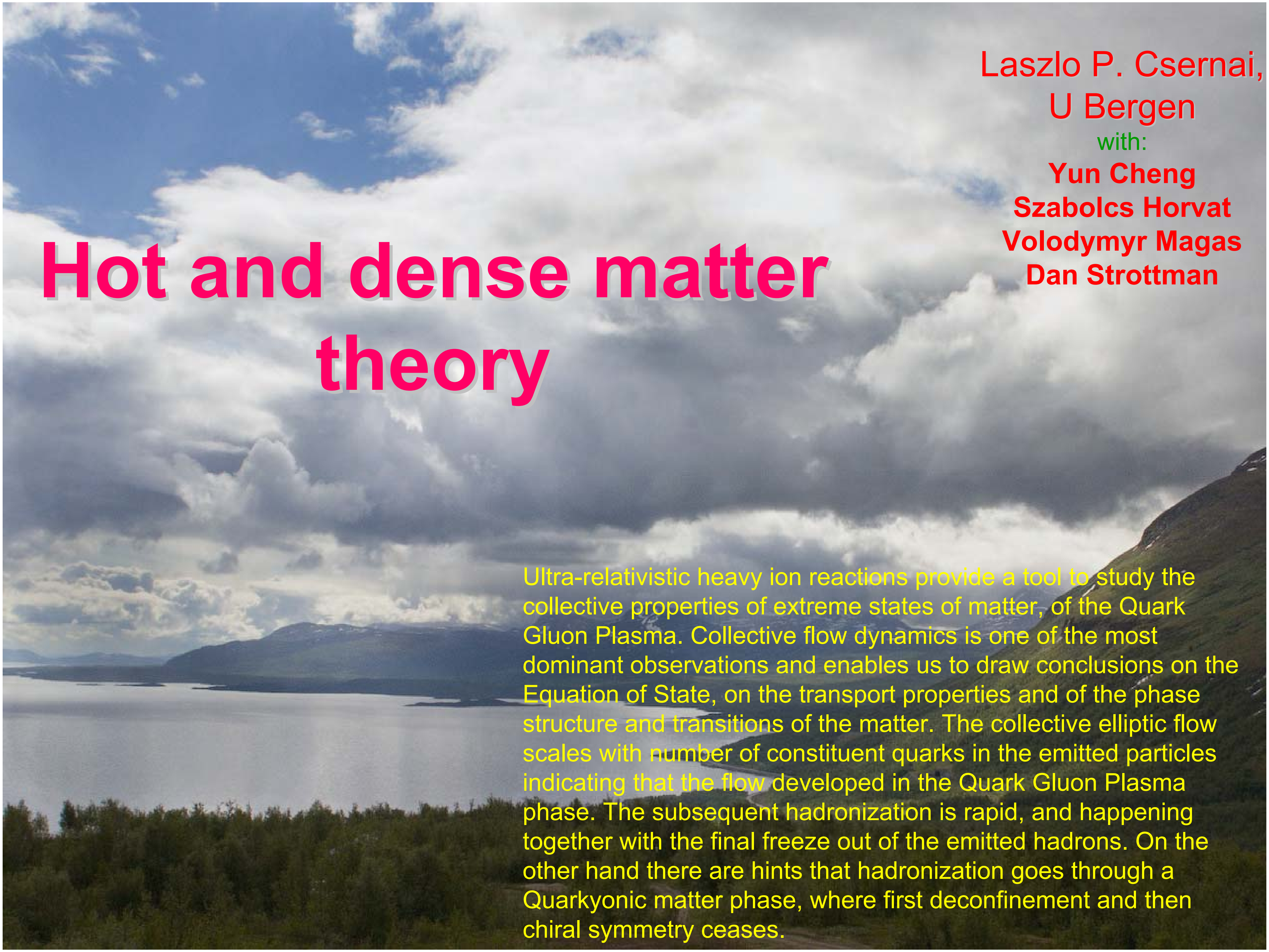


Laszlo P. Csernai,
U Bergen
with:
Yun Cheng
Szabolcs Horvat
Volodymyr Magas
Dan Strottman

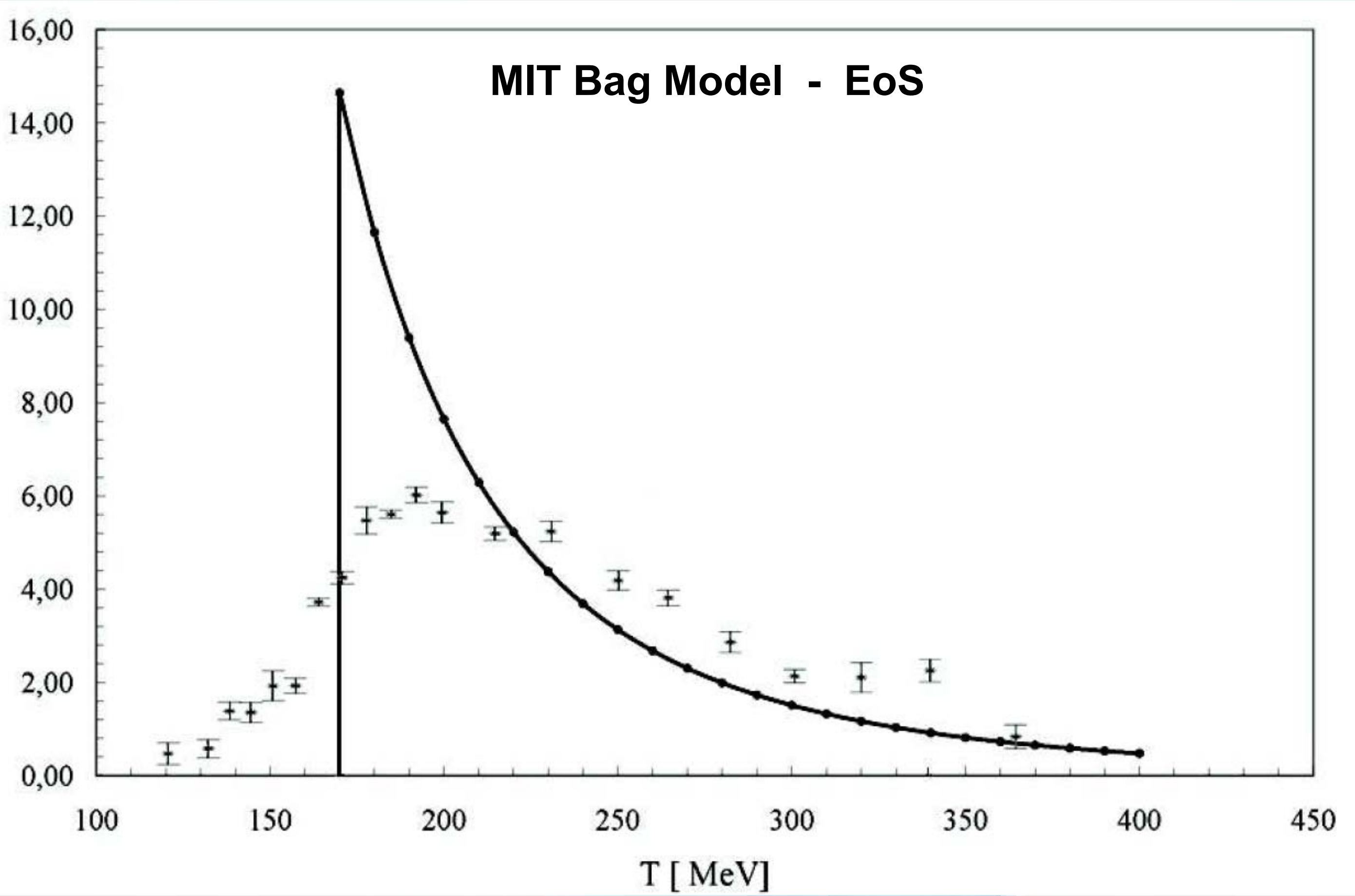
Hot and dense matter theory

Ultra-relativistic heavy ion reactions provide a tool to study the collective properties of extreme states of matter, of the Quark Gluon Plasma. Collective flow dynamics is one of the most dominant observations and enables us to draw conclusions on the Equation of State, on the transport properties and of the phase structure and transitions of the matter. The collective elliptic flow scales with number of constituent quarks in the emitted particles indicating that the flow developed in the Quark Gluon Plasma phase. The subsequent hadronization is rapid, and happening together with the final freeze out of the emitted hadrons. On the other hand there are hints that hadronization goes through a Quarkyonic matter phase, where first deconfinement and then chiral symmetry ceases.

Extreme states of matter - QGP

- Collective properties – Equation of State (EoS), new phases
  - Lattice QCD / Maxwell-constriction < problematic →
  - Hadronization from supercooled plasma
- Transport properties – viscosity, dissipation ← → EoS
  - Relativistic treatment is involved
- From collective dynamics in ultra-relativistic collisions,
  v1, v2, jets, Mach cones

Interaction Measure

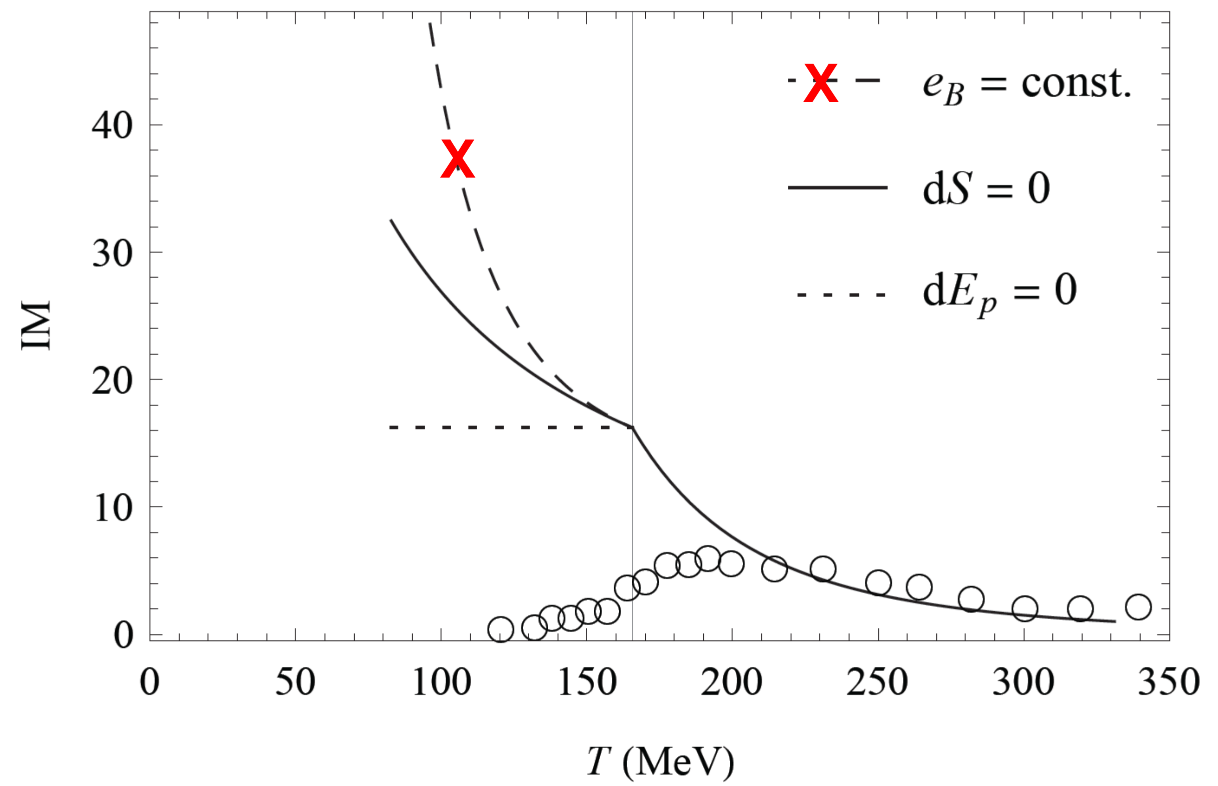


Clusterization in QGP due to dynamical streching of the plasma [Mishustin, CPOD 2007]

Dynamical viscous pressure ~ bulk stress → p<0 → cavitation ~ bubble / droplet formation [Rajogapal, Tripuraneni 2009]

Interaction measure, (e-3p)/T4 , from the MIT Bag model and from Lattice QCD [MILC]. The bag model is acceptable above T=200MeV. The bag model behaviour around Tc with a fix B leads **to negative pressure**.

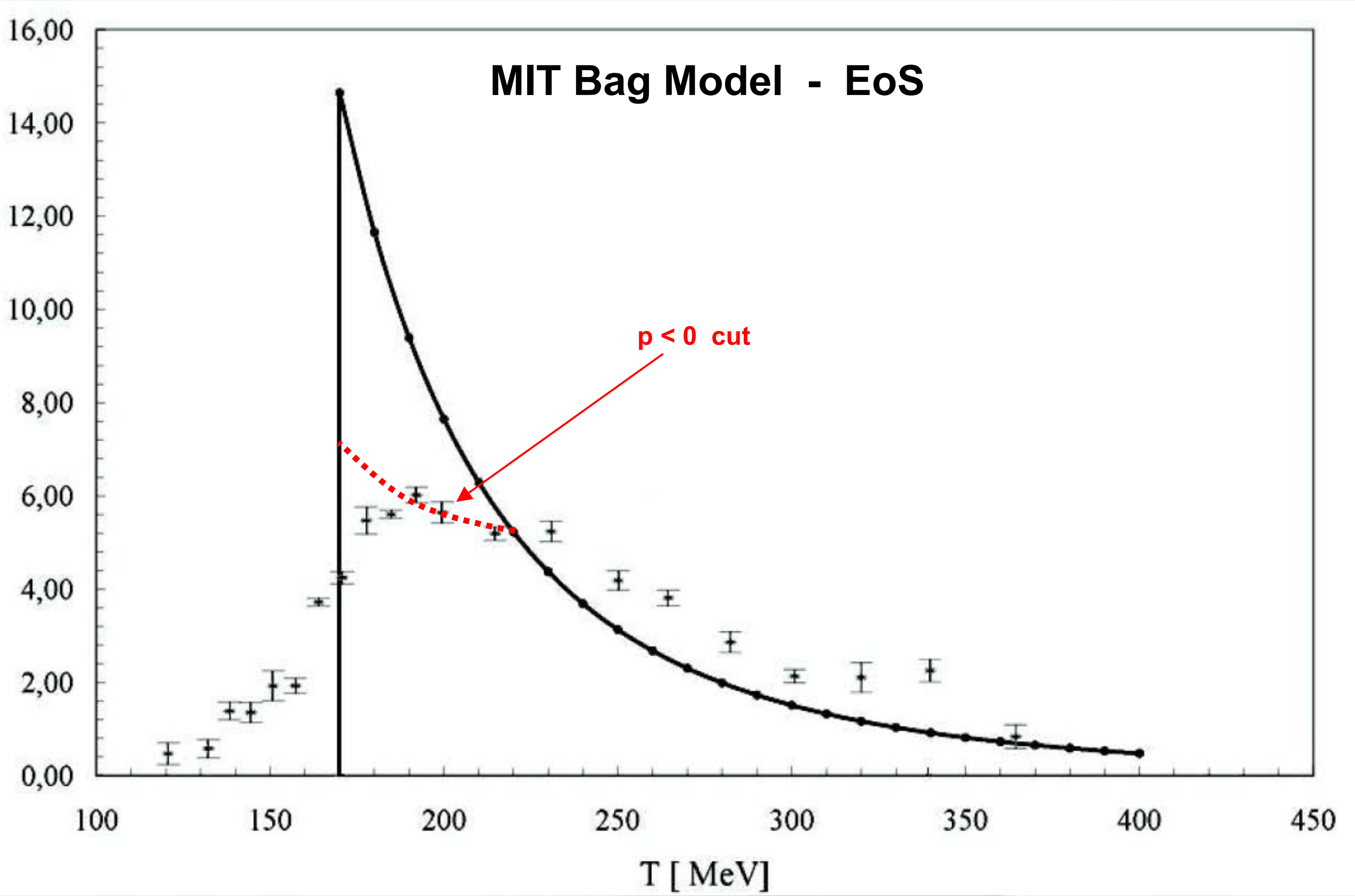
EoS – Surface of an expanding system



IM from the MIT Bag model and lattice QCD calculation (circles) [MILC 2005]. There is relatively good agreement above a temperature of 200 MeV. At T=165 MeV the pressure drops to zero. The Bag energy density must decrease, the change of T and s in adiabatic (full) and dissipative (dotted) expansion are shown.

→ Final stage EoS depends on hadronization mechanism !

Interaction Measure



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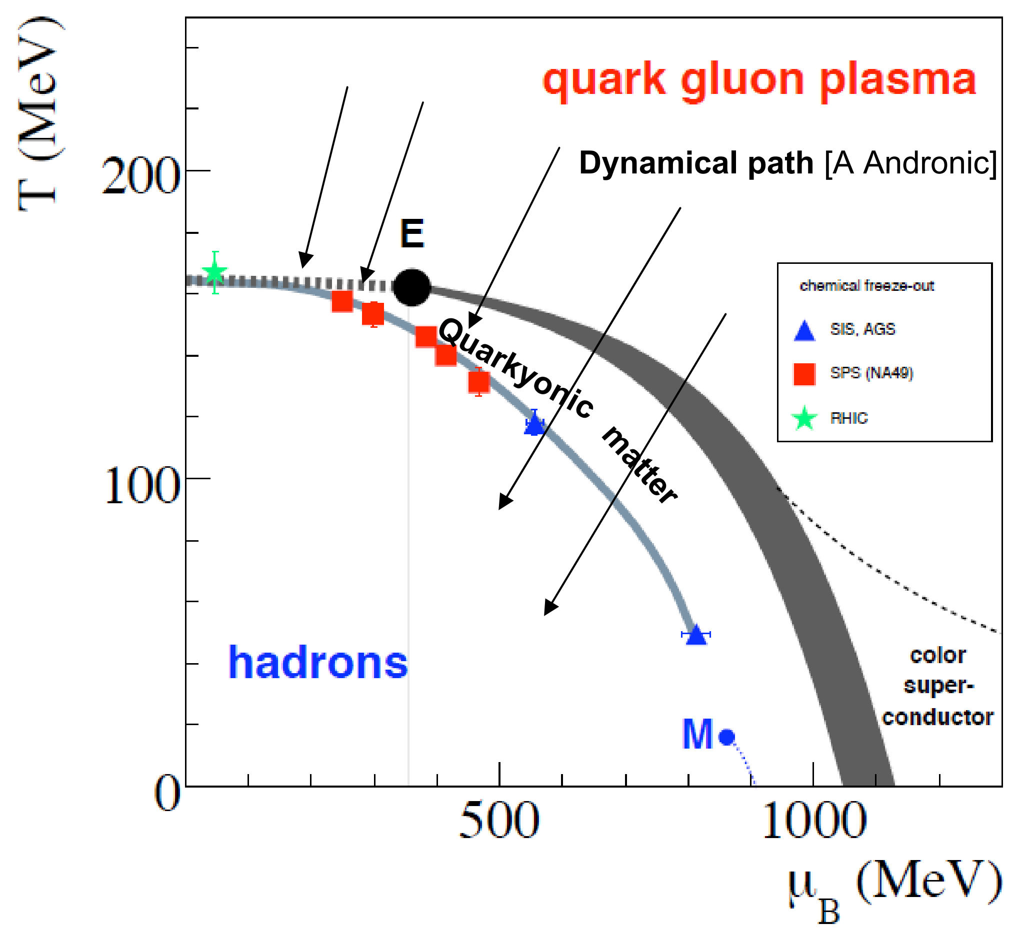
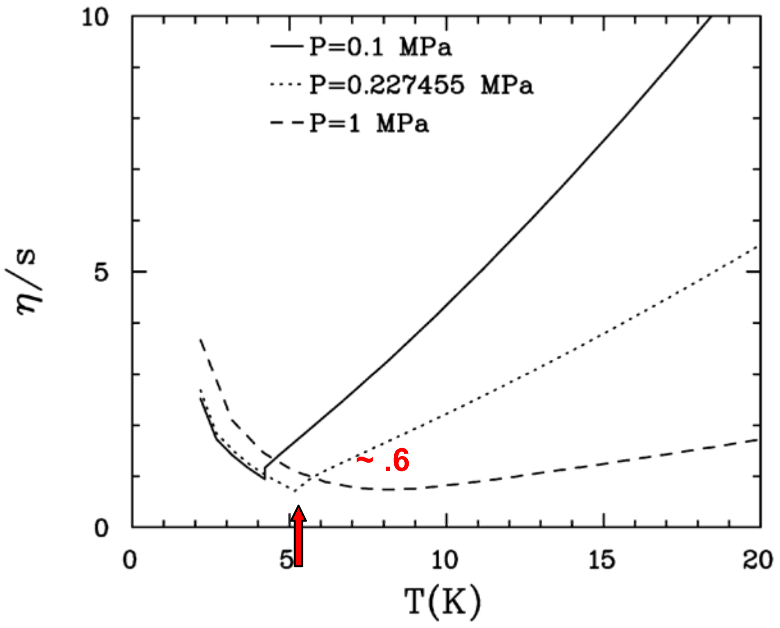
Fluid Dynamics

↔

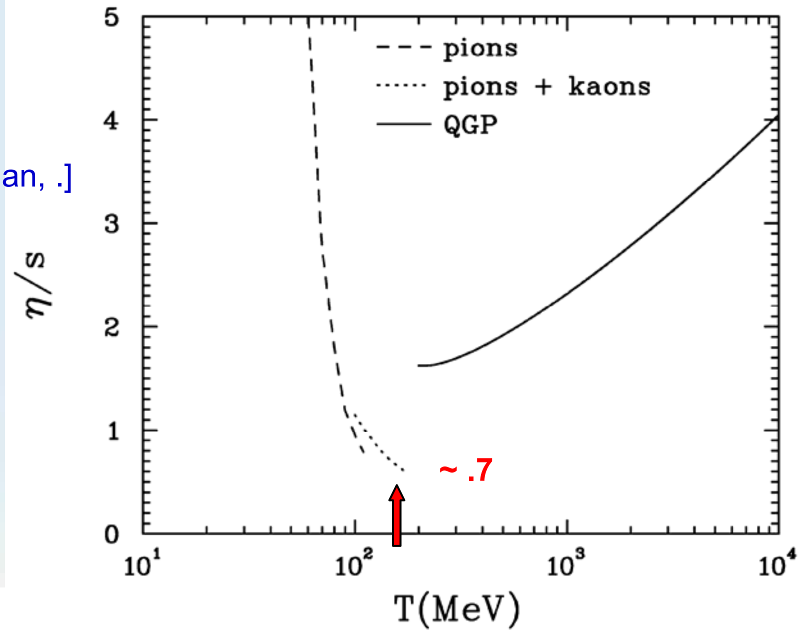
Equation of State & Transport Properties





Helium (NIST)

[Prakash, Venugopalan, .]

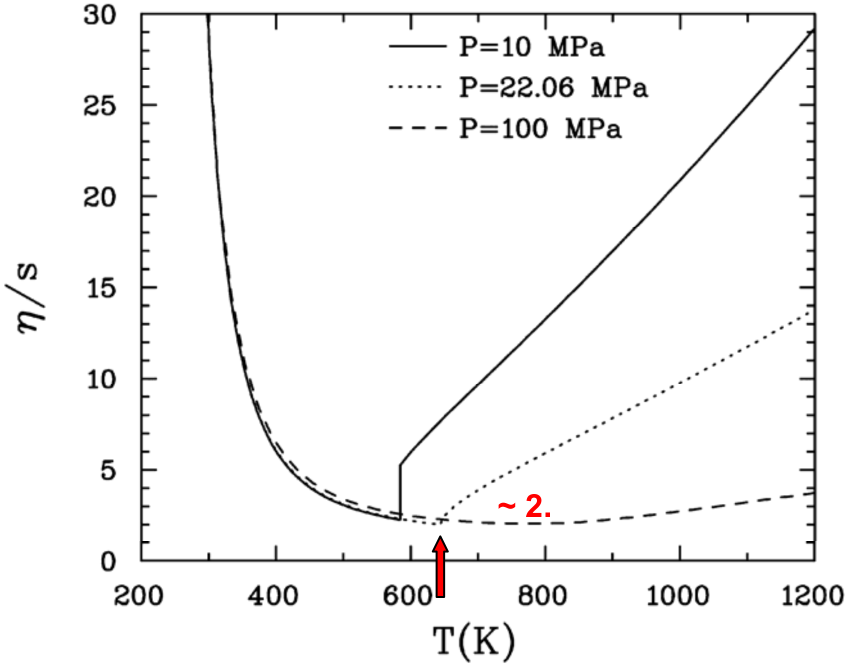


QGP (Arnold, Moore, Yaffe)

This phenomenon can help us to detect experimentally the critical point!

η can be determined from (i) fluctuation of flow parameters and from (ii) scaling properties of flow parameters.

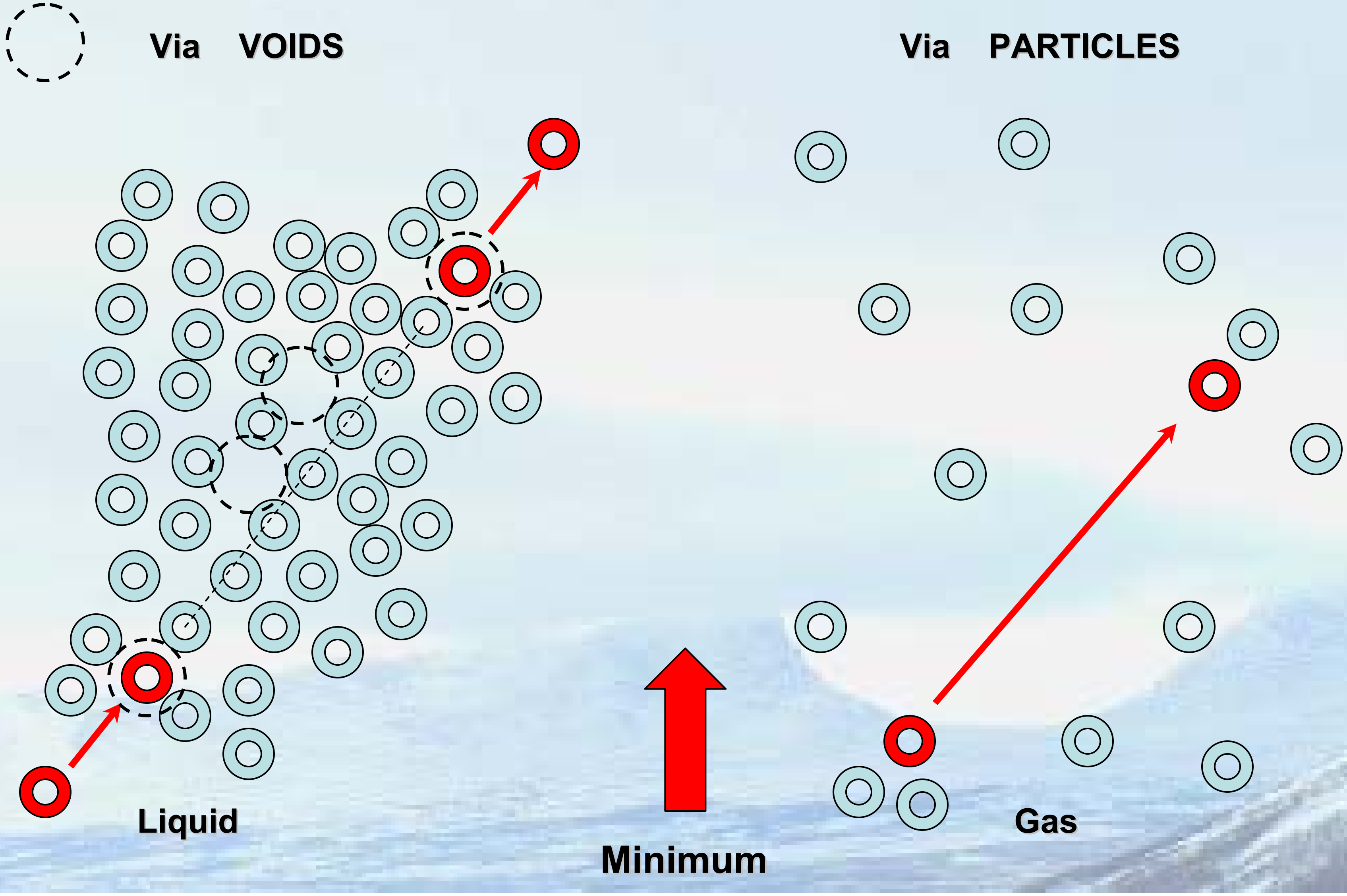
[L.P. Csernai, J.I. Kapusta, and L.D. McLerran, PRL **97,** 152303 (2006)]



Water (NIST)

Viscosity – Momentum transfer

[Enskog, 1921]



Via VOIDS

Via PARTICLES

Liquid

Minimum

Gas

Multi Module Modeling

M 1st – Initial state -- pre eq., Yang-Mills flux tube model

M 2nd – Fluid dynamics -- (near) Thermal equilibrium

M 3rd – Final Freeze-out -- simultaneous Hadronization & FO (recomb.)

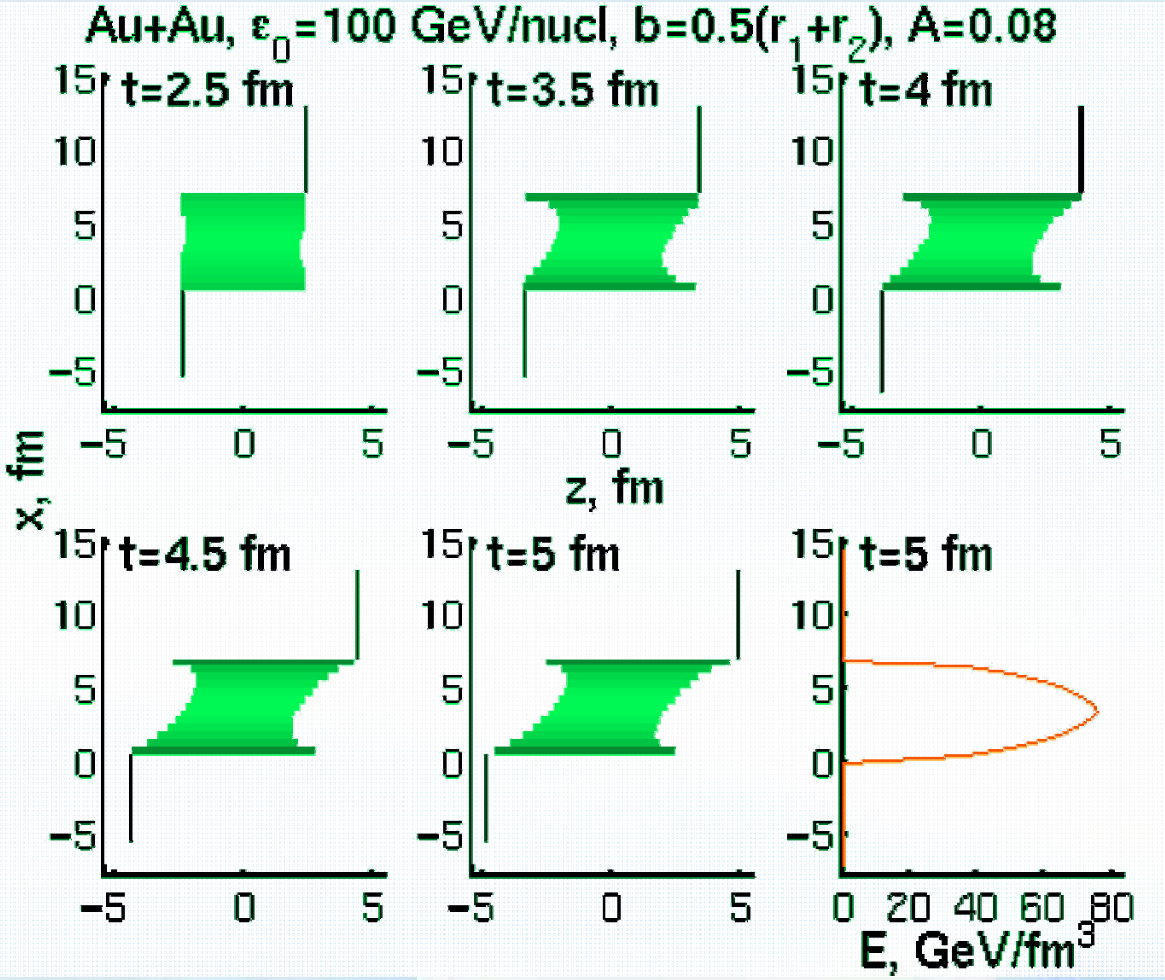
Collective dynamics → Flow observables

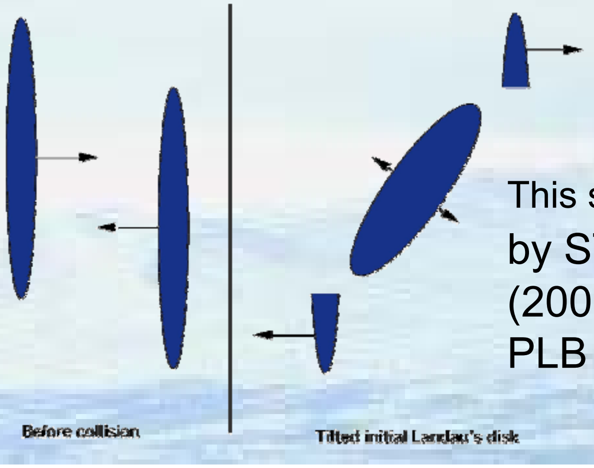
- V_1 & V_2 observed and analyzed
- CQN scaling ← Flow develops in QGP

Goal:

How these 3 stages and transport processes influence the observables

Initial State

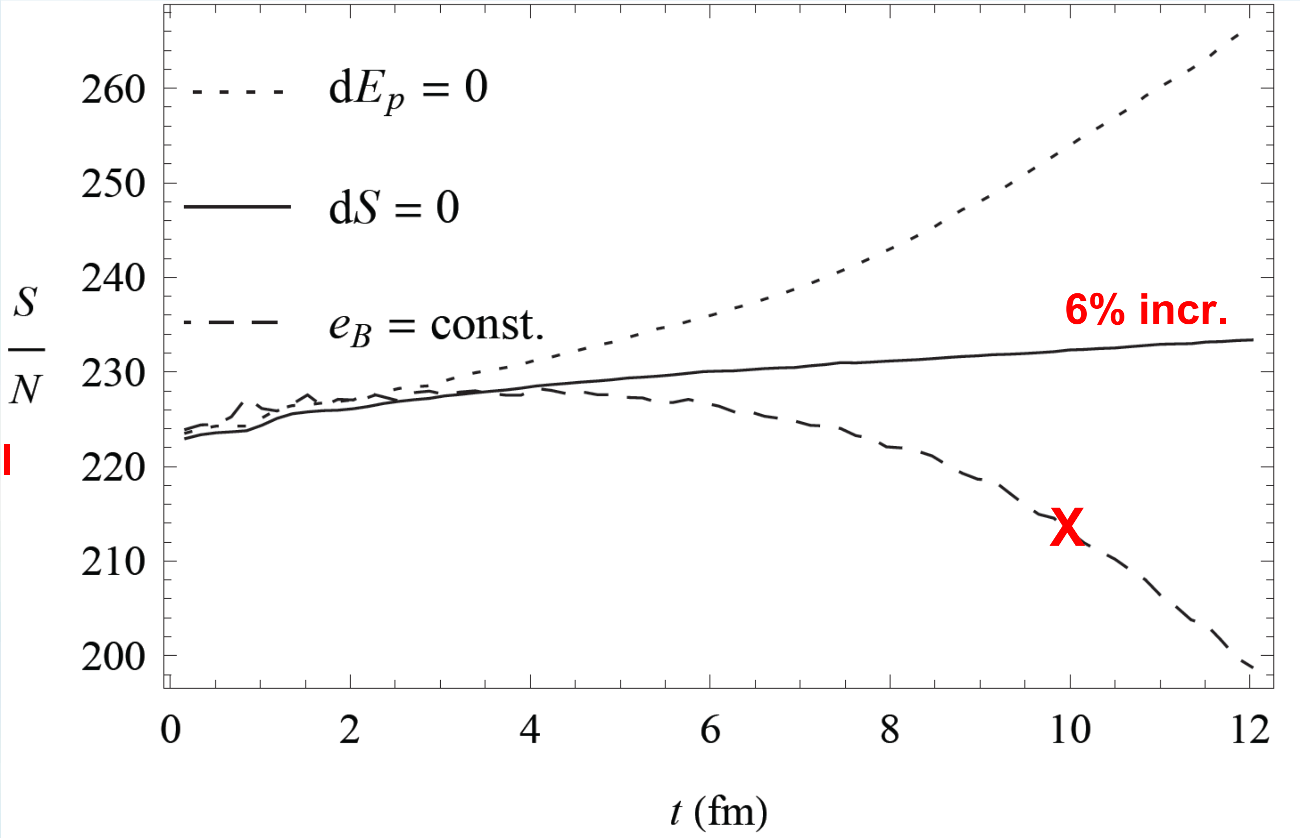




This shape is confirmed by STAR HBT: PLB496 (2000) 1; & M.Lisa &al. PLB 489 (2000) 287.

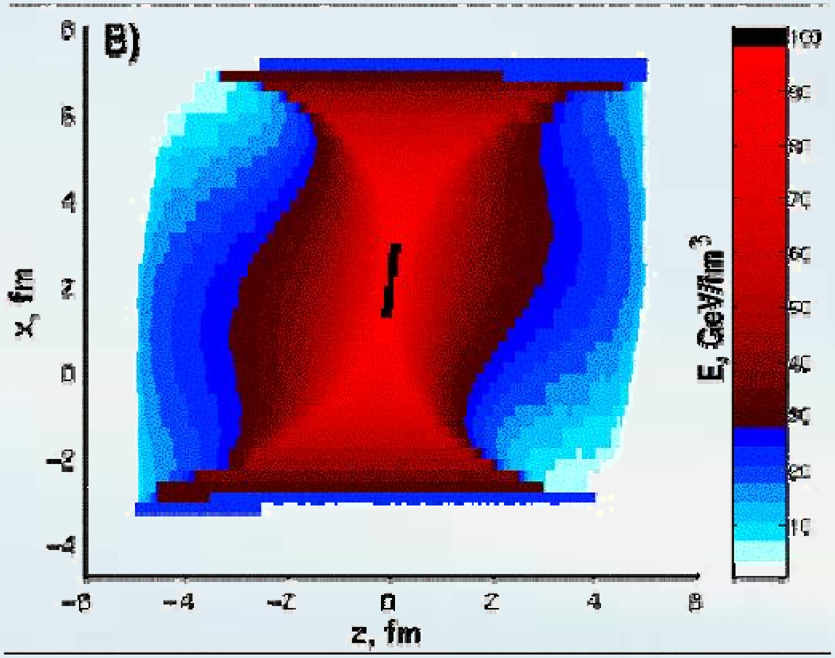
3^{rd} flow component

Entropy development in hydro

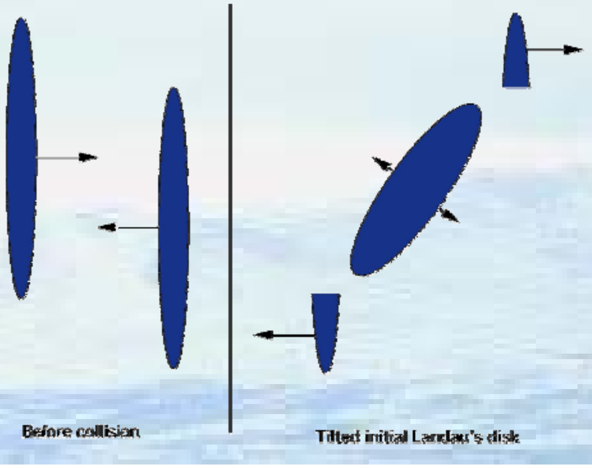


High initial entropy

Initial state – reaching equilibrium



Initial state
V. Magas, L.P. Csernai and D. Strottman
Phys. Rev. C64 (2001) 014901
Nucl. Phys. A 712 (2002) 167–204



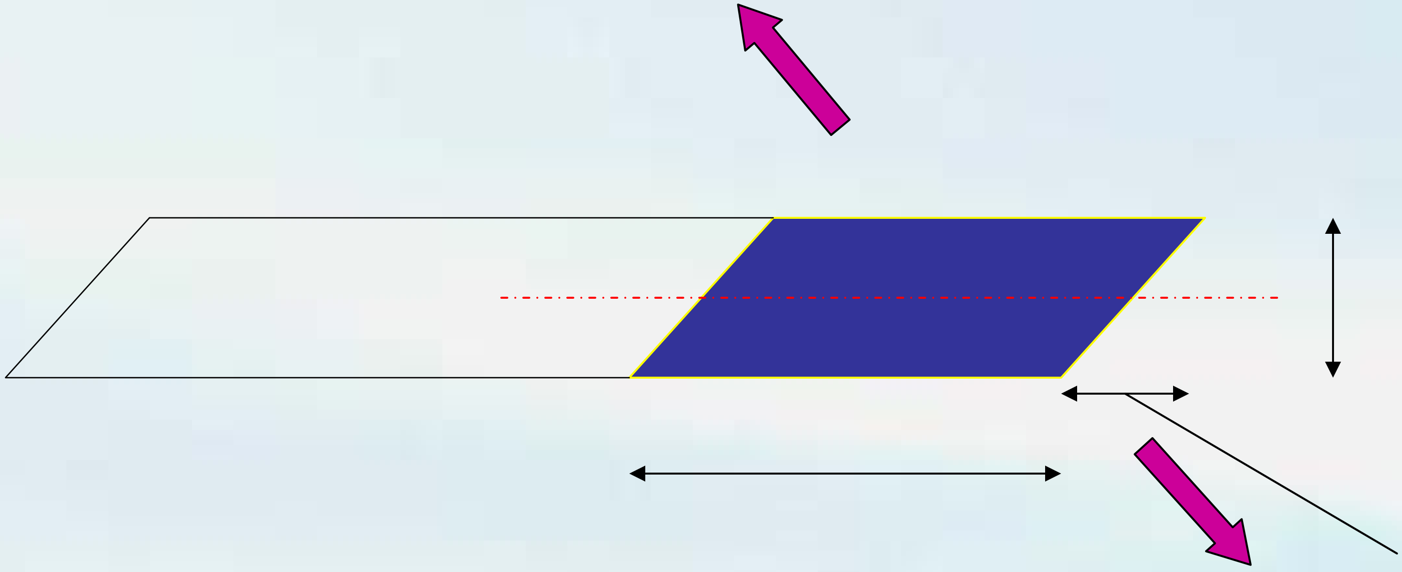
This shape is confirmed by STAR HBT: PLB496 (2000) 1; & M.Lisa &al. PLB 489 (2000) 287.

3rd flow component

M1

Flow is a *diagnostic* tool

Why should we measure v_1 ???



Impact par., **b**

Transparency – string tension, **A**

Equilibration time, **Tf**

Consequence:
$v_1(y)$, $v_2(y)$, ...

M2

Hydro

The relativistic Euler equations used are:

$$\frac{\partial N}{\partial t} + \vec{\nabla} \cdot (\vec{v} N) = 0 \,, \tag{1}$$

$$\frac{\partial M_k}{\partial t} + \frac{\partial}{\partial x_i}(v_i M_k) = -\frac{\partial P}{\partial x_k} \,, \tag{2}$$

$$\frac{\partial E}{\partial t} + \vec{\nabla} \cdot (\vec{v} E) = -\nabla \cdot (\vec{v} P) \,. \tag{3}$$

Here and in the following work, *N* is the particle number, ***M*** is the momentum, *E* is the energy and *P* is the pressure, all defined in the calculational frame. They are related to the rest frame quantities by the relations:

$$\begin{aligned} N &= \gamma n \\ \vec{M} &= \gamma^2 (\epsilon + P) \vec{v} \\ E &= \gamma^2 \epsilon + (\gamma^2 - 1) P \,. \end{aligned} \tag{4}$$

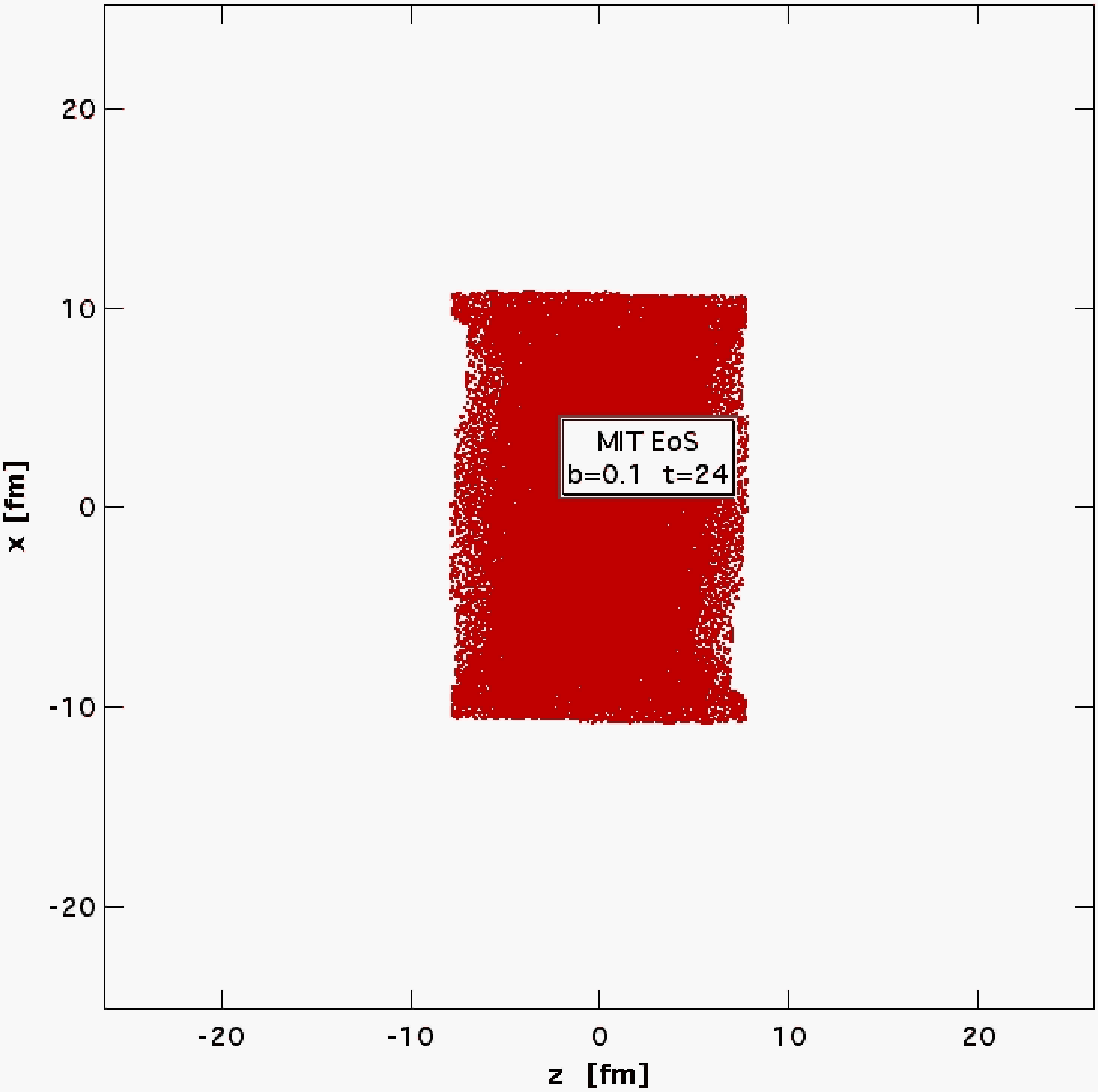
All quantities are given in the program (i.e., dimensionless) units. In the notation of Harlow *et. al (PIC code)*

$$N \rightarrow \frac{N}{n_0}$$

$$\vec{M} \rightarrow \frac{\vec{M}}{n_0 m_0}$$

$$E \rightarrow \frac{E}{n_0 m_0}$$

$$\epsilon \rightarrow \frac{\epsilon}{n_0 m_0} = y \,.$$

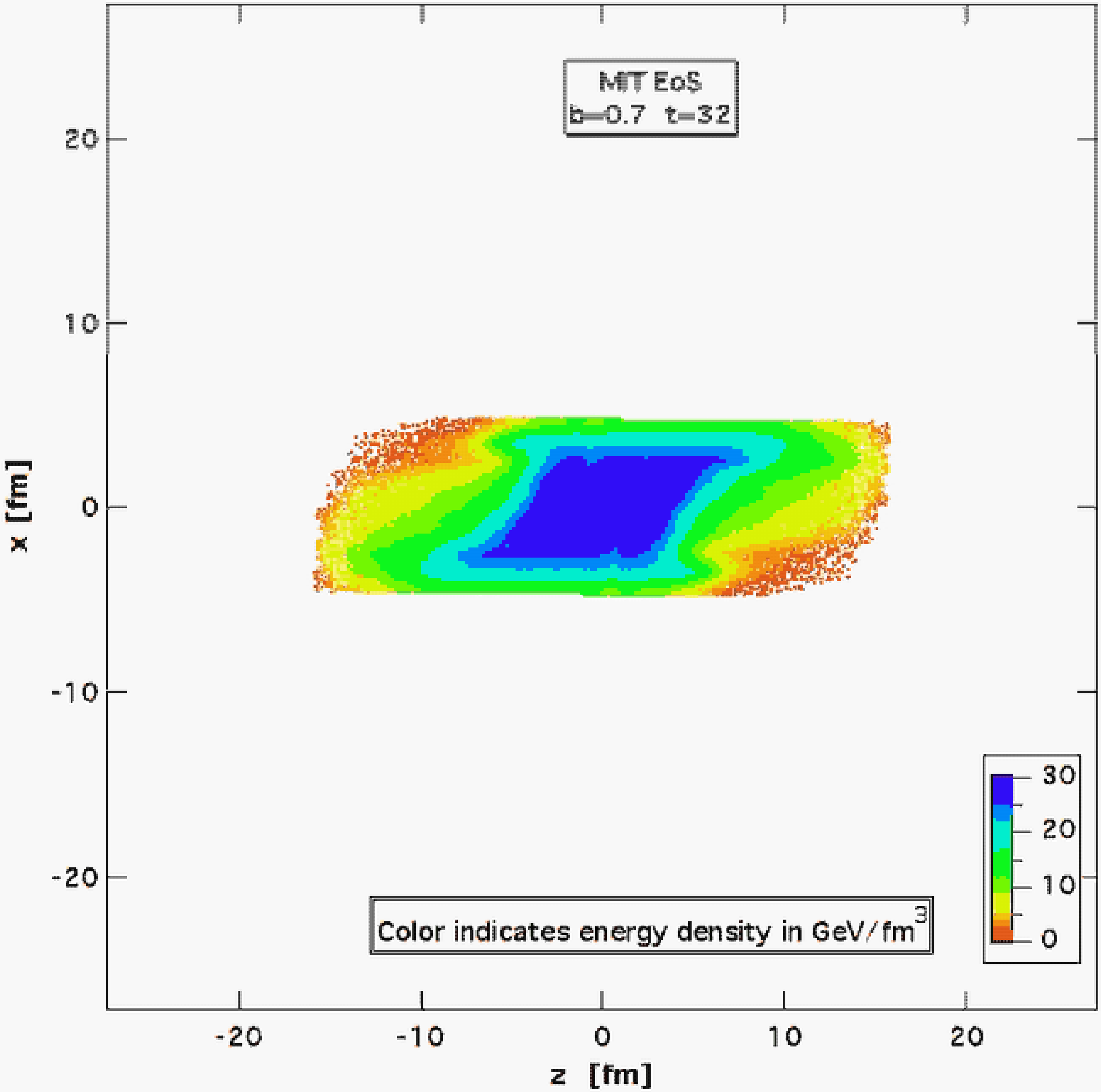


Au+Au at
65+65 A GEV,
b= 0.1 (R_p + R_t)

Plotted: positions of the lagrangian fluid cells, marker particles of the PIC method.

Cell resolution
tnc = 24

The initial structure is maintained in the expansion due to low (numerical) viscosity.



Au+Au 65+65 A GeV, b= 70 % of b_max

Lagrangian fluid cells, moving, ~ 5 mill.

MIT Bag m. EoS

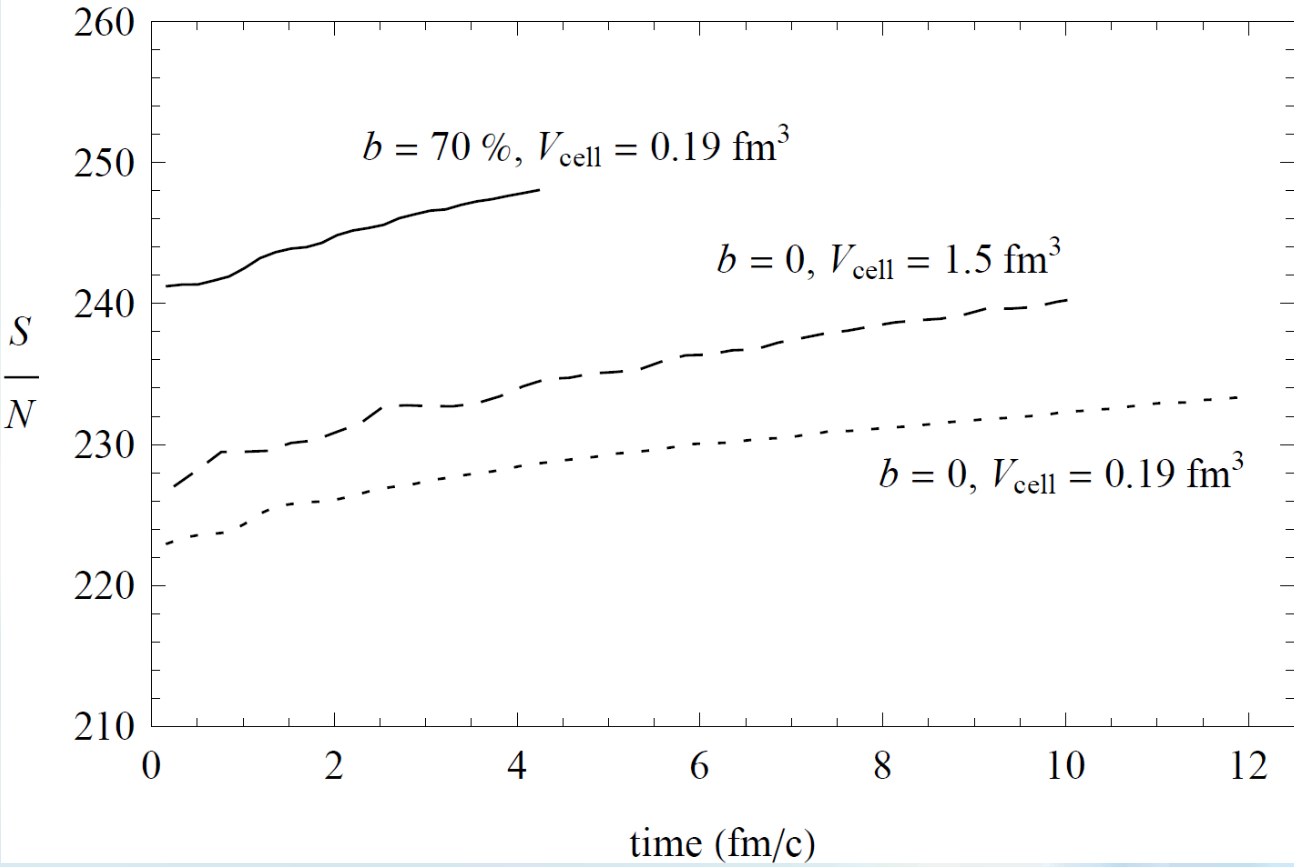
FO at T ~ 200 MeV, but calculated much longer, until pressure is zero for 90% of the cells.

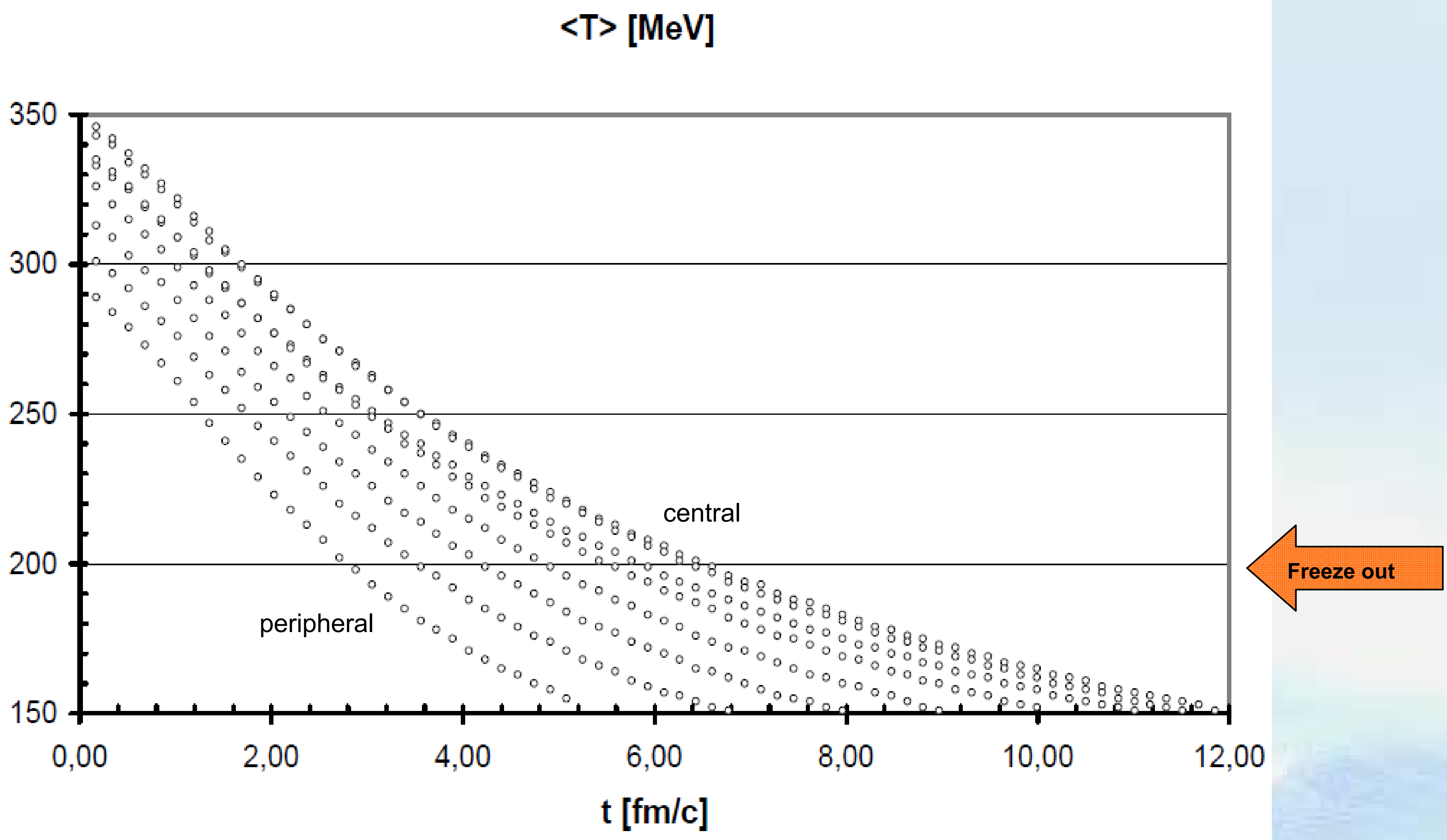
Structure and asymmetries of init. state are maintained in nearly perfect expansion.

Spatially tilted at FO, 3rd Flow component!

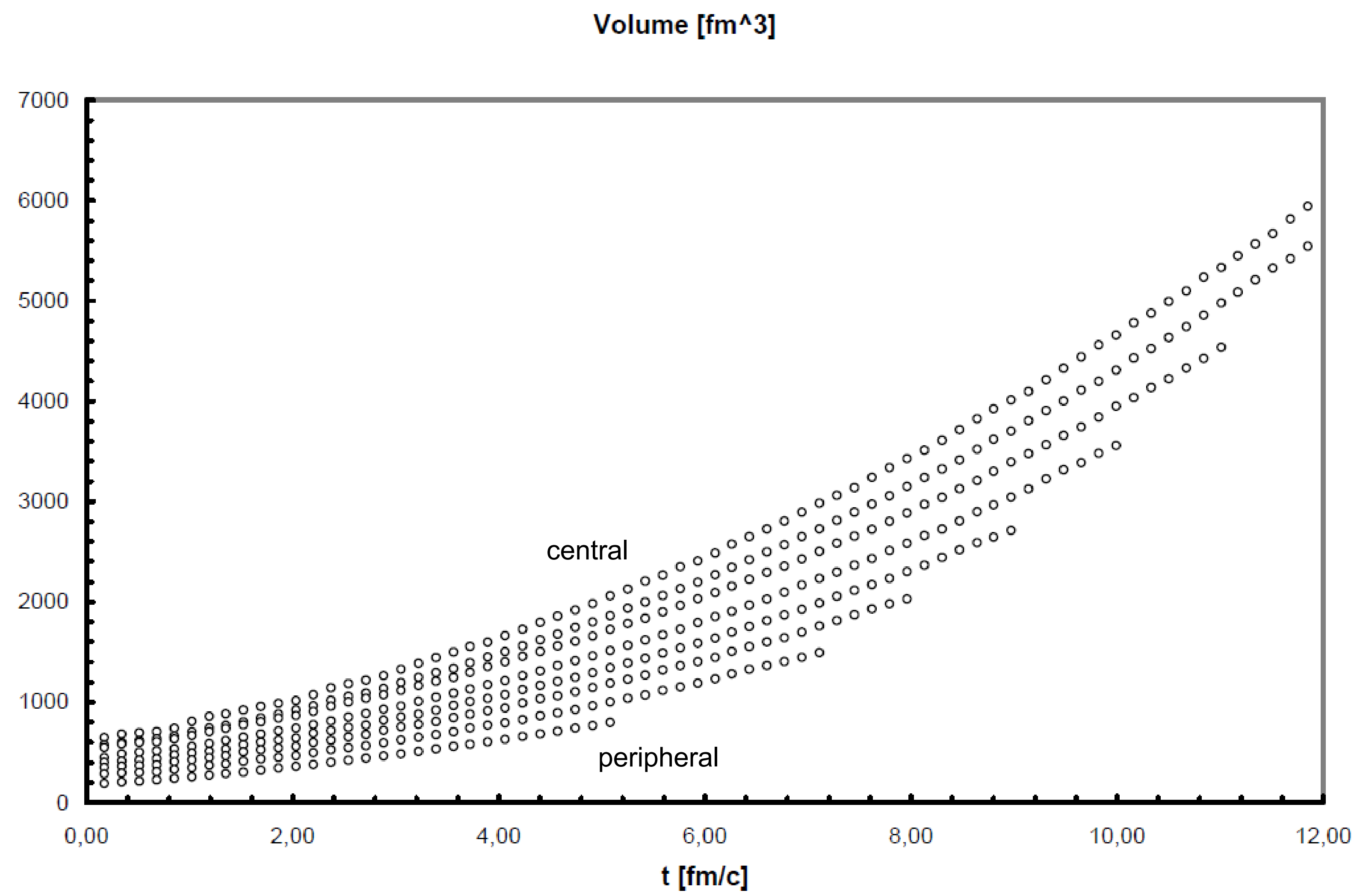
Numerical Viscosity

The expansion for central collisions shows a weak entropy increase, 5-6 %, due to the numerical viscosity, although the model considers a perfect fluid. The entropy increase due to numerical viscosity is smaller when the cell size is smaller. At late stages the entropy increase is weaker due to the smaller gradients.

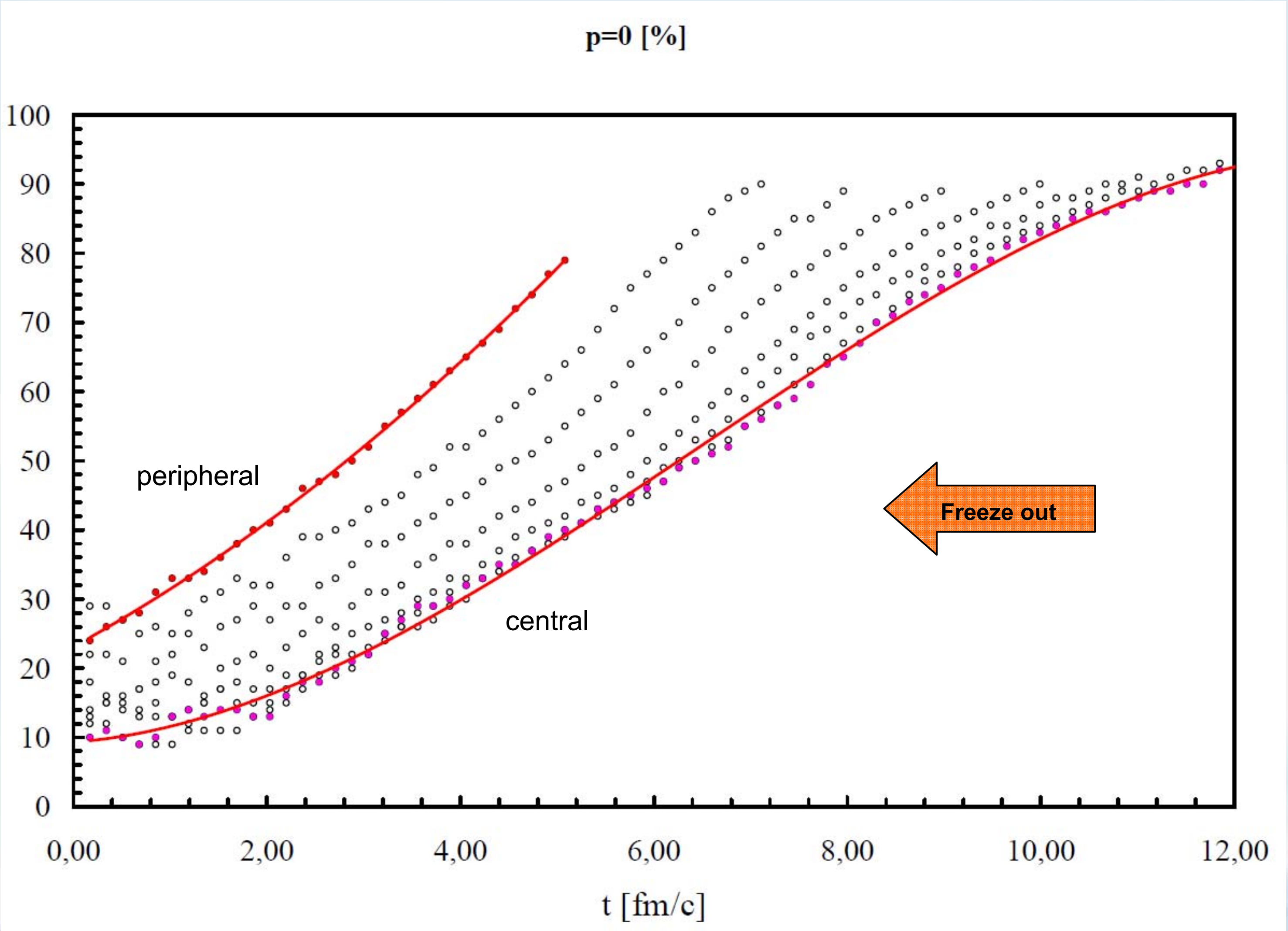




Average temperature versus time in Au+Au collisions at 65+65 AGeV, for impact parameters, b = 0, 0.1, 0.2, … 0.7 b_max from the top (0.00) down (0.7).



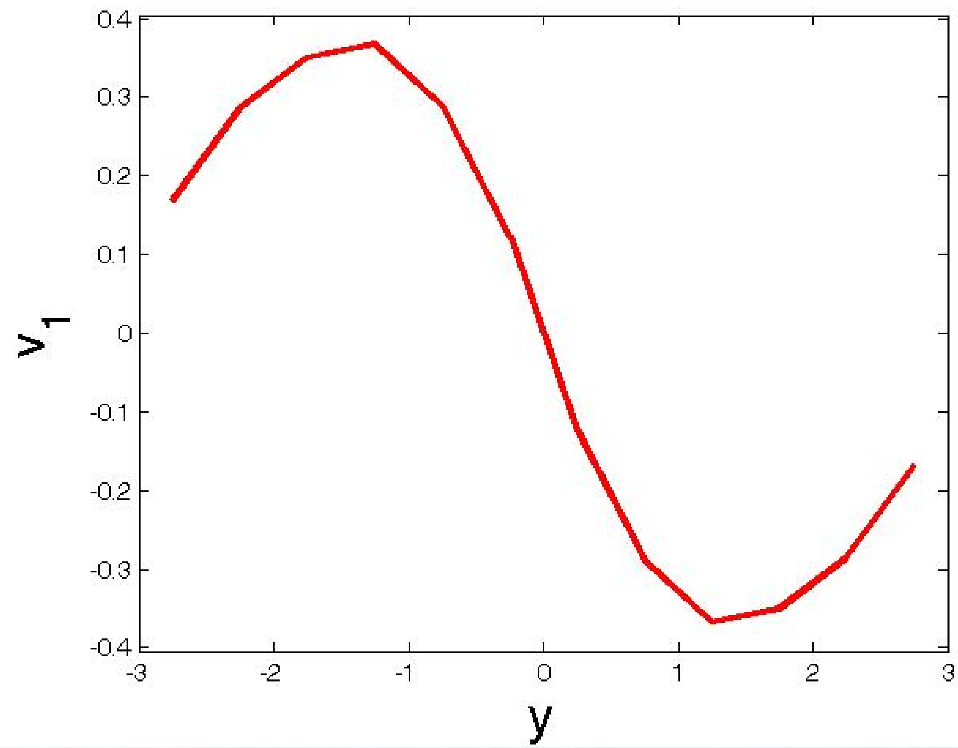
Volume of the expanding matter versus time in Au+Au collisions at 65+65 AGeV, for impact parameters, b = 0, 0.1, 0.2, … 0.7 b_max from the top (0.00) down (0.7).



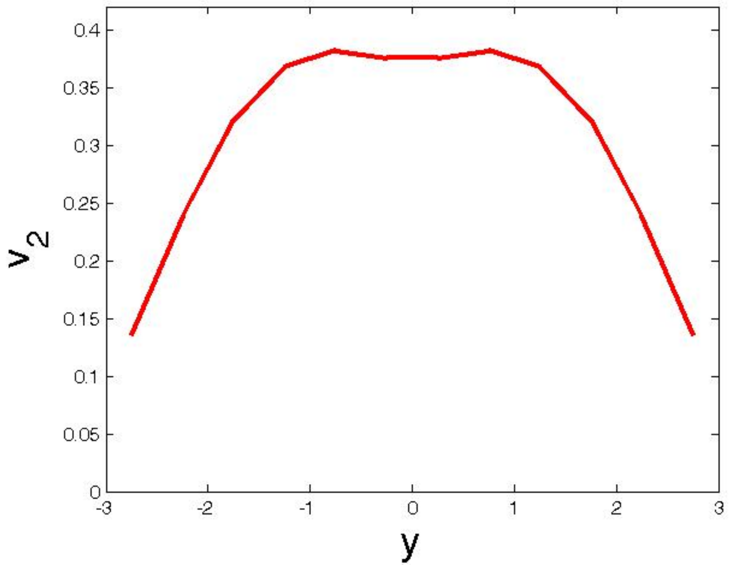
Percentage of the cells with vanishing pressure (P=0) versus time in Au+Au collisions at 65+65 AGeV, for impact parameters, b = 0, 0.1, 0.2, … 0.7 b_max. The most peripheral collision at the top (b=0.7) and the most central one (b=0.00) are indicated in red with a trend line.

Freeze Out

b=30% b-max. *Flow in hydro, after appr.(*) F.O.*

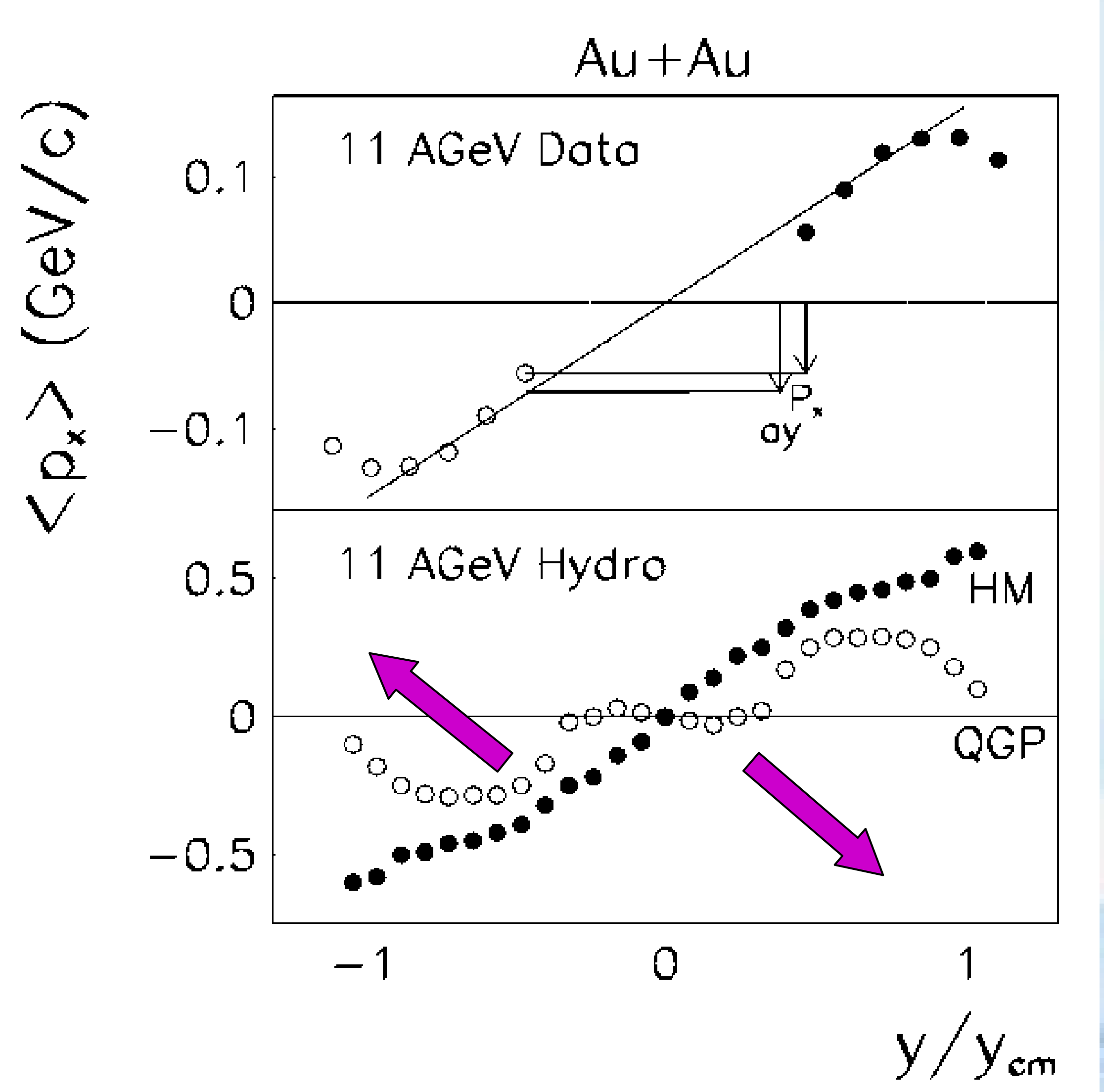


Correct FO description is of Vital Importance !



(*) Thermal smoothing in z-direction only with T_{FO} = 170 MeV and m_{FO} = 139 MeV (both fixed). Transverse smoothing would further reduce the magnitude of v1 (and v2).

„3rd flow” component



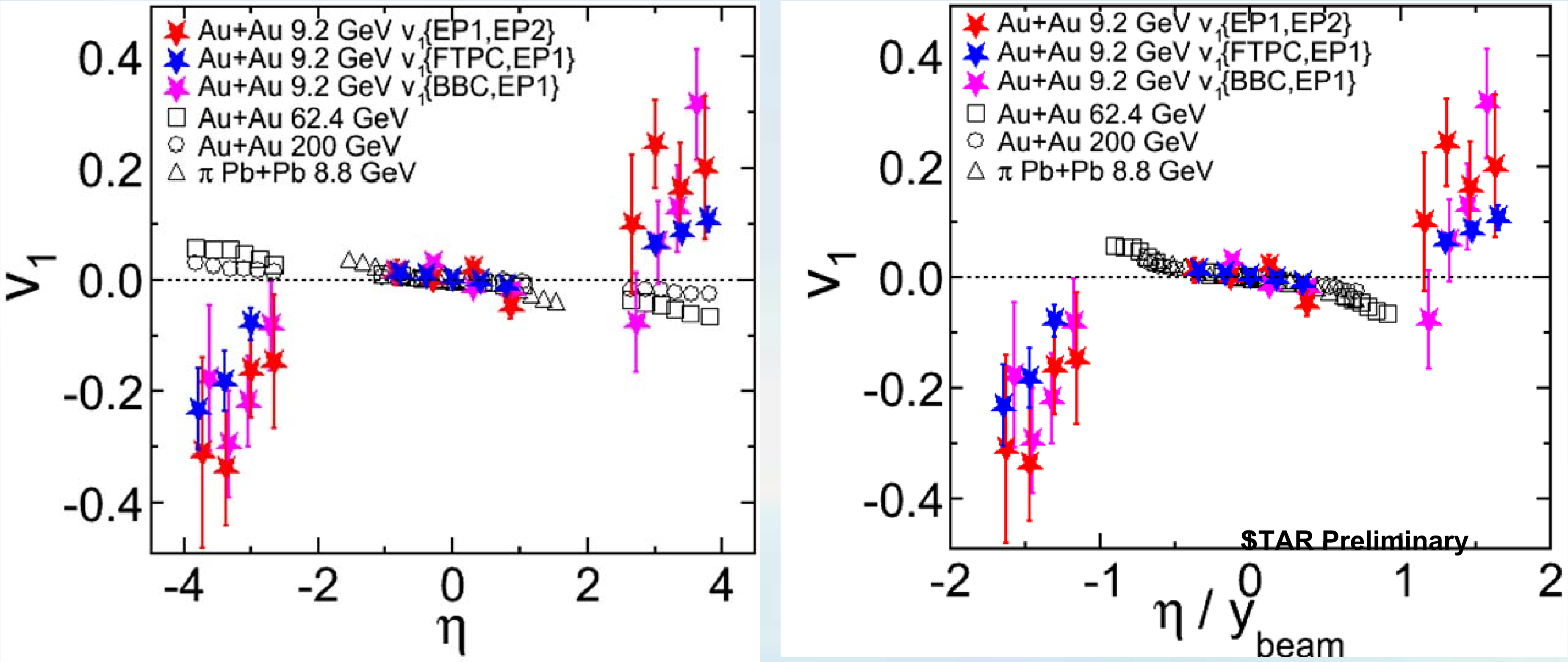
Csernai & Röhrich

[Phys.Lett. B458 **(99)** 454]

Hydro

[Csernai, HIPAGS ’93]
[Bravina, Csernai et al., PRC 50 (1994) 2161]

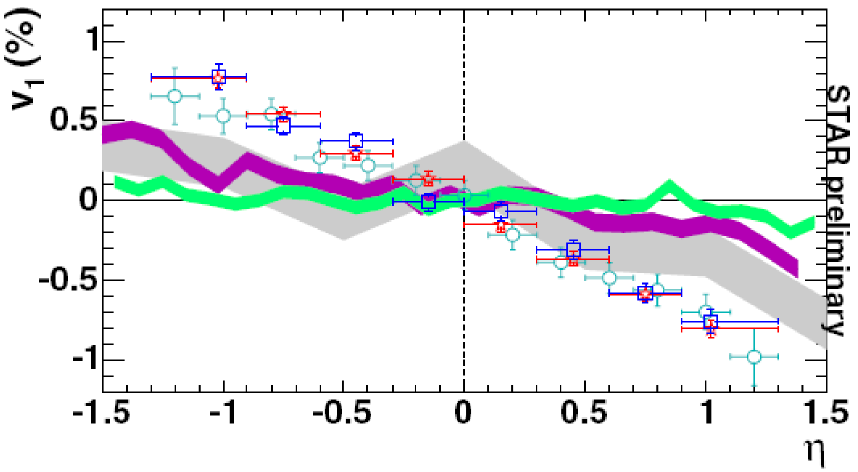
Directed Flow v_1

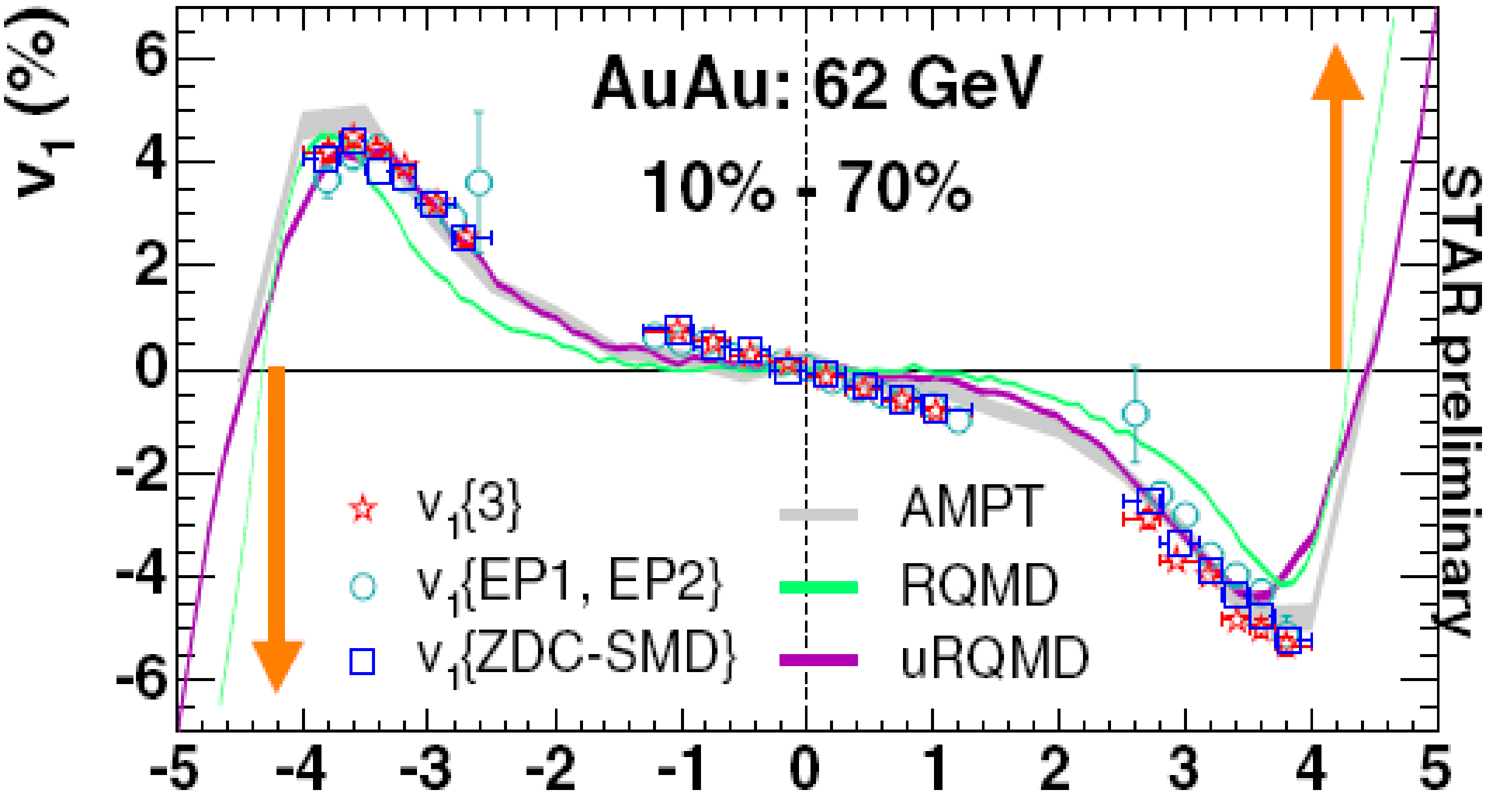


- At mid-rapidity, all the results have comparable values. At forward rapidity, the trend of v_1 from low energy is different from high energies. This is due to early longitudinal collision dynamics.
- V_1 values lie on a common trend.

STAR : PRL 92 (2004) 062301
PRL101（2008）252301
NA49: PRC68(2003)034903

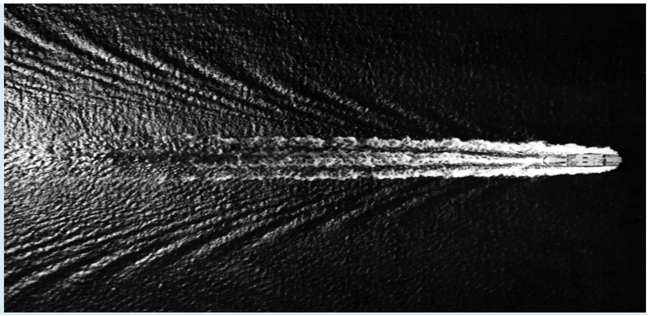
[G. Wang / STAR –
Nucl. Phys. A 774 (2006) 515–518]

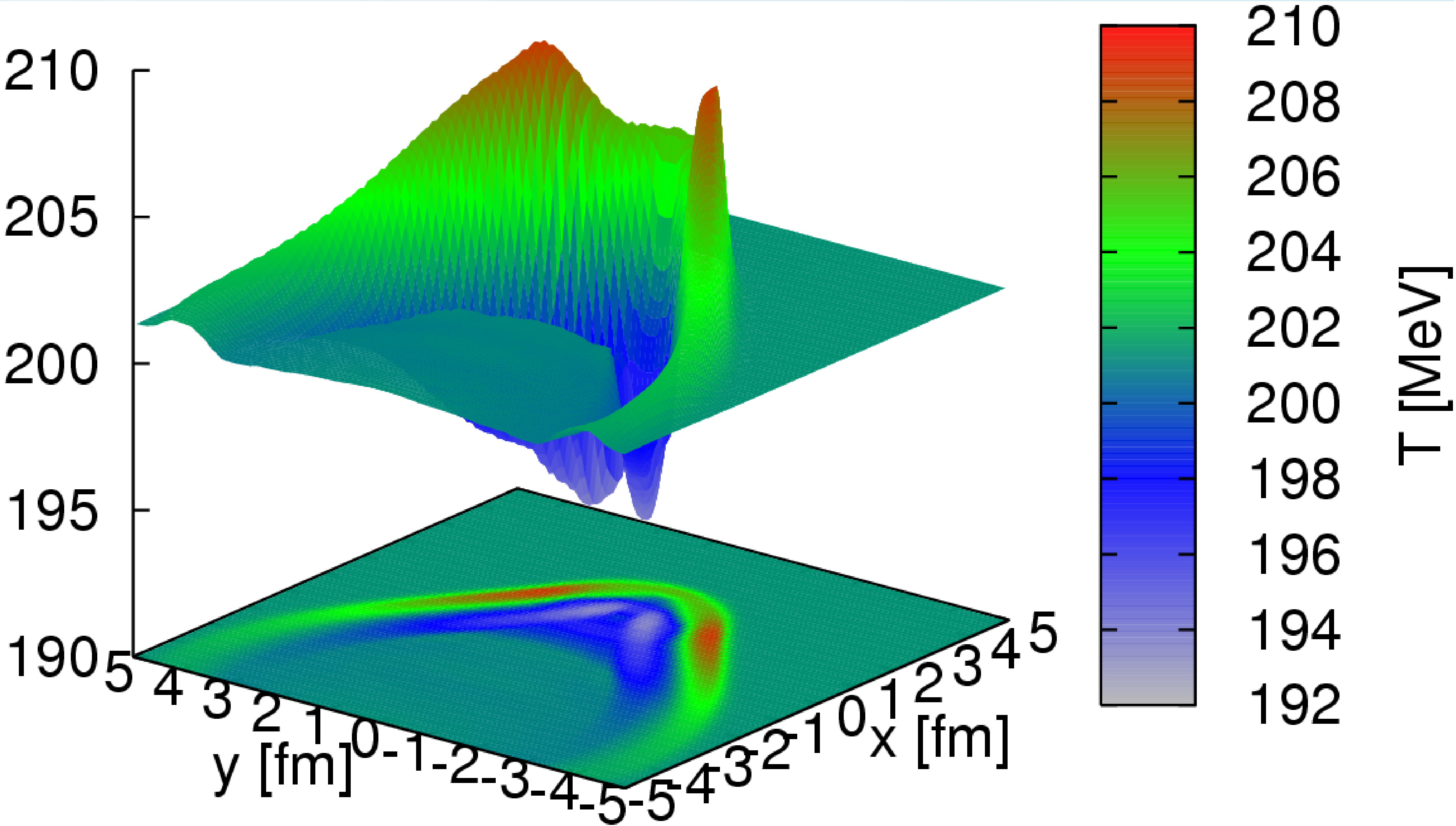




Jet quenching – Mach Shock Cone

[B. Betz, U. Frankfurt]





Freeze Out

Rapid and simultaneous FO and “hadronization”

- **Improved Cooper-Frye FO**:
- - Conservation Laws: $\left[T^{\mu\upsilon}\Lambda_{\upsilon}\right]=0,\ \left[N^{\upsilon}\Lambda_{\upsilon}\right]=0$
- - Post FO distribution: $\Theta(p^{\upsilon}\Lambda_{\upsilon})\ f(p)>0$

[L.P. Csernai,
Sov. JETP, 65 (l987) 216.]

[Cancelling Juttner or
Cut Juttner distributions.]

- **Hadronization ~ CQ-s**
- - Pre FO: Current q and $\overline{q}$, QGP
- - Post FO: Constituent q and $\overline{q}$
- - N_q and $N_{\bar{q}}$ are conserved in FO!!!

- **Choice of F.O. hyper-surface / layer**

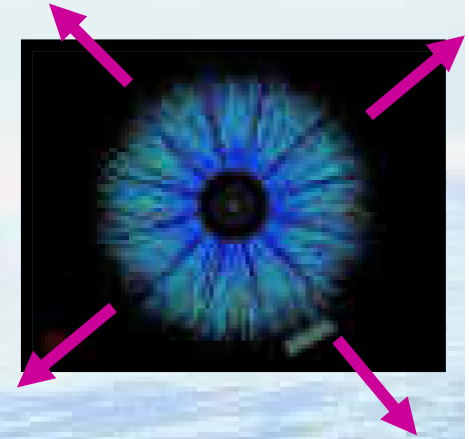
M3

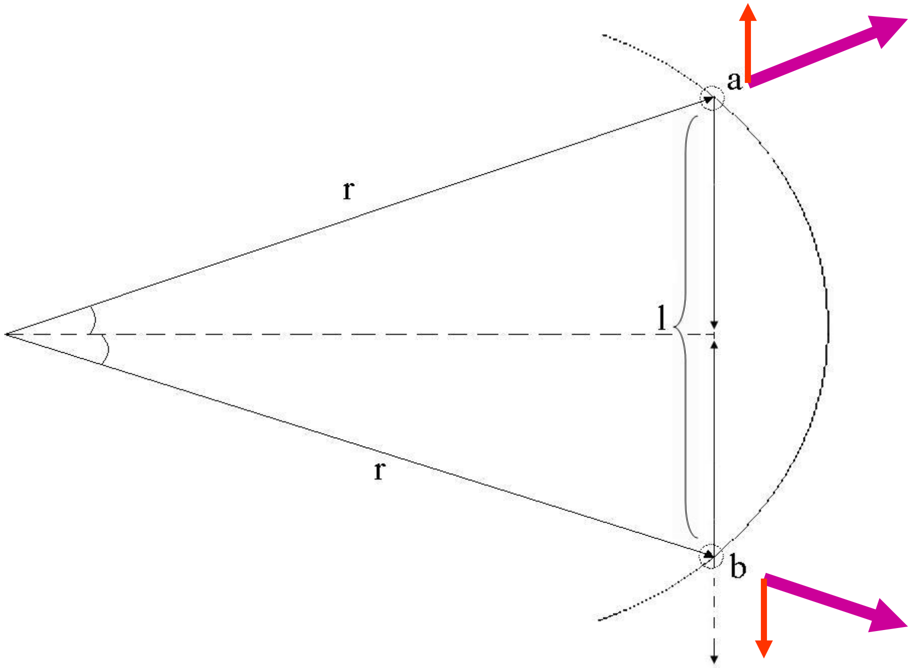
Simple geometrical FO

FO condition reads like **[Bondorf et al., NP A296 (1978) 320]**

$$\frac{l}{r} v_r = u_{ther} \,,$$

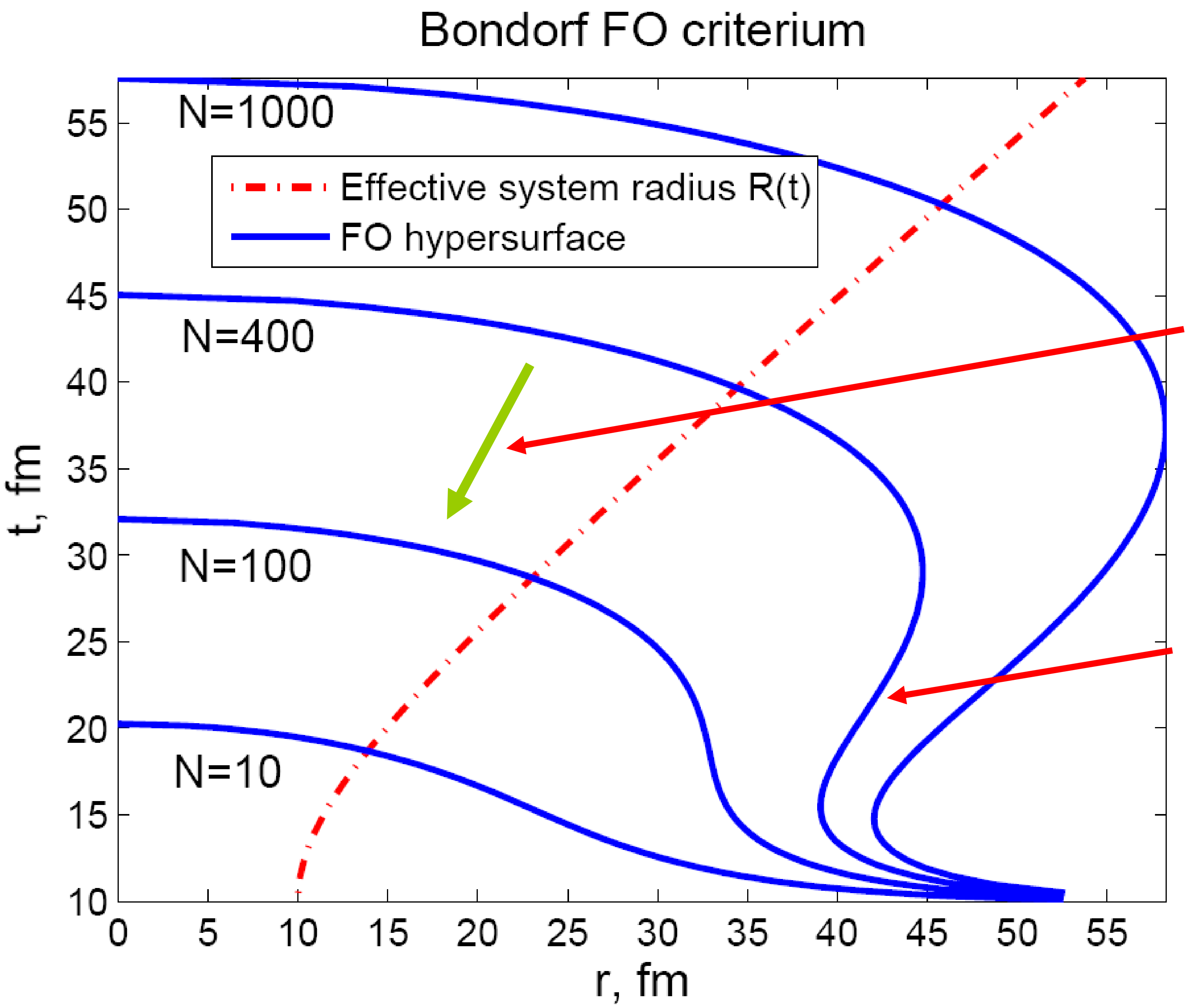
where u_{ther} is thermal velocity, v_r is r-directed component of flow velocity, l is average distance between particles.





M3

Simple geometrical FO



Recom-
bination:

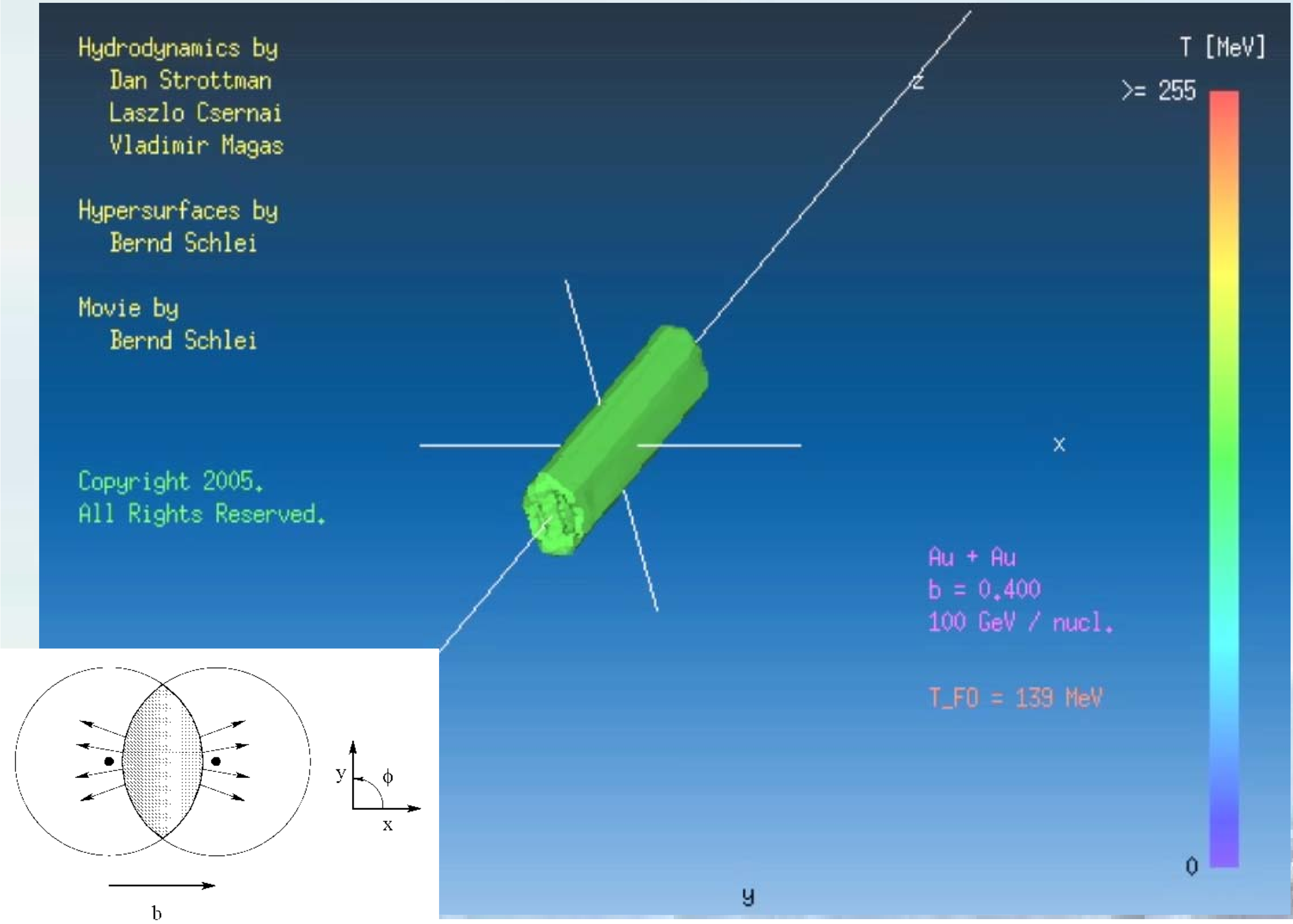
N
reduced
in FO !!!

Entropy;
bulk visc.

FAIR!

M3

FO hypersurface



Hydrodynamics by
Dan Strottman
Laszlo Csernai
Vladimir Magas

Hypersurfaces by
Bernd Schlei

Movie by
Bernd Schlei

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Au + Au
b = 0.400
100 GeV / nucl.

T_FO = 139 MeV

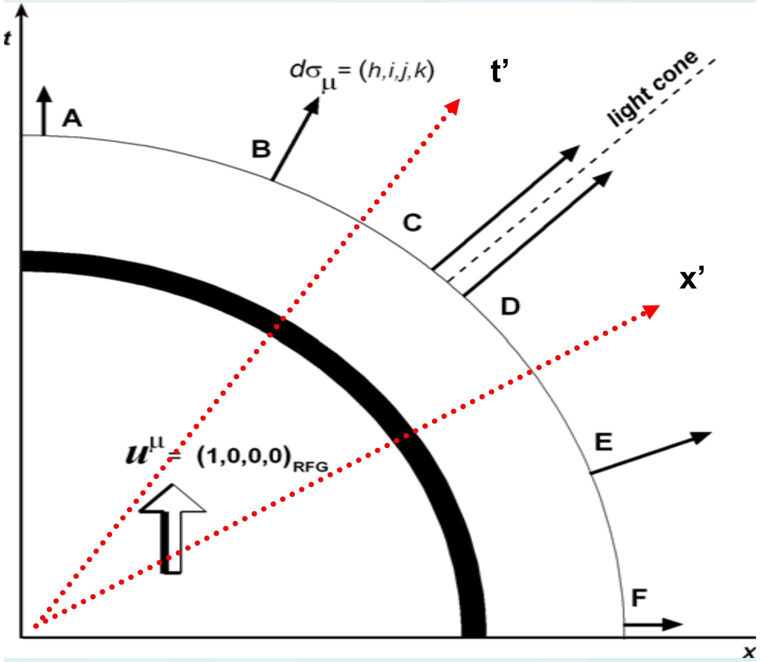
[B. Schlei,
LANL 2005]

T_c=139 MeV

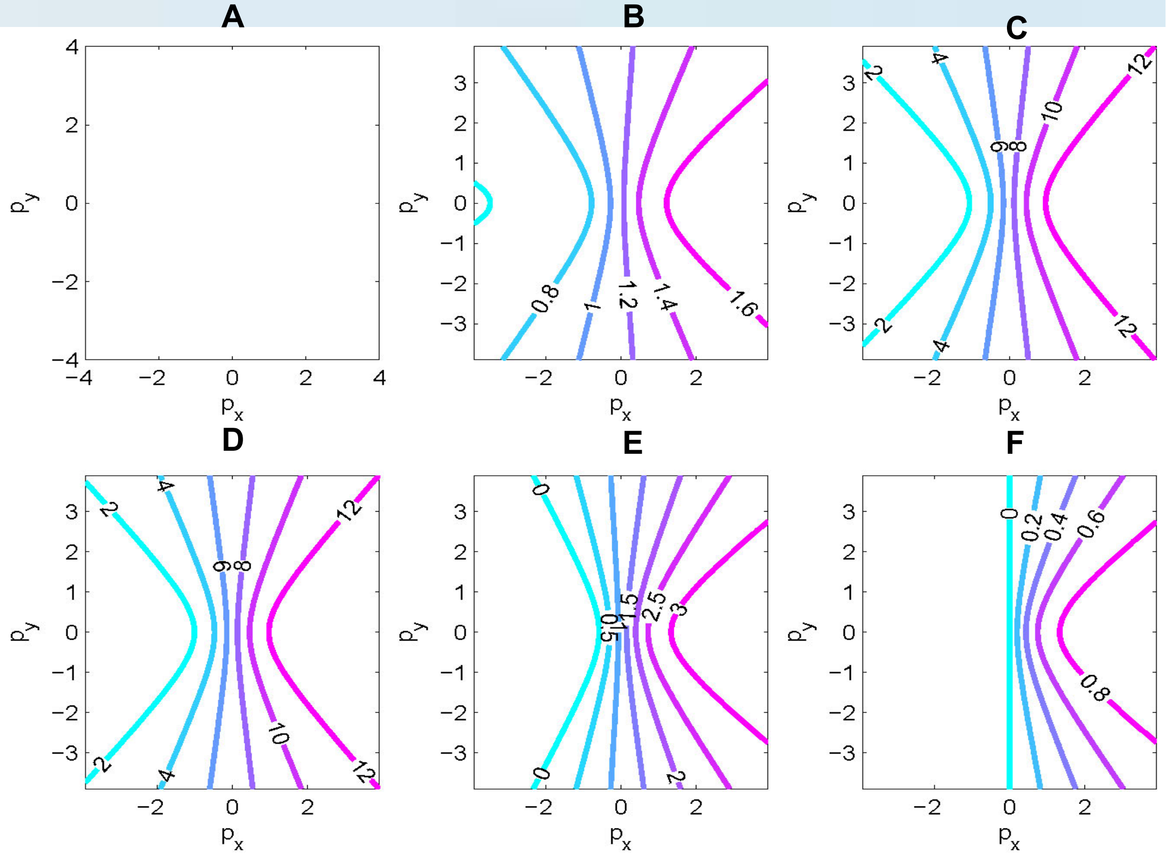
Freeze out:
V.K. Magas,
E. Molnar.

M3

The invariant " Escape" probability

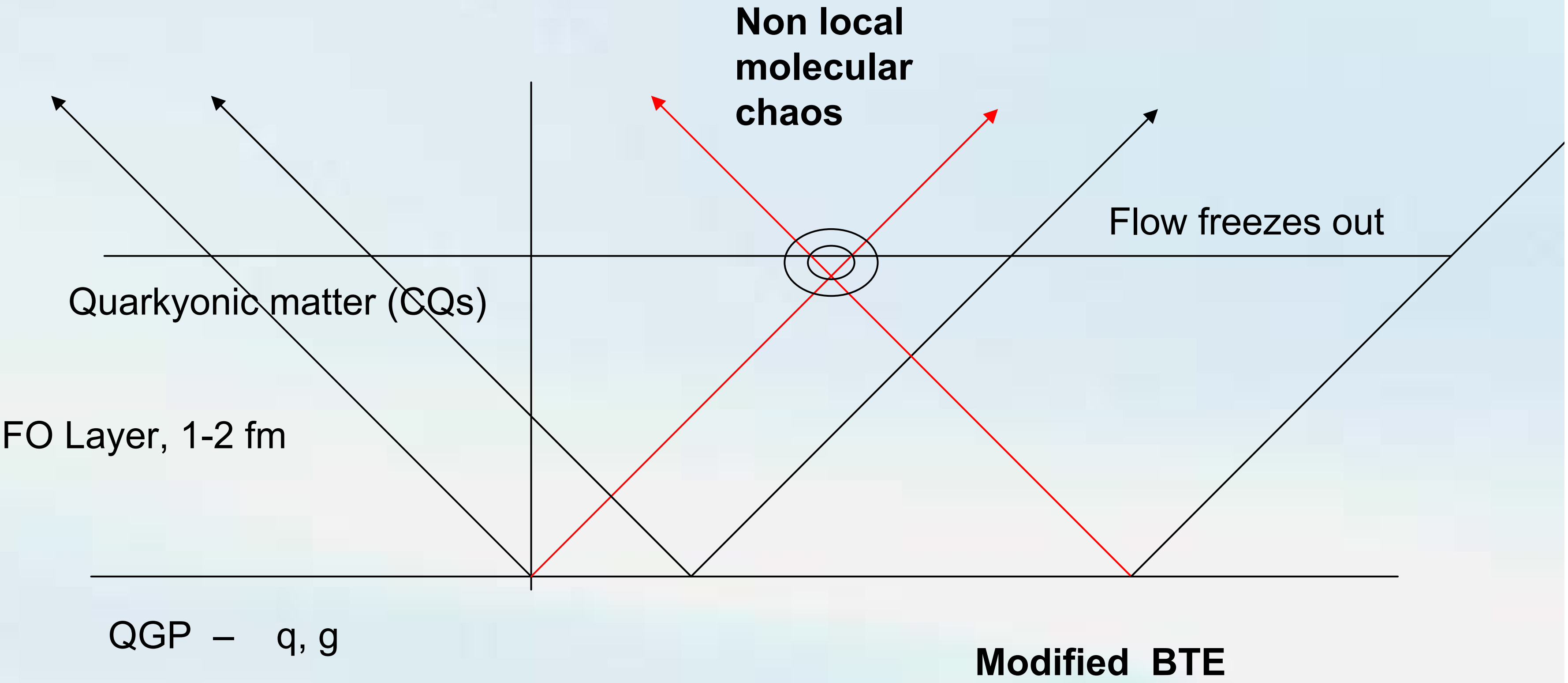


[RFG]



Escape probability factors for different points on FO hypersurface, in the RFG. Momentum values are in units of [*mc*]

$$P(p) = \frac{p^\mu d\sigma_\mu}{p^\mu u_\mu}\Theta(p^\mu d\sigma_\mu)$$



Non local molecular chaos

Flow freezes out

Quarkyonic matter (CQs)

FO Layer, 1-2 fm

QGP – q, g

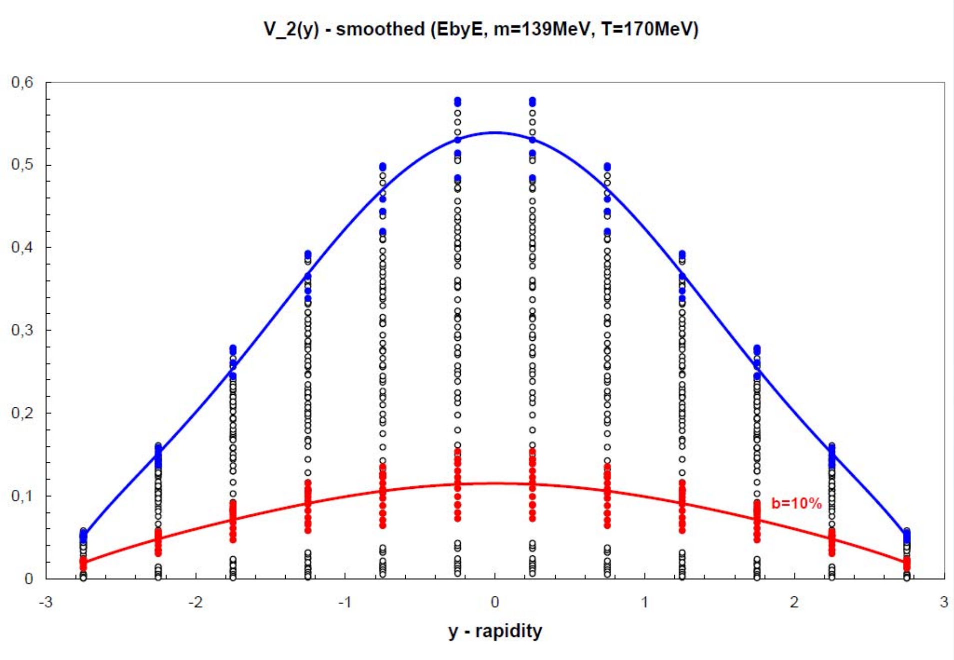
Modified BTE

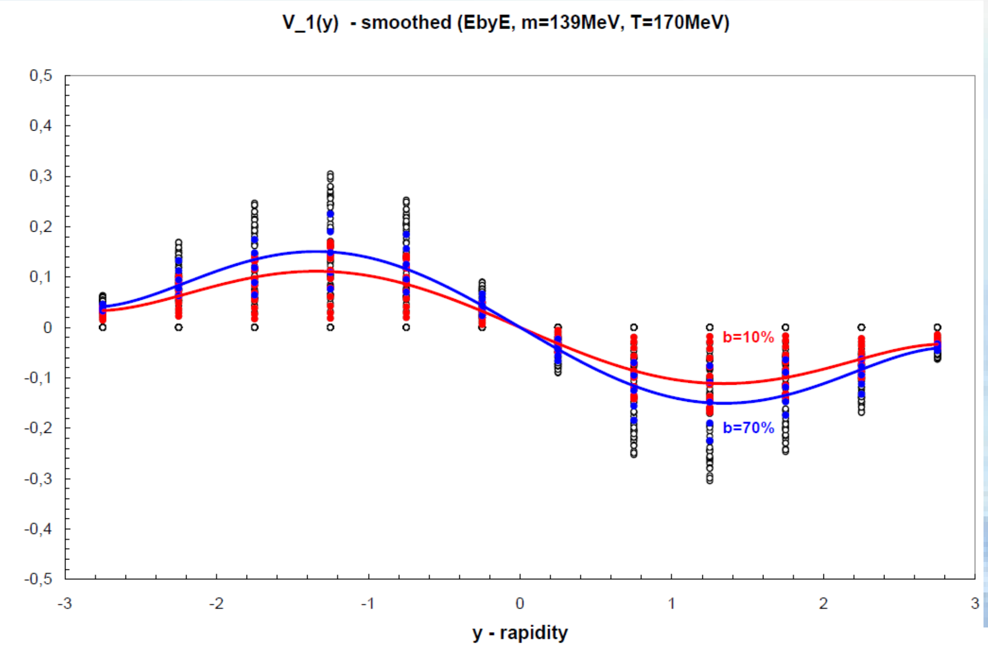
In the FO layer the main free path increases, local molecular chaos assumption does not hold, (large effective viscosity)

Current quarks are gaining mass, while gluons are absorbed, forming constituent quarks (CQs) with mass, m_o . Final flow develops with joint flow velocity, u, for all CQs.

These then recombine to hadrons, in this process E_T is conserved but, p_t and u change depending on what hadrons are formed.

Freeze Out





Pre FO:
V_1 and V_2 versus y from PIC hydro ,

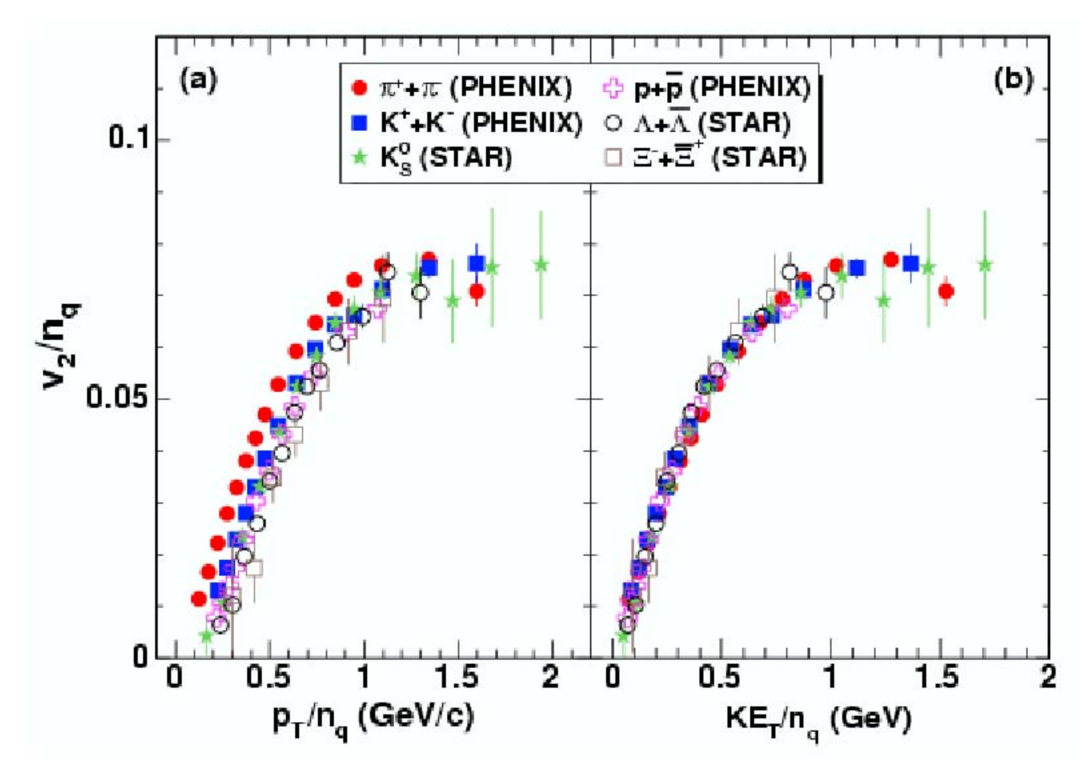
after smoothing in an FO layer considering Modified BTE with parameters m & T.

For different impact parameters, b = 10% (70%) of b_max = R_p + R_t

Before Cooper Frye FO with ‘thermal’ distributions, (with m_cq, T_cq)!

CNQ scaling

Constituent quark number scaling of v_2 (KE_T)



Collective flow of hadrons can be described in terms of constituent quarks.

Observed n_q – scaling →

Flow develops in quark phase, there is no further flow development after hadronization

R. A. Lacey (2006), nucl-ex/0608046.

NCQ - Importance of Initial State

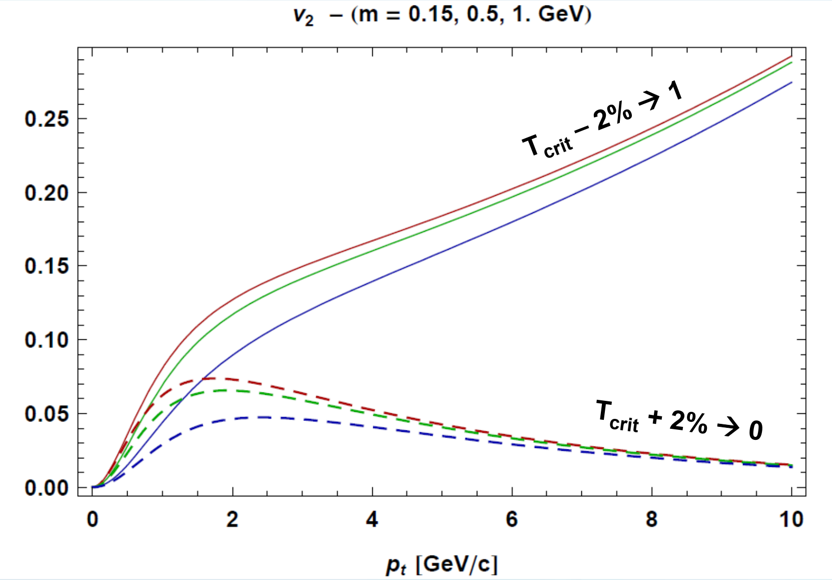
FO [w/Mishustin]

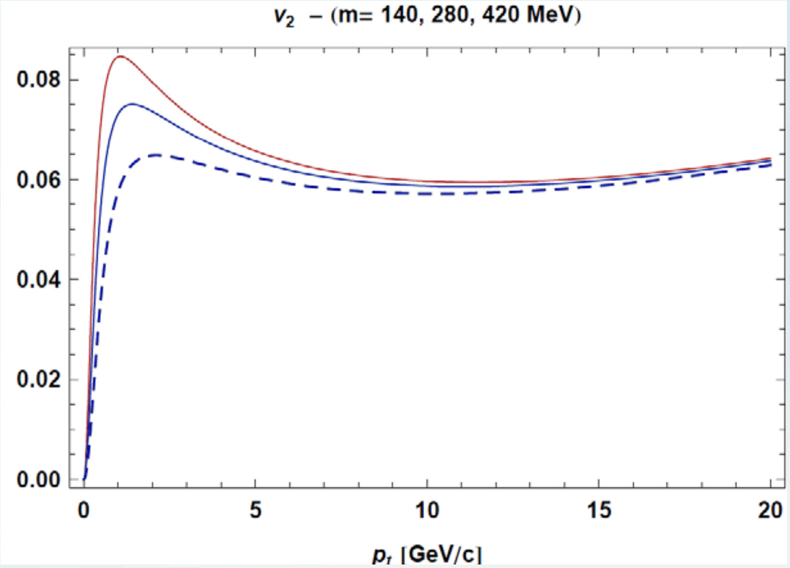
V_2 from few source models [Huovinen et al. 2001] v_2 (p_t) rises linearly at high p_t (Bjorken Model)

As Ac=
50 100

Ts=100
vx=0.2

Tcr=122





As Ac=
42 100

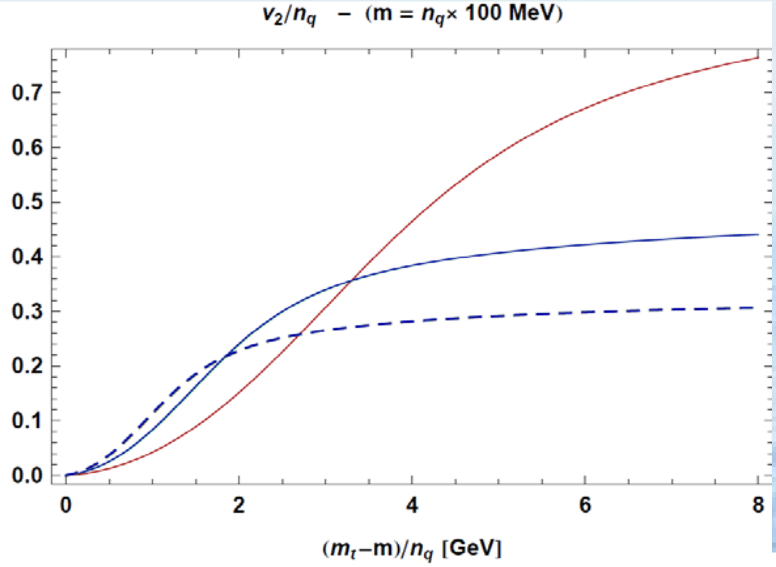
Ts=100
vx=0.5

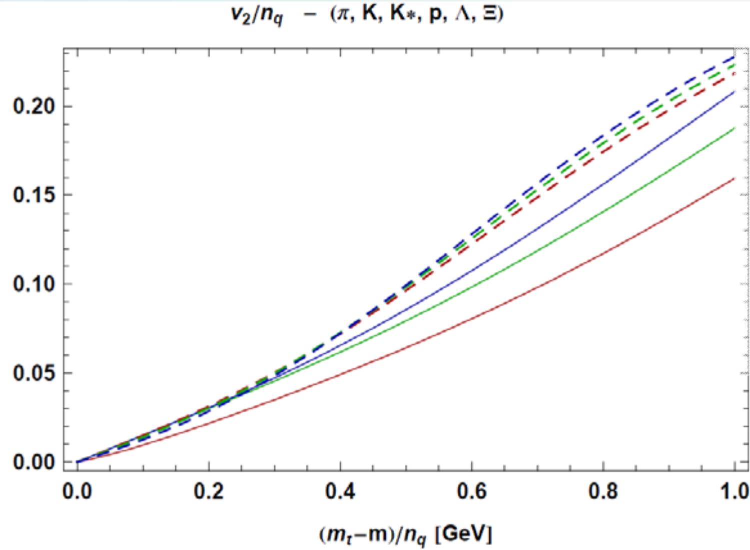
Tc=172

As Ac=
20 100

Ts=150
vx=0.25

Tc=180





T (x) ←
→ u(x)

Hadron flow does not show NCQ scaling !!

As Ac=
20 100

Ts=180
vx=0.4

Tc=150

Hadronization via recombination

Momentum distribution of mesons in simple recombination model:

$$\frac{d^3N}{dp^3} \propto \int \prod_{i=1}^{2} d^3x_i\, d^3p_i\, f_q(x_{1,}p_1) f_q(x_{2,}p_2)\, W_M(p\,, p1\,, p2\,, x1\,, x2)$$

Local $f_q(p_\mu u^\mu)$ is centered at the local u, & meson Wigner function:

momentum conservation

$$W_M(p\,, p_1, p_2, x_1, x_2) = \Phi_M(x_1 - x_2, p_1 - p_2)\ \delta(p_T - p_{T1} - p_{T2})$$

comoving quark and antiquark:

$$\Phi_M \propto \delta^3(x_1 - x_2)\,\delta^3(p_1 - p_2)$$

for the momentum distribution of mesons we get:

$$\frac{d^3N_M}{p_T\, d\, p_T\, dy\, d\,\phi} \propto \int d^3x\, f_q\left(x\,, p_t/2\right)^2$$

flow moments:

for baryons, 2 →3

$$v_n(p_T) = \frac{\displaystyle\int dy\, d\,\phi\ \ \cos n\,\phi\ \frac{d^3N}{p_T\, d\, p_T\, dy\, d\,\phi}}{\displaystyle\int dy\, d\,\phi\ \frac{d^3N}{p_T\, d\, p_T\, dy\, d\,\phi}}$$

[MolnarD-NPA774(06)257]

→ Elliptic flow of mesons:

$$v_{2,M}(p_T) = \frac{2\,v_{2,q}(p_T/2)}{1+2v_{2,q}^2(p_T/2)} \qquad \frac{v_{2,M}(p_T)}{2} = v_{2,q}(p_T/2)$$

For baryons:

$$v_{2,B}(p_T) = \frac{3\,v_{2,q}(p_T/3)+3v_{2,q}^3(p_T/3)}{1+6v_{2,q}^2(p_T/3)} \qquad \frac{v_{2,B}(p_T)}{3} = v_{2,q}(p_T/3)$$

Scaling Variables of Flow:

1st step: Flow asymmetry: V_2 / n_q → V_2 scales with n_q i.e., flow develops in QGP phase, following the common flow velocity, *u*, of all q-s and g-s. Mass here does not show up (or nearly the same mass for all constituent quarks).

Then flow asymmetry does not change any more.

In a medium p_T is not necessarily conserved, K $E_T = m_T - m$ might be conserved → scaling in the variable K E_T [J. Jia & C. Zhang, 2007]

SUMMARY

- **Initial state** is decisive and can be tested by v1 & v2
- v1 dominates in semi-central collisions
- v2 dominates in more peripheral collisions
- position of v1 peak depends on b, σ, Tf.

- **Viscosity** is important both in hydro and in the initial dynamics
- Numerical viscosity should be taken in correction

- **F.O.** : entropy condition → space like FO is weak at RHIC / LHC &
- important at FAIR
- → bulk viscosity limits space like F.O. >> FAIR
- CNQ scaling indicates QGP, simplifies F.O. description to Const. Quarks.

  This requires, however, Modified BTE description
- F.O. leads to acceleration ! (simplified approach eliminates this)

The END