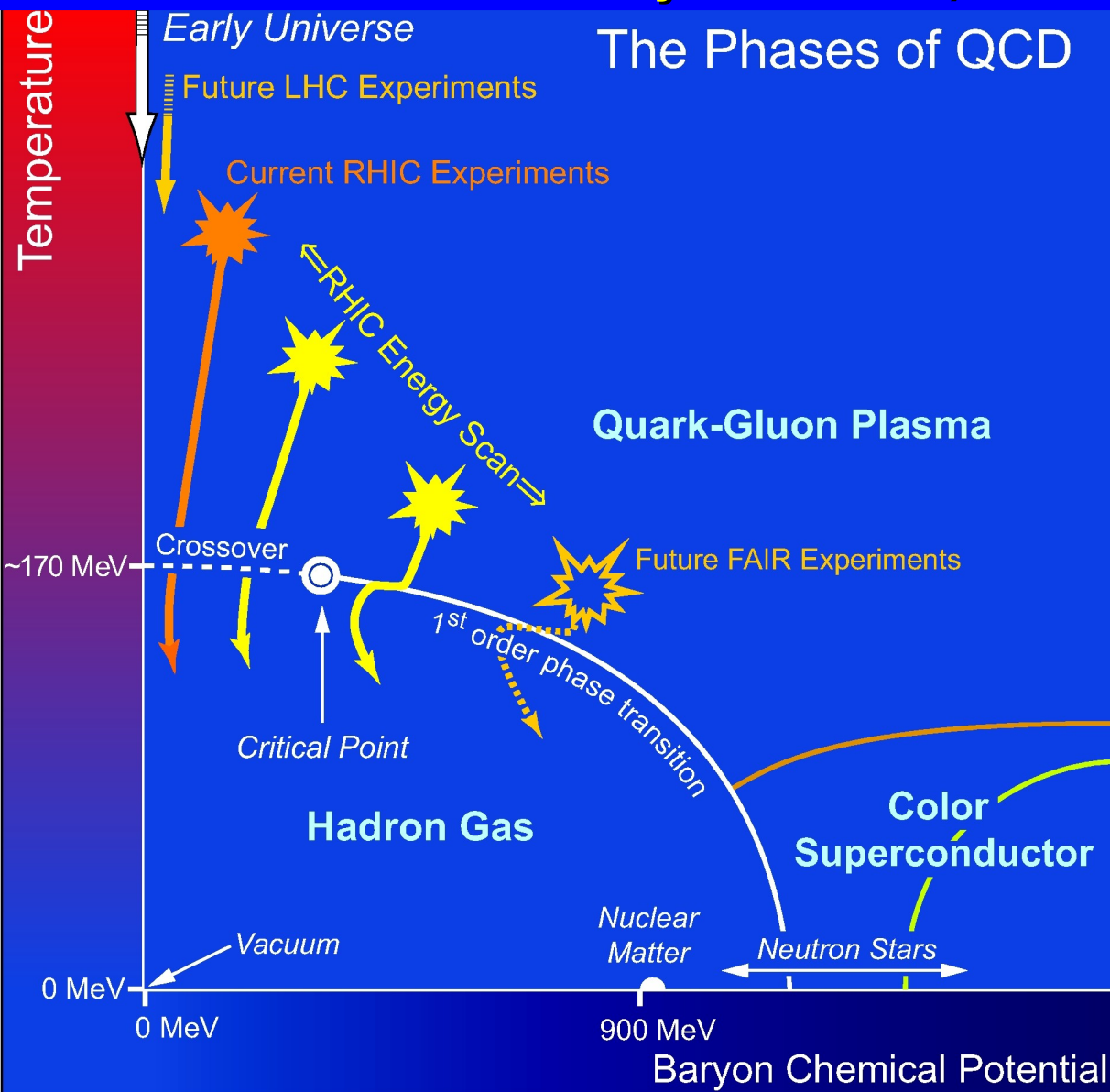


The QCD phase diagram for small values of the chemical potential

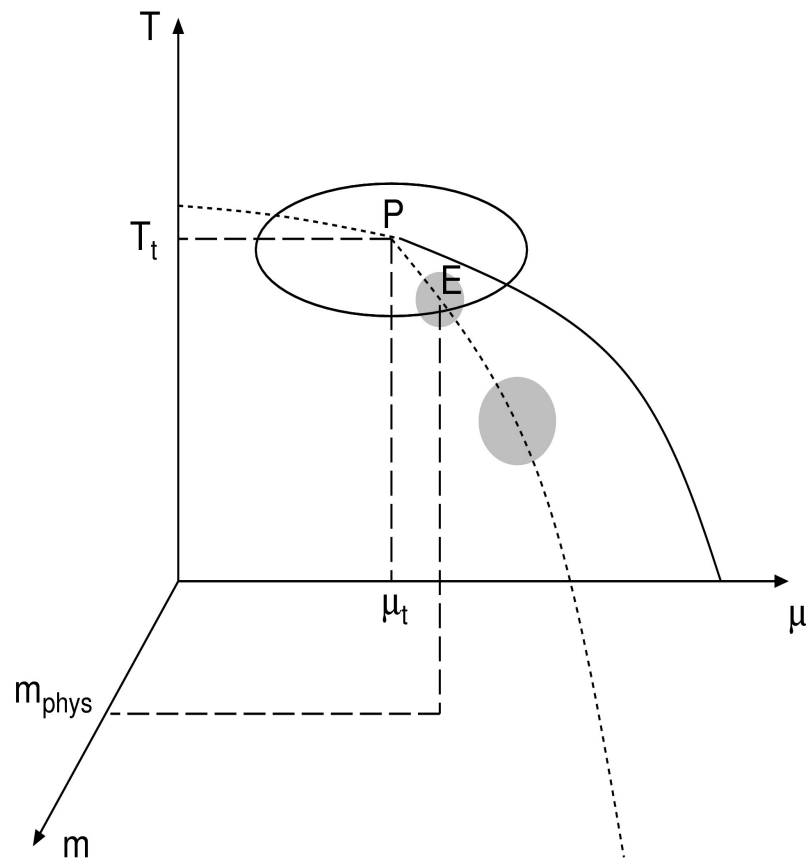
Frithjof Karsch, BNL&Bielefeld



OUTLINE:

- QCD phase diagram close to the chiral limit
- QCD close to the chiral limit, $O(N)$ scaling
- Curvature of the critical line **in the chiral limit**

Phase diagram for $\mu_B \geq 0, m_q \geq 0$



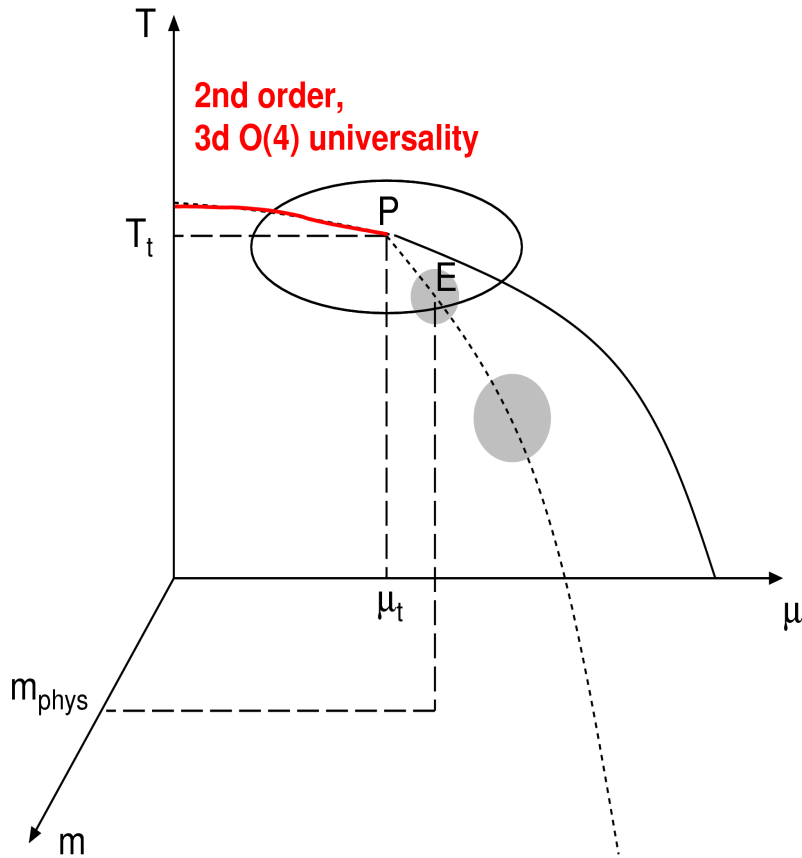
critical line at $m_q=0$

physics on crossover line controlled by universal scaling relations ?

Is it related to the experimentally determined freeze-out curve?

Y. Hatta, T. Ikeda, PRD27,014028 (2003)

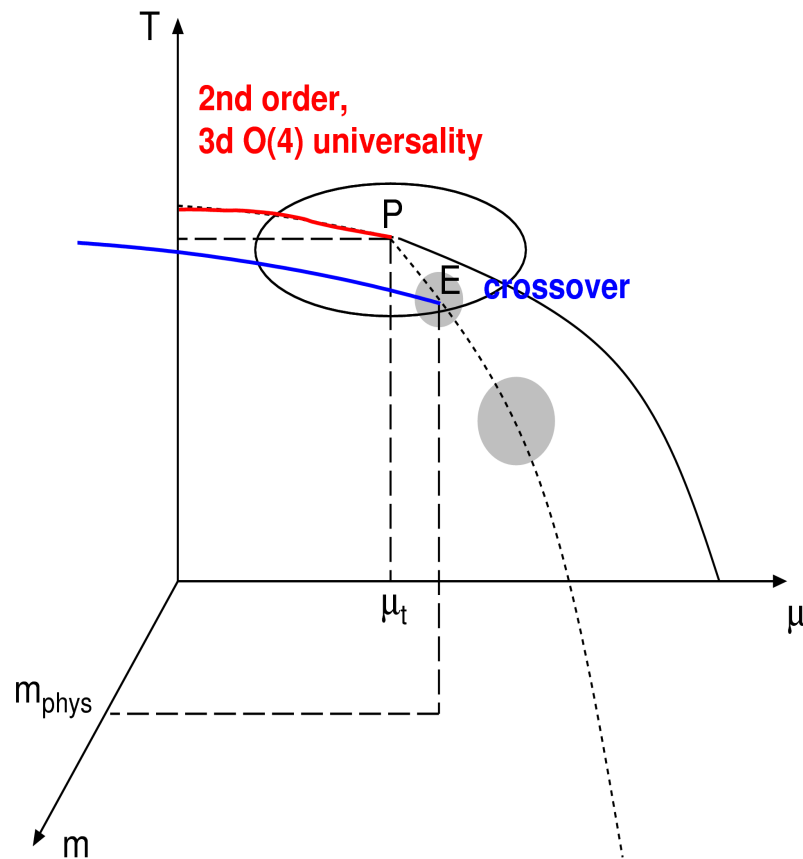
Phase diagram for $\mu_B \geq 0$, $m_q = 0$



- ❖ Is it related to the experimentally determined freeze-out curve?

Y. Hatta, T. Ikeda, PRD27,014028 (2003)

Phase diagram for $\mu_B \geq 0, m_q > 0$



◆ critical line at $m_q=0$

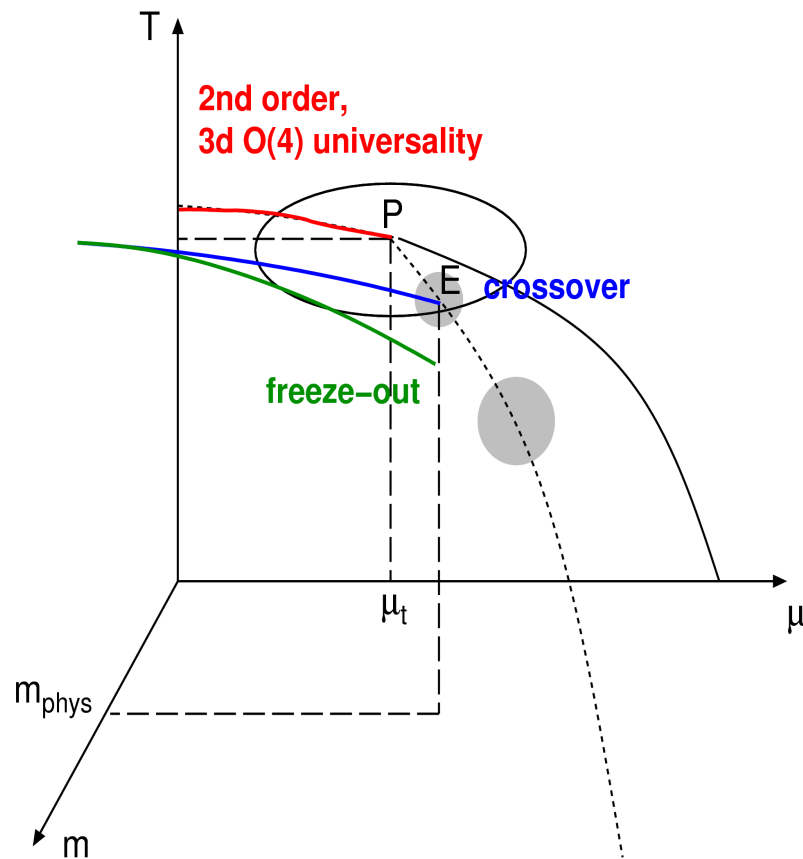
$$\frac{T_c(\mu_q)}{T_c} = 1 - \kappa_q \left(\frac{\mu_q}{T} \right)^2 - \mathcal{O}(\mu_q^4)$$

◆ physics on crossover line controlled by universal scaling relations ?

◆ Is it related to the experimentally determined freeze-out curve?

Y. Hatta, T. Ikeda, PRD27,014028 (2003)

Phase diagram for $\mu_B \geq 0, m_q > 0$



critical line at $m_q=0$

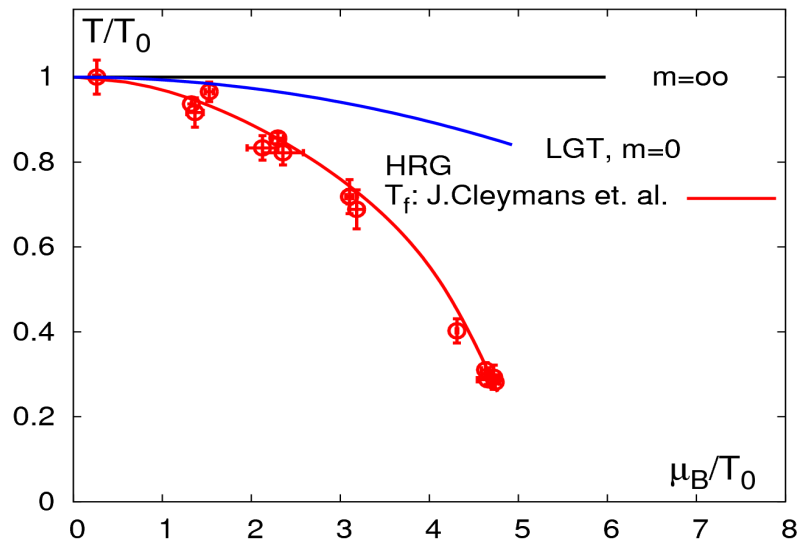
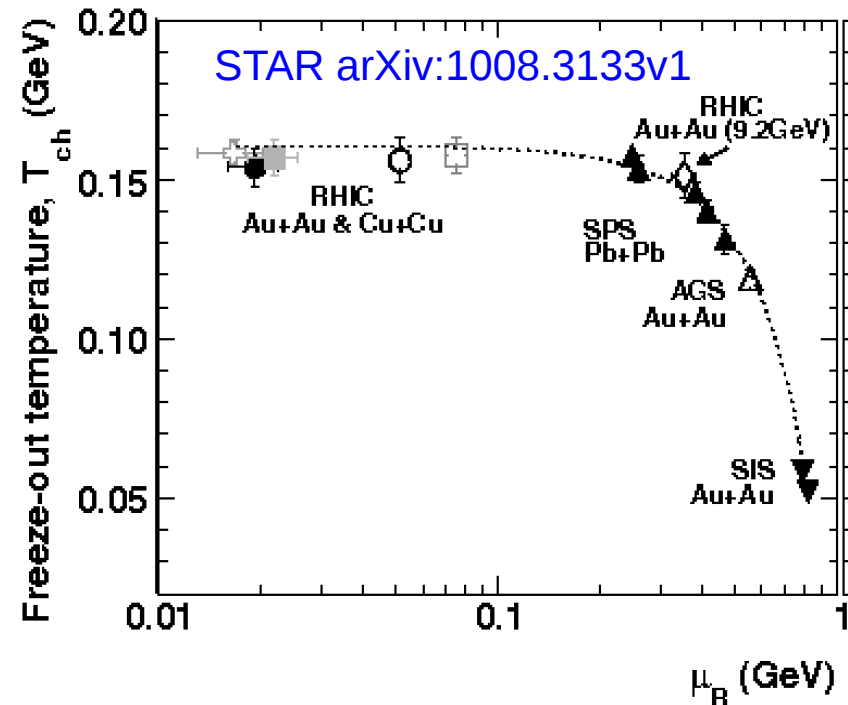
$$\frac{T_c(\mu_q)}{T_c} = 1 - \kappa_q \left(\frac{\mu_q}{T} \right)^2 - \mathcal{O}(\mu_q^4)$$

physics on crossover line controlled by universal scaling relations ?

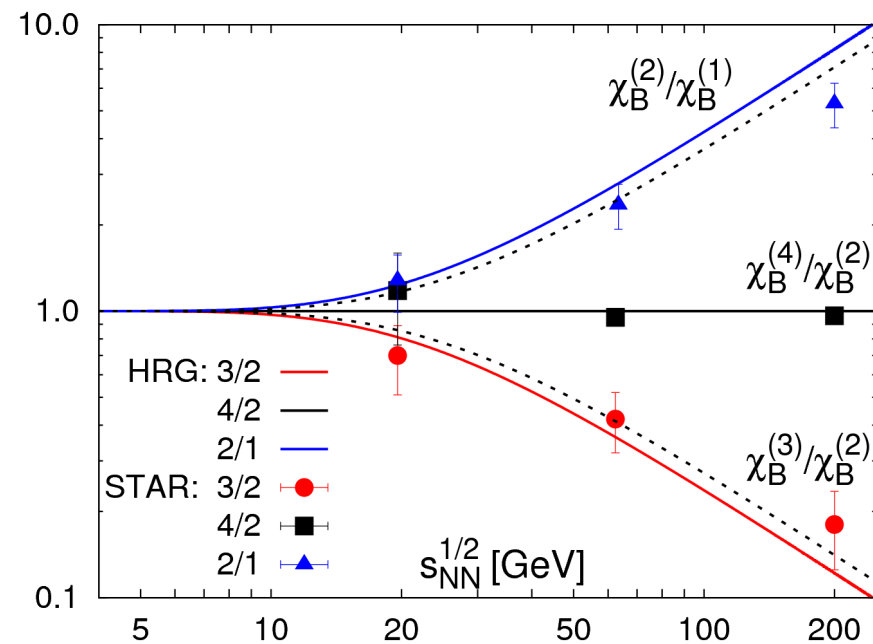
Is it related to the experimentally determined freeze-out curve?

Y. Hatta, T. Ikeda, PRD27,014028 (2003)

The RHIC low energy runs



Moments of charge fluctuations



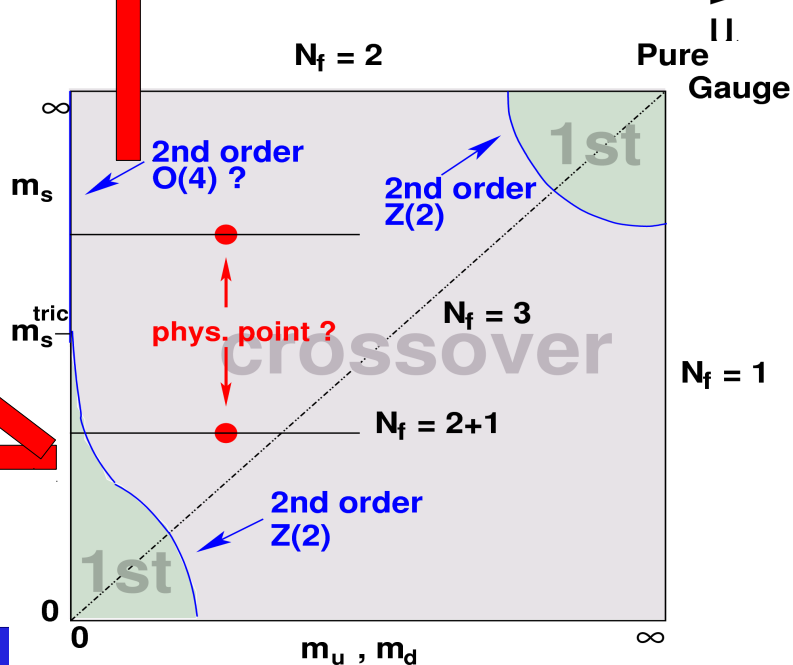
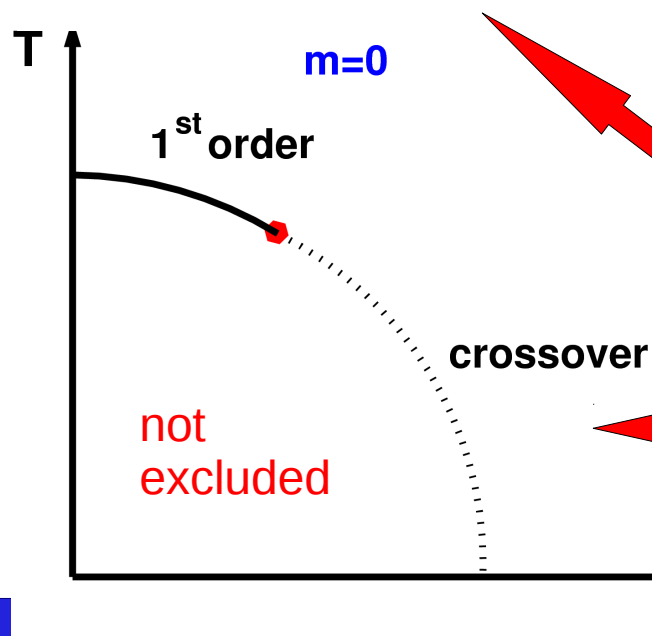
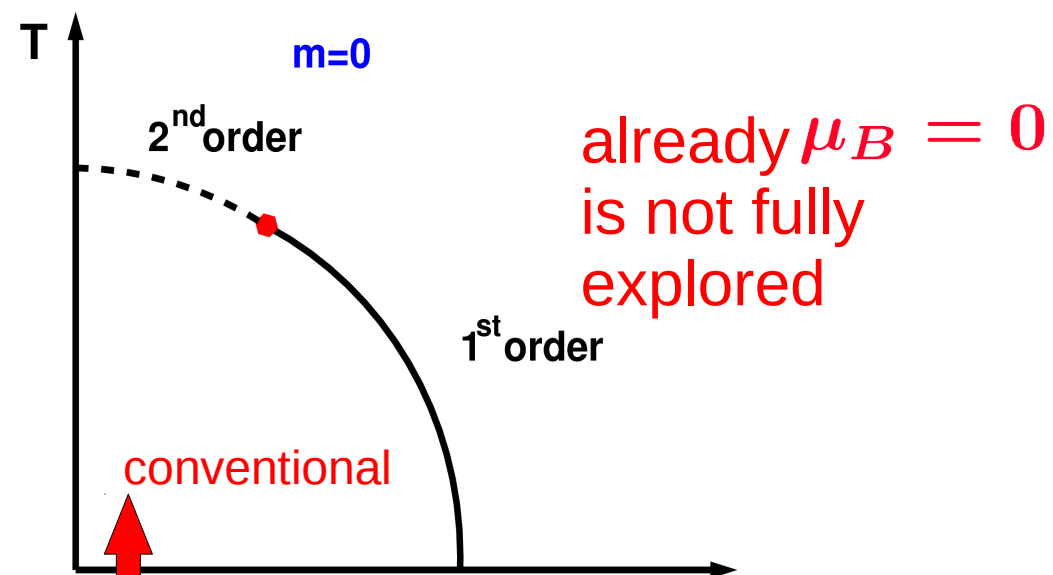
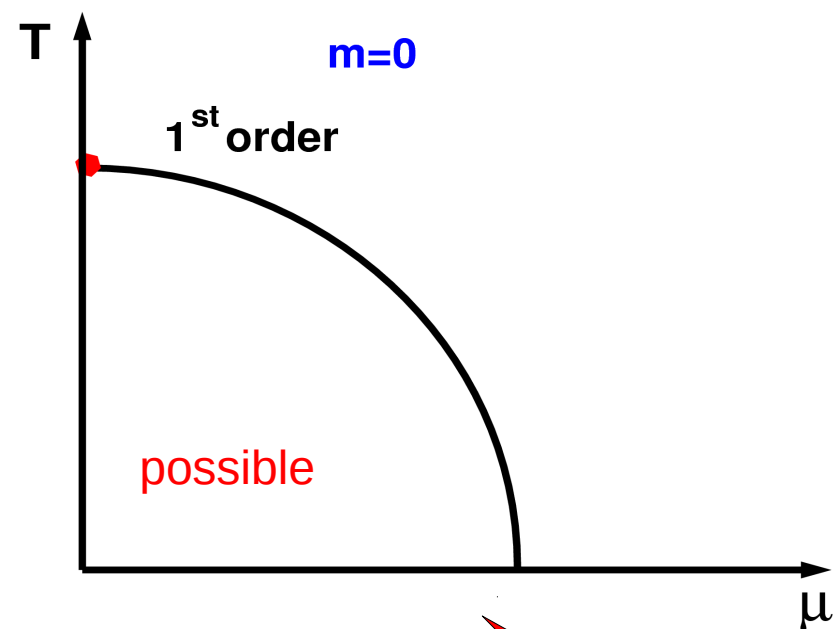
FK, K Redlich, arXiv:1007.2581
data from STAR: arXiv:1004.4959

charge fluctuations at freeze-out agree
with HRG model predictions

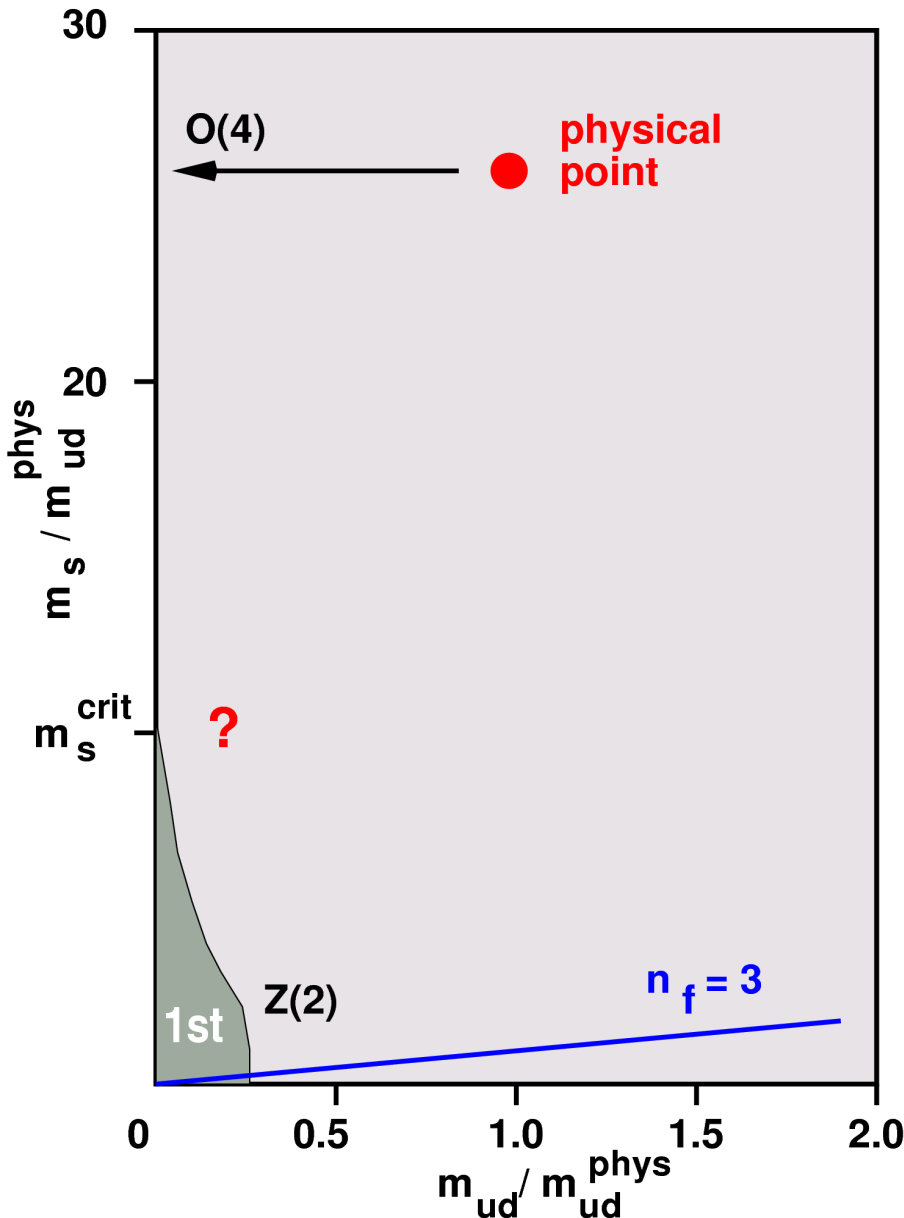
freeze-out line and chiral phase transition
BNL-Bielefeld-GSI, arXiv:1011.3130

Critical behavior in hot and dense matter

QCD phase diagram & chiral limit



Phase diagram for $\mu_B = 0$



◆ drawn to scale

Is physics at the physical quark mass point sensitive to (universal) properties of the chiral phase transition?

physical point may be above m_s^{crit}

$n_f = 3 : m_{\pi}^{crit} \lesssim 70 \text{ MeV}$

first order region starts below physical pion mass value

O(N) scaling and chiral transition

- thermodynamics in the vicinity of a critical point:

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln Z(V, T) = h^{1+1/\delta} f_s(t/h^{1/\beta\delta}) + f_r(V, T)$$

- critical behavior controlled by two relevant fields: t , h

- all couplings that do not explicitly break chiral symmetry contribute in leading order only to 't'

$$t = \frac{1}{t_0} \left(\left(\frac{T}{T_c} - 1 \right) - \kappa_q \left(\frac{\mu_q}{T} \right)^2 - \kappa_s \left(\frac{\mu_s}{T} \right)^2 - \kappa_{qs} \frac{\mu_q \mu_s}{T^2} \right)$$

$$h = \frac{1}{h_0} \frac{m_l}{m_s}$$

3 unique scales at vanishing chemical potential

3 additional, unique scales at non-vanishing chemical potential: curvature of the critical surface

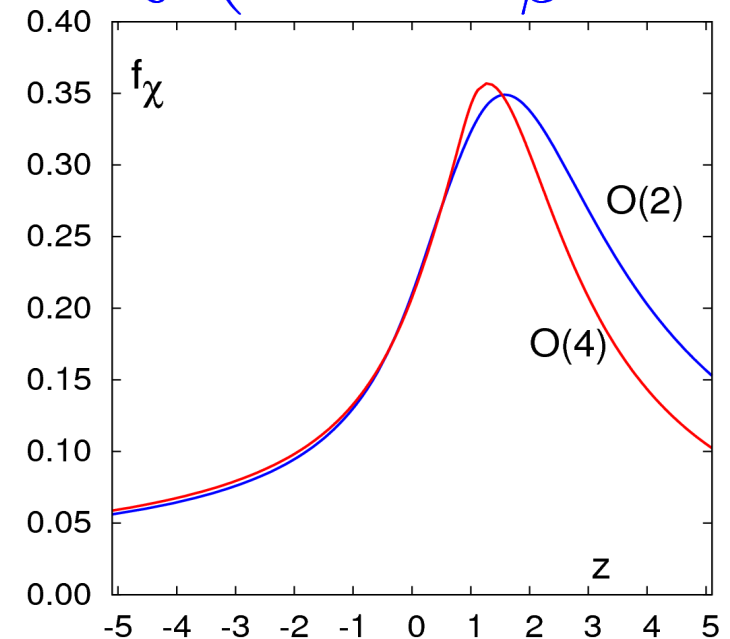
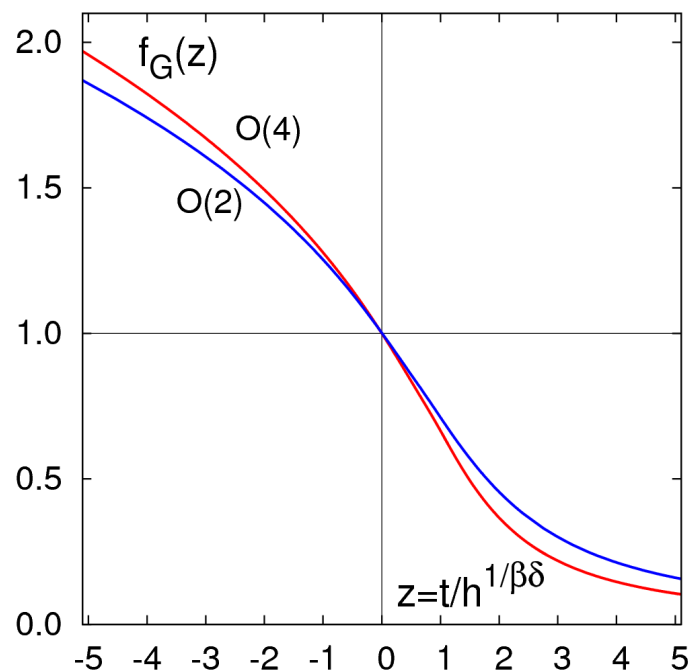
O(N) scaling and the chiral transition

- ◆ In the vicinity of $(t,h)=(0,0)$ the chiral order parameter and its susceptibility are given in terms of scaling functions

$$M = h^{1/\delta} f_G(z) \quad , \quad \chi_M = \partial M / \partial h = h^{1/\delta-1} f_\chi(z)$$

$$\chi_t = \partial M / \partial T = \frac{1}{t_0 T_c} h^{(\beta-1)/\delta\beta} f'_G(z)$$

$$f_\chi(z) = \frac{1}{\delta} \left(f_G(z) - \frac{z}{\beta} f'_G(z) \right)$$



known from 3d O(N) spin model

J. Engels et al., 2001/2003

Scaling analysis in (2+1)-flavor QCD

RBC-Bielefeld-GSI arXiv:0909.5122; hotQCD in preparation

QCD with 2 light and a “physical” strange quark mass:

- improved staggered fermions; most detailed: p4-action
extended to asqtad and hisq

- calculations have been performed on $N_\sigma^3 \times N_\tau$ lattices

$$N_\sigma = 32, 48, N_\tau = 4, 6, 8$$

- calculations with p4-action cover a wide quark mass range:

$$N_\tau = 4 : \quad 1/80 \leq m_l/m_s \leq 2/5$$

$$\Rightarrow \quad 75 \text{ MeV} \leq m_\pi \leq 320 \text{ MeV}$$

➡ evidence for O(N) scaling
expect O(2) rather than O(4) scaling with staggered fermions at non-zero lattice spacing

Goldstone modes at finite T

scaling laws at work

- ◆ In the symmetry broken phase **Goldstone modes** give the dominant contribution to the scaling functions and lead to a **divergent susceptibility** in the chiral limit

$$T < T_c \Leftrightarrow z < 0 :$$

$$\langle \bar{\psi}\psi \rangle \sim f_G(z) \quad : \quad f_G^\infty(z) = (-z)^\beta (1 + \tilde{c}_2 \beta (-z)^{-\beta\delta/2})$$

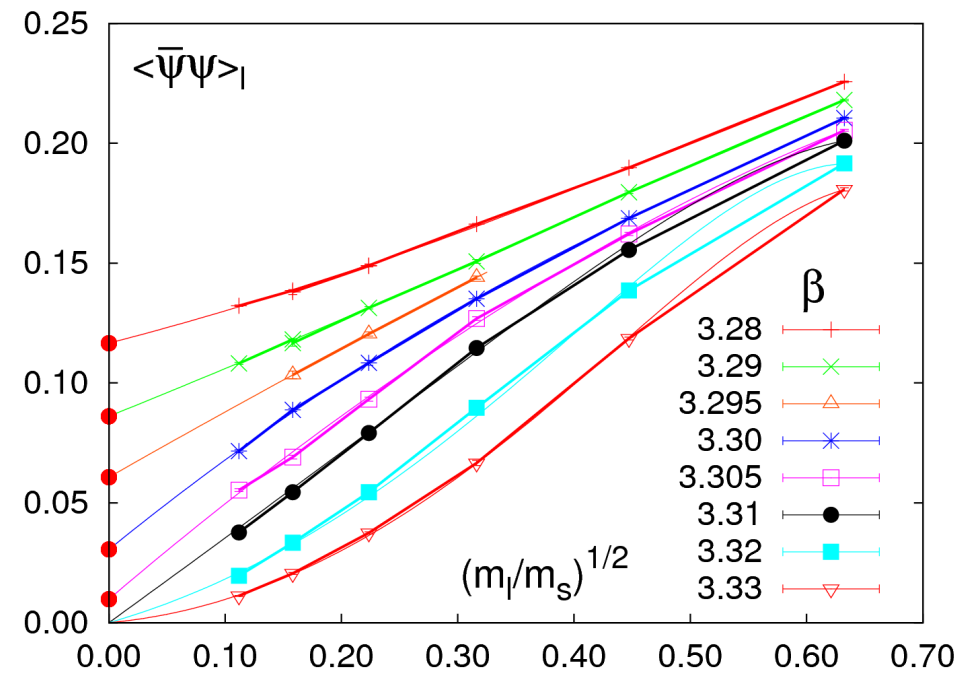
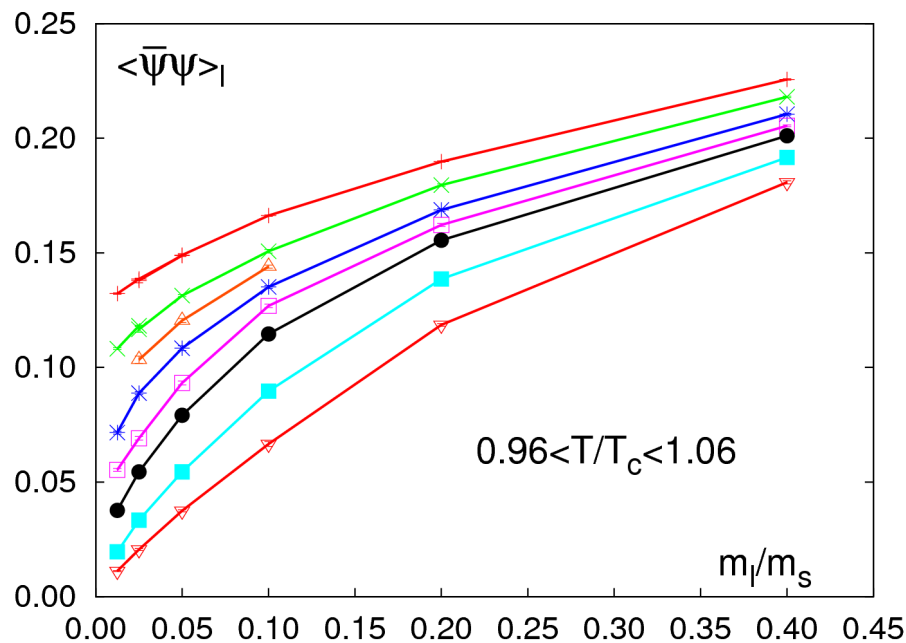
$$\Rightarrow \quad M \sim (-t)^\beta + \tilde{c}_2 \beta |t|^{\beta(1-\delta/2)} h^{1/2}$$

$$\chi_m \sim f_\chi(z) \quad : \quad (-z)^{\beta(1-\delta/2)}$$

$$\Rightarrow \quad \chi_M \sim \frac{1}{2} \tilde{c}_2 \beta |t|^{\beta(1-\delta/2)} h^{-1/2}$$

Chiral condensate (2+1)-flavor QCD

- ◆ **Goldstone modes** dominate quark mass dependence of the chiral order parameter for $T \lesssim T_c$
- ◆ analog of chiral logs at $T=0$

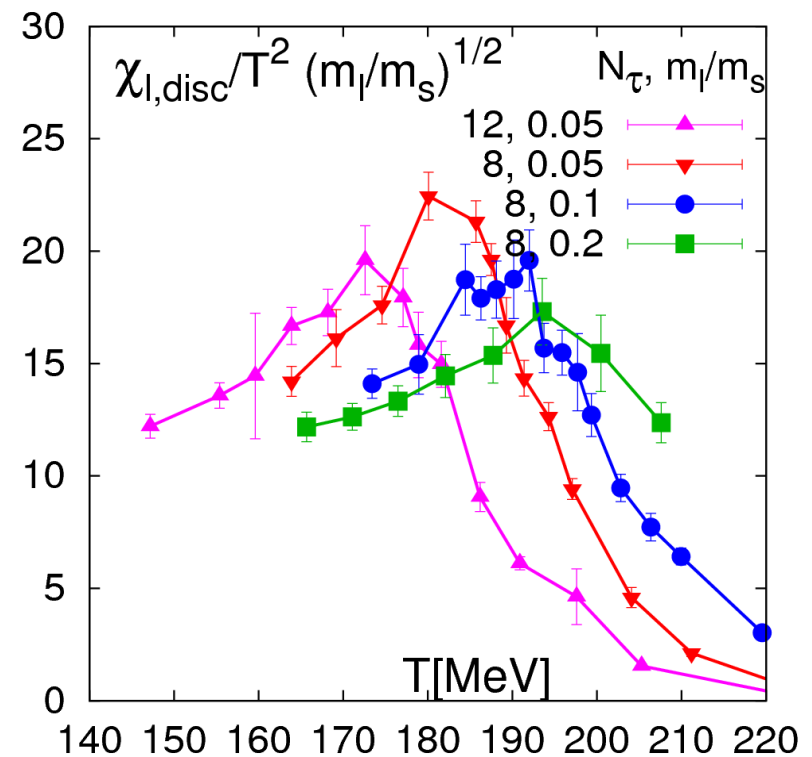
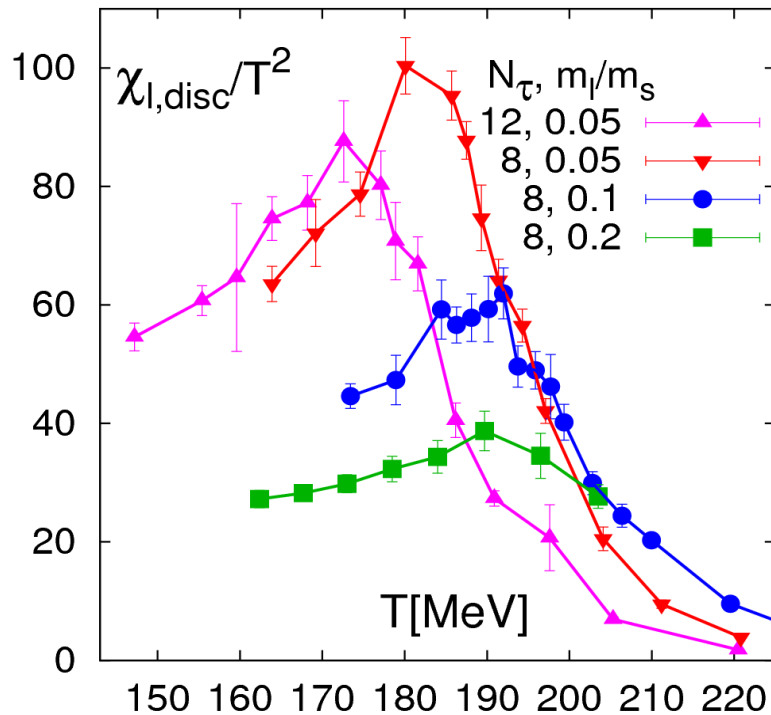


p4-action: $N_\sigma^3 \times 4$, $N_\sigma = 8 - 32$

S. Ejiri et al (BNL-Bielefeld), Phys. Rev. D80, 094505 (2009)

Disconnected chiral susceptibility (2+1)-flavor QCD

♦ chiral susceptibility: $\chi_{m,l} = \frac{\partial \langle \bar{\psi} \psi \rangle_l}{\partial m_l} = \chi_{l,disc} + 2\chi_{l,con}$



asqtad-action: $48^3 \times 8$ hotQCD preliminary

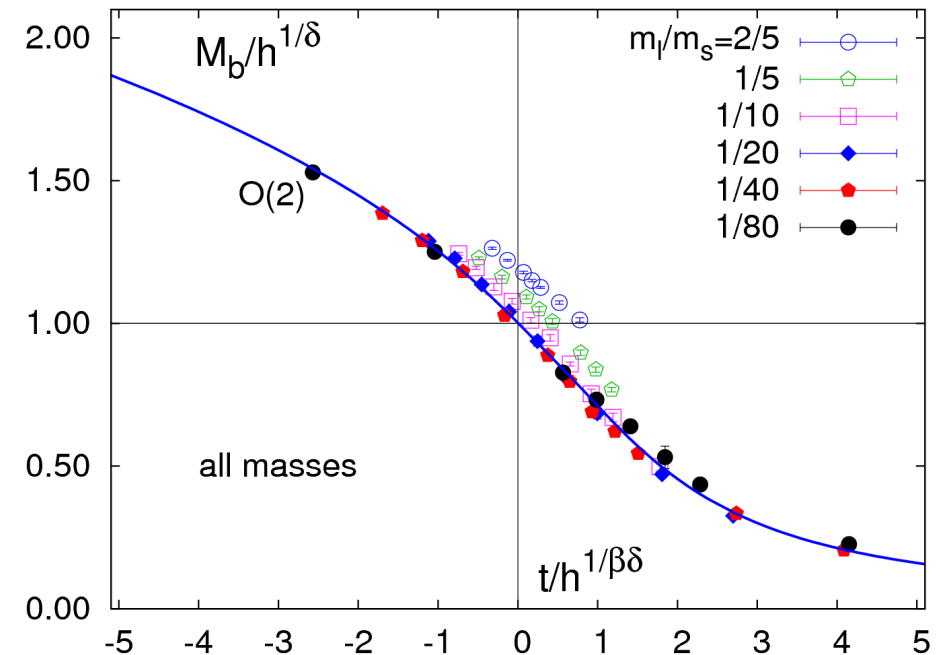
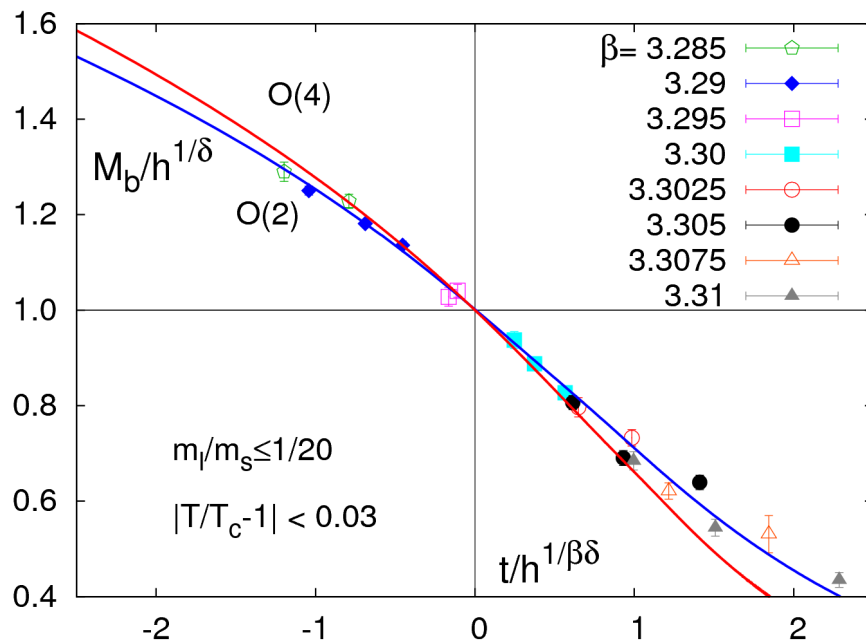
Magnetic Equation of State (2+1)-flavor QCD

chiral order parameter:

$$M_b \equiv \frac{m_s \langle \bar{\psi} \psi \rangle_l}{T^4}, \quad z \equiv t/h^{1/\beta\delta}$$

O(2) vs. O(4)

$z \rightarrow 1.2z$



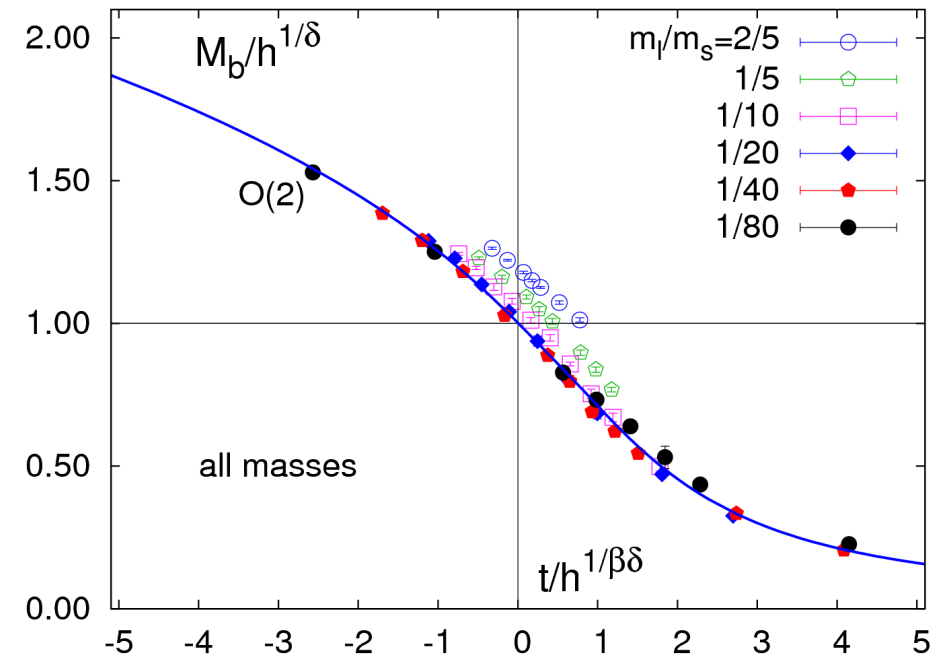
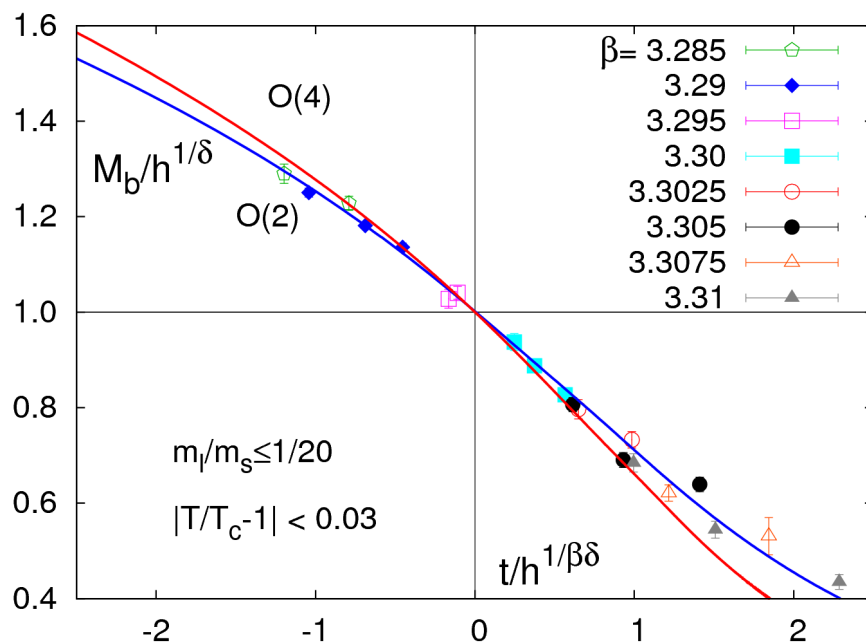
p4-action: $N_\sigma^3 \times 4$, $N_\sigma = 16, 32$

S. Ejiri et al (BNL-Bielefeld), Phys. Rev. D80, 094505 (2009)

Magnetic Equation of State (2+1)-flavor QCD

scaling violations:

$$M_b = h^{1/\delta} f_G(z) + \frac{m_l}{m_s} (a_t(T - T_c) + b_1) + \dots$$



p4-action: $N_\sigma^3 \times 4$, $N_\sigma = 16, 32$

S. Ejiri et al (BNL-Bielefeld), Phys. Rev. D80, 094505 (2009)

Scaling analysis in (2+1)-flavor QCD

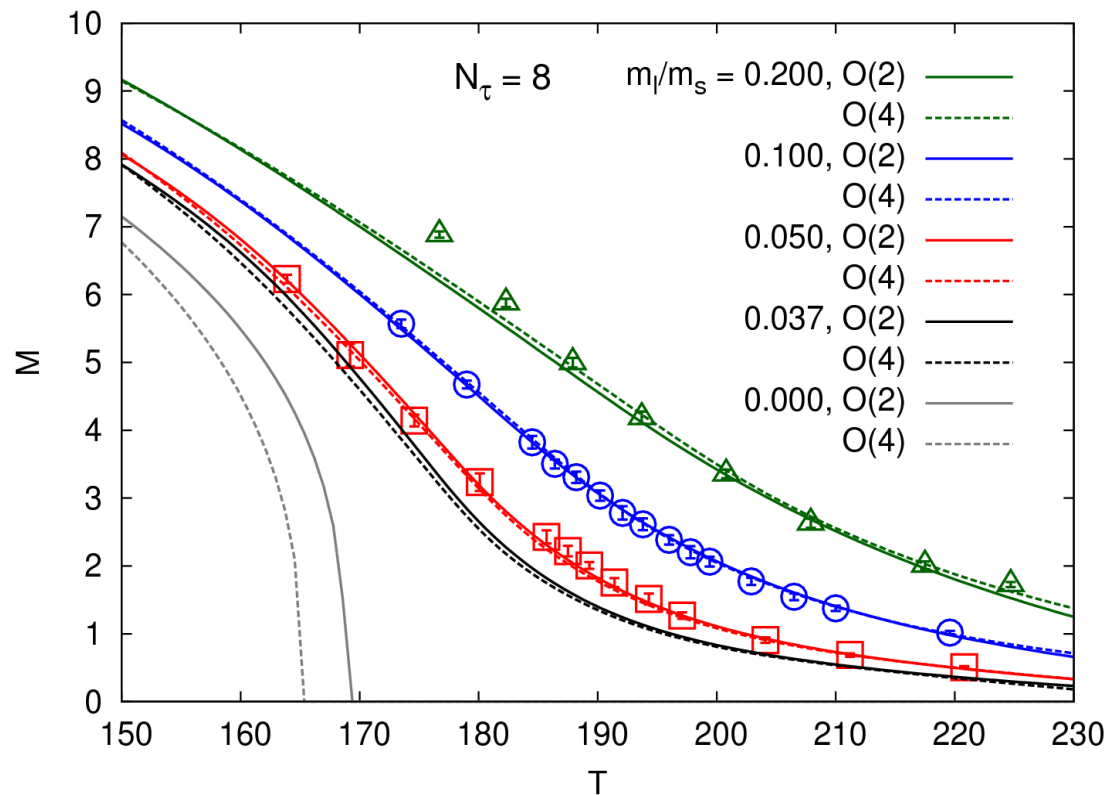
O(2) vs O(4) fits

$N_\tau = 8$

asqtad

$$M_b \equiv \frac{m_s \langle \bar{\psi} \psi \rangle_l}{T^4}$$

hotQCD preliminary



$$\Rightarrow T_c, t_0, h_0 \rightarrow z_0 = h_0^{1/\beta\delta} / t_0$$

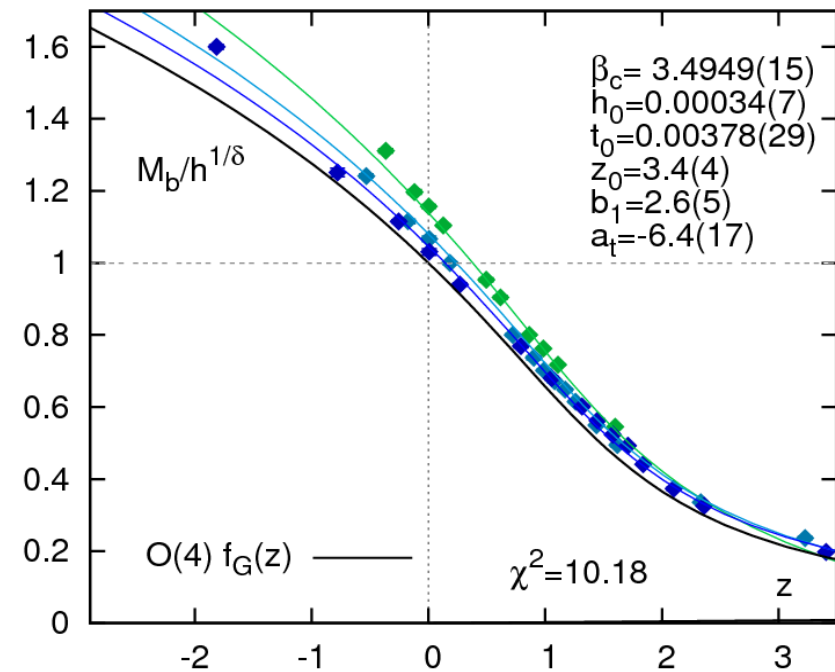
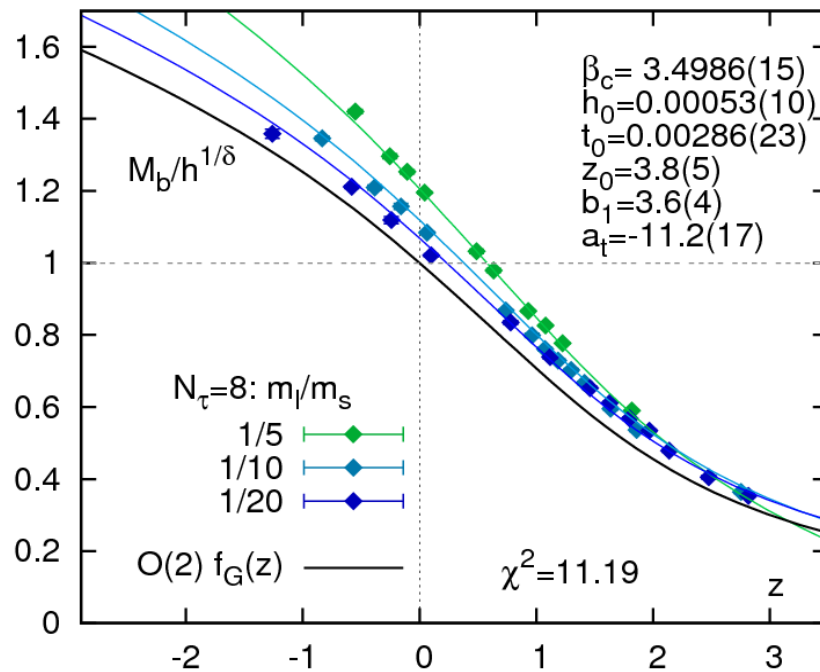
Scaling analysis in (2+1)-flavor QCD

O(2) vs O(4) fits

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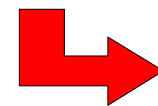
hotQCD preliminary



$$\Rightarrow T_c, t_0, h_0 \rightarrow z_0 = h_0^{1/\beta\delta} / t_0$$

W. Unger, PhD thesis, Bielefeld, September 2010

chiral phase transition



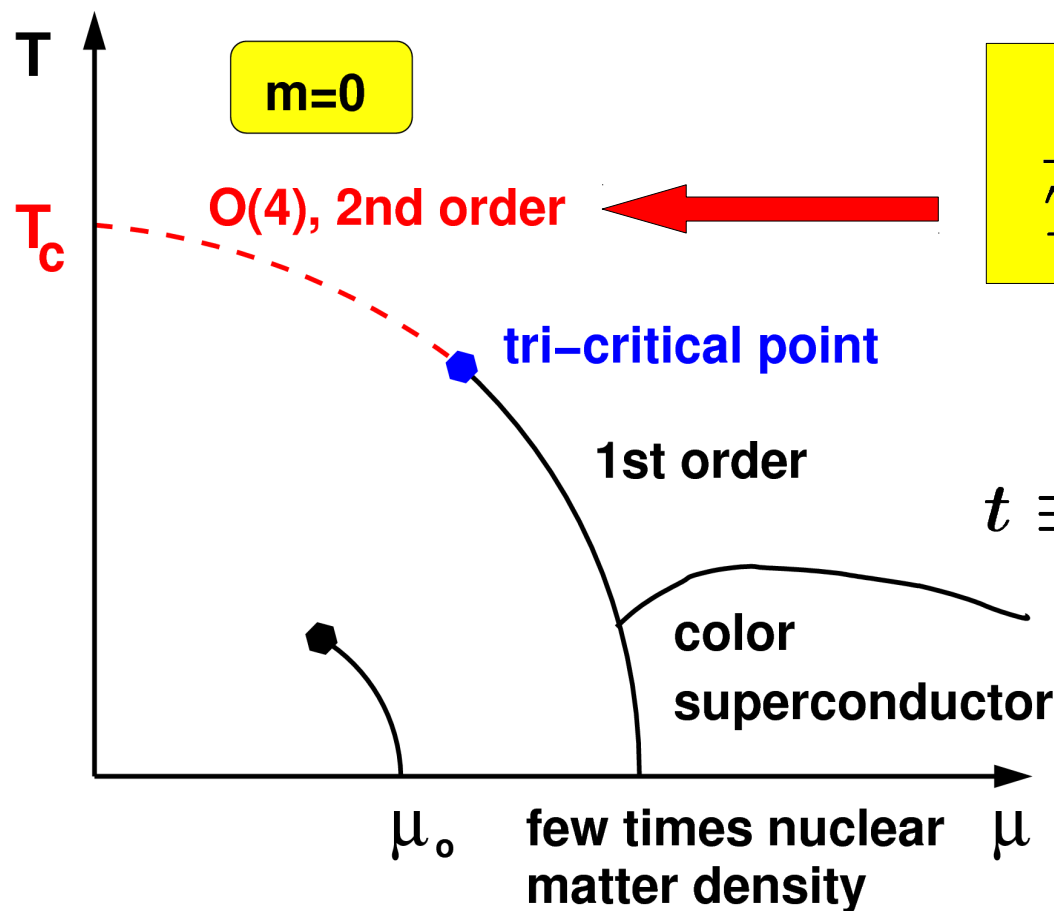
controls $T_c(m_l/m_s)$

The curvature of the critical line

BNL-Bielefeld-GSI, arXiv:1011.3130

◆ QCD, chiral limit (u,d quarks only)

$$\mu_u = \mu_d > 0, \mu_Q = \mu_s = 0$$



$$\frac{T}{T_c} = 1 - \kappa_q \left(\frac{\mu_q}{T} \right)^2$$

$$t \equiv \frac{1}{t_0} \left(\left(\frac{T}{T_c} - 1 \right) - \kappa_q \left(\frac{\mu_q}{T} \right)^2 \right)$$

scaling laws control
curvature of chiral transition
line for small μ_q/T

The curvature of the critical line

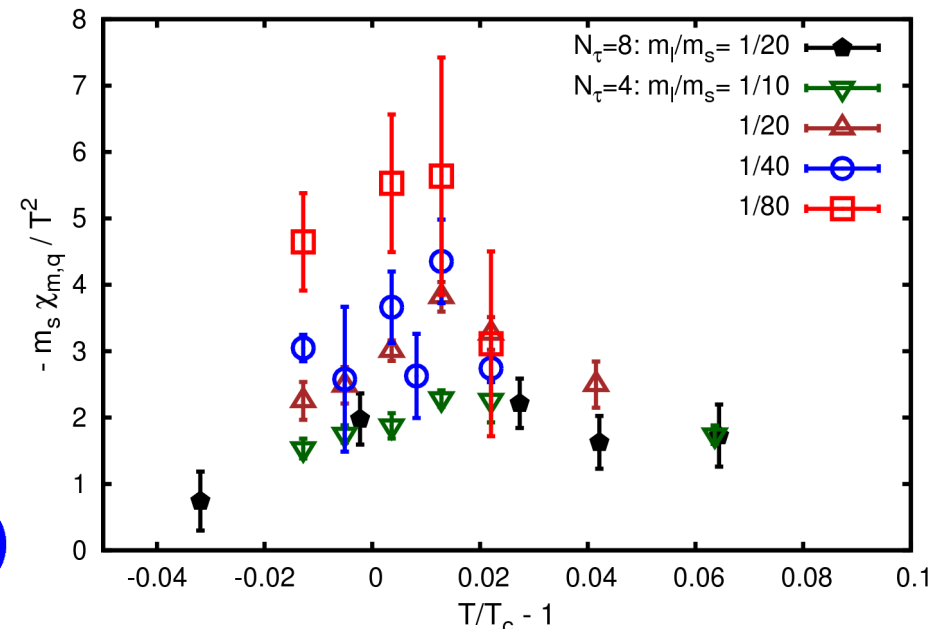
BNL-Bielefeld-GSI, arXiv:1011.3130

◆ "thermal" fluctuations of the order parameter

$$t \equiv \frac{1}{t_0} \left(\left(\frac{T}{T_c} - 1 \right) - \kappa_q \left(\frac{\mu_q}{T} \right)^2 \right), \quad z = t/h^{1/\beta\delta}$$

$$M_b \equiv \frac{m_s \langle \bar{\psi} \psi \rangle}{T^4} = h^{1/\delta} f_G(z) \quad \text{fixes } T_c, t_0, h_0$$

$$\begin{aligned} \chi_t &= \frac{\partial^2 M_b}{\partial (\mu_q/T)^2} \\ &= \frac{m_s}{T} \frac{\partial \langle \bar{\psi} \psi \rangle / T^3}{\partial (\mu_q/T)^2} \\ &= \frac{2\kappa_q}{t_0 T_c} h^{(\beta-1)/\delta\beta} f'_G(z) \end{aligned}$$

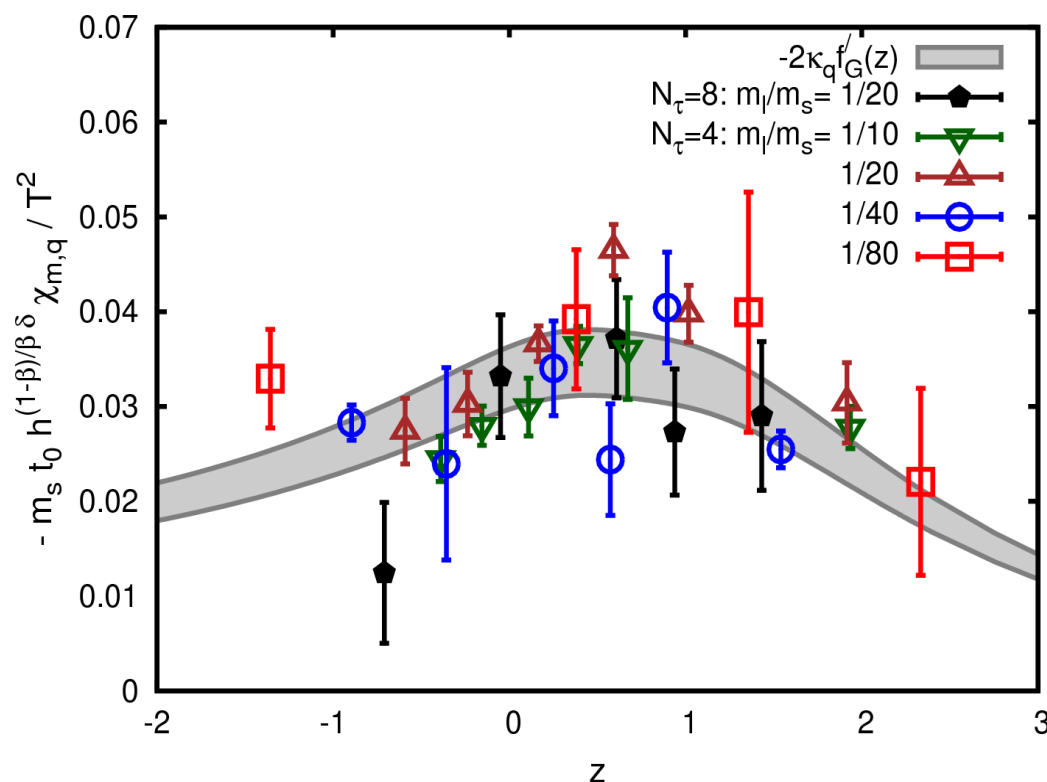


The curvature of the critical line

BNL-Bielefeld-GSI, arXiv:1011.3130

◆ "thermal" fluctuations of the order parameter

$$\chi_t = \partial^2 M_b / \partial (\mu_q / T)^2 = \frac{2\kappa_q}{t_0 T_c} h^{(\beta-1)/\delta\beta} f'(z)$$

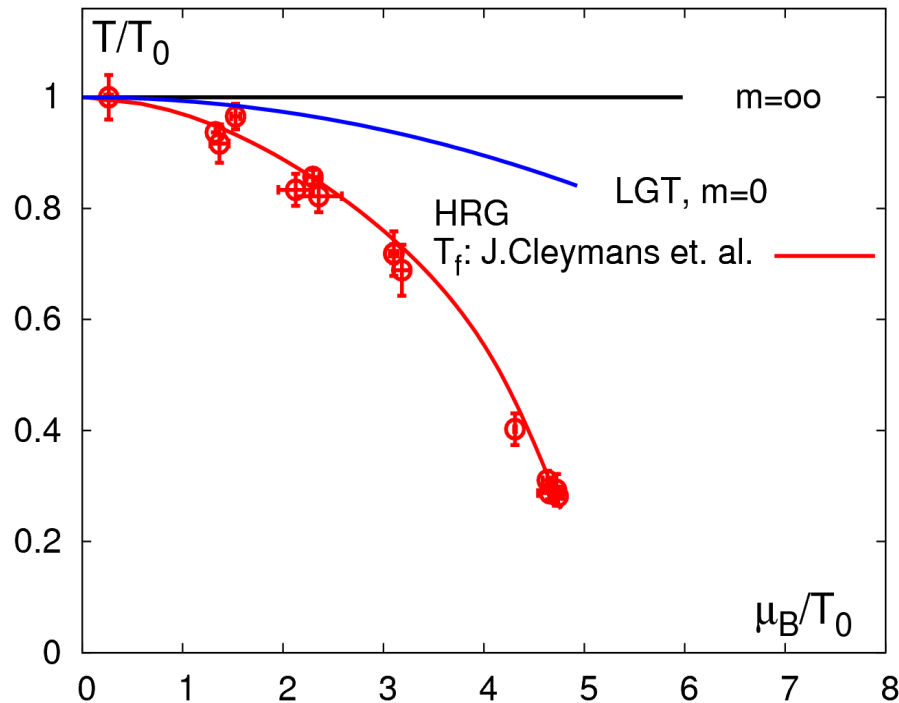


analysis for 2 values of the cut-off and 4 (1) different quark masses

$$\kappa_q = 0.059 \pm 0.006$$

compare to freeze-out curve:

Chiral Transition and Freeze-out



chiral phase transition curve:

$$\frac{T(\mu_B)}{T_c} = 1 - 0.0066(7) \left(\frac{\mu_B}{T} \right)^2 + \mathcal{O}(\mu_B^4)$$

open issues:

- continuum limit
- strangeness conservation
- non zero charge

freeze-out curve in heavy ion collisions:

$$\frac{T(\mu_B)}{T_c} = 1 - 0.023 \left(\frac{\mu_B}{T} \right)^2 - c \left(\frac{\mu_B}{T} \right)^4$$

$$\mu_B(\sqrt{s_{NN}}) = \frac{d}{1 + e\sqrt{s_{NN}}}$$

J. Cleymans et al.,
Phys.Rev. C73, 034905 (2006)

Chiral Transition and Freeze-out

sketch of a phase diagram

chiral phase transition curve at non-zero strangeness chem. pot.

$$\frac{T(\mu_B)}{T_c} = 1 - \kappa_B \left(\frac{\mu_B}{T} \right)^2 - \kappa_S \left(\frac{\mu_S}{T} \right)^2 - \kappa_{BS} \frac{\mu_B \mu_S}{T^2}$$

freeze-out curve in heavy ion collisions:

$$\frac{T(\mu_B)}{T_c} = 1 - 0.023 \left(\frac{\mu_B}{T} \right)^2 - c \left(\frac{\mu_B}{T} \right)^4$$

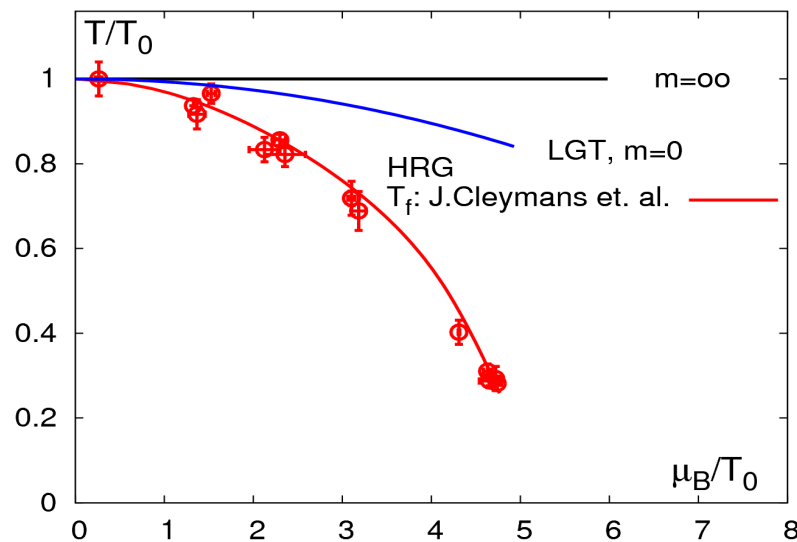
$$\mu_s \simeq \mu_B / 6$$

$$\mu_B(\sqrt{s_{NN}}) = \frac{1}{1 + e\sqrt{s_{NN}}}$$

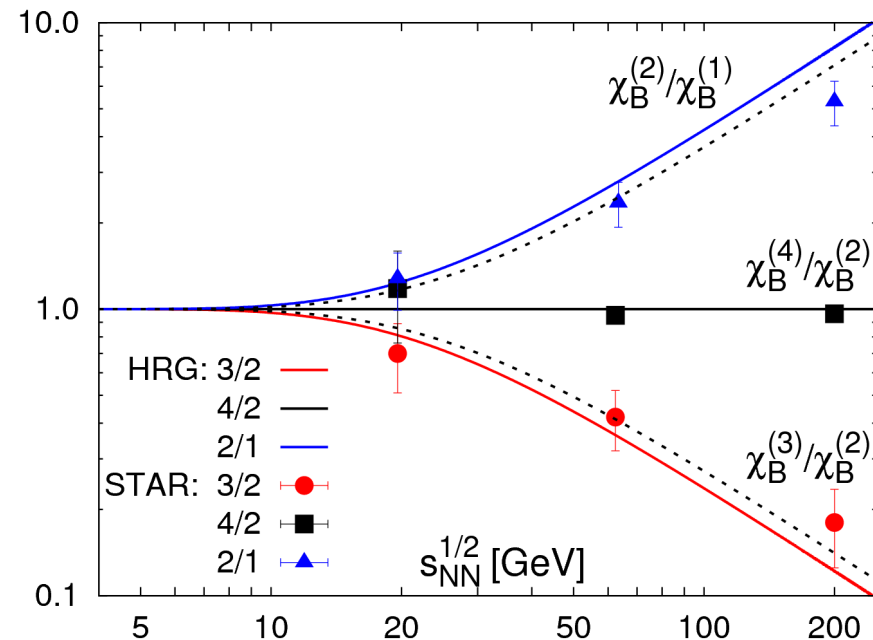
J. Cleymans et al.,
Phys.Rev. C73, 034905 (2006)

Charge fluctuations at RHIC and LHC

freeze-out line and
chiral phase transition



Moments of charge fluctuations

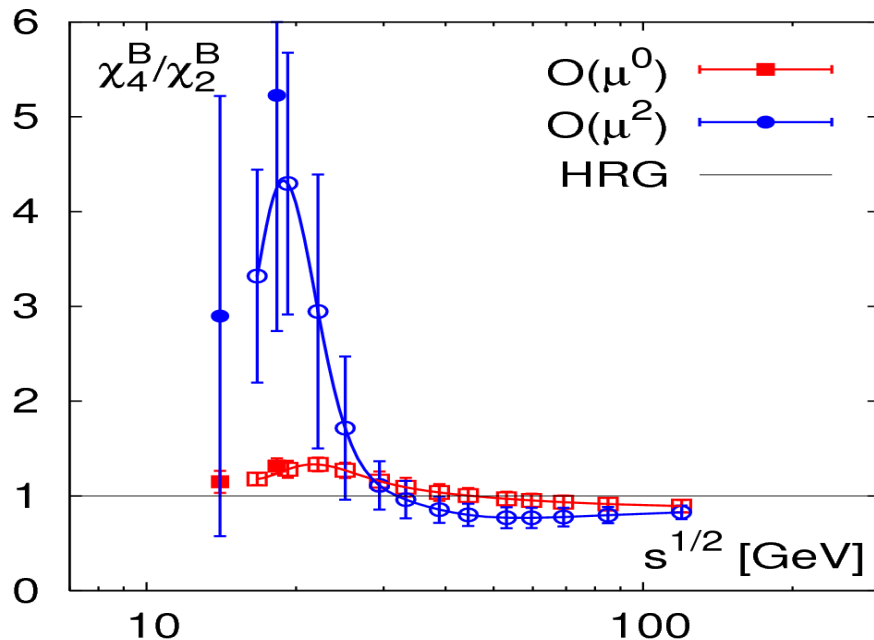


FK, K Redlich, arXiv:1007.2581
data from STAR: arXiv:1004.4959

- freeze-out occurs well in hadronic phase
- charge fluctuations at freeze-out agree with HRG model predictions
- Taylor expansions should converge well on freeze-out curve
- charge fluctuations at LHC may be sensitive to universal scaling laws

Charge fluctuations at RHIC and LHC

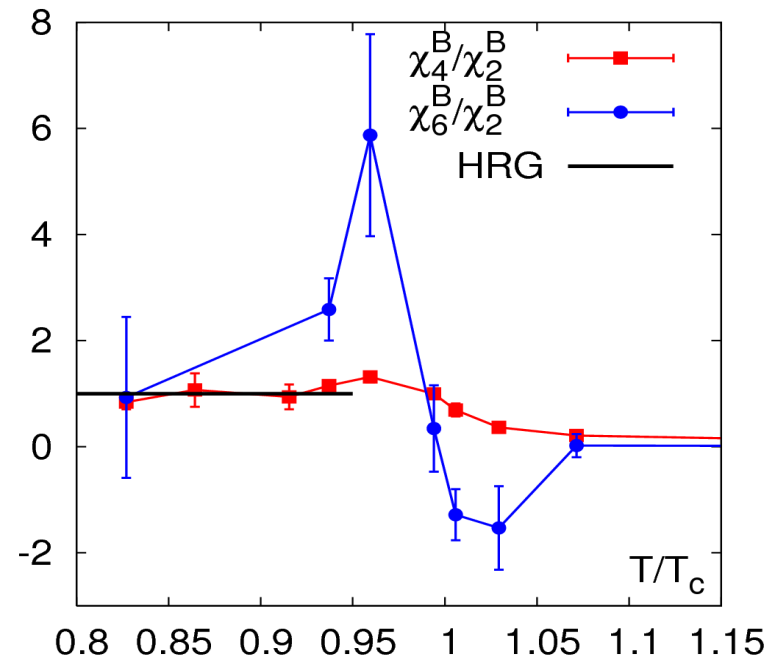
$$\mu_B > 0$$



C.Schmidt, arXiv:1007.5164

higher order Taylor expansion
needed to control fluctuations
at $\sqrt{s} \lesssim 20$ GeV

$$\mu_B = 0$$



C.Schmidt,, Jap.-Dt workshop, Mishima, Nov. 2010

LHC can probe fluctuations at $\mu_B \simeq 0$

large deviations from HRG result
should show up in 6th order cumulants

Conclusions

- QCD with physical quark masses is sensitive to universal scaling properties in the chiral limit

- the curvature of the chiral phase transition line can be determined from $O(N)$ scaling functions;

current results suggest that the curvature of the chiral transition line is smaller than that of the freeze-out curve

- the freeze-out curve will be even further away from the crossover in QCD with physical quark masses

an estimate:

$$\mu_q/T = 1 : T_{cross} - T_{freeze} \simeq 0.15T_c$$