Relativistic Physics with a Microwave Photonic Crystal Simulating Graphene

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EMMI Workshop on Strongly Coupled Systems 2010

- Something about graphene and microwave billiards
- Dirac spectrum in a photonic crystal
  - Experimental setup
  - Transmission and reflection spectra
- Extremal transmission
- Photonic crystal in a box
- Some personal remarks

Supported by DFG within SFB 634
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Nobel Prize in Physics 2010

Photo: Sergeom, Wikimedia Commons

Andre Geim

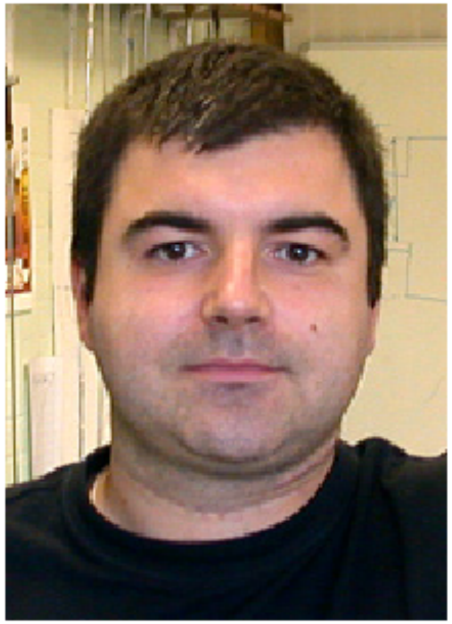


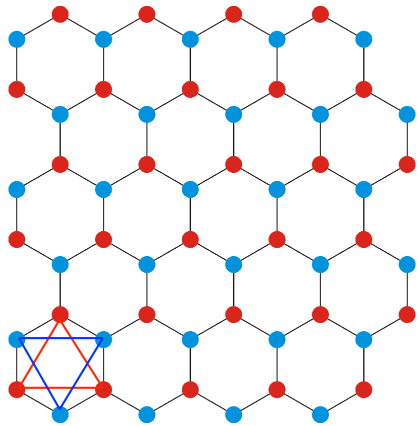
Photo: University of Manchester, UK

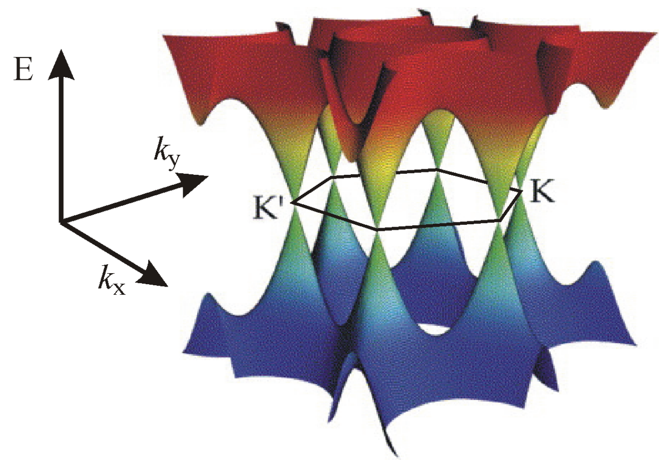
Konstantin Novoselov

The Nobel Prize in Physics 2010 was awarded jointly to Andre Geim and Konstantin Novoselov *"for groundbreaking experiments regarding the two-dimensional material graphene"*

Graphene

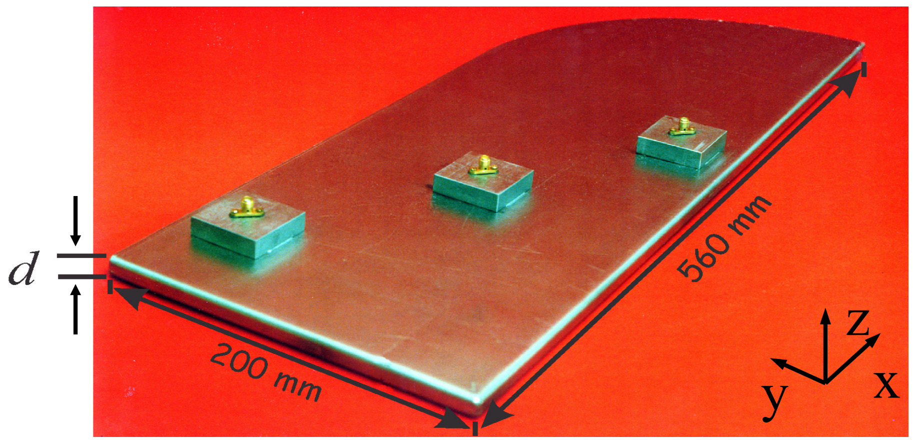
- "What makes graphene so attractive for research is that the spectrum closely resembles the Dirac spectrum for massless fermions."
  M. Katsnelson, Materials Today, 2007





- Two triangular sublattices of carbon atoms
- Near each corner of the first hexagonal Brillouin zone the electron energy E has a conical dependence on the quasimomentum
- Experimental realization of graphene in analog experiments of microwave photonic crystals

Closed Flat Microwave Billiards: Model Systems for Quantum Phenomena



vectorial Helmholtz equation
$$\left(\Delta + k_\mu^2\right)\vec{E}_\mu(\vec{r}) = 0, \quad \vec{n} \times \vec{E}_\mu(\vec{r})\Big|_{\partial G} = 0, \quad k_\mu = \frac{2\pi f_\mu}{c}$$

cylindrical resonators
$$f \le f_{\max} = \frac{c}{2d} \quad \Rightarrow \quad \vec{E}_\mu(\vec{r}) = E_\mu(x,y)\vec{e}_z$$

scalar Helmholtz equation
$$\left(\Delta + k_\mu^2\right)E_\mu(x,y) = 0, \quad E_\mu(x,y)\Big|_{\partial G} = 0$$

→ Schrödinger equation for quantum billiards

Open Flat Microwave Billiard: Photonic Crystal

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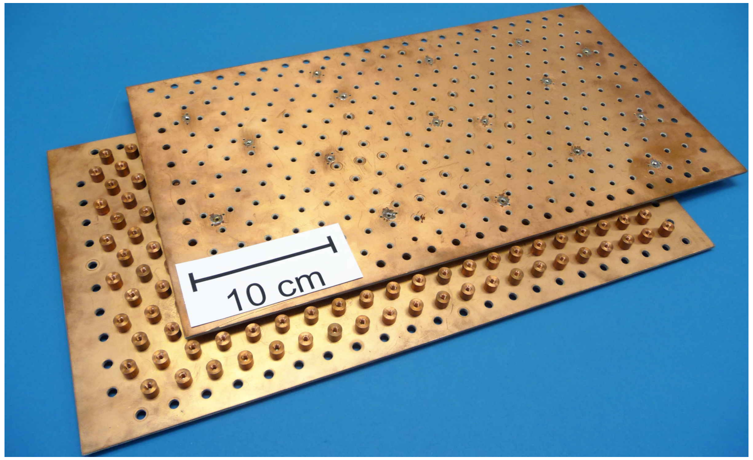
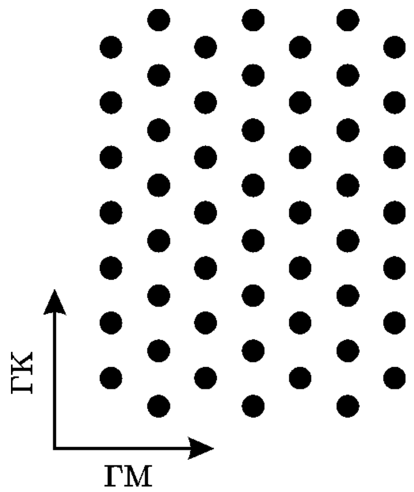
- A photonic crystal is a structure, whose electromagnetic properties vary periodically in space, e.g. an array of metallic cylinders
  → open microwave resonator

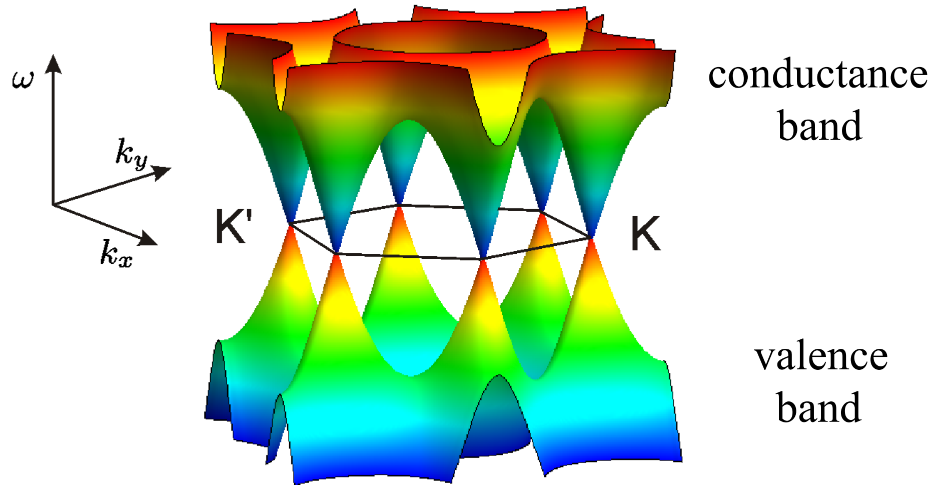


- Flat "crystal" (resonator) → E-field is perpendicular to the plates (TM_0 mode)
- Propagating modes are solutions of the scalar Helmholtz equation
  → Schrödinger equation for a quantum multiple-scattering problem
  → Numerical solution yields the band structure

Calculated Photonic Band Structure

- Dispersion relation $\omega(\vec{k})$ of a photonic crystal exhibits a band structure analogous to the electronic band structure in a solid



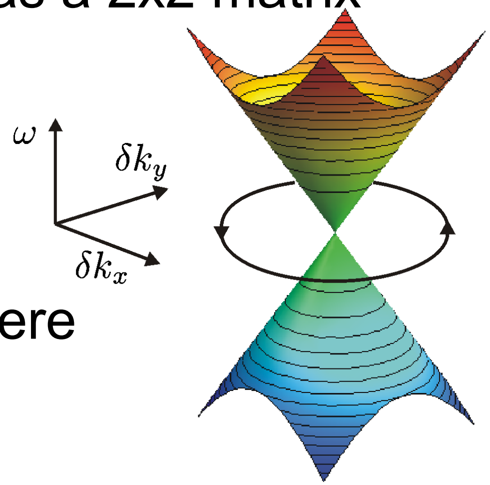


- The triangular photonic crystal possesses a conical dispersion relation
  $\rightarrow$ Dirac spectrum with a Dirac point where bands touch each other

Hamiltonian in the Vicinity of the Dirac Point

- Close to Dirac point D the Hamiltonian can be written as a 2x2 matrix

$$\hat{H} = \omega_D \mathbb{1} + v_D \left(\delta k_x \hat{\sigma}_x + \delta k_y \hat{\sigma}_y \right)$$



- $\hat{\sigma}_x, \hat{\sigma}_y$ are Pauli spin matrices, v_D is the group velocity

- Hamiltonian of this type possesses a diabolic point where the eigenvalues are degenerate

- Encircling of a diabolic point in the parameter space changes the sign of eigenvectors (Berry phase)

$$\begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \circlearrowleft - \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

- Encircling in $(\delta k_x, \delta k_y)$ plane corresponds to a 2π rotation of the coordinate frame → spinor property

Dirac Equation

- Substitution $\delta k_x \to -i\partial_x$ and $\delta k_y \to -i\partial_y$ leads to the Dirac equation

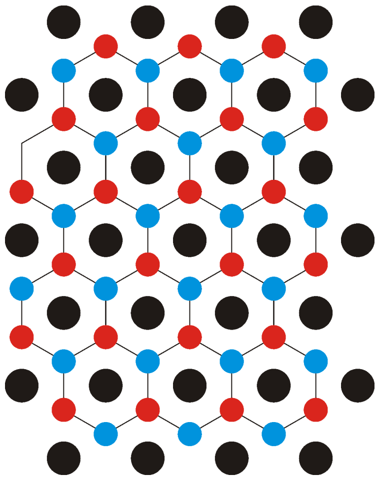
$$\begin{pmatrix} 0 & \partial_x - i\partial_y \\ \partial_x + i\partial_y & 0 \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = i\,\frac{\omega - \omega_D}{v_D} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

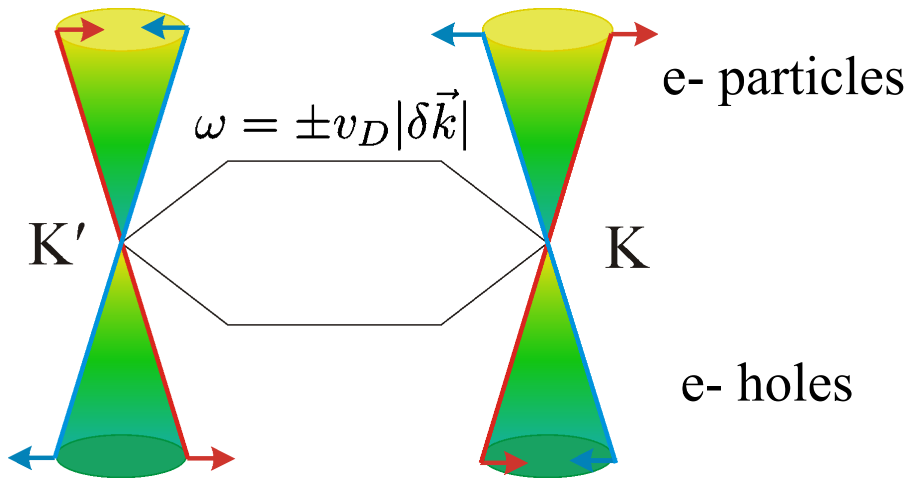
- The components ψ_1, ψ_2 of the spinor are the amplitudes of two degenerate Bloch states at the Dirac frequency ("pseudospin")

- In the vicinity of the Dirac frequency ω_D, waves in a photonic crystal can be effectively described by the Dirac equation

- The same equation holds for the spinor around the K' corner in the 1st Brillouin zone

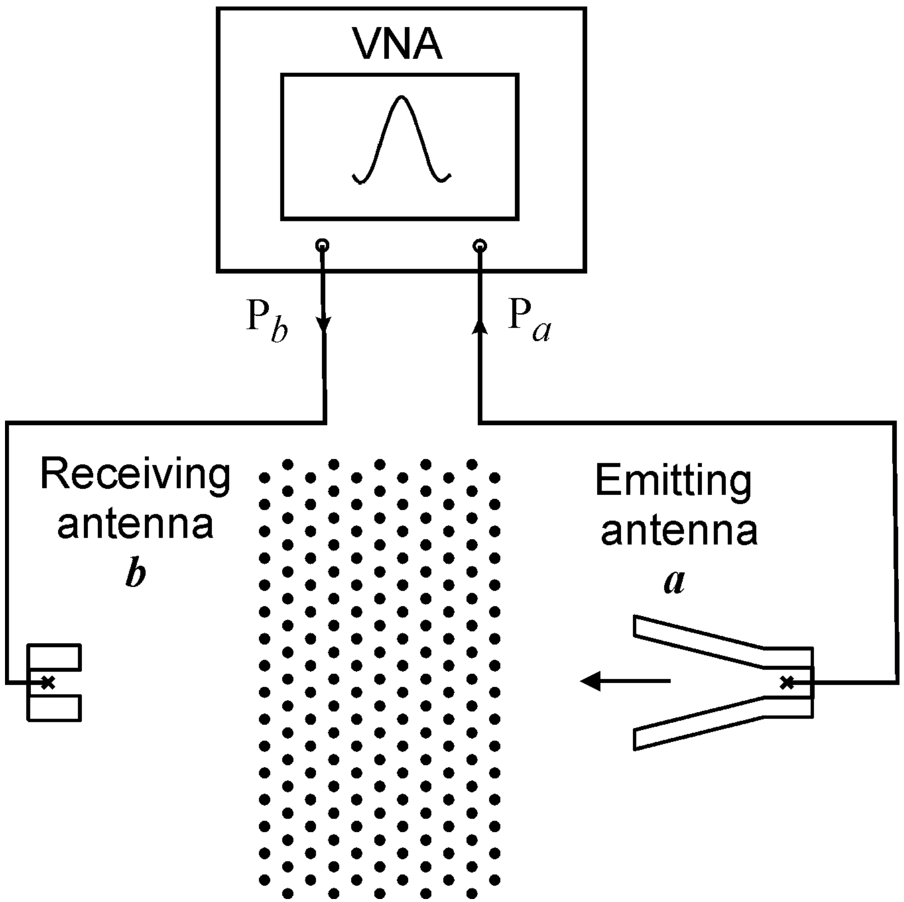
Pseudospin Interpretation





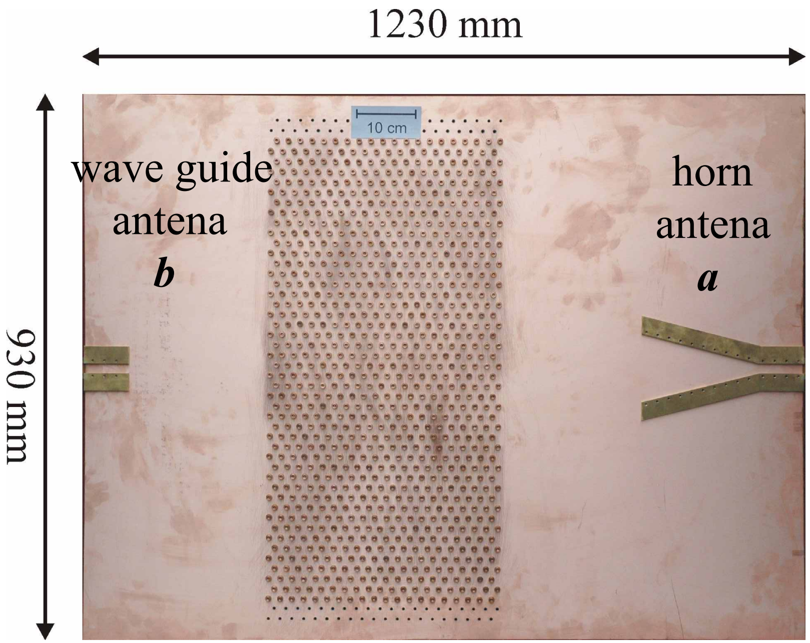
- Voids → correspond to the two triangular sublattices of carbon atoms
- Pseudospin remains fixed along the respective branches of the electronic spectrum in graphene
- In the photonic crystal we have the same degrees of freedom like in graphene → pseudospin, valley structure K' and K
- However, the anology is not complete: fermionic (electron spin) vs. bosonic (em. waves) properties, but this is not relevant for our studies
- Dispersion relation $\omega(\vec{k})$ describes the conical band structure → experiment

Scattering Experiment

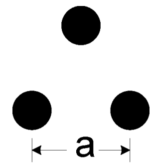


- Horn antenna emits approximately plane waves
- VNA measures the modulus of the scattering matrix given by
  $$|S_{ba}|^2 = \frac{P_b}{P_a}$$
- Transmission: $|S_{ab}|^2, |S_{ba}|^2$
- Reflection: $|S_{aa}|^2, |S_{bb}|^2$

Experimental Realization of 2D Photonic Crystal



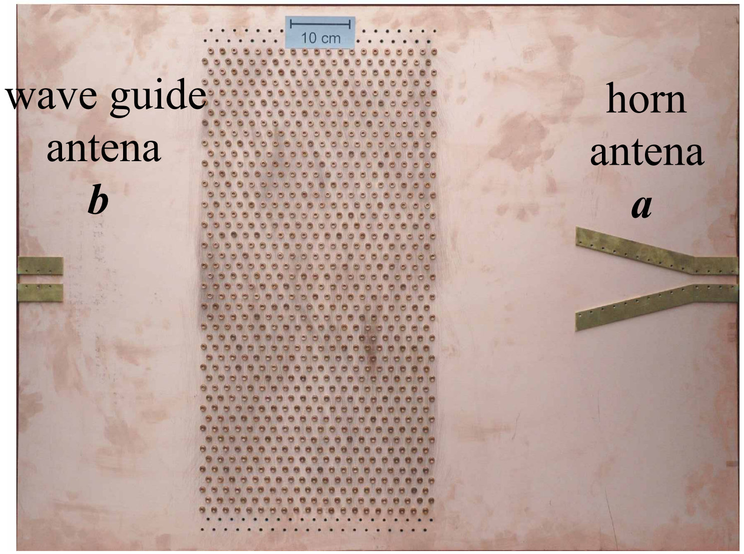
- # cylinders: 23 x 38 = 874
- Cylinder radius: R = 5 mm
- Lattice constant: a = 20 mm

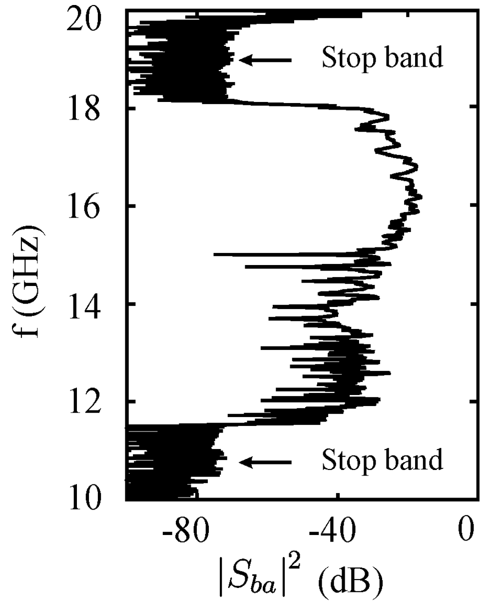


- Crystal size: 400 x 900 x 8 mm
- Frequency: f_{max}=19 GHz

- First step: experimental observation of the band structure

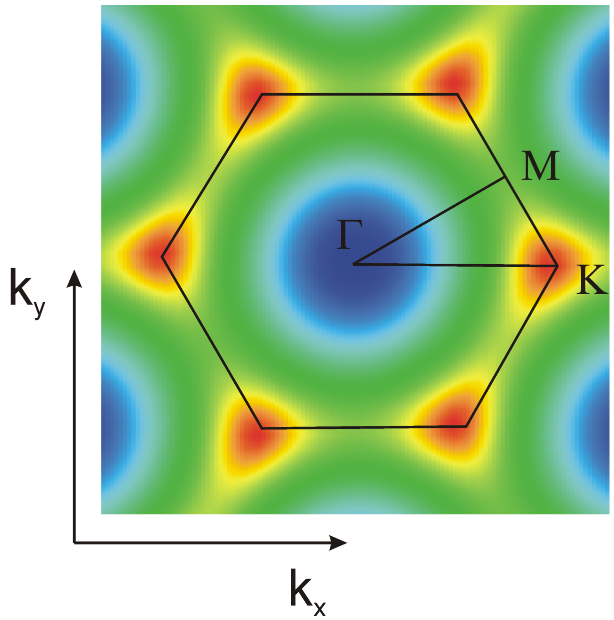
Transmission through the Photonic Crystal

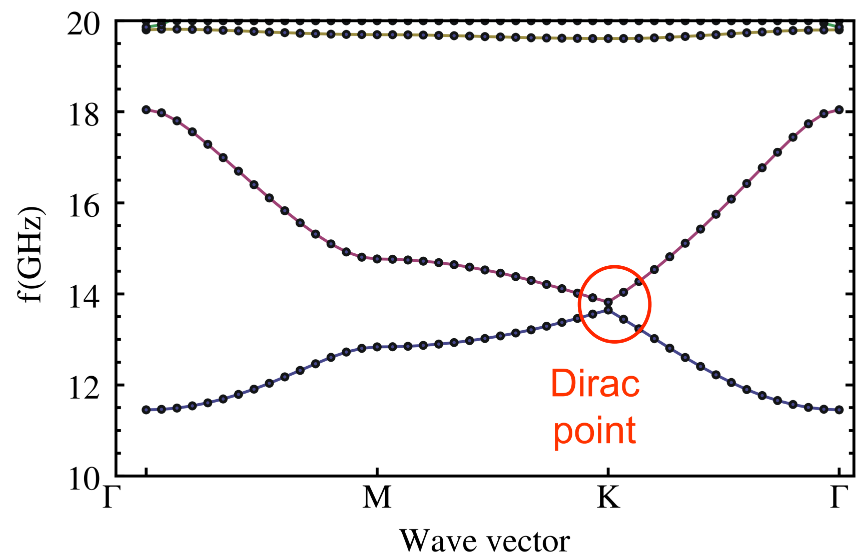




- Transmission spectrum possesses two stop bands
- Comparison with calculated band structure

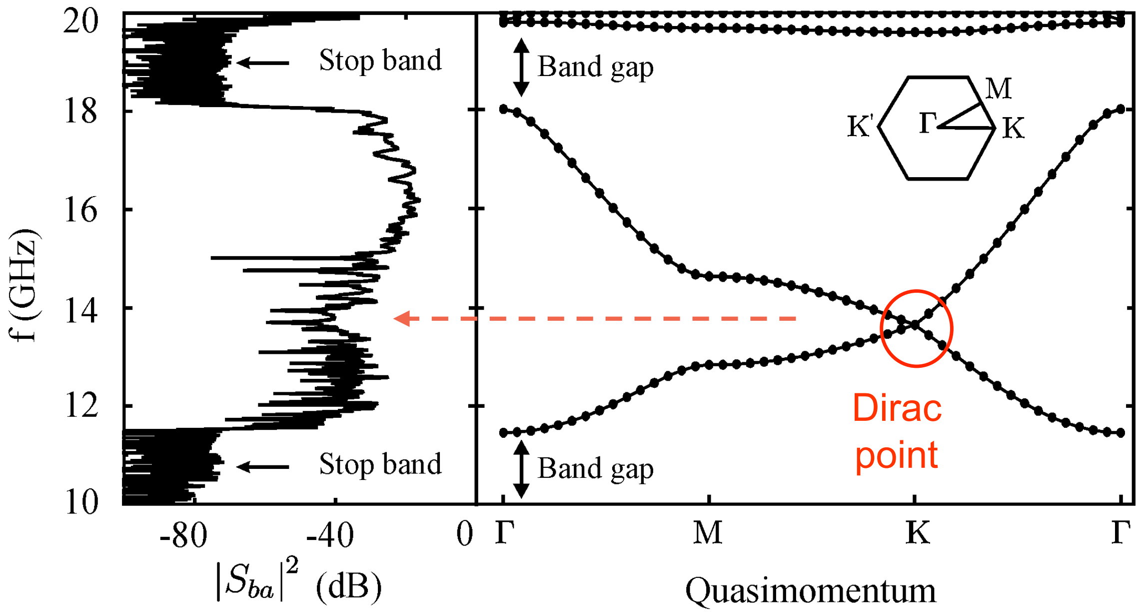
Projected Band Diagram





- The density plot of the 1st frequency band
- The projected band diagram along the irreducible Brillouin zone ΓMK
- The 1st and 2nd frequency bands touch each other at the corners of the Brillouine zone → Dirac Point

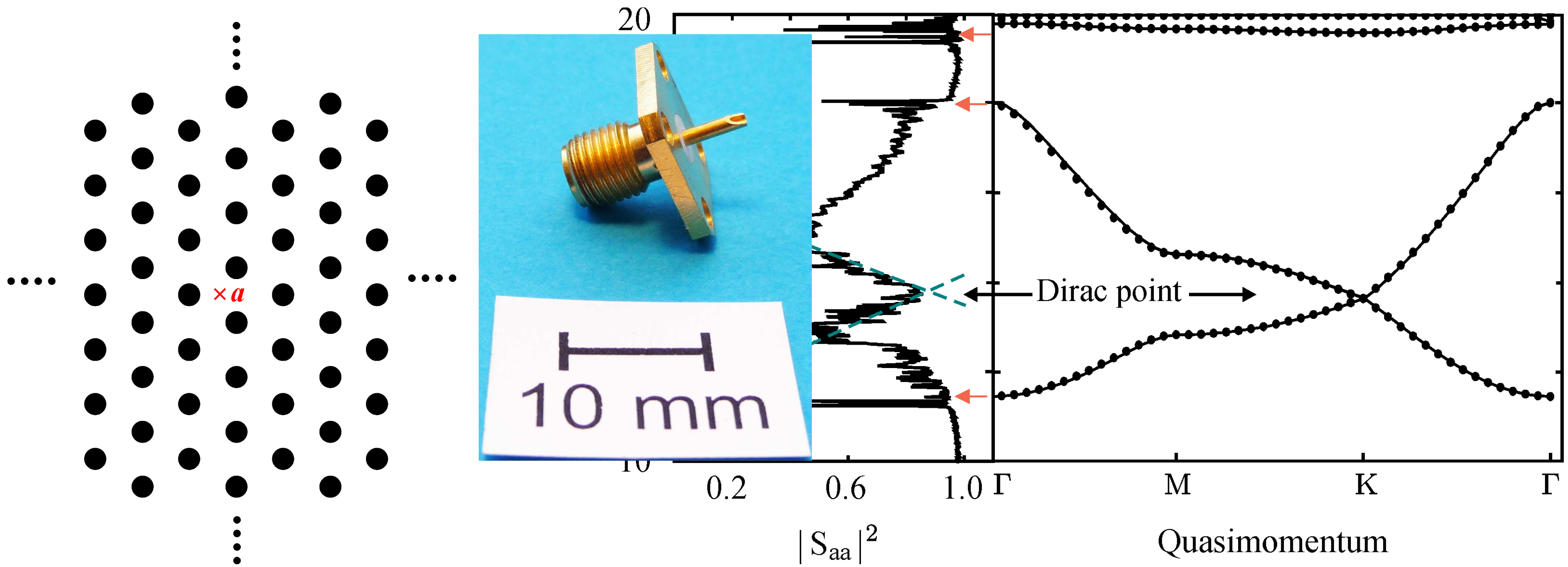
Transmission through the Photonic Crystal



- The positions of measured stop bands coincide with the calculated ones
  → lattice parameters chosen correctly
- Dirac point is not sufficiently pronounced in the transmission spectra
  → single antenna reflection measurement

Single Antenna Reflection Spectrum

- Measurement with a wire antenna *a* put through a drilling in the top plate → point like field probe



- Characteristic cusp structure around the Dirac frequency
- Van Hove singularities at the band edge $\vec{\nabla}\omega(\vec{k}) = 0$
- Next: analysis of the measured spectrum

Local Density of States and Reflection Spectrum

- The scattering matrix formalism relates the reflection spectra to the local density of states (LDOS)

$$1 - |S_{aa}(f)|^2 \propto L(\vec{r}_a, f)$$

- LDOS

$$L(\vec{r}, f) \propto \int_{BZ} |\psi(\vec{k}, \vec{r})|^2 \frac{1}{2\pi} \delta(f - f(\vec{k})) d^2k$$

- LDOS around the Dirac point (Wallace,1947)

$$L(\vec{r}_a, f) \sim \frac{\langle |\psi(\vec{r_a})|^2 \rangle}{v_D^2} |f - f_D|$$

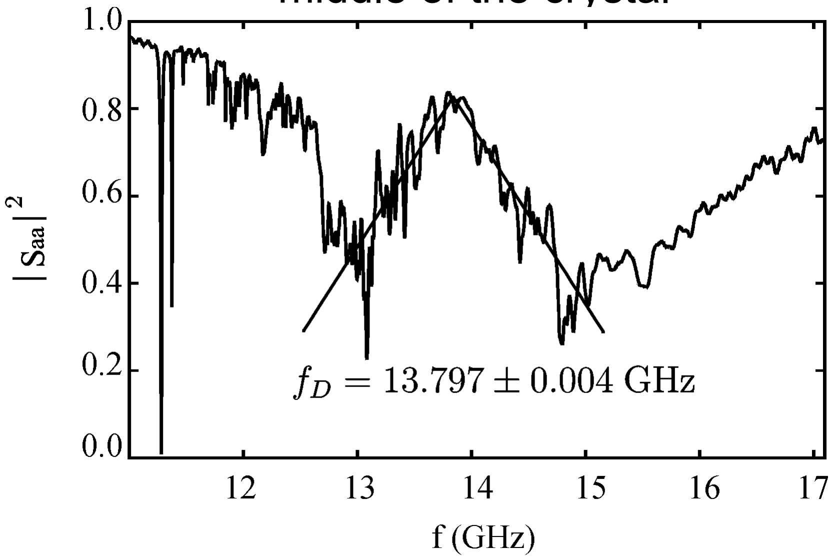
- Three parameter fit formula $|S_{aa}(f)|^2 = D - C\,|f - f_D|$

fit parameters (D, C, f_D)

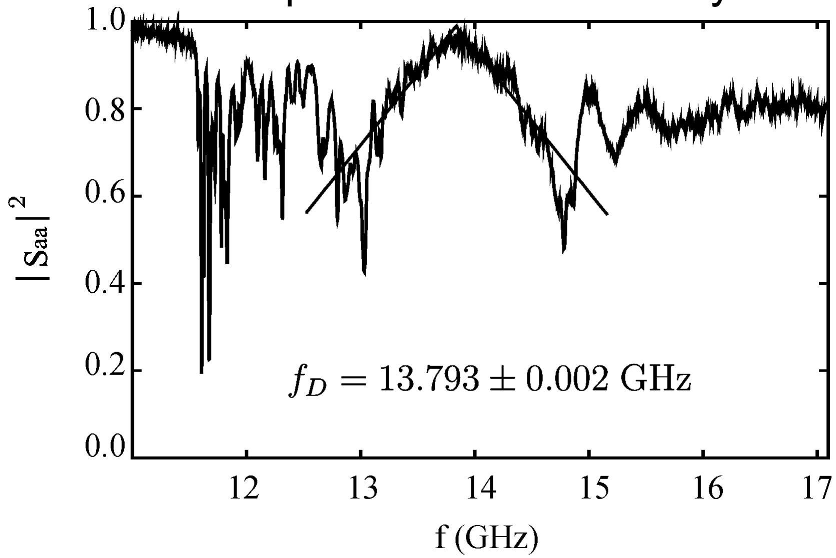
Reflection Spectra

- Description of experimental reflection spectra $|S_{aa}(f)|^2 = D - C\,|f - f_D|$

antenna ***a*** in the middle of the crystal

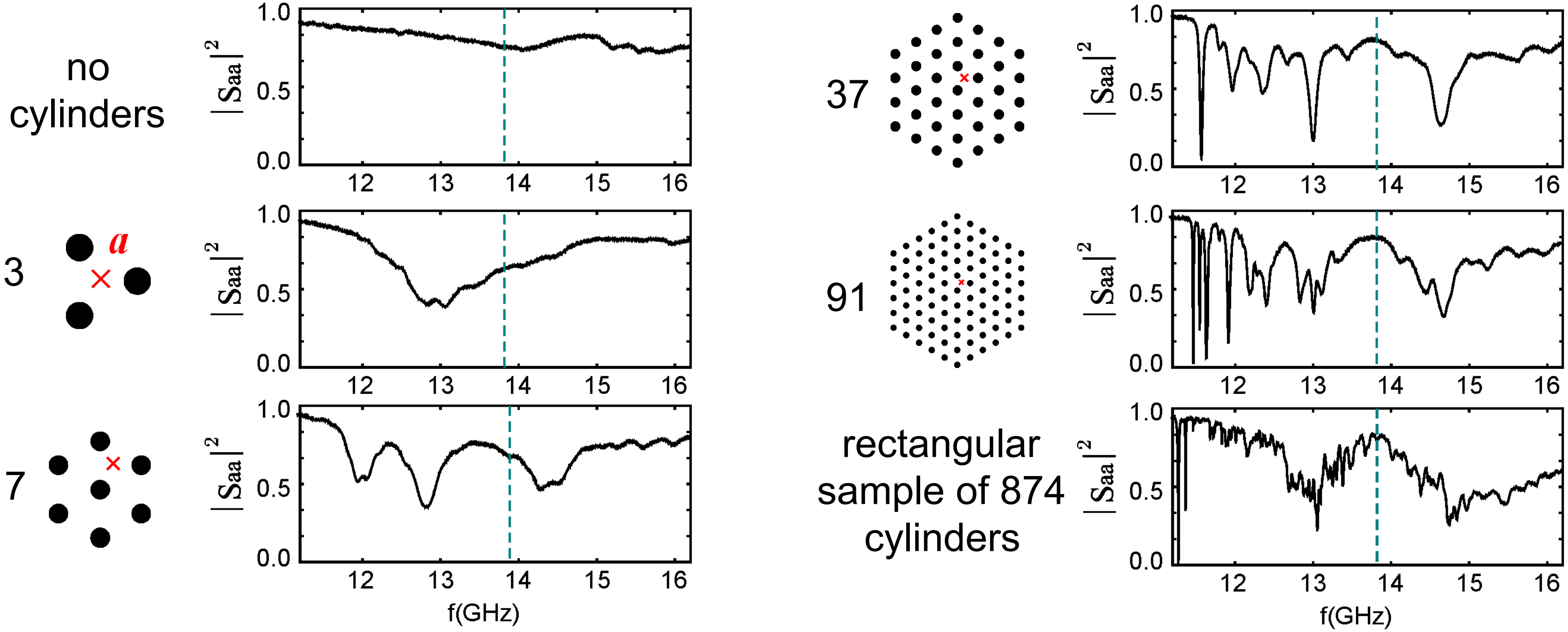


antenna ***a*** 6 rows apart from the boundary



- Experimental Dirac frequencies agree with calculated one, f_D =13.81 GHz, within the standard error of the fit
- Oscillations around the mean intensity → origin?

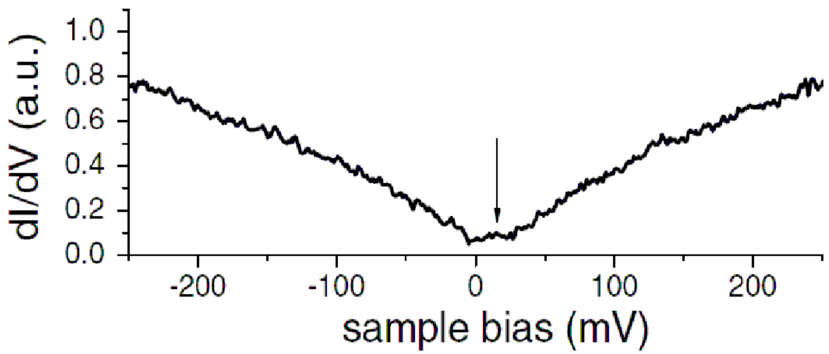
Dependence of Oscillations on Crystal Size



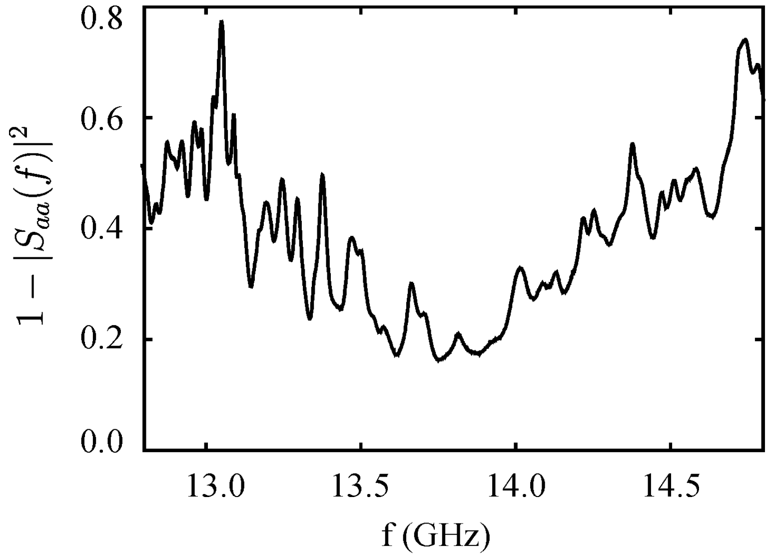
- Nature of the oscillations is a finite size effect
- Period of the oscillations is thus related to the photonic crystal size, i.e. to the density of states

Comparison with STM Measurements

graphene flake, Li *et al.* (2009)



photonic crystal



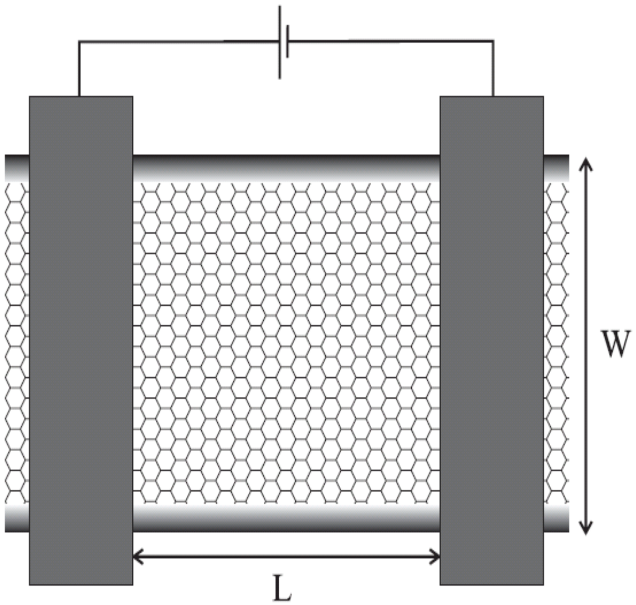
- Tunneling conductance is proportional to LDOS
- Similarity with measured reflection spectrum of the photonic crystal
- Oscillations in STM are not as pronounced due to the large sample size
- Investigation of the finestructure in the photonic crystal is underway: fluctuations (RMT)

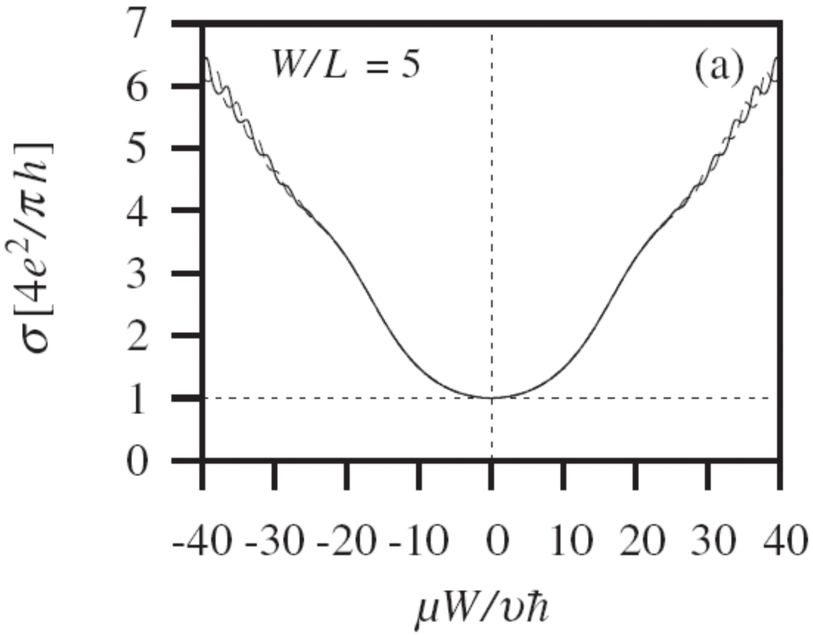
Summary I

- Connection between reflection spectra and LDOS is established
- Cusp structure in the reflection spectra is identified with the Dirac point
- Photonic crystal simulates one particle properties of graphene
- Results are published in Phys. Rev. B **82** 014301 (2010)
- Next → Transmission spectra near the Dirac Point

Transport near the Dirac Point

J. Tworzydło, B. Trauzettel, M. Titov, A. Rycerz, and C. W. J. Beenakker, PRL **96**, 246802(2006)



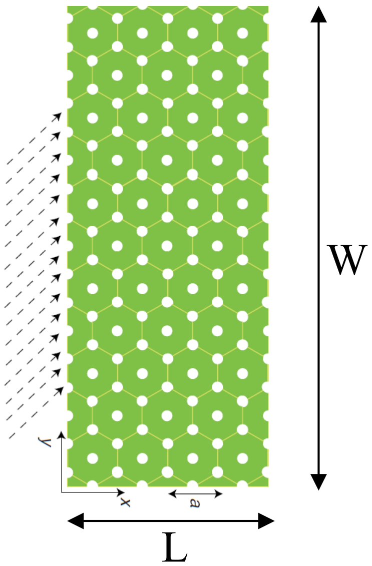


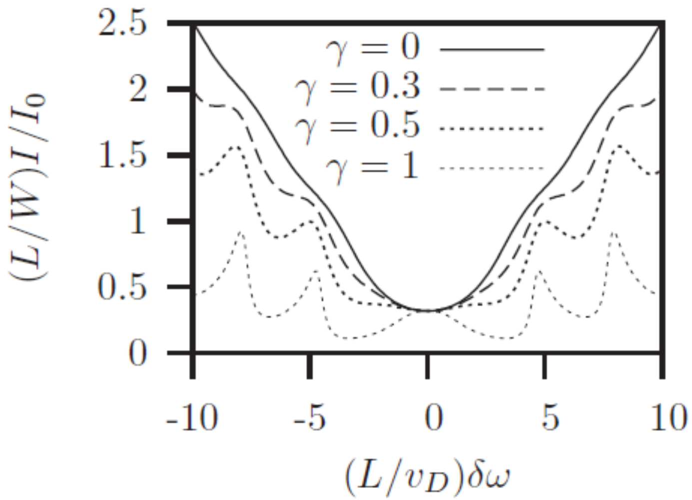
- Finite specific conductivity σ through a sample of graphene at the Dirac Point

- Conductance of a graphene ribbon scales as $G \sim \frac{e^2}{h}\frac{W}{L}$

Extremal Transmission through a Photonic Crystal

R.A. Sepkhanov, Ya.B. Bazalij and C.W.J. Beenakker (2007)

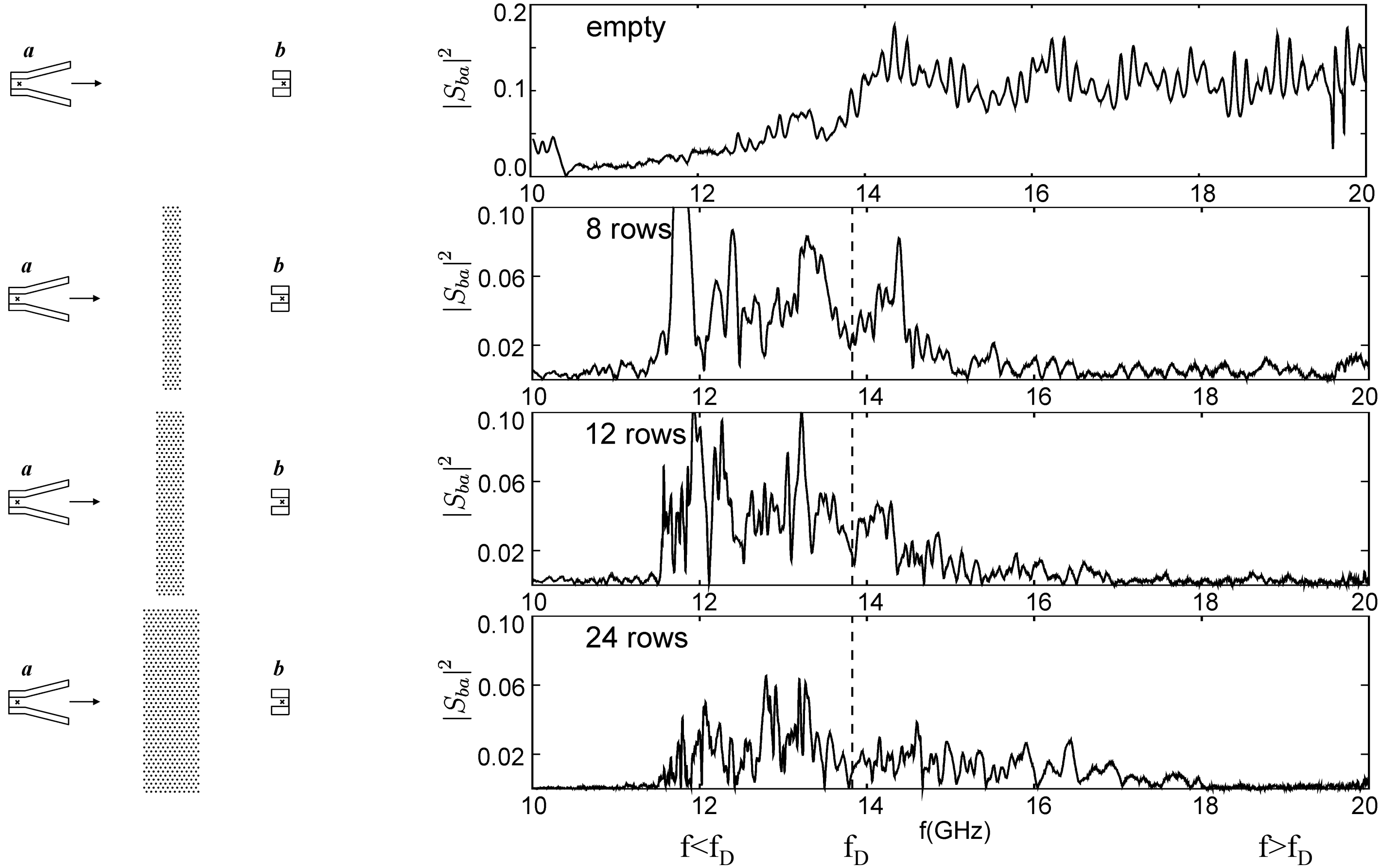




γ describes the interface

- Characteristic transmission has a minimum at the Dirac frequency
- Strong dependence on the structure of the interface of the photonic crystal
- The predicted 1/L scaling for the transmitted power is, however, independent of the details of the interface

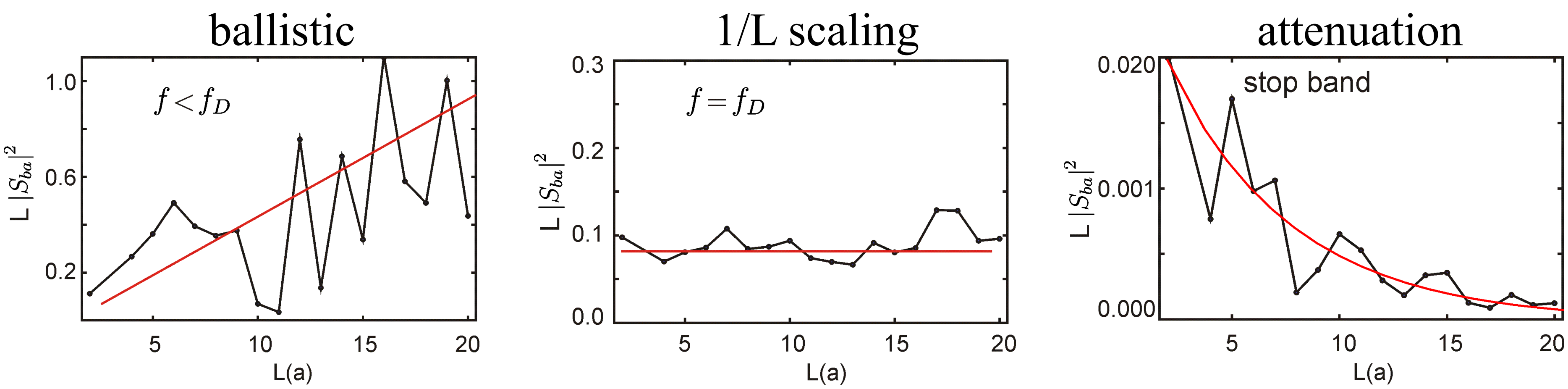
Transmission Spectra through Photonic Crystals in ΓK Direction: Some Examples



- Transmission minimum at the Dirac frequency

Extremal Transmission at the Dirac Point

- Thickness L of the photonic crystal varies from 4 to 40 layers



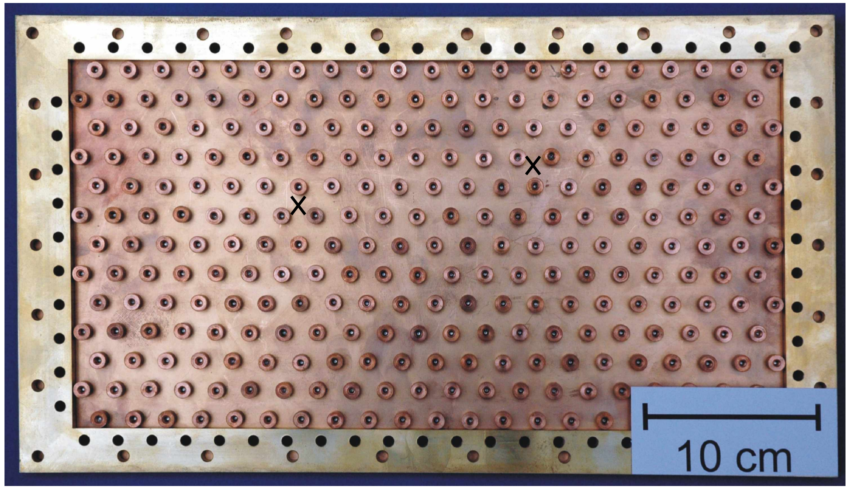
- Ballistic transport at the transmission bands
- Extremal transmission at the Dirac frequency
- Exponential attenuation at the stop band

Summary II

- Transmission spectra of photonic crystals of different thickness L were measured
- Transmission spectra of the photonic crystal near the Dirac Point show similar scaling as the conductance in graphene, i.e. 1/L behaviour
- Next → Photonic crystal in a box

“Neutrino” Billiard

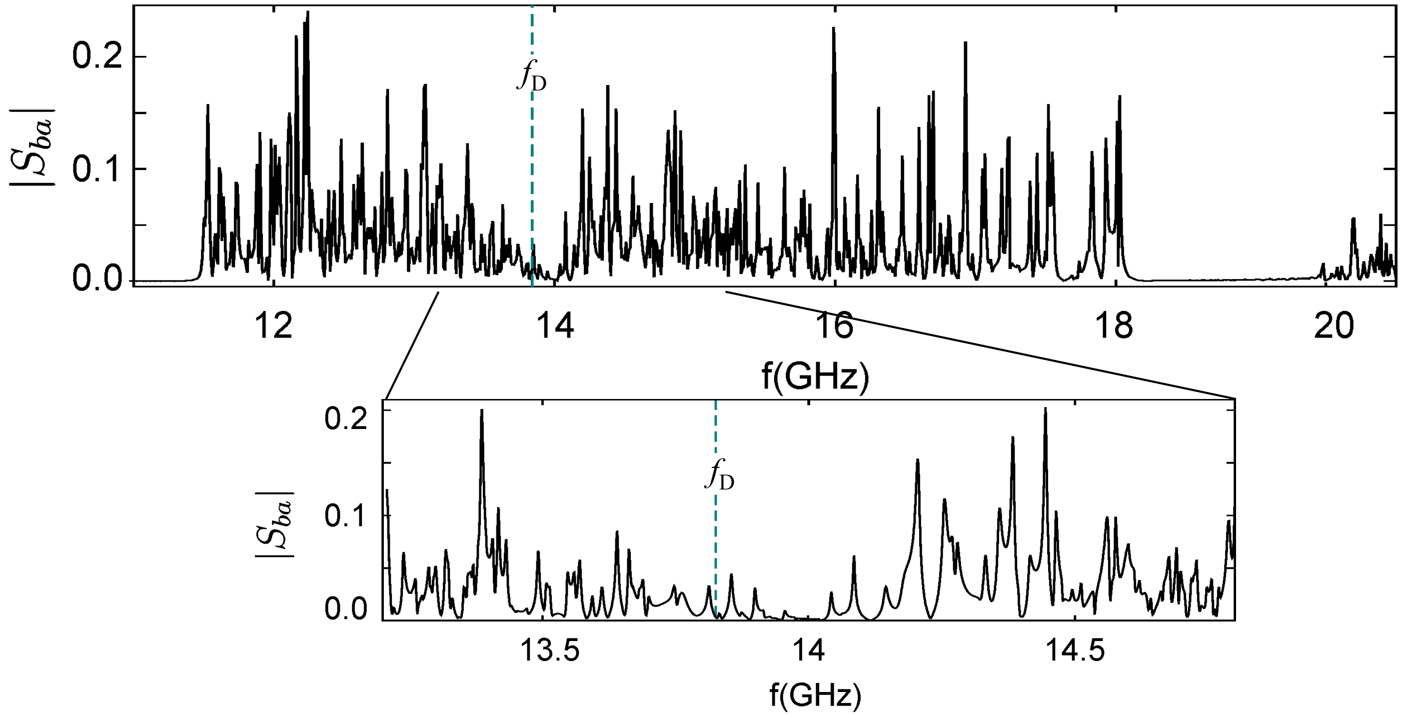
- Relativistic massless spin-half particles in a billiard
  M.V. Berry and R.J. Mondragon (1987)
- Energy spectrum is symmetric with respect to zero (Dirac) energy
- GUE statistics predicted for the energy levels
- Can we simulate the neutrino billiard with a photonic crystal in a metal box?



× antenna ***a***

× antenna ***b***

Billiard Spectrum



- Spectrum is not symmetric with respect to the Dirac frequency f_D
- Due to overlapping resonances there are missing levels
  $\rightarrow$ superconducting measurements needed to resolve all levels
- Next: intensity distributions $|E_z|^2$ $(\equiv |\psi|^2)$ at resonances near f_D

Measurement of Electric Field Intensity

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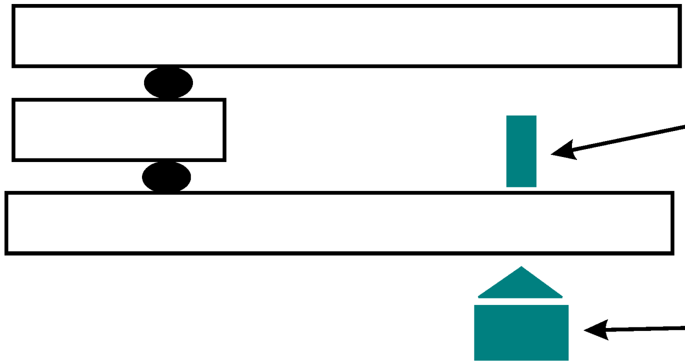
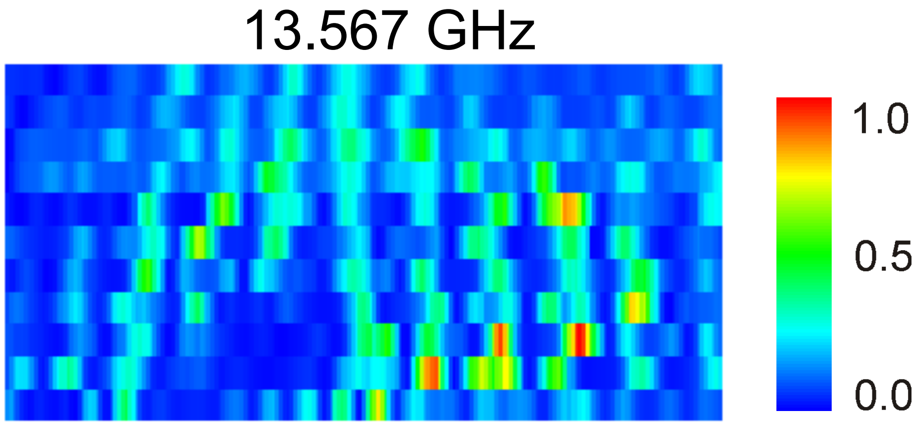


- Resonance frequency shift is related to the electric field strength

Maier and Slater(1952)

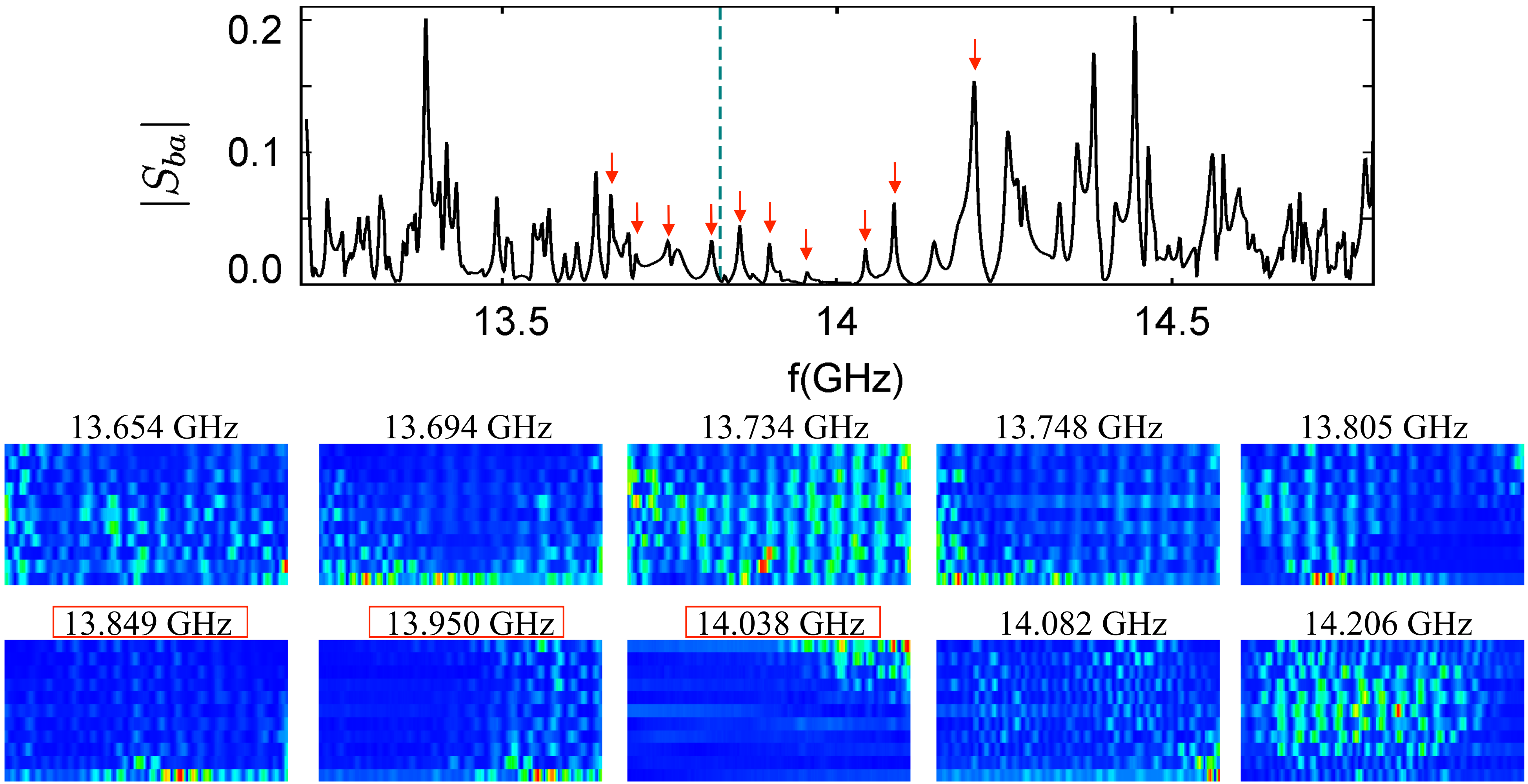
$$\delta f(x,y) = c_1 \cdot E^2(x,y)$$

Measured Intensity Distribution: Example

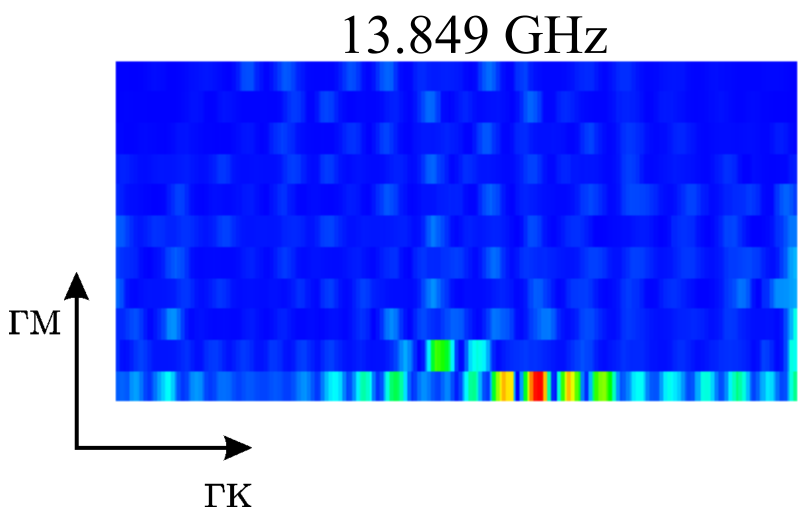


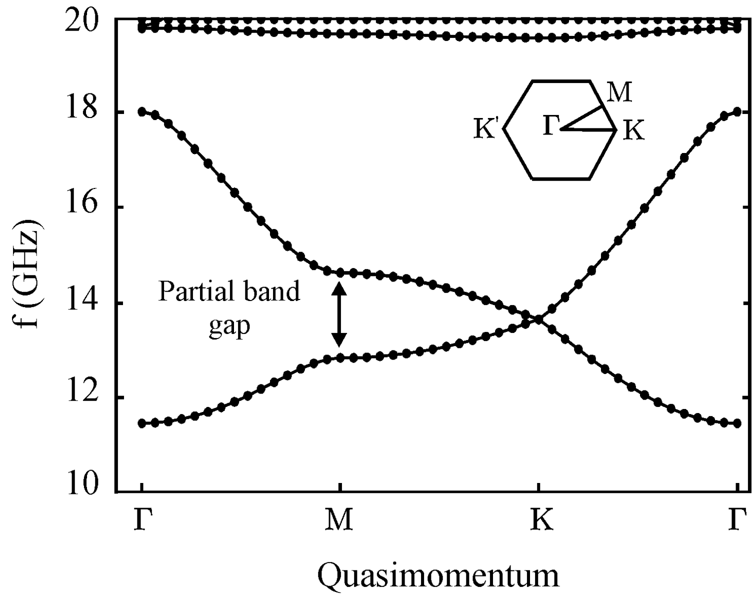
- Perturbation body moved between cylinder rows
- 5 hours for 180 intensity distributions
- Aim: identify so called edge states in graphene

Measured Electric Field Intensity Distributions



Edge States





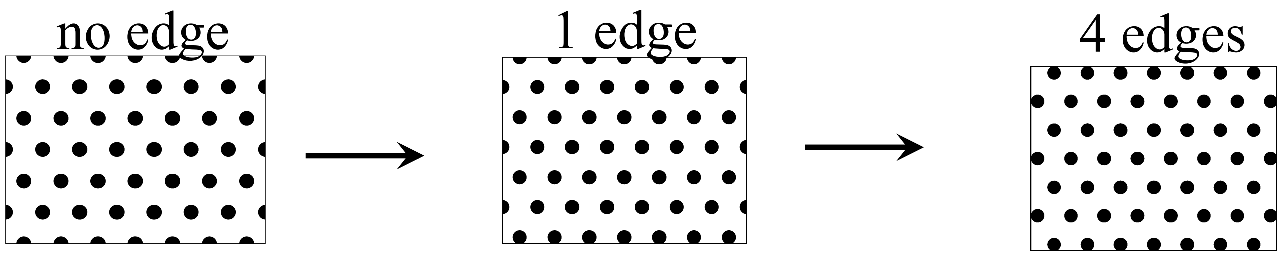
- No propagation in ΓM direction due to partial band gap
- Edge along ΓK direction corresponds to zigzag edge in graphene
- Edge along ΓM direction corresponds to armchair edge in graphene
- Observed states are analogous to edge states in graphene nanoribbons

Summary

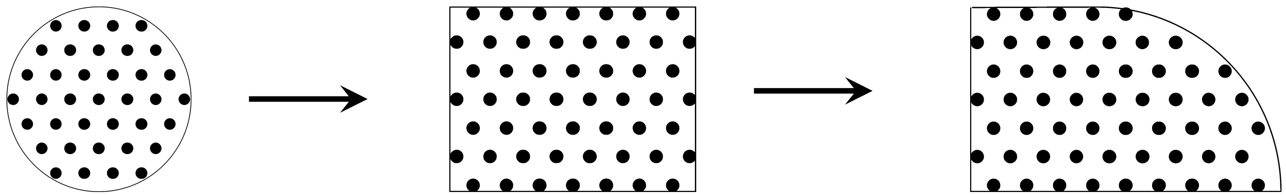
- Connection between the reflection spectra and LDOS is established
- Cusp structure in the reflection spectra is identified with the Dirac point
- Photonic crystal simulates one particle properties of graphene
- Experimental observation of extremal transmission
- Observation of edge states in the neutrino billiard

Outlook I

- Billiards family for the study of the edge states



- Superconducting billiards for study of the spectral properties



Outlook II

in Colloboration with J.Berges, C.Fischer and L. von Smekal

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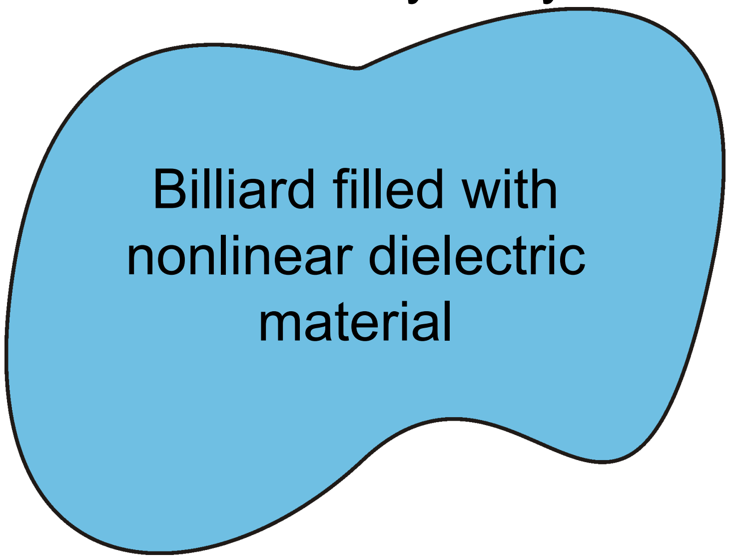
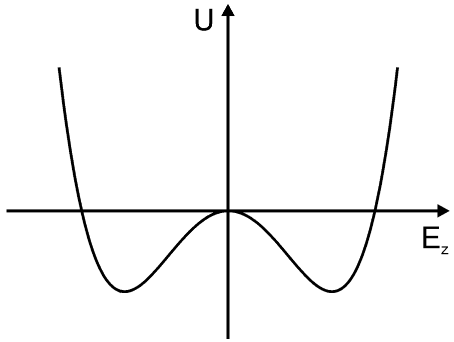
- Simulation of many body effects using RF nonlinear materials



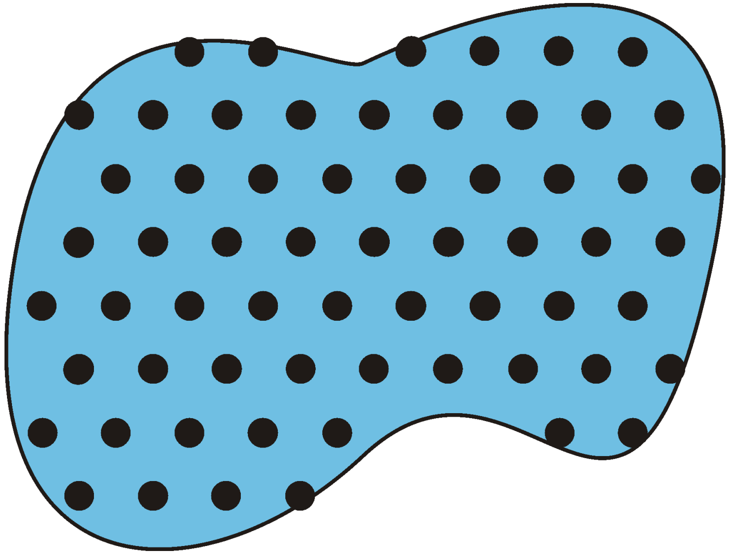


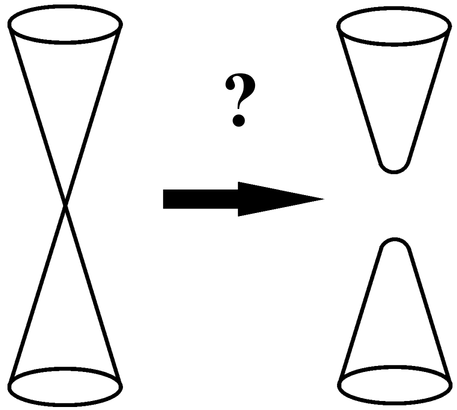
$$\Delta E_z = -k^2 E_z - \eta |E_z|^2 E_z + \gamma |E_z|^4 E_z$$

- Wave propagation described by Gross-Pitaevskii equation
- Model for interacting bosons in a hard-wall potential
- Interaction becomes observable at high RF power coupled into the resonator
- Higher-order nonlinearities produce the Mexican-hat potential

Outlook II

- Photonic crystal embedded into a nonlinear medium mimics spinor fields





- Wave propagation described by a nonlinear Dirac equation
- Interacting fermions like in a graphene flake
- Study of transport properties and semimetal-insulator phase transition

Personal Remarks

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- Jochen Wambach has served as Korreferent for 23 of my 100 doctoral students
- 18 joint publications with Jochen Wambach (1985 - present):

HIGH-RESOLUTION INELASTIC ELECTRON SCATTERING AND THE NATURE OF THE M1 TRANSITIONS TO THE $J^\pi = 1^+$ STATE AT $E_x = 5.846$ MeV IN ^{208}Pb
S. Müller, G. Küchler, A. Richter, H.P. Blok, H. Blok, C.W. de Jager, H. de Vries and J. Wambach
Phys. Rev. Letters 54, (1985) 293

FINE STRUCTURE OF THE ISOSCALAR GIANT QUADRUPOLE RESONANCE IN ^{208}Pb OBSERVED IN HIGH-RESOLUTION (e,e′) AND (p,p′) SCATTERING
S. Kamerdzhiev, J. Lisantti, P. von Neumann-Cosel, A. Richter, G. Tertychny and J. Wambach
Phys. Rev. C55, (1997) 2101

SPIN AND ORBITAL MAGNETIC QUADRUPOLE RESONANCES IN ^{48}Ca AND ^{90}Zr FROM 180° ELECTRON SCATTERING
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Personal Remarks

50 **60**

70 **60**

Many more fruitful years together !