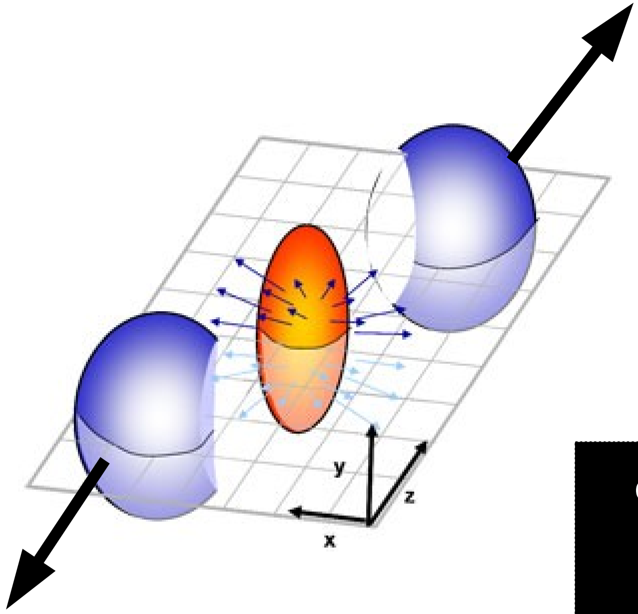
Fluidity, supercritical fluids, RHIC and LHC

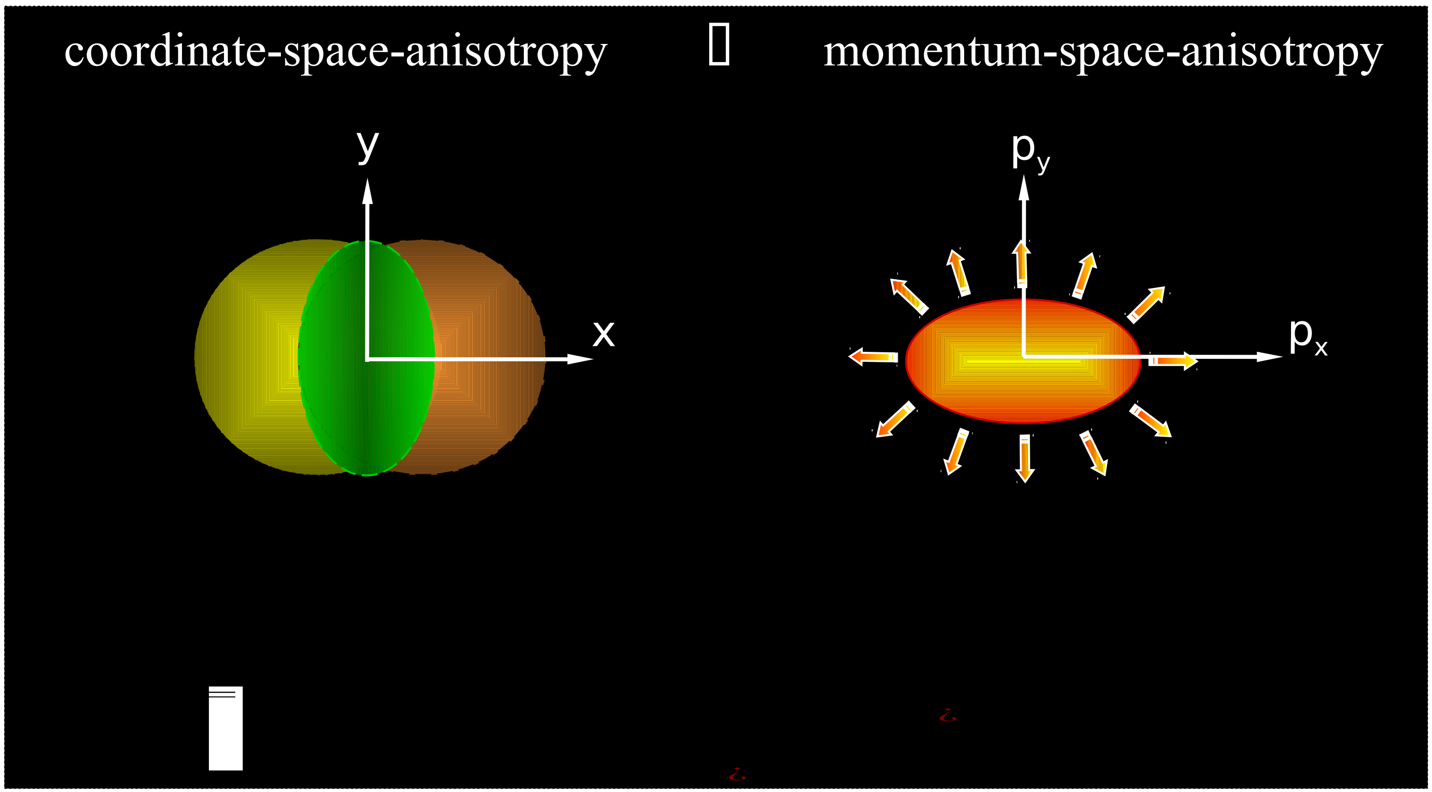
- The “Perfect Liquid” ?
- Supercritical fluids and expectations for LHC
- Connection with lattice?

Based on:
J. Liao and V.K, arXiv:0909.3105, Phys.Rev.C80:034904,2009.

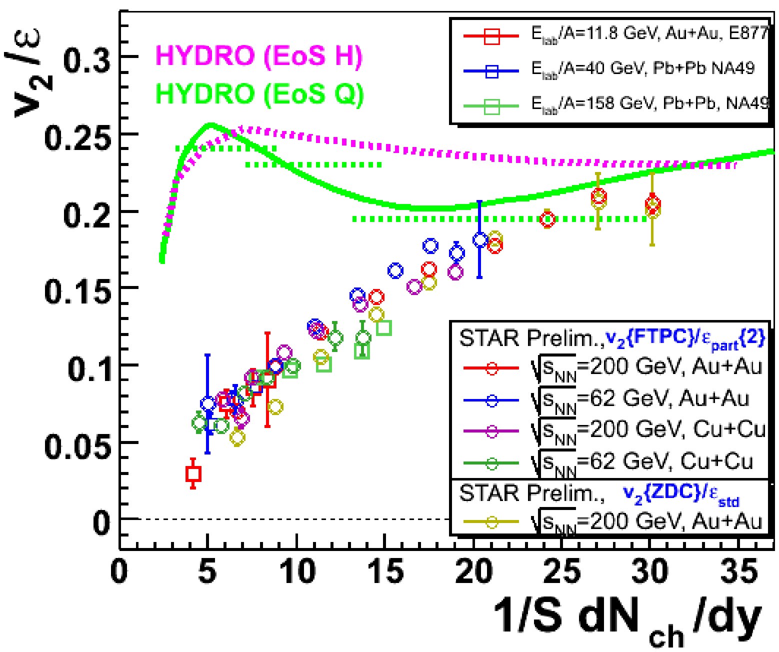
Elliptic Flow



$$\frac{dN}{d\phi\, dp_t} = \frac{dN}{d\, p_t}\left(1 + v_1(p_t)\cos(\phi) + 2\, \underline{v_2(p_t)}\cos(2\,\mathrm{phi}) + \ldots\right)$$



Integrated Elliptic Flow



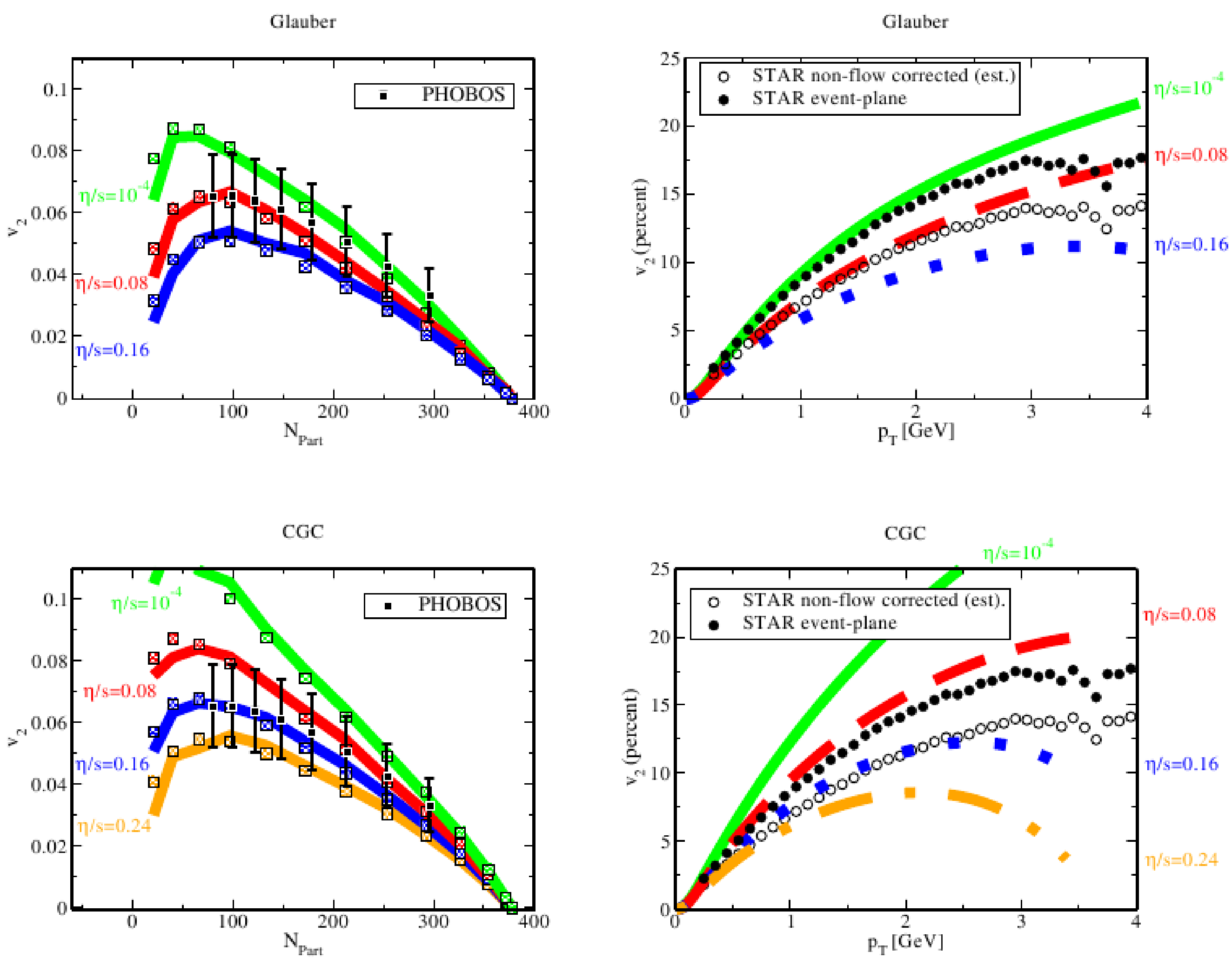
S. Voloshin, A. Poskanzer, PL ***B474****, 27(00).*
D. Teaney, et. al., nucl-th/0110037

ε=spatial eccentricity
(from centrality via Glauber)

$$v_2 = \frac{\int dp_t \frac{dN}{dp_t} v_2(p_t)}{\int dp_t \frac{dN}{dp_t}}$$

- Scales with transverse particle density!
- Scaling consistent with low density, single particle scattering
  - necessary cross-section too large, however

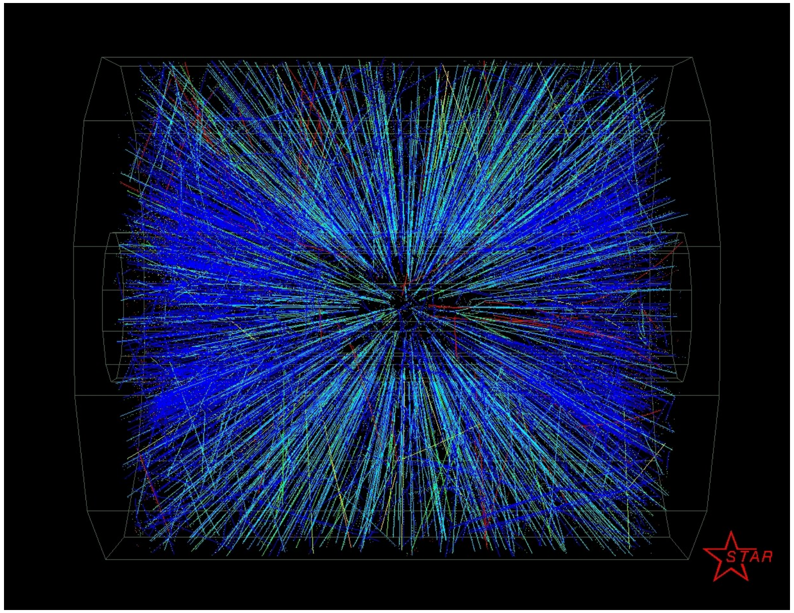
The Mantra: Viscous Hydrodynamics



Romatschke, Luzum, arXiv:0804.4015

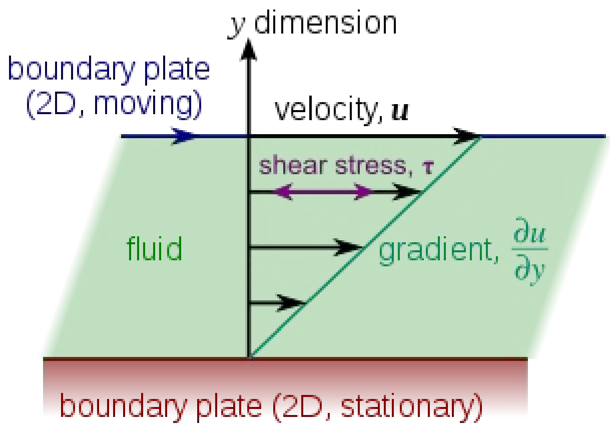
The Perfect Liquid?

VS



J. Liao and V.K, arXiv:0909.3105, Phys.Rev.C80:034904,2009.

Viscosity

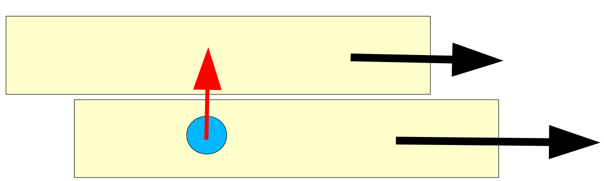


Correction to ideal fluid dynamics

Shear tensor: $\sigma_{i,j}=\eta\left(\frac{\partial v_i}{\partial x_j}+\frac{\partial v_j}{\partial x_i}-\frac{2}{3}\delta_{i,j}\frac{\partial v_i}{\partial x_i}\right)$

$\eta = shear\ viscosity$

Kinetic theory (dilute gases): Momentum exchange by particle transport



$$\eta \sim n\,\bar{v}\,m\,\lambda$$

$n = density$
$\bar{v} = thermal\ velocity$
$\lambda = mean\ free\ path$

Viscosity ~ mean free path!

Viscosity

Navier Stokes Equation: $\rho\left[\frac{\partial\vec{v}}{\partial t}+(\vec{v}\vec{\nabla})\vec{v}\right]=-\vec{\nabla}p+\eta\nabla^2\vec{v}+\ldots$

"Inertia" "Force" "Friction"

	Viscosity [kg/m s]	Kinematic Viscosity $\nu=\frac{\eta}{\rho}$ [m^2 / s]
Water	0.001	0.10
Air	0.000018	1.5

The kinematic viscosity (friction/inertia) controls how good a fluid is

Minimum viscosity

AdS/CFT correspondence:

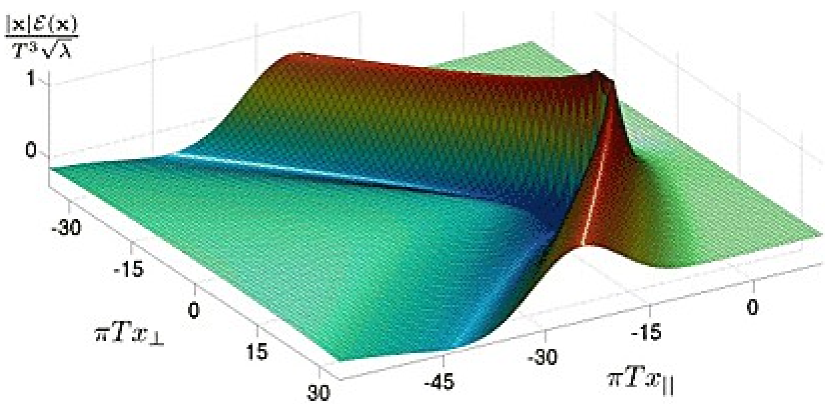
Maldacena et al, hep-th/9905111v3
Kovtun, Son, hep-th/0405231v2

$$\frac{\eta}{s} \geqslant \frac{1}{4\pi}$$

$\eta = shear\ viscosity$

$s = entropy\ density$

Holds for a large class of strongly coupled gauge theories



Kovtun, arXiv:0706.0368

Kinetic theory + waving hands:

$$\eta \sim n \bar{v}\, m\, \lambda$$

$$\bar{v}\, m = p\ , \quad p\lambda > \hbar, \quad n \sim s$$

$$\frac{\eta}{n} \sim \frac{\eta}{s} \geqslant 1$$

Quantum bound

More detailed derivation: Danielewcz and Gyulassy (85)

The perfect fluid?

- Is there a quantum bound on eta/s?
- Does the quantum bound provide a limit on fluidity?
- Has RHIC produced such a system? I assume so
- How about other substances
    - Water, Epplewoi, liquid Helium, cold quantum gases???

- How does one define fluidity?
- How do I compare systems on the atomic/molecular scale with those at quark/gluon scale?

Defining Fluidity

Hydrodynamics works for a big variety of systems:

- Liquids (Water)
- Gases (Air, sound)
- Interstellar Dust (Star formation)
- QGP ?

Problem: How to compare substances at vastly different length scales?

- Interstellar Dust: $n^{-1/3} \sim 10^{-4}$ m
- Water: $n^{-1/3} \sim 3\ 10^{-10}$ m
- Air : $n^{-1/3} \sim 3\ 10^{-9}$ m
- QGP : $n^{-1/3} < 10^{-15}$ m

Typical criterion for applicability of fluid dynamics:

Kinetic theory???

Knudsen Number : $\dfrac{Mean\ Free\ Path}{typical\ lengthscale\ of\ variation}$

Different length-scales???

Obviously not what we need

Defining Fluidity

1) Extract "effective mean free path" solely from fluid-dynamics
2) Calibrate with "inter-particle" distance

Effective mean free path:

Analyze sound modes and determine minimum wavelength

$$\omega = c_s k + \frac{i}{2} k^2 \frac{\frac{4}{3}\eta}{w/c^2}$$

Damping ~ k^2:
Hydro always works
in long wavelength limit

Enthalpy density:

$$w = \epsilon + p = Ts + \mu n \approx Ts + mn$$

$w \rightarrow mn$ Non-relativistic limit: **mass** density controls inertia

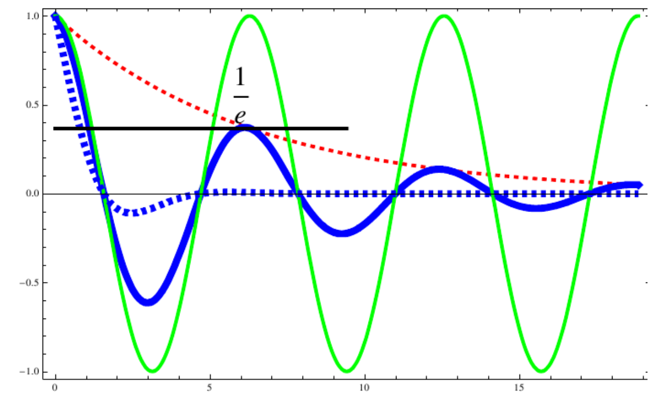
$w \rightarrow Ts$ Relativistic limit: entropy density controls inertia

$\frac{\eta}{s}$ cannot be a universal quality measure

Fluidity measure

Effective mean free path: Analyze sound modes

$$\omega = c_s k + \frac{i}{2} k^2 \frac{\frac{4}{3}\eta}{w/c^2}$$



Require: $$\frac{|\Im(\omega)|}{|\Re(\omega)|} \equiv \frac{L_\eta}{\Lambda} \ll 1$$

Provides a minimal wavelength $\Lambda = L_\eta$

Dilute (kinetic limit): $L_\eta \to \lambda_{mfp}$

$$L_\eta = \frac{\eta}{w\, c_s}$$

Enthalpy density

$$w = \epsilon + p = Ts + \mu n \approx Ts + m n$$

Fluidity measure

$$L_\eta = \frac{\eta}{w\, c_s}$$

Calibrate with "inter-particle distance" d:

$$d \Leftrightarrow \langle \epsilon(x) \epsilon(0) \rangle$$

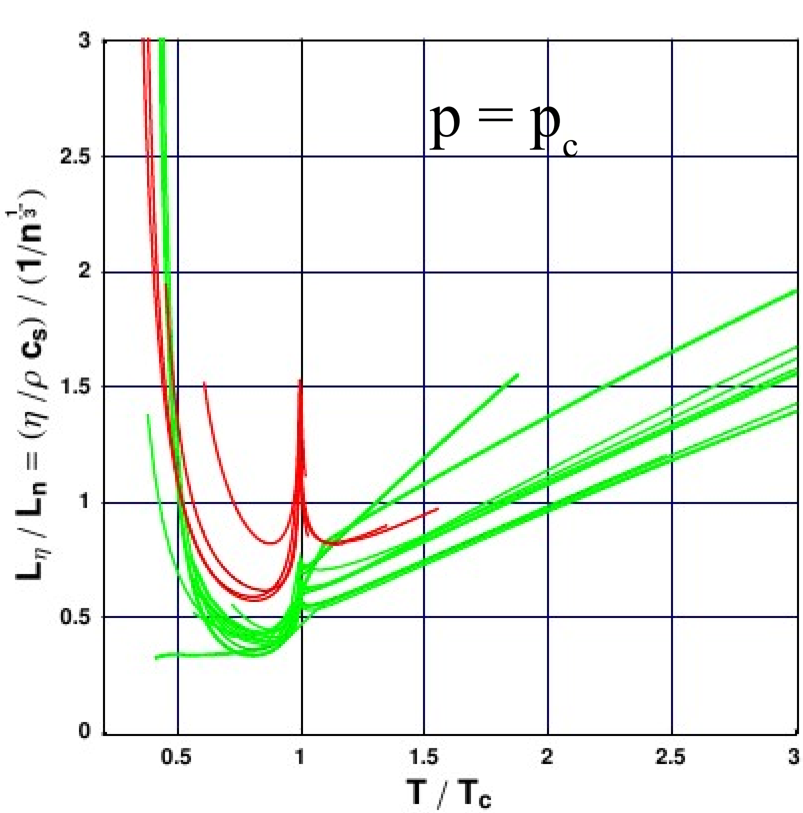
Non-relativistc systems $d = n^{(-1/3)}$

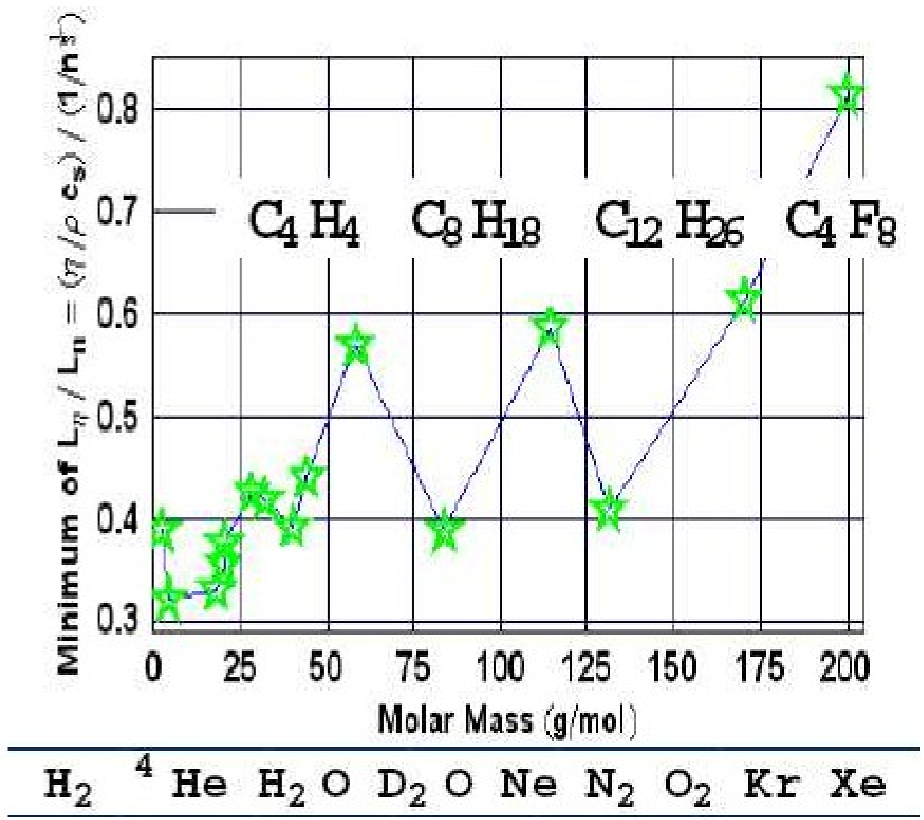
Fluidity measure: $$F = \frac{L_\eta}{d} = \frac{\eta}{w\, c_s} \frac{1}{d} = \frac{\eta}{w\, c_s} n^{1/3}$$

Depends only on *intrinsic* properties of substance
Well defined: NO kinetic theory needed!

Fluidity measure

16 substances with M_{mol}, T_c, p_c spanning 2 Orders of Magnitude

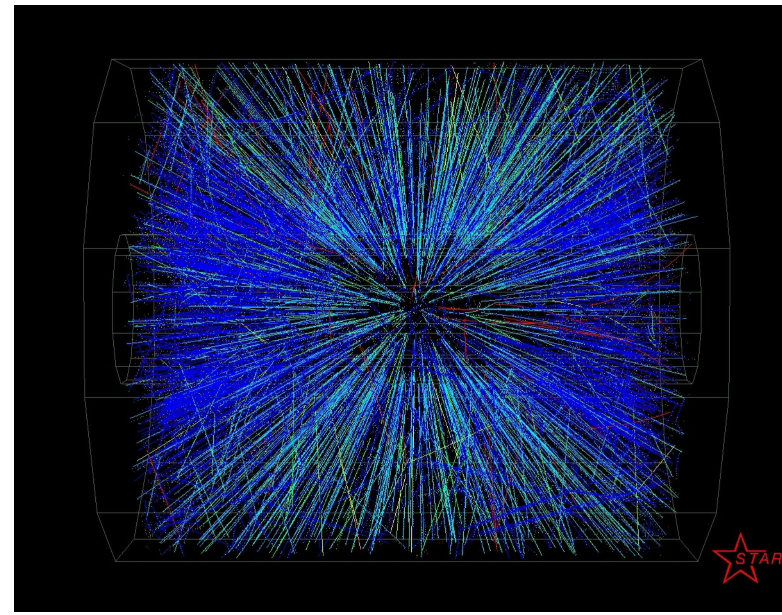




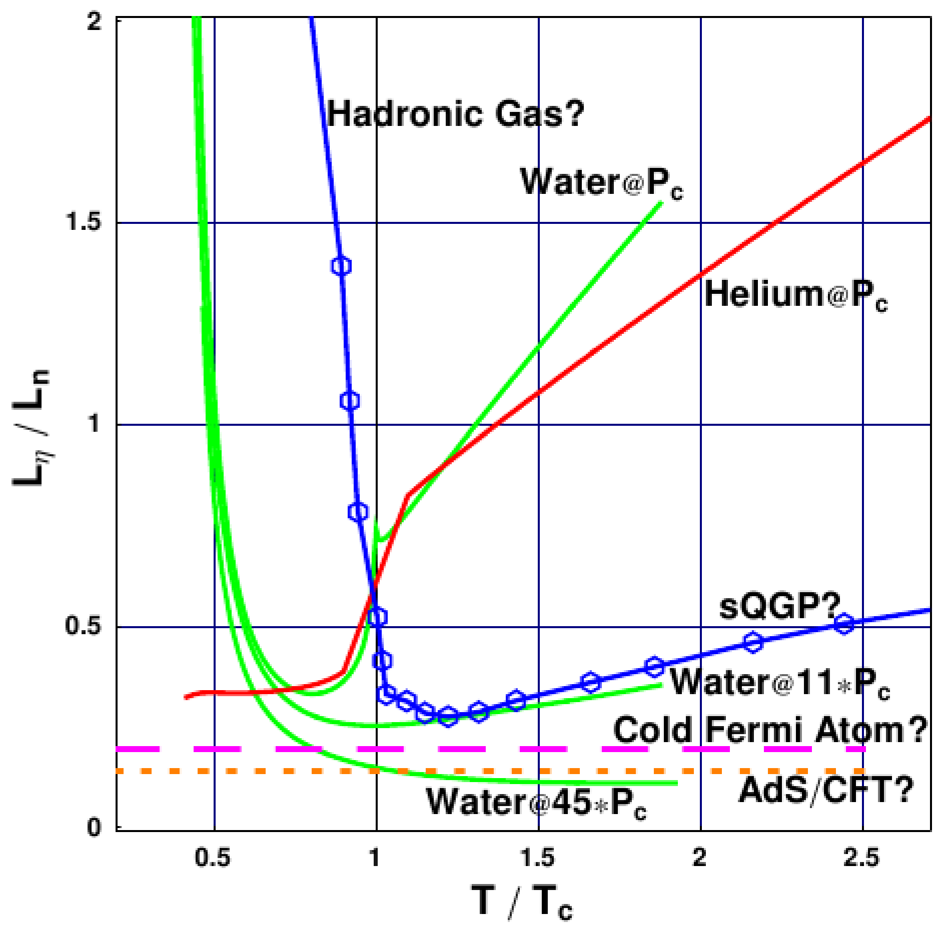
A good fluid is a good fluid!!!!!

So who is the winner?

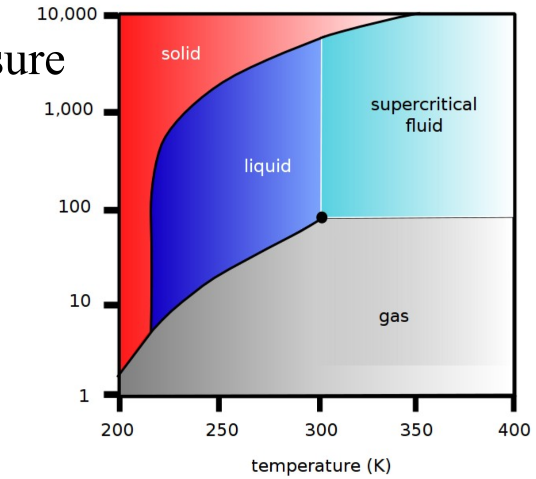
VS



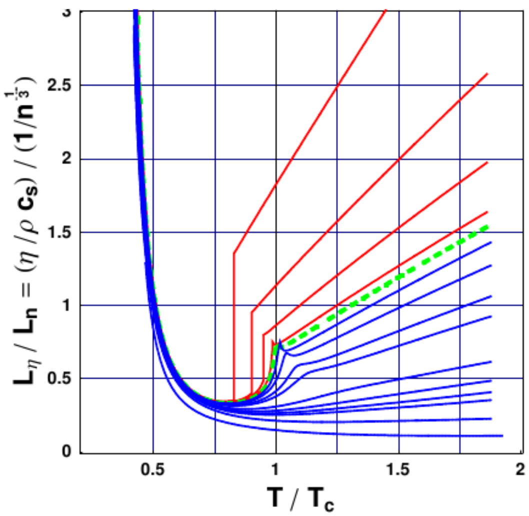
None of the above
Super-critical fluids!!!



pressure



CO_2



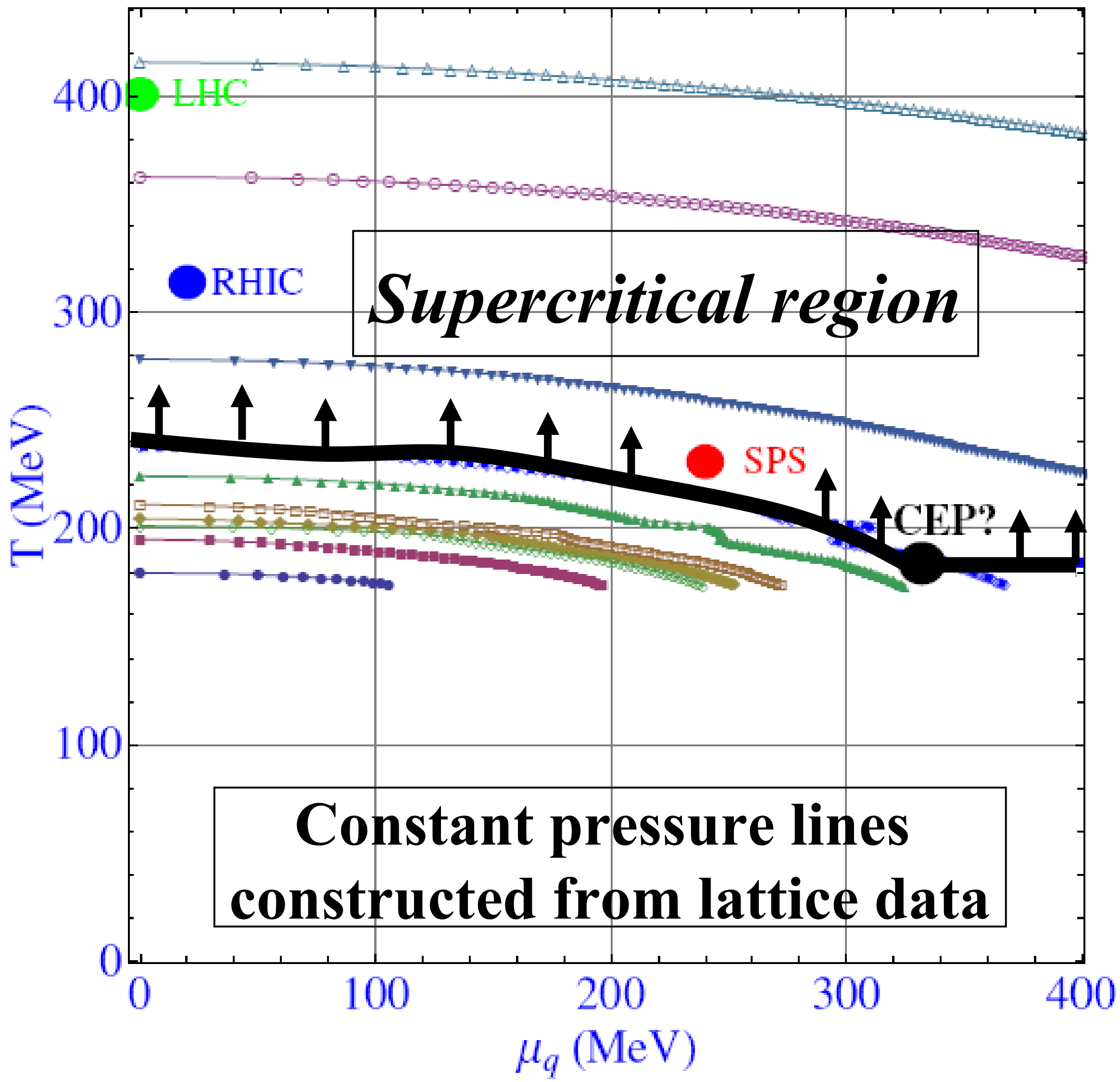
Water

Used in dry cleaning, decaffeinating coffee, ...

An the winner is...

Consequences for the QGP????

RHIC and the Dry-Cleaner

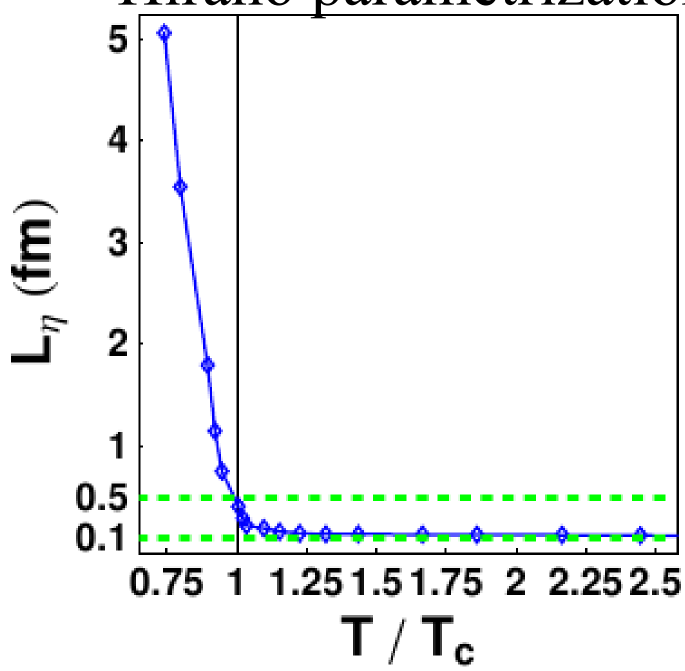


If there is a QCD critical point RHIC-QGP would be in Super critical region

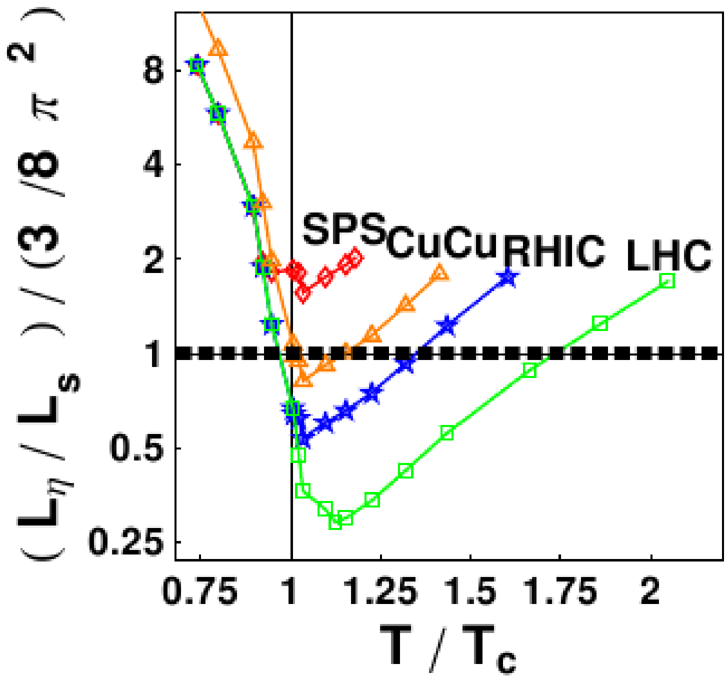
Predict: even better hydro behavior at LHC

Hydro Performance

Based on Gyulassy Hirano parametrization



Effective Knudsen Number



Simple 1-d expansion:

$$L_s = 2\tau \text{ for } \tau < R$$
$$L_s = 2R \text{ for } \tau > R$$
$$\tau_0 = 1 \text{ fm}$$

Viscosity comparison

Air: 2×10^{-5} Pa s
Water: 1×10^{-3} Pa s
Tar Pitch: 2×10^{8} Pa s
QGP: 2×10^{9} Pa s

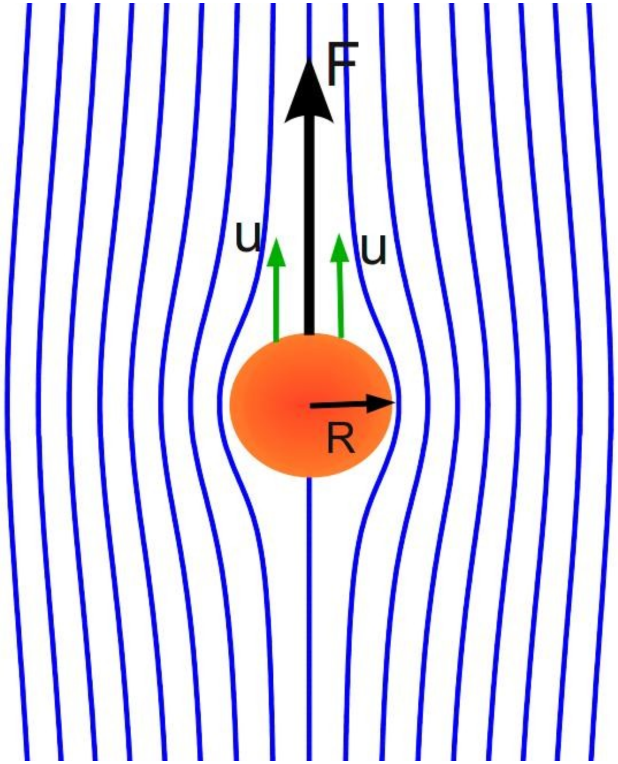
$$\eta_{\mathrm{QGP}} \simeq 10^{12}\, \eta_{\mathrm{Water}}$$

Ig-Nobel, 2005

Stirring the QGP

Stokes' Formula

$$F = 6\pi R u\,\eta$$



$$\eta_{QGP} \simeq 10^{12}\,\eta_{Water}$$

Rescale the sphere (spoon)

$$R_{QGP} \simeq 10^{-6} R_{Water}$$

$$F_{QGP} \simeq 10^{6} F_{Water}$$

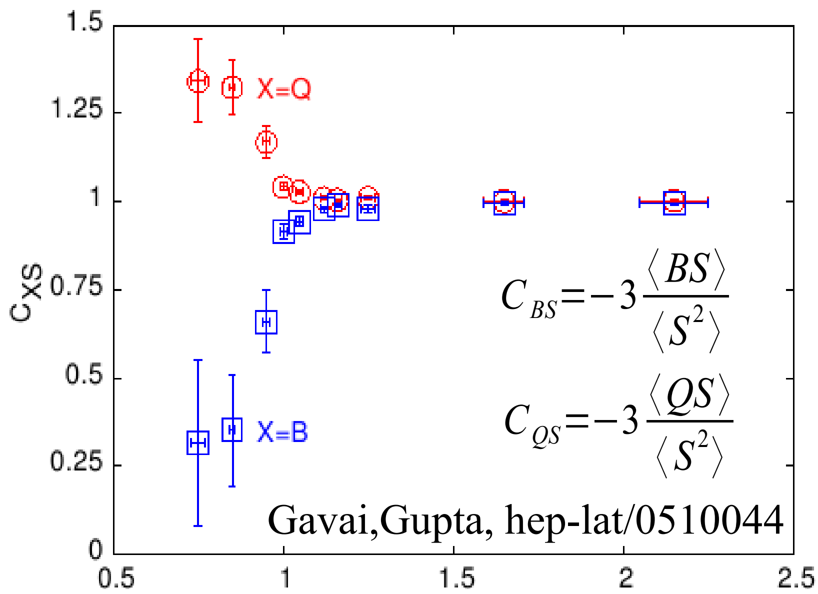
What have learned so far

- Universal Fluidity Measure $F = \frac{\eta}{w\, c_s} \frac{1}{d}$
- A good fluid is a good fluid
  - Gives F=0 for super-fluid component of ^{4}He !
- QGP nothing special
  - However, very sticky when stirred
- eta/s only meaningful for relativistic fluids without phase transition
- Supercritical fluids win the race
- QGP may be a supercritical fluid
  - Predict better hydro description at LHC

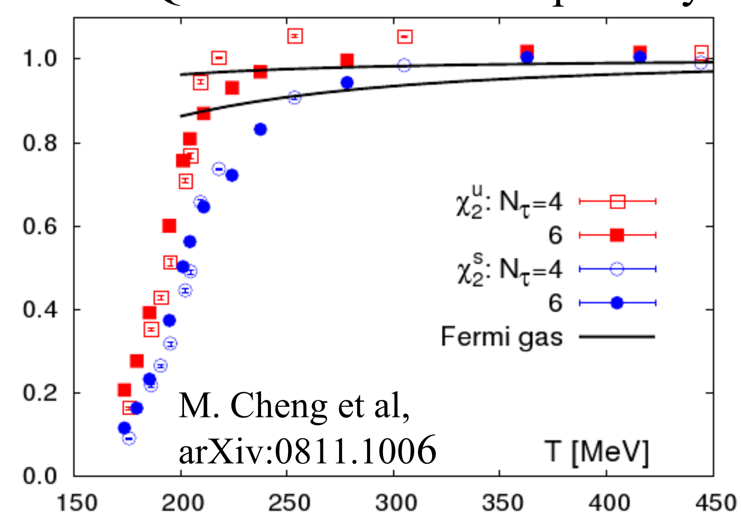
And what does Lattice QCD say?

- Mixed message
  - Shear viscosity difficult to extract (time like)
    - Requires analytic continuation to real time...
  - No or very weak correlations among quarks above T_c (weak coupling ?)
  - Some indications for liquid behavior in pure glue

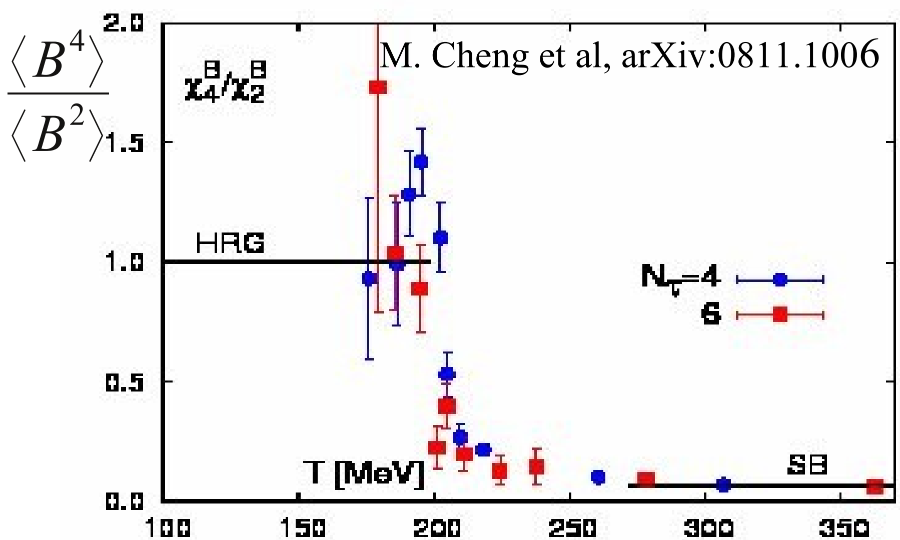
Evidence for uncorrelated flavor (quarks)



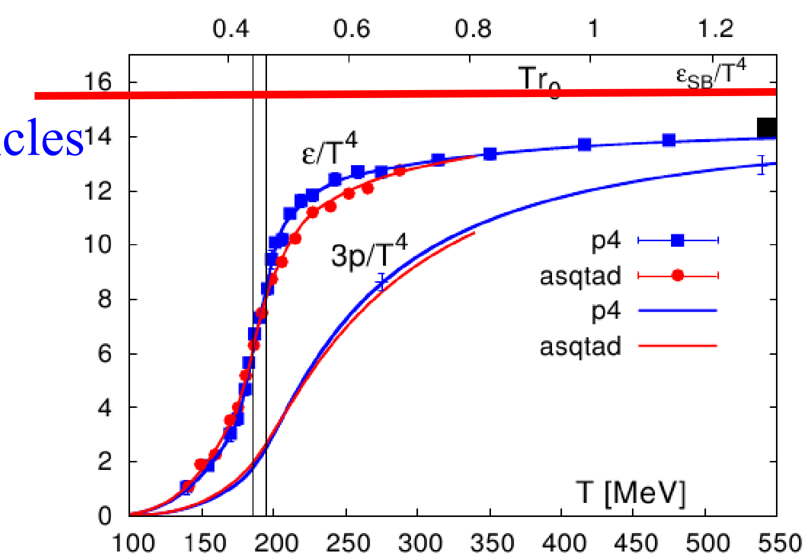
Quark Number susceptibility



Free particles



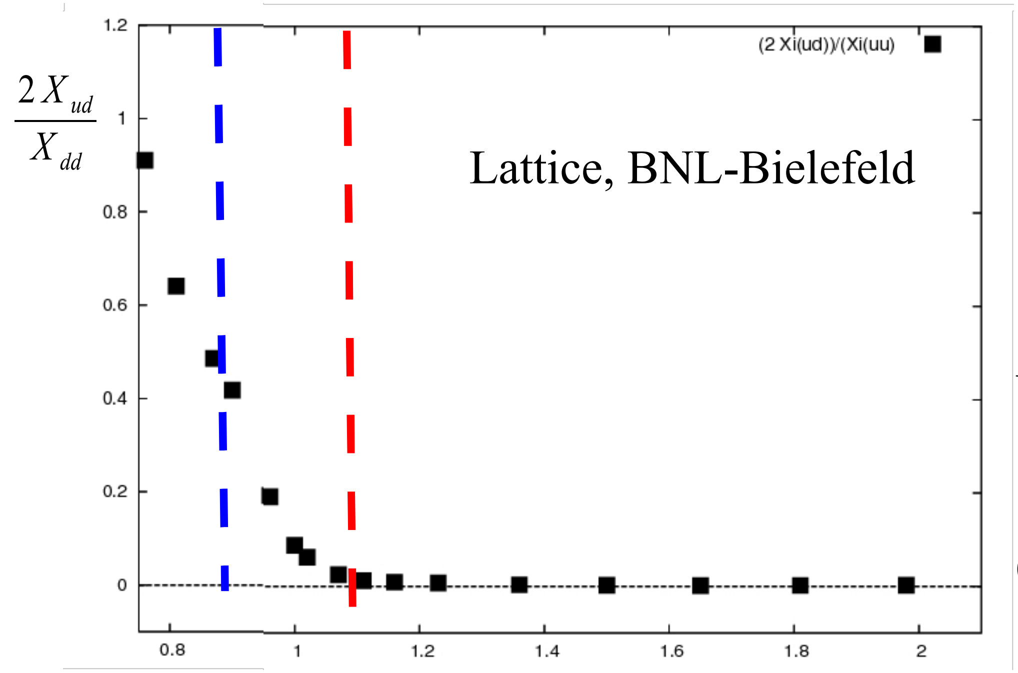
Free particles



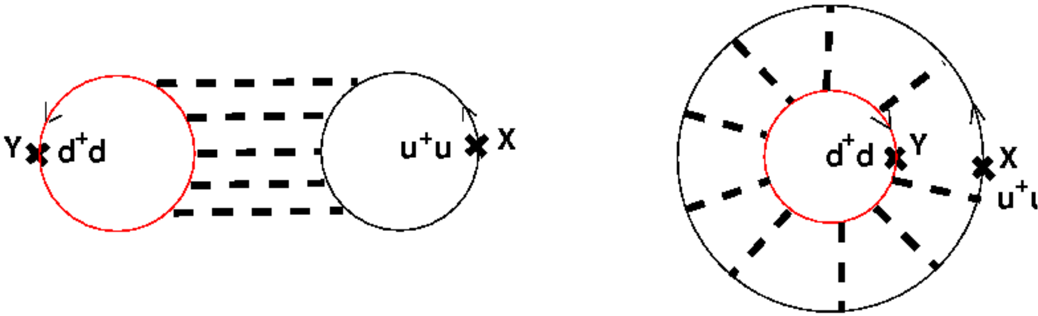
Correlations

off-diagonal Susceptibilities

$$\chi_{ud} = \langle (\delta u)(\delta d) \rangle = T^2 \frac{\partial^2}{\partial \mu_u \partial \mu_d} \log(Z) = -T \frac{\partial^2}{\partial \mu_u \partial \mu_d} F$$



1.1 T_c



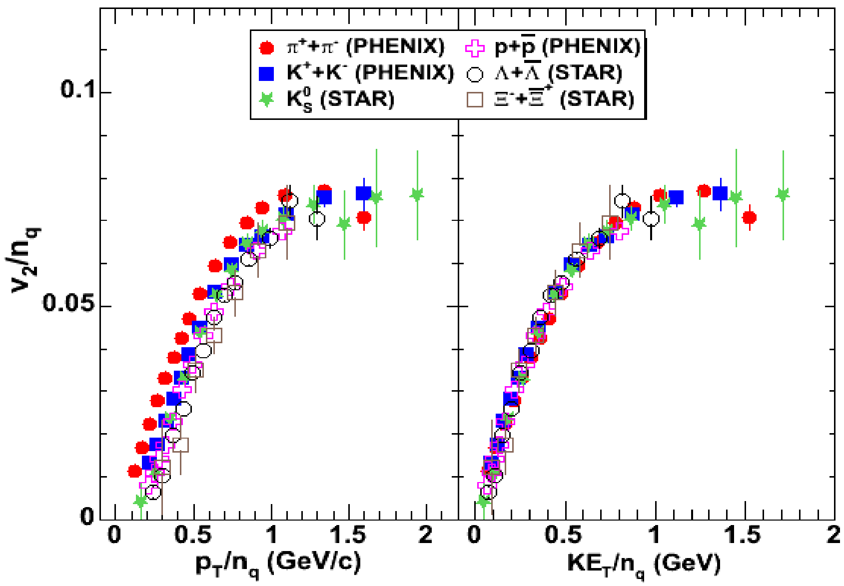
Weak Correlations above 1.1 T_c

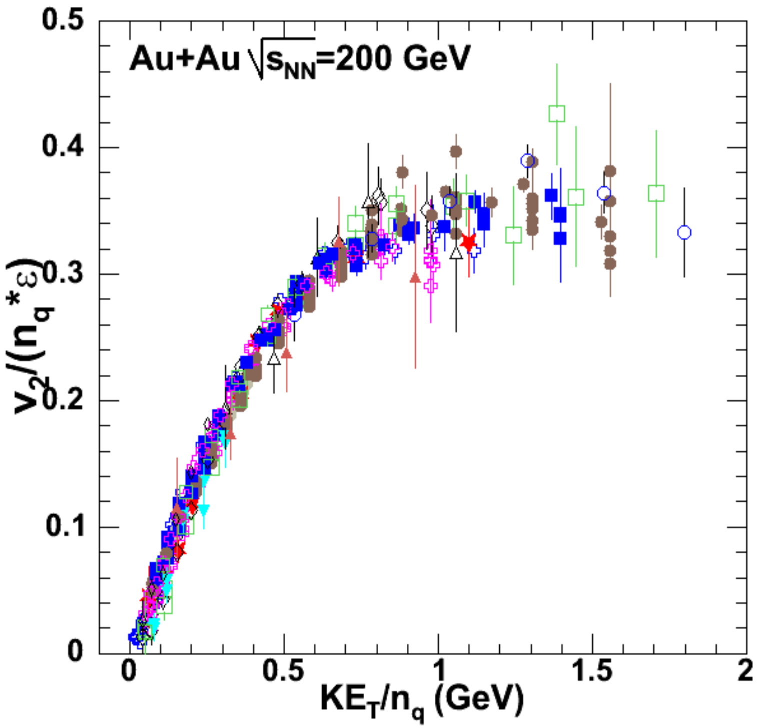
Consistent with hadron gas for $T<0.9\ T_c$

Consistent with quark number scaling of elliptic flow v_2

Elliptic Flow
(quark number scaling)

works even
for the pions???





Coalescence (recombination): $\frac{dN}{d\phi}(\bar{q}\,q)=\frac{dN}{d\phi}(\bar{q})\frac{dN}{d\phi}(q)\sim(1+2v_2\cos(\phi))(1+2v_2\cos(\phi))$

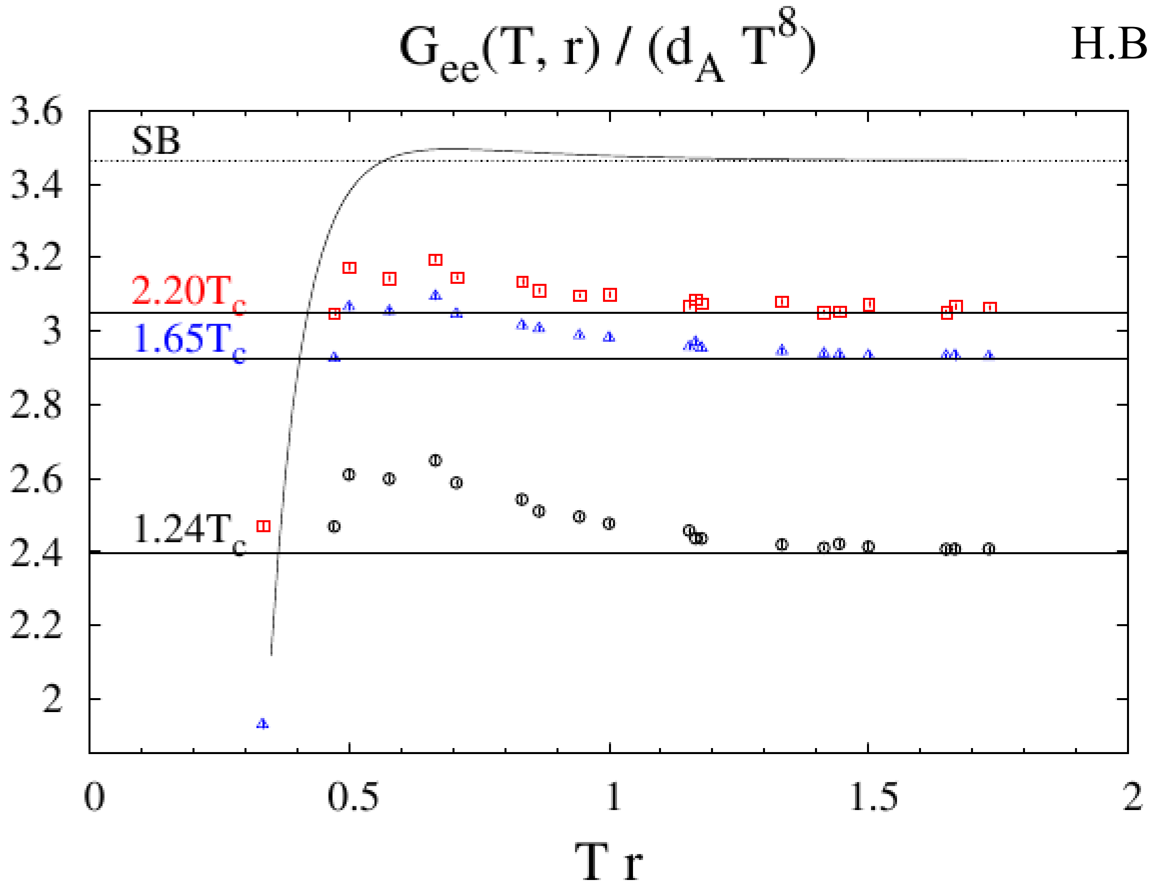
$$\sim(1+4\,v_2\cos(\phi))$$

$$\rightarrow v_2(\bar{q}\,q)=2\,v_2$$

$v_2=n_q\,v_2^{\,0}$
quark number scaling

Density-Density correlator in pure Glue

H.B. Meyer, arXiv:0808.1950



Not inconsistent with liquid like behaviour...

Summary

- The RHIC fluid
  - Fluidity of the RHIC “fluid” is nothing special
    - However very sticky when stirred
  - A good fluid is a good fluid
  - Is RHIC fluid in the super-critical regime?
    - If so, hydro should work even better at LHC

  - Lattice input
    - Very weak flavor correlations just above T_c .
      - Strong coupling???
    - Difficult to extract viscosity
    - Density-Density correlator for pure glue show some structure

The End

Things I don't understand

- Strong coupling vs. independent flavors as seen on Lattice and in v_2-scaling?
  - Repulsive force would do!
  - But QGP is compressible?
- Why do we see quark-number susceptibilities to be Stefan-Boltzmann right above T_c?
  - Do Quarks decouple?
  - If so, how come we see chemical equilibrium?
  - Where do the quarks come from if we start from gluons?

How many of these observations can be understood within AdS/CFT????

Rescale

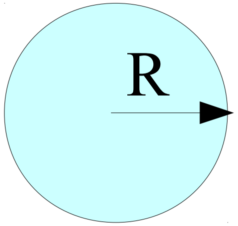
$$F = 6\pi R u\, \eta$$

$$R = \tilde{R}\, d \qquad u = \tilde{u} c_s \qquad F_\eta = \frac{\eta\, d}{w c_s}$$

$$F = 6\pi \frac{\tilde{u}}{\tilde{R}} F_\eta \; (c_s^2 R^2 w) \qquad \frac{F}{F_0} = 6\pi \frac{\tilde{u}}{\tilde{R}} F_\eta$$

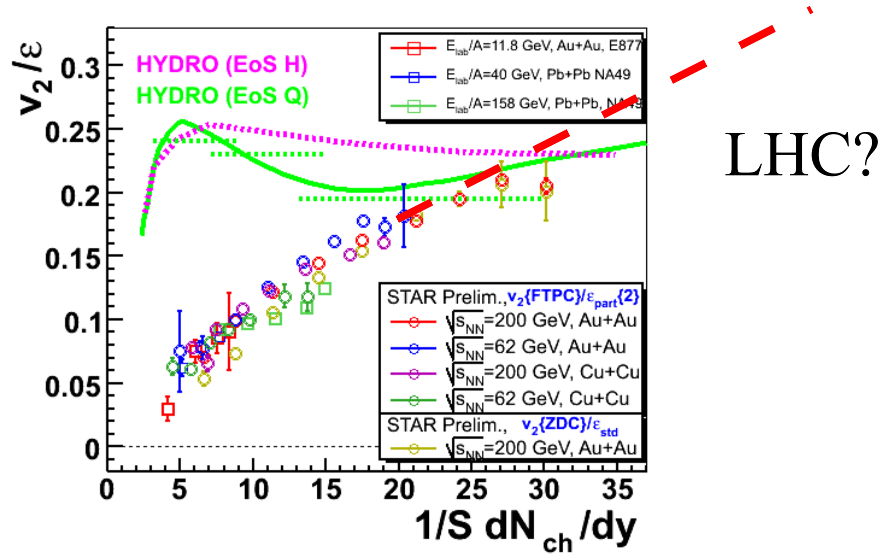
$$F_0 \equiv (c_s^2 R^2 w)$$

Force needed to accelerate sphere of radius R filled with substance to speed of sound over distance R



How to make progress

- Multiplicities, Elliptic Flow, viscosity, recombination
  - Vary energy: LHC and RHIC

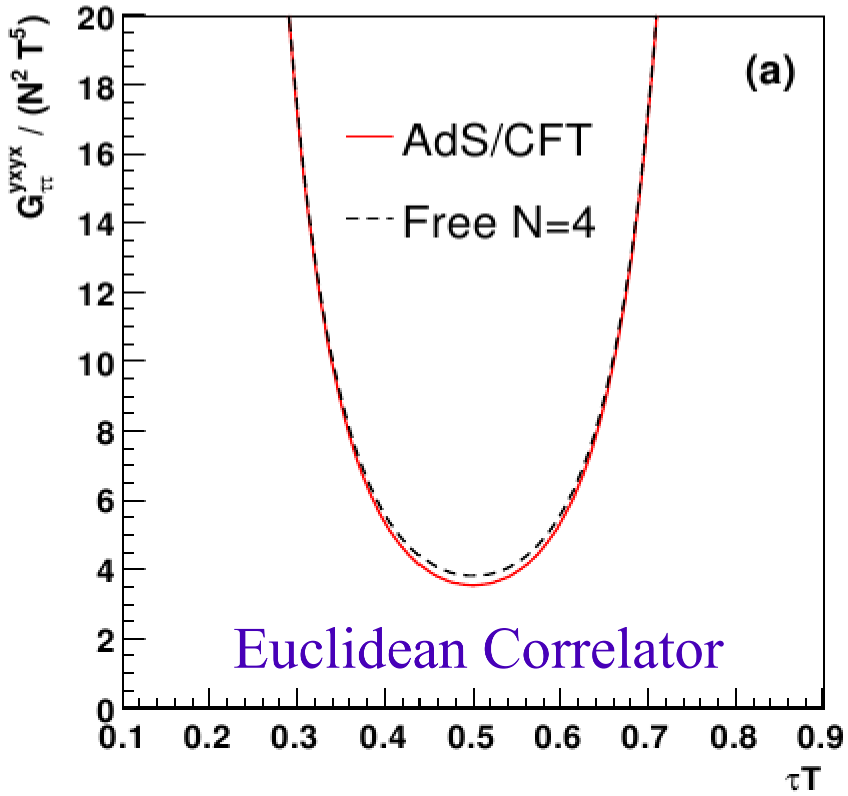


Euclidean Space Correlator

$$\eta = -\frac{d}{d\omega} Im\Pi^R|_{\omega=0^+} \qquad \Pi^R \sim \int e^{-i\omega t}\langle[T_{xy}(t),T_{xy}(0)]\rangle$$

Viscosity results from time-like correlator

$$\langle T_{xy}(\tau)T_{xy}(0)\rangle$$



D. Teaney
Phys.Rev.D74:045025,2006.
hep-ph/0602044

Difficult to distinguish between FREE and Strongly interacting in Euclidean Space

Quantum bound....

Super Fluids:
Most definitely quantum

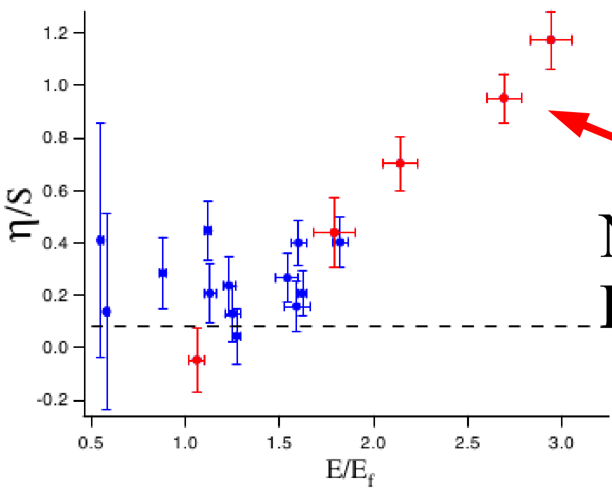
$$\eta \rightarrow 0, \quad s \rightarrow 0\,; \quad \frac{\eta}{s} \rightarrow ??$$

AdS/CFT

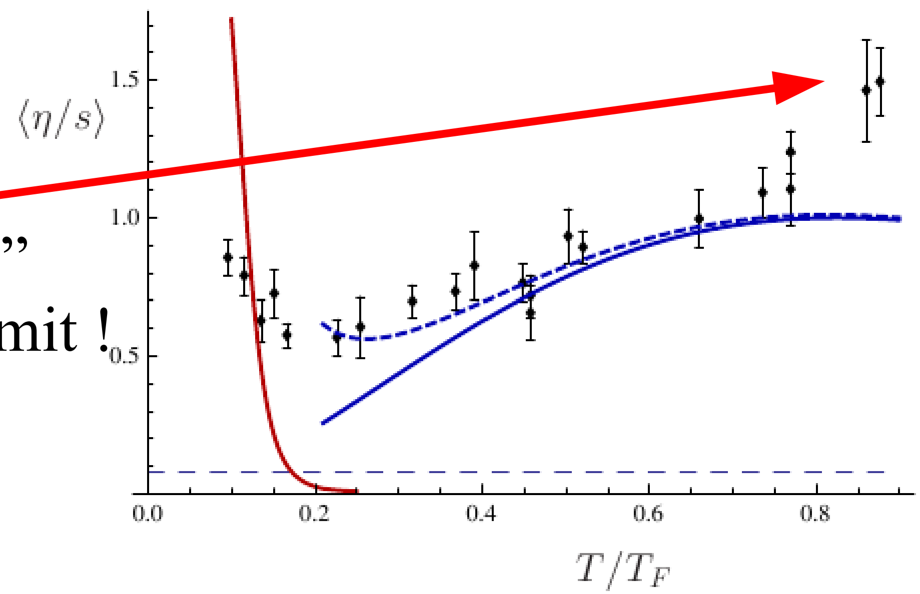
$$\eta \rightarrow \infty, \quad s \rightarrow \infty\,; \quad \frac{\eta}{s} \rightarrow \frac{1}{4\pi}$$

Where is the "quantum" at T=250 MeV????

Cold (Unitary) Gases:



Not "quantum"
But Unitary limit !



J.E. Thomas, QM09

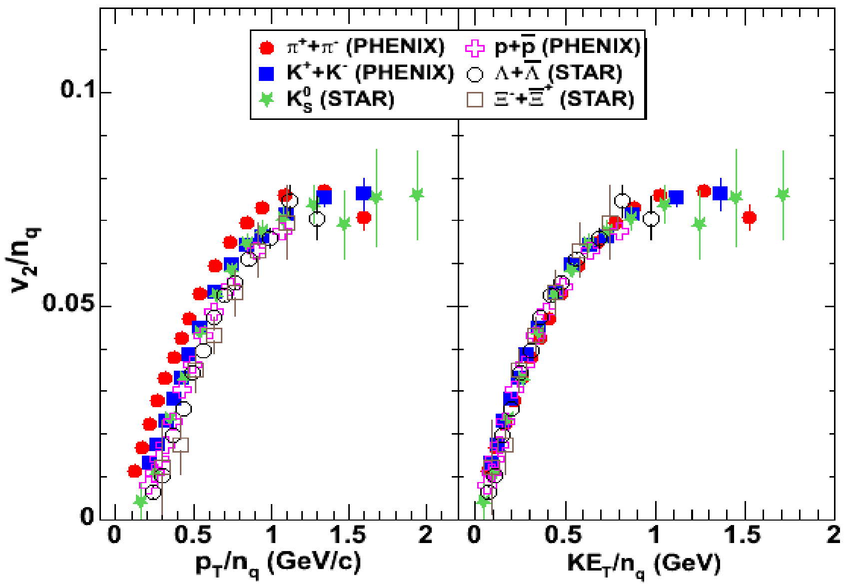
T. Schäfer, Phys. Rev. A 76, 063618 (2007)

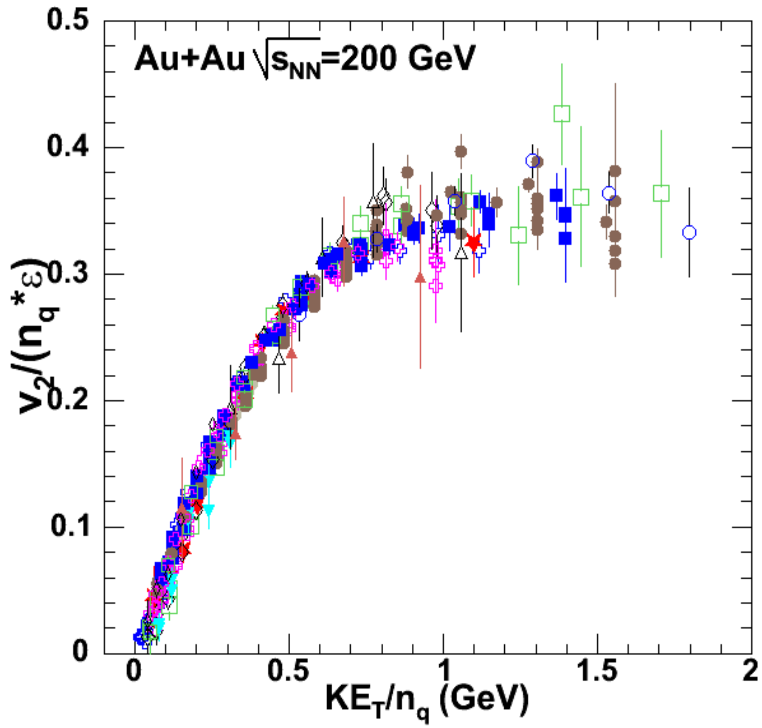
Reynolds Number

$$Re = \frac{\rho u R}{\eta} = \frac{w\tilde{u}\tilde{R}c_s d}{\eta} = \frac{\tilde{u}\tilde{R}}{F_\eta}$$

Elliptic Flow
(quark number scaling)

works even
for the pions???





Coalescence (recombination): $\frac{dN}{d\phi}(\bar{q}q) = \frac{dN}{d\phi}(\bar{q})\frac{dN}{d\phi}(q) \sim (1+2v_2\cos(\phi))(1+2v_2\cos(\phi))$

$$\sim (1+4v_2\cos(\phi))$$

$$\to v_2(\bar{q}q) = 2v_2$$

$v_2 = n_q v_2^0$
quark number scaling

Deconfinement??