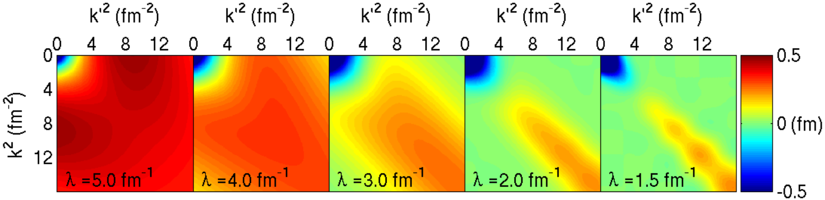
Atomic Nuclei at Low Resolution

Dick Furnstahl

Department of Physics
Ohio State University

EMMI Workshop on Strongly Coupled Systems
Happy Birthday, Jochen!
November, 2010



Outline

Overview: Low-energy nuclear physics

Lowering the resolution with RG

Survey of calculations at low resolution

Outlook

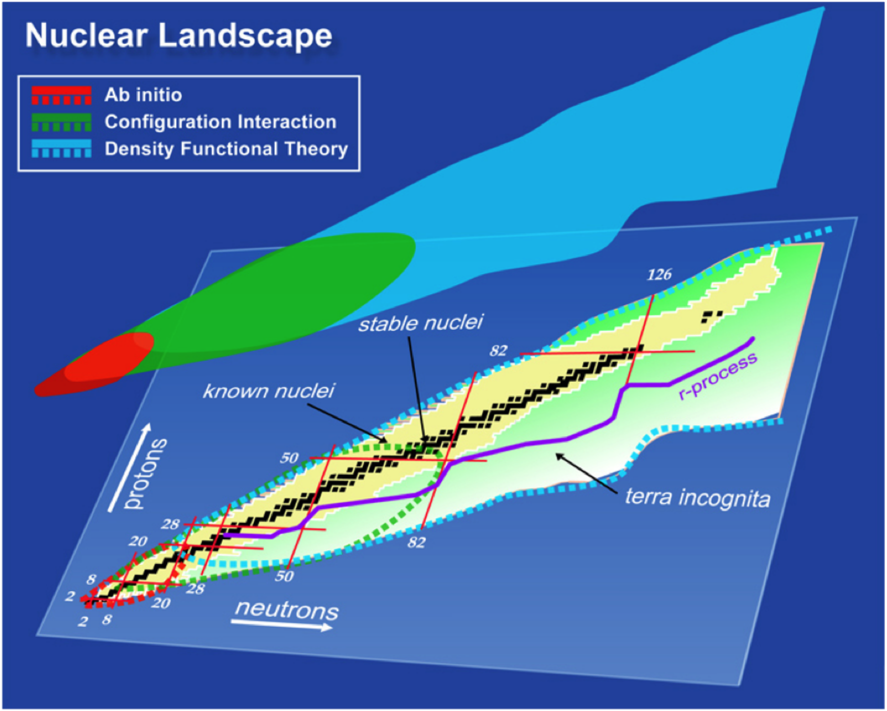
Outline

Overview: Low-energy nuclear physics

Lowering the resolution with RG

Survey of calculations at low resolution

Outlook



Nuclear Landscape
Ab initio
Configuration Interaction
Density Functional Theory
stable nuclei
known nuclei
r-process
terra incognita
protons
neutrons
126
82
50
28
20
8
2
2
8
20
28
50
82

Extremes in low-energy nuclear physics

- Extremes of nuclear existence: driplines, superheavies, . . .
- Extremes in the heavens: supernovae, neutron stars, . . .
- We want to extrapolate reliably with error estimates, connect to and exploit known microscopic physics

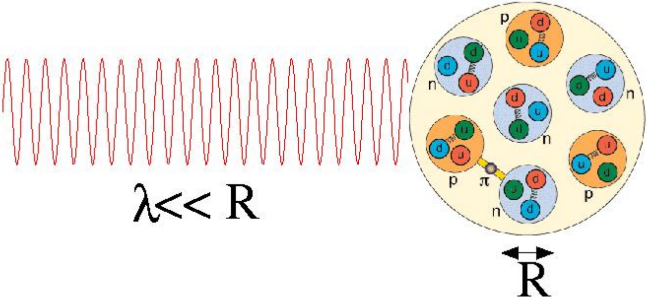
- Shakespeare's Othello (Act 5, Scene 2)

  I pray you, in your letters,
  When you shall these unlucky deeds relate,
  Speak of me as I am; nothing extenuate,
  Nor set down aught in malice. Then must you speak
  Of one that lov'd not wisely but too well;
  Of one not easily jealous, but being wrought,
  Perplex'd in the extreme . . .

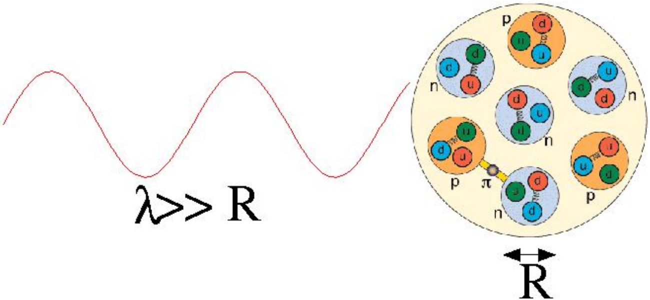
- To avoid being "perplex'd" $\Longrightarrow$ go to low resolution!

EMMI

Principle of *any* effective low-energy description

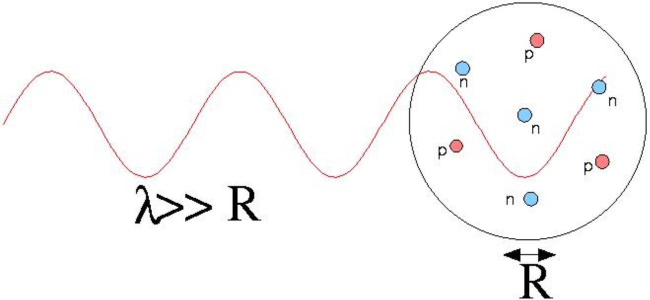


Principle of *any* effective low-energy description



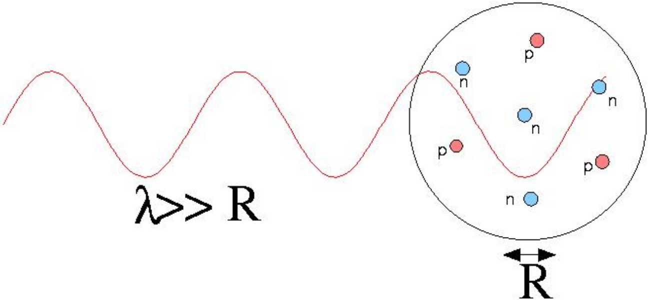
- If system is probed at low energies, fine details not resolved

Principle of *any* effective low-energy description



- If system is probed at low energies, fine details not resolved
  - Use low-energy variables for low-energy processes
  - Short-distance structure can be replaced by something simpler without distorting low-energy observables
  - *Physics interpretation can change with resolution!*
- Could be a model or systematic (e.g., effective field theory)

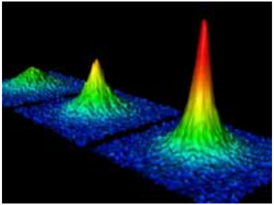
Principle of *any* effective low-energy description

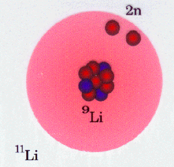


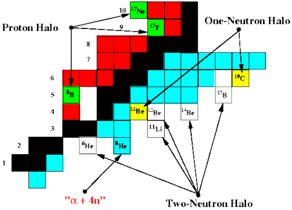
- If system is probed at low energies, fine details not resolved
  - Use low-energy variables for low-energy processes
  - Short-distance structure can be replaced by something simpler without distorting low-energy observables
  - *Physics interpretation can change with resolution!*
- Could be a model or systematic (e.g., effective field theory)
- Low density ⇔ low interaction energy ⇔ low resolution (?)

Nuclei at *very* low resolution

- If separation of scales is sufficient, then EFT with pointlike interactions is efficient (e.g., $kR \ll 1$)
- Universal properties (large a_s)
  - connect to cold atom physics
  - low-density neutron matter
  - e.g., Efimov physics
- Pionless EFT
  - e.g., $np \to d\gamma$ with $E_{\text{typ}} \approx 0.02$–$0.2\,\text{MeV}$
- Halo EFT
  - $B_{\text{valence}} \ll B_{\text{core}}, E_{ex}$
  - $n\alpha$-system (Bedaque et al.) or $\alpha\alpha$-system (Higa et al.) or ...

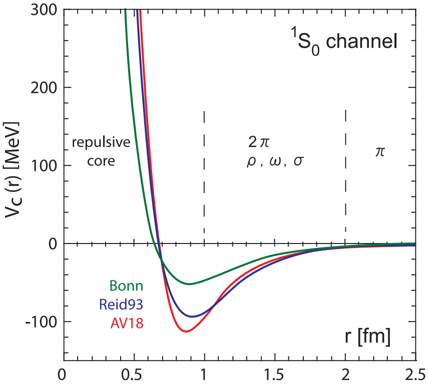


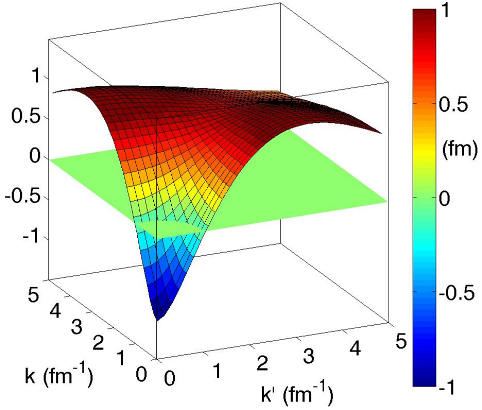




Here: focus on systems where pion exchange is resolved

S-wave NN potential in momentum space

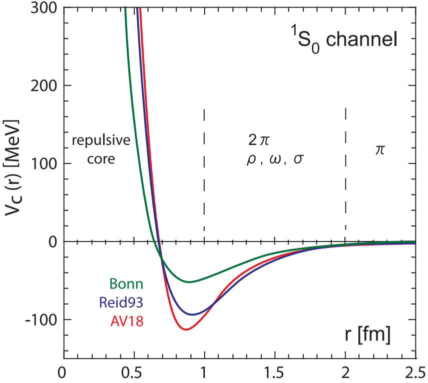


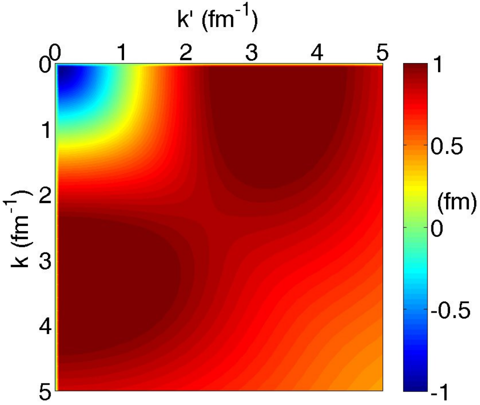


$$V_{L=0}(k,k') = \int d^3r\, j_0(kr)\, V(r)\, j_0(k'r) = \langle k | V_{L=0} | k' \rangle \implies V_{kk'} \text{ matrix}$$

- Momentum units ($\hbar = c = 1$): typical relative momentum in large nucleus $\approx 1\,\text{fm}^{-1} \approx 200\,\text{MeV}$ but …
- Repulsive core $\implies$ large high-k ($\gtrsim 2\,\text{fm}^{-1}$) components

S-wave NN potential in momentum space

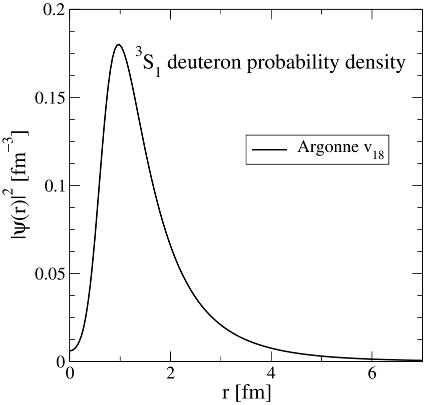


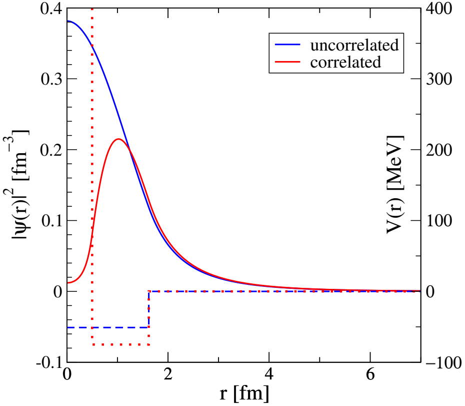


$$V_{L=0}(k,k') = \int d^3r\, j_0(kr)\, V(r)\, j_0(k'r) = \langle k|V_{L=0}|k'\rangle \implies V_{kk'} \text{ matrix}$$

- Momentum units ($\hbar = c = 1$): typical relative momentum in large nucleus $\approx 1\,\text{fm}^{-1} \approx 200\,\text{MeV}$ but ...
- Repulsive core $\implies$ large high-k ($\gtrsim 2\,\text{fm}^{-1}$) components

Consequences of a repulsive core

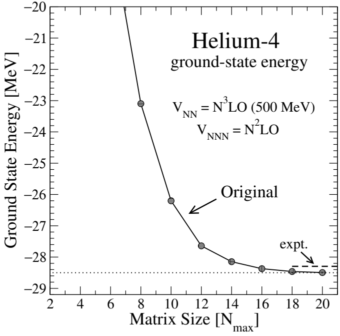


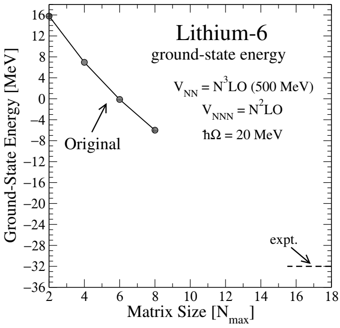


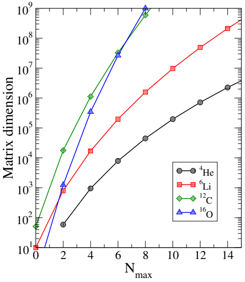
- Probability at short separations suppressed $\Longrightarrow$ "correlations"
- Short-distance structure $\Leftrightarrow$ high-momentum components
- Greatly complicates expansion of many-body wave functions

Many short wavelengths ⟹ Large matrices

- Harmonic oscillator basis with $N_{\max}$ shells for excitations
- Graphs show convergence for *soft* chiral EFT potential (although not at optimal $\hbar\Omega$ for ^{6}Li)







- Factorial growth of basis with A ⟹ limits calculations
- Problem: mismatch of scales/dof's. Solution: use RG.

S. Weinberg on the Renormalization Group

- From "Why the Renormalization Group is a good thing"

  "The method in its most general form can I think be understood as a way to arrange in various theories that the degrees of freedom that you're talking about are the relevant degrees of freedom for the problem at hand."

- Third Law of Progress in Theoretical Physics:
  "You may use any degrees of freedom you like to describe a physical system, but if you use the wrong ones, you'll be sorry!"

S. Weinberg on the Renormalization Group

- From "Why the Renormalization Group is a good thing"

  "The method in its most general form can I think be understood as a way to arrange in various theories that the degrees of freedom that you're talking about are the relevant degrees of freedom for the problem at hand."

- Third Law of Progress in Theoretical Physics:

  "You may use any degrees of freedom you like to describe a physical system, but if you use the wrong ones, you'll be sorry!"

- Improving perturbation theory in high-energy physics
  - Mismatch of energy scales can generate large logarithms
  - Shift between couplings and loop integrals to reduce logs
- Universality in critical phenomena
  - Filter out short-distance degrees of freedom
- Simplifying calculations of nuclear structure/reactions
  - Make nuclear physics look more like quantum chemistry!
  - Like other RG applications, can seem like magic

Outline

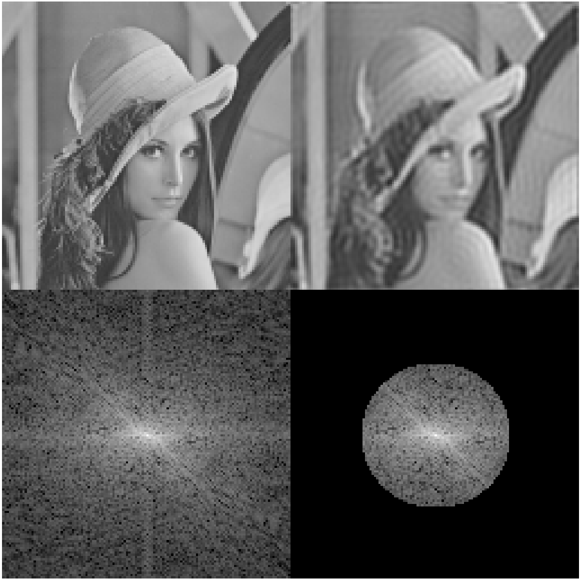
Overview: Low-energy nuclear physics

Lowering the resolution with RG

Survey of calculations at low resolution

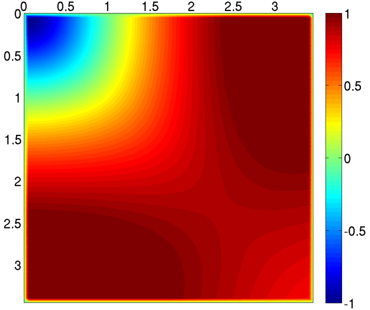
Outlook

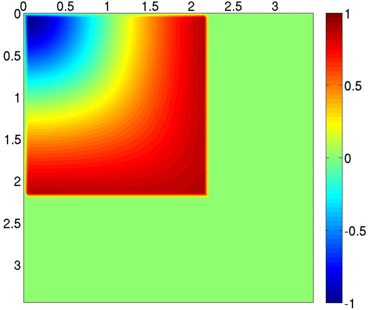
Low-pass filter on an image

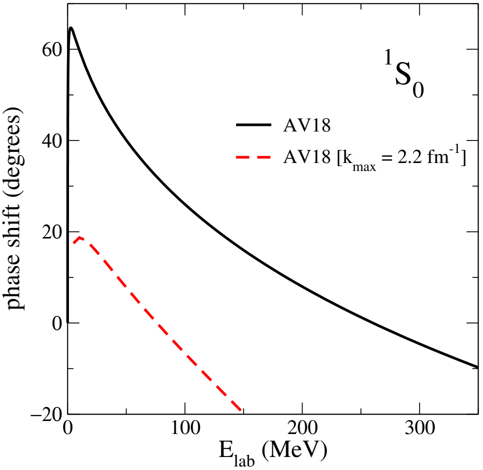


- Much less information needed
- Long-wavelength info is preserved
- Could also lower resolution by "block spinning"

Effect of low-pass filter on observables







Why did our low-pass filter fail?

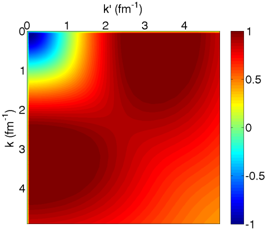
- Basic problem: low k and high k are coupled (wrong dof's!)
- E.g., perturbation theory for (tangent of) phase shift:

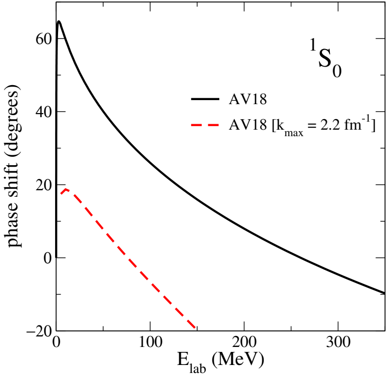
$$\langle k|V|k\rangle + \sum_{k'} \frac{\langle k|V|k'\rangle\langle k'|V|k\rangle}{(k^2 - k'^2)/m} + \cdots$$

- Solution: Unitary transformation of the H matrix $\Longrightarrow$ decouple!

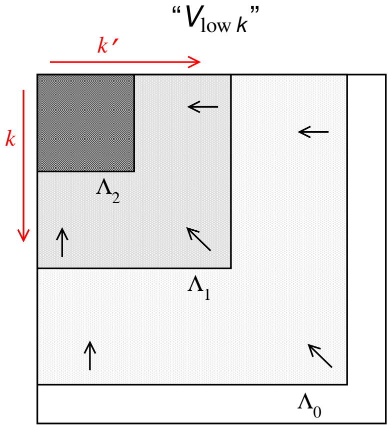
$$\begin{aligned} E_n &= \langle \Psi_n|H|\Psi_n\rangle \qquad U^\dagger U = 1 \\ &= (\langle \Psi_n|U^\dagger)UHU^\dagger(U|\Psi_n\rangle) \\ &= \langle \widetilde{\Psi}_n|\widetilde{H}|\widetilde{\Psi}_n\rangle \end{aligned}$$

- Here: Decouple using RG

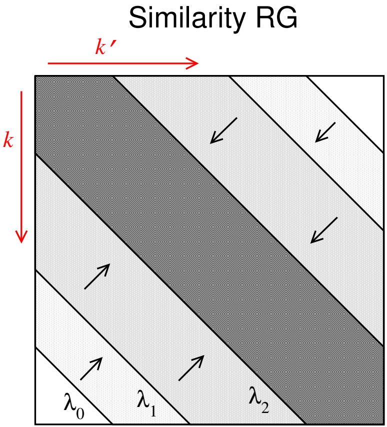




Two ways to decouple with RG equations



- Lower a cutoff Λ_i in k, k', e.g., demand $dT(k, k'; k^2)/d\Lambda = 0$



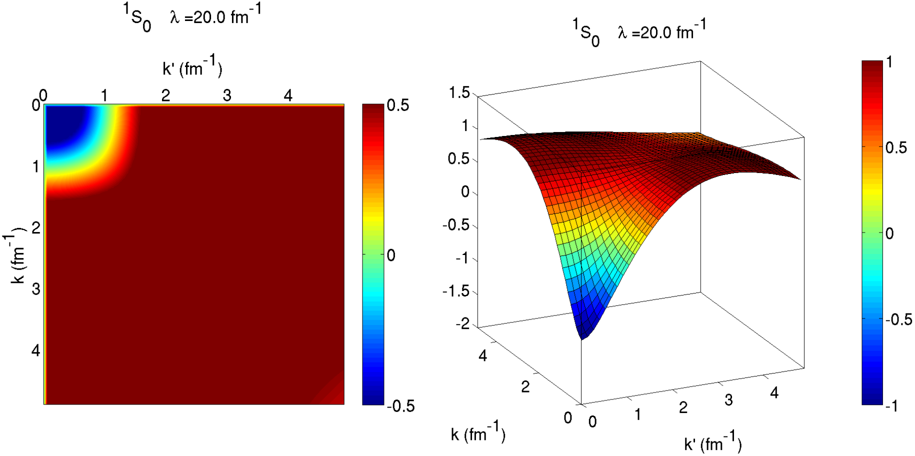
- Drive the Hamiltonian toward diagonal with "flow equation" [Wegner; Glazek/Wilson (1990's)]

$\Longrightarrow$ Both tend toward universal low-momentum interactions!

Flow equations in action: NN only [arXiv:0912.3688]

- In each partial wave with $\epsilon_k = \hbar^2 k^2/M$ and $\lambda^2 = 1/\sqrt{s}$

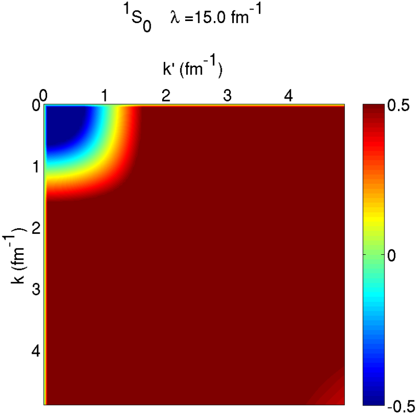
$$\frac{dV_\lambda}{d\lambda}(k,k') \propto -(\epsilon_k - \epsilon_{k'})^2 V_\lambda(k,k') + \sum_q (\epsilon_k + \epsilon_{k'} - 2\epsilon_q) V_\lambda(k,q) V_\lambda(q,k')$$

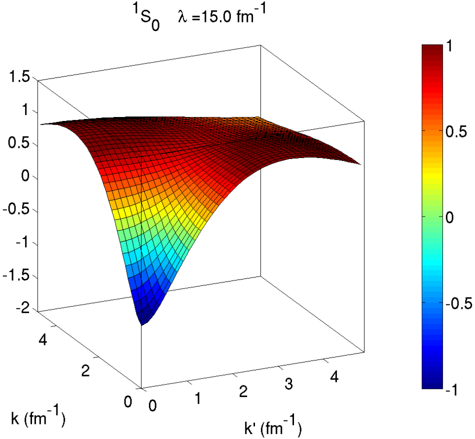


Flow equations in action: NN only [arXiv:0912.3688]

- In each partial wave with $\epsilon_k = \hbar^2 k^2/M$ and $\lambda^2 = 1/\sqrt{s}$

$$\frac{dV_\lambda}{d\lambda}(k,k') \propto -(\epsilon_k - \epsilon_{k'})^2 V_\lambda(k,k') + \sum_q (\epsilon_k + \epsilon_{k'} - 2\epsilon_q) V_\lambda(k,q) V_\lambda(q,k')$$

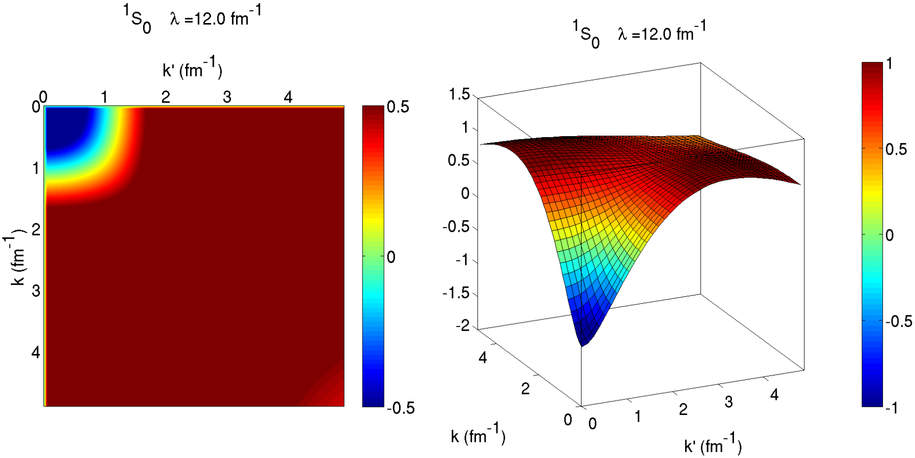




Flow equations in action: NN only [arXiv:0912.3688]

- In each partial wave with $\epsilon_k = \hbar^2 k^2/M$ and $\lambda^2 = 1/\sqrt{s}$

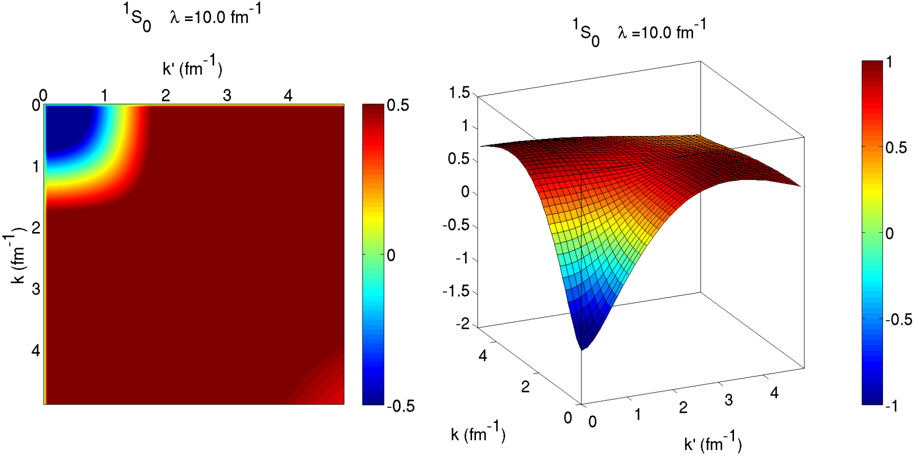
$$\frac{dV_\lambda}{d\lambda}(k,k') \propto -(\epsilon_k - \epsilon_{k'})^2 V_\lambda(k,k') + \sum_q (\epsilon_k + \epsilon_{k'} - 2\epsilon_q) V_\lambda(k,q) V_\lambda(q,k')$$



Flow equations in action: NN only [arXiv:0912.3688]

- In each partial wave with $\epsilon_k = \hbar^2 k^2/M$ and $\lambda^2 = 1/\sqrt{s}$

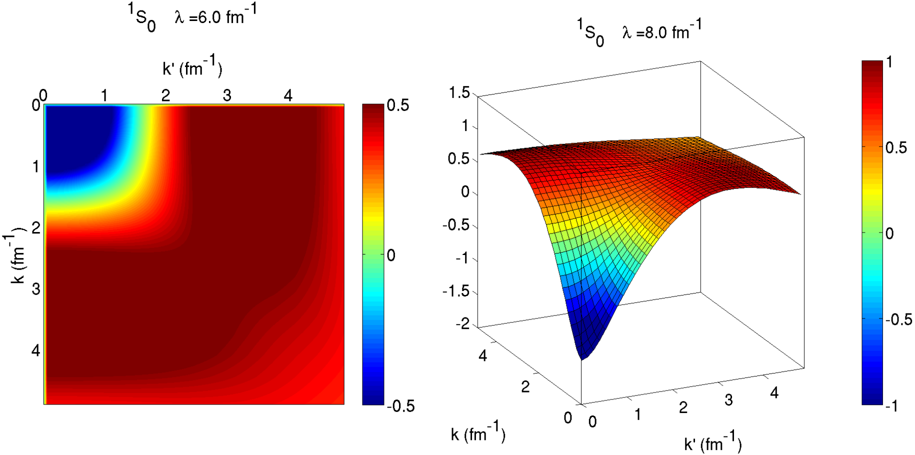
$$\frac{dV_\lambda}{d\lambda}(k,k') \propto -(\epsilon_k - \epsilon_{k'})^2 V_\lambda(k,k') + \sum_q (\epsilon_k + \epsilon_{k'} - 2\epsilon_q) V_\lambda(k,q) V_\lambda(q,k')$$



Flow equations in action: NN only [arXiv:0912.3688]

- In each partial wave with $\epsilon_k = \hbar^2 k^2/M$ and $\lambda^2 = 1/\sqrt{s}$

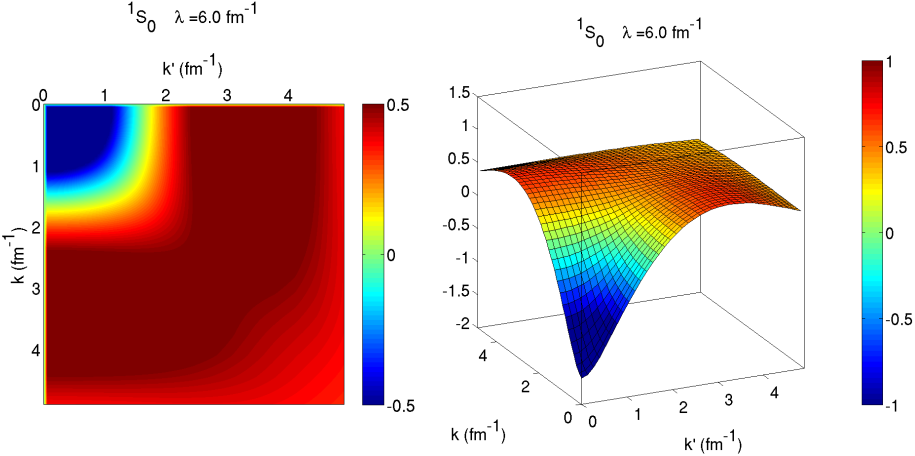
$$\frac{dV_\lambda}{d\lambda}(k,k') \propto -(\epsilon_k - \epsilon_{k'})^2 V_\lambda(k,k') + \sum_q (\epsilon_k + \epsilon_{k'} - 2\epsilon_q) V_\lambda(k,q) V_\lambda(q,k')$$



Flow equations in action: NN only [arXiv:0912.3688]

- In each partial wave with $\epsilon_k = \hbar^2 k^2/M$ and $\lambda^2 = 1/\sqrt{s}$

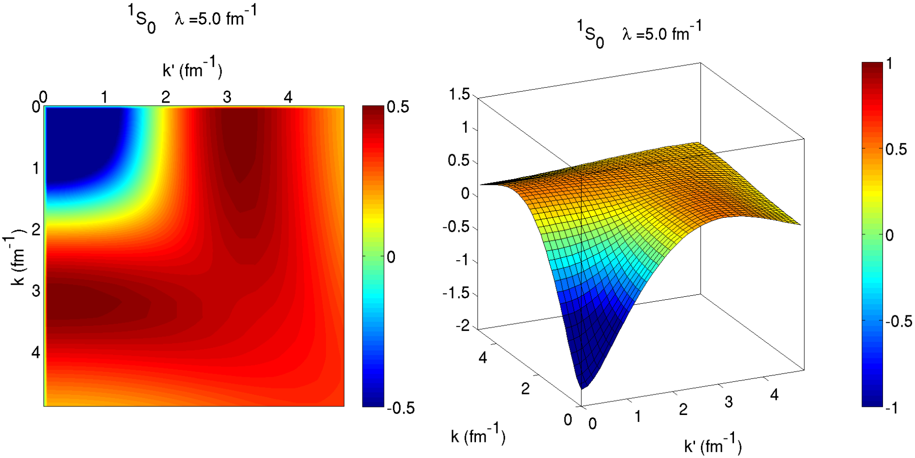
$$\frac{dV_\lambda}{d\lambda}(k,k') \propto -(\epsilon_k - \epsilon_{k'})^2 V_\lambda(k,k') + \sum_q (\epsilon_k + \epsilon_{k'} - 2\epsilon_q) V_\lambda(k,q) V_\lambda(q,k')$$



Flow equations in action: NN only [arXiv:0912.3688]

- In each partial wave with $\epsilon_k = \hbar^2 k^2/M$ and $\lambda^2 = 1/\sqrt{s}$

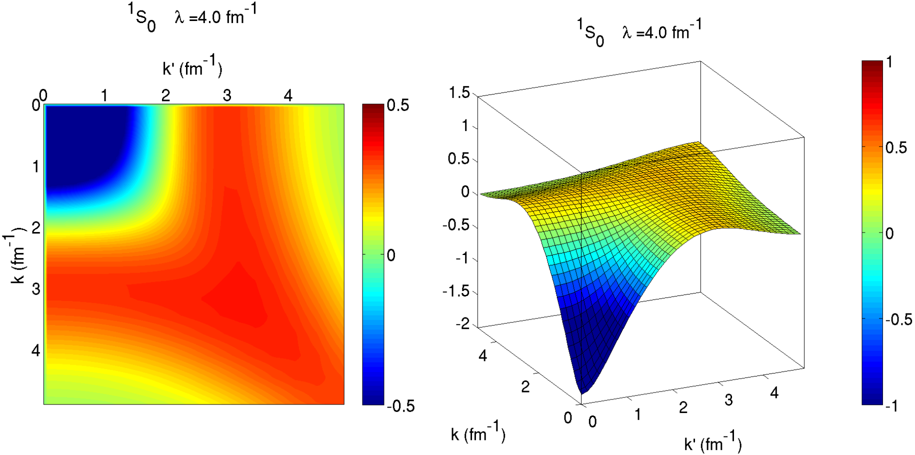
$$\frac{dV_\lambda}{d\lambda}(k,k') \propto -(\epsilon_k - \epsilon_{k'})^2 V_\lambda(k,k') + \sum_q (\epsilon_k + \epsilon_{k'} - 2\epsilon_q) V_\lambda(k,q) V_\lambda(q,k')$$



Flow equations in action: NN only [arXiv:0912.3688]

- In each partial wave with $\epsilon_k = \hbar^2 k^2/M$ and $\lambda^2 = 1/\sqrt{s}$

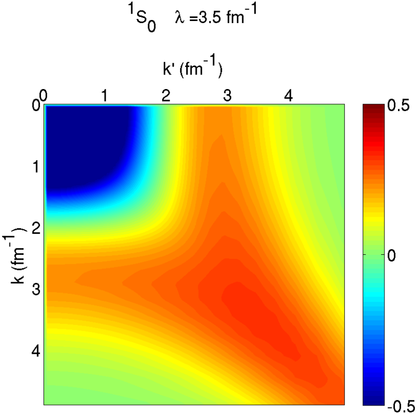
$$\frac{dV_\lambda}{d\lambda}(k,k') \propto -(\epsilon_k - \epsilon_{k'})^2 V_\lambda(k,k') + \sum_q (\epsilon_k + \epsilon_{k'} - 2\epsilon_q) V_\lambda(k,q) V_\lambda(q,k')$$

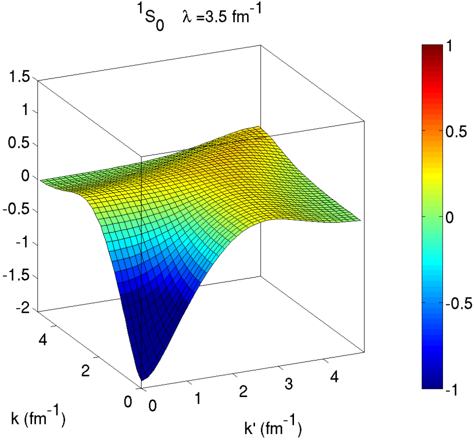


Flow equations in action: NN only [arXiv:0912.3688]

- In each partial wave with $\epsilon_k = \hbar^2 k^2/M$ and $\lambda^2 = 1/\sqrt{s}$

$$\frac{dV_\lambda}{d\lambda}(k,k') \propto -(\epsilon_k - \epsilon_{k'})^2 V_\lambda(k,k') + \sum_q (\epsilon_k + \epsilon_{k'} - 2\epsilon_q) V_\lambda(k,q) V_\lambda(q,k')$$

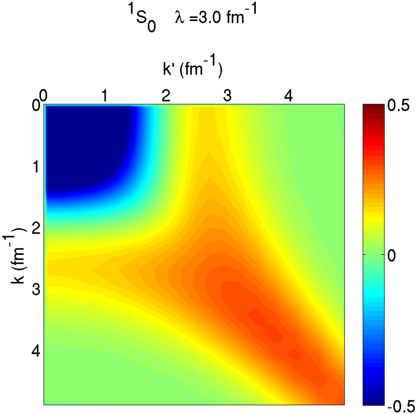


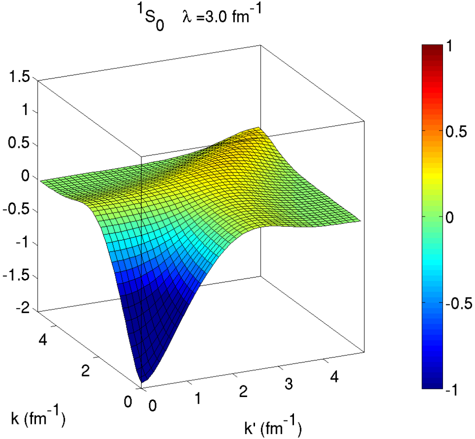


Flow equations in action: NN only [arXiv:0912.3688]

- In each partial wave with $\epsilon_k = \hbar^2 k^2/M$ and $\lambda^2 = 1/\sqrt{s}$

$$\frac{dV_\lambda}{d\lambda}(k,k') \propto -(\epsilon_k - \epsilon_{k'})^2 V_\lambda(k,k') + \sum_q (\epsilon_k + \epsilon_{k'} - 2\epsilon_q) V_\lambda(k,q) V_\lambda(q,k')$$

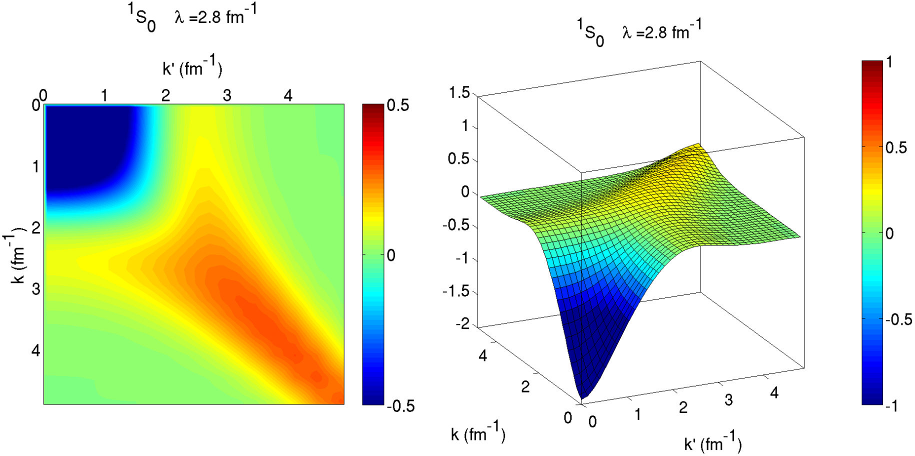




Flow equations in action: NN only [arXiv:0912.3688]

- In each partial wave with $\epsilon_k = \hbar^2 k^2/M$ and $\lambda^2 = 1/\sqrt{s}$

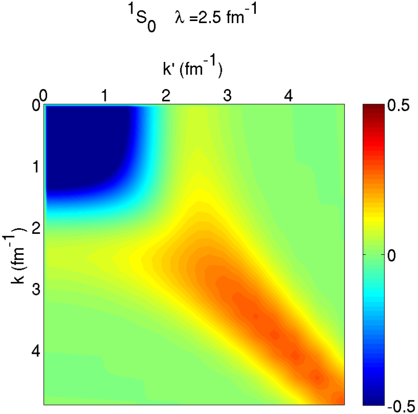
$$\frac{dV_\lambda}{d\lambda}(k,k') \propto -(\epsilon_k - \epsilon_{k'})^2 V_\lambda(k,k') + \sum_q (\epsilon_k + \epsilon_{k'} - 2\epsilon_q) V_\lambda(k,q) V_\lambda(q,k')$$

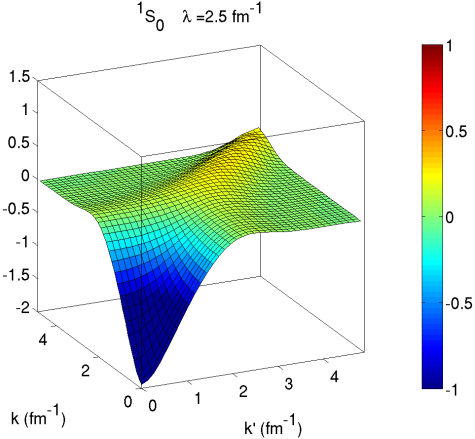


Flow equations in action: NN only [arXiv:0912.3688]

- In each partial wave with $\epsilon_k = \hbar^2 k^2/M$ and $\lambda^2 = 1/\sqrt{s}$

$$\frac{dV_\lambda}{d\lambda}(k,k') \propto -(\epsilon_k - \epsilon_{k'})^2 V_\lambda(k,k') + \sum_q (\epsilon_k + \epsilon_{k'} - 2\epsilon_q) V_\lambda(k,q) V_\lambda(q,k')$$

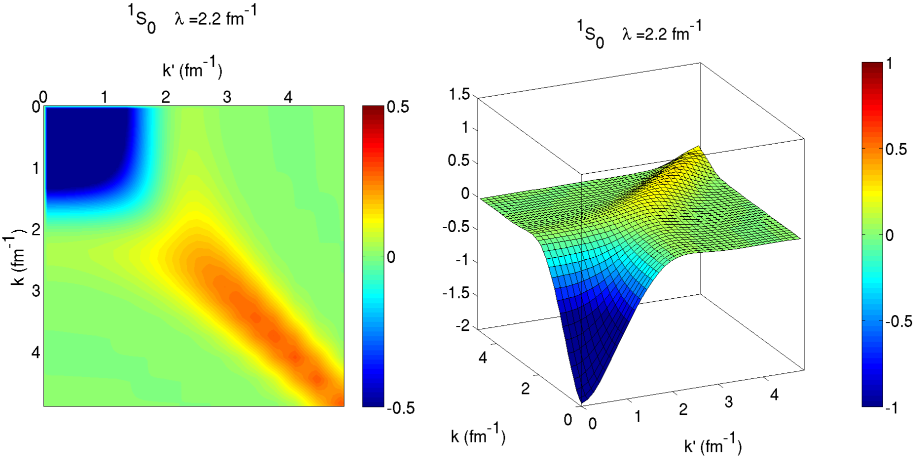




Flow equations in action: NN only [arXiv:0912.3688]

- In each partial wave with $\epsilon_k = \hbar^2 k^2/M$ and $\lambda^2 = 1/\sqrt{s}$

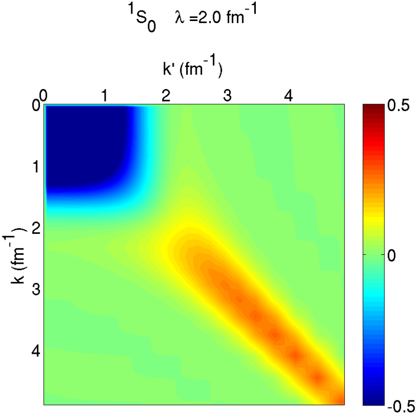
$$\frac{dV_\lambda}{d\lambda}(k,k') \propto -(\epsilon_k - \epsilon_{k'})^2 V_\lambda(k,k') + \sum_q (\epsilon_k + \epsilon_{k'} - 2\epsilon_q) V_\lambda(k,q) V_\lambda(q,k')$$

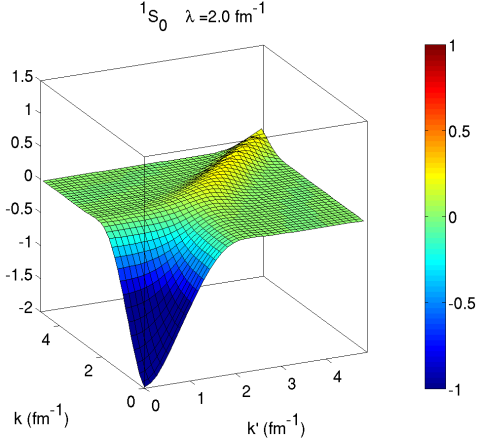


Flow equations in action: NN only [arXiv:0912.3688]

- In each partial wave with $\epsilon_k = \hbar^2 k^2/M$ and $\lambda^2 = 1/\sqrt{s}$

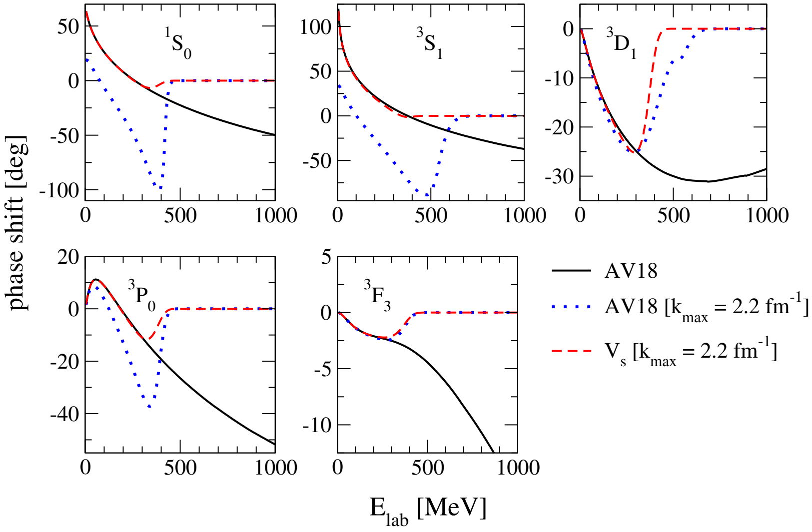
$$\frac{dV_\lambda}{d\lambda}(k,k') \propto -(\epsilon_k - \epsilon_{k'})^2 V_\lambda(k,k') + \sum_q (\epsilon_k + \epsilon_{k'} - 2\epsilon_q) V_\lambda(k,q) V_\lambda(q,k')$$



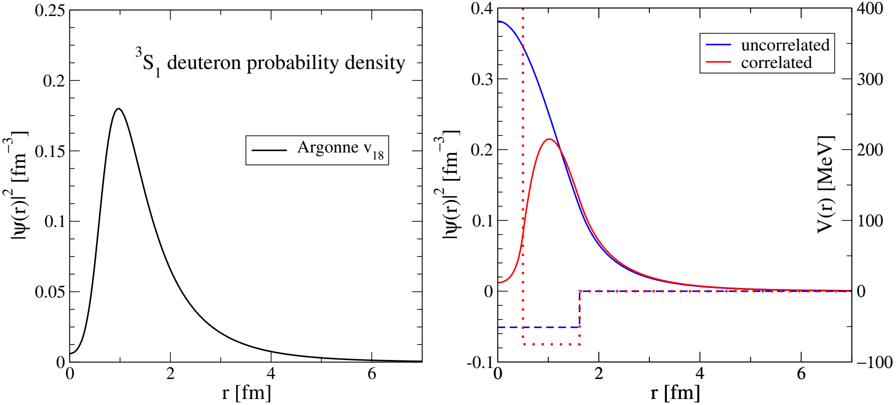


Low-Pass Filters Work! [Jurgenson et al., (2008)]

- Phase shifts with $V_s(k, k') = 0$ for $k, k' > k_{\text{max}}$

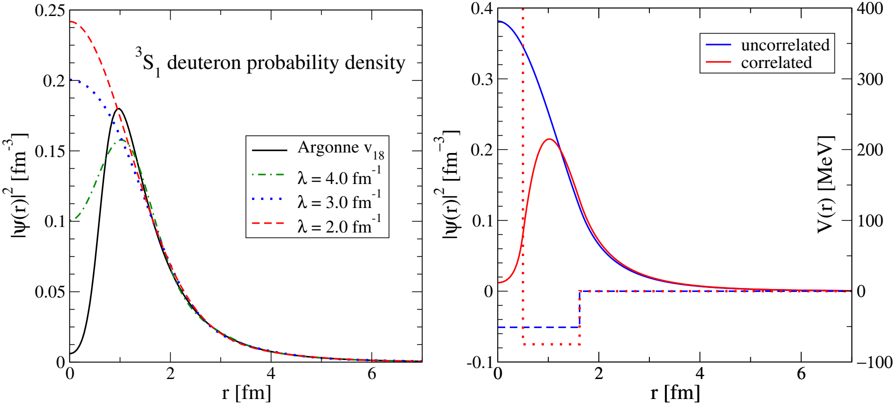


Consequences of a Repulsive Core Revisited



- Probability at short separations suppressed $\Longrightarrow$ "correlations"
- Greatly complicates expansion of many-body wave functions
- Short-distance structure $\Leftrightarrow$ high-momentum components

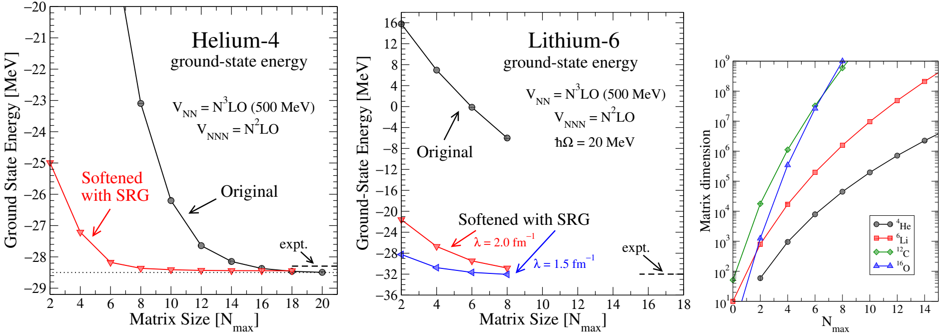
Consequences of a Repulsive Core Revisited



- Transformed potential $\Longrightarrow$ no short-range correlations in wf!
- Potential is now non-local: $V(\mathbf{r})\psi(\mathbf{r}) \longrightarrow \int d^3\mathbf{r}'\ V(\mathbf{r},\mathbf{r}')\psi(\mathbf{r}')$
  - A problem for Green's Function Monte Carlo approach
  - Not a problem for many-body methods using HO matrix elements

Many short wavelengths $\Longrightarrow$ Large matrices

- Harmonic oscillator basis with $N_{\max}$ shells for excitations
- Graphs show convergence for *soft* chiral EFT potential and evolved SRG potentials (including NNN)



- Better convergence, but rapid growth of basis still a problem $\Longrightarrow$ see talk by R. Roth

Basics: SRG flow equations [arXiv:0912.3688]

- Transform an initial hamiltonian, $H = T + V$:

$$H_s = U_s H U_s^\dagger \equiv T + V_s \ ,$$

where s is the *flow parameter*. Differentiating wrt s:

$$\frac{dH_s}{ds} = [\eta_s, H_s] \qquad \text{with} \qquad \eta_s \equiv \frac{dU_s}{ds} U_s^\dagger = -\eta_s^\dagger \ .$$

- η_s is specified by the commutator with “generator” G_s:

$$\eta_s = [G_s, H_s] \ ,$$

which yields the flow equation (T held fixed),

$$\frac{dH_s}{ds} = \frac{dV_s}{ds} = [[G_s, H_s], H_s] \ .$$

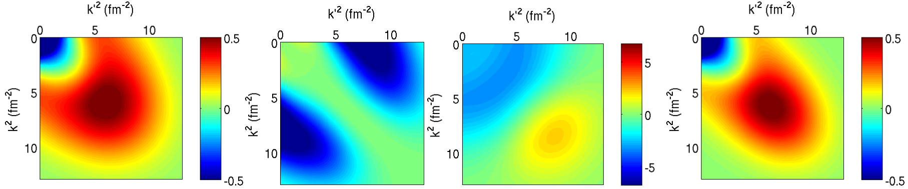
- G_s determines flow $\Longrightarrow$ many choices (T, H_D, H_{BD}, …)

Flow in momentum basis with $G_s = T$

- For $A = 2$, project on rel. momentum states $|k\rangle$, but generic

$$\frac{dV_s}{ds} = [[T_{\rm rel}, V_s], H_s] \qquad \text{with} \quad T_{\rm rel}|k\rangle = \epsilon_k |k\rangle \quad \text{and} \quad \lambda^2 = 1/\sqrt{s}$$

$$\frac{dV_\lambda}{d\lambda}(k,k') \propto -(\epsilon_k - \epsilon_{k'})^2 V_\lambda(k,k') + \sum_q (\epsilon_k + \epsilon_{k'} - 2\epsilon_q) V_\lambda(k,q) V_\lambda(q,k')$$



$V_{\lambda=3.0}(k,k')$ 1st term 2nd term $V_{\lambda=2.5}(k,k')$

- First term drives $^1S_0$ V_λ toward diagonal:

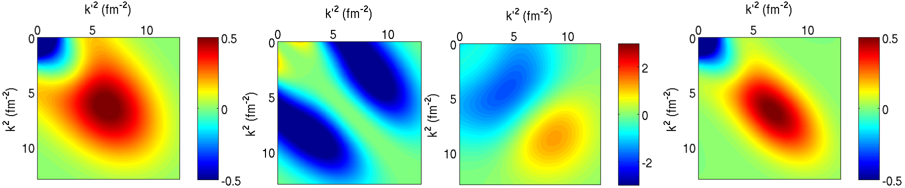
$$V_\lambda(k,k') = V_{\lambda=\infty}(k,k')\, e^{-[(\epsilon_k - \epsilon_{k'})/\lambda^2]^2} + \cdots$$

Flow in momentum basis with $G_s = T$

- For NN only, project on rel. momentum states $|k\rangle$, but generic

$$\frac{dV_s}{ds} = [[T_{\rm rel}, V_s], H_s] \qquad \text{with} \quad T_{\rm rel}|k\rangle = |\epsilon_k\rangle \quad \text{and} \quad \lambda^2 = 1/\sqrt{s}$$

$$\frac{dV_\lambda}{d\lambda}(k,k') \propto -(\epsilon_k - \epsilon_{k'})^2 V_\lambda(k,k') + \sum_q (\epsilon_k + \epsilon_{k'} - 2\epsilon_q) V_\lambda(k,q) V_\lambda(q,k')$$



$V_{\lambda=2.5}(k,k')$ 1st term 2nd term $V_{\lambda=2.0}(k,k')$

- First term drives $^1S_0$ V_λ toward diagonal:

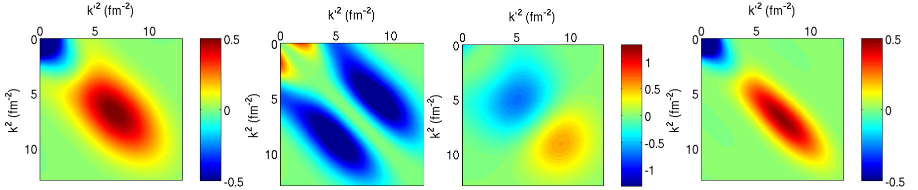
$$V_\lambda(k,k') = V_{\lambda=\infty}(k,k')\, e^{-[(\epsilon_k - \epsilon_{k'})/\lambda^2]^2} + \cdots$$

Flow in momentum basis with $G_s = T$

- For NN only, project on rel. momentum states $|k\rangle$, but generic

$$\frac{dV_s}{ds} = [[T_{\rm rel}, V_s], H_s] \qquad \text{with} \quad T_{\rm rel}|k\rangle = |\epsilon_k\rangle \quad \text{and} \quad \lambda^2 = 1/\sqrt{s}$$

$$\frac{dV_\lambda}{d\lambda}(k,k') \propto -(\epsilon_k - \epsilon_{k'})^2 V_\lambda(k,k') + \sum_q (\epsilon_k + \epsilon_{k'} - 2\epsilon_q) V_\lambda(k,q) V_\lambda(q,k')$$



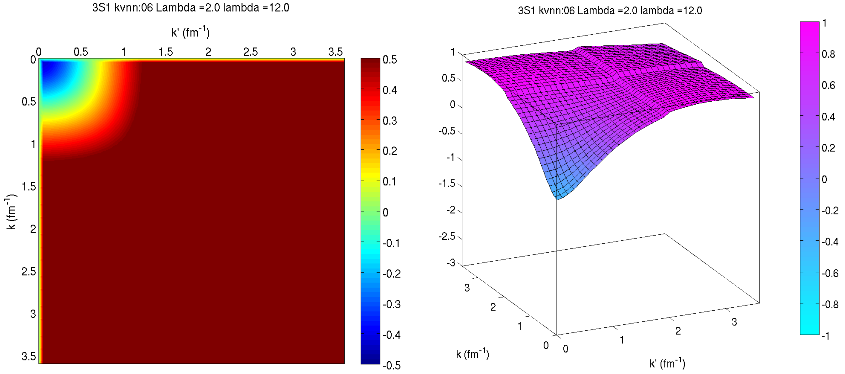
$V_{\lambda=2.0}(k,k')$ | 1st term | 2nd term | $V_{\lambda=1.5}(k,k')$

- First term drives $^1S_0$ V_λ toward diagonal:

$$V_\lambda(k,k') = V_{\lambda=\infty}(k,k')\, e^{-[(\epsilon_k - \epsilon_{k'})/\lambda^2]^2} + \cdots$$

Block Diagonalization Via SRG $[G_s = H_{BD}]$

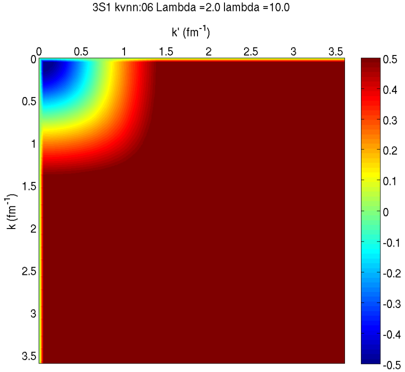
- Can we get a $\Lambda = 2\,\text{fm}^{-1}$ $V_{\text{low}\,k}$-like potential with SRG?
- Yes! Use $\frac{dH_s}{ds} = [[G_s, H_s], H_s]$ with $G_s = \begin{pmatrix} PH_sP & 0 \\ 0 & QH_sQ \end{pmatrix}$

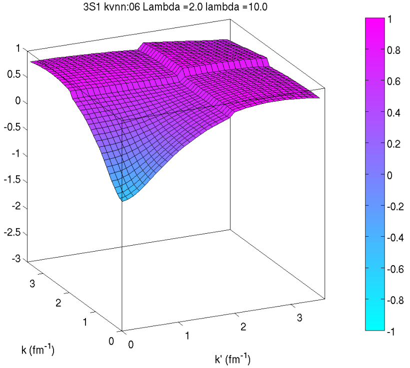


- Best generators for nuclear applications?

Block Diagonalization Via SRG $[G_s = H_{BD}]$

- Can we get a $\Lambda = 2\,\text{fm}^{-1}$ $V_{\text{low}\,k}$-like potential with SRG?
- Yes! Use $\frac{dH_s}{ds} = [[G_s, H_s], H_s]$ with $G_s = \begin{pmatrix} PH_sP & 0 \\ 0 & QH_sQ \end{pmatrix}$

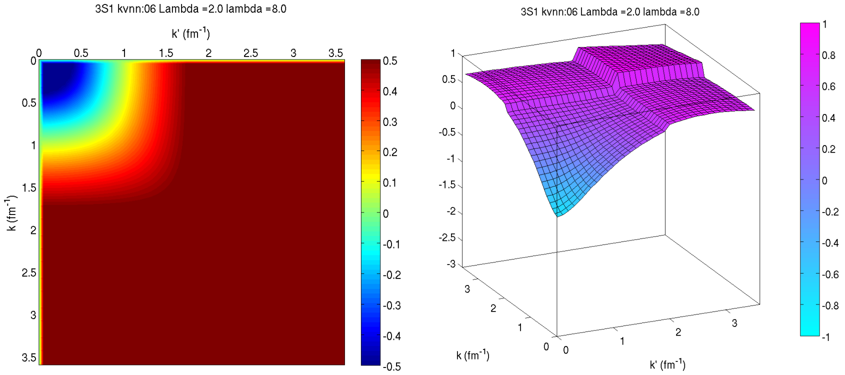




- Best generators for nuclear applications?

Block Diagonalization Via SRG $[G_s = H_{BD}]$

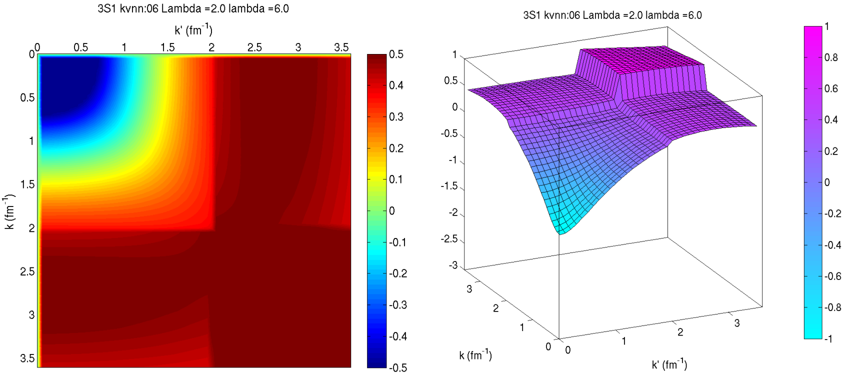
- Can we get a $\Lambda = 2\,\mathrm{fm}^{-1}$ $V_{\mathrm{low}\,k}$-like potential with SRG?
- Yes! Use $\frac{dH_s}{ds} = [[G_s, H_s], H_s]$ with $G_s = \begin{pmatrix} PH_sP & 0 \\ 0 & QH_sQ \end{pmatrix}$



- Best generators for nuclear applications?

Block Diagonalization Via SRG [$G_s = H_{BD}$]

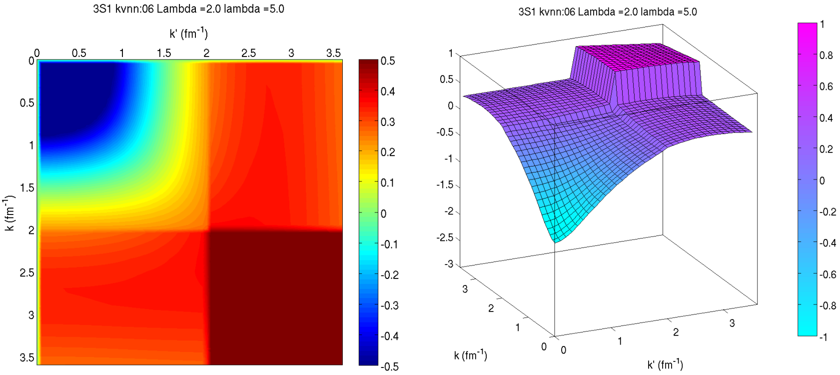
- Can we get a $\Lambda = 2\,\text{fm}^{-1}$ $V_{\text{low}\,k}$-like potential with SRG?
- Yes! Use $\frac{dH_s}{ds} = [[G_s, H_s], H_s]$ with $G_s = \begin{pmatrix} PH_sP & 0 \\ 0 & QH_sQ \end{pmatrix}$



- Best generators for nuclear applications?

Block Diagonalization Via SRG [$G_s = H_{BD}$]

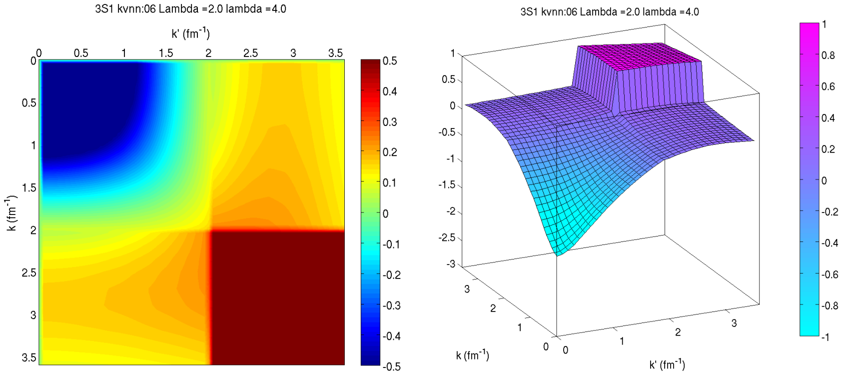
- Can we get a $\Lambda = 2\,\text{fm}^{-1}$ $V_{\text{low}\,k}$-like potential with SRG?
- Yes! Use $\frac{dH_s}{ds} = [[G_s, H_s], H_s]$ with $G_s = \begin{pmatrix} PH_sP & 0 \\ 0 & QH_sQ \end{pmatrix}$



- Best generators for nuclear applications?

Block Diagonalization Via SRG $[G_s = H_{BD}]$

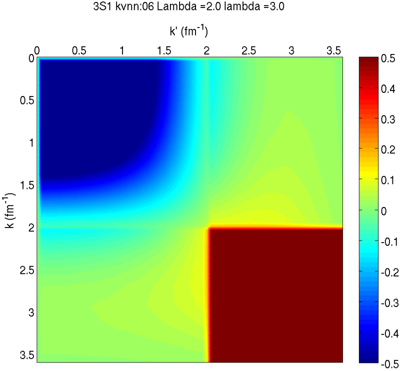
- Can we get a $\Lambda = 2\,\text{fm}^{-1}$ $V_{\text{low}\,k}$-like potential with SRG?
- Yes! Use $\frac{dH_s}{ds} = [[G_s, H_s], H_s]$ with $G_s = \begin{pmatrix} PH_sP & 0 \\ 0 & QH_sQ \end{pmatrix}$

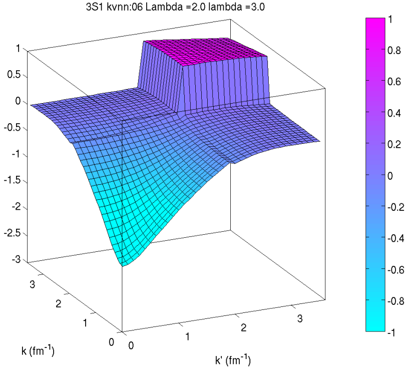


- Best generators for nuclear applications?

Block Diagonalization Via SRG [$G_s = H_{BD}$]

- Can we get a $\Lambda = 2\,\text{fm}^{-1}$ $V_{\text{low}\,k}$-like potential with SRG?
- Yes! Use $\frac{dH_s}{ds} = [[G_s, H_s], H_s]$ with $G_s = \begin{pmatrix} PH_sP & 0 \\ 0 & QH_sQ \end{pmatrix}$

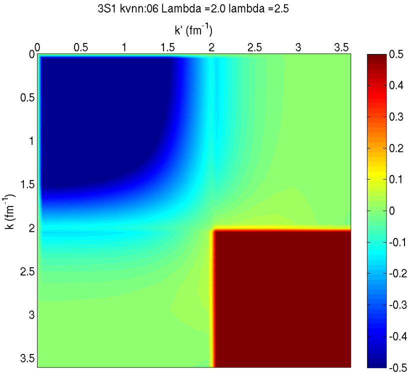


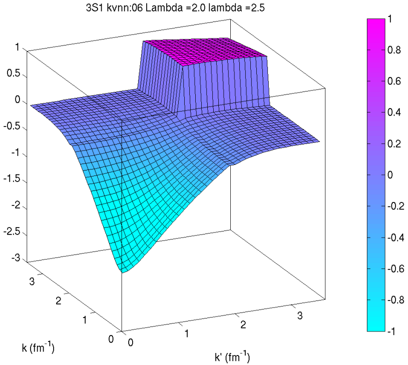


- Best generators for nuclear applications?

Block Diagonalization Via SRG [$G_s = H_{BD}$]

- Can we get a $\Lambda = 2\,\text{fm}^{-1}$ $V_{\text{low}\,k}$-like potential with SRG?
- Yes! Use $\frac{dH_s}{ds} = [[G_s, H_s], H_s]$ with $G_s = \begin{pmatrix} PH_sP & 0 \\ 0 & QH_sQ \end{pmatrix}$

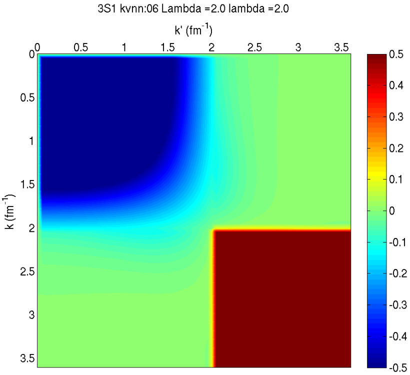


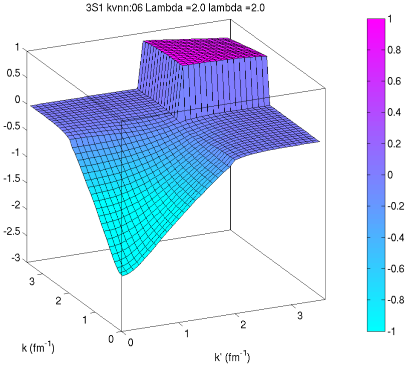


- Best generators for nuclear applications?

Block Diagonalization Via SRG [$G_s = H_{BD}$]

- Can we get a $\Lambda = 2\,\text{fm}^{-1}$ $V_{\text{low}\,k}$-like potential with SRG?
- Yes! Use $\frac{dH_s}{ds} = [[G_s, H_s], H_s]$ with $G_s = \begin{pmatrix} PH_sP & 0 \\ 0 & QH_sQ \end{pmatrix}$

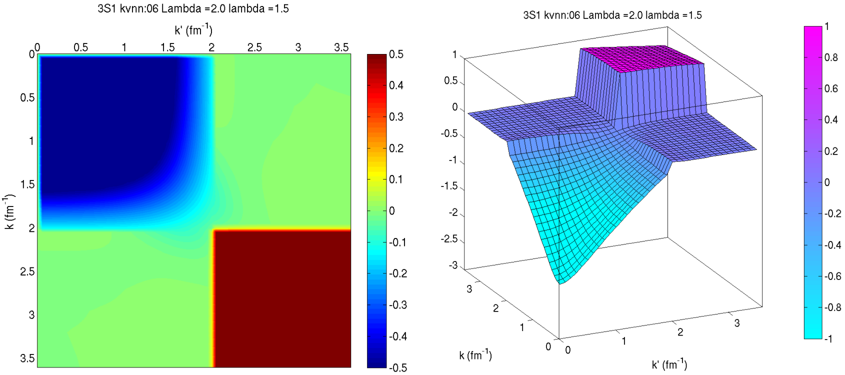




- Best generators for nuclear applications?

Block Diagonalization Via SRG $[G_s = H_{BD}]$

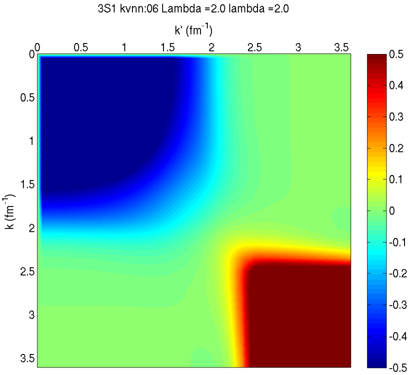
- Can we get a $\Lambda = 2\,\text{fm}^{-1}$ $V_{\text{low}\,k}$-like potential with SRG?
- Yes! Use $\frac{dH_s}{ds} = [[G_s, H_s], H_s]$ with $G_s = \begin{pmatrix} PH_sP & 0 \\ 0 & QH_sQ \end{pmatrix}$

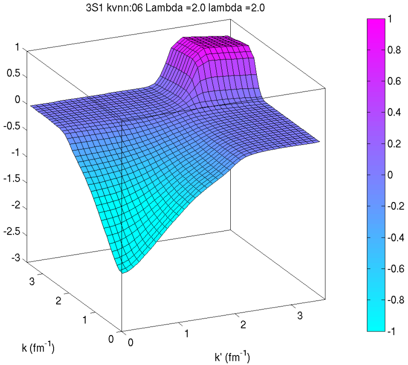


- Best generators for nuclear applications?

Block Diagonalization Via SRG $[G_s = H_{BD}]$

- Can we get a $\Lambda = 2\,\mathrm{fm}^{-1}$ $V_{\mathrm{low}\,k}$-like potential with SRG?
- Yes! Use $\frac{dH_s}{ds} = [[G_s, H_s], H_s]$ with $G_s = \begin{pmatrix} PH_sP & 0 \\ 0 & QH_sQ \end{pmatrix}$

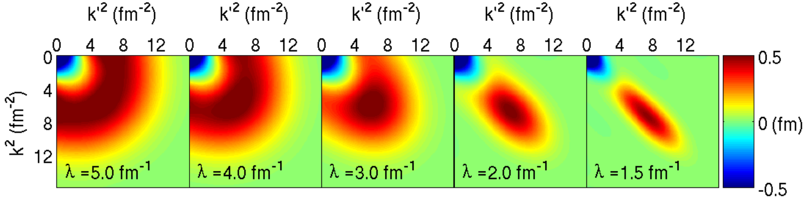




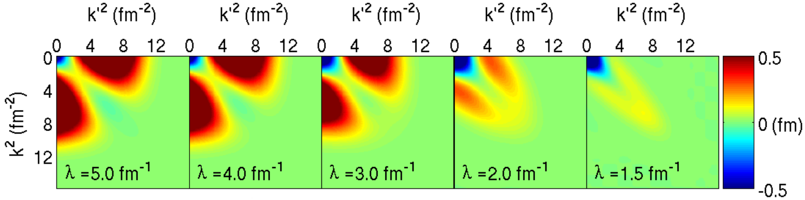
- Best generators for nuclear applications?

Flow of N^{3}LO chiral EFT potentials

- $^1S_0$ from N^{3}LO (500 MeV) of Entem/Machleidt



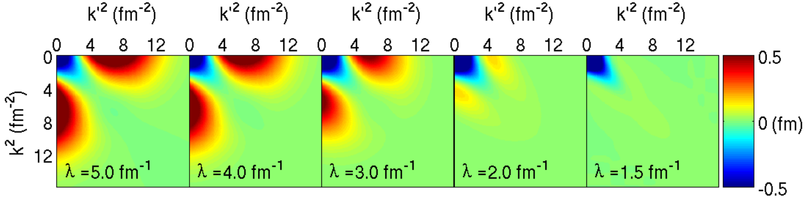
- $^1S_0$ from N^{3}LO (550/600 MeV) of Epelbaum et al.



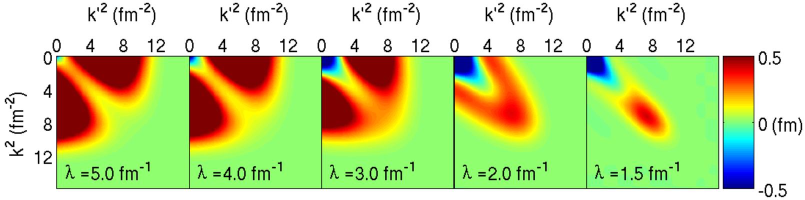
- Significant decoupling even for “soft” EFT interaction

Flow of N^{3}LO chiral EFT potentials

- $^3S_1$ from N^{3}LO (500 MeV) of Entem/Machleidt

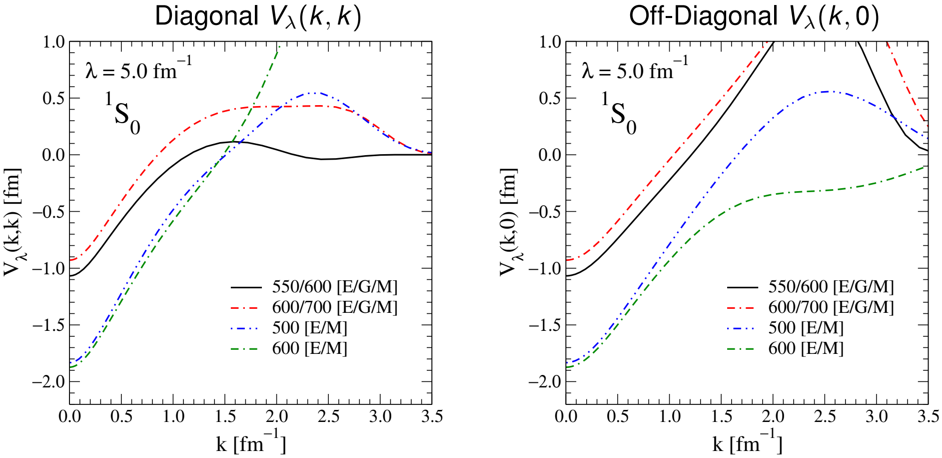


- $^3S_1$ from N^{3}LO (550/600 MeV) of Epelbaum et al.



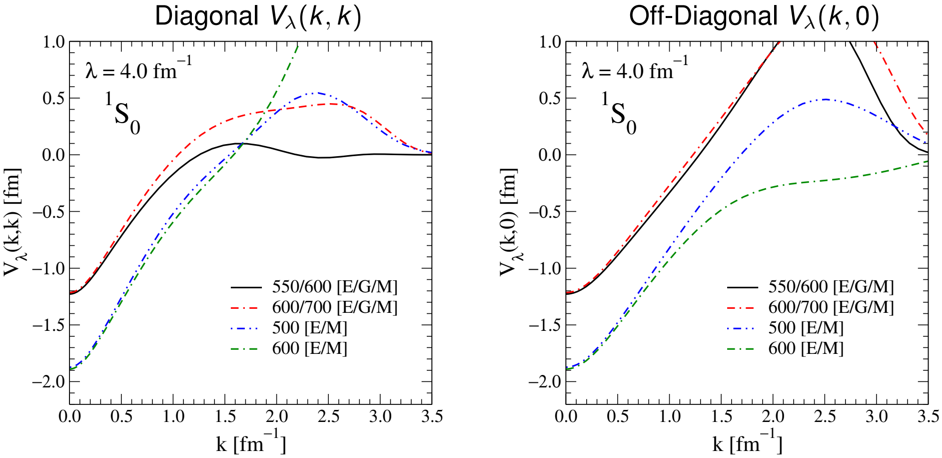
- Significant decoupling even for “soft” EFT interaction

Filter by running to lower λ via SRG $\Longrightarrow$ $\approx$Universal



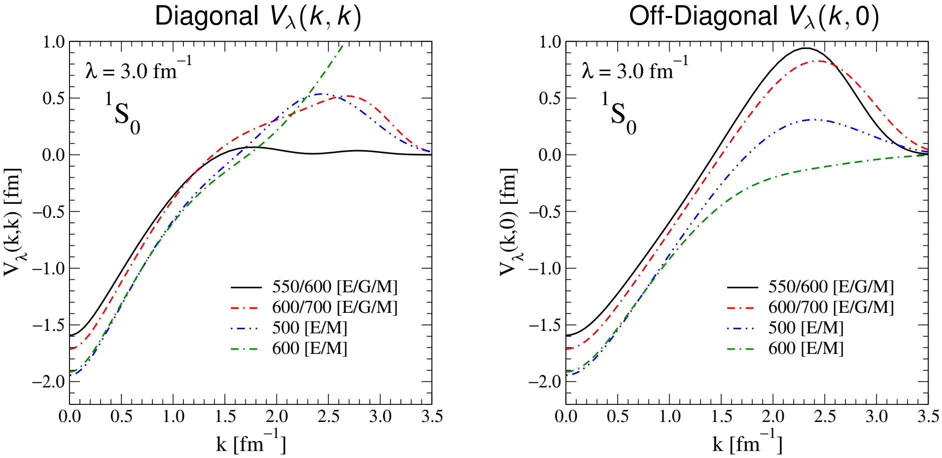
- Consistent with inverse scattering when S-matrices agree
- Will evolved NNN interactions be universal?

Filter by running to lower λ via SRG $\Longrightarrow$ $\approx$Universal



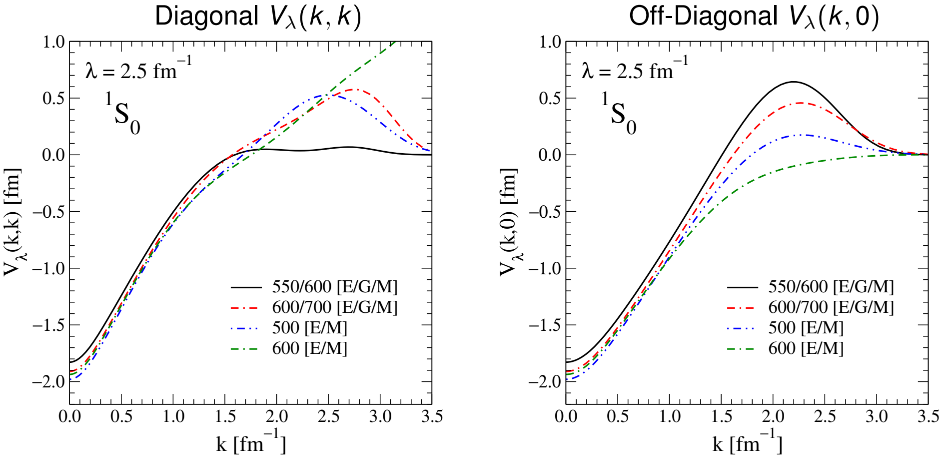
- Consistent with inverse scattering when S-matrices agree
- Will evolved NNN interactions be universal?

Filter by running to lower λ via SRG $\Longrightarrow$ $\approx$Universal



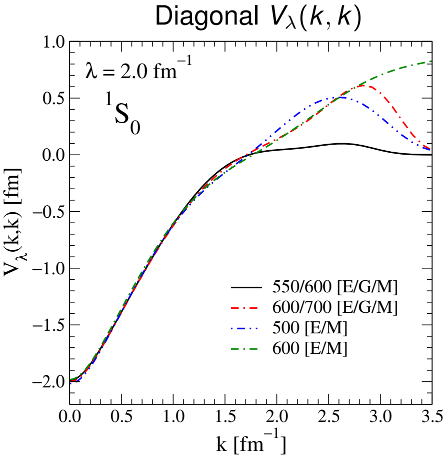
- Consistent with inverse scattering when S-matrices agree
- Will evolved NNN interactions be universal?

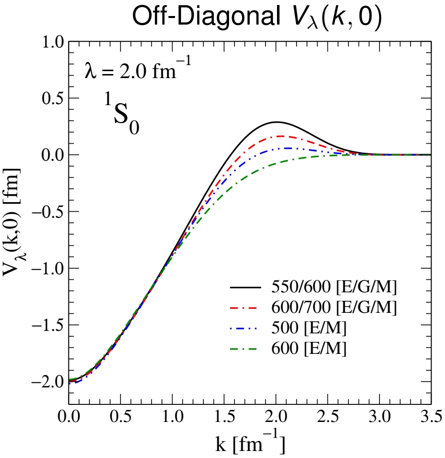
Filter by running to lower λ via SRG $\Longrightarrow$ $\approx$Universal



- Consistent with inverse scattering when S-matrices agree
- Will evolved NNN interactions be universal?

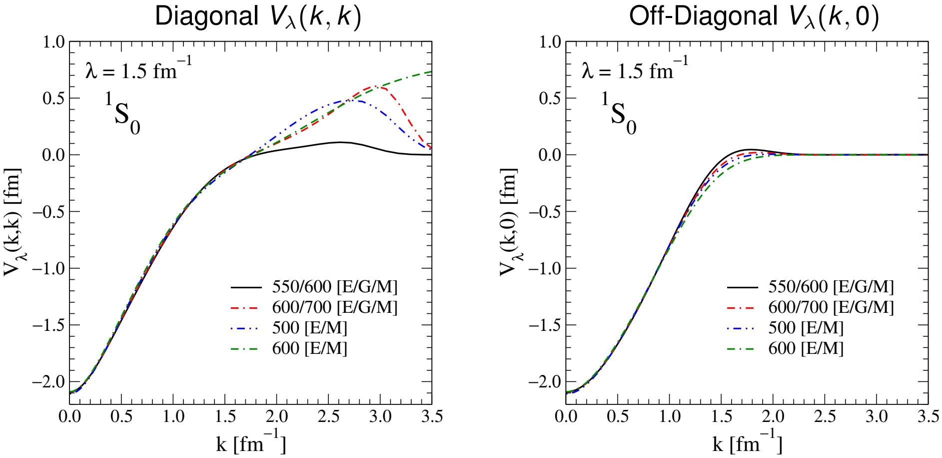
Filter by running to lower λ via SRG $\Longrightarrow$ ≈Universal





- Consistent with inverse scattering when S-matrices agree
- Will evolved NNN interactions be universal?

Filter by running to lower λ via SRG $\Longrightarrow$ $\approx$Universal



- Consistent with inverse scattering when S-matrices agree
- Will evolved NNN interactions be universal?

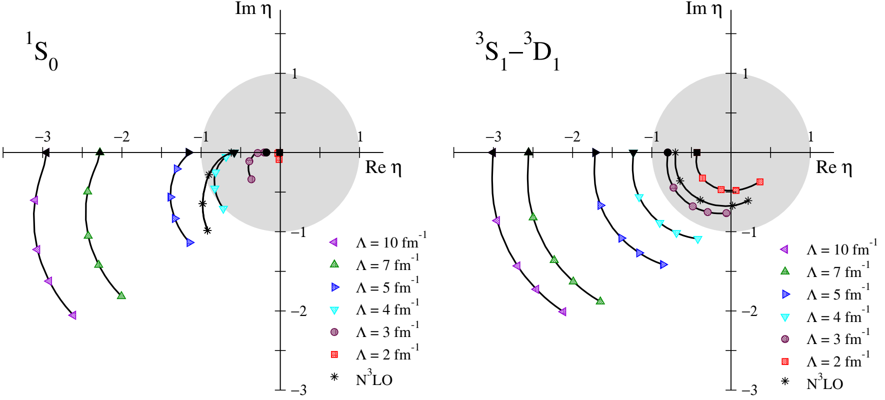
Lowering resolution increases "perturbativeness"

Born Series: $T(E) = V + V\frac{1}{E-H_0}V + V\frac{1}{E-H_0}V\frac{1}{E-H_0}V + \cdots$

- For fixed E, find (complex) eigenvalues $\eta_\nu(E)$ [Weinberg]

$$\frac{1}{E-H_0}V|\Gamma_\nu\rangle = \eta_\nu|\Gamma_\nu\rangle \quad \Longrightarrow \quad T(E)|\Gamma_\nu\rangle = V|\Gamma_\nu\rangle(1+\eta_\nu+\eta_\nu^2+\cdots)$$

$\Longrightarrow$ T diverges if *any* $|\eta_\nu(E)| \geq 1$ [Bogner et al. (2006)]



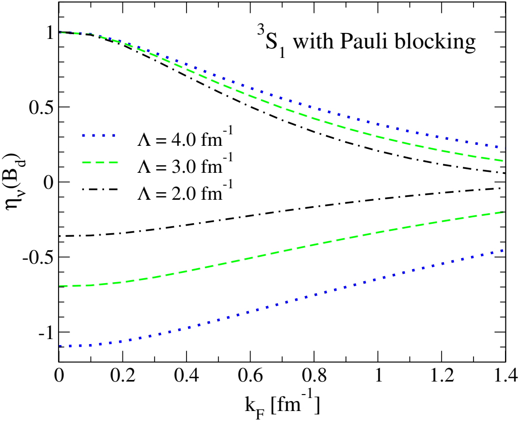
Lowering resolution increases "perturbativeness"

Born Series: $T(E) = V + V\frac{1}{E-H_0}V + V\frac{1}{E-H_0}V\frac{1}{E-H_0}V + \cdots$

- For fixed E, find (complex) eigenvalues $\eta_\nu(E)$ [Weinberg]

$$\frac{1}{E-H_0}V|\Gamma_\nu\rangle = \eta_\nu|\Gamma_\nu\rangle \quad \Longrightarrow \quad T(E)|\Gamma_\nu\rangle = V|\Gamma_\nu\rangle(1 + \eta_\nu + \eta_\nu^2 + \cdots)$$

$\Longrightarrow$ T diverges if *any* $|\eta_\nu(E)| \geq 1$ [Bogner et al. (2006)]



Flow equations lead to many-body operators

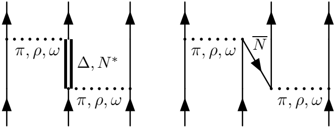
- Consider a's and $a^\dagger$'s wrt s.p. basis and reference state:

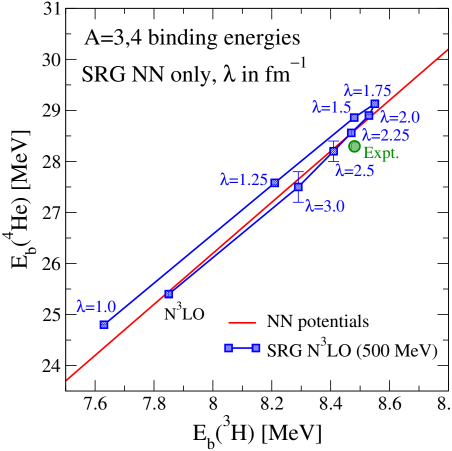
$$\frac{dV_s}{ds} = \Big[\Big[\sum \underbrace{a^\dagger a}_{G_s}, \sum \underbrace{a^\dagger a^\dagger aa}_{\text{2-body}}\Big], \sum \underbrace{a^\dagger a^\dagger aa}_{\text{2-body}}\Big] = \cdots + \sum \underbrace{a^\dagger a^\dagger a^\dagger aaa}_{\text{3-body!}} + \cdots$$

  so there will be A-body forces (and operators) generated
- Is this a problem?
  - Ok if "induced" many-body forces are same size as natural ones
- Nuclear 3-body forces already needed in unevolved potential
  - In fact, there are A-body forces (operators) initially
  - Natural hierarchy from chiral EFT
    $\Longrightarrow$ stop flow equations before unnatural or tailor G_s to suppress
  - Still needed: analytic bounds on A-body growth
- SRG is a tractable method to evolve many-body operators
- Alternative: choose a non-vacuum reference state
  $\Longrightarrow$ in-medium SRG (e.g., HF reference state)

Observations on three-body forces

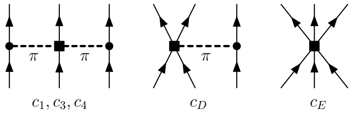
- Three-body forces arise from eliminating/decoupling dof's
  - excited states of nucleon
  - relativistic effects
  - high-momentum intermediate states
- Omitting 3-body forces leads to model dependence
  - observables depend on Λ/λ
  - cutoff dependence as tool

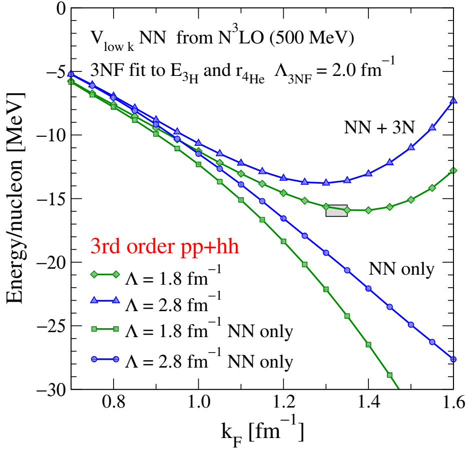




Observations on three-body forces

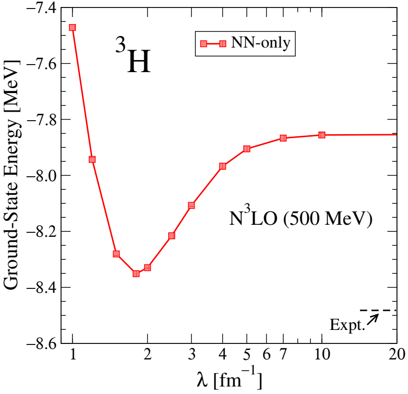
- Three-body forces arise from eliminating/decoupling dof's
  - excited states of nucleon
  - relativistic effects
  - high-momentum intermediate states
- Omitting 3-body forces leads to model dependence
  - observables depend on Λ/λ
  - cutoff dependence as tool
- NNN at different Λ/λ must be fit or evolved to χEFT
  - NNN contribution is important at low resolution (e.g., nuclear matter)
  - how large is 4-body?

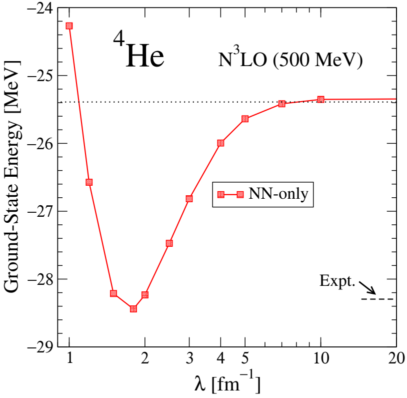




3D SRG evolution with $T_{\rm rel}$ in a Jacobi HO basis

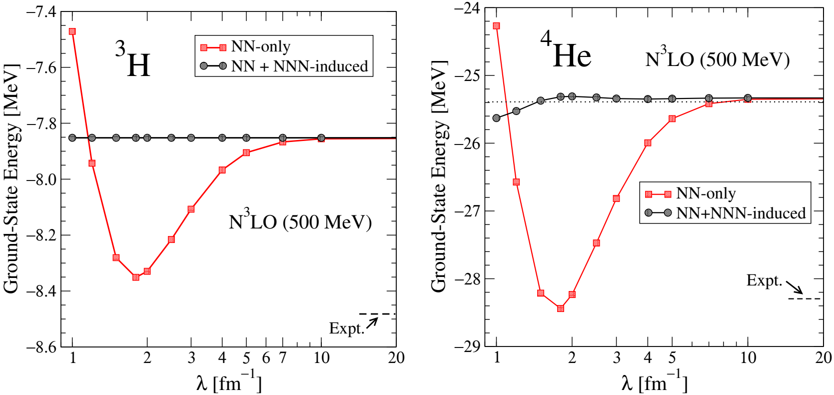
- Evolve in *any* basis [E. Jurgenson, P. Navrátil, rjf (2009)]
  - Here: use anti-symmetric Jacobi HO basis from NCSM
  - Directly obtain SRG matrix elements in HO basis
  - Separate 3-body evolution not needed
- Compare 2-body only to full $2 + 3$-body evolution:





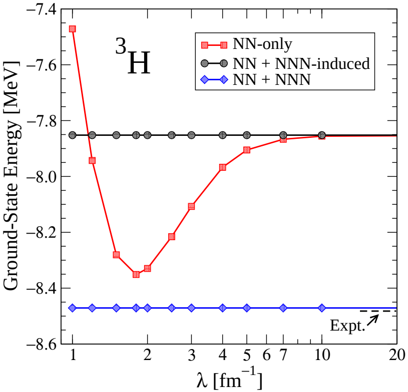
3D SRG evolution with T_{rel} in a Jacobi HO basis

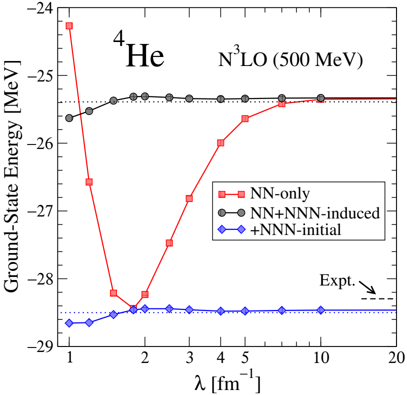
- Evolve in *any* basis [E. Jurgenson, P. Navrátil, rjf (2009)]
  - Here: use anti-symmetric Jacobi HO basis from NCSM
  - Directly obtain SRG matrix elements in HO basis
  - Separate 3-body evolution not needed
- Compare 2-body only to full $2 + 3$-body evolution:



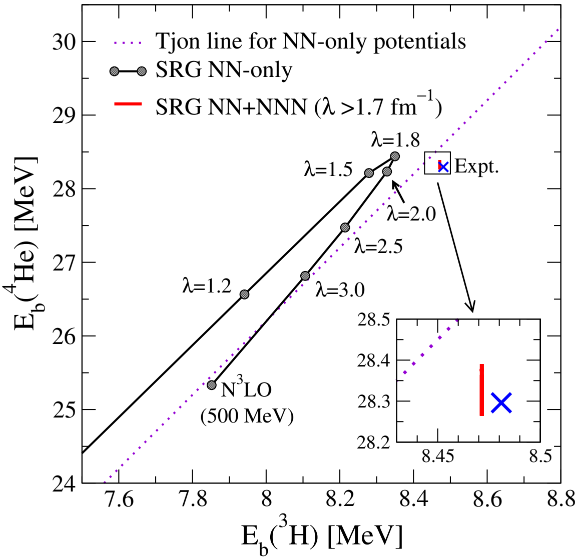
3D SRG evolution with $T_{\rm rel}$ in a Jacobi HO basis

- Evolve in *any* basis [E. Jurgenson, P. Navrátil, rjf (2009)]
  - Here: use anti-symmetric Jacobi HO basis from NCSM
  - Directly obtain SRG matrix elements in HO basis
  - Separate 3-body evolution not needed
- Compare 2-body only to full $2 + 3$-body evolution:



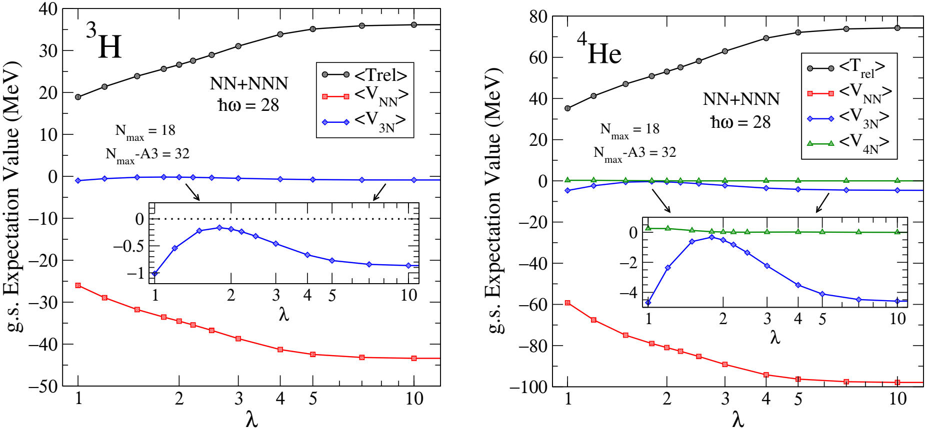


Tjon line revisited



Contributions to the ground-state energy

- Look at ground-state matrix elements of KE, NN, 3N, 4N



- Clear hierarchy, but also strong cancellations at NN level
- What about the *A* dependence? [See R. Roth talk]

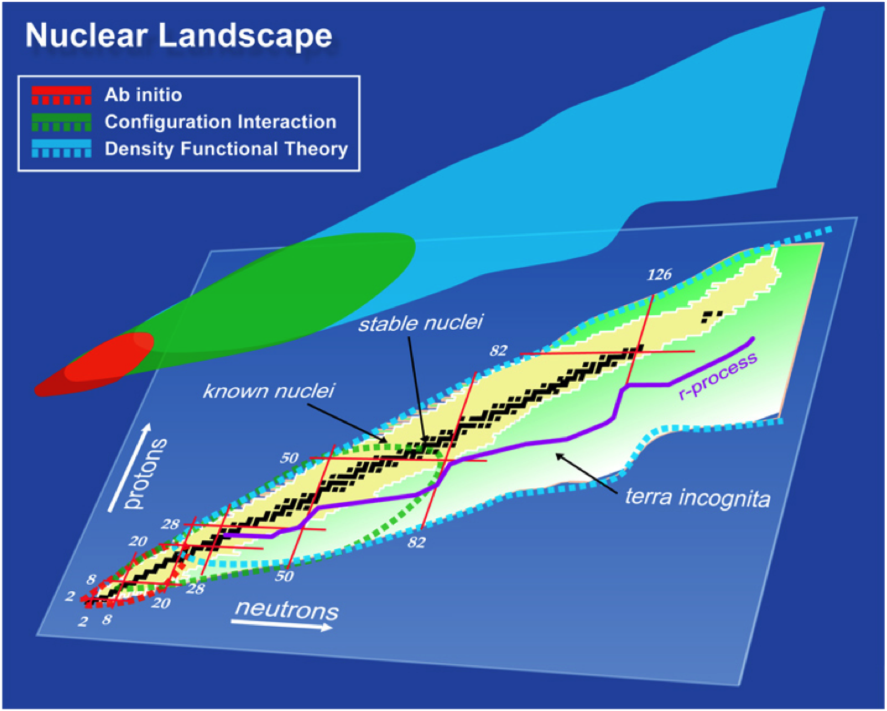
Outline

Overview: Low-energy nuclear physics

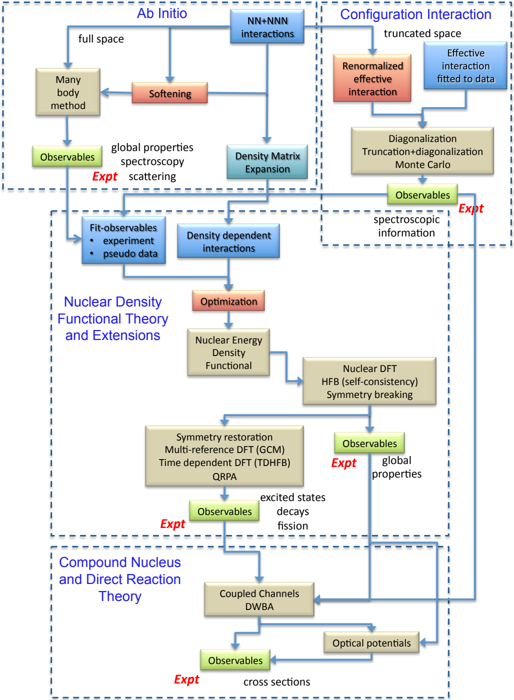
Lowering the resolution with RG

Survey of calculations at low resolution

Outlook


Nuclear Landscape
Ab initio
Configuration Interaction
Density Functional Theory
stable nuclei
known nuclei
r-process
terra incognita
protons
neutrons
126
82
50
28
20
8
2
82
50
28
20
8
2

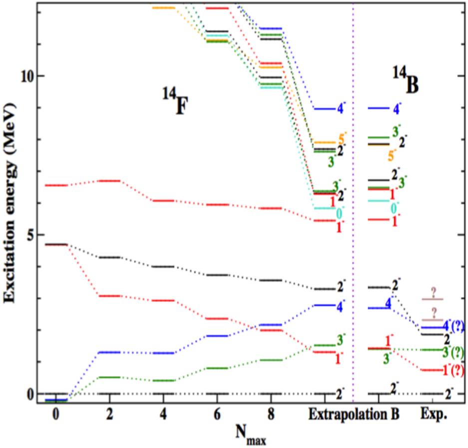
- SciDAC UNEDF project
- Universal Nuclear Energy Density Functional
- Collaboration of physicists, applied mathematicians, and computer scientists
- US funding but international collaborators also
- See unedf.org for highlights



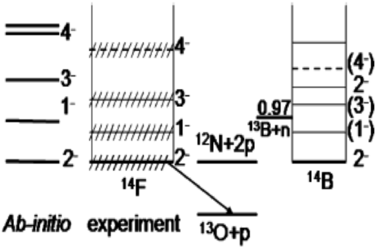
Unstable "proton-dripping" fluorine-14

- Ab initio calculation using low-*k* inverse-scattering potential
- Theory preceded recent experimental measurement

P. Maris et al., PRC **81**, 021301(R) (2010)



V.Z. Goldberg et al., Phys. Lett. B **692**, 307 (2010)

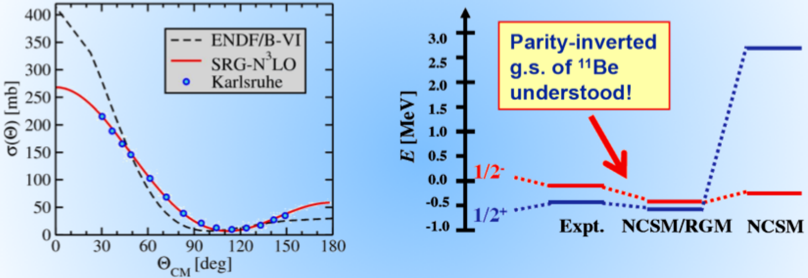


- Matrix dimension 2×10^9, 2.5 hours on 30,000 cores

Ab initio approach to light-ion reactions

- NCSM/RGM using low momentum SRG NN interactions
- See, e.g., Navrátil, Roth, Quaglioni, PRC **82**, 034609 (2010) for nucleon-nucleus scattering

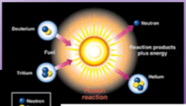
S.Quaglioni and P. Navrátil, PRL101, 092501 (2008); PRC79, 044606 (2009)

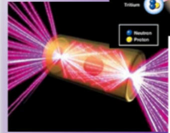


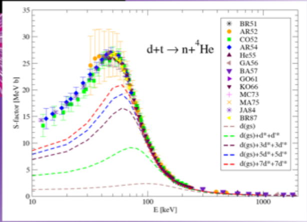
The n-^{4}He differential cross section for 17 MeV neutrons (left) and ^{11}Be bound spectrum (right) obtained within the NCSM/RGM compared to experimental data

Ab initio approach to light-ion reactions

- Applications to fusion energy systems and stellar evolution

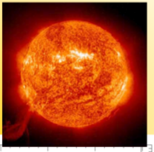


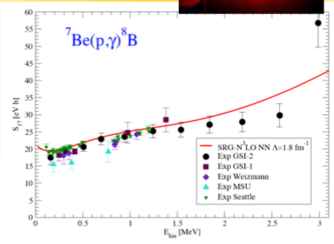




First principles calculations for d+T reaction can lead to a fundamental understanding of thermonuclear fusion

Thermonuclear reactions power stars, ${}^7Be(p,\gamma){}^8B$ is the principal source of observed solar neutrinos



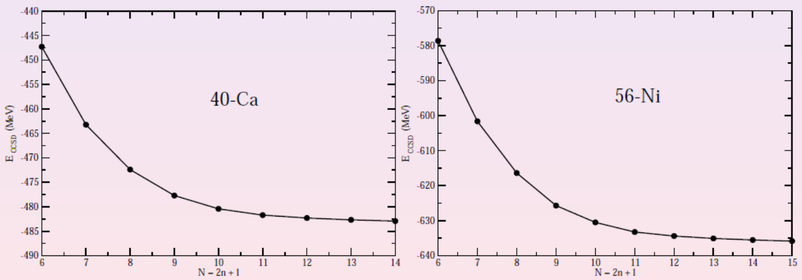


- Still to do: including SRG-evolved NNN interactions

More perturbative $\Longrightarrow$ like quantum chemistry

- Powerful coupled cluster method works! (figure from G. Hagen)

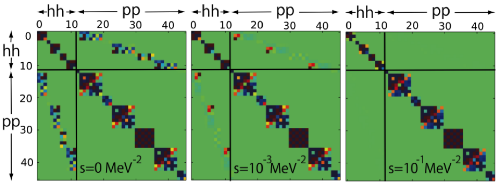
Converged results for ^{40}Ca and ^{56}Ni, using N^{3}LO evolved down to $\lambda = 2.5\mathrm{fm}^{-1}$ from similarity renormalization group theory.

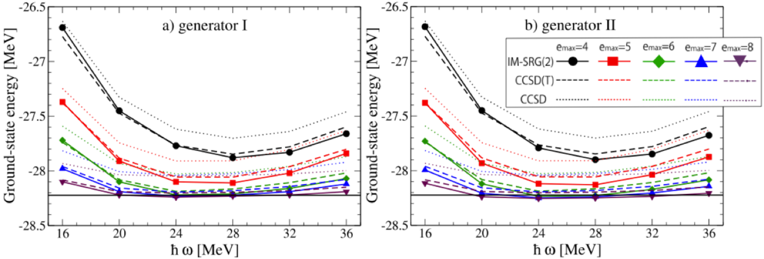


- Improved convergence with low-resolution SRG potentials but also with “bare” chiral EFT potentials [T. Pappenbrock]
- CC extended to 3-body forces by Hagen et al.

In-medium SRG for nuclei [Tsukiyama, Bogner, Schwenk]

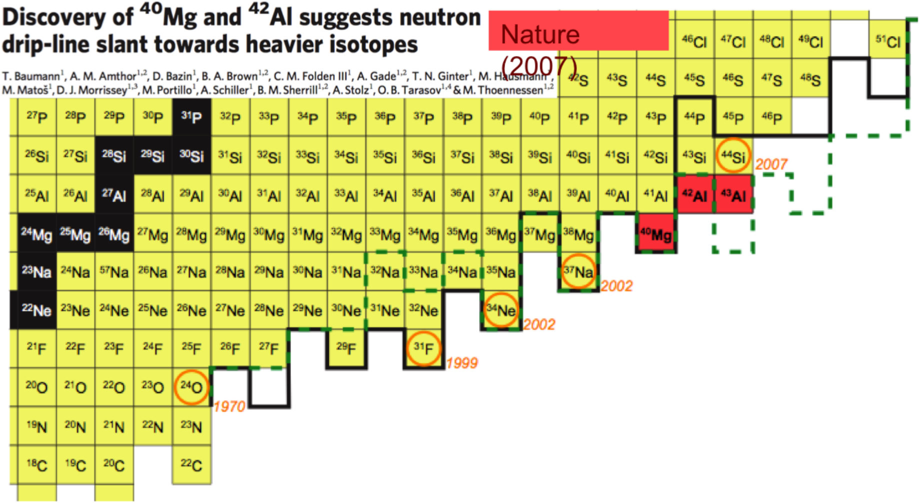
- SRG in *A*-body system using normal ordering
- Decouple 1p1h, 2p2h, . . . sectors from (HF) reference state; approximate N-body





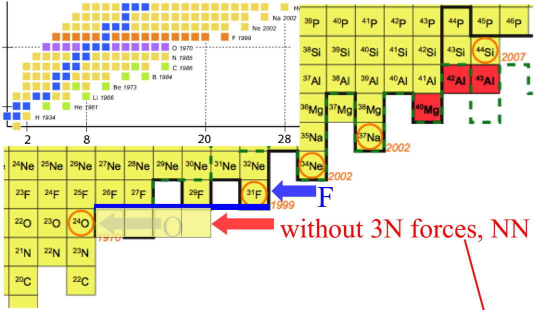
- Promising results for closed-shell nuclei: ^{4}He, ^{16}O, ^{40}Ca
- Energies between coupled cluster CCSD and CCSD(T)
- Non-perturbative valence shell-model effective interactions

Finding the “driplines” — limits of existence!



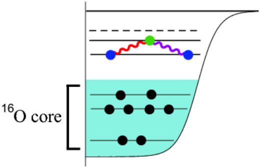
- Oxygen-24 is double magic $\Longrightarrow$ Why is it at limit of stability?

The oxygen anomaly - not reproduced without 3N forces

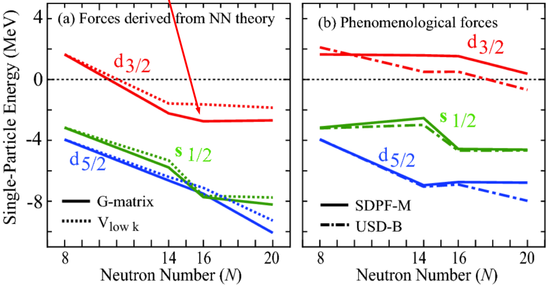


without 3N forces, NN interactions too attractive

many-body theory based on two-nucleon forces:
drip-line incorrect at ^{28}O



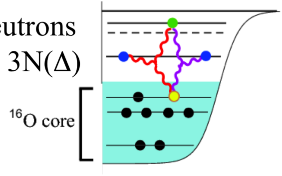
fit to experiment



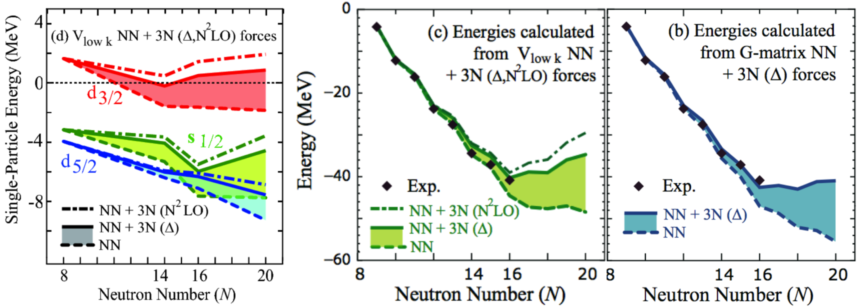
The oxygen anomaly - impact of 3N forces

include “normal-ordered” 2-body part of 3N forces (enhanced by core A)

leads to repulsive interactions between valence neutrons
can understand partly based on Pauli principle for 3N(Δ)



$d_{3/2}$ orbital remains unbound from ^{16}O to ^{28}O



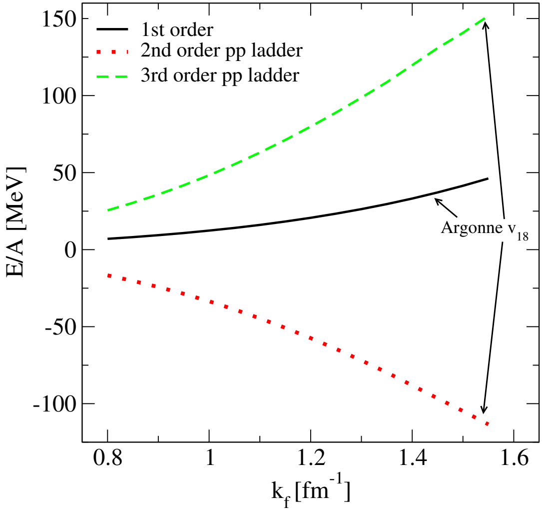
first microscopic explanation of the oxygen anomaly

Otsuka, Suzuki, Holt, Schwenk, Akaishi (2010)

Low resolution $\Longrightarrow$ MBPT is feasible!

- MBPT $\equiv$ Many-Body Perturbation Theory

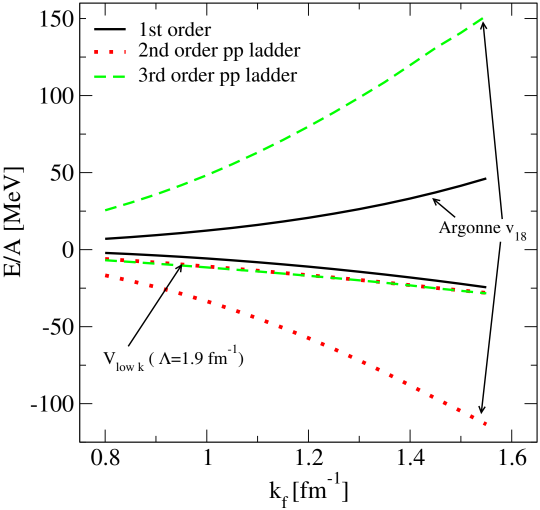
- Compare high resolution to low resolution
- MBPT converges!
- R. Roth et al. $\Longrightarrow$ apply to finite nuclei (4NF?)



Low resolution $\Longrightarrow$ MBPT is feasible!

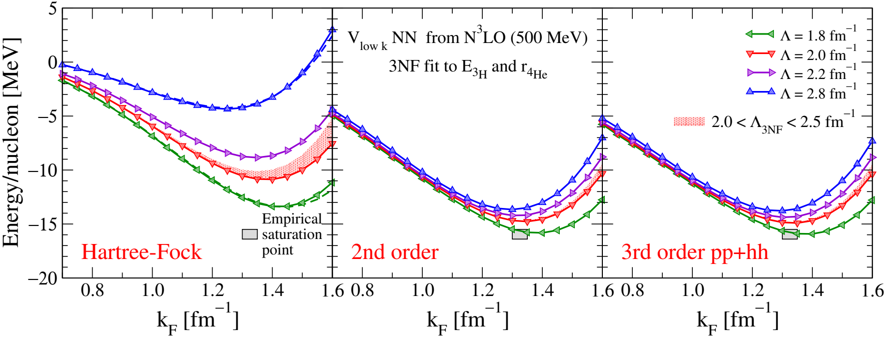
- MBPT $\equiv$ Many-Body Perturbation Theory

- Compare high resolution to low resolution
- MBPT converges!
- R. Roth et al. $\Longrightarrow$ apply to finite nuclei (4NF?)
- Need 3-body force for saturation (evolved or fit)



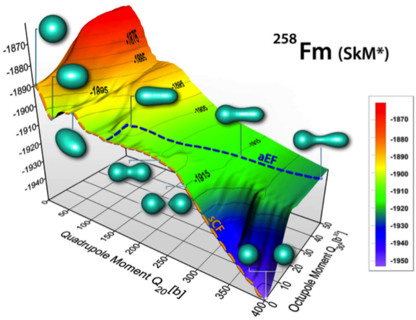
One of the paths to microscopic nuclear DFT

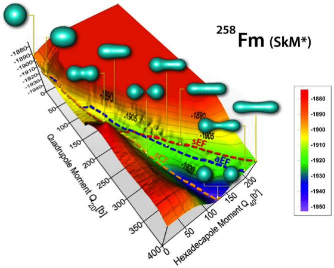
- Construct a chiral EFT to a given order (N^3LO at present)
- Evolve Λ down with RG (to $\Lambda \approx 2\,\text{fm}^{-1}$ for ordinary nuclei)
  - NN interactions fully, NNN interactions approximately
- Generate density functional in MBPT
  - Hartree-Fock plus "≈ second order", use "DME" in *k*-space

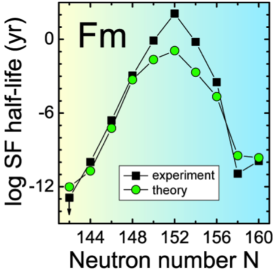


Bogner et al., (2009), Hebeler et al., (2010), Gebremariam et al., (2010)

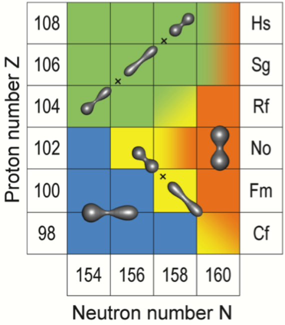
Spontaneous fission: Energy surfaces from DFT





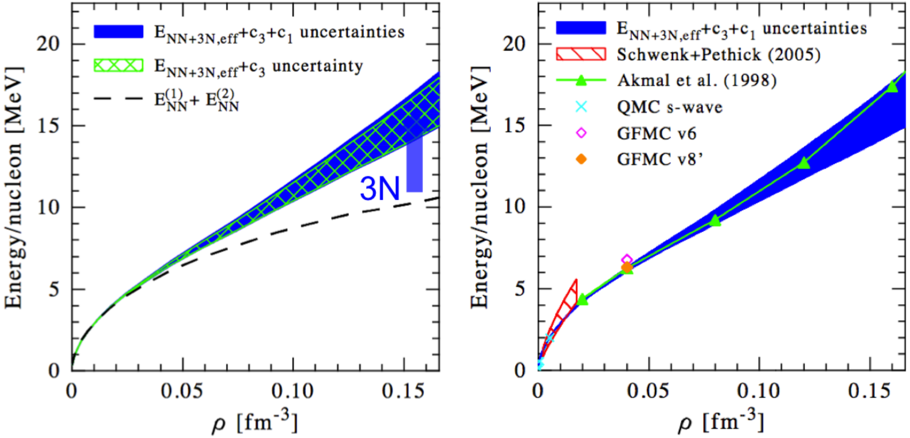


A. Staszczak et al.,
PRC 80, 014309 (2009)



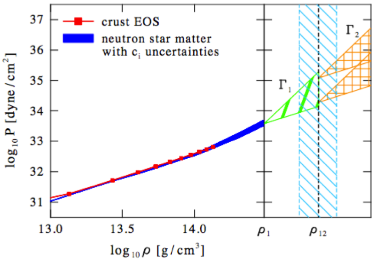
Low resolution calculations of neutron matter

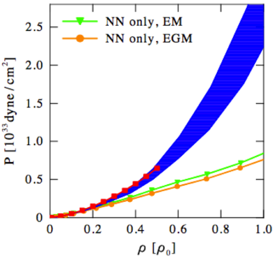
- Evolve NN to low momentum, fit NNN to $A = 3, 4$
- Neutron matter in perturbation theory [Hebeler, Schwenk (2010)]



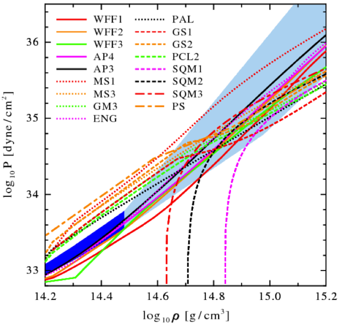
- Use cutoff dependence to estimate many-body uncertainty
- Uncertainties from long-range NNN constants are greatest

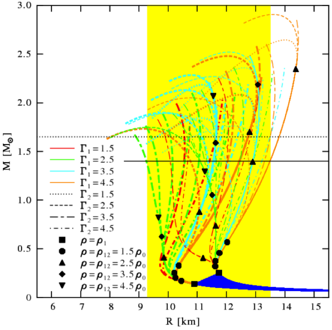
Constraining neutron stars: $R = 9.7$–13.9 km for $1.4\,M_{\rm sun}$





Hebeler, Lattimer, Pethick, Schwenk (2010)





- Extrapolate EOS to higher density
- Solve for M vs. R $\Longrightarrow$ yellow band constrains radius

Outline

Overview: Low-energy nuclear physics

Lowering the resolution with RG

Survey of calculations at low resolution

Outlook

Summary: Atomic Nuclei at Low Resolution

- Strategy: Lower the resolution and track dependence on it
  - High resolution $\Longrightarrow$ high momenta can be painful!
    (“It hurts when I do this.” “Then don’t do that.”)
  - Correlations in wave functions reduced dramatically
  - Non-local potentials and many-body operators “induced”

Summary: Atomic Nuclei at Low Resolution

- Strategy: Lower the resolution and track dependence on it
  - High resolution $\Longrightarrow$ high momenta can be painful!
    ("It hurts when I do this." "Then don't do that.")
  - Correlations in wave functions reduced dramatically
  - Non-local potentials and many-body operators "induced"
- Flow equations (SRG) achieve low resolution by decoupling
  - Band (or block) diagonalizing Hamiltonian matrix (or . . .)
  - Unitary transformations: observables don't change
    but physics interpretation may change!
  - Nuclear case: evolve until few-body forces/operators
    start to explode or use in-medium SRG

Summary: Atomic Nuclei at Low Resolution

- Strategy: Lower the resolution and track dependence on it
  - High resolution $\Longrightarrow$ high momenta can be painful!
    ("It hurts when I do this." "Then don't do that.")
  - Correlations in wave functions reduced dramatically
  - Non-local potentials and many-body operators "induced"
- Flow equations (SRG) achieve low resolution by decoupling
  - Band (or block) diagonalizing Hamiltonian matrix (or . . .)
  - Unitary transformations: observables don't change
    but physics interpretation may change!
  - Nuclear case: evolve until few-body forces/operators
    start to explode or use in-medium SRG
- Applications to nuclei and beyond
  - CI, coupled cluster, HH, . . . converge faster $\Longrightarrow$ new possibilities
  - Microscopic shell model $\Longrightarrow$ role of 3-body forces
  - MBPT works $\Longrightarrow$ constructive nuclear density functional theory

Some open questions and issues

- Power counting for evolved many-body interactions
  - Need analytic estimates plus more numerical tests
- Operator issues (SRG evolves operators, too!)
  - Scaling of many-body operators
  - Technical issues (e.g., boosting)
  - Factorization for many-body systems
- Can different choices for G_s . . .
  - control the growth of many-body forces?
  - improve convergence in HO basis?
  - drive a non-local potential to local form?
- Use of different basis for SRG evolution
  - Need momentum-space implementation
  - Hyperspherical coordinates? (also for visualization)
- Do many-body interactions flow to universal form?
- Can the SRG help with constructing/analyzing EFT's?

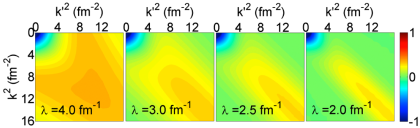
Thanks: collaborators and others at low resolution

- Darmstadt: R. Roth, A. Schwenk
- INT / Chalmers: L. Platter
- Iowa State: P. Maris, J. Vary
- Michigan State: S. Bogner, H. Hergert
- LLNL: E. Jurgenson, N. Schunck
- Los Alamos: J. Drut
- Ohio State: E. Anderson, M. Bettencourt, W. Li, R. Perry, K. Wendt
- ORNL/UofT: M. Kortelainen, W. Nazarewicz, M. Stoitsov
- TRIUMF: P. Navratil
- UNEDF
- Warsaw: S. Glazek

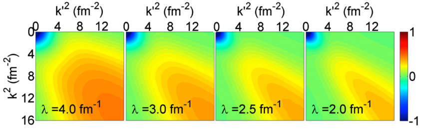
Flow equations and the SRG: History

- In the early 1970's, Ken Wilson and Franz Wegner
  $\Longrightarrow$ critical phenomena and renormalization group (RG)
- Twenty years later, Wilson and Wegner innovate again
  - Unitary RG flow to make many-particle Hamiltonians increasingly energy diagonal
  - Glazek and Wilson, "Renormalization of Hamiltonians" (1993)
    $\Longrightarrow$ SRG for QCD on the light front
  - Wegner, "Flow Equations for Hamiltonians" (1994)
    $\Longrightarrow$ condensed matter problems
- S. Kehrein, "Flow-Equation Approach to Many-Particle Systems"
  - Dissipative quantum systems to correlated electron physics to non-equilibrium problems to . . .
- Particularly well suited for low-energy nuclear physics!
  - Only applied in last few years [arXiv:0912.3688]
  - Technically simpler and more versatile than other methods

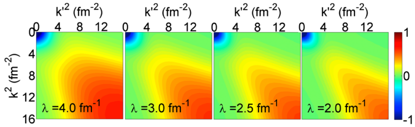
Novel generators: $G_s = f(T)$ [Shirley Li, OSU physics major]



$$G_s = T$$

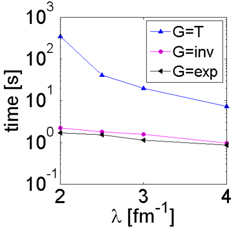


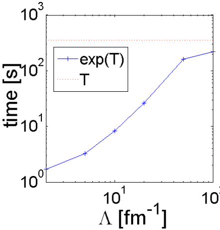
$$G_s = -\Lambda^2/(1 + T/\Lambda^2) \approx c + T + \cdots$$



$$G_s = -\Lambda^2 e^{-T/\Lambda^2} \approx c + T + \cdots$$

- For $\Lambda = 2\,\mathrm{fm}^{-1}$, low E part of V still decoupled
- Much less evolution at high E
  $\Longrightarrow$ much faster!

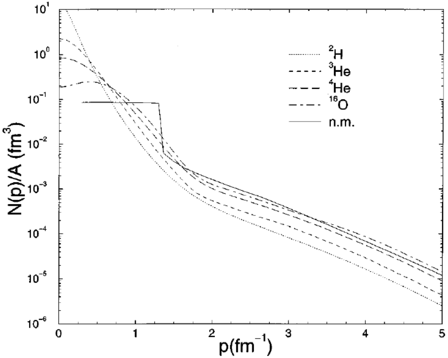




- Allows evolution to low λ
- Application to $A > 2$ evolution
- Other useful generators?

Factorization in few-body nuclei: $n(k)$ at large k

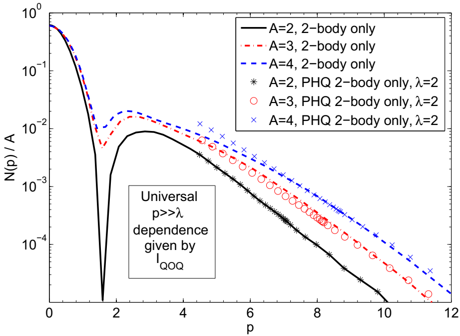
- AV14 NN with VMC



From Pieper, Wiringa, and Pandharipande (1992).

- Conventional explanation: Dominance of NN potential and short-range correlations (Frankfurt et al.)

- *A* bosons in 1D model



- Alternative: *factorization*

$$\int_0^\lambda \int_0^\lambda \psi_\lambda^\dagger(k') \, [I_{QOQ} K_\lambda(k') K_\lambda(k)] \, \psi_\lambda(k)$$

- universal p dependence from I_{QOQ}
- norm. factor from low-energy m.e.