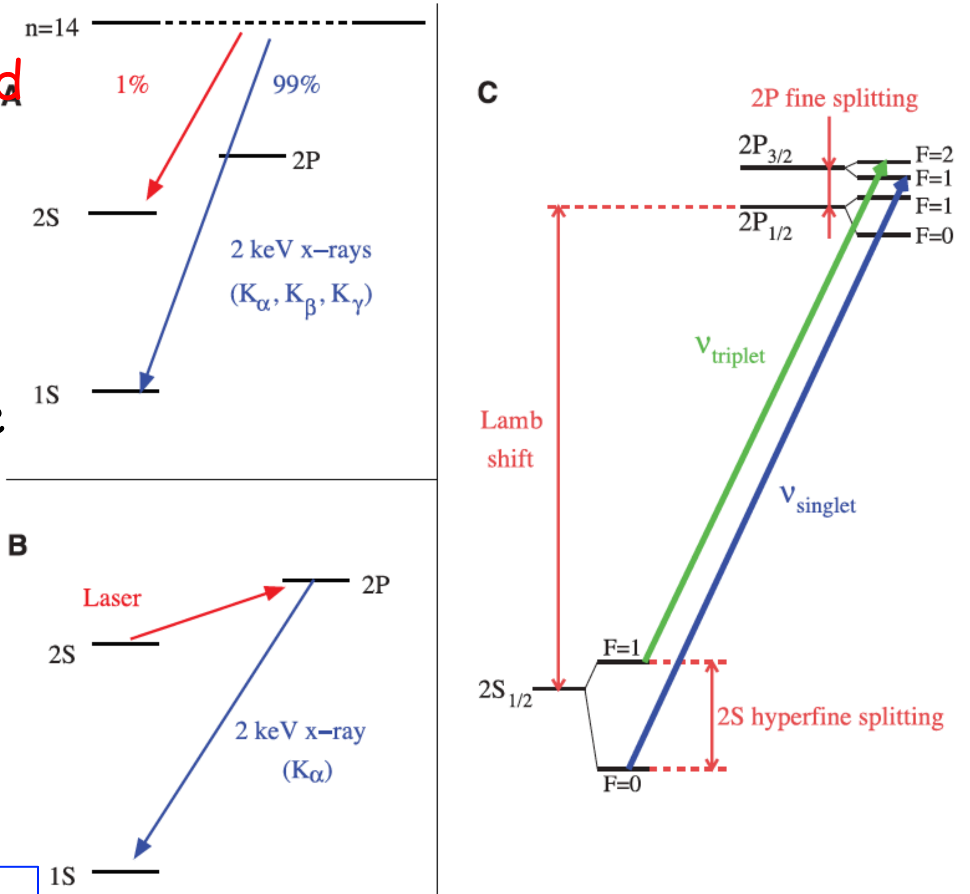
A Antognini[1,2], F D Amaro[3], F Biraben[4], J M R Cardoso[3],
D S Covita[5], A Dax[6], S Dhawan[6], L M P Fernandes[3], A Giesen[7],
T Graf[8], T W Hänsch[1,9], P Indelicato[4], L Julien[4], C-Y Kao[10],
P Knowles[11], F Kottmann[2], E-O Le Bigot[4], Y-W Liu[10],
ulhauser[11],
s[3],
13,



Abstract. B… rogen (μ^-p) we have
measured the … Hz [1]. By comparing
this measurem… on bound-state QED
we have detern… w value differs by 5.0
standard devia… 3 standard deviation
from the e-p sc… ncy may arise from a
computational … oblem in bound-state
QED, an unkn… nental error.

Lamb shift and hyperfine splitting (1)

Negative μ beams at PSI are stopped in H_2 gas target at 1 hPa and 20°C



A) Formation of *μp* atoms in highly excited states. 1% populates the 2S state (τ=1 μs).

B) Laser excitation of 2S-2P transition

C) 2S and 2P energy levels.

ν_s and ν_p: measured transitions

$$\frac{1}{4}h\nu_s + \frac{3}{4}h\nu_t = \Delta E_L + 8.8123(2)\text{meV}$$

$$h\nu_s - h\nu_t = \Delta E_{HFS} - 3.2480(2)\text{meV}$$

$$\Delta E_L^{exp} = 202.3706(23)\ \text{meV}$$

$$\Delta E_{HFS}^{exp} = 22.8089(51)\ \text{meV}$$

Lamb shift and hyperfine splitting (1)

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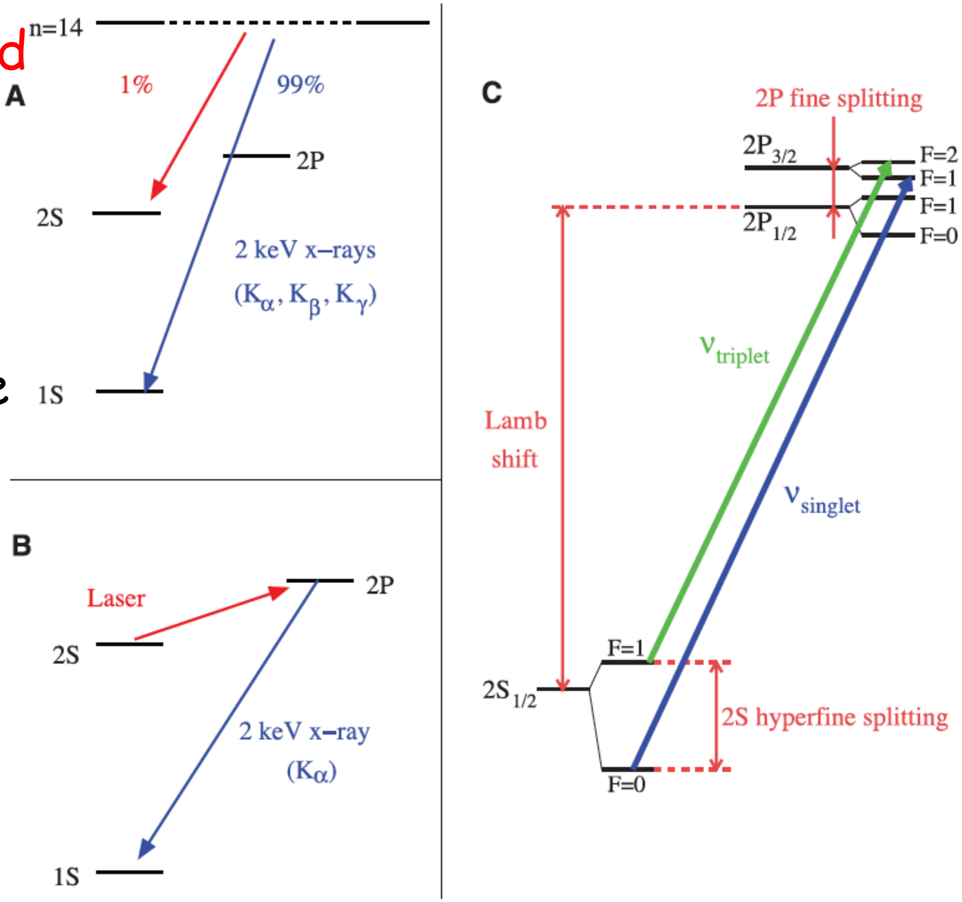


A) Formation of μp atoms in highly excited states. 1% populates the 2S state (τ=1 μs).

B) Laser excitation of 2S-2P transition

C) 2S and 2P energy levels.

ν_s and ν_p: measured transitions

An electron in S state has some probability to be inside the proton.
The electric field (charge distribution) is modified by the proton size.
The ν_s and ν_p transitions are affected by the proton size (few %)

Lamb shift and hyperfine splitting (II)

$$\Delta E_{\text{finite size}} = \frac{2\pi Z\alpha}{3} r_E^2 |\Psi(0)|^2$$

Atomic wave function at the origin

$|\Psi(0)|^2 \approx m_r^3$, $m_r(\mu p \text{ system}) \cong 186\, m_e$

H radius : 60000 x p radius

μH Bohr radius is ≈ 200 times smaller: larger sensitivity!

$$\frac{1}{4} h\nu_s + \frac{3}{4} h\nu_t = \Delta E_L + 8.8123(2)\text{meV}$$

$$h\nu_s - h\nu_t = \Delta E_{HFS} - 3.2480(2)\text{meV}$$

$$\Delta E_L^{exp} = 202.3706(23)\text{ meV}$$

$$\Delta E_{HFS}^{exp} = 22.8089(51)\text{ meV}$$

$$\Delta E_L^{th} = 206.0336(15) - 5.2275(10) r_E^2 + \Delta E_{TPE}$$

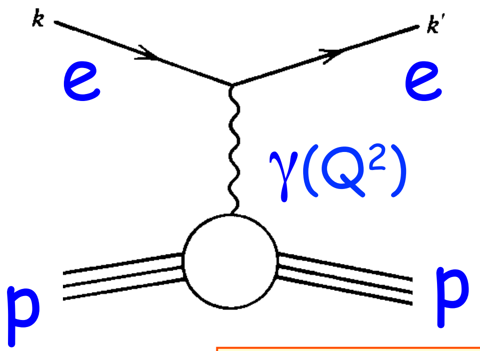
$$\Delta E_{TPE} = 0.0332(20)\text{ meV}$$

$$r_E = 0.84087(26)^{exp}(29)^{th}\text{ fm} = 0.84087(39)\text{ fm}$$

Small radius

Hadron physics: e-p scattering

ep-elastic scattering : Rosenbluth separation

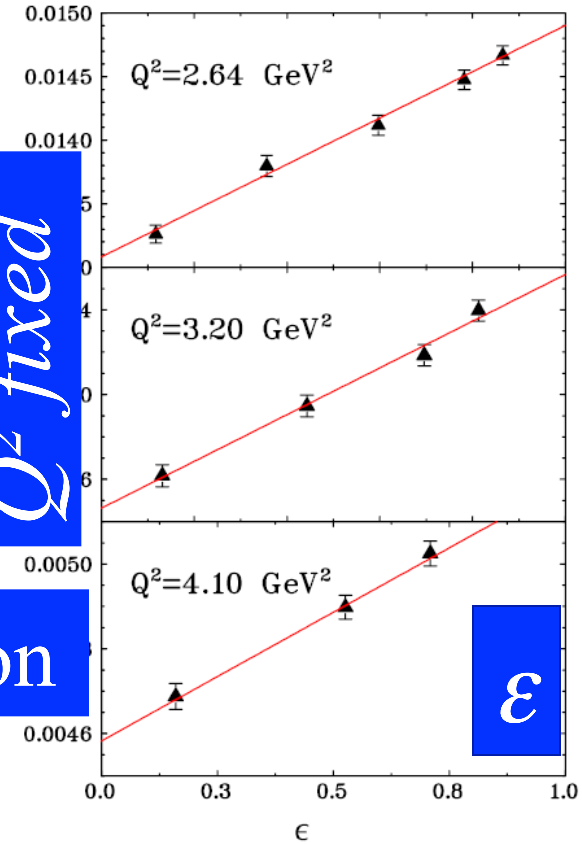


$$\frac{d\sigma}{d\Omega}=\left(\frac{d\sigma}{d\Omega}\right)_{Mott}\frac{1}{(1+\tau)}\left(G_E^2(Q^2)+\frac{\tau}{\varepsilon}G_M^2(Q^2)\right)$$

1950

$$\varepsilon=\left(1+2(1+\tau)\tan^2\left(\frac{\theta_e}{2}\right)\right)^{-1},\quad \tau=\frac{Q^2}{4M^2}$$

$$\sigma_R=\varepsilon G_E^2+\tau G_M^2$$



PRL 94, 142301 (2005)

Linearity of the reduced cross section

→ $\tan^2\theta_e$ dependence

→ Holds for 1γ exchange only

Root mean square radius

$$F(q) = \frac{\int_\Omega d^3\vec{x} e^{i\vec{q}\cdot\vec{x}} \rho(\vec{x})}{\int_\Omega d^3\vec{x} \rho(\vec{x})}.$$

In *non-relativistic approach*
(and also in relativistic but in *Breit frame*)
FFs are Fourier transform of the density

density $\rho(r)$	Form factor $F(q^2)$	r.m.s. $< r_c^2 >$	comments
δ	1	0	pointlike
e^{-ar}	$\frac{a^4}{(q^2+a^2)^2}$	$\frac{12}{a^2}$	dipole
$\frac{e^{-ar}}{r}$	$\frac{a^2}{q^2+a^2}$	$\frac{6}{a^2}$	monopole
$\frac{e^{-ar^2}}{r^2}$	$e^{-q^2/(4a^2)}$	$\frac{1}{2a}$	gaussian
ρ_0 for $x \leq R$ 0 for $r \geq R$	$\frac{3(\sin X - X\cos X)}{X^3}$ $X = qR$	$\frac{3}{5}R^2$	square well

Root mean square radius

$$F(q) = \frac{\int_\Omega d^3\vec{x} e^{i\vec{q}\cdot\vec{x}} \rho(\vec{x})}{\int_\Omega d^3\vec{x} \rho(\vec{x})}.$$

$$< r_c^2 >= \frac{\int_0^\infty x^4 \rho(x) dx}{\int_0^\infty x^2 \rho(x) dx}.$$

Expanding in Taylor series:

$$F(q) \sim 1 - \frac{1}{6} q^2 < r_c^2 > + O(q^2),$$

$$\langle r_{E/M}^2 \rangle = -\frac{6\hbar^2}{G_{E/M}(0)} \frac{dG_{E/M}(Q^2)}{dQ^2} \Big|_{Q^2=0}.$$

RMS is *the limit of the* ***form factor derivative*** *for* $Q^2 \to 0$

Think……

The elastic cross section diverges as $1/(Q^2)^2$ when $Q^2 \to 0$

When $Q^2 \to 0$? $Q^2=-4EE'\sin^2(\theta/2)$

1) $E'=0$: capture process,
compound hydrogen atom ->
the scattering formalism does not apply

OR

2) $\theta=0$: the incident electron does not 'feel' the target

The extrapolation of electron to photon induced processes
is generally not meaningful

PRL 105, 242001 (2010) PHYSICAL REVIEW LETTERS week ending 10 DECEMBER 2010

High-Precision Determination of the Electric and Magnetic Form Factors of the Proton

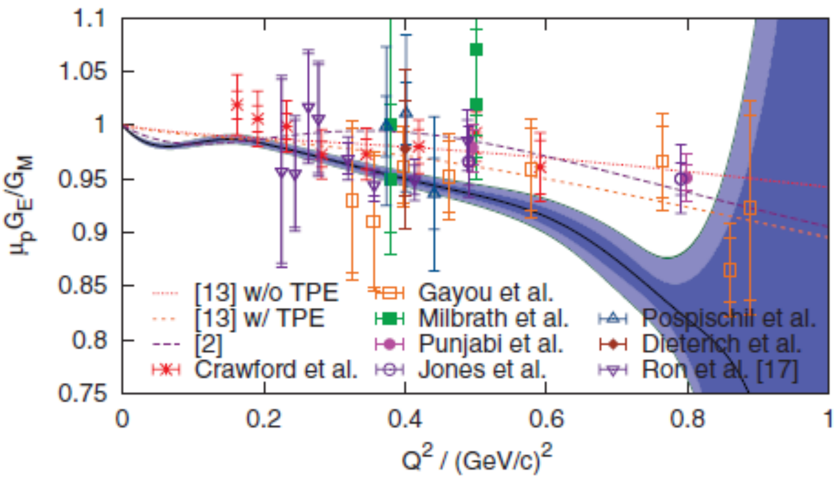
J. C. Bernauer,[1,*] P. Achenbach,[1] C. Ayerbe Gayoso,[1] R. Böhm,[1] D. Bosnar,[2] L. Debenjak,[3] M. O. Distler,[1,†] L. Doria,[1] A. Esser,[1] H. Fonvieille,[4] J. M. Friedrich,[5] J. Friedrich,[1] M. Gómez Rodríguez de la Paz,[1] M. Makek,[2] H. Merkel,[1] D. G. Middleton,[1] U. Müller,[1] L. Nungesser,[1] J. Pochodzalla,[1] M. Potokar,[3] S. Sánchez Majos,[1] B. S. Schlimme,[1] S. Širca,[6,3] Th. Walcher,[1] and M. Weinriefer[1]

Mainz, A1 collaboration (1400 points)

$Q^2 > 0.004\ \mathrm{GeV}^2$

- Radiative corrections
- Two photon exchange
- Coulomb corrections

$$\langle r_E^2\rangle^{1/2} = 0.879(5)_{\rm stat}(4)_{\rm syst}(2)_{\rm model}(4)_{\rm group}\ \mathrm{fm},$$
$$\langle r_M^2\rangle^{1/2} = 0.777(13)_{\rm stat}(9)_{\rm syst}(5)_{\rm model}(2)_{\rm group}\ \mathrm{fm}.$$

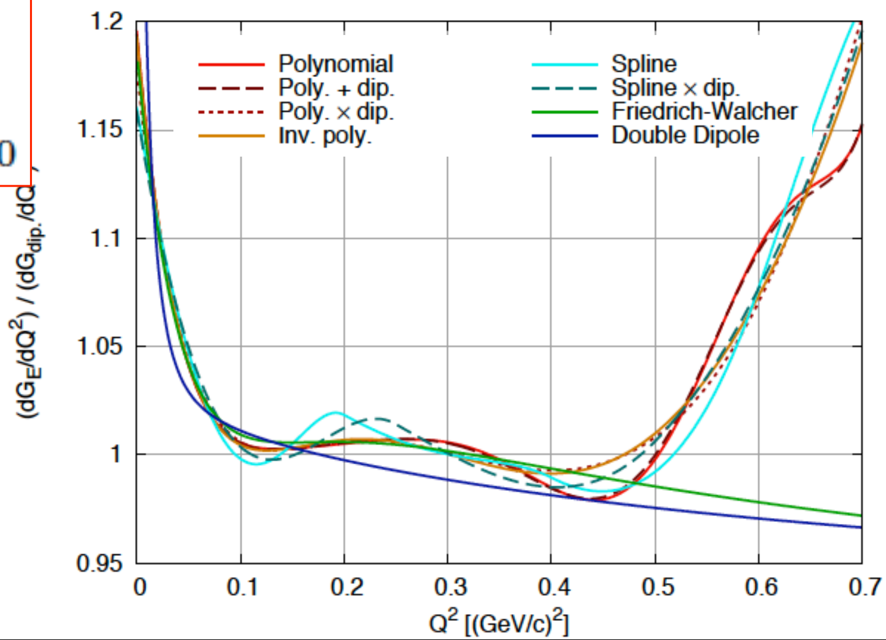


What about extrapolation to $Q^2 \to 0$?

G.I. Gakh, A. Dbeyssi, E.T-G, D. Marchand, V.V. Bytev, Phys.Part.Nucl.Lett. 10 (2013) 393, Phys.Rev. C84 (2011) 015212

Mainz ep elastic scattering

$$\left\langle r^2_{E/M} \right\rangle = -\frac{6\hbar^2}{G_{E/M}(0)} \frac{\mathrm{d}G_{E/M}(Q^2)}{\mathrm{d}Q^2}\Bigg|_{Q^2=0}$$



1) Rosenbluth extraction

2) Direct extraction
(assuming a function for FFs)

Spline

$$\left\langle r_E^2 \right\rangle^{\frac{1}{2}} = 0.875(5)_{\mathrm{stat.}}(4)_{\mathrm{syst.}}(2)_{\mathrm{model}}\,\mathrm{fm},$$

$$\left\langle r_M^2 \right\rangle^{\frac{1}{2}} = 0.775(12)_{\mathrm{stat.}}(9)_{\mathrm{syst.}}(4)_{\mathrm{model}}\,\mathrm{fm}$$

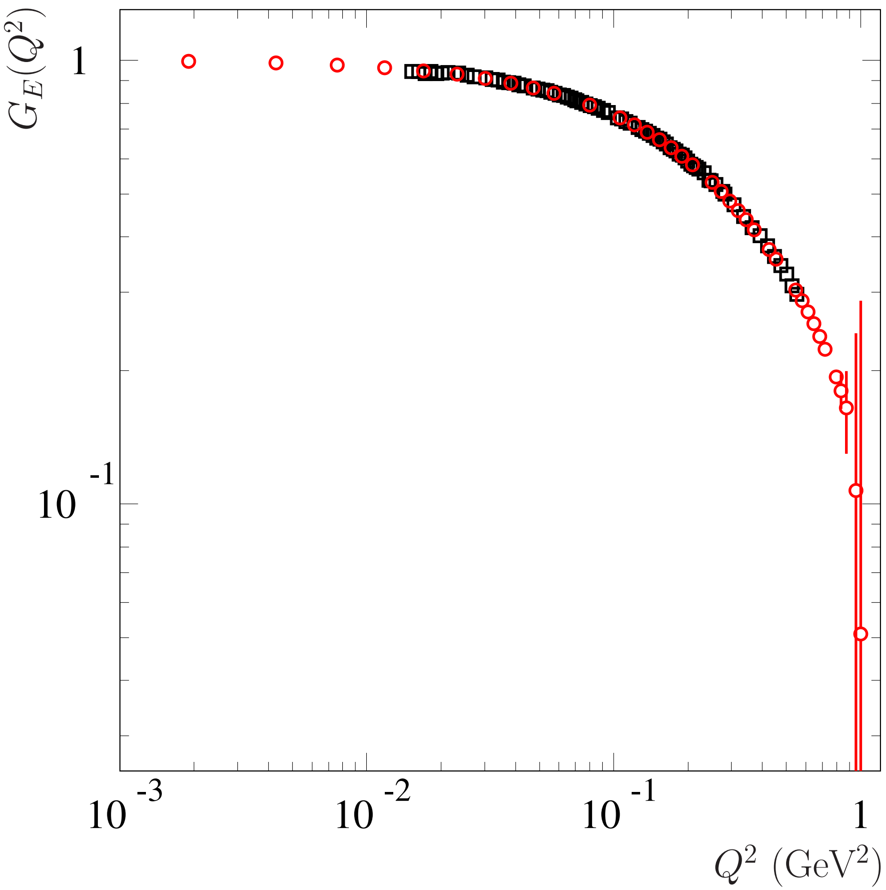
Polynomial

$$\left\langle r_E^2 \right\rangle^{\frac{1}{2}} = 0.883(5)_{\mathrm{stat.}}(5)_{\mathrm{syst.}}(3)_{\mathrm{model}}\,\mathrm{fm},$$

$$\left\langle r_M^2 \right\rangle^{\frac{1}{2}} = 0.778(^{+14}_{-15})_{\mathrm{stat.}}(10)_{\mathrm{syst.}}(6)_{\mathrm{model}}\,\mathrm{fm}.$$

J.C. Bernauer, PhD, Mainz

Mainz ep elastic scattering



<u>Spline:</u> Q2>0.0005 GeV^2

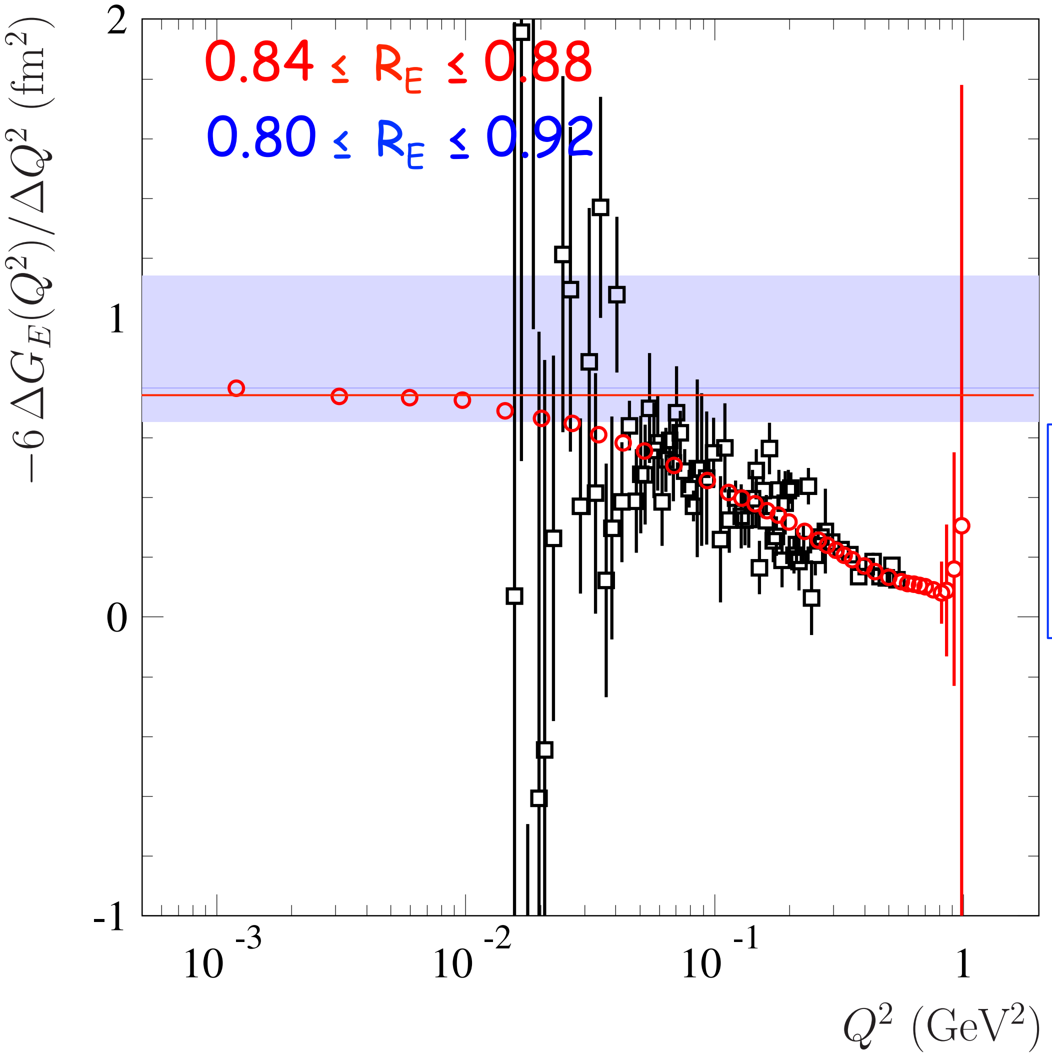
G_E from a global fit of $\sigma(Q^2,\varepsilon)$, based on a pre-defined function

<u>Rosenbluth:</u> Q2>0.0152 GeV^2

G_E and G_M from the slope and intercept of σ_{red} (ε), at fixed (Q^2,ε). (larger errors, Q^2 interval)

The choice of a pre-defined function imposes serious constraints to the radius through the derivative!

Mainz ep elastic scattering-derivative



Rosenbluth

Spline

$$\Delta G_{E,j}^{S,R} = \frac{G_{E,j+1}^{S,R} - G_{E,j}^{S,R}}{Q_{j+1}^{2S,R} - Q_j^{2S,R}},$$

$$\delta\Delta G_{E,j}^{S,R} = \frac{\sqrt{\left(\delta G_{E,j+1}^{S,R}\right)^2 + \left(\delta G_{E,j}^{S,R}\right)^2}}{Q_{j+1}^{2S,R} - Q_j^{2S,R}}$$

$$\overline{Q}_j^{2S,R} = \frac{Q_{j+1}^{2S,R} + Q_j^{2S,R}}{2}$$

Mainz Data – Fitting Procedure

- <u>4 sets of data:</u>

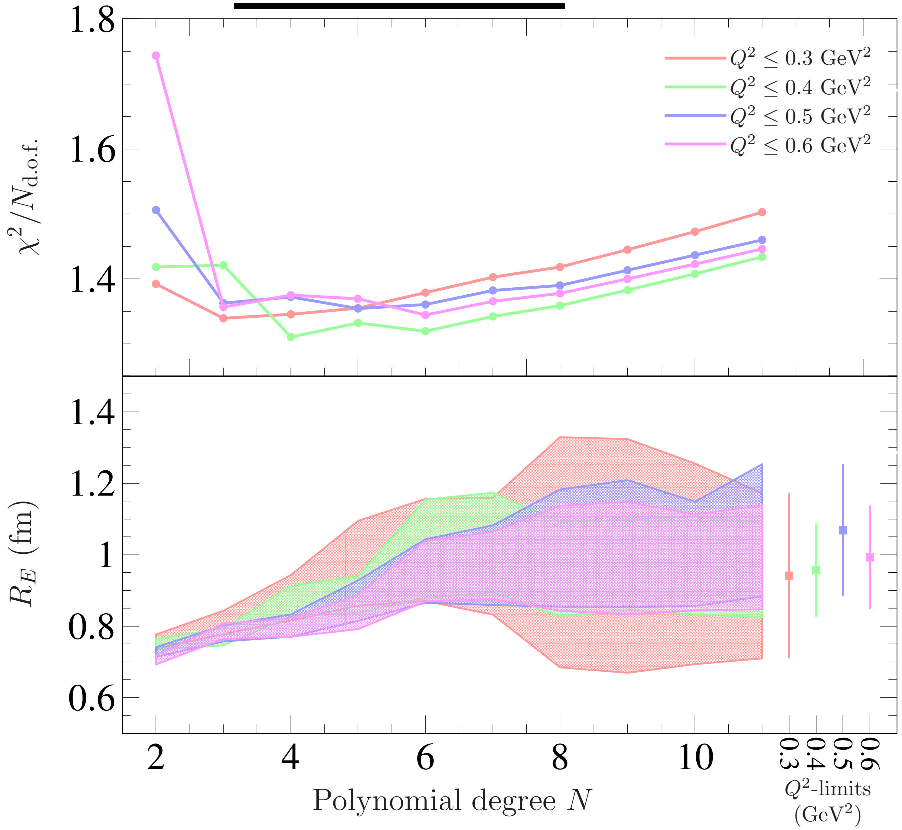
  - 2 G_E data: Rosenbluth and Spline
  - 2 discrete derivatives

$$\left\{\overline{Q}_j^{2S}, \Delta G_{E,j}^{S}, \delta\Delta G_{E,j}^{S}\right\}_{j=1}^{N_S-1} \qquad \left\{\overline{Q}_j^{2R}, \Delta G_{E,j}^{R}, \delta\Delta G_{E,j}^{R}\right\}_{j=1}^{N_R-1}$$

- 4 Q^2 ranges,
- polynomes up to 12 degree

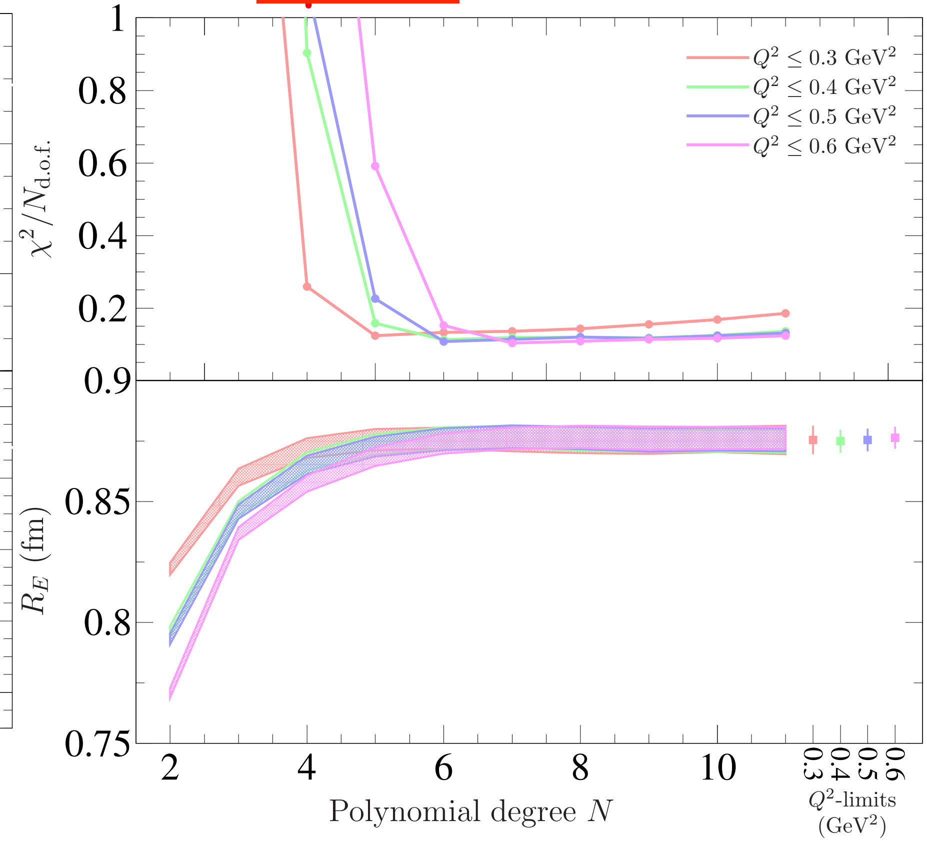
Radius – Fitting dG_E (R &S)

Rosenbluth



Large errors

Spline

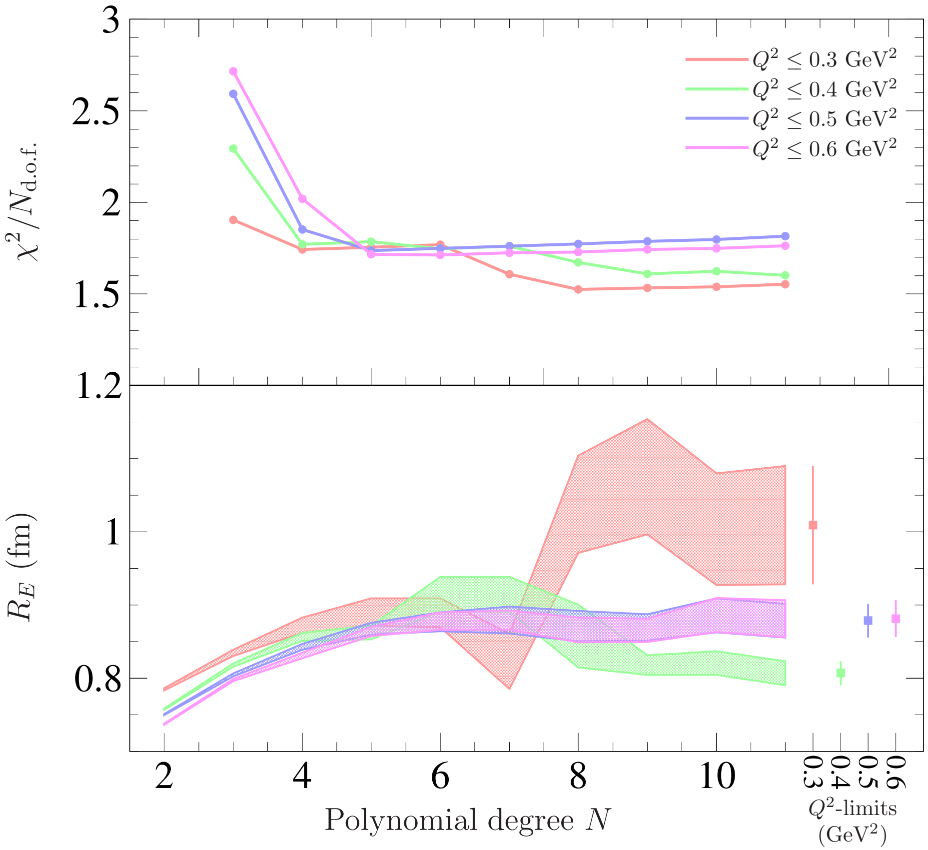


Stability of the results

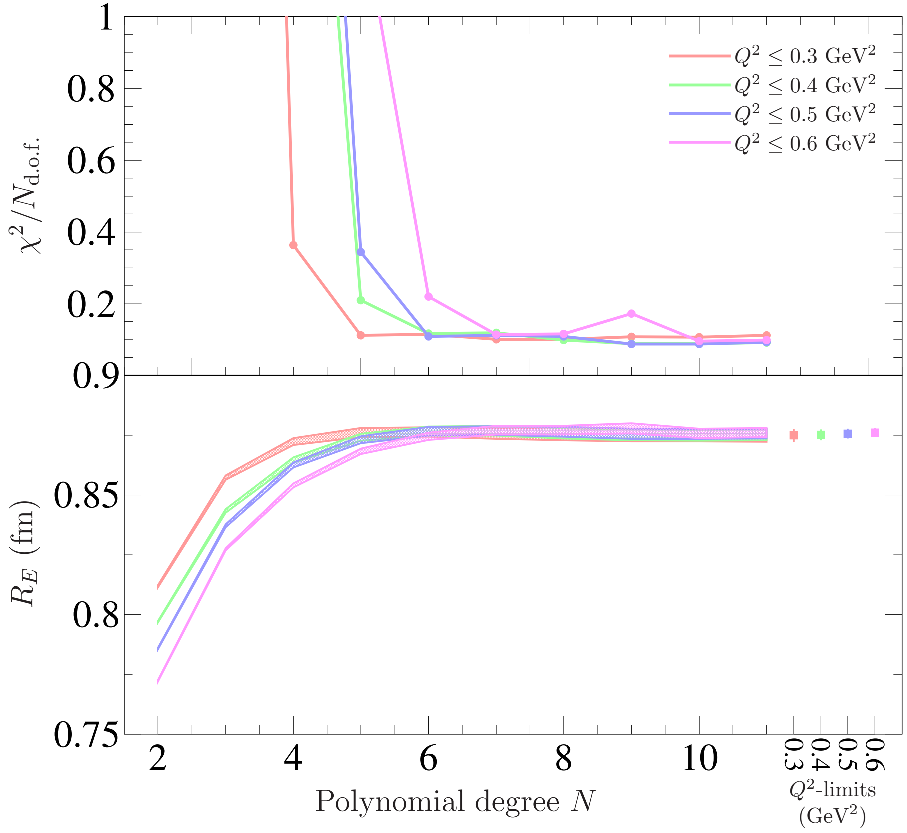
Very small $\chi 2$

Radius – Fitting G_E & dG_E (R &S)

Rosenbluth



Spline

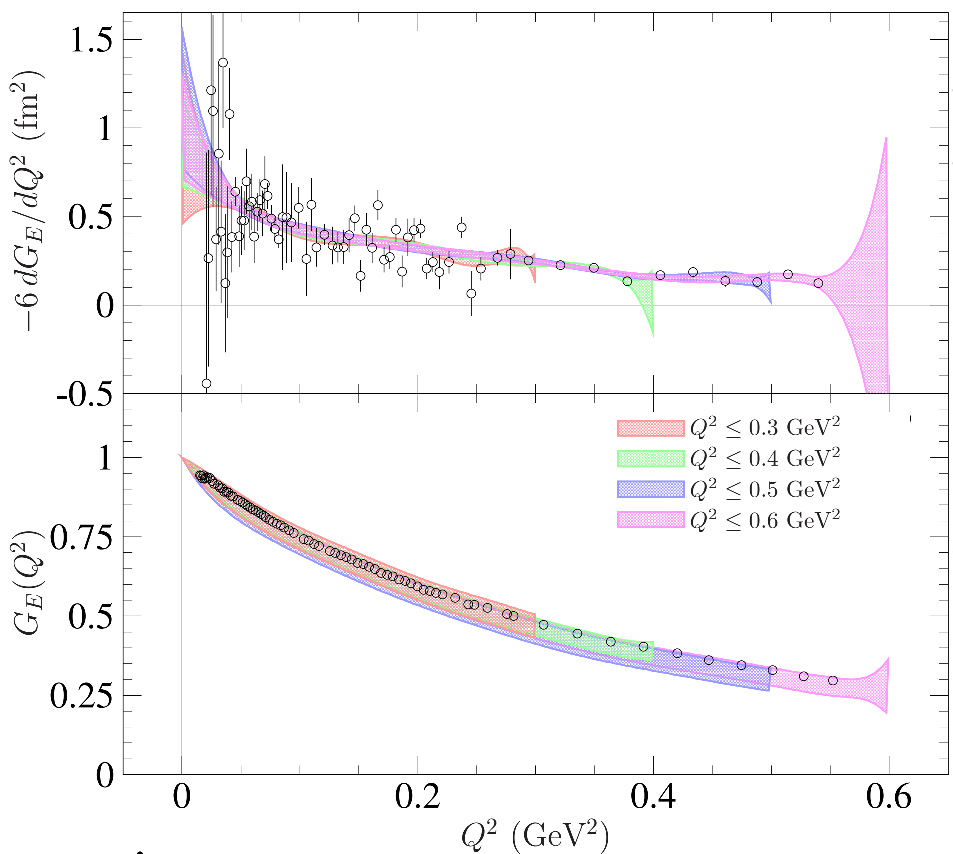


Large errors

Stability of the results
Very small errors
Very small $\chi 2$

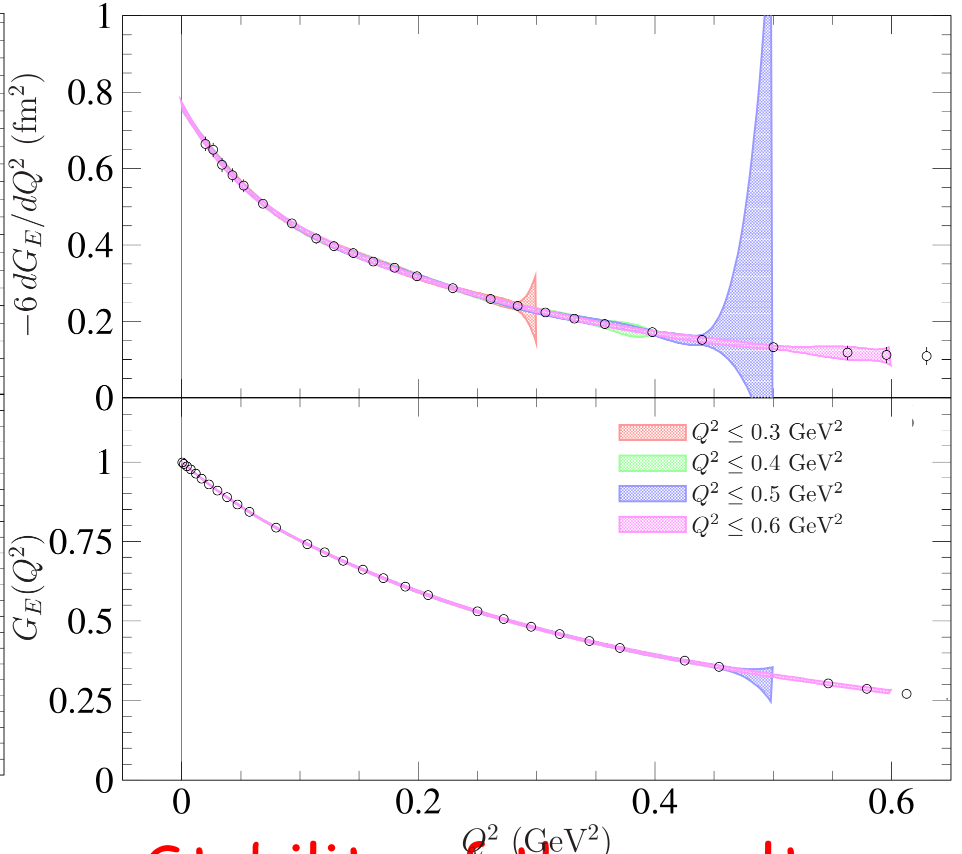
Functions – dG_E (R &S)

Rosenbluth



Large errors

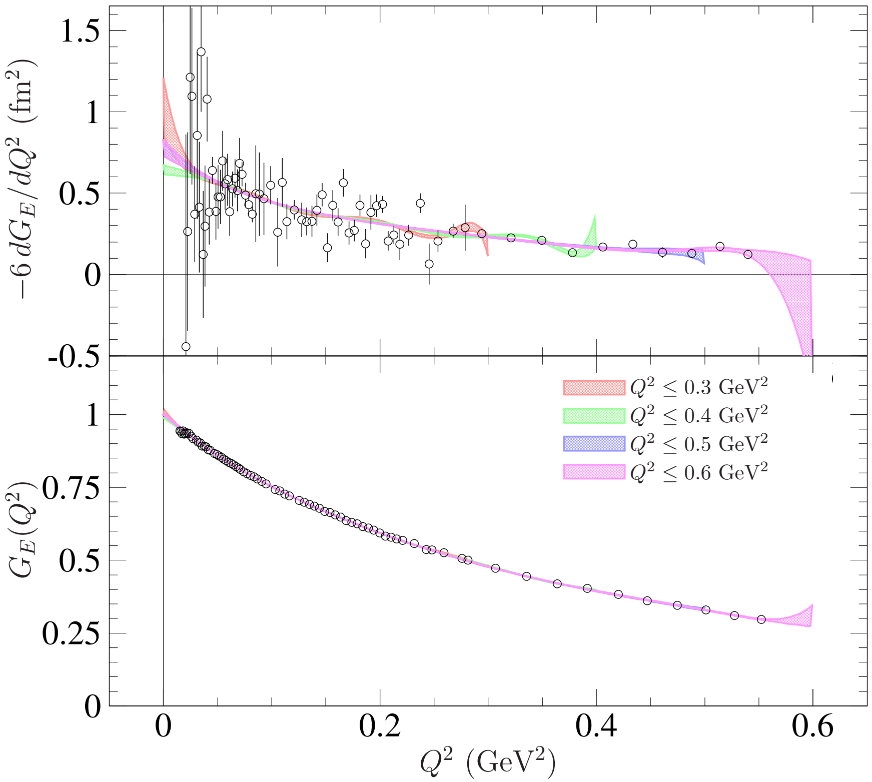
Spline



Stability of the results
Very small errors
Very small $\chi 2$

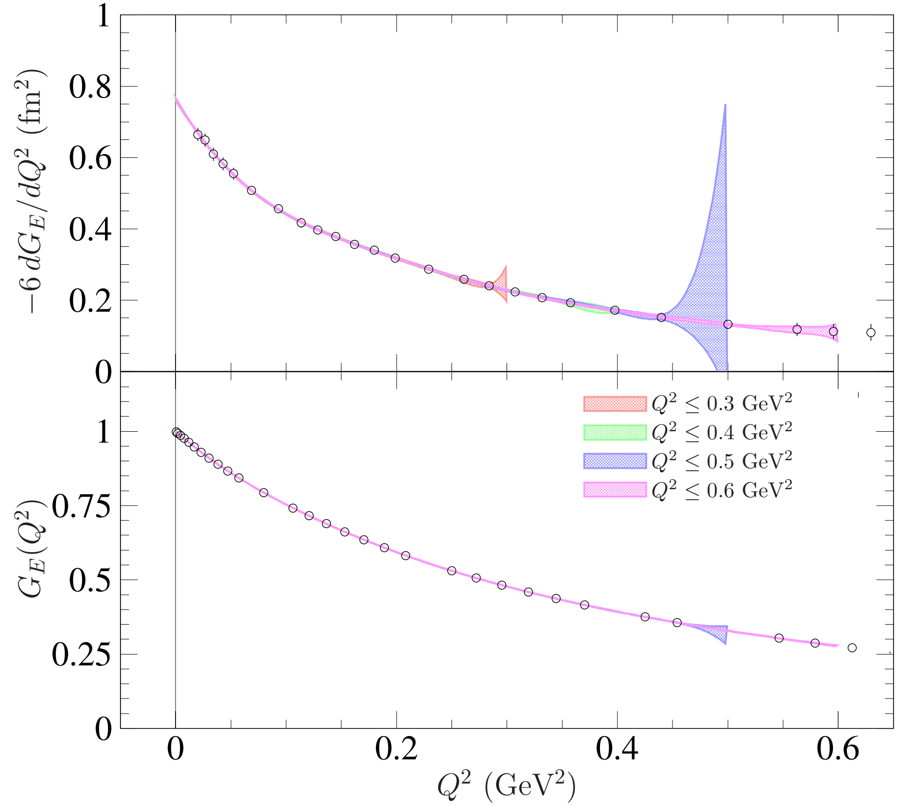
Functions – Fitting G_E & dG_E(R &S)

Rosenbluth



Large errors

Spline



Stability of the results

Very small $\chi 2$

Mainz – Fitting Procedure

		Rosenbluth		Spline	
		$\chi^2/N_{\mathrm{d.o.f.}}$	R_E (fm)	$\chi^2/N_{\mathrm{d.o.f.}}$	R_E (fm)
$Q^2 \leq 0.3$ GeV^2	dG_E/dQ^2	1.50	0.9411 ± 0.2310	0.19	0.8754 ± 0.0059
	$G_E \cup dG_E/dQ^2$	1.55	1.0088 ± 0.0809	0.11	0.8749 ± 0.0026
$Q^2 \leq 0.4$ GeV^2	dG_E/dQ^2	1.43	0.9568 ± 0.1309	0.14	0.8749 ± 0.0048
	$G_E \cup dG_E/dQ^2$	1.60	0.8070 ± 0.0164	0.09	0.8751 ± 0.0023
$Q^2 \leq 0.5$ GeV^2	dG_E/dQ^2	1.46	1.0681 ± 0.1848	0.13	0.8754 ± 0.0047
	$G_E \cup dG_E/dQ^2$	1.82	0.8786 ± 0.0229	0.09	0.8756 ± 0.0020
$Q^2 \leq 0.6$ GeV^2	dG_E/dQ^2	1.45	0.9927 ± 0.1453	0.12	0.8763 ± 0.0046
	$G_E \cup dG_E/dQ^2$	1.76	0.8811 ± 0.0253	0.10	0.8761 ± 0.0019

- S- Errors << R-data (x 5-10)
- S- Values very stable, R-values depend on fitting scheme
- Discrepancy on the central R- and S- values
- Very small $\chi 2$ and stability of S-results derive from the large constraint due to the pre-imposed function

Final values from Mainz data

Rosenbluth

$$R_E^{R,1C} = 0.99 \pm 0.15\,\mathrm{fm}\,,$$

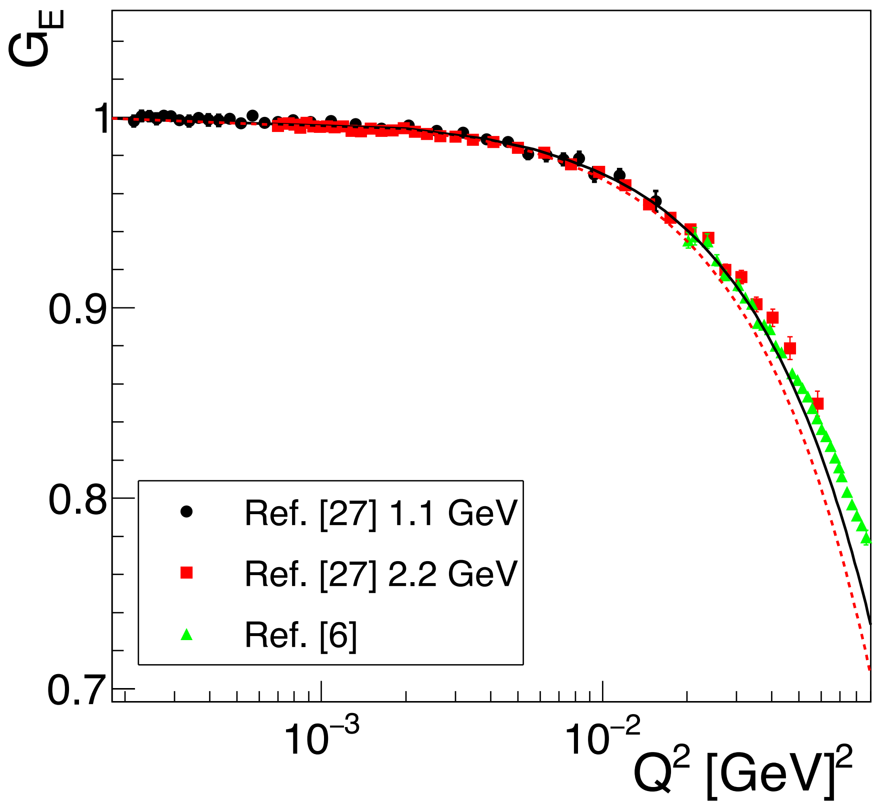
$$R_E^{R,2C} = 0.88 \pm 0.03\,\mathrm{fm}\,,$$

Spline

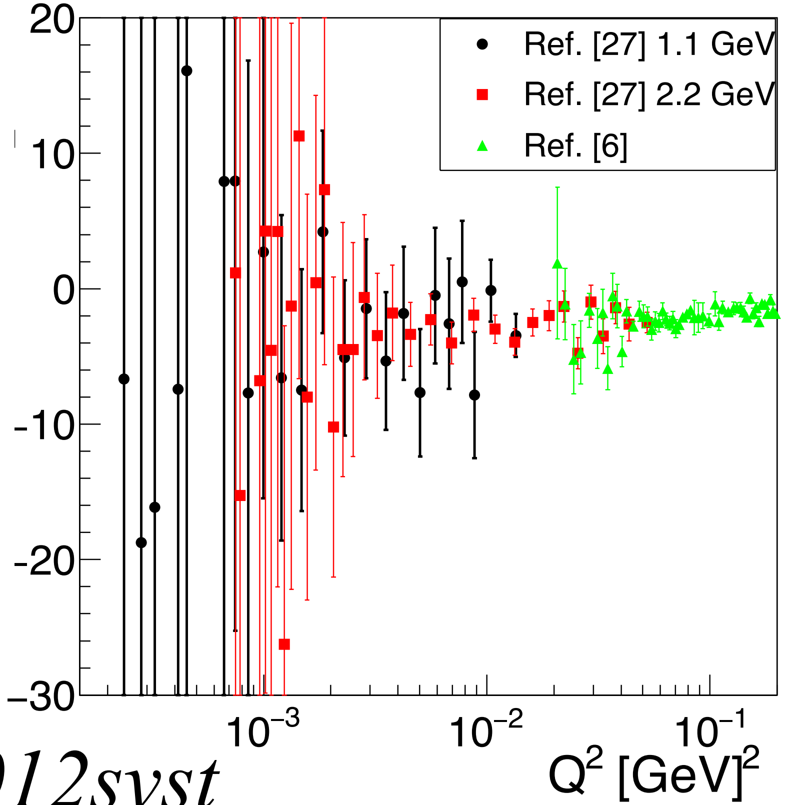
$$R_E^{S,1C} = 0.876 \pm 0.005\,\mathrm{fm}\,,$$

$$R_E^{S,2C} = 0.876 \pm 0.002\,\mathrm{fm}\,,$$

Mainz & CLAS11



Plateau: visual for log scale!

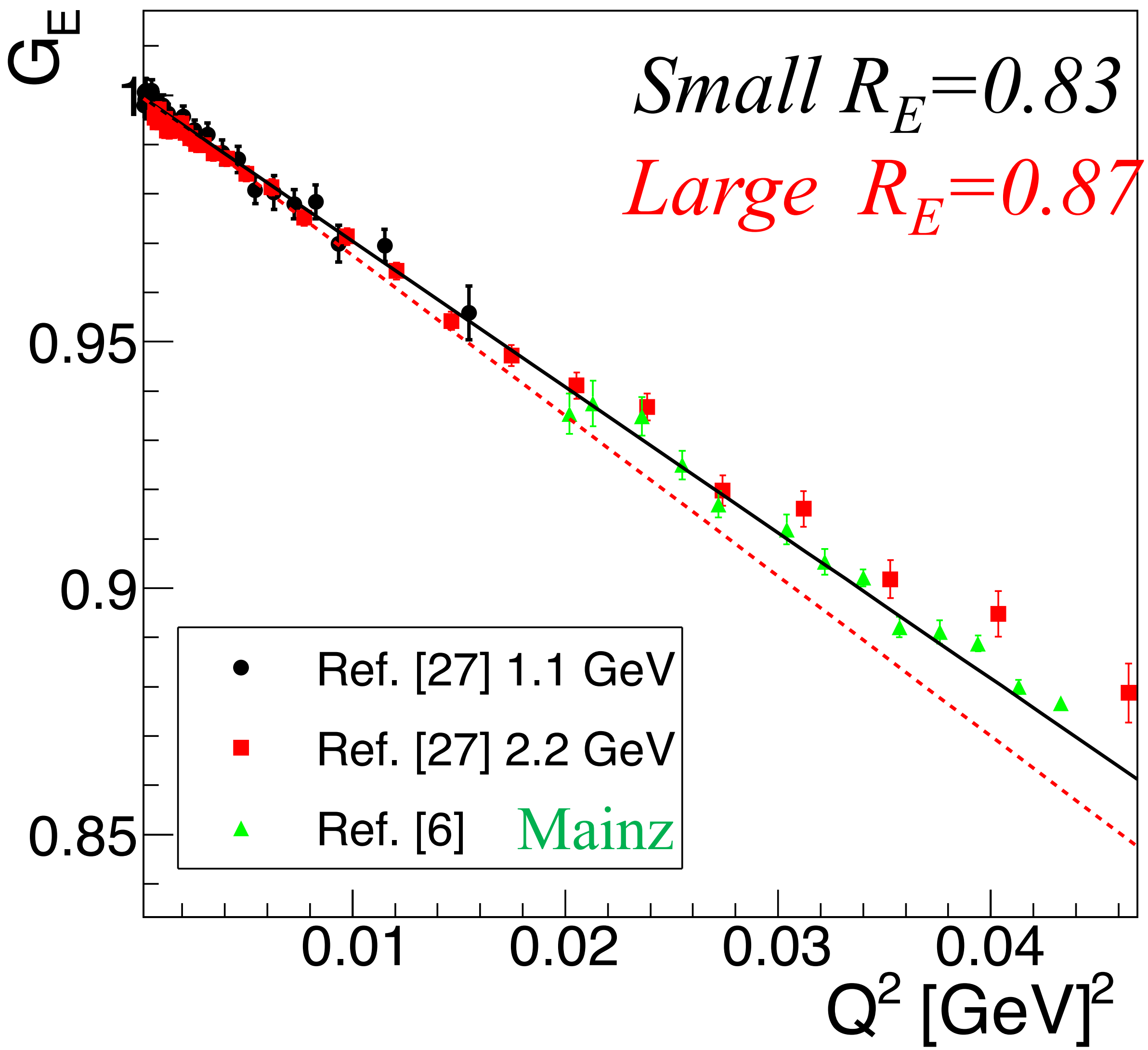


CLAS 11: Small radius!

$$R_E=0.831 \pm 0.007 stat \pm 0.012 syst$$

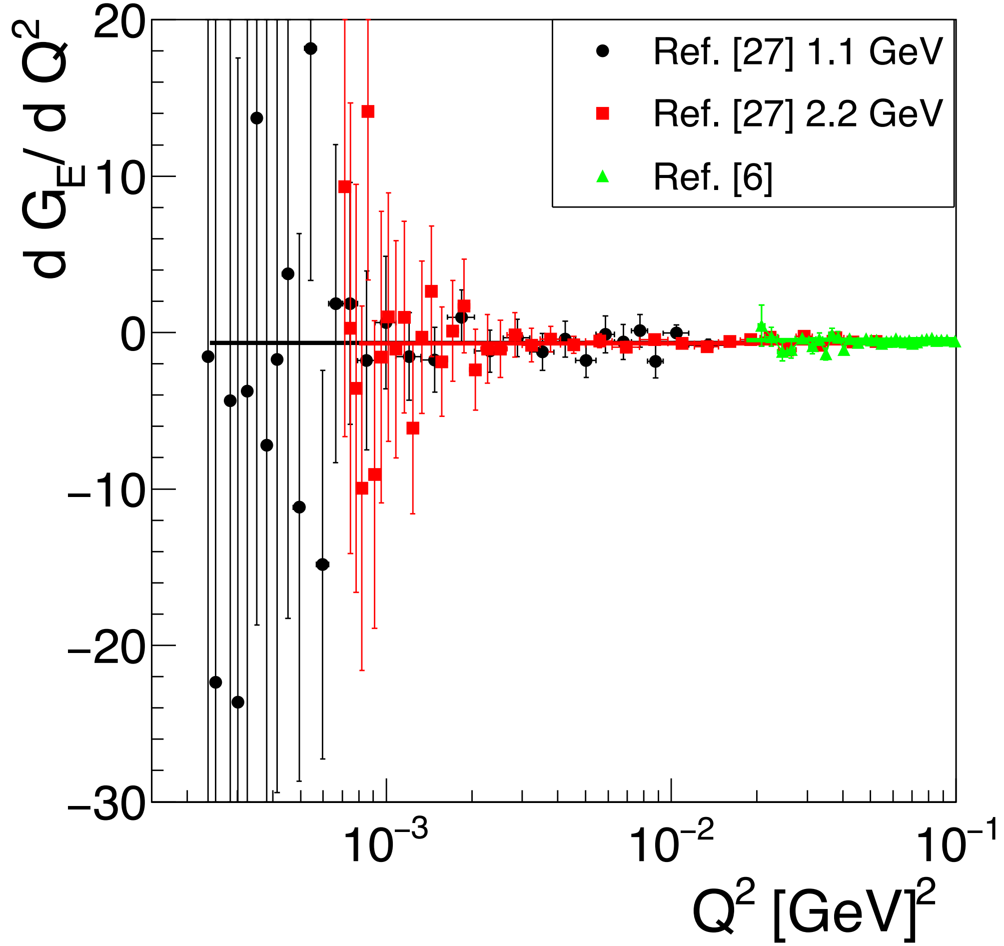
Smaller Q2, larger the error on the derivative

Mainz & CLAS11 Constrained Linear Fit



Mainz & CLAS11- at first sight

Rough estimation from a constrained linear fit



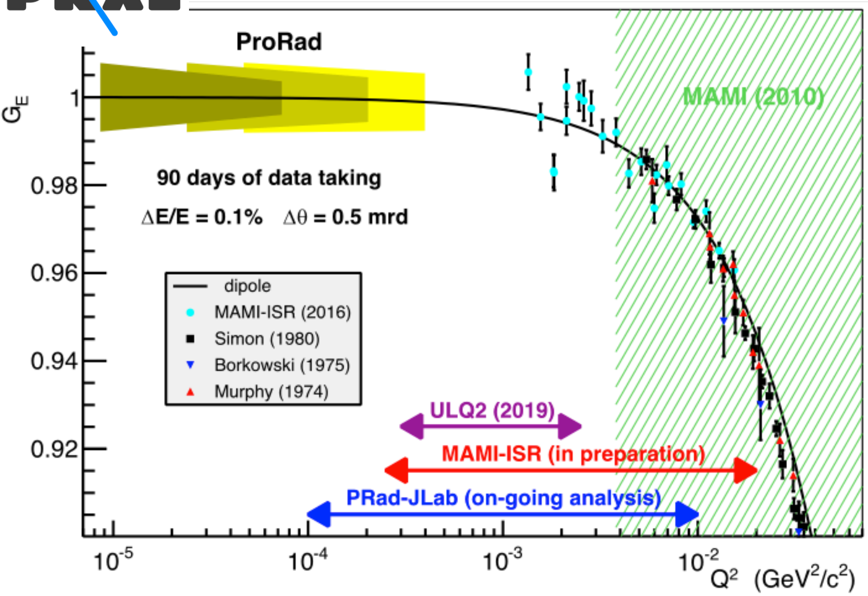
$R_E = 0.81 \pm 0.08$

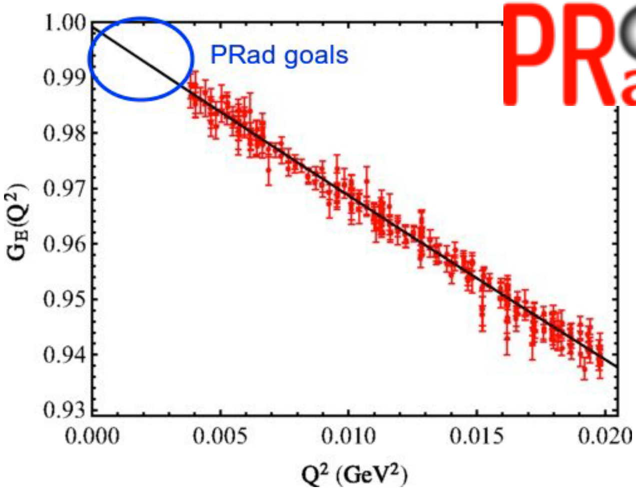
$R_E = 0.82 \pm 0.09$

and from Mainz data:

$R_E = 0.7 \pm 0.02$

Planned ep experiments



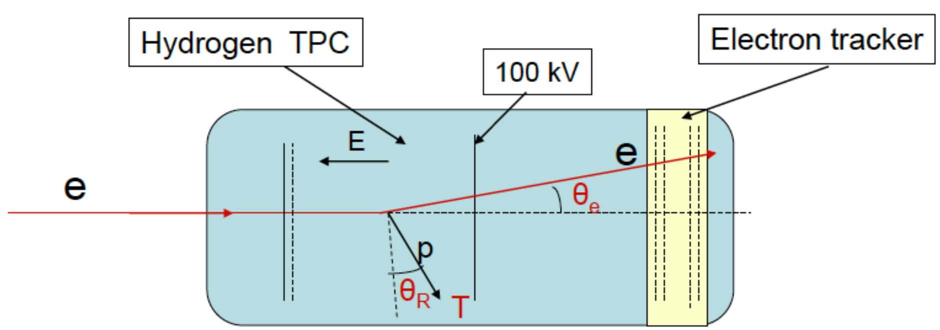


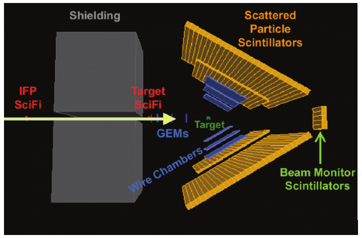
Systematics at sub%
Abs Norm by Moller

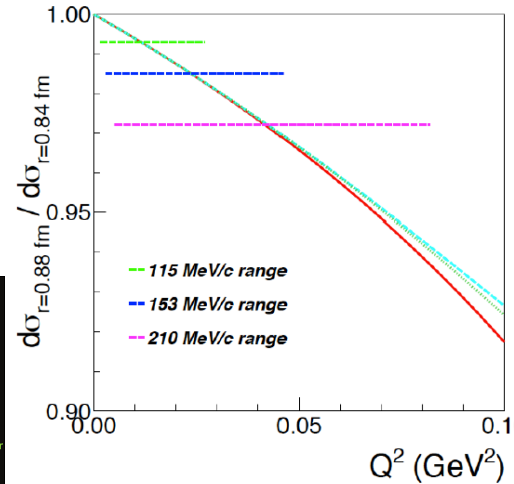
MUSE@PSI: muon beam

PNPI@MAMI: e and p detection

Combined recoiled proton@forward tracker detector







Conclusions

Discrepancy between the determination of the proton radius:

- CODATA (ep scattering & H) and muonic hydrogen
- ep elastic scattering and μH
- Recent and previous Hydrogen Lamb shift experiments
- Tension between analysis of ep-scattering: extrapolation to $Q^2=0$!!!

Our suggestion: work on derivatives

- *The cross section is measured, but the radius is related to the derivative!*
- *extrapolation of the derivative*
- *errors blow up at low Q^2*

- Similar problem in the high energy side, for the form factor ratio (polarized versus unpolarized)!