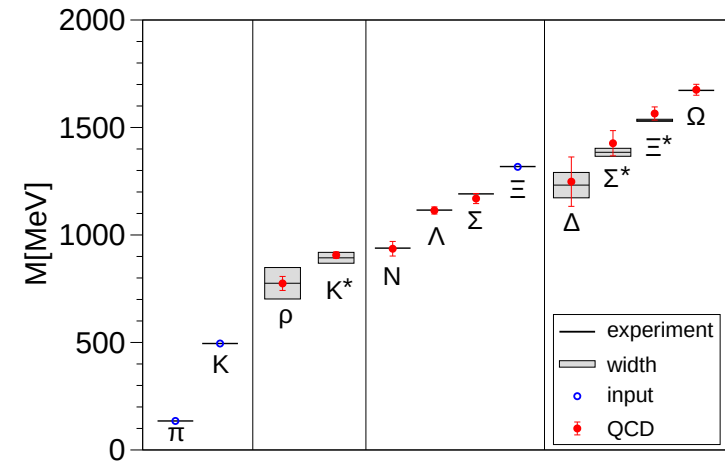


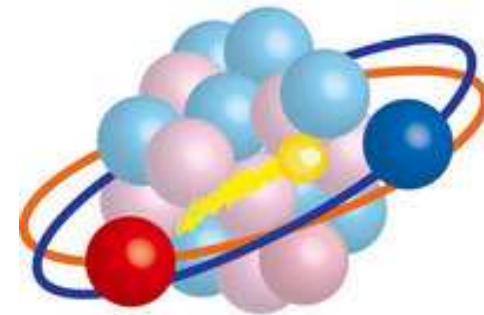
Introduction

FACETS of STRONG QCD

- quarks and gluons form hadrons
 - ⇒ **lattice QCD**
 - ⇒ **exploring the strong color force**
- nucleons and mesons form nuclei
 - ⇒ **nuclear physics**
 - ⇒ **exploring the residual color force**



BMW collaboration (2008)

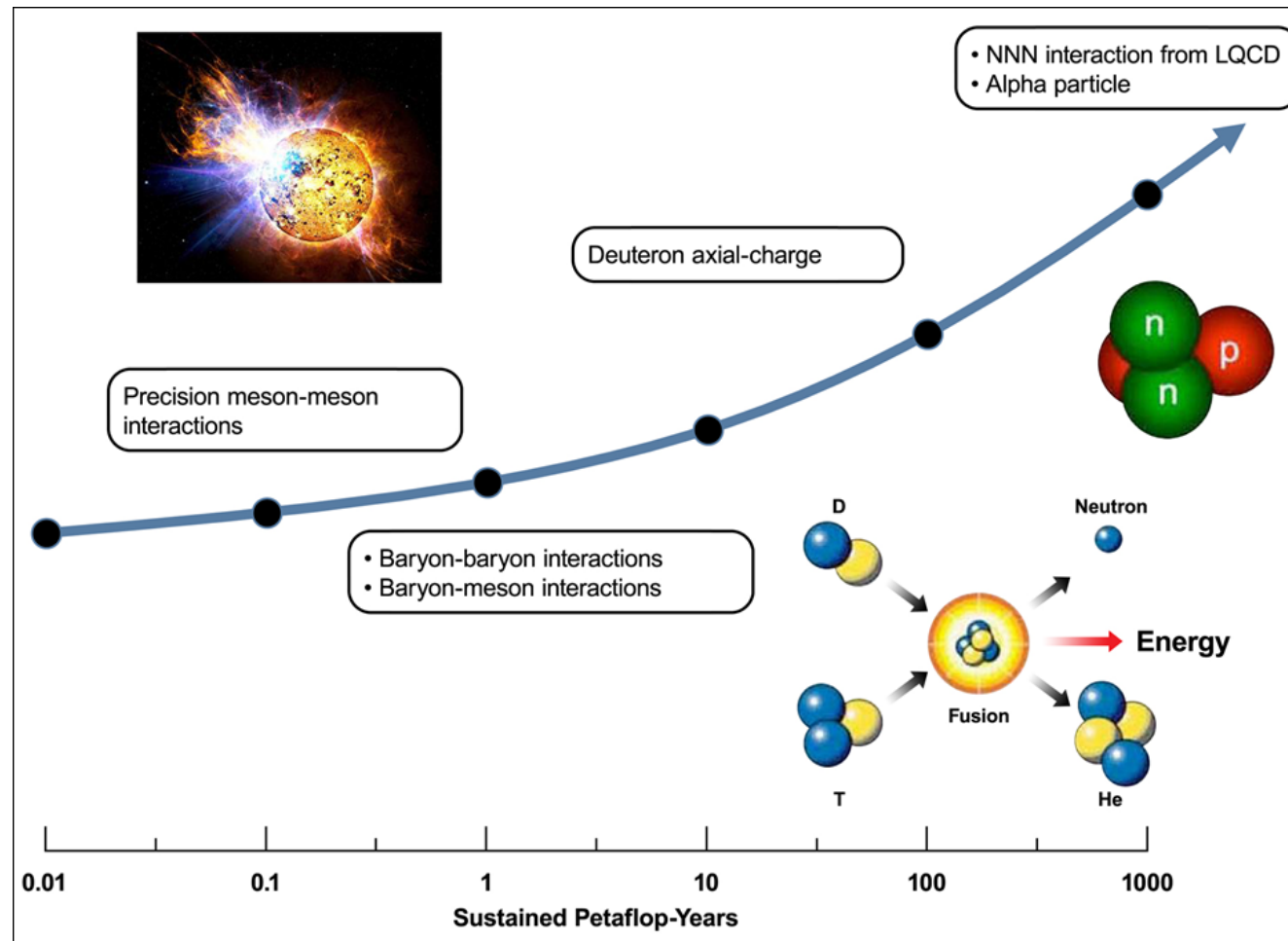


NUCLEAR PHYSICS from LATTICE QCD

5

- LQCD requires hundreds of sustained petaflop-yrs — not quite there yet

USDOE, Computing at the Extreme Scale, 2009



THE NUCLEAR LANDSCAPE: AIMS & METHODS

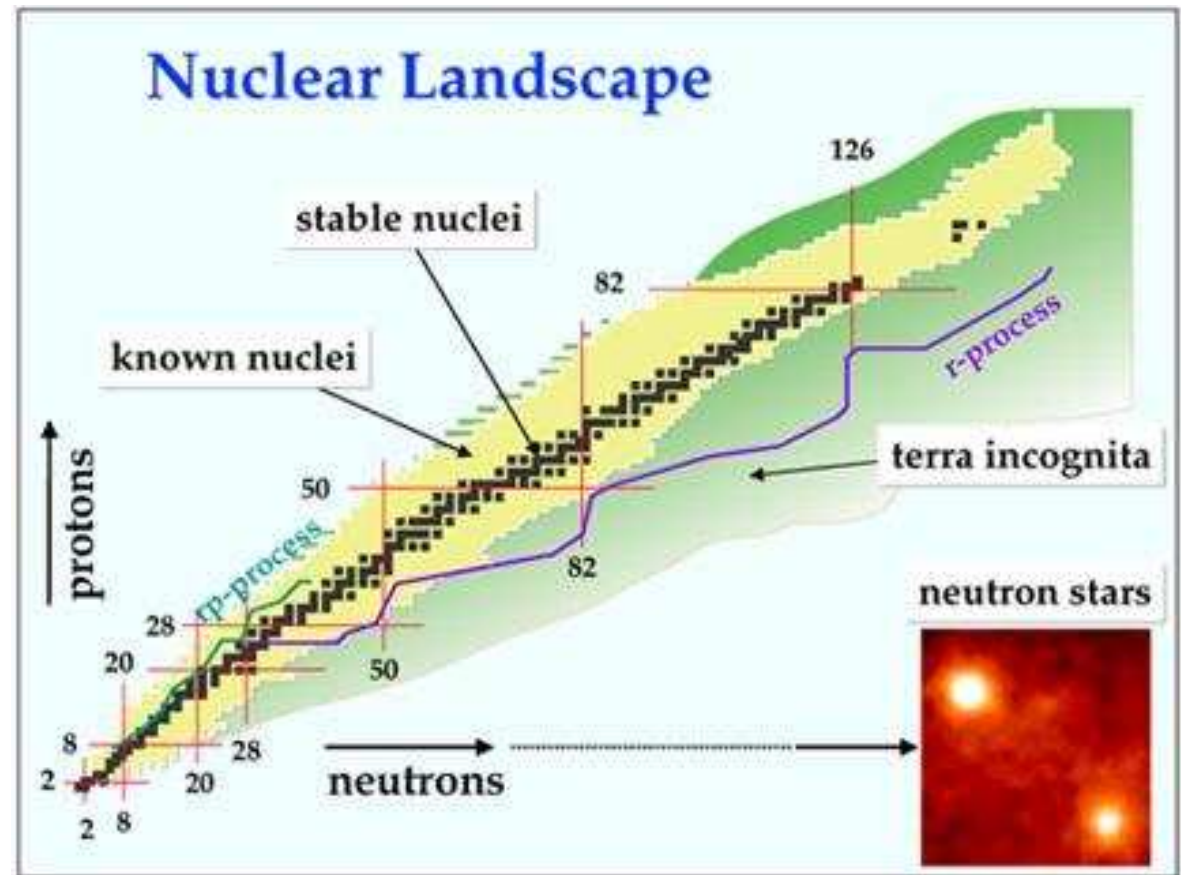
6

- Theoretical methods:

- Lattice QCD: $A = 0, 1, 2, \dots$
- NCSM, Faddeev-Yakubowsky, GFMC, ... :
 $A = 3 - 16$
- coupled cluster, ... : $A = 16 - 100$
- density functional theory, ... : $A \geq 100$

- Chiral EFT:

- provides accurate NN and 3N forces
- successfully applied in light nuclei
with $A = 2, 3, 4$
- *combine with simulations to go to larger A*



⇒ Nuclear Lattice Simulations

Effective Field Theory for Nuclear Physics

- Scales in nuclear physics:

Natural: $\lambda_\pi = 1/M_\pi \simeq 1.5 \text{ fm}$

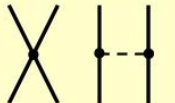
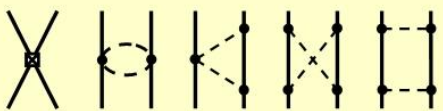
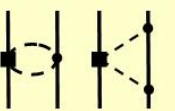
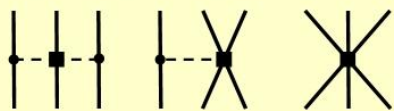
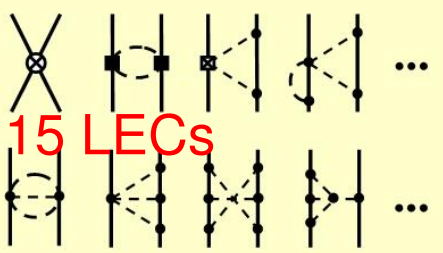
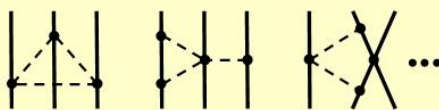
Unnatural: $|a_{np}(^1S_0)| = 23.8 \text{ fm}$, $a_{np}(^3S_1) = 5.4 \text{ fm} \gg 1/M_\pi$

- this can be analyzed in a suitable EFT based on

$$\mathcal{L}_{\text{QCD}} \rightarrow \mathcal{L}_{\text{EFF}} = \mathcal{L}_{\pi\pi} + \mathcal{L}_{\pi N} + \mathcal{L}_{NN} + \dots$$

- pion and pion-nucleon sectors are perturbative in $Q/\Lambda_\chi \rightarrow$ chiral perturbation th'y
- $\mathcal{L}_{NN}$ collects short-distance contact terms, to be fitted
- NN interaction requires non-perturbative resummation
 $\rightarrow$ chirally expand V_{NN} , use in regularized LS equation

CHIRAL POTENTIAL and NUCLEAR FORCES

	Two-nucleon force	Three-nucleon force	Four-nucleon force	
LO	 2 LECs			$\mathcal{O}((Q/\Lambda_\chi)^0)$
NLO	 7 LECs			$\mathcal{O}((Q/\Lambda_\chi)^2)$
N ² LO		 2 LECs		$\mathcal{O}((Q/\Lambda_\chi)^3)$
N ³ LO	 15 LECs			$\mathcal{O}((Q/\Lambda_\chi)^4)$

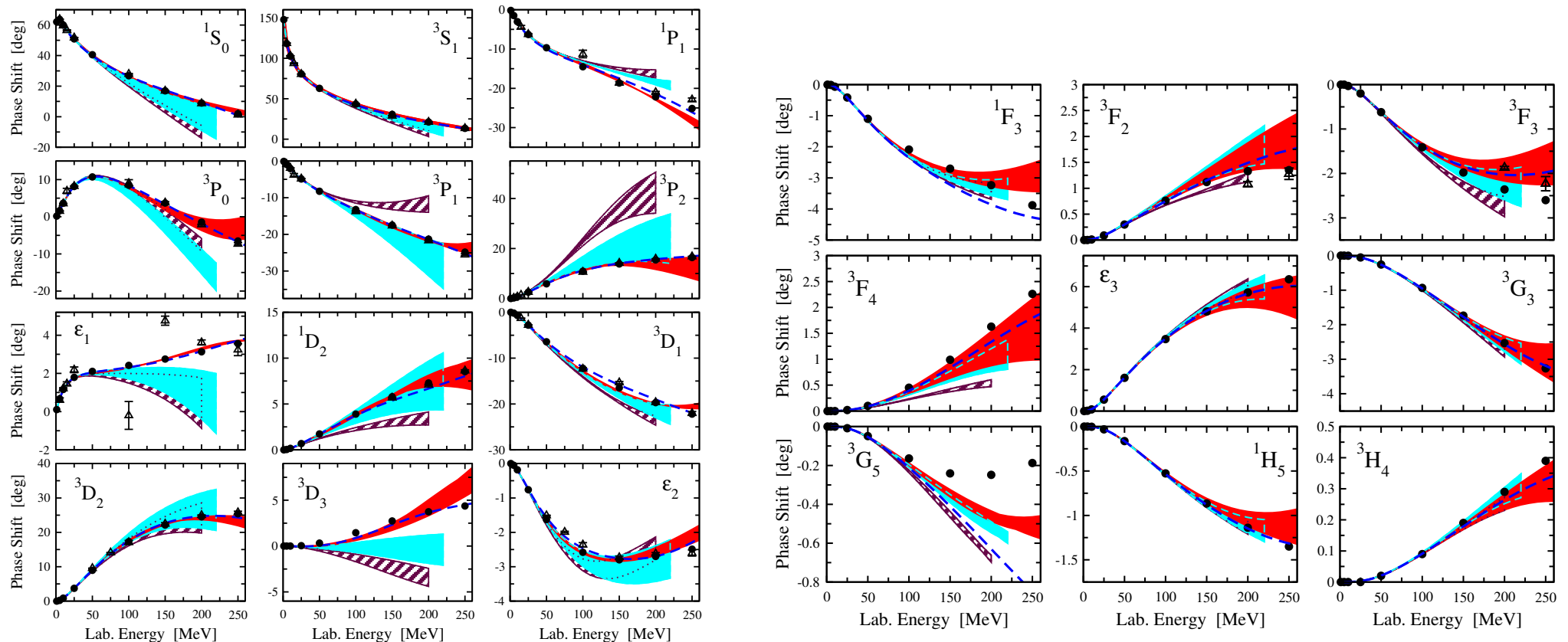
- Isospin breaking (strong and electromagnetic) and deltas also included
- explains naturally the observed hierarchy of nuclear forces

Results for few-nucleon systems

NB: Just an appetizer → details in
E. Epelbaum, H.-W. Hammer, UGM, Rev. Mod. Phys. **81** (2009) 1773
[arXiv:0811.1338 [nucl-th]]

Epelbaum, Glöckle, M., Nucl. Phys. A **747** (2005) 362 [nucl-th/0405048]; Entem, Machleidt, Phys. Rev. C **68** (2003) 041001 [nucl-th/0304018]

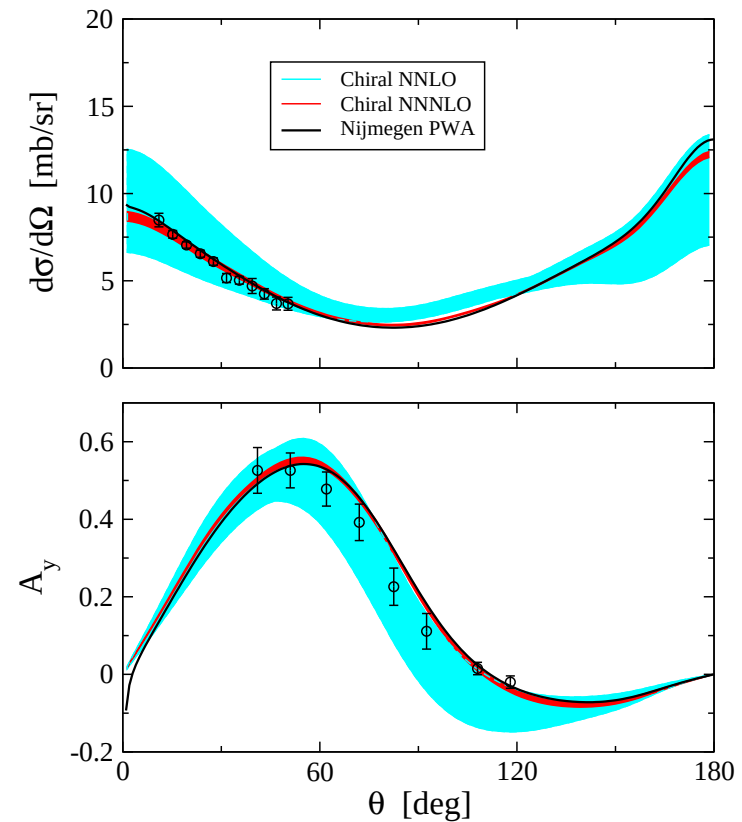
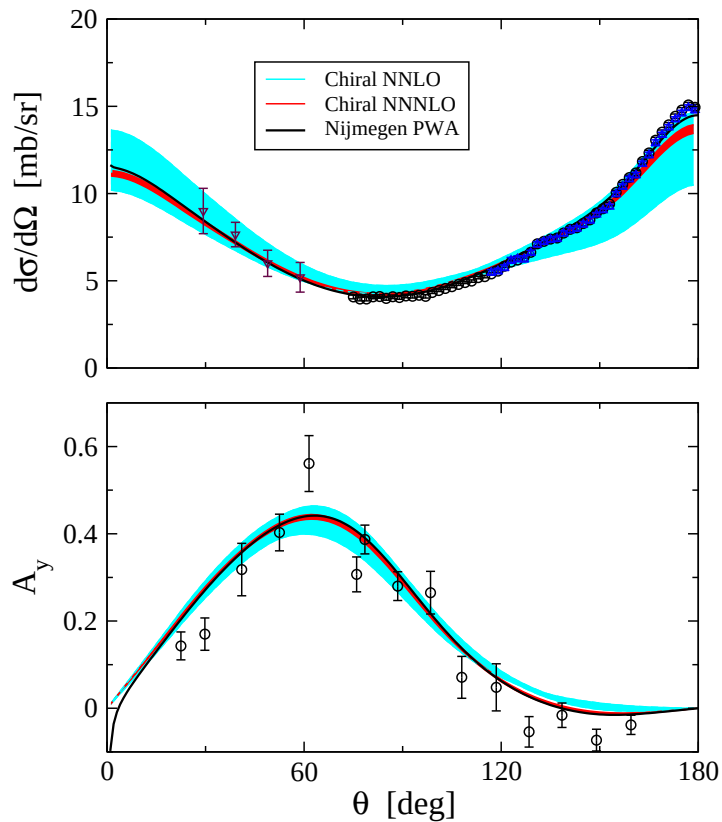
- Results: S-, P-, D- and selected peripheral partial waves



- legend: ■ NLO ■ N²LO ■ N³LO
- excellent description & good convergence $\Rightarrow$ high-precision approach

NN INTERACTION at N³LO: OBSERVABLES

- Some results for np scattering, $E_{\text{lab}} = 96$ & 143 MeV



— N³LO — N²LO — Nijm PWA

3N INTERACTION to N²LO: PREDICTIONS

13

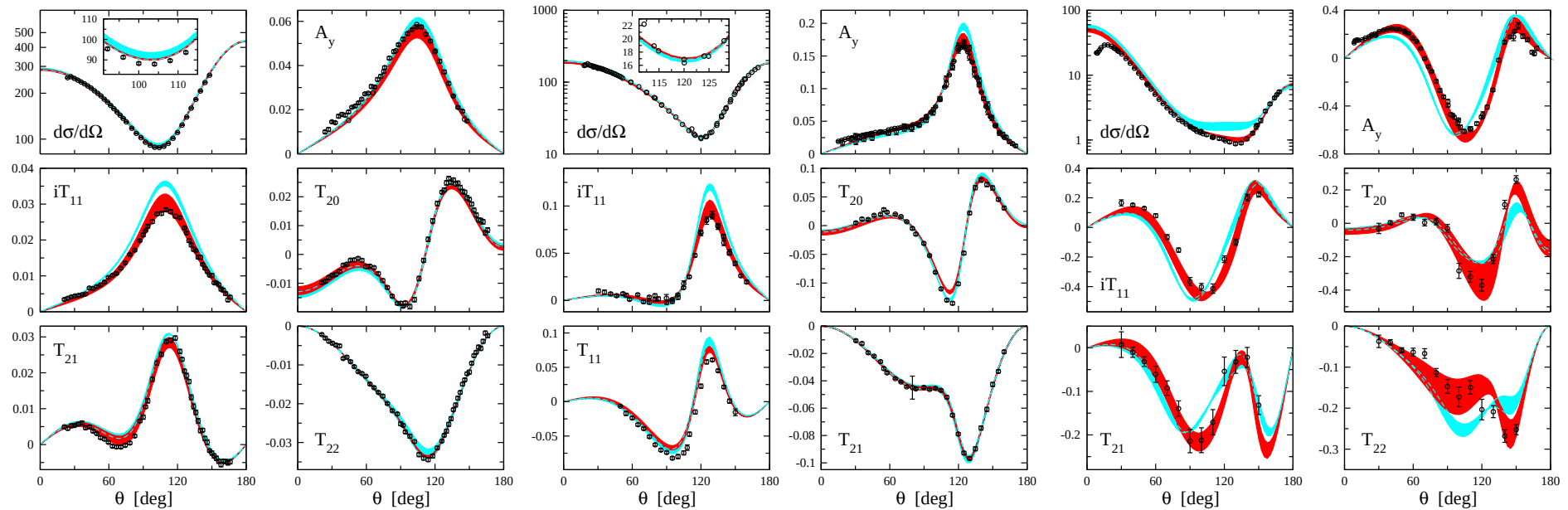
Epelbaum, Prog. Part. Nucl. Phys. **57** (2006) 654

- 2 parameters fixed from triton BE & nd doublet scattering length → predictions
- nd scattering: consider diff. XS, vector and tensor analyzing powers

$$E_n = 3 \text{ MeV}$$

$$E_n = 10 \text{ MeV}$$

$$E_n = 65 \text{ MeV}$$



— N²LO
— NLO

⇒ Need full N³LO calculation + Coulomb!

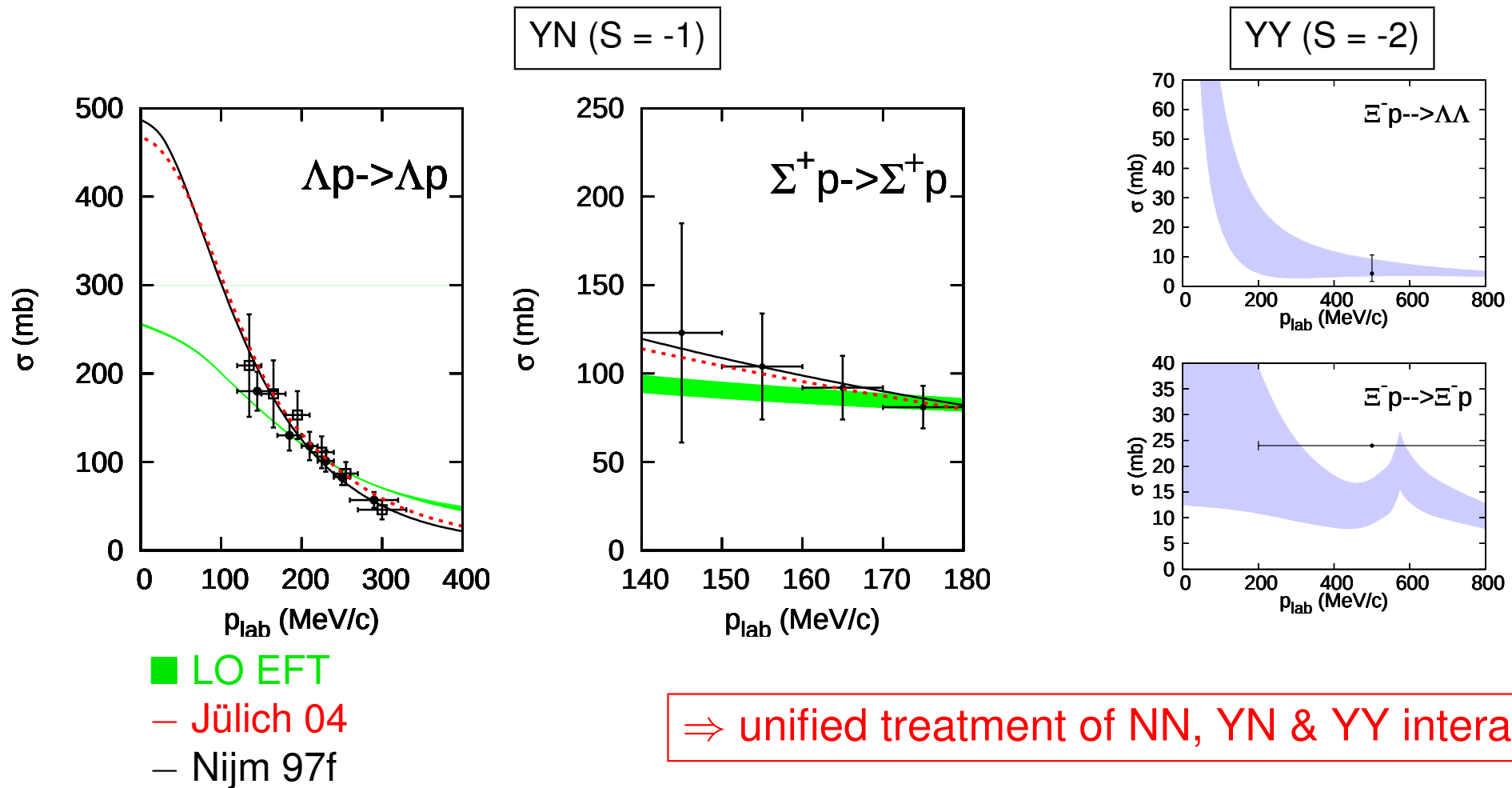
Bernard et al.
Deltuva et al.
Glöckle et al.

EXTENSION TO YN and YY INTERACTIONS

14

Haidenbauer, Polinder, M., Nucl. Phys. A **779** (2006) 244; Phys. Lett. B **653** (2007) 29; Haidenbauer, M. Phys. Lett. B **684** (2010) 275

- Hyper-nuclear physics at FAIR, J-PARC, JLab, . . . : YN & YY interactions ($Y = \Lambda, \Sigma, \Xi$)
- Already leading order calculation quite successful (5+1 LECs) $\rightarrow$ more natural
- First time: theoretical error bars + bound hypertriton



Nuclear lattice simulations – Formalism –

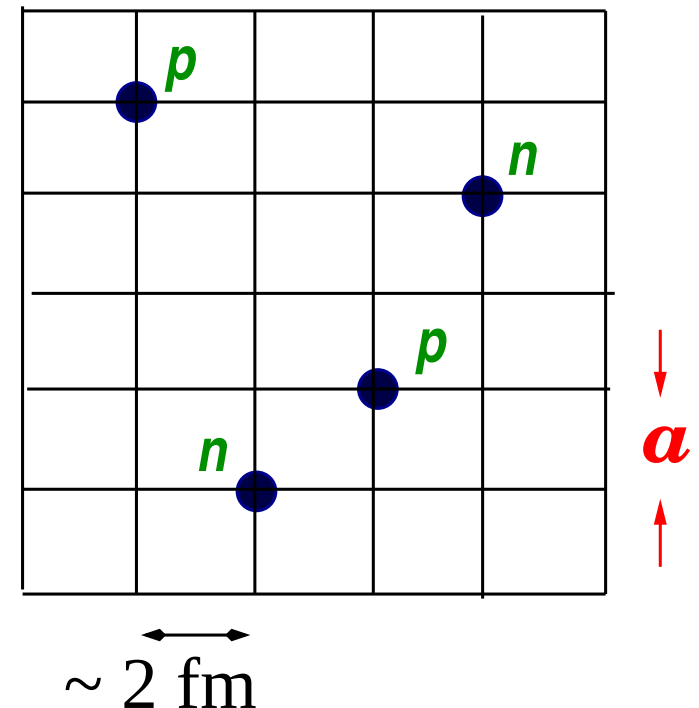
NUCLEAR LATTICE SIMULATIONS

Borasoy, Krebs, Lee, M., Nucl. Phys. **A768** (2006) 179
 Borasoy, Epelbaum, Krebs, Lee, M., Eur. Phys. J. **A31** (2007) 105

- nucleons are point-like fields on the sites
- discretized chiral potential with point-like instantaneous pions
- typical lattice parameters

$$\Lambda = \pi/a \simeq 300 \text{ MeV [UV cutoff]}$$

$$L_s \simeq 10 \dots 20 \text{ fm}$$



- strong suppression of sign oscillations due to approximate Wigner SU(4) symmetry
- hybrid Monte Carlo & transfer matrix (similar to LQCD)

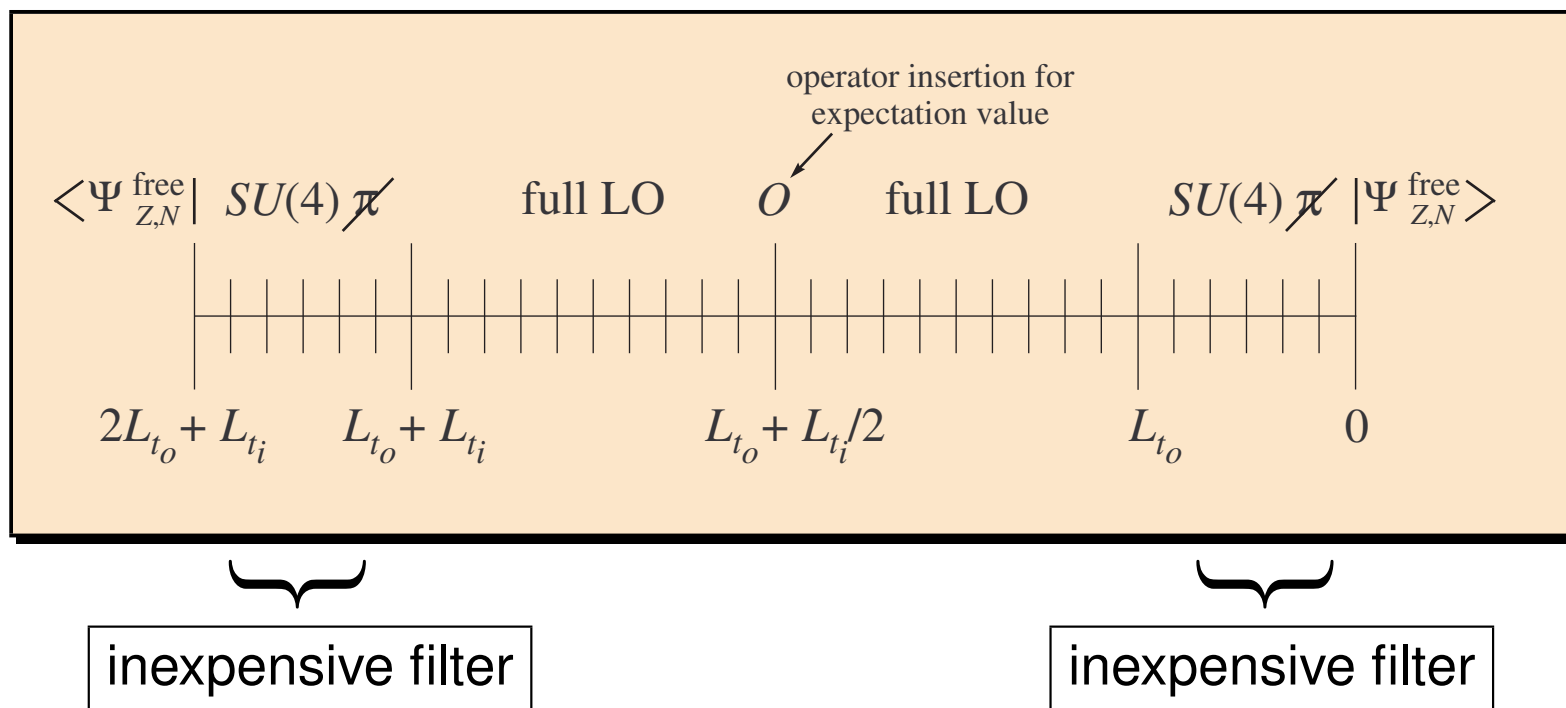
J. W. Chen, D. Lee and T. Schäfer, Phys. Rev. Lett. **93** (2004) 242302

TRANSFER MATRIX CALCULATION

- Expectation value of any normal-ordered operator $\mathcal{O}$

$$\langle \Psi_A | \mathcal{O} | \Psi_A \rangle = \lim_{t \rightarrow \infty} \frac{\langle \Psi_A | \exp(-tH/2) \mathcal{O} \exp(-tH/2) | \Psi_A \rangle}{\langle \Psi_A | \exp(-tH) | \Psi_A \rangle}$$

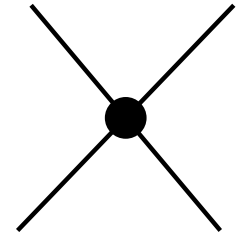
- Anatomy of the transfer matrix



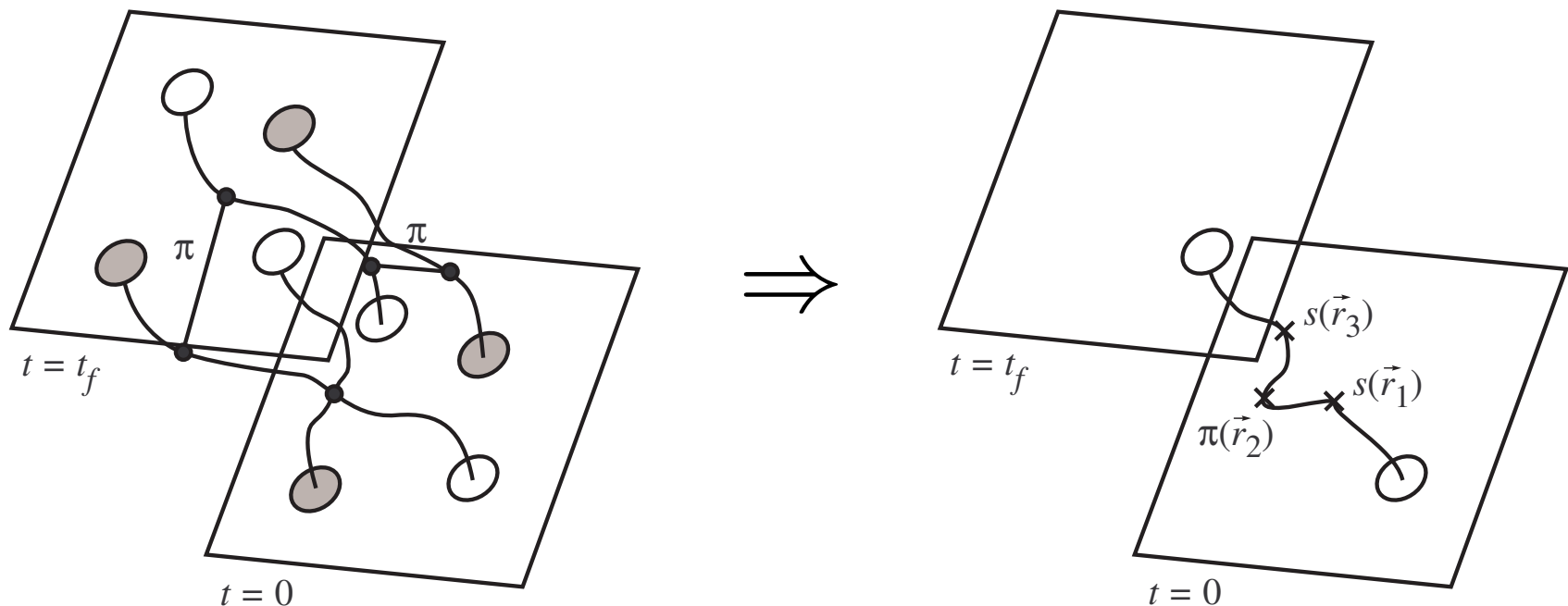
MONTE CARLO with AUXILIARY FIELDS

- Contact interactions represented by auxiliary fields s, s_I

$$\exp(\rho^2/2) \propto \int_{-\infty}^{+\infty} ds \exp(-s^2/2 - s\rho), \quad \rho \sim N^\dagger N$$



- Correlation function = path-integral over pions & auxiliary fields



SCATTERING in a FINITE VOLUME

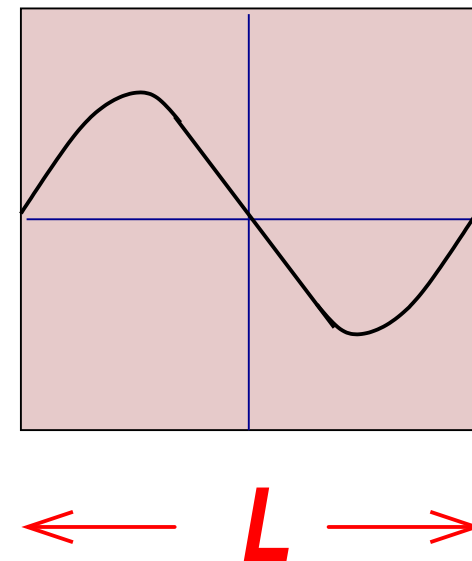
Lüscher, Comm. Math. Phys. **104** (1986) 177; **105** (1986) 153; Nucl. Phys, B **354** (1991) 531

- cubic lattice: rotation group $SO(3)$ broken to $SO(3, Z)$
- 5 irreducible representations (A_1, T_1, E, T_2, A_2) include definite J modulo 4
- Lüscher's formula for phase shifts (large L)

$$\exp(2i\delta_0) = \frac{Z_{00}(1; q^2) + i\pi^{3/2}q}{Z_{00}(1; q^2) - i\pi^{3/2}q}$$

$$q = 2\pi n/L, \quad n \in \mathbb{Z}^3$$

$$Z_{00}(s; q^2) = \frac{1}{\sqrt{4\pi}} \sum_{n \in \mathbb{Z}^3} \frac{1}{(n^2 - q^2)^s}$$



- standard method in lattice QCD, see e.g. NPLQCD on hadron-hadron scattering
Beane, Orginos, Savage, Int. J. Mod. Phys. E **17** (2008) 1517
- however: not appropriate for partial-wave mixing, need a different formalism

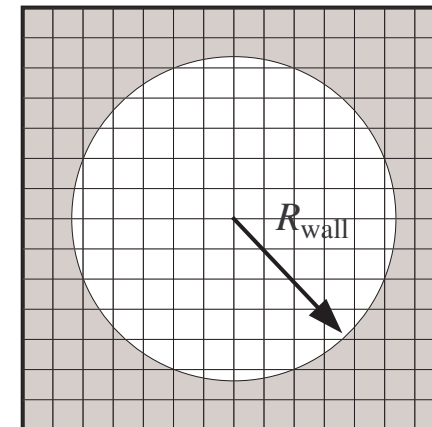
MEASURING PHASE SHIFTS on the LATTICE

Borasoy, Epelbaum, Krebs, Lee, M., Eur. Phys. J. **A 34** (2007) 185

- Spherical wall method: place a wall at sufficiently large R
 $\rightarrow$ standing wave allows to extract phases δ_L & mixings ϵ_L

$$\Psi(\vec{r}) = [\cos \delta_L j_L(kr) - \sin \delta_L y_L(kr)] Y_{L,m}(\theta, \phi)$$

$$\Psi(R) = 0 \Rightarrow \boxed{\tan \delta_L = \frac{j_L(kR)}{y_L(kR)}} \quad (\text{similar for triplet case})$$

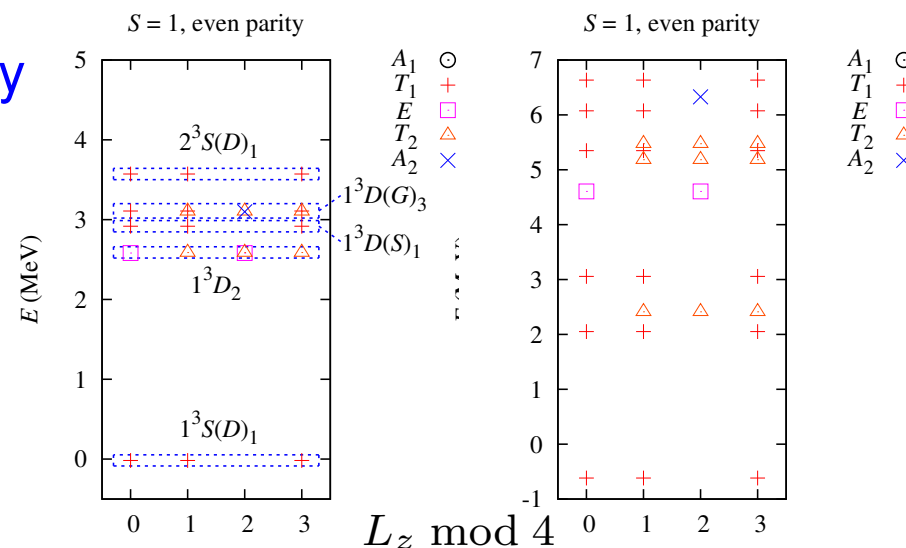


- Toy model: attractive Gaussian potential w/ central & tensor forces

reproduces continuum phase shifts accurately

extra copies of the 2-particle interaction
 due to periodic b.c. removed

much better than standard boxes
 (cf. $R = 10 + \epsilon$ vs $V = 12^3$)



COMPUTATIONAL EQUIPMENT

- Present & future = JUGENE (BlueGene/P/Q)



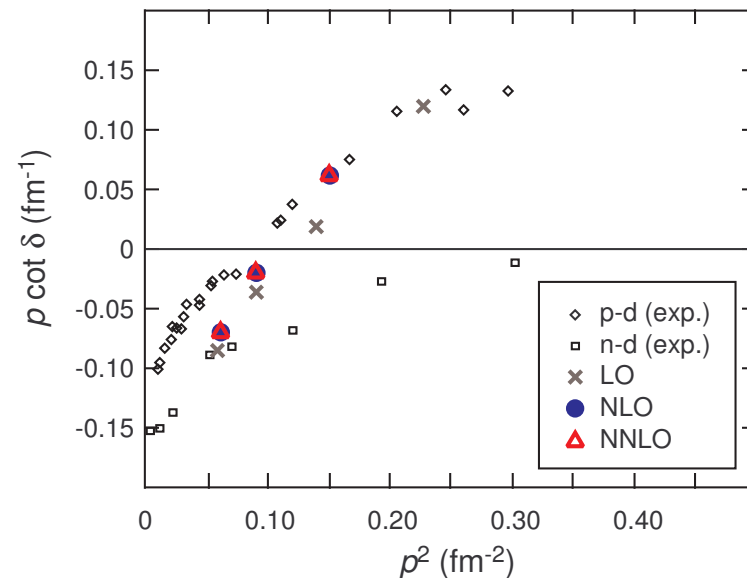
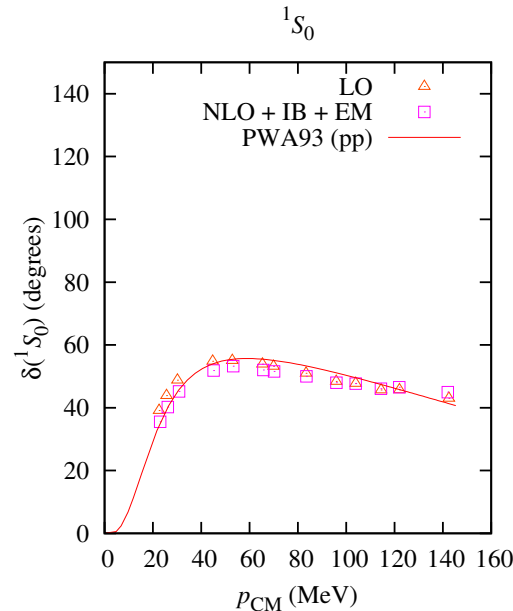
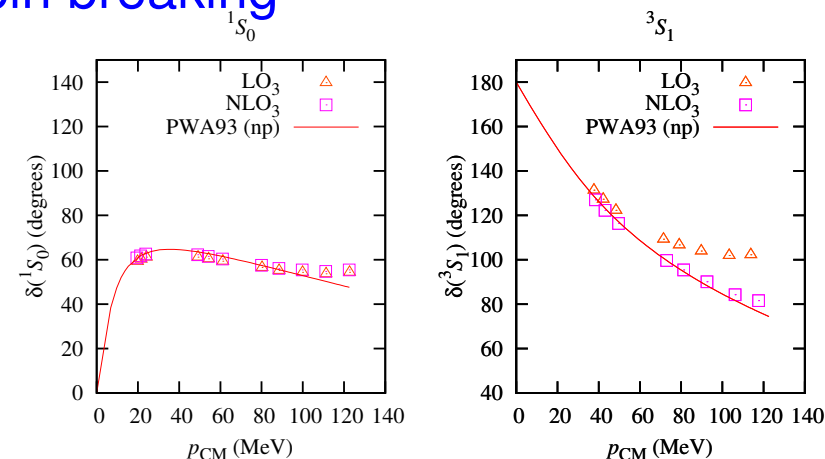
A 3D visualization of a cubic lattice, represented by a blue wireframe grid. Inside the lattice, there are several blue spheres, each with a black arrow pointing in a specific direction, representing magnetic moments. The spheres are distributed across the lattice, with arrows pointing in various directions (up, down, left, right, and diagonally), illustrating the complex magnetic ordering of the material.

FIXING PARAMETERS & FIRST PREDICTIONS

- work at NNLO including strong and em isospin breaking
- 9 NN LECs from np scattering and Q_d
- 2 LECs for isospin-breaking (np , pp , nn)
- 2 LECs D , E related to the leading 3NF

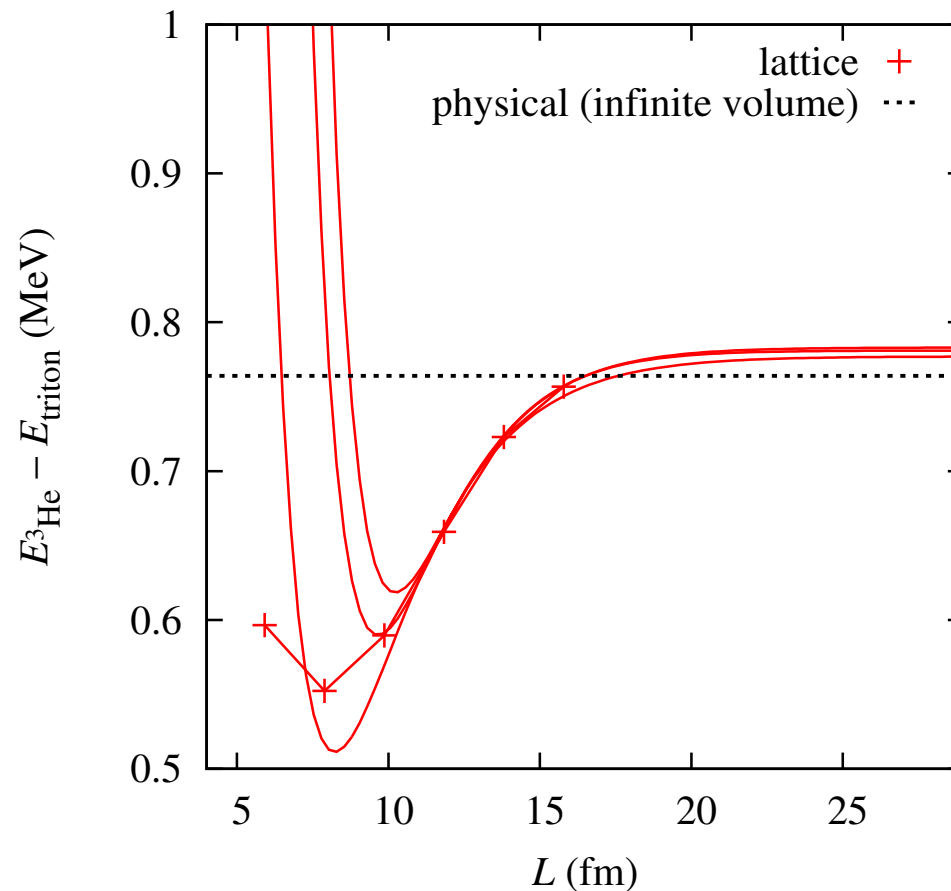
⇒ make predictions

- pp vs np scattering
- nd spin-3/2 quartet channel
- . . .



PREDICTIONS: TRITON & HELIUM-3

- binding energies of 3N systems: $E(L) = \text{B.E.} - \frac{a}{L} \exp(-bL)$
- ⇒ predict the energy difference $E(^3\text{He}) - E(^3\text{H})$

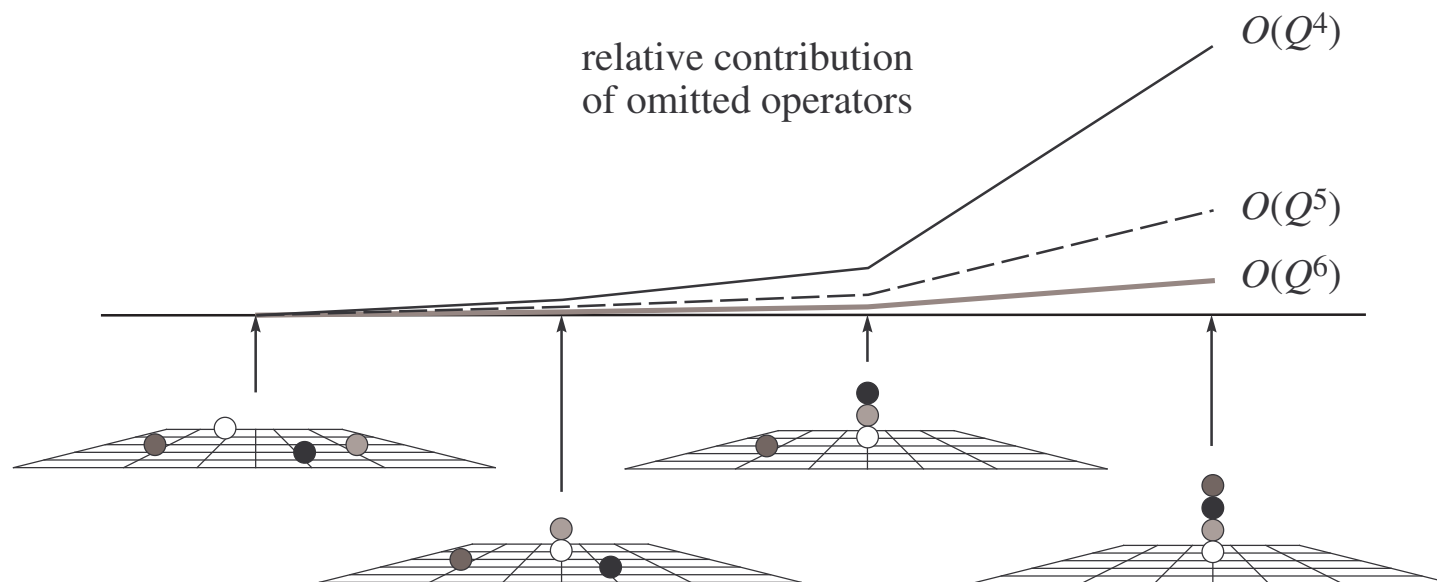


0.78(5) MeV
0.76 MeV [exp.]

SYSTEMS with 4 or MORE NUCLEONS

Epelbaum, Krebs, Lee, M., Phys. Rev. Lett. **104** (2010) 142501; Eur. Phys. J. **A 45** (2010) 335

- relative contribution of the omitted operators:

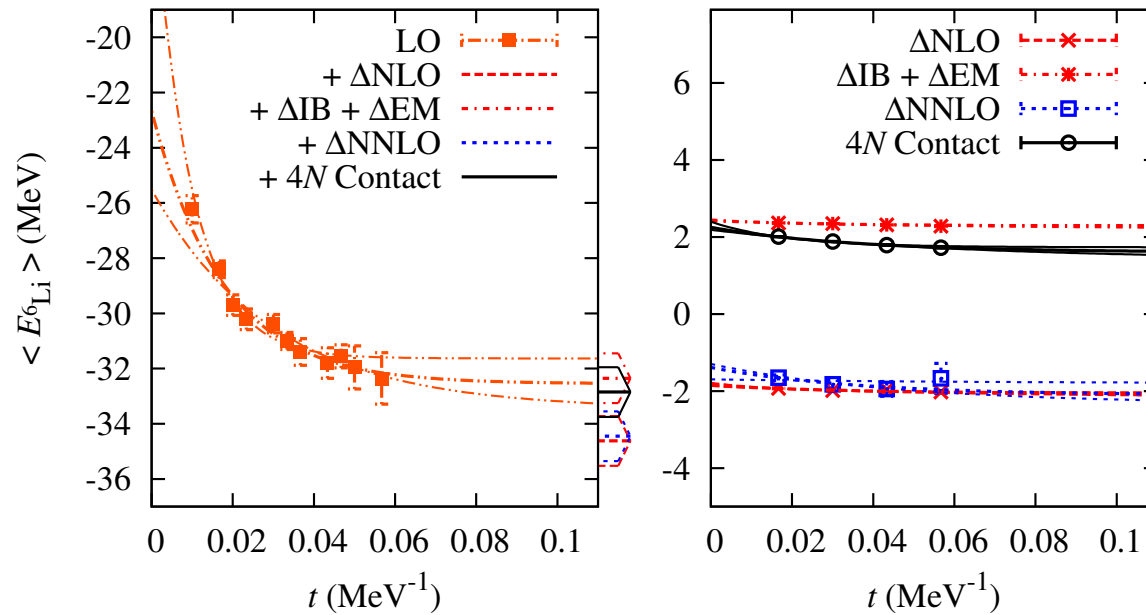


- largest error when 4 nucleons are in close proximity → *universal behaviour*
- test this hypothesis by adding an effective 4N interaction → *fixed in ${}^4\text{He}$*

GROUNDSTATE ENERGY of ${}^6\text{Li}$

26

- In a box of size $L = 9.9$ fm



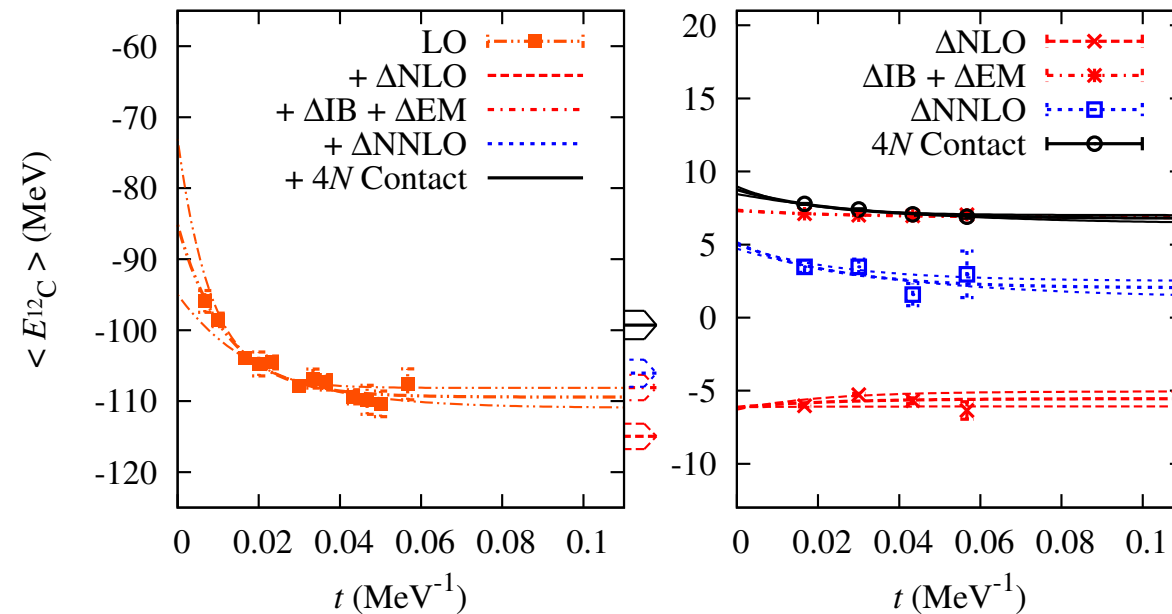
energy @ order	MeV
LO	-32.6(9)
NLO	-34.6(9)
NLO + IB + EM	-32.4(9)
NNLO + IB + EM	-34.5(9)
NNLO + IB + EM + 4N	-32.9(9)
physical	-32.0

- still to be done: infinite volume extrapolation

GROUNDSTATE ENERGY of ^{12}C

27

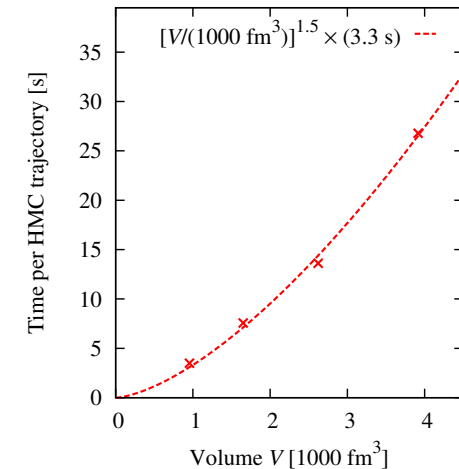
- In a box of size $L = 13.8$ fm



energy @ order	MeV
LO	-109(2)
NLO	-115(2)
NLO + IB + EM	-108(2)
NNLO + IB + EM	-106(2)
NNLO + IB + EM + 4N	-99(2)
physical	-92.2

- still to be done: infinite volume extrapolation

- $$X_{12C}^{\text{CPU}} = 5 \times 10^{-5} \text{ PFlop-yr}$$
- $$X_{12C}^{\text{storage}} = 0.14 \text{ TB}$$



- $$X^{\text{CPU}} \approx X_{12\text{C}}^{\text{CPU}} \times \left(\frac{A}{12}\right)^{3.2} \exp[0.10(A - 12)] \quad [\text{e.g. } \times 30.5 \text{ for } {}^{24}\text{Mg}]$$

$$X^{\text{storage}} \approx X_{12C}^{\text{storage}} \times \left(\frac{A}{12}\right)^2 \exp[0.10(A - 12)] \quad [\text{e.g. } \times 13.3 \text{ for } {}^{24}\text{Mg}]$$

• ○ ◁ < ^ ▽ ▷ ●

DILUTE NEUTRON MATTER at NLO

30

Borasoy et al., Eur. Phys. J. **A 35** (2008) 357; Epelbaum et al., Eur. Phys. J. **A 40** (2009) 199

- put $N = 8, 12, 16$ neutrons in a box with $V = 4^3, 5^3, 6^3, 7^3$

$$k_F = (3\pi^2 N)^{1/3} / L$$

⇒ 2- 11% of nuclear matter density

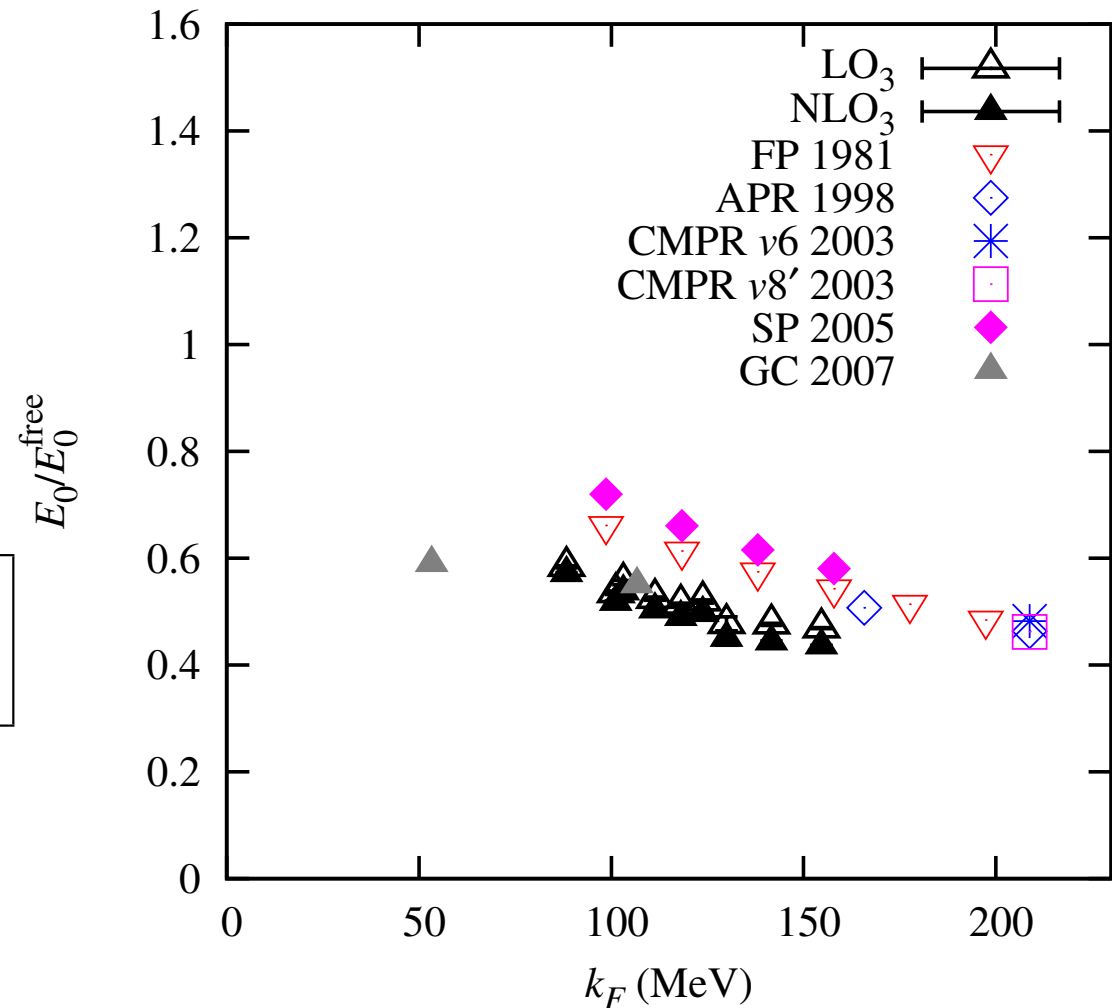
- close to the unitary limit

$$\frac{E_0}{E_0^{\text{free}}} = \xi - \frac{\xi_1}{k_F a} + 0.15 k_F r_{\text{eff}}$$

$$\xi = 0.31(1) \quad \xi_1 \simeq 0.81$$

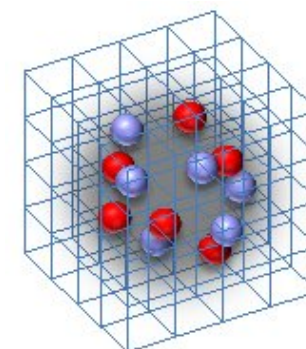
- cf. GFMC (AV-8'): $\xi = 0.42(1)$

Carlson et al.



SUMMARY & OUTLOOK

- Nuclear forces from chiral EFT: direct relation to QCD
 - systematic and precise approach based on chiral $\mathcal{L}_{\text{eff}}$
 - consistent three-nucleon forces & external currents
 - many tests in few-nucleons systems based on exact calc's
 - extension to hyperon-nucleon interactions
- Nuclear lattice simulations
 - formulate continuum EFT on space-time lattice
 - new method to extract phase shifts & mixing angles
 - promising results for $A = 2, 3, 4, 6, 12$ at NNLO
 - computational scaling $\rightarrow$ medium nuclei in reach
 - neutron matter around the unitary limit, P-wave pairing



$\Rightarrow$ larger A and higher precision

SPARES etc.

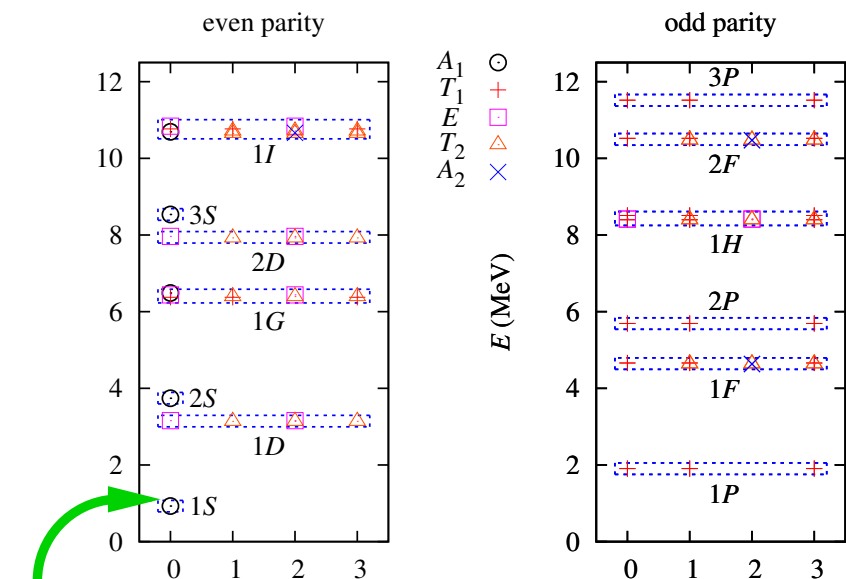
WHY NUCLEAR LATTICE SIMULATIONS?

- lattice QCD can not deal with nuclei (few simulations with $A = 2, 3$)
- exact few-body methods apply only to systems with $A \leq 4$ (Faddeev-Yakubowsky)
- need a different method to solve the nuclear many-body problem
 - Monte-Carlo methods already shown to be quite successful with phen. forces
 - extend the nuclear EFT to a discretized space-time and solve via MC
- early works (not complete):
 - momentum lattice study of nuclear matter (QHD) Brockmann, Frank, 1992
 - lattice EFT study of infinite nuclear matter at finite T, ρ Müller et al., 2000
 - absence of sign oscillations for non-zero μ in the Hubbard model Chen, Kaplan, 2004
 - lattice non-lin. realization of chiral sym. w/ static nucleons Chandrasekharan et al., 2004
- recent review: Lee, Prog. Nucl. Part. Phys.**63** (2009) 117 [arXiv:0804.3501]

MEASURING PHASE SHIFTS on the LATTICE II

- Free particle spectrum for $R = 10 + \epsilon$

- Interacting spectrum for $S = 0$



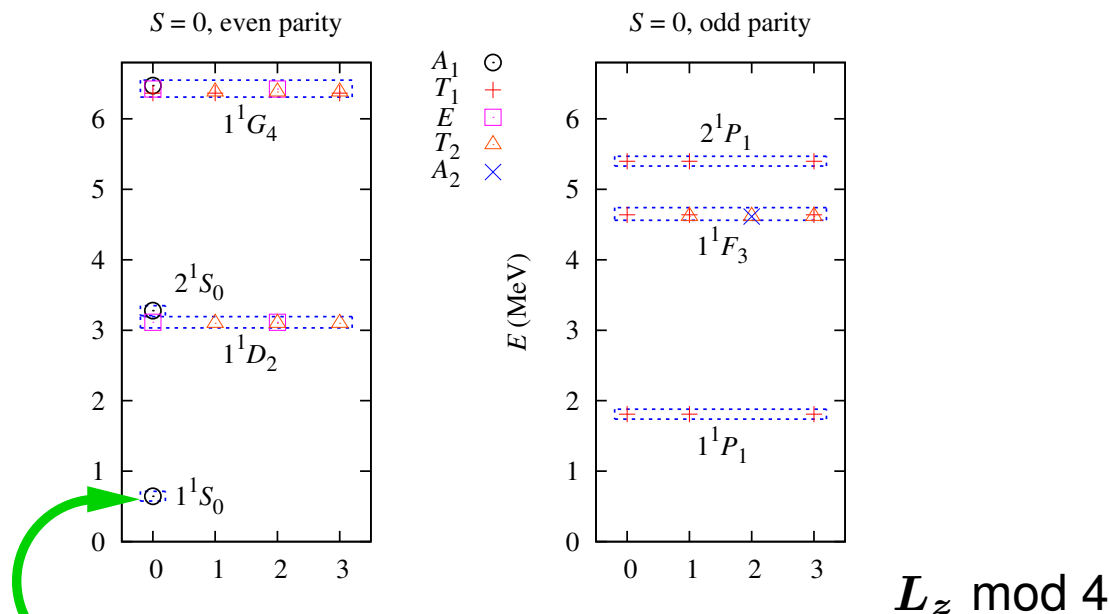
1^1S_0 energy = 0.9280 MeV



$$k_{\text{free}} = 29.52 \text{ MeV}, \quad j_0(k_{\text{free}}R) = 0$$



$$R = \frac{\pi}{k_{\text{free}}} = 0.1064 \text{ MeV}^{-1}$$



1^1S_0 energy = 0.6445 MeV



$$k = 24.60 \text{ MeV}$$



$$\delta(^1S_0) = \tan^{-1} \left[\frac{j_0(kR)}{y_0(kR)} \right] = 30^\circ$$

LATTICE ACTIONS - FREE NUCLEONS and PIONS

Borasoy, Lee, Schäfer, Phys. Rev. C **70** (2004) 014007

- $\mathcal{O}(a^4)$ improvement of free nucleon lattice Hamiltonian

$$H_{\text{free}} = \frac{1}{m} \sum_{k=0}^3 \sum_{\vec{n}_s, l_s, i, j} f_k \left[a_{i,j}^\dagger(\vec{n}_s) \left[a_{i,j}(\vec{n}_s + k\hat{l}_s) + a_{i,j}(\vec{n}_s - k\hat{l}_s) \right] \right]$$

with: $f_{1,2,3,4} = \frac{49}{2}, -\frac{3}{4}, \frac{3}{40}, -\frac{1}{180}, [a_{0,0}, a_{1,0}, a_{0,1}, a_{1,1}] = [a_{\uparrow,p}, a_{\downarrow,p}, a_{\uparrow,n}, a_{\downarrow,n}]$

$\vec{n}_{(s)} = \text{integer-valued (spatial) lattice vector}, \hat{l}_s = \hat{1}, \hat{2}, \hat{3} = \text{unit lattice vectors}$

- free lattice pions with instantaneous interactions:

$$S_{\pi\pi}(\pi_I) = \frac{\alpha_t}{2} \sum_{I=1}^3 \sum_{\vec{n}} \pi_I(\vec{n}) (-\Delta + M_\pi^2) \pi_I(\vec{n}), \quad \alpha_t = a_t/a$$

- auxiliary field contributions: $S_{ss}(s, s_I) = \frac{1}{2} \sum_{\vec{n}} s(\vec{n})^2 + \frac{1}{2} \sum_{I=1,2,3} \sum_{\vec{n}} s_I(\vec{n})^2$

LATTICE ACTIONS – INTERACTIONS

- Consider three different lattice actions (suppression of finite cut-off errors)
- LO1: naive discretization → multi-particle clustering at coarse a

$$\mathcal{A}(V_{\text{LO1}}) = C + C_I \vec{\tau}_1 \cdot \vec{\tau}_2 + \mathcal{A}(V^{\text{OPEP}})$$

- LO2: Gaussian smearing → no clustering but too attractive P-waves

$$\mathcal{A}(V_{\text{LO2}}) = C f(\vec{q}^2) + C_I f(\vec{q}^2) \vec{\tau}_1 \cdot \vec{\tau}_2 + \mathcal{A}(V^{\text{OPEP}})$$

- LO3: Gaussian smearing and spin-isopin projections
→ combines advantages of LO1 and LO2

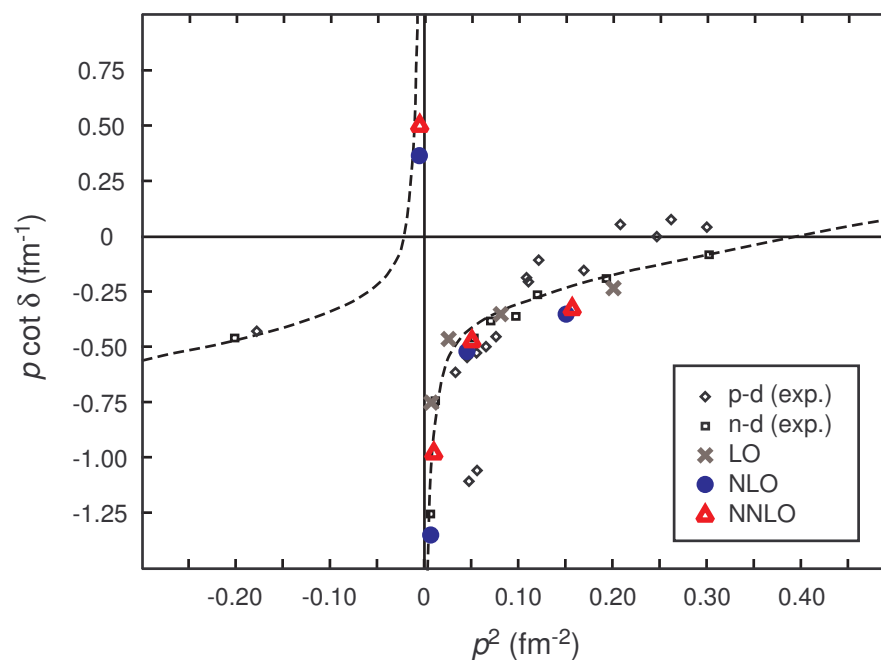
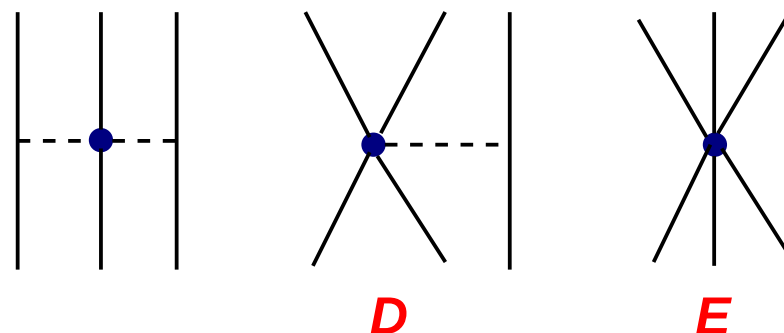
$$\begin{aligned} \mathcal{A}(V_{\text{LO3}}) = & C_{1S0} f(\vec{q}^2) \left(\frac{1}{4} - \frac{1}{4} \vec{\sigma}_1 \cdot \vec{\sigma}_2 \right) \left(\frac{3}{4} + \frac{1}{4} \vec{\tau}_1 \cdot \tau_2 \right) \\ & + C_{3S1} f(\vec{q}^2) \left(\frac{3}{4} + \frac{1}{4} \vec{\sigma}_1 \cdot \vec{\sigma}_2 \right) \left(\frac{1}{4} - \frac{1}{4} \vec{\tau}_1 \cdot \tau_2 \right) \\ & + \mathcal{A}(V^{\text{OPEP}}) \end{aligned}$$

$$\text{with: } f(\vec{q}^2) \propto \exp \left[-b \sum_{l_s} (1 - \cos ql_s) \right], \quad b \simeq 0.6$$

THREE-NUCLEON FORCES AT NNLO

Epelbaum, Krebs, Lee, M., Eur. Phys. J. A **41** (2009) 125

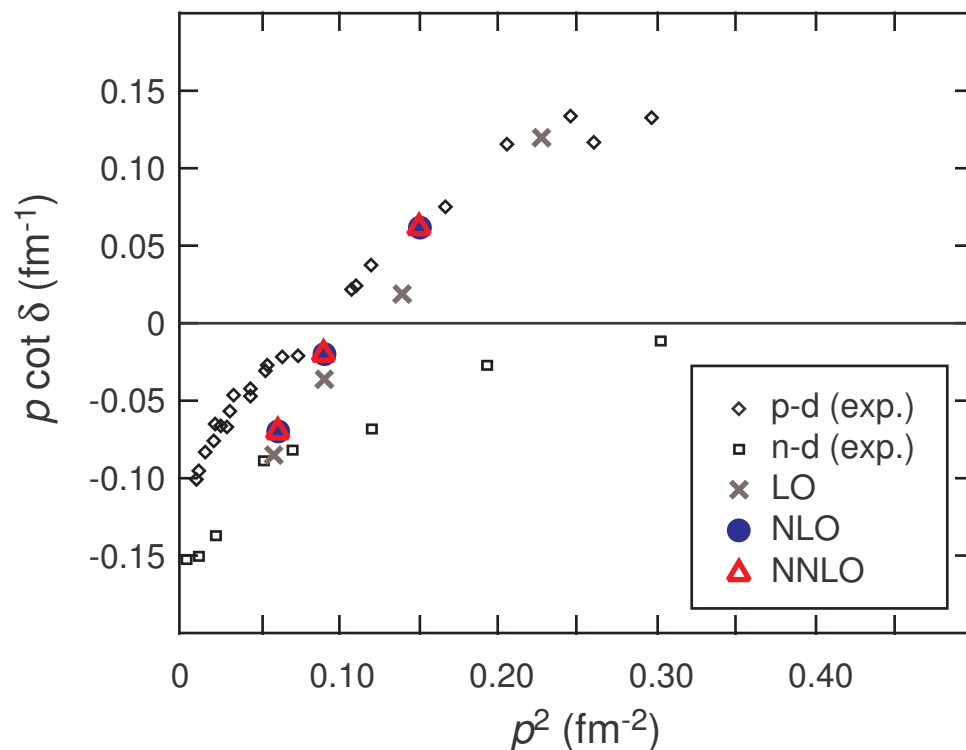
- 3N forces at NNLO:
 - three topologies
 - two parameters D and E
- determine D and E from fit to the triton binding energy and doublet nd scattering
- ⇒ make predictions
- D can also be determined in pion production experiments



PREDICTIONS at NNLO

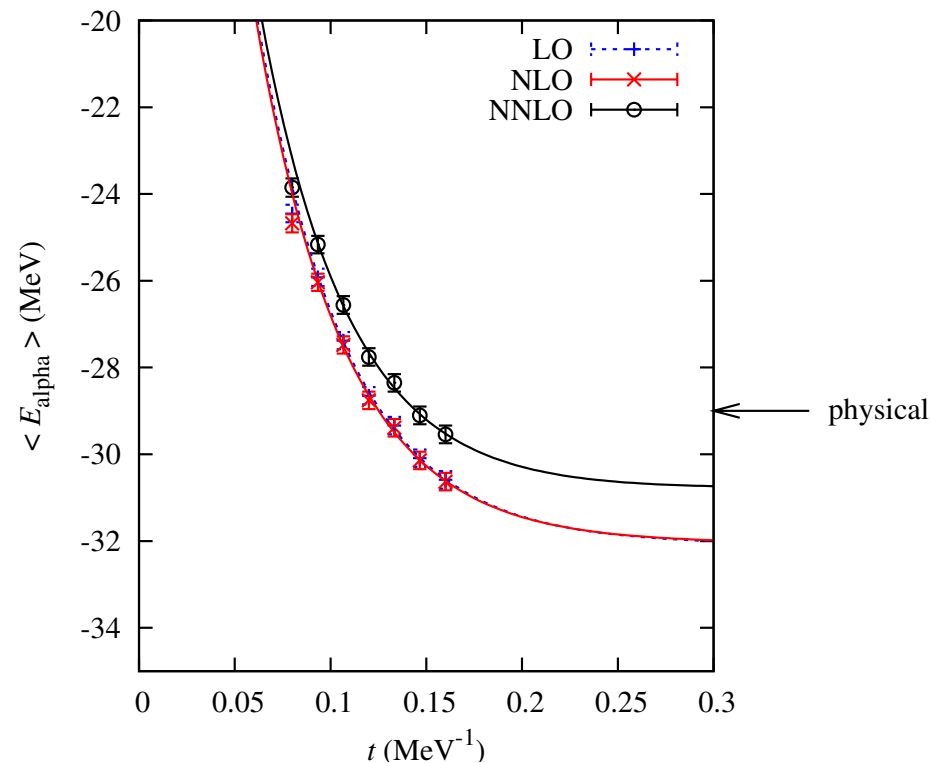
Epelbaum, Krebs, Lee, M., Eur. Phys. J. A **41** (2009) 125

- nd spin-3/2 quartet channel



- small NNLO corrections
- consistent w/ pd & nd data

- α -particle BE



- $\text{BE}(^4\text{He}) = 30.7(5) \text{ MeV}$
- 5% overbinding

ISOSPIN BREAKING & COULOMB EFFECTS

Epelbaum, Krebs, Lee, M., Phys. Rev. Lett. **104** (2010) 142501; Eur. Phys. J. **A 45** (2010) 335

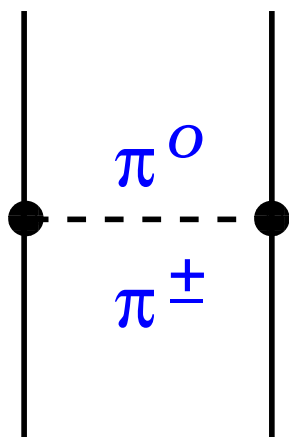
- considered by various groups for continuum EFT

Friar, van Kolck 1999, Walzl, M, Epelbaum 2001, ...

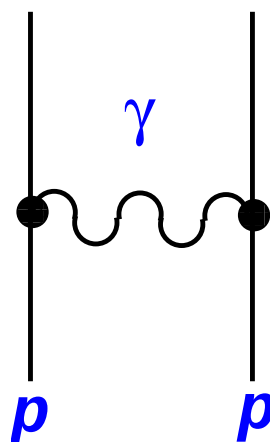
- main effects:

pion mass difference

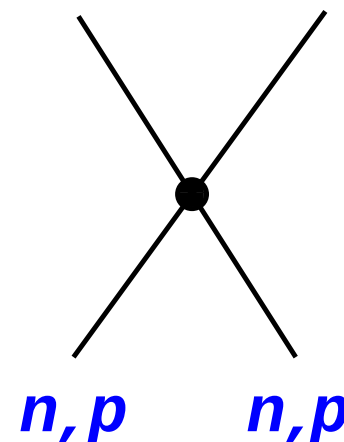
$$M_{\pi^{\pm}} \neq M_{\pi^0}$$



Coulomb potential



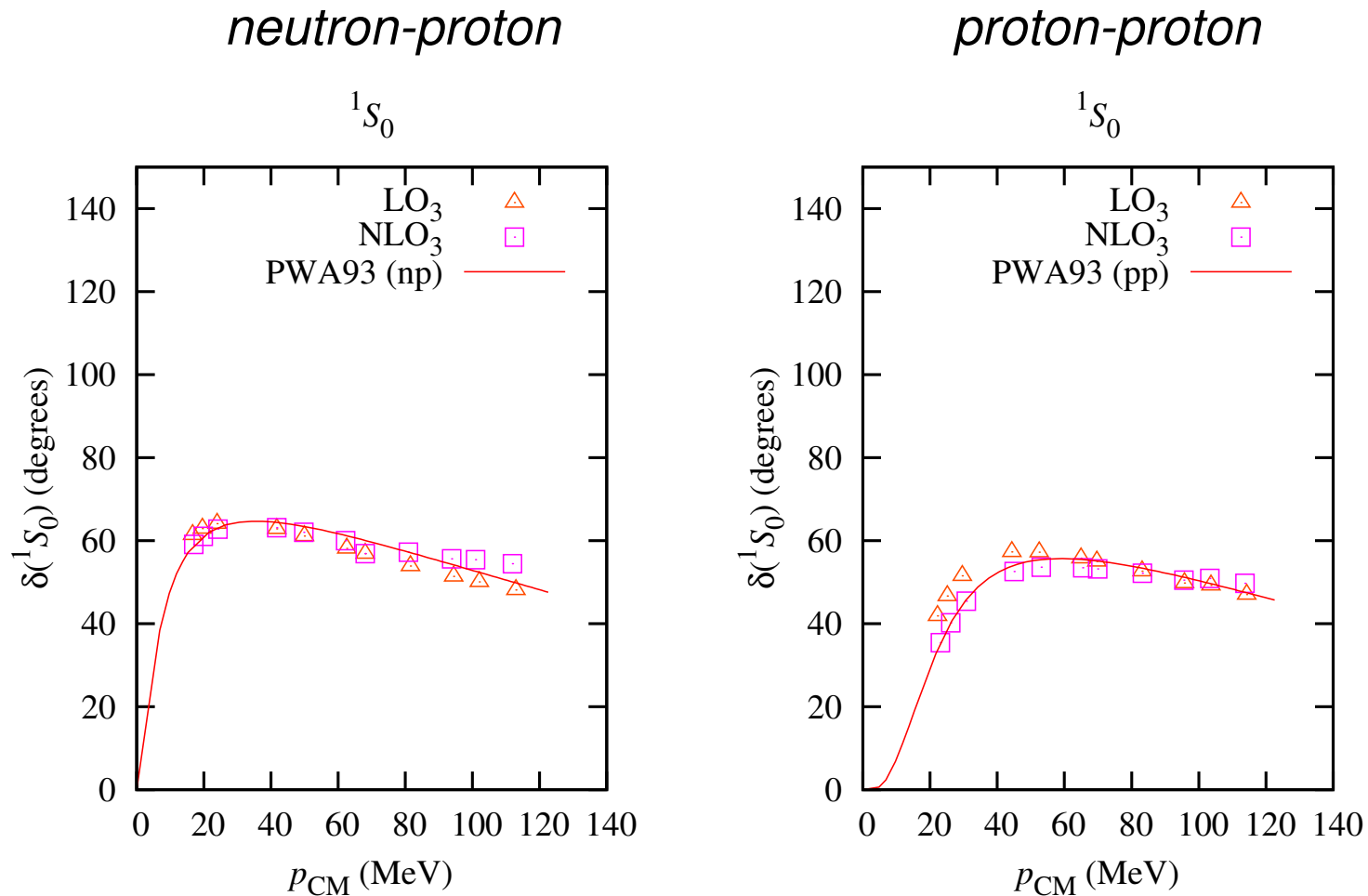
IV contact terms



→ 2 new LECs to be fitted

NUCLEON-NUCLEON INTERACTION

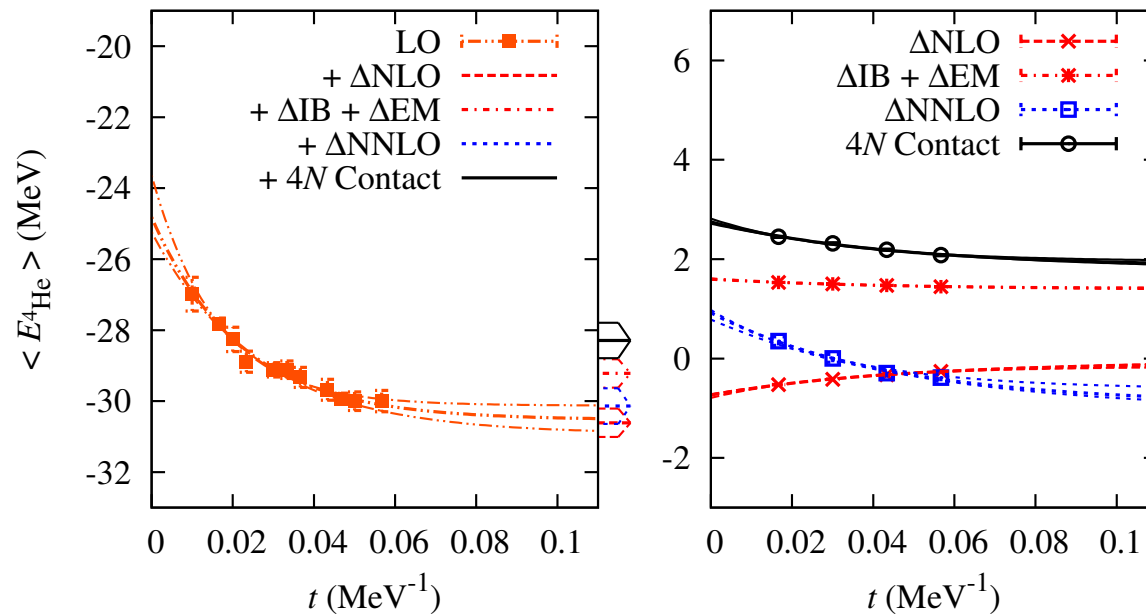
- fit three LECs to pp , np and nn scattering lengths



GROUNDSTATE ENERGY of ${}^4\text{He}$

41

- In a box of size $L = 9.9 \text{ fm}$



energy @ order	MeV
LO	-30.5(4)
NLO	-30.6(4)
NLO + IB + EM	-29.2(4)
NNLO + IB + EM	-30.1(5)
NNLO + IB + EM + 4N	-28.3(5)
physical	-28.3

- still to be done: infinite volume extrapolation

RESONANCE SATURATION of the 4N LECs

Epelbaum, M., Glöckle, Elster, Phys. Rev. C65 (2002) 044001 [nucl-th/0106007]

- Basic idea:

Expand meson-exchange in powers of t/M_R^2
and map on 4N operators

$$\frac{g^2}{t - M_R^2} = -\frac{g^2}{M_R^2} - \frac{g^2 t}{M_R^4} + \dots$$

with $\bullet \propto t^0 \propto \vec{\nabla}^0$

$\blacksquare \propto t \propto \vec{\nabla}^2$

- works amazingly well

all LECs of natural size

(in units of $1/F_\pi^n \Lambda_\chi^m$)

