Radiative corrections to $\bar{p}+p \rightarrow e^{+}+e^{-}$

Cross sections and MC generator

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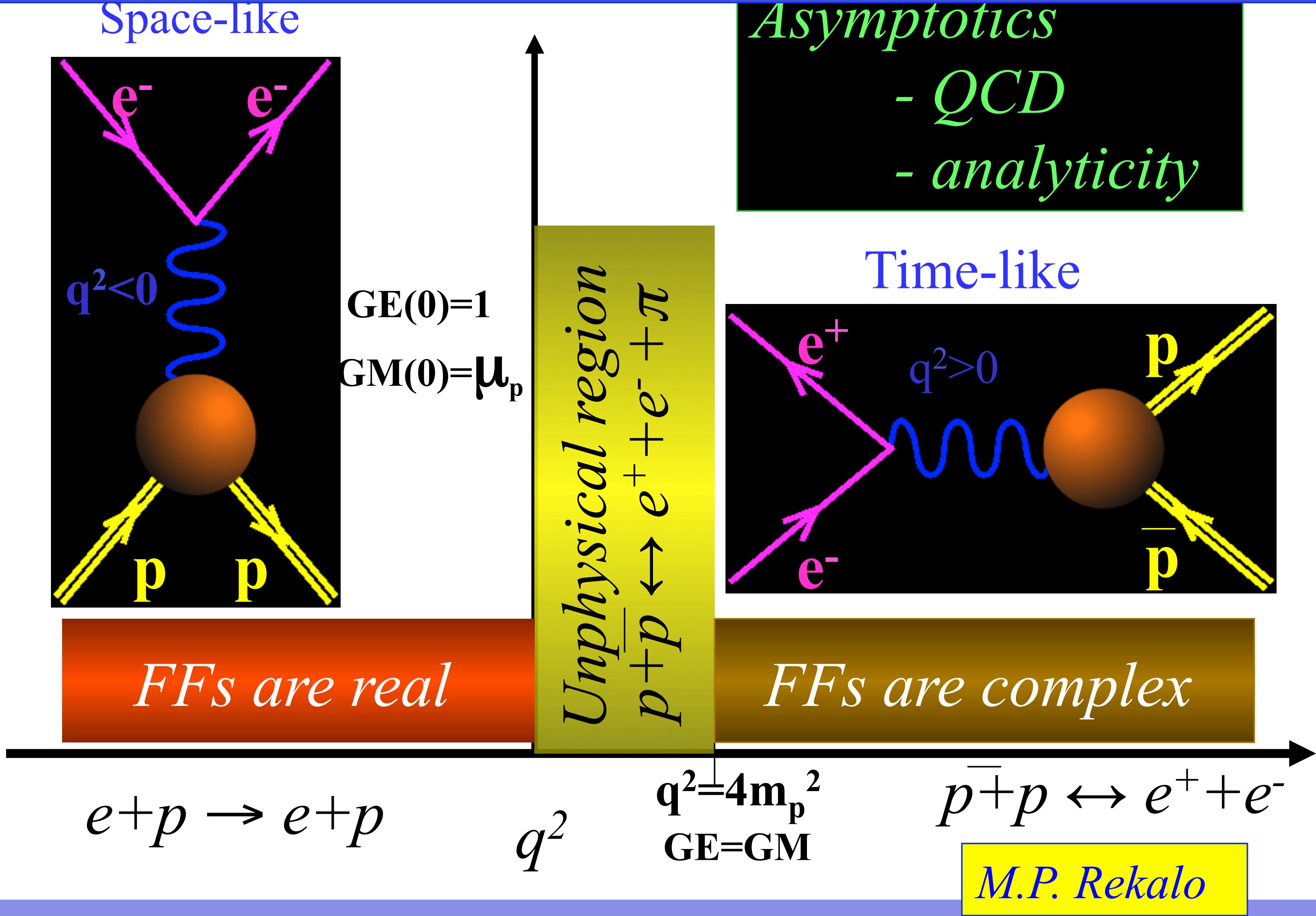
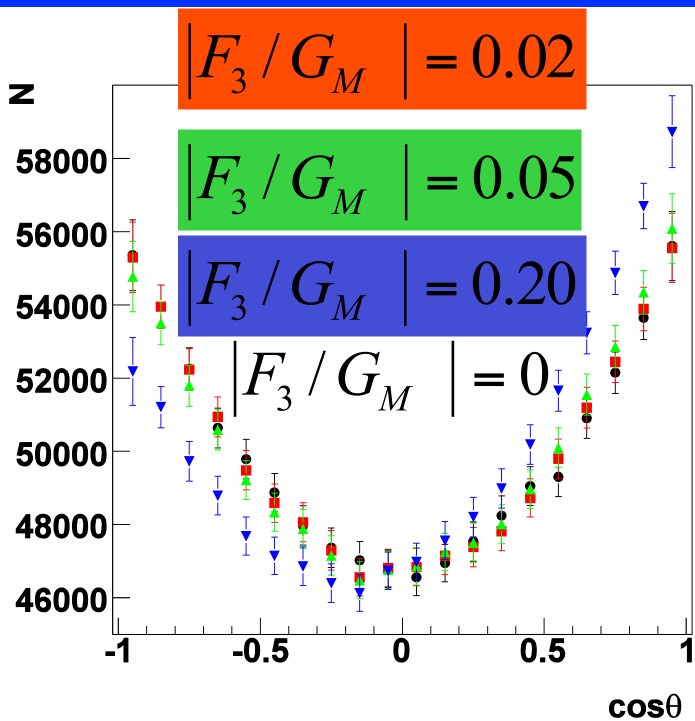
Electromagnetic Form Factors



QED Radiative Corrections

- Must be applied to any process involving charged particles, in particular electrons
- Modify the absolute value of the experimental observables and their dependence from the relevant kinematical variables



-The cross section for $\bar{p} + p \to e^+ + e^-$ (1 γ-exchange):

$$\frac{d\sigma}{d(\cos\theta)} = \frac{\pi\alpha^2}{8m^2\sqrt{\tau - 1}}\left[\tau|G_M|^2(1+\cos^2\theta) + |G_E|^2\sin^2\theta\right]$$

θ: angle between e^- and $\bar{p}$ in cms.

Adding odd cosθ term

Angular distribution

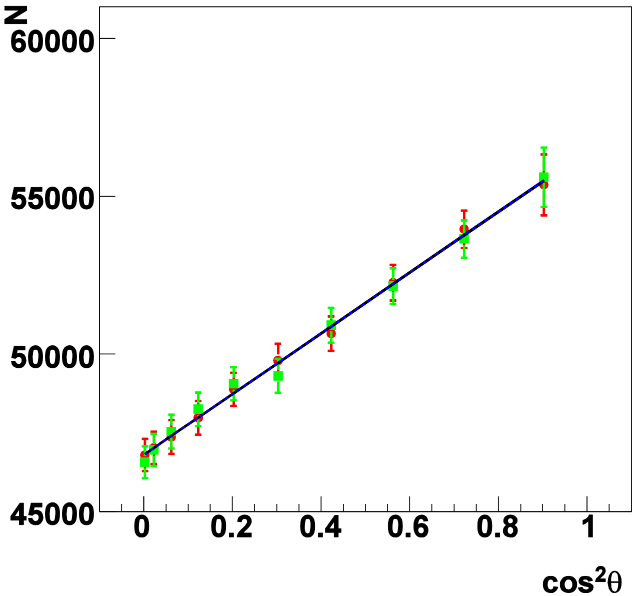
$$\frac{d\sigma}{d(\cos\theta)} = \sigma_0 [1 + \mathcal{A}\cos^2\theta]$$

$$\mathcal{A} = \frac{\tau|G_M|^2 - |G_E|^2}{\tau|G_M|^2 + |G_E|^2} = \frac{\tau - \mathcal{R}^2}{\tau + \mathcal{R}^2}.$$

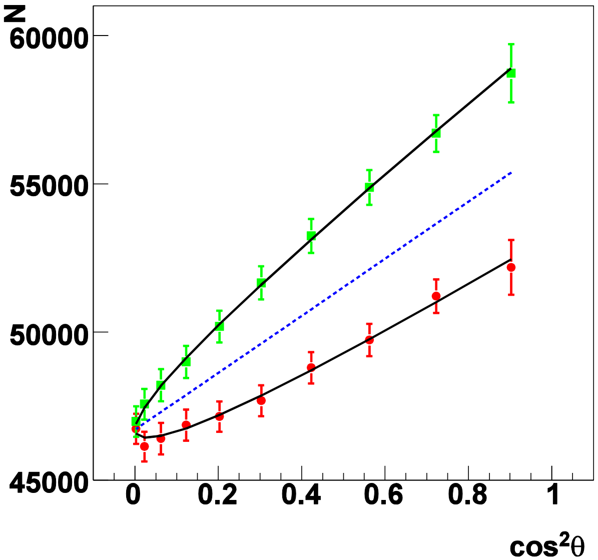
$$\mathcal{R} = |G_E|/|G_M|$$

$$\sigma_0 = \frac{\alpha^2}{4q^2}\sqrt{\frac{\tau}{\tau-1}}\left(|G_M|^2 + \frac{1}{\tau}|G_E|^2\right)$$

Angular asymmetry

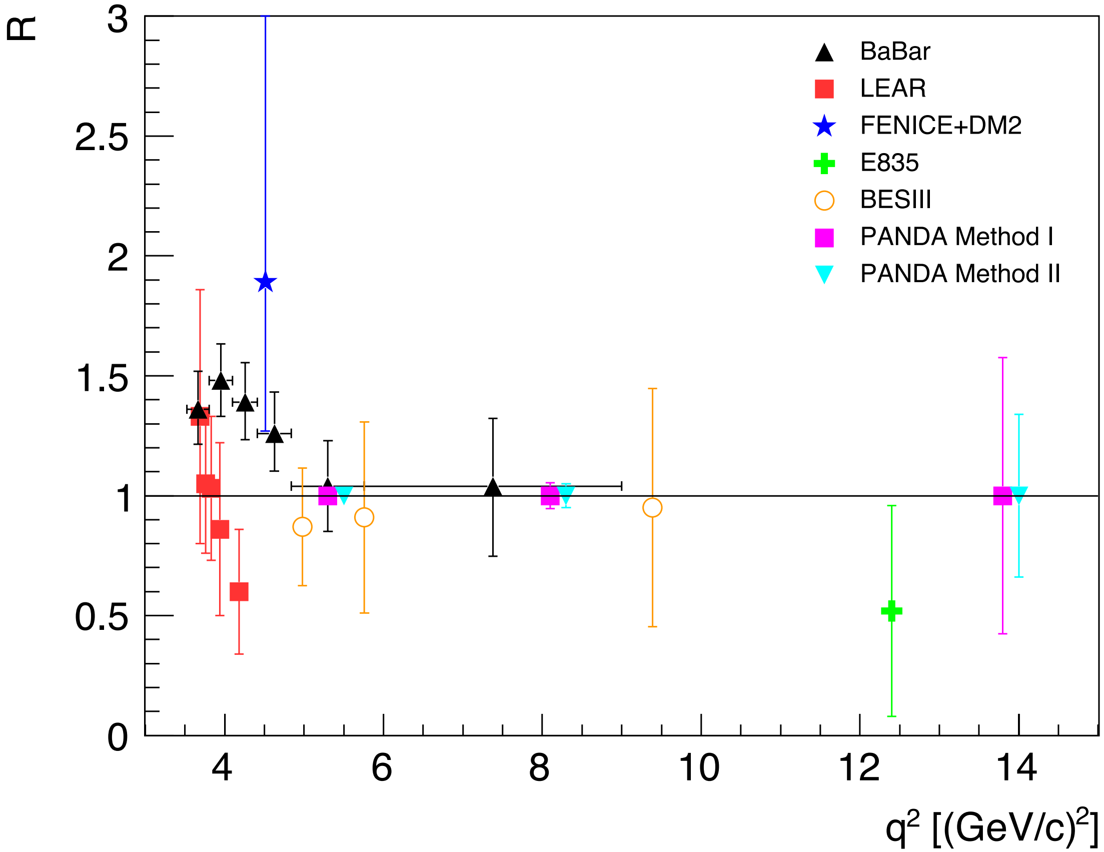


Adding odd cosθ term



E.T.-G. and M.P. Rekalo, PLB 504(2001) 291

PandaROOT simulations on R=GE/GM



D. Khaneft, A. Dbeyssi, et al., EPJA,52 (2016) 325

Few percent error on G_E and G_M implies radiative corrections at percent level !

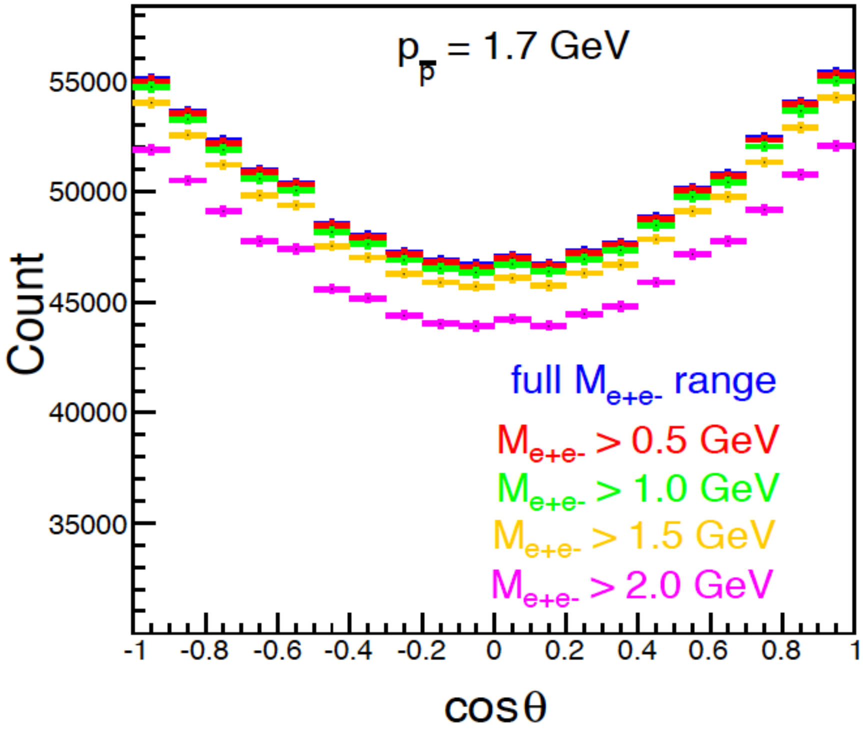
$$\bar{p}(p_-) + p(p_+) \rightarrow e^+(q_+) + e^-(q_-) + \gamma(k).$$

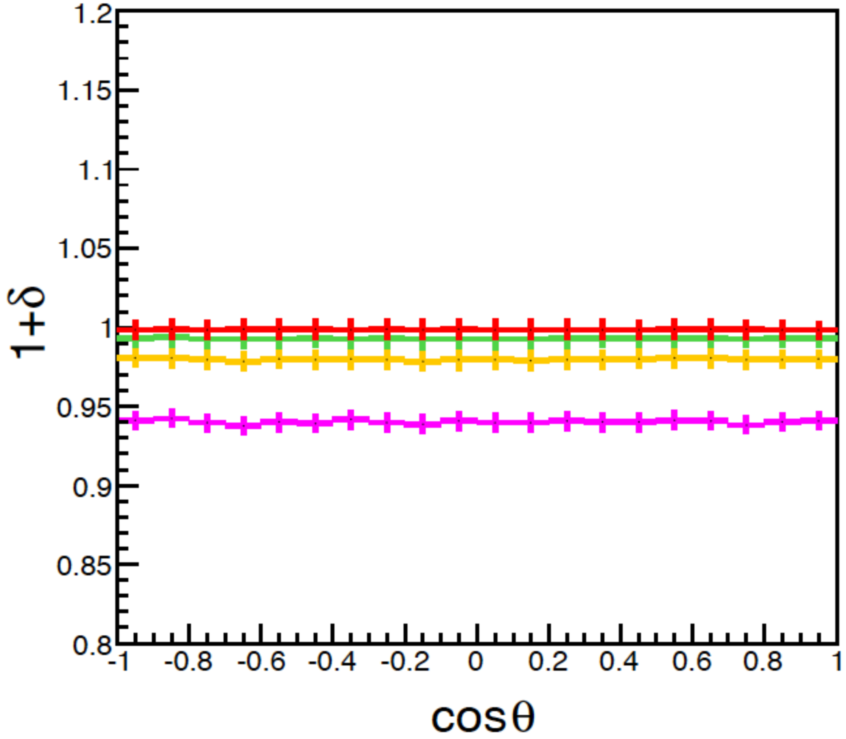
PHOTOS MC

A. Dbeyssi, PhD

- Include radiation from final electron
- No interference

$$\frac{d\sigma^c}{d\Omega} = \frac{d\sigma^B}{d\Omega}(1+\delta),$$



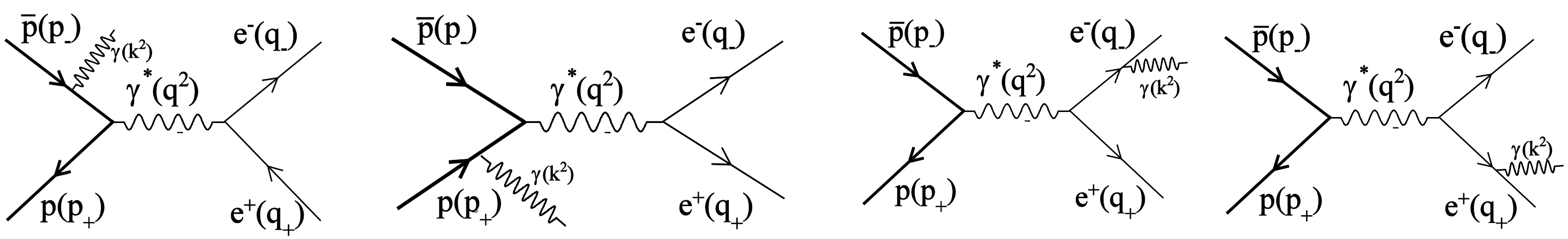


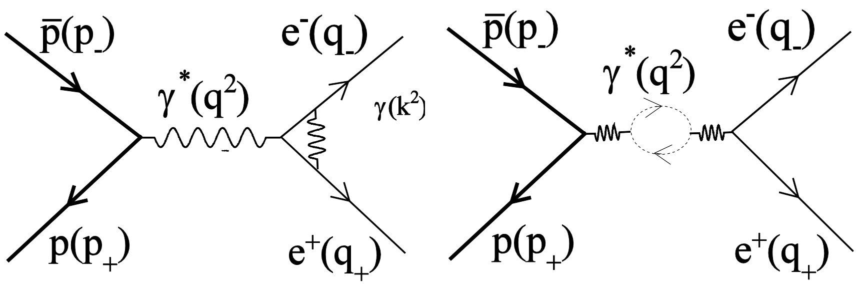
PHYSICAL REVIEW D 82, 094016 (2010)

Radiative proton-antiproton annihilation to a lepton pair

A. I. Ahmadov,[1,2,*] V. V. Bytev,[1,†] E. A. Kuraev,[1,‡] and E. Tomasi-Gustafsson[3,§]

- *Virtual and soft photon emission*





$$p(p_+) + \bar{p}(p_-) \to e^+(q_+) + e^-(q_-)$$

- *Hard photon emission*

$$\bar{p}(p_-) + p(p_+) \to e^+(q_+) + e^-(q_-) + \gamma(k).$$

- *LLA- Structure Functions* Kuraev,Fadin (1985)

Radiative corrections

at NLO, we have two different (i.e. non-interfering) final states:

$* \; e^+e^-$: **born, virtual corrections (vacuum polarization, ...)**

$* \; e^+e^-\gamma$: **real corrections (initial state radiation, final state radiation)**

$$\mathcal{O}(\alpha) \quad \mathcal{O}(\alpha^2) \quad \mathcal{O}(\alpha^{3/2})$$

$$\Rightarrow d\sigma \sim |\mathcal{M}_B + \sum_i \mathcal{M}_i|^2 + |\mathcal{M}_{ISR} + \mathcal{M}_{FSR}|^2 \qquad \text{truncated at } \mathcal{O}(\alpha^3)$$

$$\sim |\mathcal{M}_B|^2 + \sum_i 2\mathrm{Re}(\mathcal{M}_\mathrm{B}\mathcal{M}_\mathrm{i}^*) + |\mathcal{M}_\mathrm{ISR}|^2 + |\mathcal{M}_\mathrm{FSR}|^2 + 2\mathrm{Re}(\mathcal{M}_\mathrm{ISR}\mathcal{M}_\mathrm{FSR}^*)$$

$$\sim |\mathcal{M}_B|^2 \left\{ 1 + \sum_i \frac{2\mathrm{Re}(\mathcal{M}_\mathrm{B}\mathcal{M}_\mathrm{i}^*)}{|\mathcal{M}_B|^2} + \frac{|\mathcal{M}_{ISR}|^2}{|\mathcal{M}_B|^2} + \frac{|\mathcal{M}_{FSR}|^2}{|\mathcal{M}_B|^2} + \frac{2\mathrm{Re}(\mathcal{M}_\mathrm{ISR}\mathcal{M}_\mathrm{FSR}^*)}{|\mathcal{M}_B|^2} \right\}$$

$$\sim |\mathcal{M}_B|^2 \left\{ 1 + \delta_\mathrm{vacuum} + \delta_\text{e-vertex} + \delta_\text{p-vertex} + \delta_\mathrm{box} + \underbrace{\delta_\mathrm{ISR} + \delta_\mathrm{FSR} + \delta_\text{interference ISR/FSR}}_{\equiv \delta_\gamma} \right\}$$

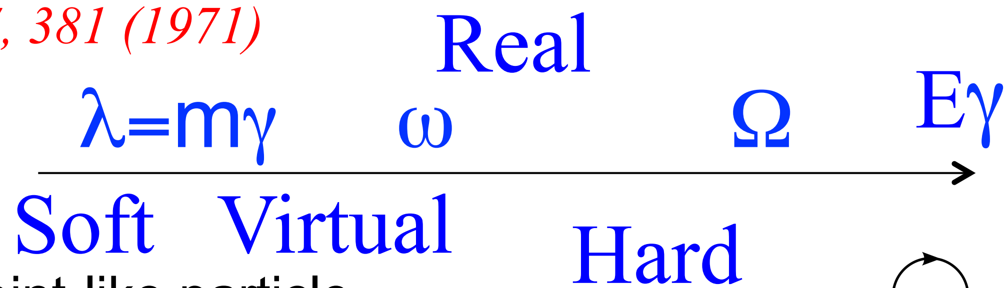
therefore, at $\mathcal{O}(\alpha^3)$, we write

$$d\sigma = d\sigma_B \left\{ 1 + \underbrace{\delta_\mathrm{vacuum} + \delta_\text{e-vertex} + \delta_\text{p-vertex} + \delta_\mathrm{box}}_{\text{interference born, virtual diagrams}} + \delta_\gamma \right\}$$

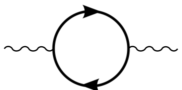
ISR, FSR, interference ($\leftarrow \delta_\gamma$)

Radiative corrections

A.I. Ahmadov et al. Phys. Rev. D 82, 094016 (2010)
G. Bonneau et al. Nucl. Phys. B 27, 381 (1971)



Virtual Corrections:



- The proton is treated as a point-like particle
- all leptons (e, μ, τ), and point-like pion contribute to vacuum polarization

Real Corrections: *the formalism depends on the photon kinematics:*

soft photon regime

- $E\gamma<\omega$ no experimental detection of the photon: the threshold ω is determined by theory/experiment
- The cross section is integrated over the photon degrees of freedom

hard photon regime

$E\gamma>\omega$ the photon is detected in the experiment
the experiment is sensitive to the photon kinematics

Radiative corrections

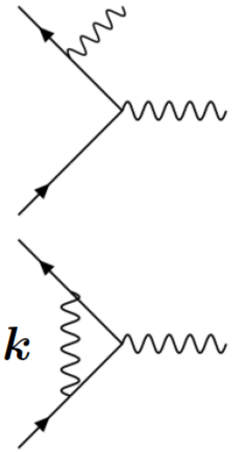
- **real corrections (ISR, FSR, interference) have been completely re-calculated**
  [Ahmadov et al. does not separate explicitly ISR/FSR terms;
  [kinematic regime $s, t, u \gg M^2, m^2$ assumed in interference term not adequate for PANDA

following F. A. Berends et al. Nucl. Phys. B **57** 381 (1973)
F. A. Berends et al. Nucl. Phys. B **63** 381 (1973) on $e^+e^- \to \mu^+\mu^-$

in the soft photon limit: $E_\gamma < \omega = b\sqrt{s}/2$ $\quad b < 10^{-2}$

- **infrared divergences regularized by giving the photon a small mass λ (infrared cutoff)**

  * **real diagrams:** $E_\gamma > \lambda = a\sqrt{s}$ $\quad 10^{-6} < a < 10^{-3}$
  * **virtual diagrams:** $\int_0 dk^2 \to \int_{\lambda^2} dk^2$

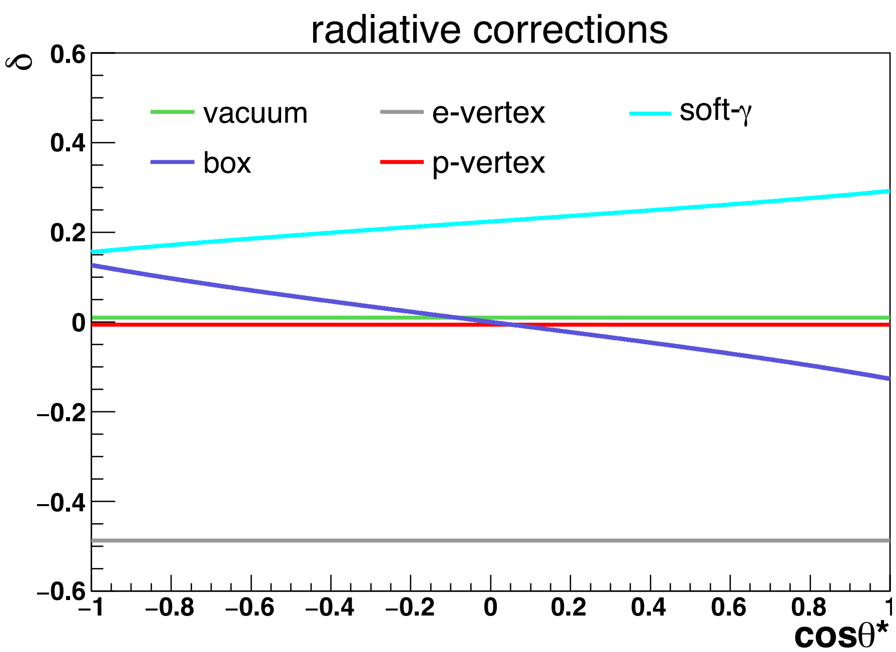


- **cancellation of divergences:**

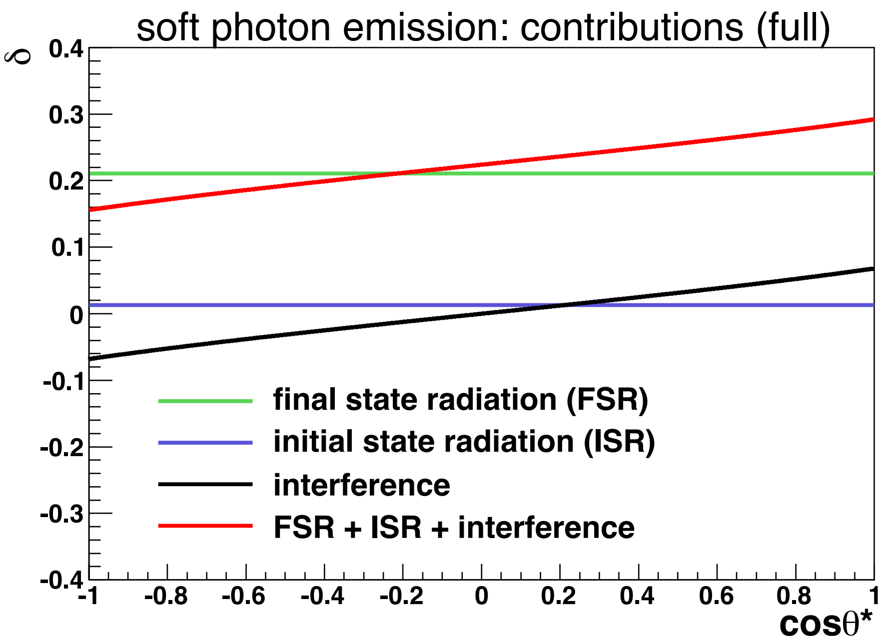
$$\delta_{\text{e-vertex}} + \delta_{\text{FSR}} = \text{finite}$$
$$\delta_{\text{p-vertex}} + \delta_{\text{ISR}} = \text{finite}$$
$$\delta_{\text{box}} + \delta_{\text{interference ISR/FSR}} = \text{finite}$$

- **full cross section is λ-independent (more accurately, λ-stable)**

Soft and Virtual Photon Emission

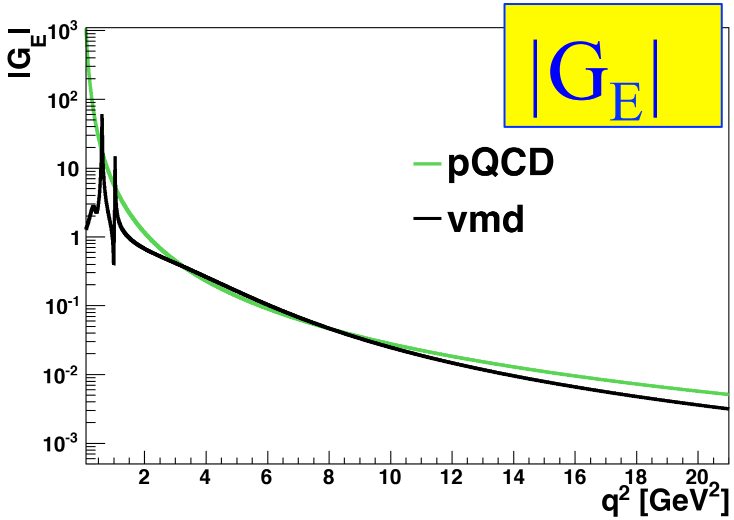


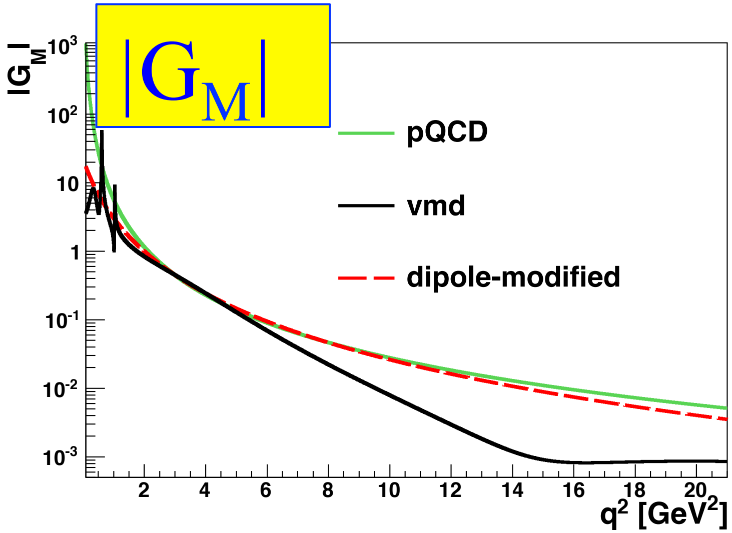
$$s = 5.08\ \mathrm{GeV}^2$$

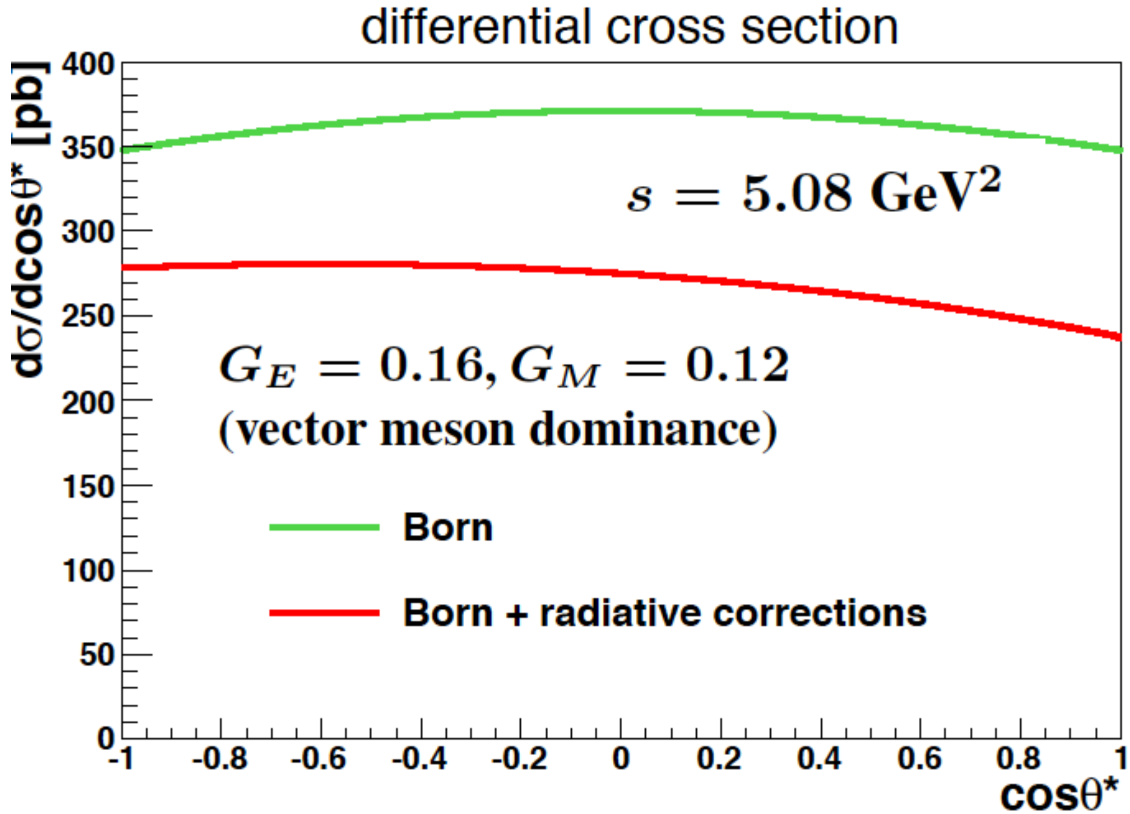


Radiative cross section(soft and virtual RC)

$$d\sigma/d\cos\theta^* = (d\sigma_B/d\cos\theta^*)(1+\delta\_)$$







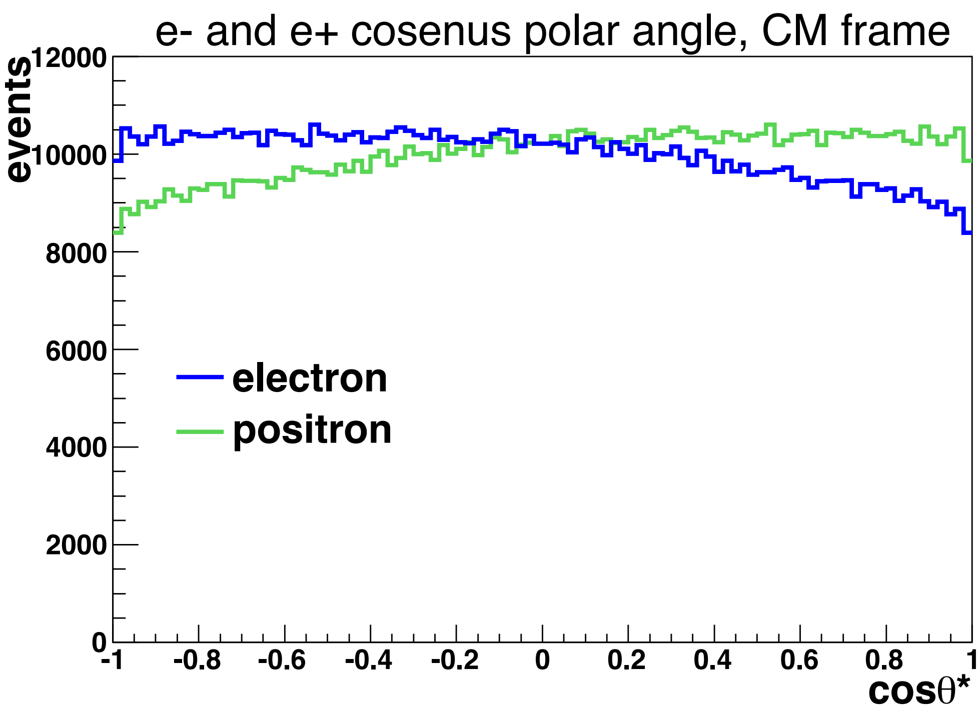
$$\sigma = \int d\cos\theta^* \frac{d\sigma}{d\cos\theta^*} = 434\ \mathrm{pb}$$

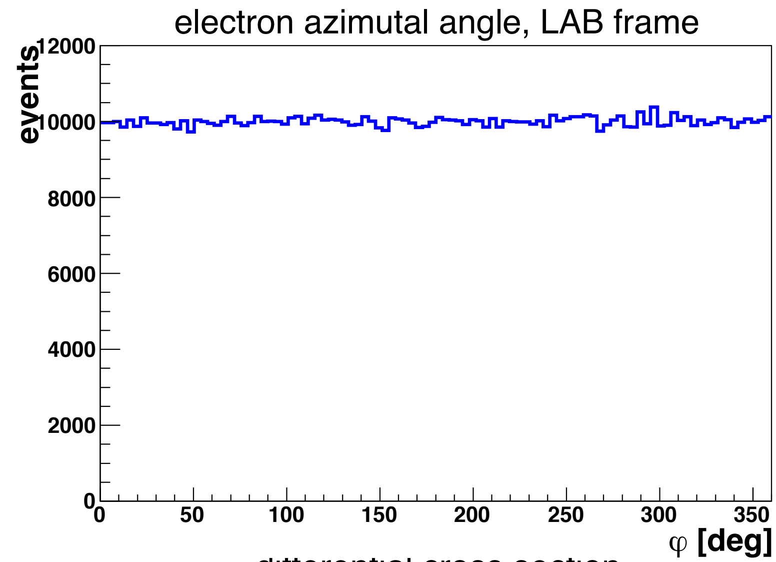
$$-0.8 < \cos\theta^* < 0.8$$

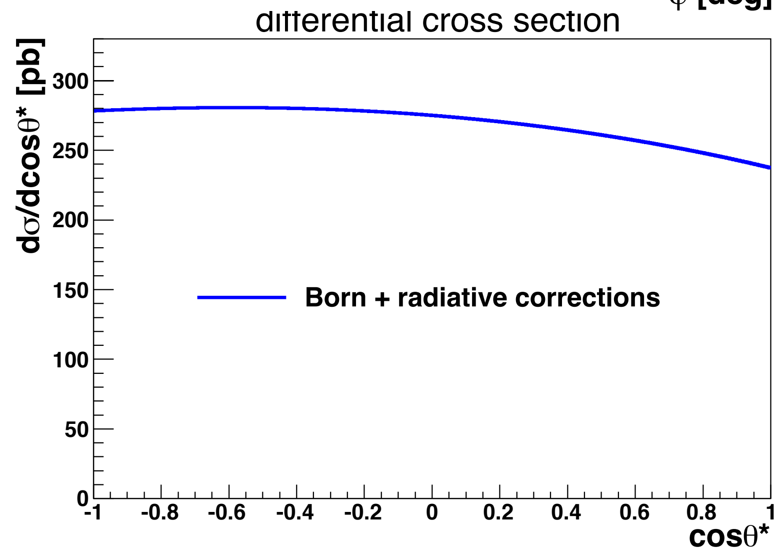
Event Generator (soft and virtual RC)

$$s = 5.08\ \mathrm{GeV}^2$$

MC: N=10^6 generated events





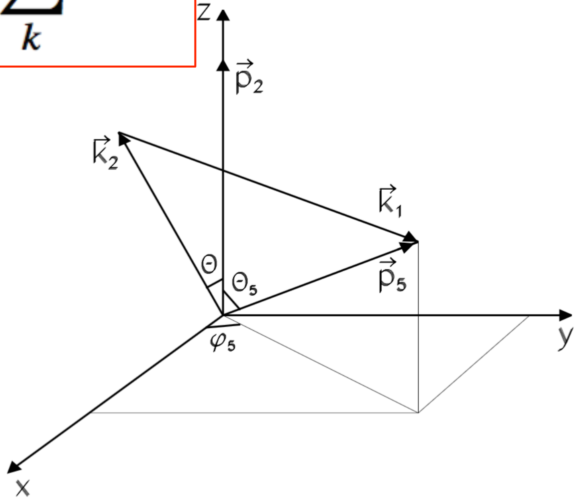


Hard Photon Emission

Fully differential cross section as function **of invariants**

$$d\sigma_H = \frac{\alpha^3}{\pi^2 s}\int (M_{\rm ISR}+M_{\rm FSR})(M_{\rm ISR}+M_{\rm FSR})^+ \cdot \theta_\omega \cdot \theta_P \cdot d\Phi_3,$$

$$(M_{\rm ISR}+M_{\rm FSR})(M_{\rm ISR}+M_{\rm FSR})^+ = R_{\rm ISR}+R_{\rm FSR}+R_{\rm INT} = \sum_k R_k\,.$$



Energy limits on the hard photon

$$\theta_\omega = \theta(p_0-\omega)$$

$$\theta_P \equiv \theta(\Omega - p_0)$$

Phase space

$$d\Phi_3 = \delta(p_1+p_2-k_1-k_2-p)\frac{d^3k_1}{2k_{10}}\frac{d^3k_2}{2k_{20}}\frac{d^3p}{2p_0}$$

Hard Photon Emission

New calculation of hard photon emission:

A.I. Ahmadov et al. Phys. Rev. D 82, 094016 (2010)

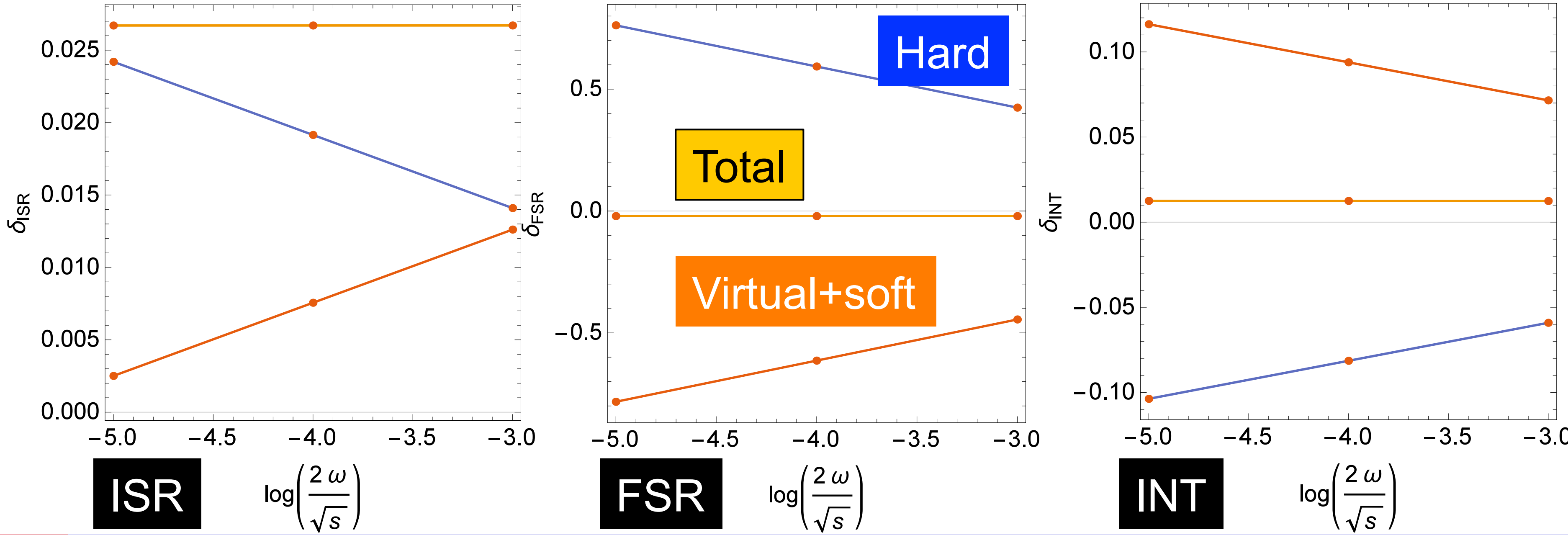
- The proton is treated as a point-like particle, exact for leptons

J. Van de Wiele and S. Ong, Eur. Phys. J. A49, 18 (2013).

- Proton FFs are taken into account, but inconsistently (p* virtuality)

Unified treatement of soft and hard photon

A. G. Aleksejevs, S. G. Barkanova, and V. A. Zykunov, Phys. Atom. Nucl. 79, 78 (2016)



Invariants and physical variables

Lorentz-invariant	$\bar{p}+p \to e^- + e^+$	$\bar{p}+p \to e^- + e^+ + \gamma$
$s=(p_1+p_2)^2$	Eq. (2)	Eq. (2)
$t=(p_2-k_2)^2$	Eq. (3)	Eq. (44)
$u=(p_1-k_2)^2$	Eq. (4)	Eq. (46)
$z_1=2pp_1$	0	$z+v-v_1$
$v_1=2pp_2$	0	independent
$z=2pk_1$	0	independent
$v=2pk_2$	0	independent
$s_1=(k_1+k_2)^2$	s	$s-z-v$
$t_1=(p_1-k_1)^2$	t	$t+v-v_1$
$u_1=(p_2-k_1)^2$	u	$u+v_1-z$

$$s+t+u=2m^2+2M^2 \approx M^2.$$

$$s = (p_1+p_2)^2 = 4E^2, \quad (2)$$

$$t = (p_2-k_2)^2 = -\frac{s}{4}(1+\beta^2-2\beta\cos\theta), \quad (3)$$

$$u = (p_1-k_2)^2 = -\frac{s}{4}(1+\beta^2+2\beta\cos\theta), \quad (4)$$

$$s+t+u=z+2m^2+2M^2.$$

$$t=\frac{1}{2}\left(2M^2+2m^2-s+z+\cos\theta\cdot\beta\sqrt{(s-z)^2-4m^2s}\right). \quad (44)$$

$$u=\frac{1}{2}\left(2M^2+2m^2-s+z-\cos\theta\cdot\beta\sqrt{(s-z)^2-4m^2s}\right). \quad (46)$$

Cross section (invariants)

$$\sigma_H = \frac{\alpha^3}{8\pi s} \iiint \frac{dv\,dz\,dv_1}{\sqrt{R}} \frac{s-z}{s} \sum_k R_k \cdot \theta(R) \cdot \theta_\omega \cdot \theta_P .$$

$$d\Phi_3 = \frac{\pi}{16\sqrt{\lambda(s, M^2, M^2)}} \frac{dt\,dv\,dz\,dv_1}{\sqrt{-\Delta_4}}$$

$$\Delta_4 = -R/16$$

$$R = -\det \begin{pmatrix} 2p_1p_1 & 2p_2p_1 & 2k_1p_1 & 2k_2p_1 \\ 2p_1p_2 & 2p_2p_2 & 2k_1p_2 & 2k_2p_2 \\ 2p_1k_1 & 2p_2k_1 & 2k_1k_1 & 2k_2k_1 \\ 2p_1k_2 & 2p_2k_2 & 2k_1k_2 & 2k_2k_2 \end{pmatrix}$$

$$\theta_P \equiv \theta(\Omega - p_0) = \theta\left(\Omega - \frac{v+z}{2\sqrt{s}}\right),$$

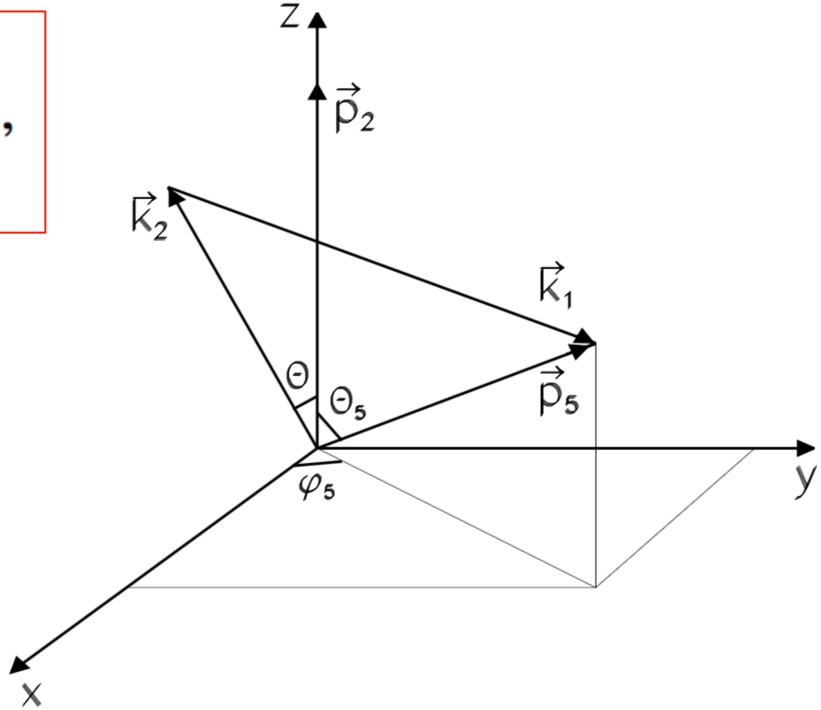
$$\theta_\omega = \theta(p_0 - \omega)$$

Invariants and Physical Variables

$$\frac{d\sigma_R}{d\cos\theta} = \frac{\alpha^3}{4\pi s}\int_\lambda^\Omega dp_0|\vec{p}|\int_0^\pi d\theta_p \sin\theta_p \int_0^{2\pi} d\varphi_p \frac{|\vec{k}_2|}{k_{10}g(x_0)}\sum_k R_k\theta_P.$$

$$g(x) = 1 + \frac{x(1-|\vec{p}|A(x^2-m^2)^{-1/2})}{\sqrt{x^2-2|\vec{p}|A\sqrt{x^2-m^2}+|\vec{p}|^2}},$$

$$v_1 = 2p_0p_{20} + 2|\vec{p}||\vec{p}_2|\cos\theta_5,$$
$$z_1 = 2p_0p_{10} - 2|\vec{p}||\vec{p}_1|\cos\theta_5,$$
$$v = 2p_0k_{20} + 2|\vec{p}||\vec{k}_2|A,$$
$$z = 2p_0(\sqrt{s}-k_{20}) - 2|\vec{p}||\vec{k}_2|A.$$

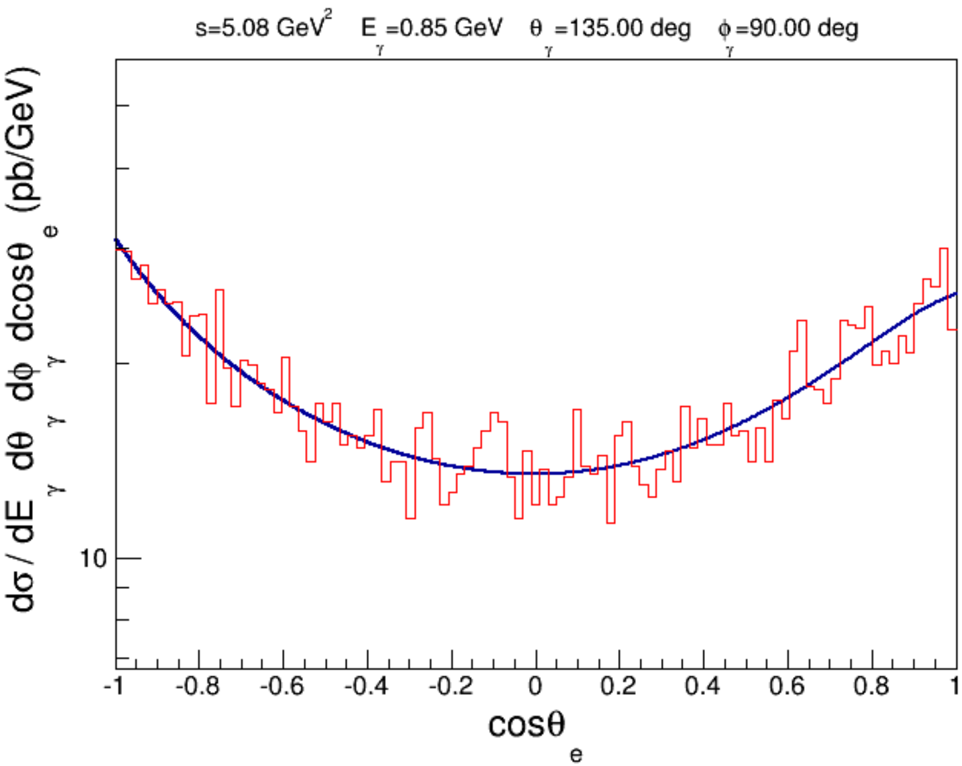


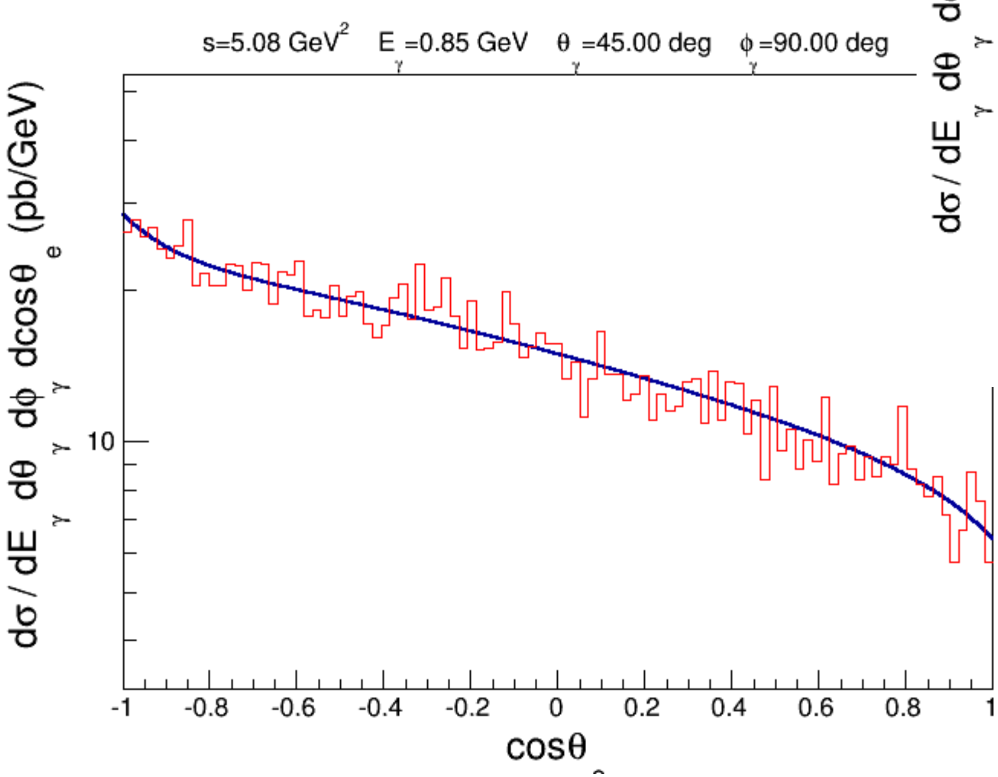
$$A = \cos(\widehat{\vec{p}_5,\vec{k}_2}) = \sin\theta\sin\theta_5\cos\varphi_5 + \cos\theta\cos\theta_5.$$

$$\frac{d\sigma}{ds\,d\cos\theta_\gamma\,dE_\gamma\,d\phi_\gamma\,d\cos\theta_e}$$

physical variables

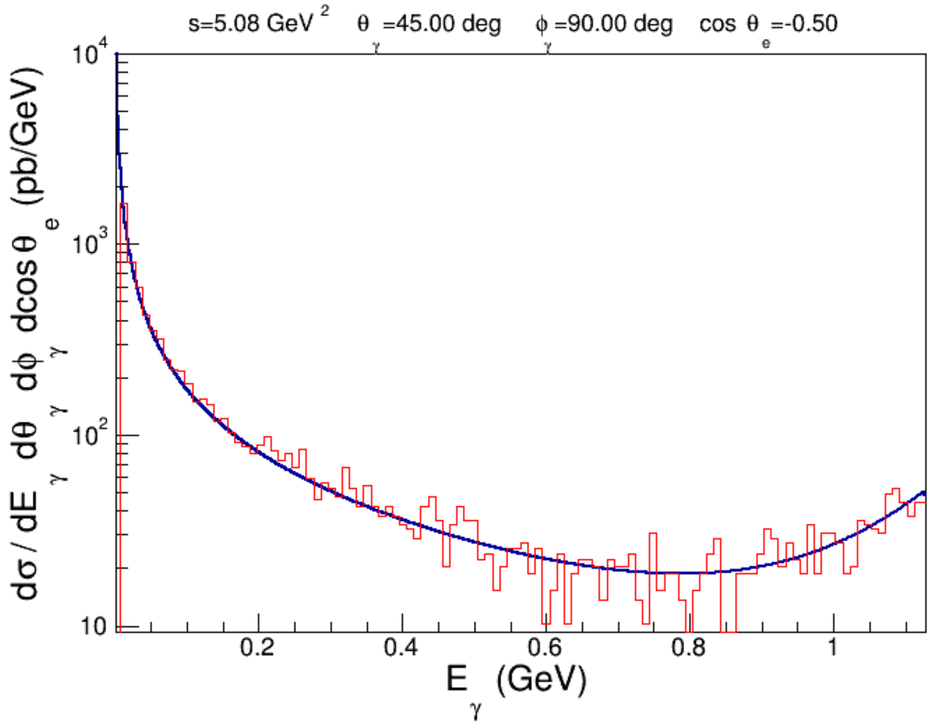
$\mathrm{Cos}\theta_e$



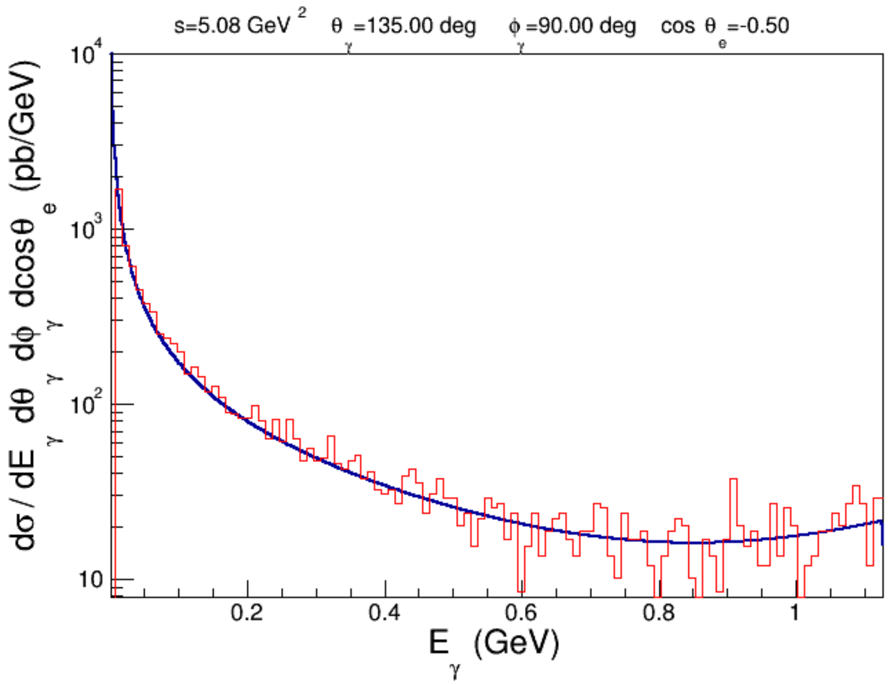


$$\frac{d\sigma}{ds\, d\cos\theta_\gamma\, dE_\gamma\, d\phi_\gamma\, d\cos\theta_e}$$

physical variables



Eγ



Conclusions

Status:

- A stand-alone MC generator is under test for

$$\bar{p}(p_-) + p(p_+) \to e^+(q_+) + e^-(q_-) + \gamma(k).$$

- Soft, virtual, real, hard photon emission

What is next:

- Full understanding of the 3-particle final state distributions
- Study of numerical singularities
- Projections of physical spectra in the relevant variables

- *Replace PHOTOS in PANDARoot for our reaction!*