Spin Correlations in Hot Dense Matter and Ultracold Atomic Gases

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Main themes

- Clock shifts in atomic gases
- Neutrino mean free paths in collapsing stars

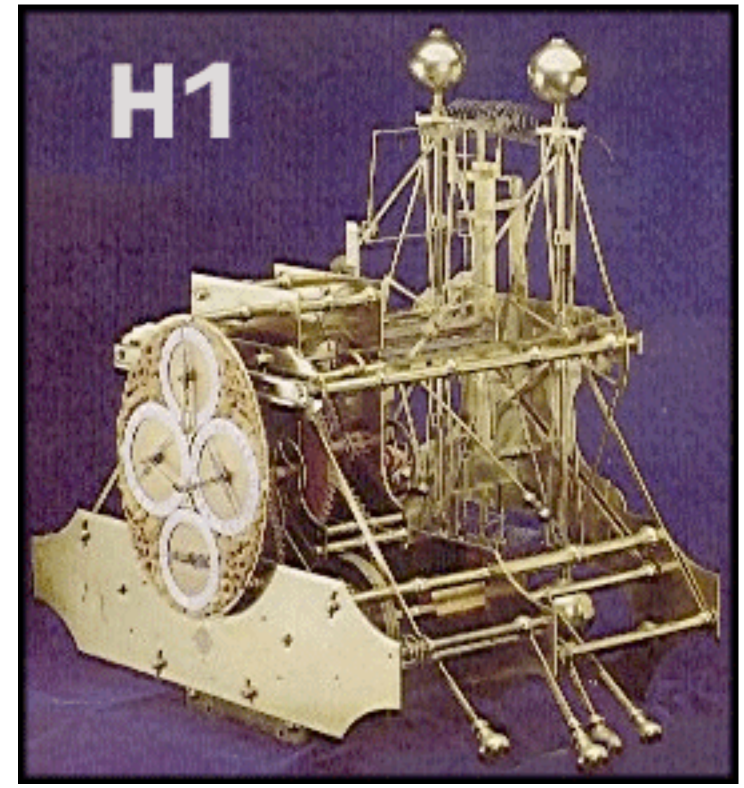
Conservation laws and symmetry principles

Clocks important

Technology.

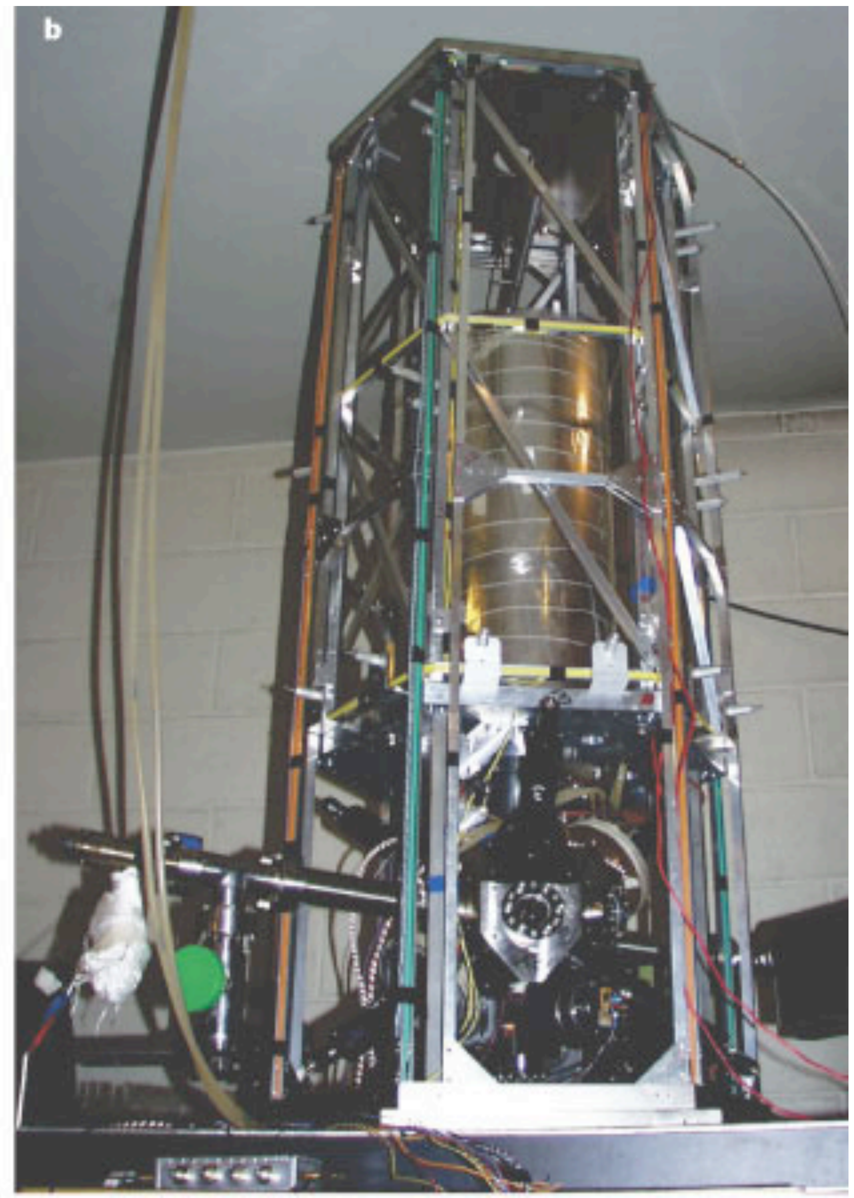
- Measuring longitude
  (John Harrison (1693-1776))

- Modern applications
  (e.g. GPS navigation)



Basic science

- Time dependence of fundamental constants



The first atomic clock (Zacharias, 1953)

A rubidium fountain atomic clock (C. Solomon et al., ENS)

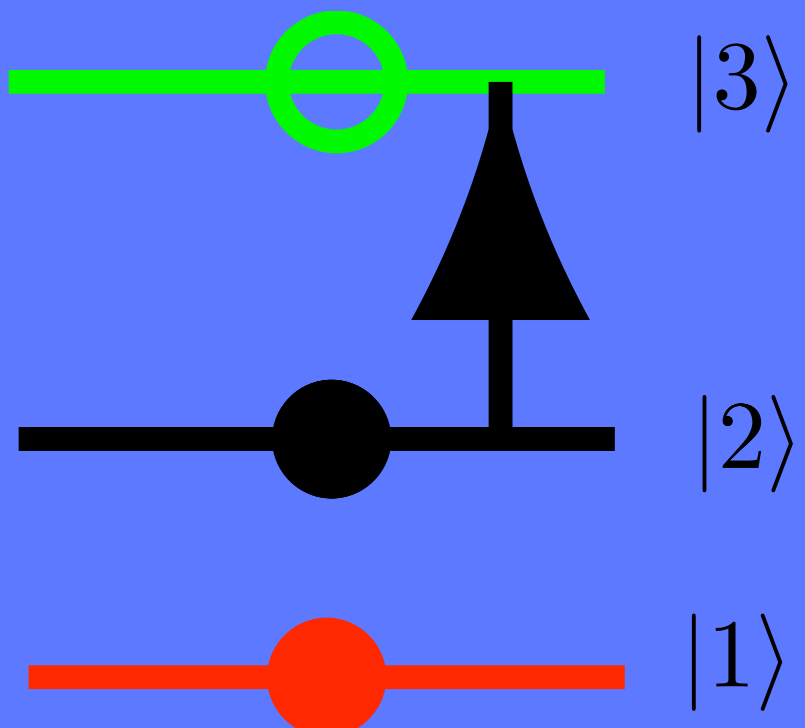
Basic idea

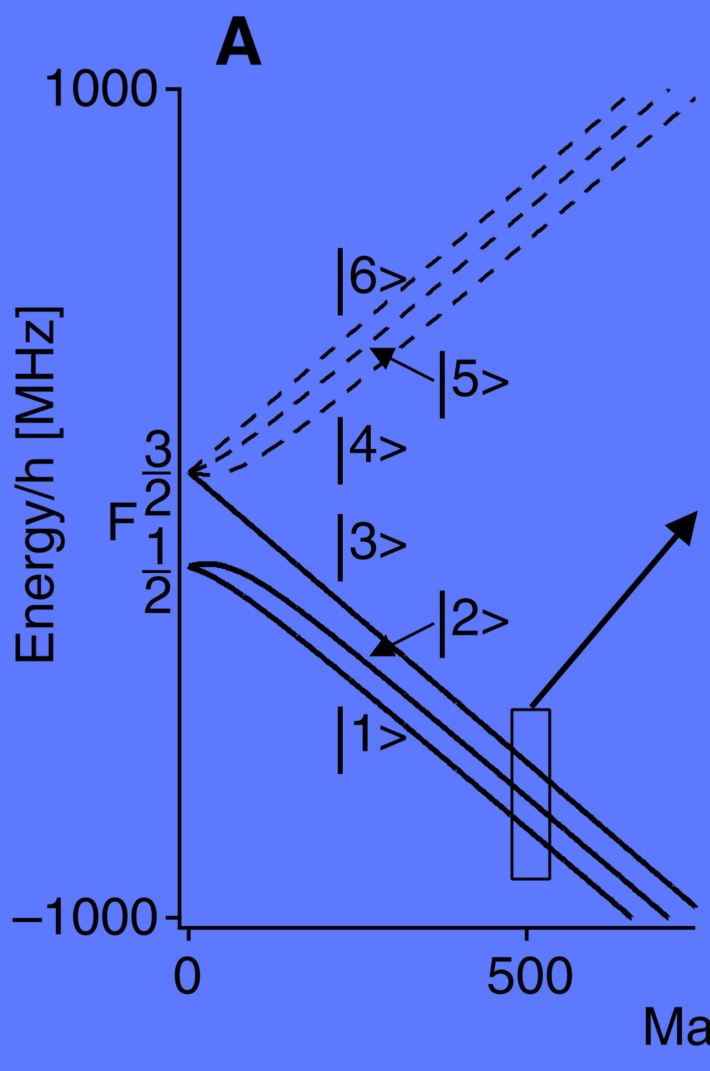
- Change state of one atom
- Interactions with other atoms depend on the state
- Spectral line is shifted. Mean field energy
- "Clock" shift, or collisional shift
- Size: $\Delta E = n\,4\pi\hbar^2 a/m \sim \epsilon_F\, n^{1/3} a$.
  a – scattering length, n – density,
  ϵ_F – Fermi energy, k_F – Fermi wavenumber
- 1 μK corresponds to 20,000 Hz
- Current clocks: want precision of 1 in 10^{16}
- Optical transition. Want clock shift $\lesssim 1$ Hz

Fermi gases with resonant interactions

- Can make interactions strong
- Probe correlations by rf spectroscopy

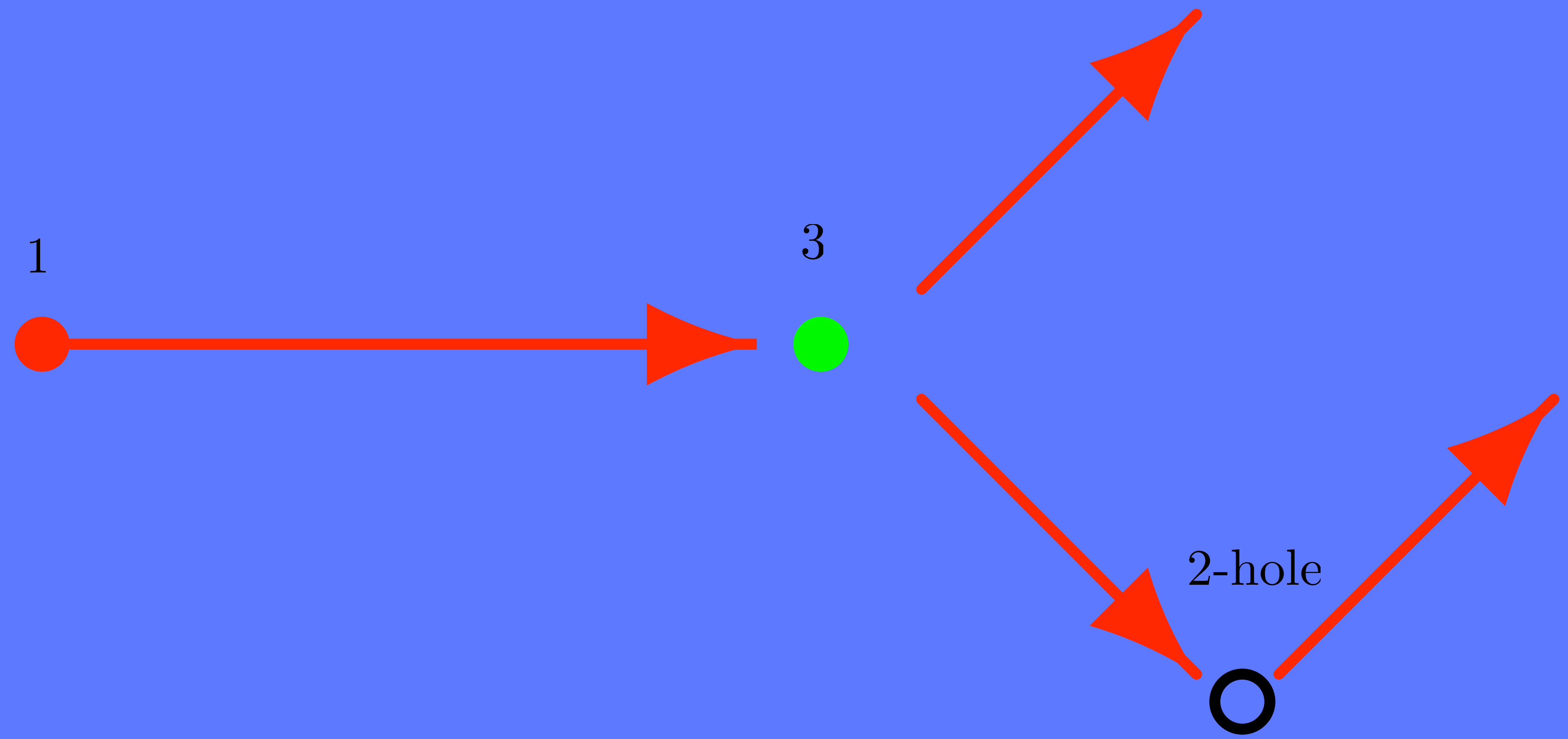
^{6}Li (Gupta *et al.*, 2003)





Scattering from correlated pairs

- Excitation creates a 3-atom 2-atom-hole pair
- Shift due to interaction with 1-atoms
- Similar to problem of pion-deuteron scattering (Glauber, 1955)



Theory of shifts in dilute gases

- Naive theory. Energy shift of atom is

$$\Delta E_i = -n_1 \frac{4\pi\hbar^2}{m} \frac{\sin 2\delta_{1i}}{2k} \to n_1 \frac{4\pi\hbar^2}{m} a_{1i}$$

$$\text{Line shift } \Delta E_{23} = n_1 \frac{4\pi\hbar^2}{2mk} [\sin 2\delta_{12} - \sin 2\delta_{13}] \to n_1 \frac{4\pi\hbar^2}{m} (a_{13} - a_{12})$$

- Standard theory of line shape in a dilute gas gives (Koelman *et al.*, Phys. Rev. **38**, 2525 (1988), Dickerscheid and Stoof (unpublished)) for the self energy

$$\Sigma = -\frac{4\pi\hbar^2 n_1}{2mik} \left[\mathrm{e}^{2\mathrm{i}(\delta_{13}-\delta_{12})} - 1 \right]$$

$$\Delta E_{23} \propto \sin(2\delta_{12} - 2\delta_{13}) = \sin 2\delta_{13}(1 - 2\sin^2 \delta_{12}) - \sin 2\delta_{12}(1 - 2\sin^2 \delta_{13})$$

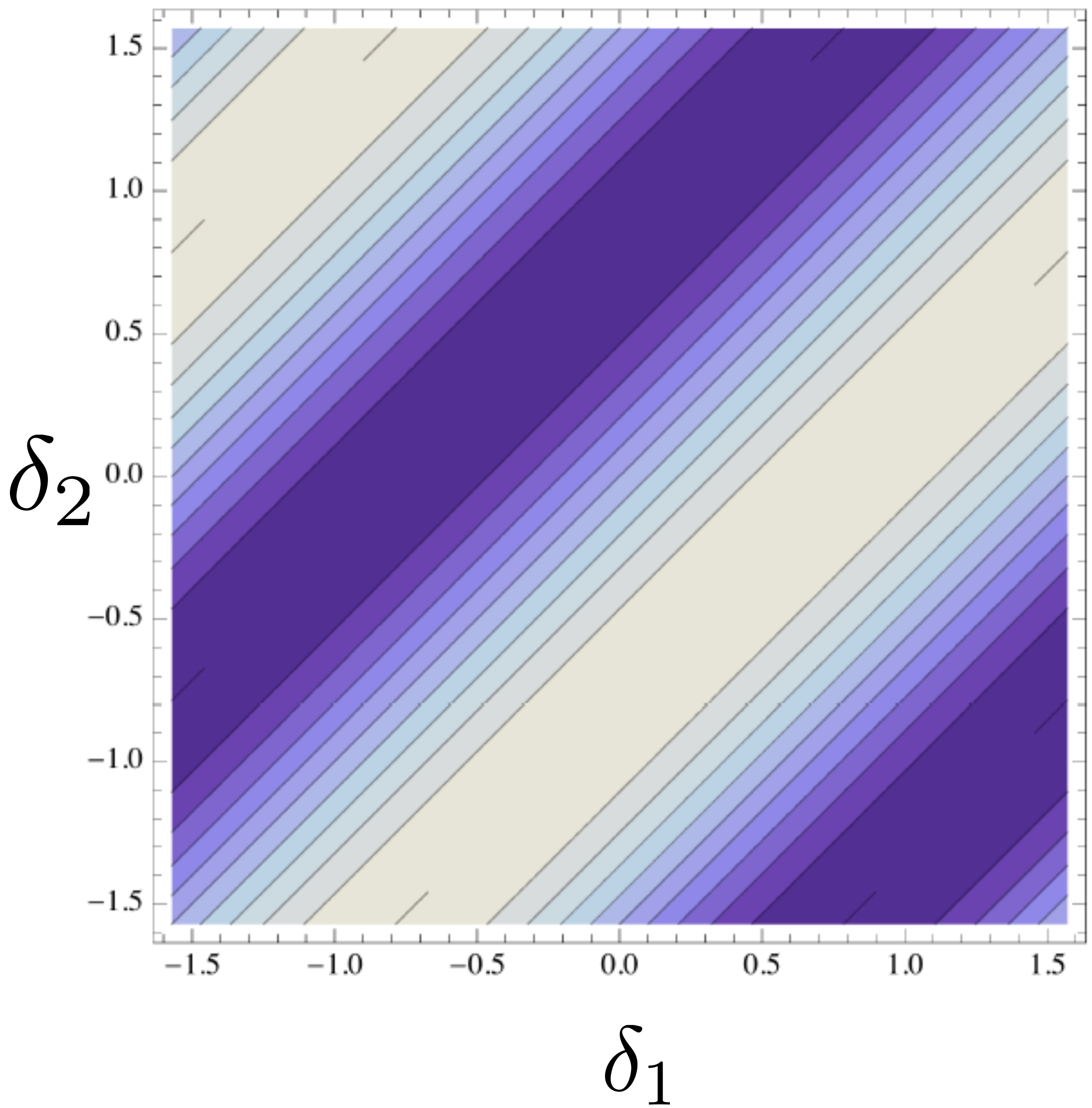
Linewidth $\mathrm{Im}\Sigma \propto \sin^2(\delta_{12} - \delta_{13})$
S-matrix $\propto \mathrm{e}^{2\mathrm{i}\delta}$

- Like "shadowing" in imaginary part.

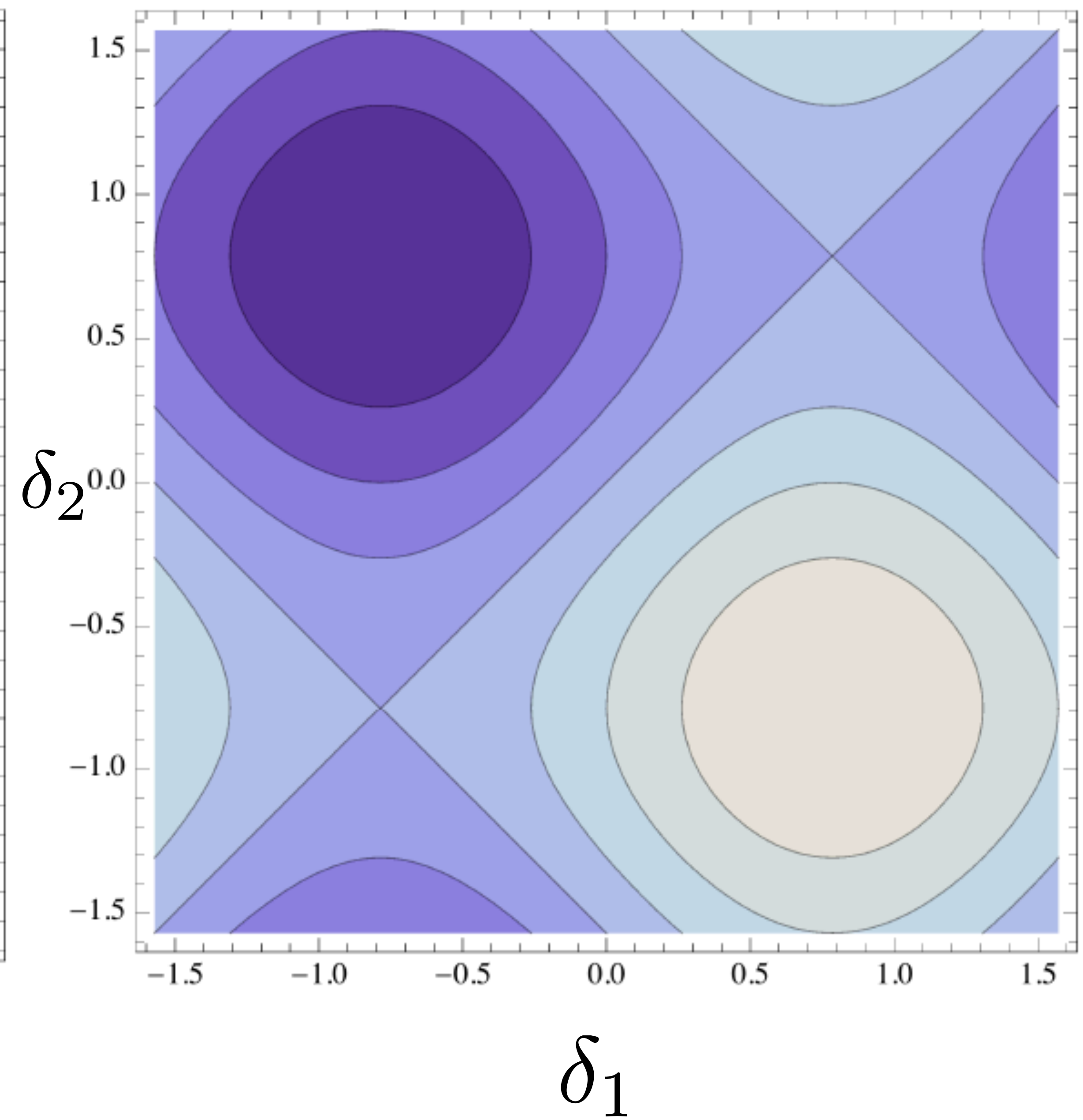
- Shifts finite for resonant scattering.

Line shifts

With vertex corrections



Without vertex corrections

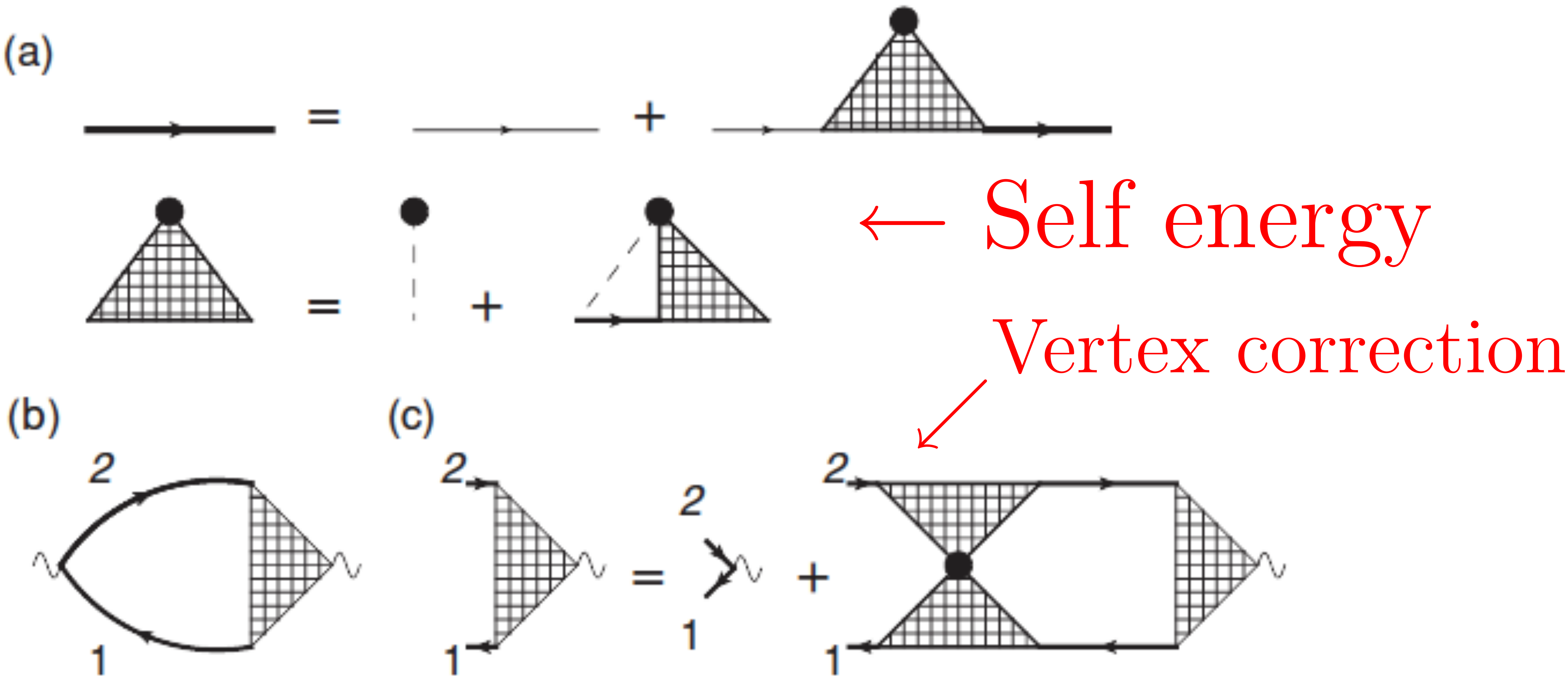


Exactly soluble model

- Take $m_1 \gg m_2 = m_3$.
- 1-atoms can be treated as static impurities.

G. M. Bruun, C. J. P., and Z. Yu,
Phys. Rev. A **81**, 033621 (2010).

Calculating the correlation function



Line shapes

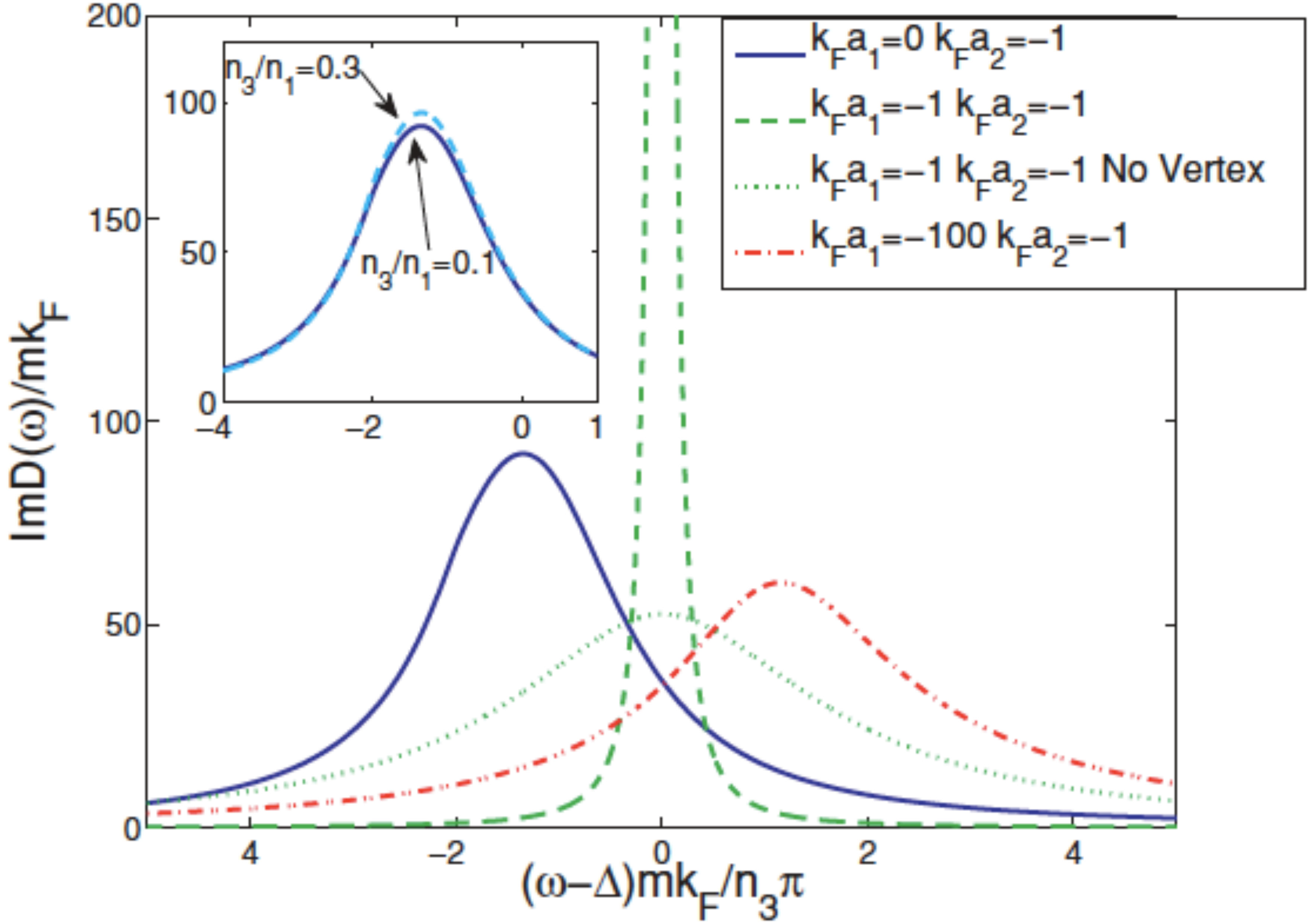
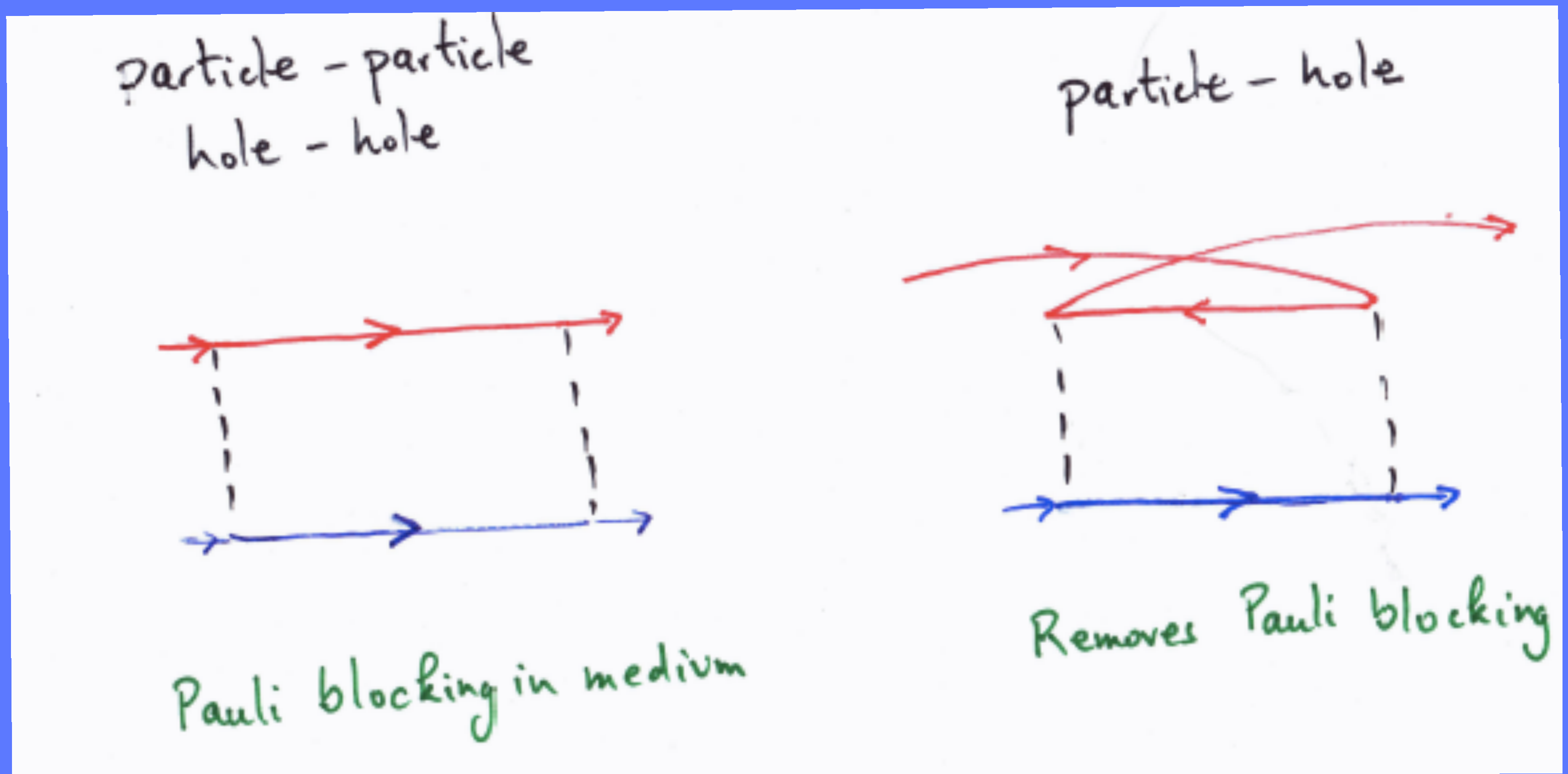


FIG. 2. (Color online) The transition rate as a function of frequency and interaction. The inset shows the transition rate for varying impurity concentration.

G. M. Bruun, C. J. P., Zhenhua Yu, Phys. Rev. A **81**, 033621 (2010)

Important lessons

- Vertex corrections. Can change sign of shift
- Shifts not large for resonant interactions.
- To obtain correct result for phase shift in a medium, it is necessary to include particle–hole scattering (bubbles) in addition to particle–particle and hole–hole scattering (ladders)



Making better clocks

- Use fermions. Interaction effects reduced by Pauli principle
- Work in an optical lattice (no Doppler shift)
- Work with optical transition

G. K. Campbell et al., Science **324**, 360 (2009).
^{87}Sr ^{1}S$_0$-^{3}P$_0$ optical transition (4.3×10^{14} Hz).
Shift (~ 1 Hz) seen even though atoms are fermions.

Surprise!

Why is there a shift?

- How identical do fermions need to be to obey Pauli principle?
- Think in terms of two level system ($^1S_0$ and $^3P_0$). Pseudospin.
- Two atoms. Rotationally invariant interaction $\propto \mathbf{S_1} \cdot \mathbf{S_2}$.
- Precession of total spin unaltered: $d\mathbf{S}_1/dt \propto \mathbf{S_2} \times \mathbf{S_1}$ and $d\mathbf{S}_2/dt \propto \mathbf{S_1} \times \mathbf{S_2}$. Thus $d(\mathbf{S}_1 + \mathbf{S_2})/dt = 0$.

Solution: Inhomogeneity of magnetic field

- Spins in different states in the optical lattice precess at different rates.
- Initially all spins point in same direction ($^1S_0$).
- Spins in different motional states precess and get out of step. Triplet admixture.
- With further precession, a change in the total spin results.

K. Gibble, PRL **103**, 113202 (2009)
A. M. Rey, A. V. Gorshkov, and C. Rubbo, PRL **103**, 260402 (2009)
Z. Yu and CJP, PRL **104**, 010801 (2010).

Supernova 1987A, Hubble Space Telscope

General considerations and history

- Neutrinos seen from SN 1987A
- Early calculations. Energy transport by neutrinos. Colgate and White (1966)
- 1970s. Discovery of weak neutral currents ($\nu + N \to \nu + N$), in addition to charged currents ($p + e^- \to n + \nu_e$).
- Improved equations of state.
- Failure of direct explosion mechanism.
- Shock revival by neutrino heating (Bethe, Wilson).
- Still no agreed mechanism for generating explosion. (Convection, sound waves, ...)
- *Need improved estimates of neutrino processes*

Neutrino processes

- Rates important but not frequency shifts. (cf. cold gases)
- Scattering from nucleons $N + \nu \rightarrow N + \nu$
- Initial and final state interactions $N + N + \nu \rightarrow N + N + \nu$
  Landau-Pomeranchuk-Migdal effect (Raffelt, Seckel, Strobel, Sigl, Hannestad, Sedrakian, Dieperinck,...)
- Neutrino pair bremsstrahlung and annihilation $N + N \rightleftharpoons N + N + \nu + \bar{\nu}$
- Similar processes in neutron star cooling (also charged currents)

Complications.

Neutrino flavours

Inhomogeneity of matter (coherent scattering)

Basic formalism

- Treat weak interactions using Golden Rule.

$$\text{Rates} \propto G_{\mathrm{F}}^2 \int d\omega d^3q \ldots S(q,\omega)$$

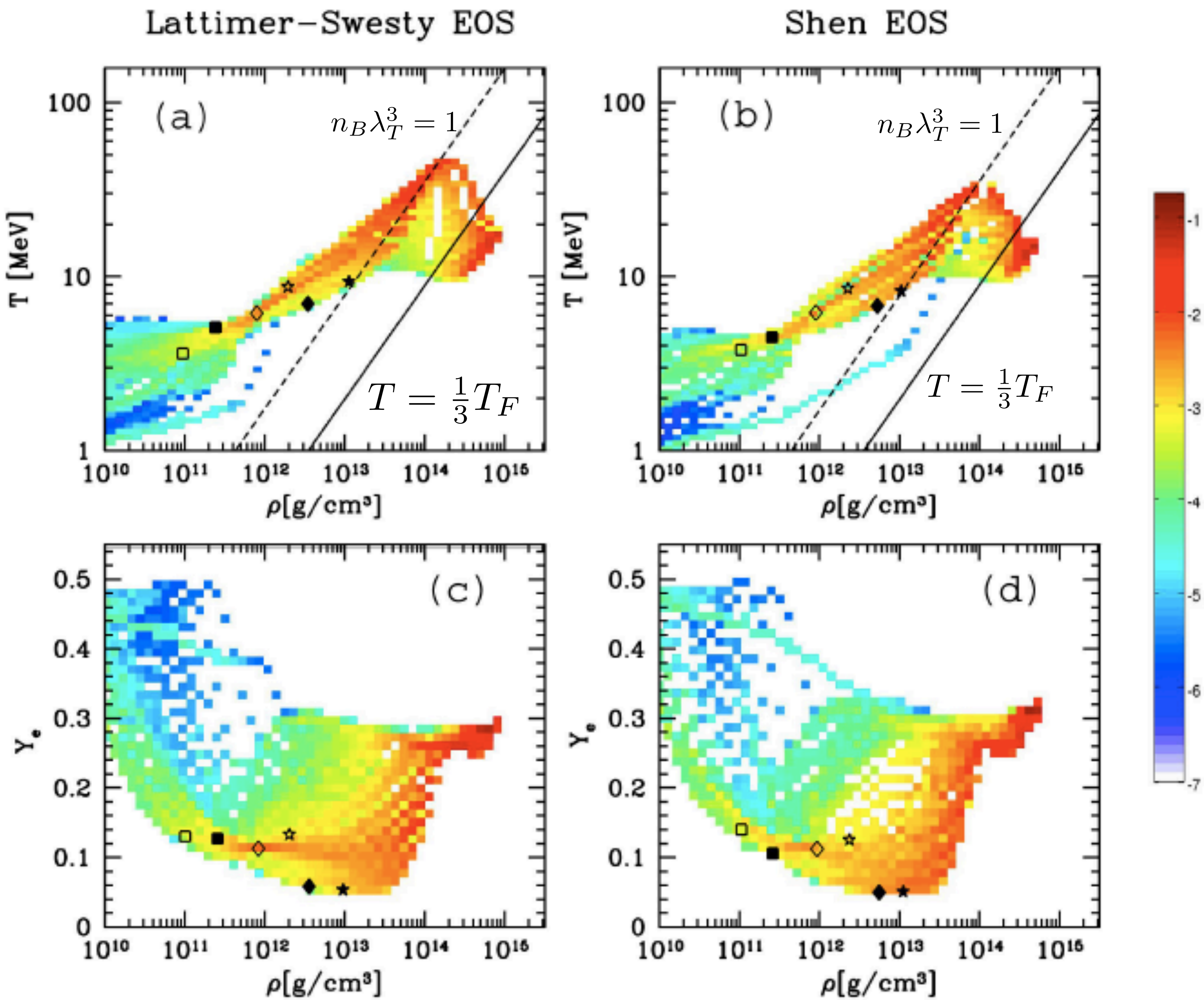
- Information about medium encoded in $S(q,\omega)$ – density-density (vector) or spin-spin (axial vector) dynamical structure factors.

- Related to the corresponding correlation functions:

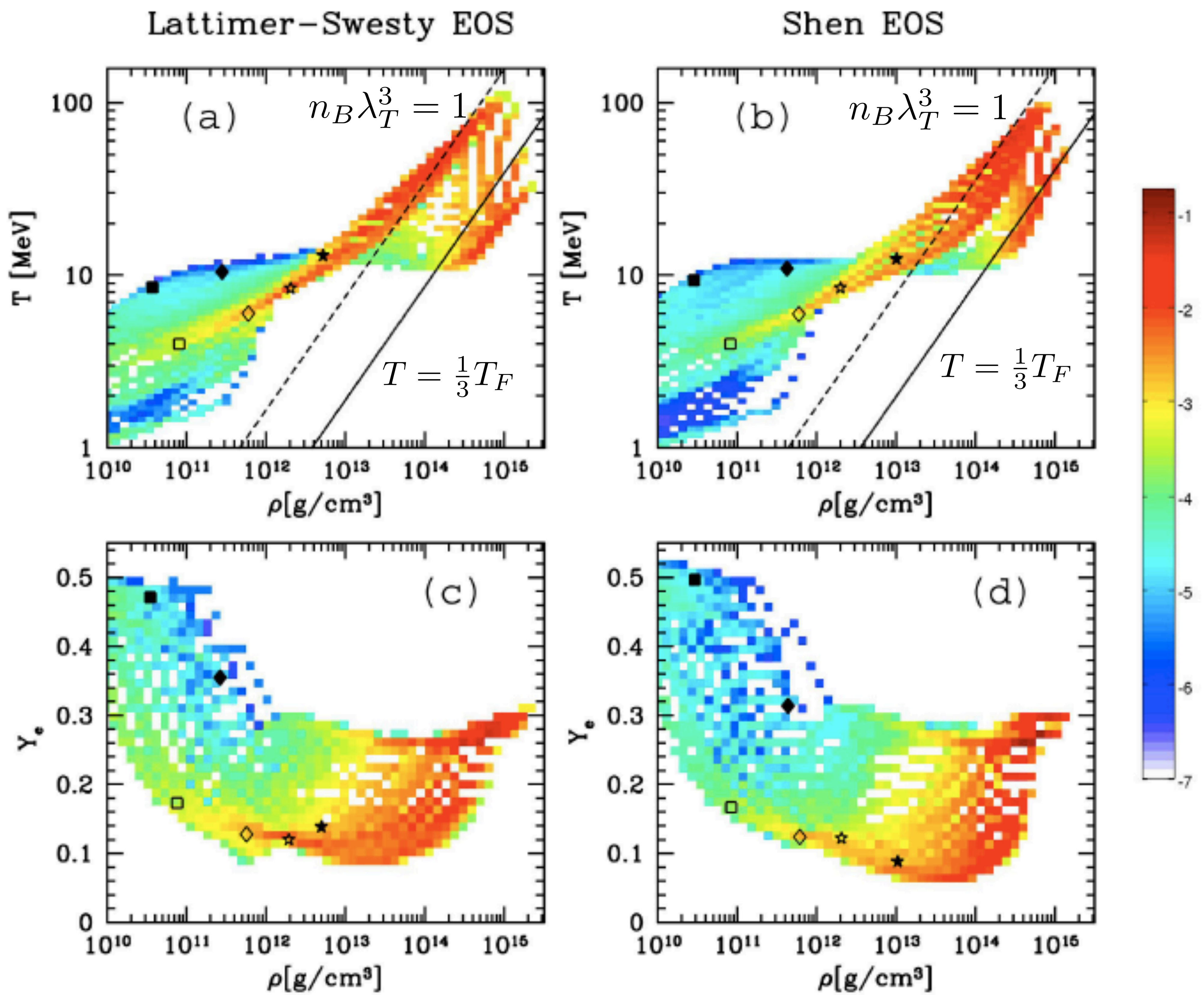
$$S(q,\omega) = \frac{1}{\pi n} \frac{1}{1 - \mathrm{e}^{-\omega/T}} \mathrm{Im}\chi(\omega, \mathbf{q})$$

- Crucial densities seem to be somewhat below nuclear matter density. (High densities – neutrinos trap. Low densities – few interactions)

15 $M_\odot$ progenitor (M. Liebendörfer and A. Perego)



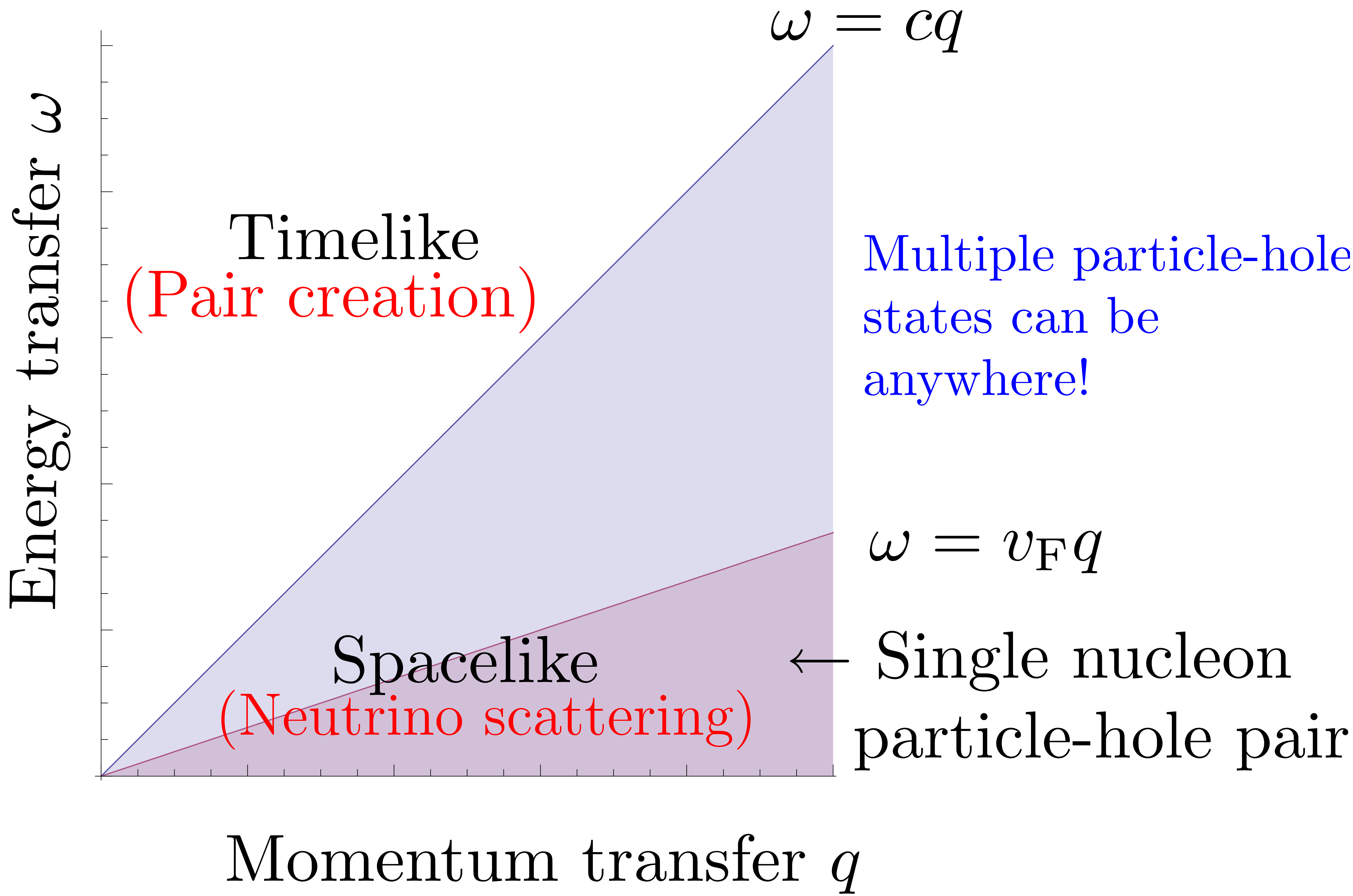
40 $M_\odot$ progenitor (M. Liebendörfer and A. Perego)



Phase Space

- Neutrino scattering. $(\omega, \mathbf{q})$ spacelike.
- Neutrino pair production or annihilation. $(\omega, \mathbf{q})$ timelike.
- Single nucleon particle-hole pair. $|\omega| \leq v_{\mathrm{F}} q$, i.e. spacelike.
- Need collisions between nucleons for the latter processes.
  Two or more particle-hole pair creation.
  Both timelike and spacelike.
- Also affects scattering.
- Quasiparticles damped. Landau-Pomeranchuk-Migdal effect.
  (Raffelt and coworkers.)

Phase Space



$\omega = cq$
Energy transfer ω
Timelike
(Pair creation)
Multiple particle-hole
states can be
anywhere!
$\omega = v_{\mathrm{F}} q$
Spacelike
(Neutrino scattering)
← Single nucleon
particle-hole pair
Momentum transfer q

Effect of Interactions

- Generally axial vector interaction most important. Factor 3. Vector important for coherent scattering from nuclei.

- Density response. Particle number conserved.

$$\omega_{m0}(\rho_{\mathbf{q}})_{m0} = -\mathbf{q} \cdot (\mathbf{j}_{\mathbf{q}})_{m0}, \qquad (\rho_{\mathbf{q}})_{m0} = -\frac{\mathbf{q} \cdot (\mathbf{j}_{\mathbf{q}})_{m0}}{\omega_{m0}}$$

For $q \to 0$, no matrix elements to states with nonzero excitation energy (e.g., multipair states).

- Spin response. Total spin not conserved. (cf. liquid ^{3}He) Nonzero matrix elements to multipair states.

- Effects well known to nuclear physicists. $(g_A \neq 1)$

Spin response function

- Kinetic equation.
- Mean field. Collisionless Landau–Boltzmann equation. Single pairs.

$$\chi_\sigma = N(0)/[1 + g_0 N(0)]$$

- Add collisions. This includes two-pair states.

$$\chi_\sigma = X^0/[1 + g_0 X^0]$$

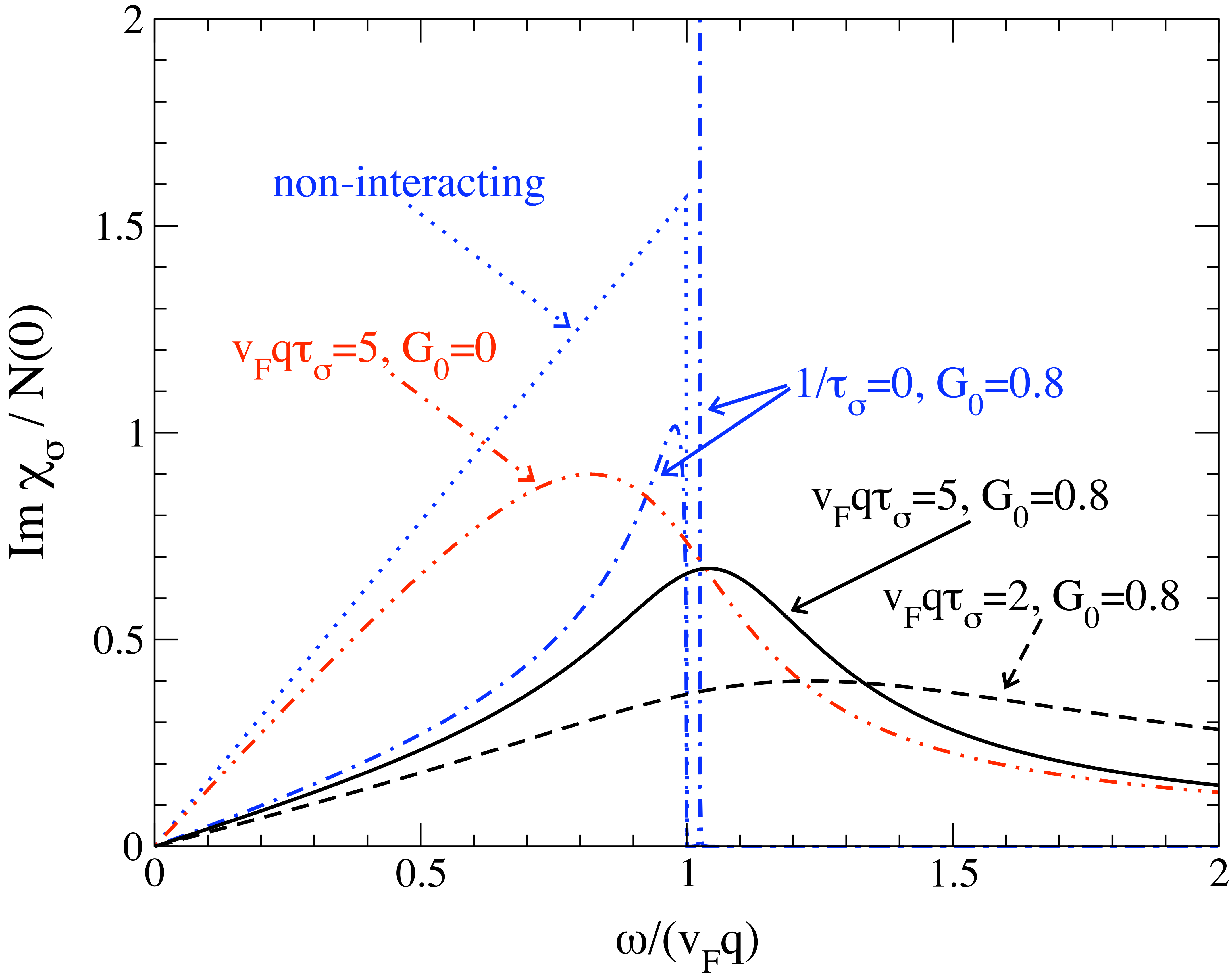
X^0 includes effects of collisions.

- Includes effects of nonzero q and mean field.
- Long wavelengths, $q \to 0$

$$\chi_\sigma = \frac{N(0)}{1 + G_0 - \mathrm{i}\omega\tau_\sigma}$$

Usual relaxation form. τ_σ – spin relaxation time.

In- and out-scattering terms in Boltzmann equation. Vertex corrections.



Spin dynamical structure factor

Collision Rates

Degenerate matter $(T \ll T_F)$

- Collision rate

$$\frac{1}{\tau} = C[T^2 + (\omega/2\pi)^2]$$

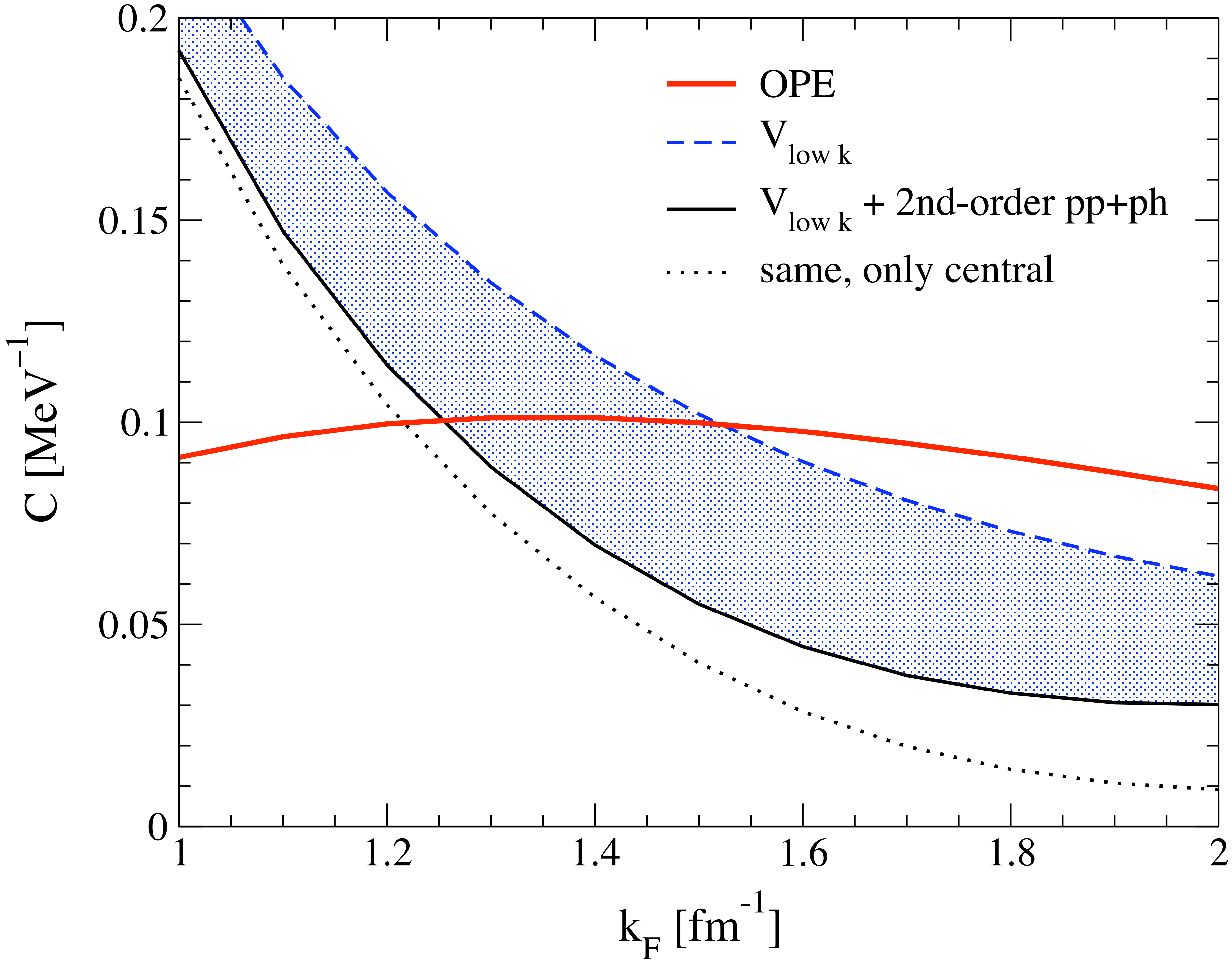
- Spin relaxation rate

$$\frac{1}{\tau_\sigma} = C_\sigma[T^2 + (\omega/2\pi)^2]$$

Non-central, especially tensor forces essential!

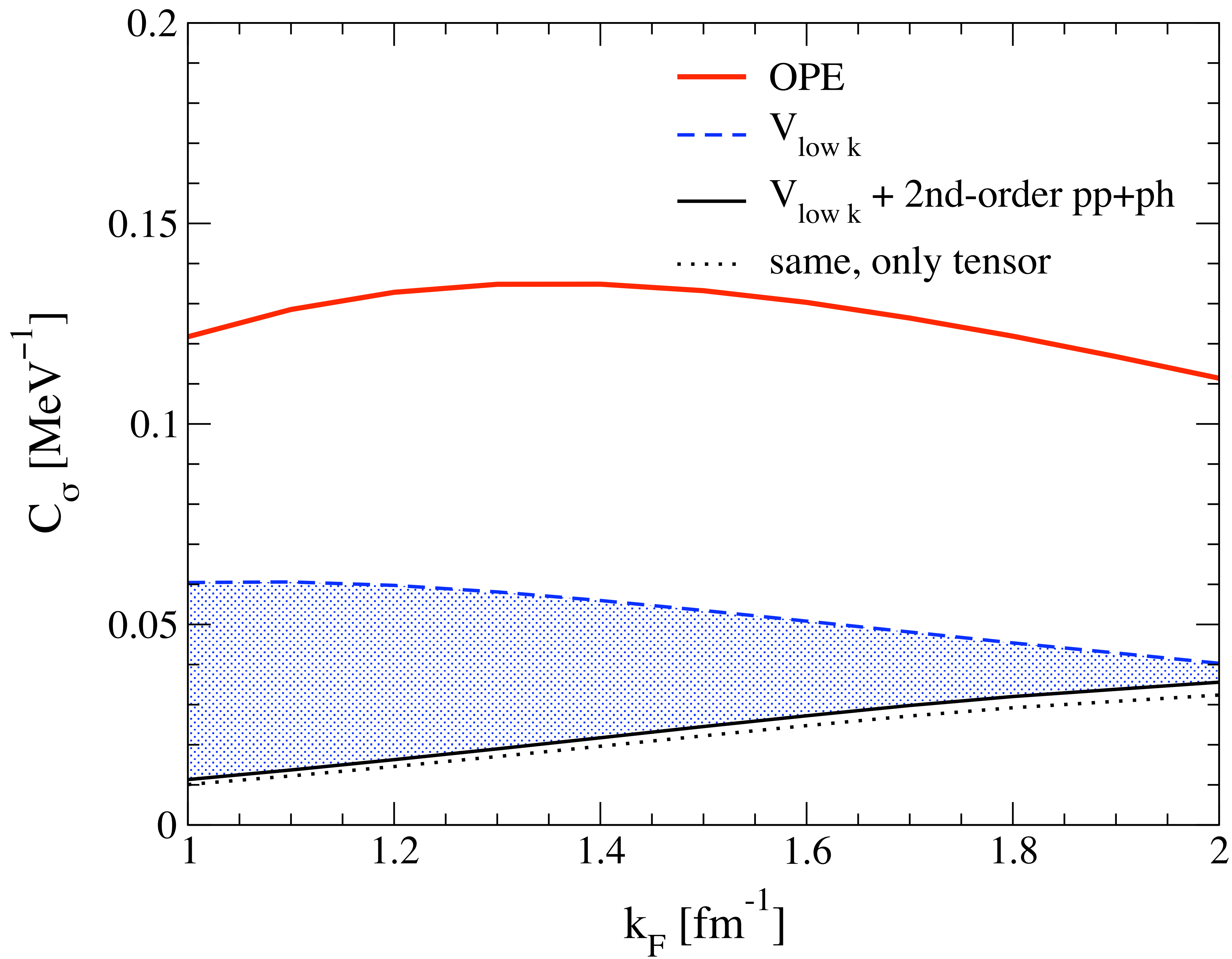
Nucleon–nucleon interactions

- One-pion exchange. $\uparrow + \uparrow \;\; \rightarrow \;\; \downarrow + \downarrow$
- Potentials fitted to scattering data (Paris, Argonne, ...)
- V_{lowk}. (Schwenk, Brown, Friman)
  - Build in effects of short-range correlations by introducing cut-off.
  - Put in medium effects by perturbation theory.
- Chiral effective field theory.



Quasiparticle relaxation rate

$$\frac{1}{\tau} = C\left[T^2 + \left(\frac{\omega}{2\pi}\right)^2\right]$$



Spin relaxation rate reduced compared with OPE

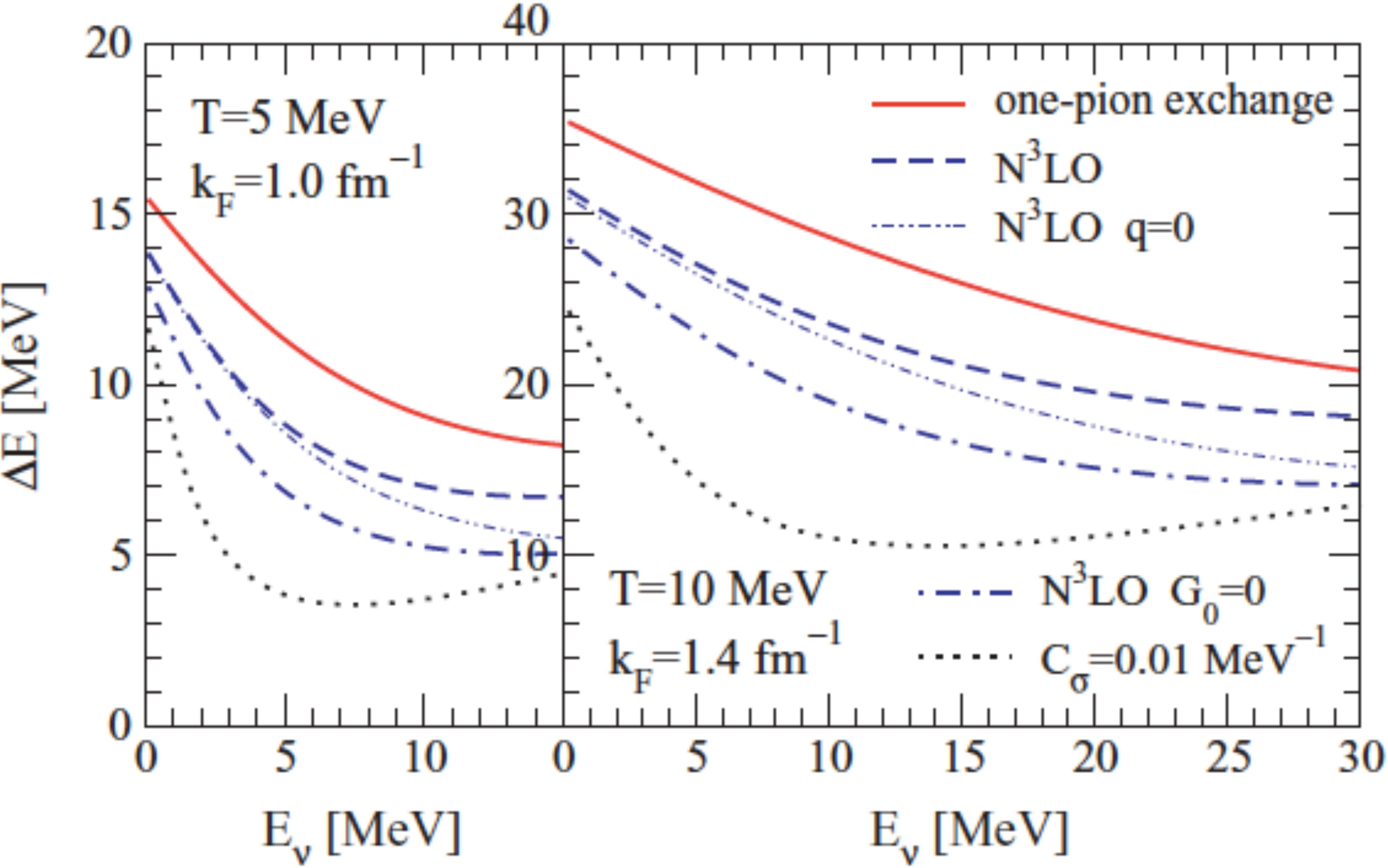
$$\frac{1}{\tau_\sigma} = C_\sigma \left[T^2 + \left(\frac{\omega}{2\pi} \right)^2 \right]$$

Collisions affect energy transfer to and from neutrinos

- No collisions. Energy transfer $\sim v_F q$.
- With collisions. Energy transfer $\sim \hbar/\tau_\sigma$ even for $q = 0$.

RMS Energy Transfer in Scattering

(Chiral effective field theory) S. Bacca, K. Hally, C. J. Pethick, and A. Schwenk, Phys. Rev. C **80**, 032802 (2009).



Thanks to:

Emma Olsson, Pawel Haensel, Gennadi Lykasov,
Achim Schwenk, Sonia Bacca, Kathy Hally

E. Olsson and C. P., Phys. Rev. C **66**, 065803 (2002).
E. Olsson, P. Haensel, and C. P., Phys. Rev. C **70**, 025804 (2004).
G. Lykasov, E. Olsson, and C. P., Phys. Rev. C **72**, 025802 (2005).
G. Lykasov, C. P., and A. Schwenk, Phys. Rev. C **78**, 045803 (2008).

For the future

- Mixtures of neutrons and protons
- Other components (Clusters – α particles, other nuclei).
- Extend to less degenerate regime.
- Prepare tables for numerical codes. Modular codes.
- Sensitivity tests. Where to put calculating effort.
- Better estimates of in-medium effects on scattering.

Final remarks

- Similar effects in diverse areas of physics
- Powerful effects of conservation laws
- Plenty of work to do!

Til lykke! Jochen

“Hip, Hip, Hurrah!”
(P. S. Krøyer)