

Observational Constraints on Neutron Stars

James M. Lattimer

Department of Physics & Astronomy
Stony Brook University

Outline

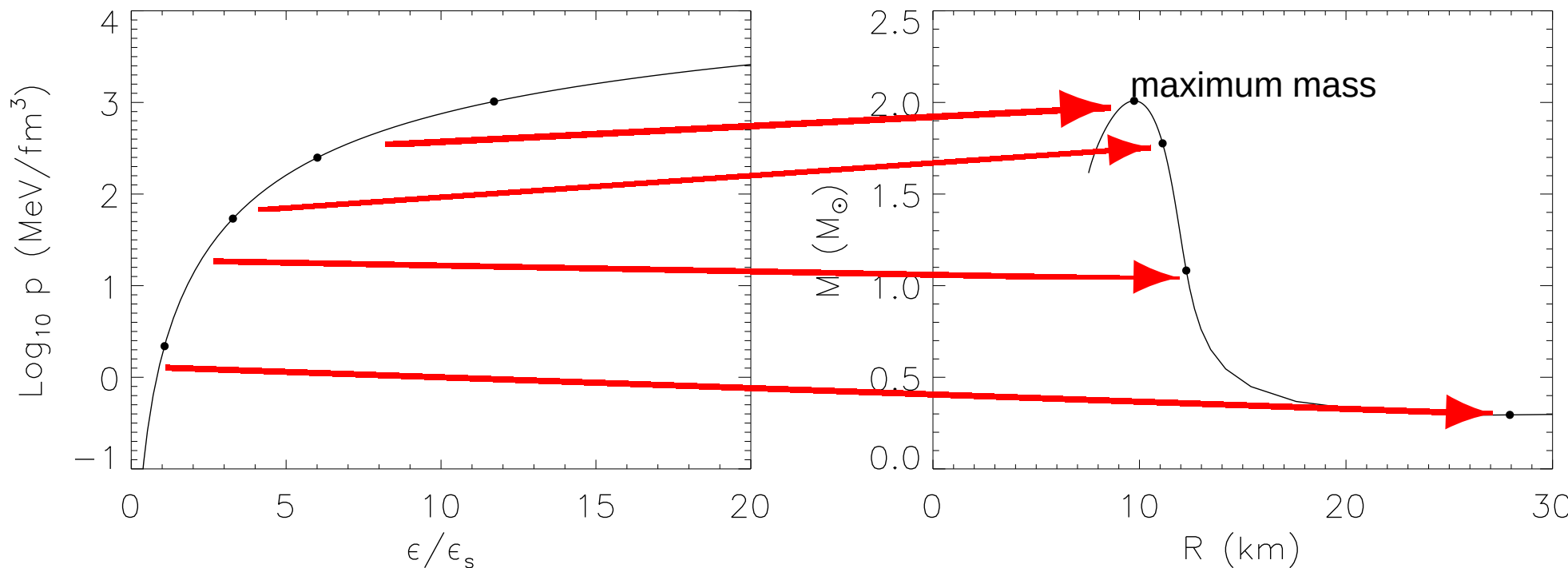
- Neutron Star Structure
 - Neutron Star Maximum Mass
 - Quark Star Maximum Mass
- Mass Measurements
 - $2 M_{\odot}$ Neutron Stars?
- Neutron Star Radii
 - Photospheric Radius Expansion X-Ray Bursters
 - Thermal Emission from Cooling Neutron Stars
 - Constraints on the EOS and Symmetry Energy
 - Consistency with Neutron Matter Expectations
- Thermal Evolution of Neutron Stars
 - Cooling of Cas A and Measurement of the n^3P_2 Gap

Neutron Star Structure

Tolman-Oppenheimer-Volkov equations

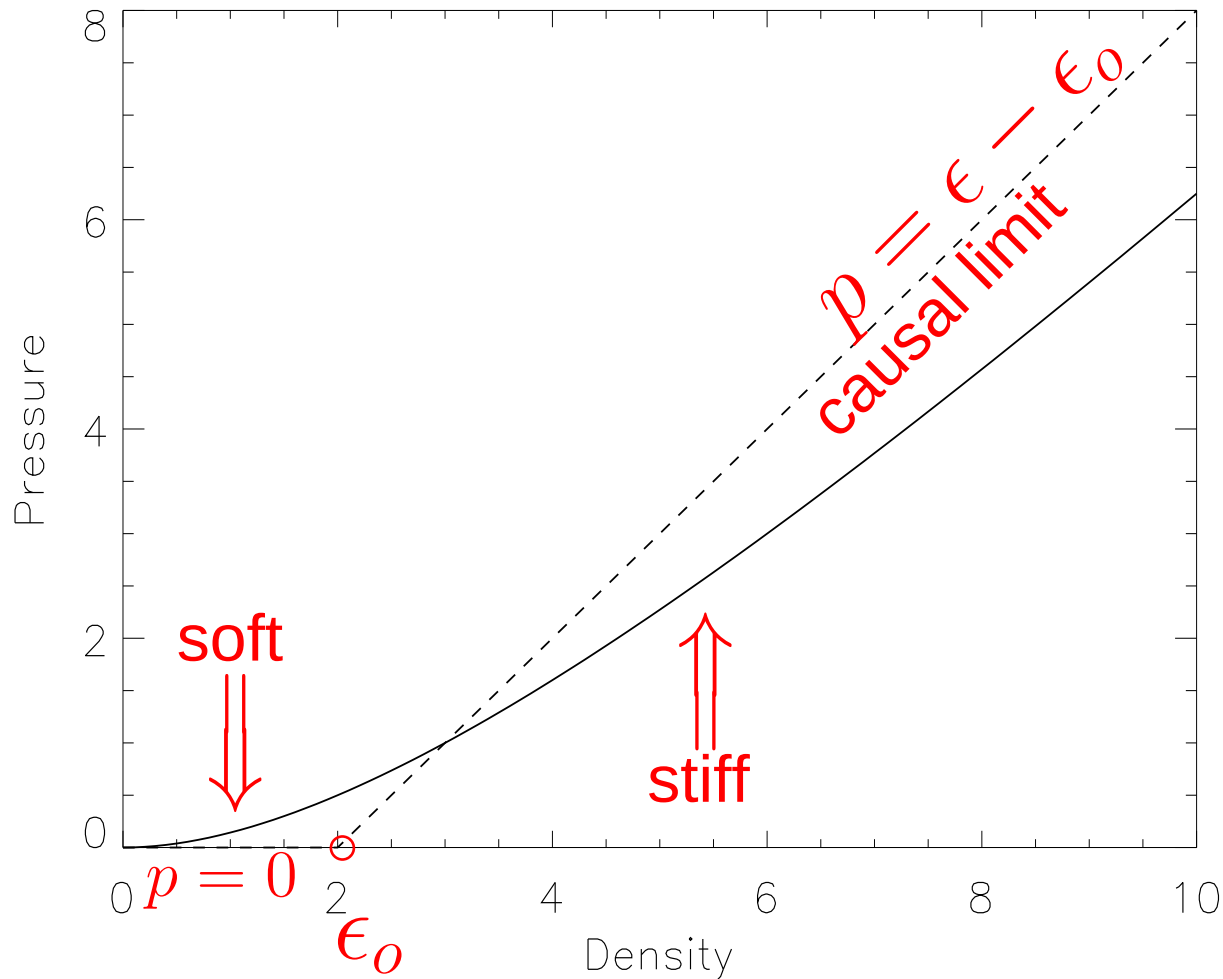
$$\frac{dp}{dr} = -\frac{G}{c^2} \frac{(m + 4\pi p r^3)(\epsilon + p)}{r(r - 2Gm/c^2)}$$

$$\frac{dm}{dr} = 4\pi \frac{\epsilon}{c^2} r^2$$



Extreme Properties of Neutron Stars

- The most compact and massive configurations occur when the low-density equation of state is "soft" and the high-density equation of state is "stiff".



ϵ_0 is the only EOS parameter

The TOV solutions scale with ϵ_0

Extreme Properties of Neutron Stars

- $M_{max} = 4.1 (\varepsilon_s/\varepsilon_0)^{1/2} M_\odot$
- $M_{B,max} = 5.41 (m_B c^2/\mu_o)(\varepsilon_s/\varepsilon_0)^{1/2} M_\odot \geq 2.63 M_\odot$
- $R_{min} = 2.82 GM/c^2 = 4.3 (M/M_\odot) \text{ km} \geq 8.5 \text{ km}$
- $\mu_{B,max} = 2.09 \text{ GeV}$
- $\varepsilon_{c,max} = 50 (M_\odot/M_{largest})^2 \varepsilon_s \leq 13.1 \varepsilon_s$
- $p_{c,max} = 33 (M_\odot/M_{largest})^2 \varepsilon_s \leq 8.6 \varepsilon_s$
- $n_{B,max} = 38 (M_\odot/M_{largest})^2 n_s \leq 9.8 n_s$
- $BE_{max} = 0.34 M$
- $P_{min} = 0.74 (M_\odot/M_{sph})^{1/2} (R_{sph}/10 \text{ km})^{3/2} \text{ ms} \geq 0.43 \text{ ms}$

Extreme Properties of Quark Stars

Maximally compact quark EOS: $p = \frac{1}{3}(\varepsilon - \varepsilon_0)$

$\varepsilon_0 = 4B/3$ in massless non-interacting MIT bag model

- $M_{max} = 2.5 (\varepsilon_s/\varepsilon_0)^{1/2} M_\odot$
- $M_{B,max} = 3.0 (m_B c^2/\mu_o)(\varepsilon_s/\varepsilon_0)^{1/2} M_\odot \geq 2.40 M_\odot$
- $\mu_{B,max} = 1.46 \text{ GeV}$
- $\varepsilon_{c,max} = 30 (M_\odot/M_{largest})^2 \varepsilon_s \leq 7.7 \varepsilon_s$
- $p_{c,max} = 7.9 (M_\odot/M_{largest})^2 \varepsilon_s \leq 2.0 \varepsilon_s$
- $n_{B,max} = 27 (M_\odot/M_{largest})^2 n_s \leq 7.0 n_s$
- $BE_{max} = 0.21 M$

The effects of adding QCD corrections, finite strange quark mass and CFL gap makes the EOS more attractive and less compact.

Mass-Radius Diagram and Theoretical Constraints

GR:

$$R > 2GM/c^2$$

$P < \infty$:

$$R > (9/4)GM/c^2$$

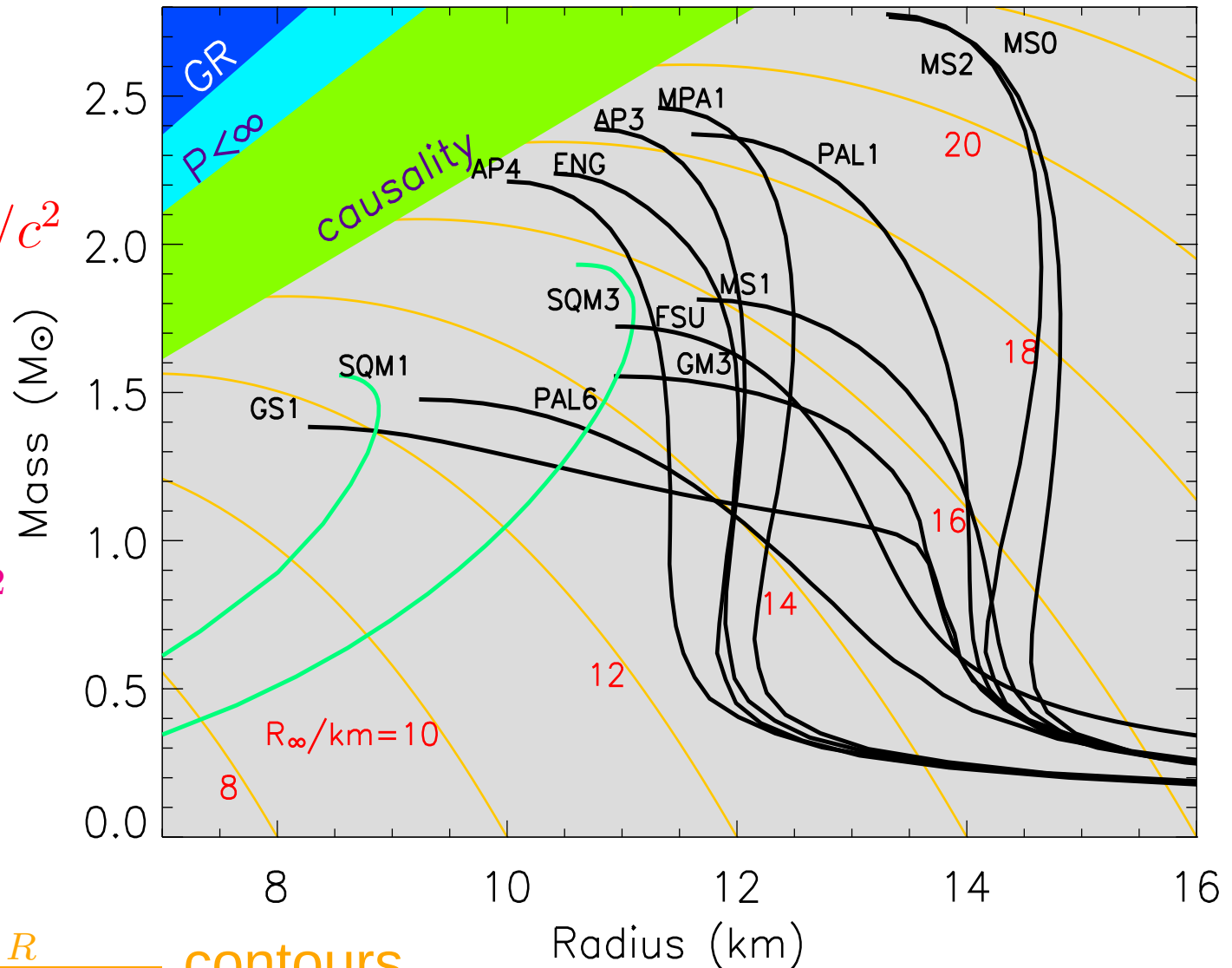
causality:

$$R \gtrsim 2.9GM/c^2$$

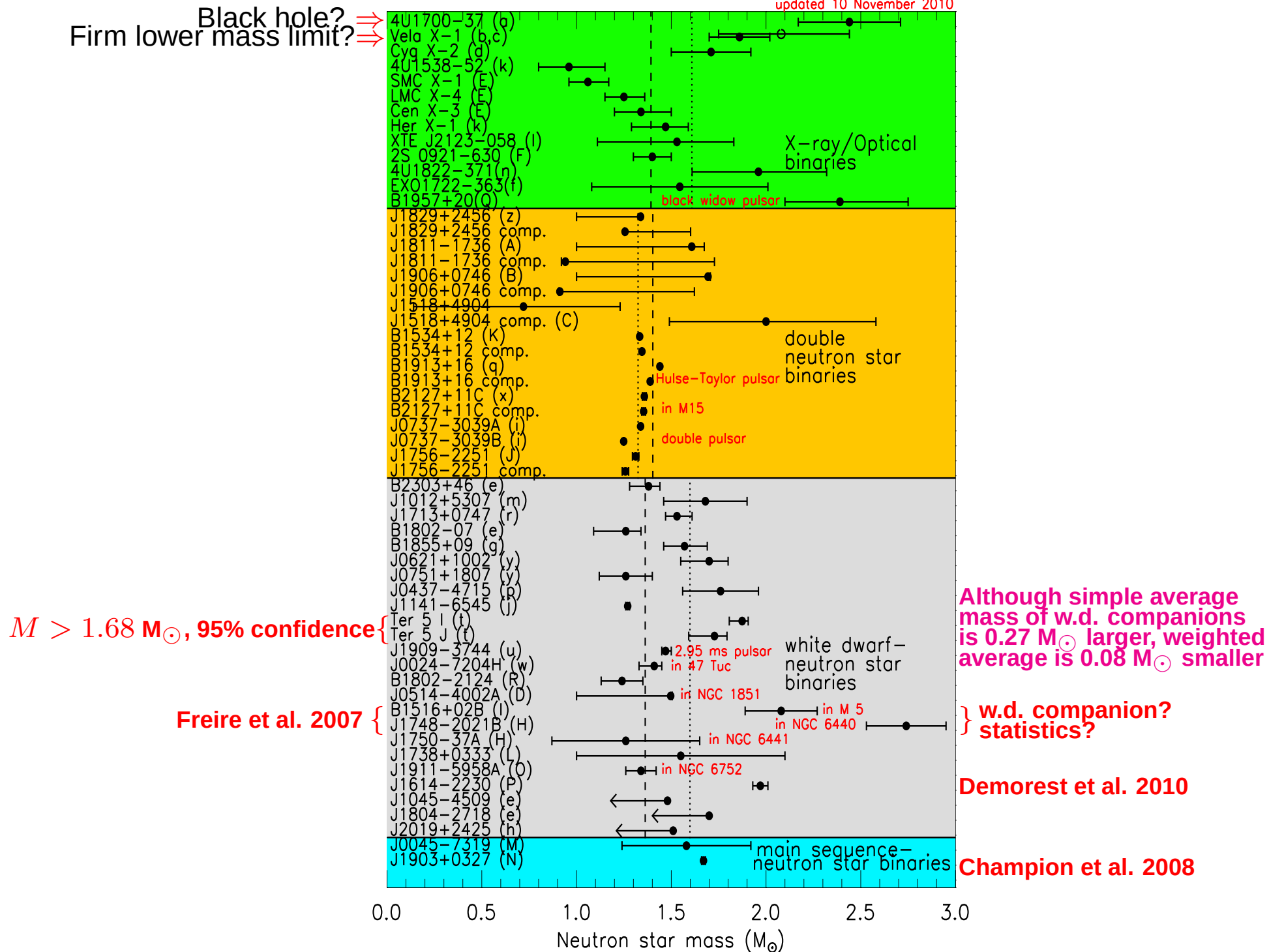
— normal NS

— SQS

$$— R_\infty = \frac{R}{\sqrt{1-2GM/Rc^2}} \text{ contours}$$



Black hole? $\Rightarrow$
Firm lower mass limit? $\Rightarrow$

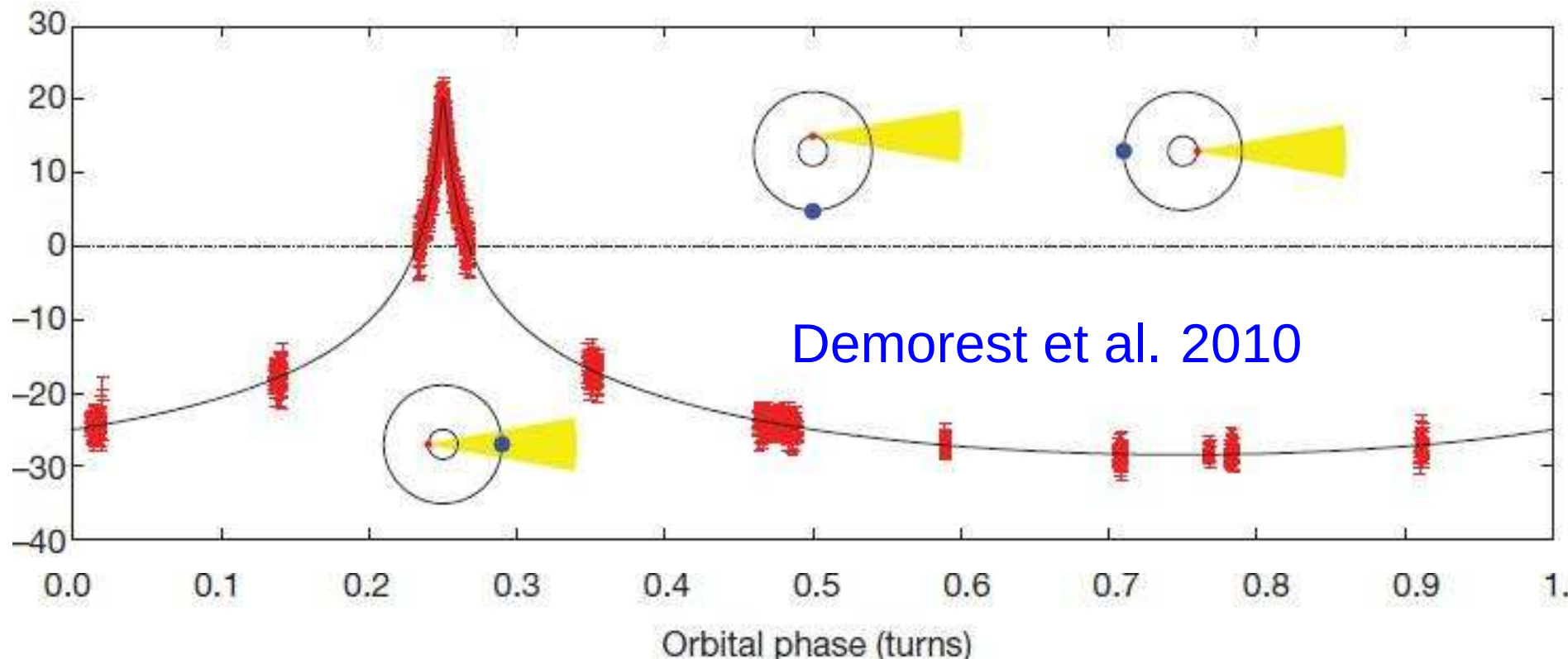


PSR J1614-2230

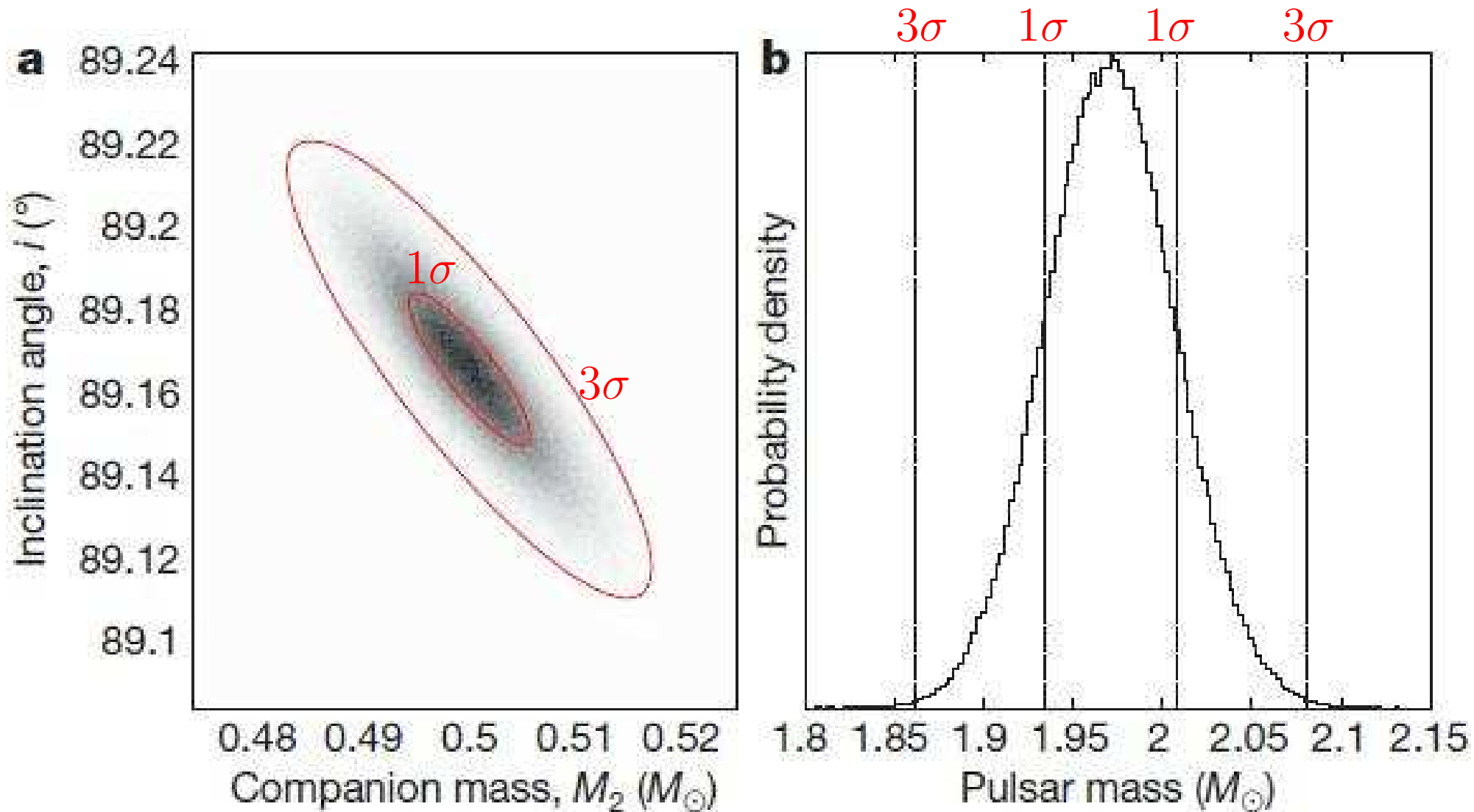
A 3.15 ms pulsar in an 8.69d orbit with an $0.5 M_{\odot}$ white dwarf companion.

Shapiro delay yields edge-on inclination: $\sin i = 0.99984$

Distance > 1 kpc, $B \simeq \times 10^8$ G



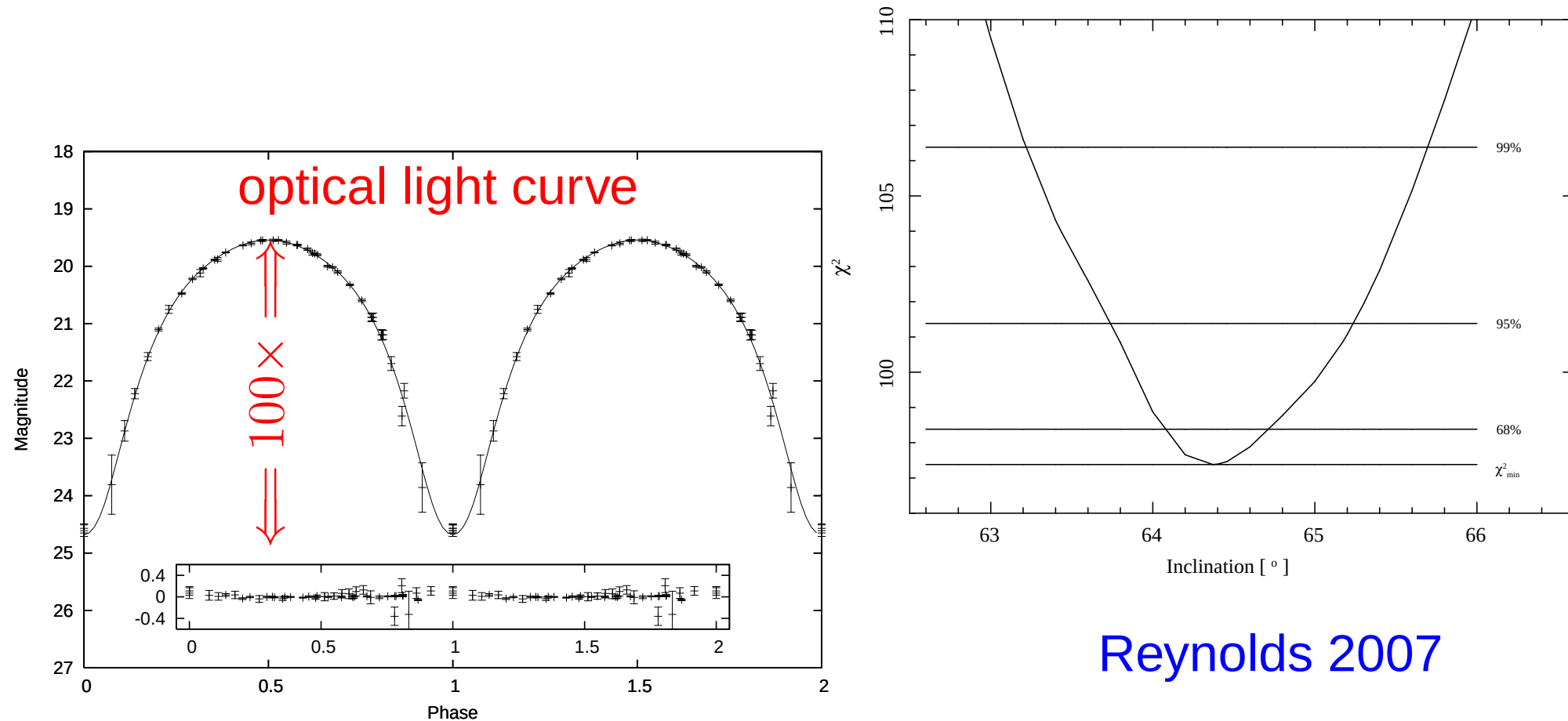
PSR J1614-2230



Demorest et al. 2010

Black Widow Pulsar PSR B1957+20

A 1.6ms pulsar in a circular 9.17h orbit with a low-mass ($\sim 0.03 M_{\odot}$) companion which is bloated and strongly irradiated by pulsar. Light curve shows enormous brightness variations.

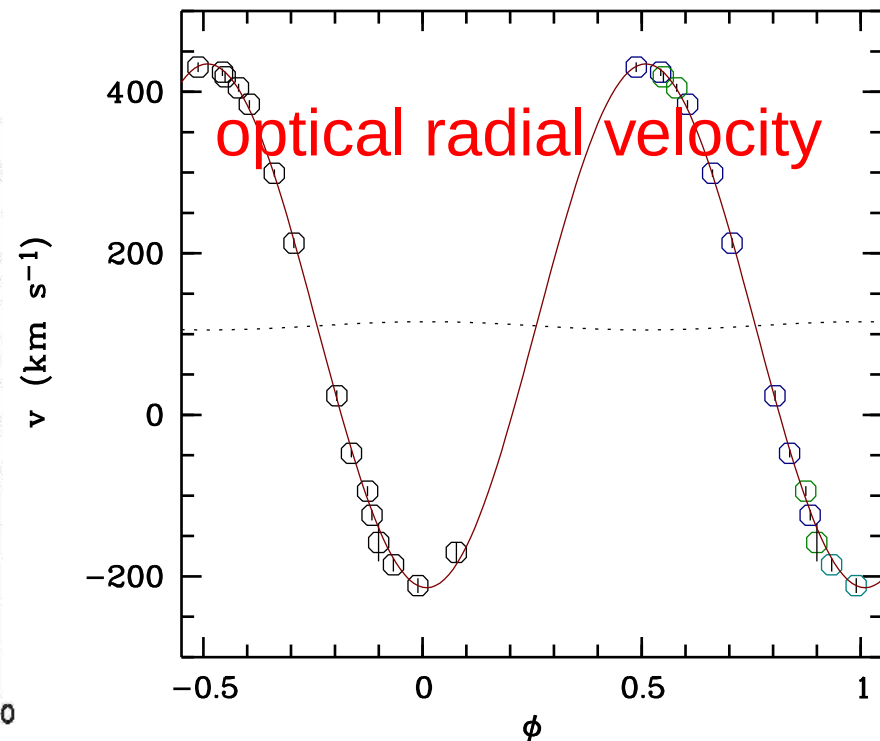
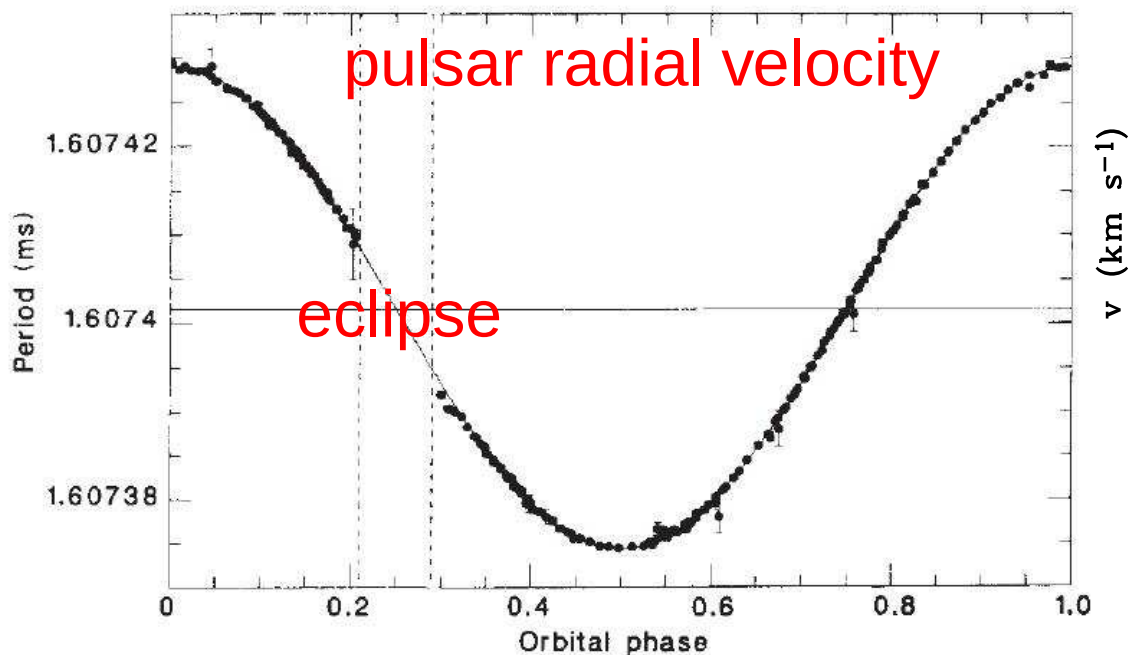


Reynolds 2007

Black Widow Pulsar PSR B1957+20

Pulsar is eclipsed for 50-60 minutes each orbit; eclipsing plasma has a volume much larger than Roche lobe radius of companion.

Companion is ablated by pulsar leading to mass loss and eventual disappearance of companion. Companion nearly fills its Roche lobe.



van Kerkwijk 2010

Black Widow Pulsar PSR B1957+20

$$K_i = 2\pi \frac{a_i \sin i}{P}$$

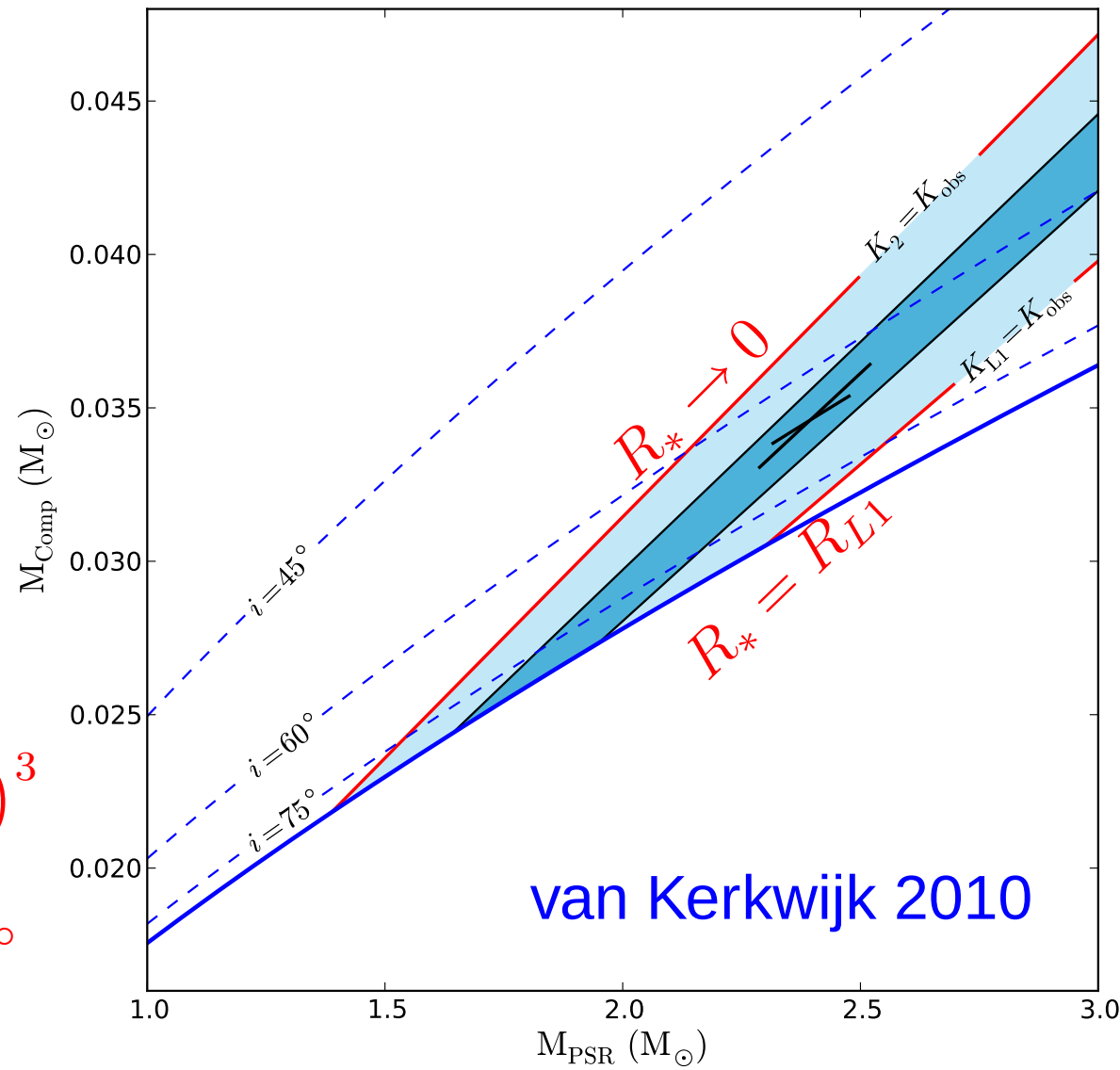
$$q = \frac{M_P}{M_*} = \frac{a_*}{a_P} = \frac{K_*}{K_P}$$

$$K_* = K_{obs} \left(1 + \frac{R_*}{a_*} \right)$$

Radiated surface has
smaller orbit than
center of mass.

$$M_P = q(1 + q)^2 \frac{P}{2\pi G} \left(\frac{K_P}{\sin i} \right)^3$$

$$M_P > 1.8 M_\odot, \sin i < 66^\circ$$



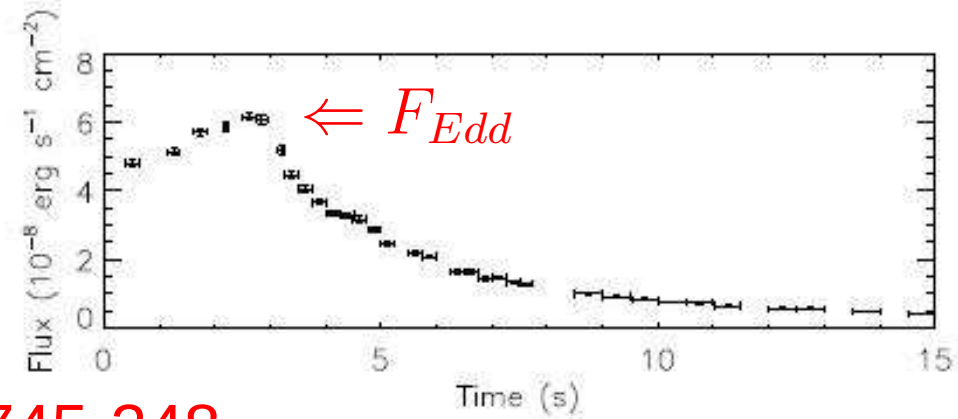
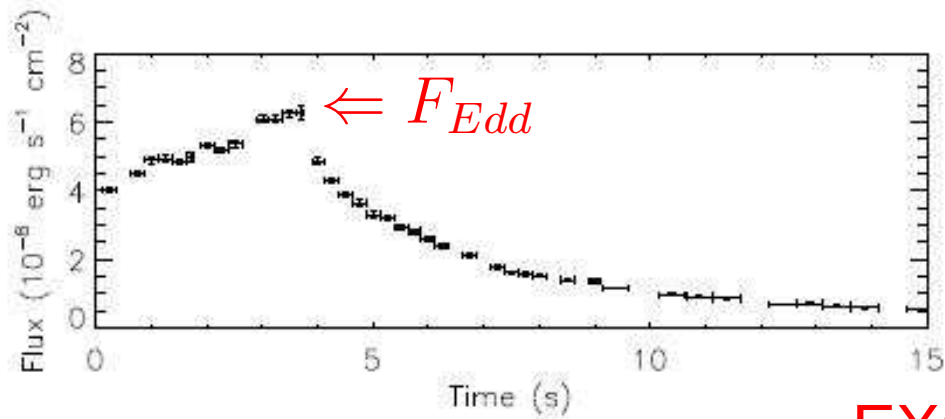
Radiation Radius

- Combination of flux and temperature measurements yields apparent angular diameter (pseudo-BB):

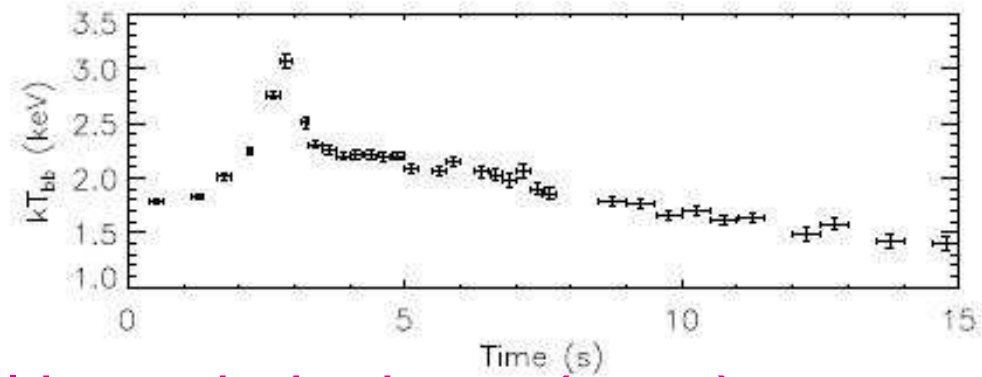
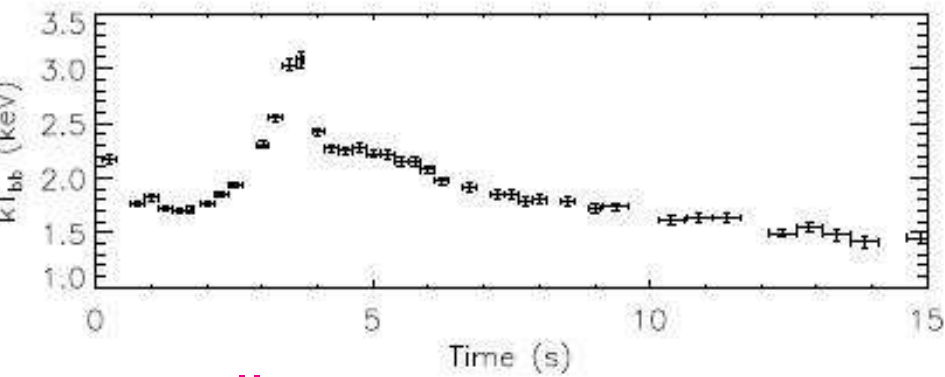
$$\frac{R_\infty}{d} = \frac{R}{d} \frac{1}{\sqrt{1 - 2GM/Rc^2}}$$

- Observational uncertainties include distance, interstellar H absorption (hard UV and X-rays), atmospheric composition
- Best chances for accurate radii are from
 - Nearby isolated neutron stars (parallax measurable)
 - Quiescent X-ray binaries in globular clusters (reliable distances, low *B* H-atmospheres)

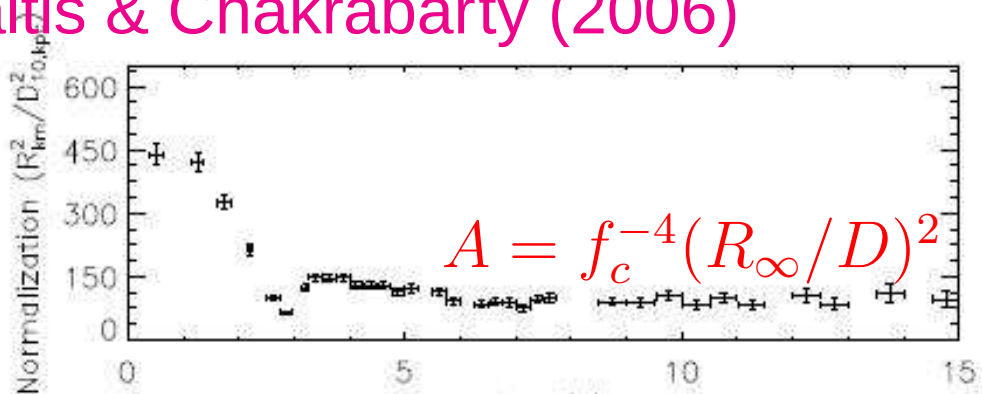
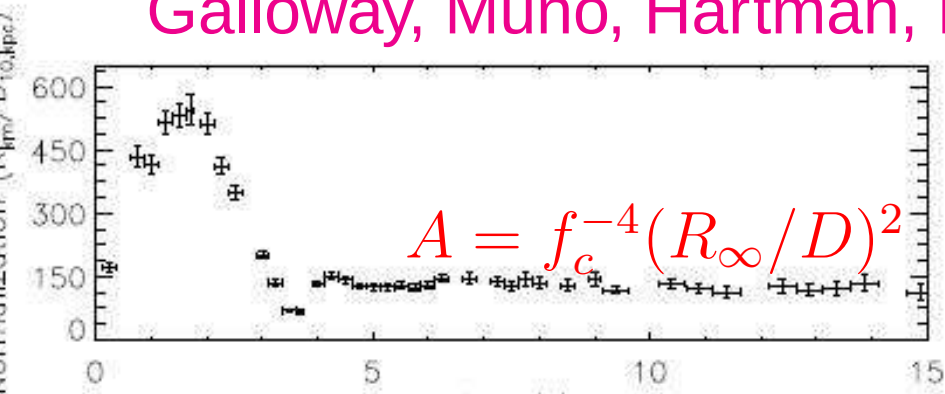
Photospheric Radius Expansion X-Ray Bursts



EXO 1745-248



Galloway, Munro, Hartman, Psaltis & Chakrabarty (2006)



Systematics with $R_{ph} = R$

$$F_{Edd} = \frac{GMc}{\kappa D^2} \sqrt{1 - 2\frac{GM}{R_{ph}c^2}} = \frac{GMc}{\kappa D^2} \sqrt{1 - 2\beta}$$

$$\kappa \simeq 0.2(1 + X) \text{ cm}^2 \text{g}^{-1}$$

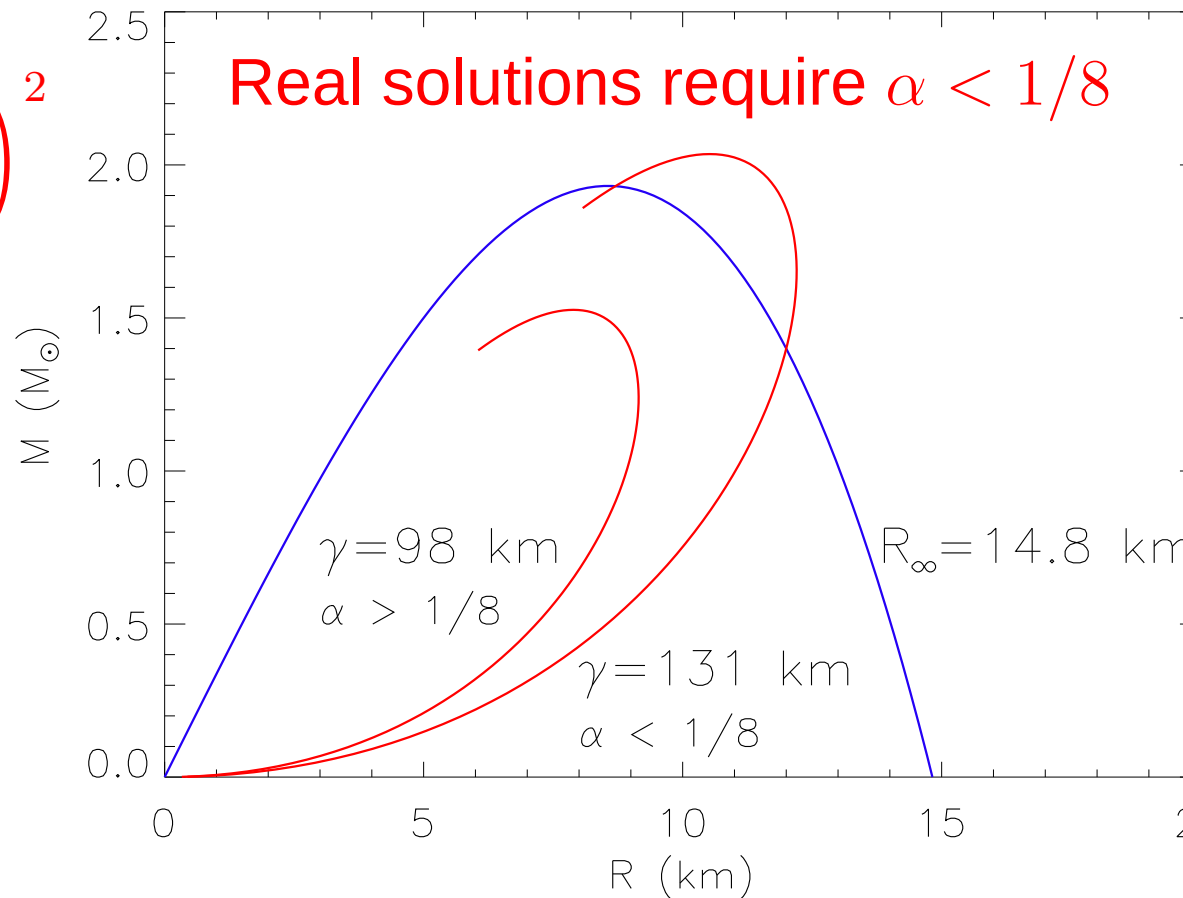
$$A = \frac{F_\infty}{\sigma T_\infty^4} = f_c^{-4} \left(\frac{R_\infty}{D} \right)^2$$

$$\alpha = \frac{F_{Edd}}{\sqrt{A}} \frac{\kappa D}{c^3 f_c^2} = \beta(1 - 2\beta)$$

$$\gamma = \frac{Ac^3 f_c^4}{F_{Edd}\kappa} = \frac{R}{\beta(1 - 2\beta)^{3/2}}$$

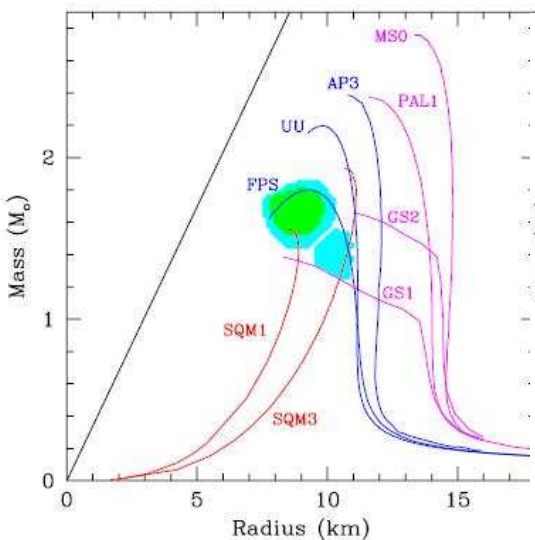
$$\beta = \frac{1}{4} \pm \frac{1}{4} \sqrt{1 - 8\alpha}$$

$$R_\infty = \alpha\gamma$$

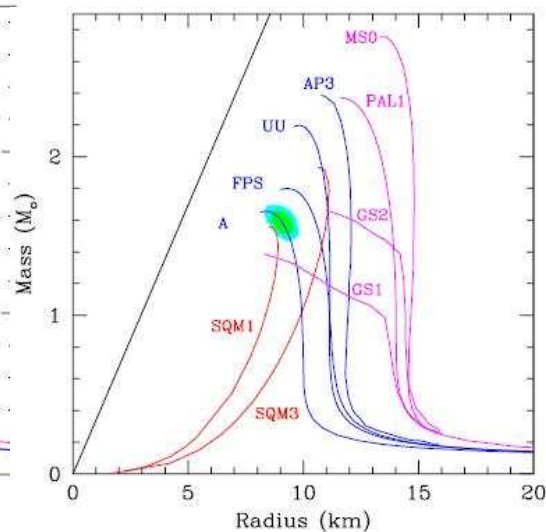


$$R_{ph} = R$$

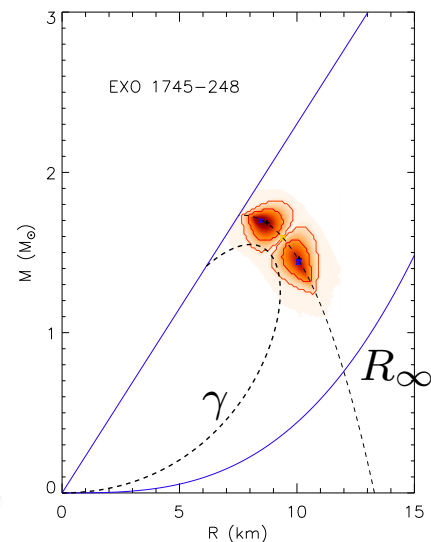
EXO 1745-248
 $\alpha = 0.14 \pm 0.02$



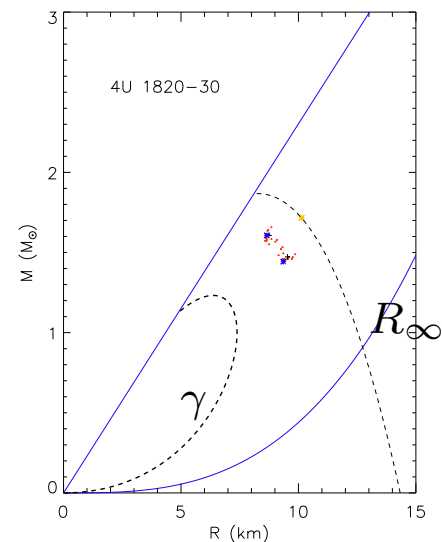
4U 1820-30
 $\alpha = 0.18 \pm 0.02$



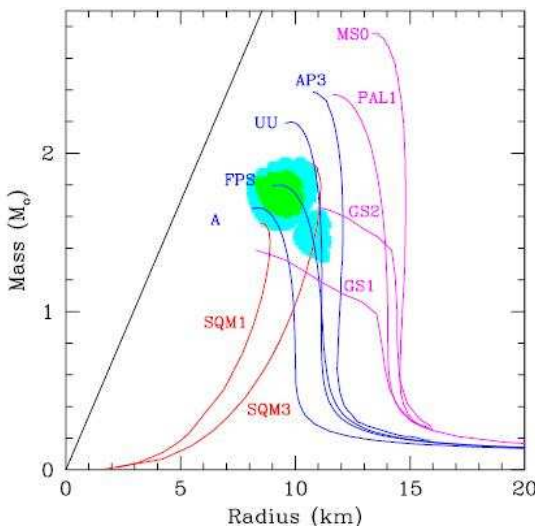
EXO 1745-248
 $\alpha = 0.14 \pm 0.02$



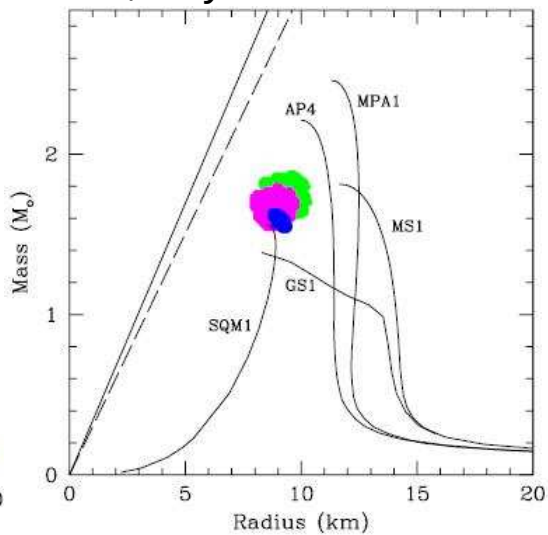
4U 1820-30
 $\alpha = 0.18 \pm 0.02$



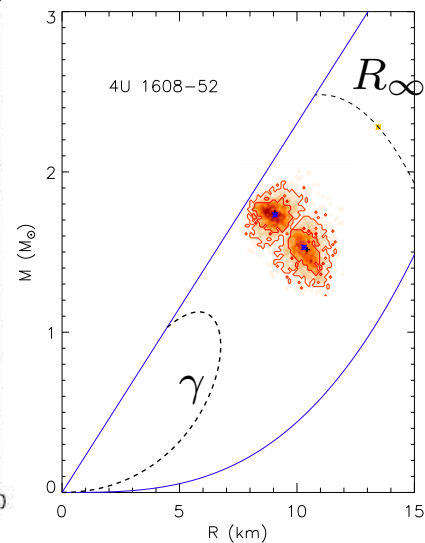
4U 1608-52
 $\alpha = 0.26 \pm 0.11$



Ozel, Baym & Güver 2010



4U 1608-52
 $\alpha = 0.26 \pm 0.11$



Systematics with $R_{ph} \gg R$

$$F_{Edd} = \frac{GMc}{\kappa D^2} \sqrt{1 - 2 \frac{GM}{R_{ph} c^2}} \simeq \frac{GMc}{\kappa D^2}$$

$$\kappa \simeq 0.2(1 + X) \text{ cm}^2 \text{ g}^{-1}$$

$$A = \frac{F_{\infty}}{\sigma T_{\infty}^4} = f_c^{-4} \left(\frac{R_{\infty}}{D} \right)^2$$

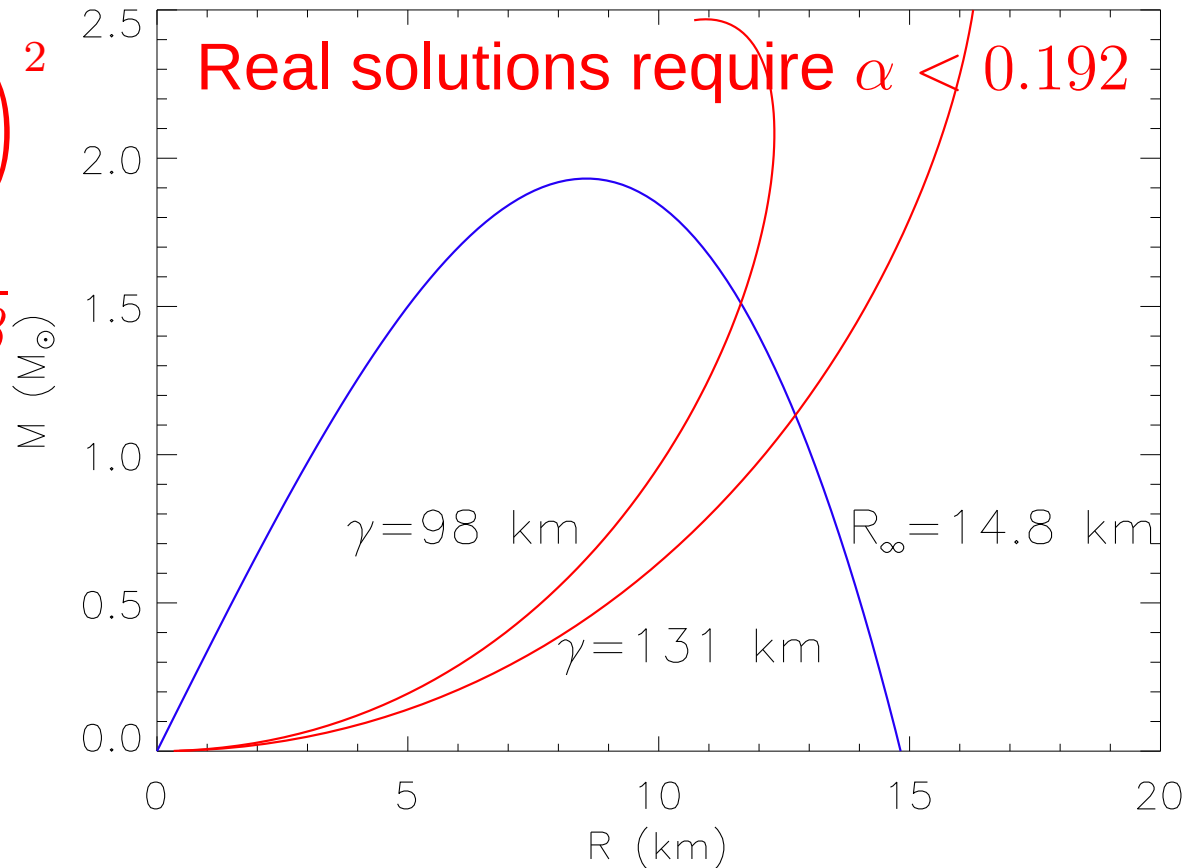
$$\alpha = \frac{F_{Edd}}{\sqrt{A}} \frac{\kappa D}{c^3 f_c^2} = \beta \sqrt{1 - 2\beta}$$

$$\gamma = \frac{Ac^3 f_c^4}{F_{Edd} \kappa} = \frac{R}{\beta(1 - 2\beta)}$$

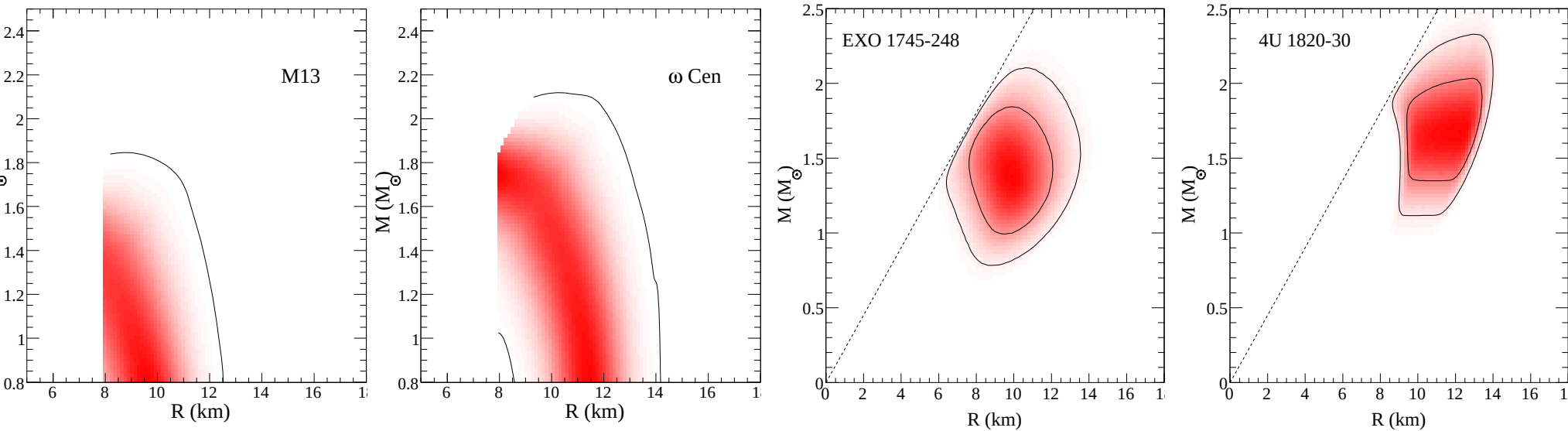
$$\beta = \frac{1}{6} \left[1 + \sqrt{3} \sin \theta - \cos \theta \right]$$

$$\theta = \frac{1}{3} \cos^{-1}(1 - 54\alpha^2)$$

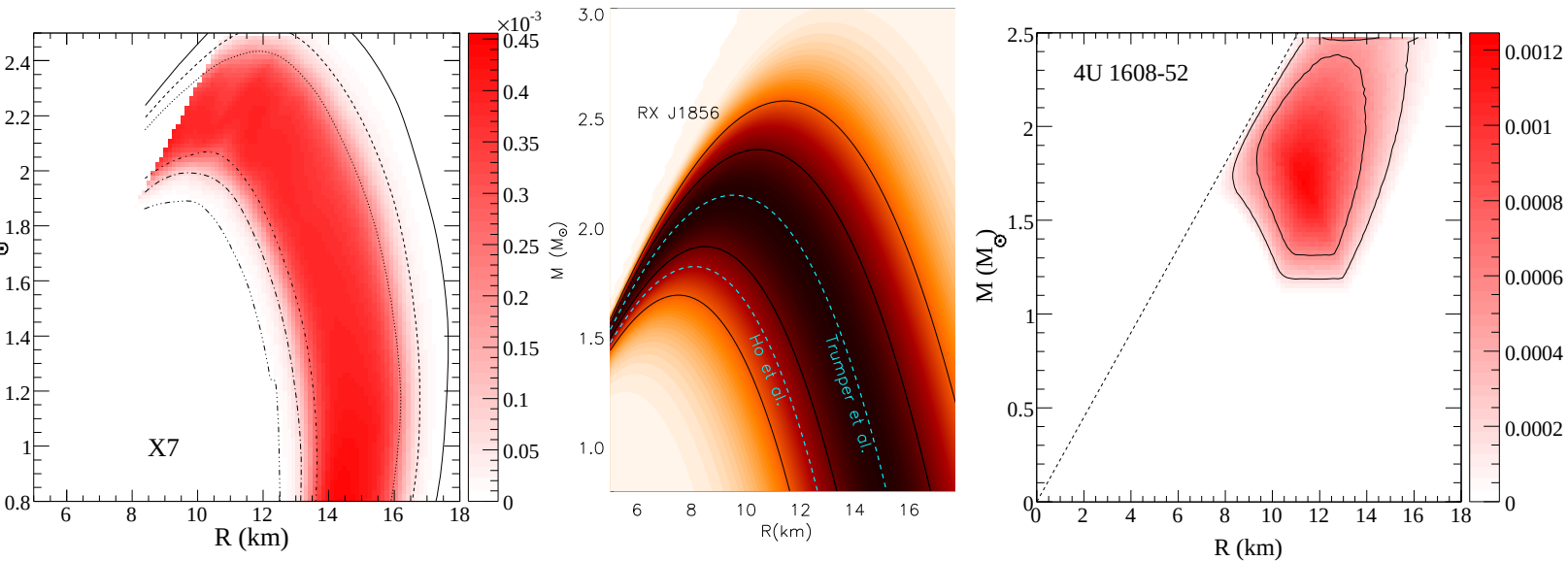
$$R_{\infty} = \alpha \gamma$$



M-R Probability Distributions



Steiner, Lattimer & Brown 2010



Bayesian Analysis

$P(\mathcal{M}_i|D)$: conditional probability of the model $\mathcal{M}_i$ given the data D (what we want).

$$P(\mathcal{M}_i|D) = \frac{P(D|\mathcal{M}_i)P(\mathcal{M}_i)}{\sum_k P(D|\mathcal{M}_k)P(\mathcal{M}_k)} \propto P(D|\mathcal{M}_i).$$

$P(\mathcal{M}_i)$: prior probability of the model $\mathcal{M}_i$ without any information from the data D (assumed uniform).

$P(D)$: prior probability of the data D ($M - R$ probabilities).

$P(D|\mathcal{M}_i)$: conditional probability of the data D given the model $\mathcal{M}_i$. i is the EOS parameter set and the set of specific masses j .

$$P(D|\mathcal{M}_i) \propto \prod_{k,j} \mathcal{D}_{k,j} |_{M=M_j, R=R_k(M_j)}$$

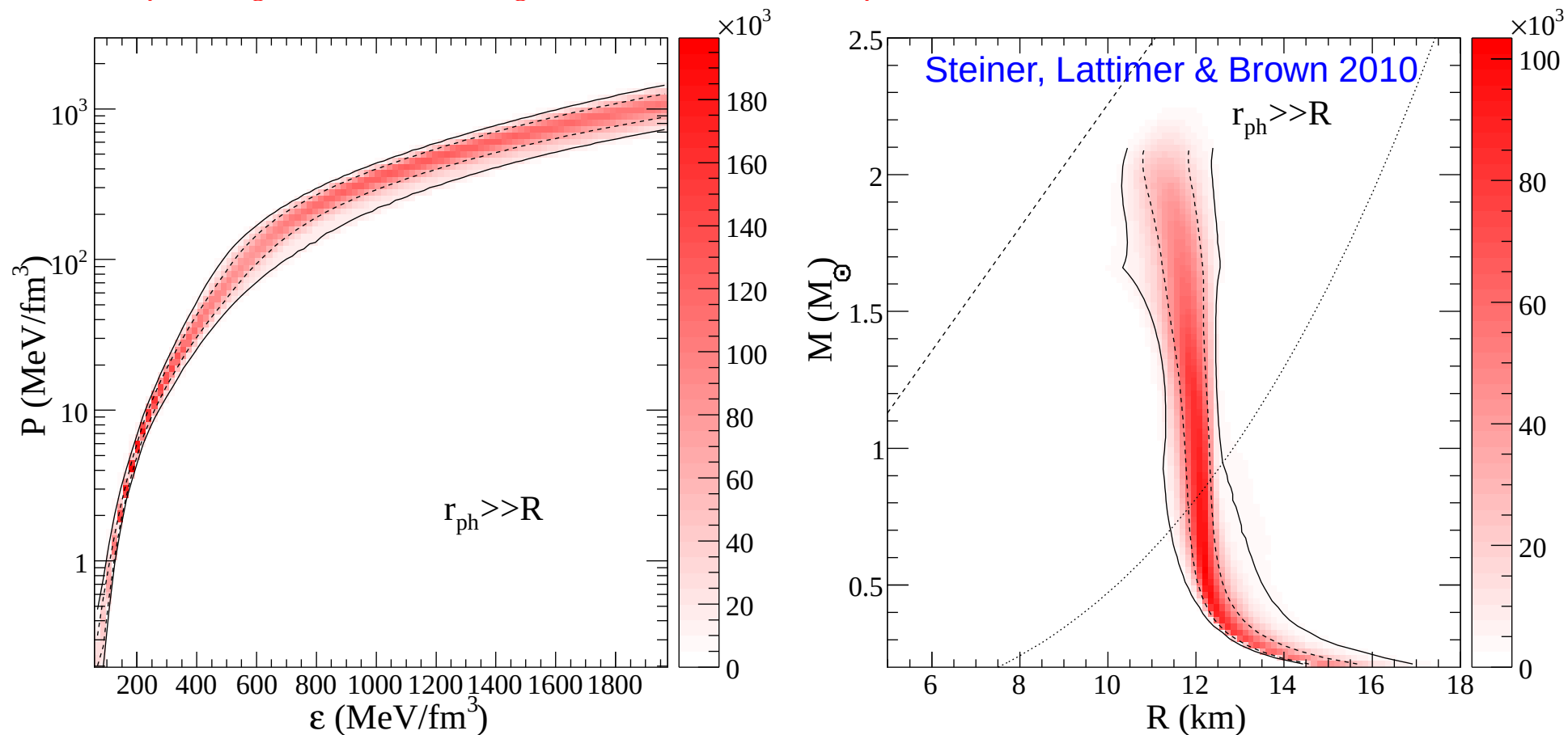
$\mathcal{D}_{k,j}$: probability of observing the radius R_j given a mass M_j in observation k .

The posterior probability for a model parameter p_i is

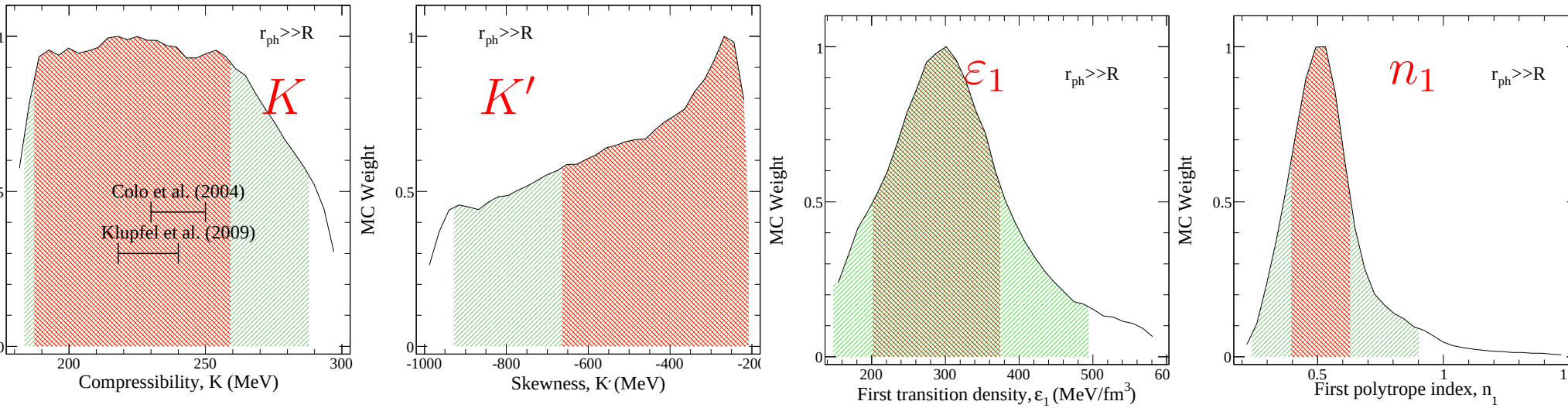
$$P(p_i|D) \propto \int P(D|\mathcal{M}_i) dp_1 dp_2 \dots dp_{i-1} dp_{i+1} \dots dp_{N_p} dM_1 dM_2 \dots dM_{N_M}$$

Bayesian TOV Inversion

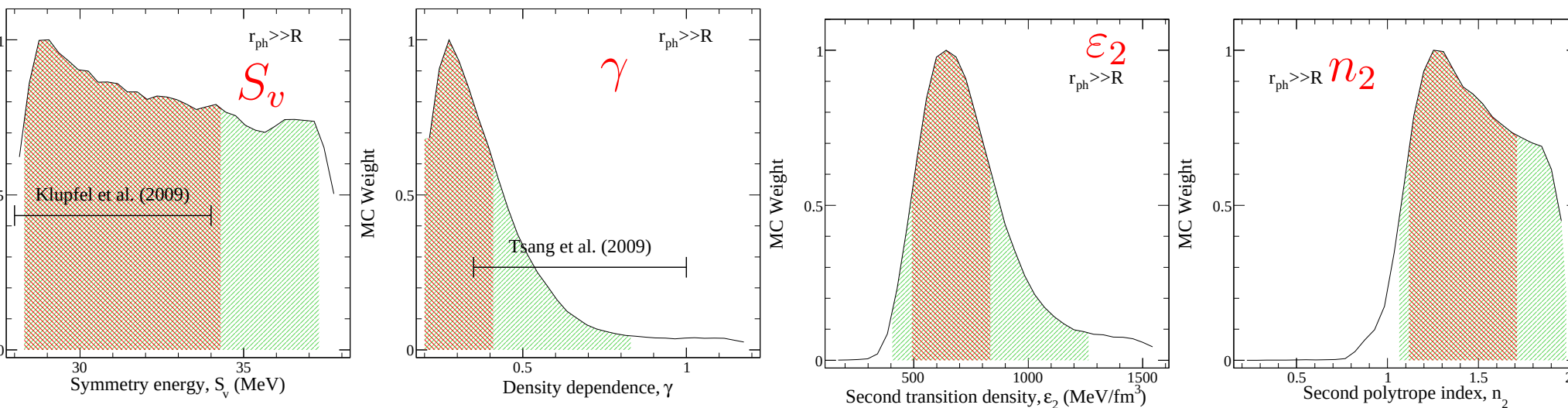
- $\varepsilon < 0.5\varepsilon_0$: EOS from BBP and NV
- $0.5\varepsilon_0 < \varepsilon < \varepsilon_1$: EOS parametrized by K, K', S_v, γ
- $\varepsilon_1 < \varepsilon < \varepsilon_2$: EOS is polytrope with n_1 ; $\varepsilon > \varepsilon_2$: EOS is polytrope with n_2
- A-priori EOS parameters ($K, K', S_v, \gamma, \varepsilon_1, n_1, \varepsilon_2, n_2$) uniformly distributed
- M and R probability distributions for 7 neutron stars treated equally ($0.8 M_\odot < M < 2.5 M_\odot$; $5 \text{ km} < R < 18 \text{ km}$)



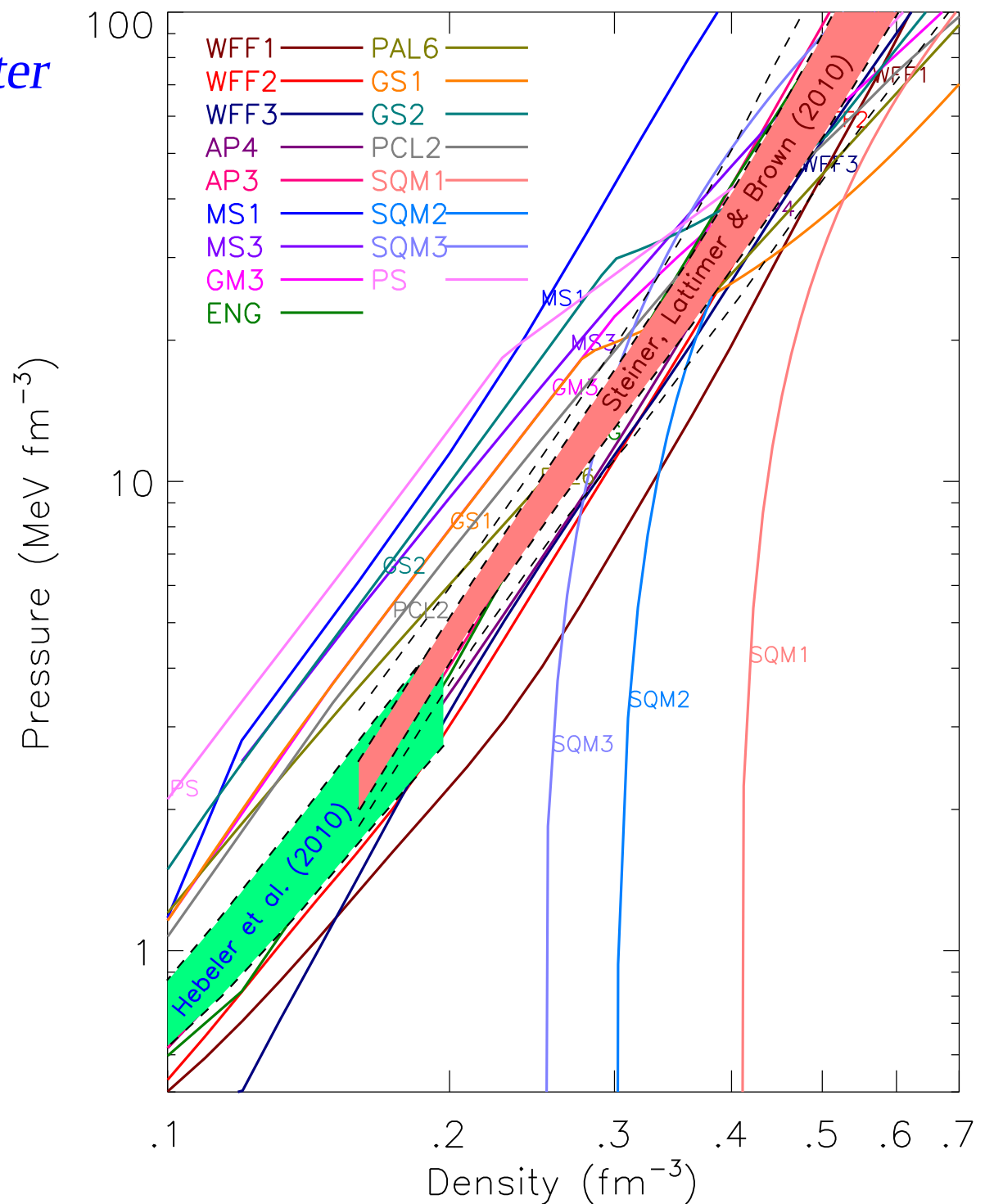
Inferred Model EOS Parameters



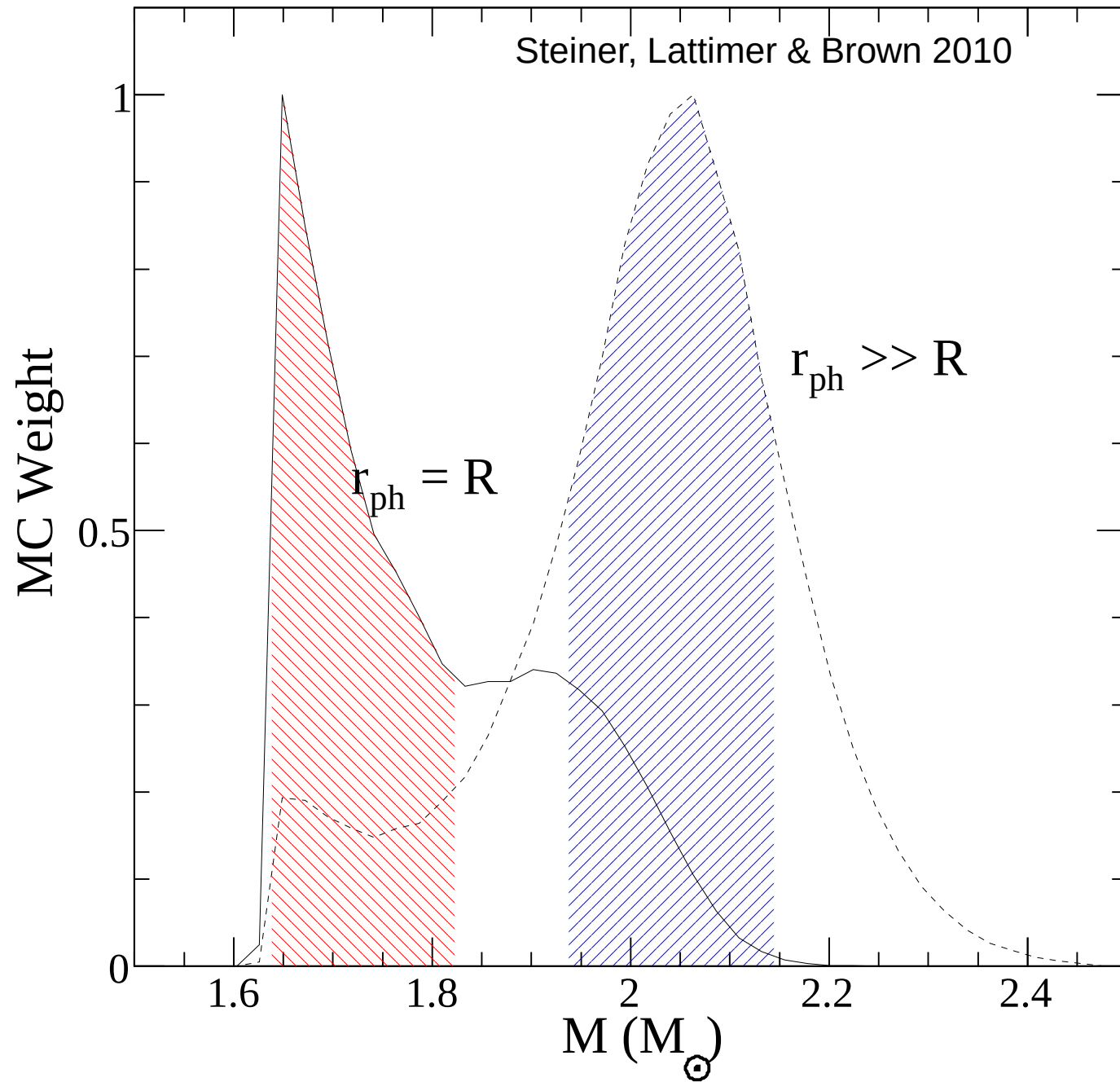
Steiner, Lattimer & Brown 2010



Consistency with Neutron Matter Calculations



Maximum Mass Probability Distributions



Neutron Star Cooling

Gamow & Schönberg proposed the direct Urca process: nucleons at the top of the Fermi sea beta decay.



Energy conservation guaranteed by beta equilibrium

$$\mu_n - \mu_p = \mu_e$$

Momentum conservation requires $|k_{Fn}| \leq |k_{Fp}| + |k_{Fe}|$.

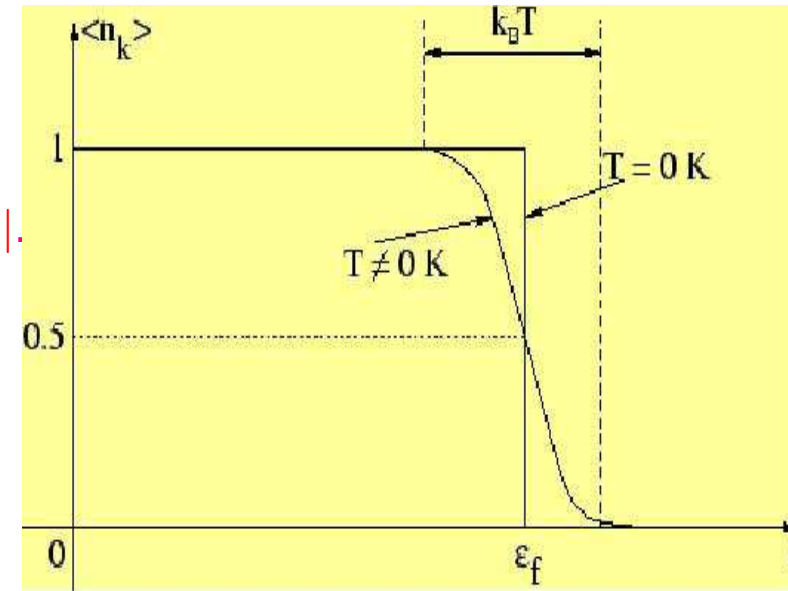
Charge neutrality requires $k_{Fp} = k_{Fe}$,

therefore $|k_{Fn}| \geq 2|k_{Fn}|$.

Degeneracy implies $n_i \propto k_{Fi}^3$, thus $x \geq x_{DU} = 1/9$.

With muons ($n > 2n_s$), $x_{DU} = \frac{2}{2+(1+2^{1/3})^3} \simeq 0.148$

If $x < x_{DU}$, bystander nucleons needed:
modified Urca process is then dominant.



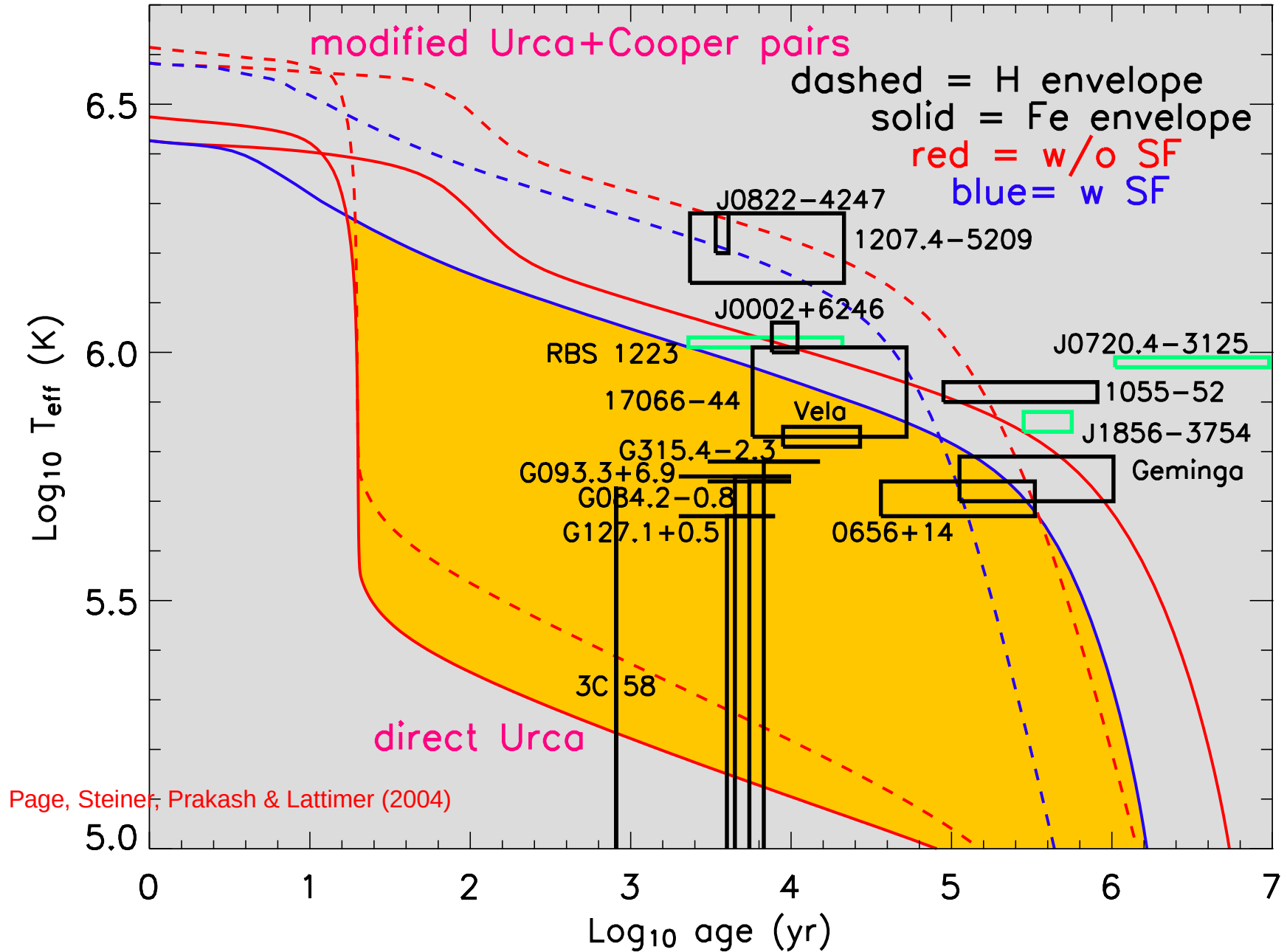
Neutrino emissivities:

$$\dot{\epsilon}_{MU} \simeq \left(\frac{T}{\mu_n} \right)^2 \dot{\epsilon}_{DU}.$$

Beta equilibrium composition:

$$x_\beta \simeq (3\pi^2 n)^{-1} \left(\frac{4E_{sym}}{\hbar c} \right)^3 \simeq 0.04 \left(\frac{n}{n_s} \right)^{0.5-2}.$$

Neutron Star Cooling



Transient Rapid Cooling

MURCA emissivity $\dot{\epsilon}_{MU} \propto T^8$

PBF emissivity $\dot{\epsilon}_{PBF} \propto F(T) T^7$
 $\sim T^8 \simeq f \epsilon_{MU}$

Specific heat $C_V \propto T$

Neutrino dominated
cooling

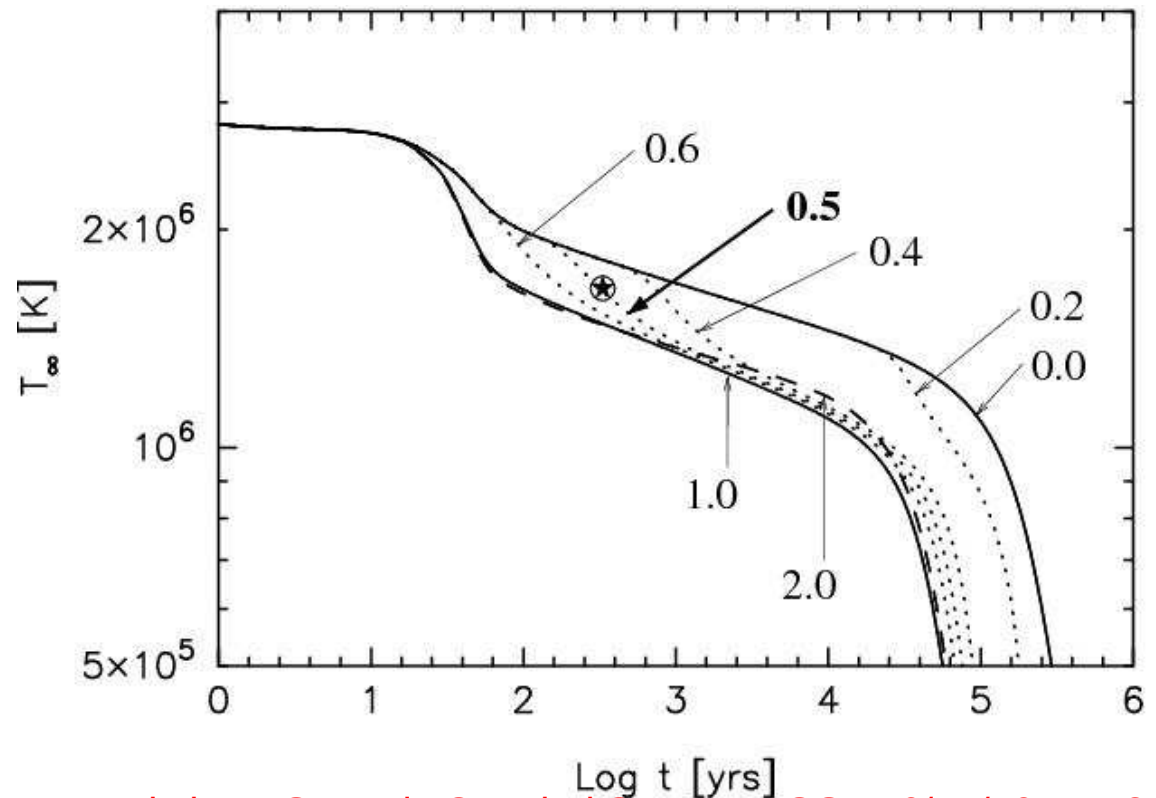
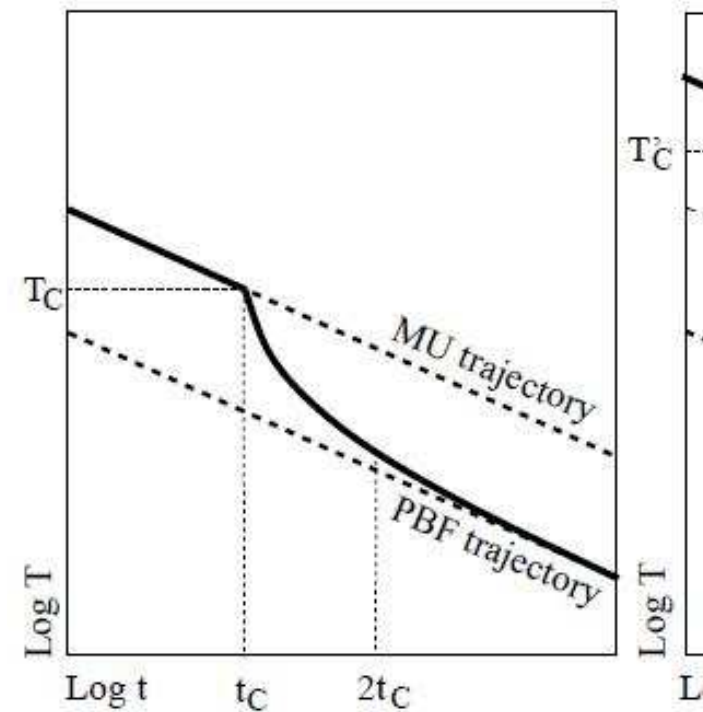
$$C_V dT/dt = -L_\nu$$

$$T \propto (t/\tau)^{-1/6}$$

$$\tau_{PBF} = \tau_{MU}/f$$

$$(d \ln T / d \ln t)_{transient} \simeq (1 - 25) (d \ln T / d \ln t)_{MU}$$

Very sensitive to T_C



Cas A

Remnant of Type IIb
(gravitational collapse,
no H envelope) SN in
1680 (Flamsteed).

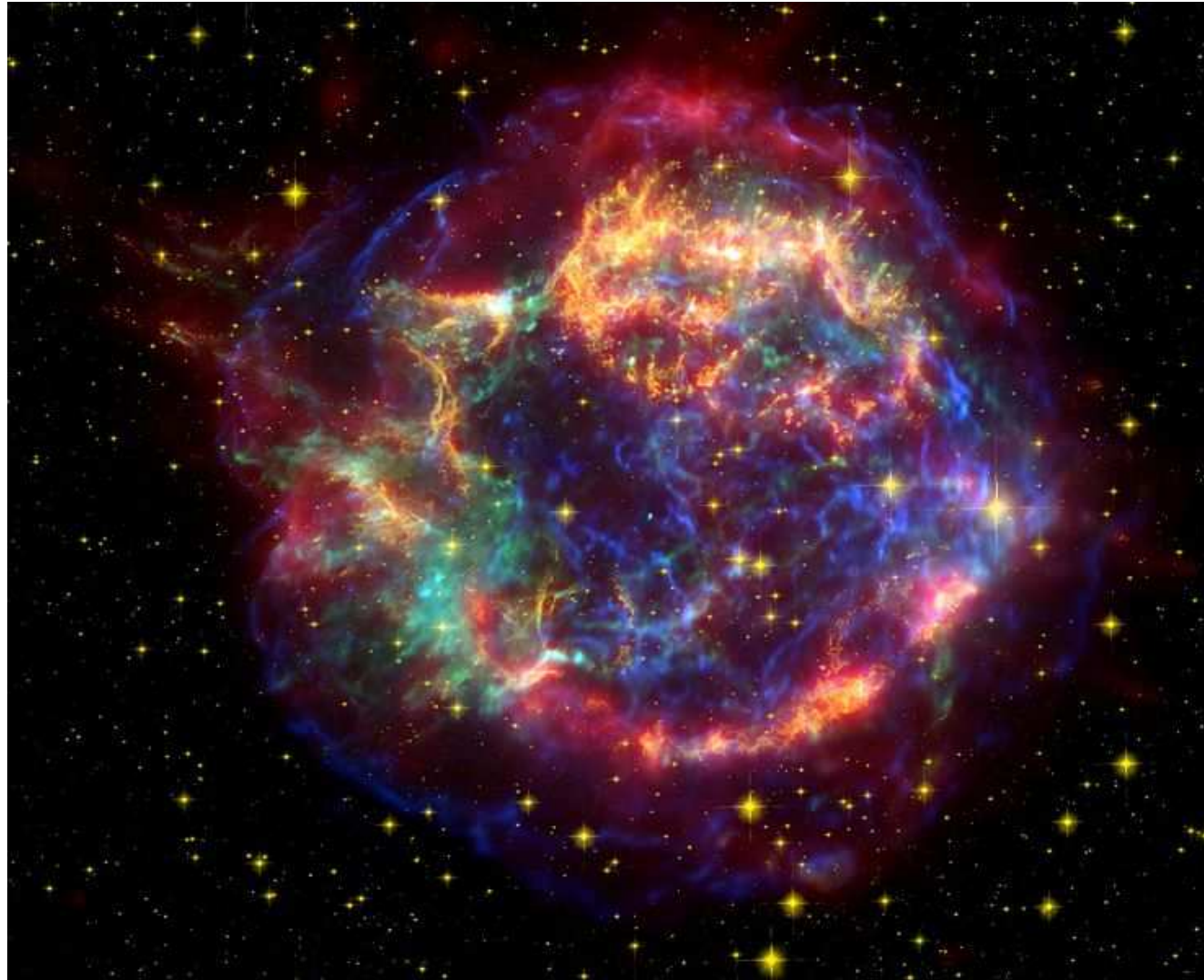
3.4 kpc distance

3.1 pc diameter

Strongest radio source
outside solar system,
discovered in 1947.

X-ray source detected
(Aerobee flight, 1965)

X-ray point source detected
(Chandra, 1999)



Spitzer, Hubble, Chandra

Cas A

X-ray spectrum
indicates thin C
atmosphere
(Ho & Heinke 2009)

10 years of X-ray
data show

cooling at rate

$$\frac{d \ln T_e}{d \ln t} = -1.25 \pm 1.0$$

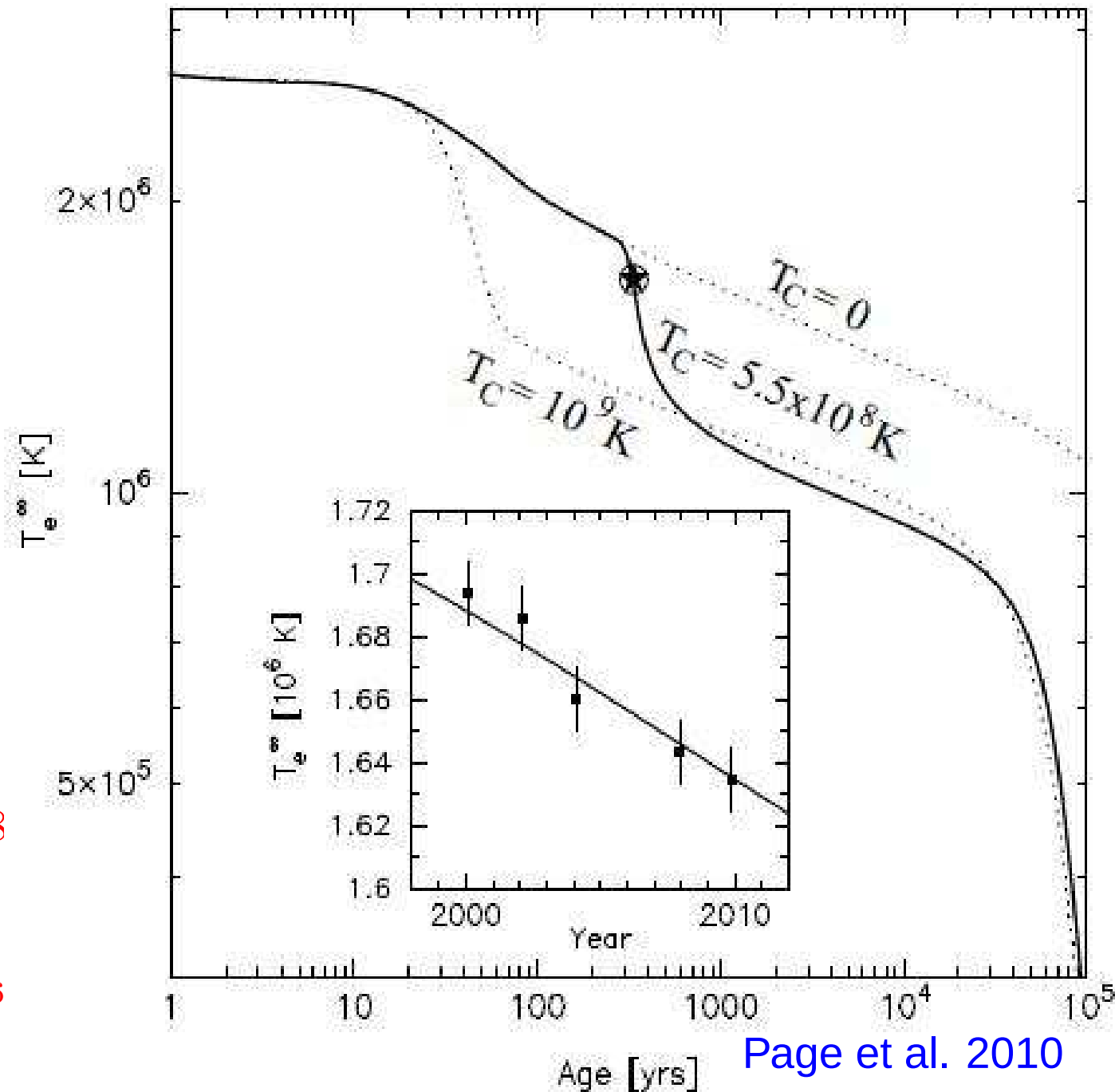
(Heinke & Ho 2010)

Modified Urca:

$$\left(\frac{d \ln T_e}{d \ln t} \right)_{MU} \simeq -0.08$$

$$T_C \simeq 5 \pm 1 \times 10^8 \text{ K}$$

$$T_C \propto (t_C L / C_V)^{-1/6}$$



Page et al. 2010

Conclusions

- Maximum neutron star mass may be above $2 M_{\odot}$ based on pulsar mass measurements and inferred from astrophysical observations of photospheric radius expansion bursts and thermal emissions.
- Quark stars or quark cores in neutron stars can be ruled out if $M_{max} \geq 2.4 M_{\odot}$.
- In spite of large estimated errors for individual stars, taken as an ensemble, they predict a remarkably tight pressure-density relation.
- Astrophysical observations are consistent with predictions of neutron matter calculations.
- Predictions:
 - A very soft nuclear symmetry energy, $\gamma = 0.3 \pm 0.1$.
 - Smallish radii of $1.4 M_{\odot}$ neutron stars, 11.3 ± 0.3 km.
 - Small neutron skin thickness of ^{208}Pb , $\delta R \simeq 0.15 \pm 0.02$ fm.
 - A small n $^3\text{P}_2$ gap, $T_C \simeq 5 \times 10^8$ K.