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Convergence of density-matrix expansions for nuclear interactions

B. Gillis Carlsson Jyväskylä University, Finland

in collaboration with Jacek Dobaczewski and the FIDIPRO team

Team

The FiDiPro team:



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Plan for this talk

Generalized Skyrme functionals with higher order derivatives

- How to construct them [1]
- New computer code (HOSPHE) [2,3]

Density matrix expansions, method to derive the coupling constants

- Test of DMEs using the Gogny force [4,5]
- Direct term (Hartree) Gradient expansion
- Exchange term (Fock) quasi-local expansion
- First test of convergence properties (beyond 2nd order)
- New DME method proposed
- B.G. Carlsson, J. Dobaczewski and M. Kortelainen et. al. Phys. Rev. C78, 044326 (2008)
 B.G. Carlsson, J. Dobaczewski, J. Toivanen and P. Vesely, To appear in Comp. Phys. Comm., arXiv:0912.3230 (2010)
 J. Toivanen, et al., To appear in PRC, arXiv:0912.3234v1
 J. Dobaczewski, B.G. Carlsson and M. Kortelainen, Submitted to J. Phys. G,arXiv:1002.3646v1 (2010)
 B.G. Carlsson and J. Dobaczewski, submitted to PRL, arXiv:1003.2543v1 (2010)

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Ansatz for a nuclear density functional

The EDF is assumed to have the form

$$\mathcal{E} = \int \mathrm{d}^3 \mathbf{r} \mathcal{H}_E(\mathbf{r}),$$

where

$$\mathcal{H}_E(\mathbf{r}) = \frac{\hbar^2}{2m} \tau_0 + \mathcal{H}(\mathbf{r}).$$

we focus on the potential energy density $\mathcal{H}(\mathbf{r})$

- expansion in derivatives of the one-body density
- consistent with symmetries of the nucleon-nucleon force

One-body density matrix

Separation into standard scalar and vector parts,

$$\rho(\mathbf{r}\sigma,\mathbf{r}'\sigma') = \frac{1}{2}\rho(\mathbf{r},\mathbf{r}')\,\delta_{\sigma\sigma'} + \frac{1}{2}\sum_{a} \langle\sigma|\sigma_{a}|\sigma'\rangle\,s_{a}(\mathbf{r},\mathbf{r}')\,,$$

Building blocks in the spherical representation:

$$\begin{aligned} \rho_{00}(\mathbf{r},\mathbf{r}') &= \rho(\mathbf{r},\mathbf{r}'), \\ \mathbf{s}(\mathbf{r},\mathbf{r}') &= -i\left\{\frac{1}{\sqrt{2}}\left(s_{x}(\mathbf{r},\mathbf{r}')-is_{y}(\mathbf{r},\mathbf{r}')\right),s_{z}(\mathbf{r},\mathbf{r}'), \\ &\qquad -\frac{1}{\sqrt{2}}\left(s_{x}(\mathbf{r},\mathbf{r}')+is_{y}(\mathbf{r},\mathbf{r}')\right)\right\}, \\ \nabla &= -i\left\{\frac{1}{\sqrt{2}}\left(\nabla_{x}-i\nabla_{y}\right),\nabla_{z},\frac{-1}{\sqrt{2}}\left(\nabla_{x}+i\nabla_{y}\right)\right\}, \\ k &= -i\left\{\frac{1}{\sqrt{2}}\left(k_{x}-ik_{y}\right),k_{z},\frac{-1}{\sqrt{2}}\left(k_{x}+ik_{y}\right)\right\}. \end{aligned}$$

where $\mathbf{k} = \frac{\mathbf{\nabla}_{\mathbf{r}'} - \mathbf{\nabla}_{\mathbf{r}''}}{2i}$

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Higher-order derivative operators

Independent derivative operators to 6th order

No.	tensor D _{nL}	order <i>n</i>	rank <i>L</i>
1	1	0	0
2	∇	1	1
3	$[\nabla \nabla]_0$	2	0
4	$[\nabla \nabla]_2$	2	2
5	$[\nabla \nabla]_0 \overline{\nabla}$	3	1
6	$[\nabla [\nabla \nabla]_2]_3$	3	3
7	$[\nabla \nabla]_0^2$	4	0
8	$[\nabla\nabla]_0[\nabla\nabla]_2$	4	2
9	$[\nabla [\nabla [\nabla \nabla]_2]_3]_4$	4	4
10	$[\nabla\nabla]_0^2 \nabla$	5	1
11	$[\nabla\nabla]_0 [\nabla[\nabla\nabla]_2]_3$	5	3
12	$\left[\nabla \left[\nabla \left[\nabla \left[\nabla \nabla \right]_2\right]_3\right]_4\right]_5$	5	5
13	$[\nabla \nabla]_0^3$	6	0
14	$[\nabla \nabla]_0^2 [\nabla \nabla]_2$	6	2
15	$[\nabla \nabla]_0 [\nabla [\nabla \nabla \nabla]_2]_3]_4$	6	4
16	$[\nabla [\nabla [\nabla [\nabla [\nabla \nabla]_2]_3]_4]_5]_6$	6	6

Replacing $\nabla \leftrightarrow \mathbf{k}$ gives K_{nL}

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Local densities

formed by acting several times on the scalar and vector non-local densities with the relative momentum operator and taking limit $\mathbf{r}'=\mathbf{r}$

$$\rho_{nL\nu J}(\mathbf{r}) = \left\{ [K_{nL}\rho_{\nu}(\mathbf{r},\mathbf{r}')]_J \right\}_{\mathbf{r}'=\mathbf{r}},$$

where the *n*th-order and rank-*L* relative derivative operator K_{nL} acts on the scalar (v = 0) or vector (v = 1) Labeled using time-reversal (*T*) and space-inversion (*P*) parities defined as,

$$T = (-1)^{n+\nu},$$

 $P = (-1)^{n}.$

Local-densities from the scalar density matrix

No.			$ \rho_{nLvJ} = \text{density} $	п	L	V	J	Т	Р
1	*	•	$ ho_{0000} = \left[ho ight]_0$	0	0	0	0	1	1
2	*		$ \rho_{1101} = [k\rho]_1 $	1	1	0	1	-1	$^{-1}$
3	*	•	$\rho_{2000} = [[kk]_0 \rho]_0$	2	0	0	0	1	1
4	*	•	$\rho_{2202} = [[kk]_2 \rho]_2$	2	2	0	2	1	1
5	*		$\rho_{3101} = [[kk]_0 k \rho]_1$	3	1	0	1	$^{-1}$	$^{-1}$
6	*		$\rho_{3303} = [[k[kk]_2]_3\rho]_3$	3	3	0	3	$^{-1}$	$^{-1}$
7	*	٠	$\rho_{4000} = [[kk]_0^2 \rho]_0$	4	0	0	0	1	1
8	*	•	$\rho_{4202} = [[kk]_0 [kk]_2 \rho]_2$	4	2	0	2	1	1
9			$\rho_{4404} = [[k[k[kk]_2]_3]_4^{-}\rho]_4$	4	4	0	4	1	1
10	*		$\rho_{5101} = [[kk]_0^2 k \rho]_1$	5	1	0	1	-1	-1
11			$\rho_{5303} = [[kk]_0 [k[kk]_2]_3 \rho]_3$	5	3	0	3	$^{-1}$	$^{-1}$
12			$\rho_{5505} = [[k[k[kk]_2]_3]_4]_5\rho]_5$	5	5	0	5	$^{-1}$	$^{-1}$
13	*	•	$\rho_{6000} = [[kk]_0^3 \rho]_0$	6	0	0	0	1	1
14			$\rho_{6202} = [[kk]_0^2 [kk]_2 \rho]_2$	6	2	0	2	1	1
15			$\rho_{6404} = [[kk]_0[k[k[kk]_2]_3]_4\rho]_4$	6	4	0	4	1	1
16			$\rho_{6606} = [[k[k[k[kk]_2]_3]_4]_5]_6\rho]_6$	6	6	0	6	1	1

Terms in the energy density

Terms in the EDF are required to be

- quadratic in densities
- time reversal invariant
- space-inversion invariant
- invariant with respect to rotations
- A general term can be written

$$T_{mI,nLvJ,Q}^{m'I',n'L'v'J'}(\mathbf{r}) = C[[D_{m'I'}\rho_{n'L'v'J'}(\mathbf{r})]_Q [D_{mI}\rho_{nLvJ}(\mathbf{r})]_Q]_0,$$

With density-independent coupling-constants P.I gives

$$T_{ml,nL\nu J}^{n'L'\nu'J'}(\mathbf{r}) = C[\rho_{n'L'\nu'J'}(\mathbf{r})[D_{ml}\rho_{nL\nu J}(\mathbf{r})]_{J'}]_{0}$$

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Skyrme terms

Second order expansion is equivalent to the Skyrme functional

No.		C ^{n' L' v' J'} mI , nLvJ	Spherical		Cartesian
1	٠	$C_{00,0000}^{0000}$	$[\rho\rho]_0$	=	ρ^2
2		$C_{00,0011}^{0011}$	[<i>ss</i>] ₀	-	$\frac{1}{\sqrt{3}}s^2$
3	•	$C^{0000}_{20,0000}$	$[\rho[[\nabla\nabla]_0\rho]_0]_0$	=	$\frac{1}{\sqrt{3}}\rho\Delta\rho$
4	•	$C_{00,2000}^{0000}$	$[\rho[[kk]_0\rho]_0]_0$	=	$\frac{1}{\sqrt{3}}\left(ho au-rac{1}{4} ho\Delta ho ight)$
5		$C_{00,1110}^{1110}$	$[[ks]_0[ks]_0]_0$	=	$\frac{1}{3} \left(J^{(0)} \right)^2$
6	•	$C_{00,1111}^{1111}$	$[[ks]_1[ks]_1]_0$	=	$\frac{1}{\sqrt{12}}$ J ²
7		$C_{00,1112}^{1112}$	$[[ks]_2[ks]_2]_0$	=	$\frac{1}{\sqrt{5}} \sum_{ab} J^{(2)}_{ab} J^{(2)}_{ab}$
8	•	C ⁰⁰⁰⁰ 11,1111	$[\rho[\nabla[ks]_1]_0]_0$	=	$\frac{1}{\sqrt{6}}\rho \nabla \cdot \mathbf{J}$
9		$C_{00,1101}^{1101}$	$[[k\rho]_1[k\rho]_1]_0$	=	$\frac{1}{\sqrt{3}}\mathbf{j}^2$
10		$C_{20,0011}^{0011}$	$[s[[\nabla\nabla]_0 s]_1]_0$	=	$\frac{1}{3}$ s Δ s
11		$C_{22,0011}^{0011}$	$[s[[\nabla\nabla]_2 s]_1]_0$	=	$rac{-1}{\sqrt{5}}\left((\boldsymbol{\nabla}\cdot\mathbf{s})^2+rac{1}{3}\mathbf{s}\Delta\mathbf{s} ight)$
12		$C_{00,2011}^{0011}$	$[s[[kk]_0s]_1]_0$	=	$\frac{1}{3}\left(\mathbf{s}\cdot\mathbf{T}-\frac{1}{4}\mathbf{s}\Delta\mathbf{s}\right)$
13		$C_{00,2211}^{0011}$	$[s[[kk]_2s]_1]_0$	=	$\frac{1}{\sqrt{5}} \left(\mathbf{s} \cdot \mathbf{F} + \frac{1}{4} \left(\boldsymbol{\nabla} \cdot \mathbf{s} \right)^2 \right)$
14		$C_{11,0011}^{1101}$	$[[k\rho]_1[\nabla s]_1]_0$	=	$\frac{-\frac{1}{3}\mathbf{s}\cdot\mathbf{T}+\frac{1}{12}\mathbf{s}\Delta\mathbf{s}}{\frac{1}{\sqrt{6}}\mathbf{j}\cdot\boldsymbol{\nabla}\times\mathbf{s}}$

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Time-even 4th order terms

No.		C ^{n'L'} v'J' CmI,nLvJ	ρ _n 'L'v'J'	D _{ml}	, PnLvJ
1		c0000	lime-even	12212	l ime-even
1	•	-0000	[P]0	[V V]0	[0]0
2	•	C20,2000	[<i>P</i>]0	$[\nabla \nabla]_0$	$\left[\left[kk\right]_{0}\rho\right]_{0}$
3	•	$C_{22,2202}^{0000}$	$[\rho]_0$	$[\nabla \nabla]_2$	$[[kk]_2 \rho]_2$
4	٠	$C_{00,4000}^{0000}$	[<i>P</i>]0	1	$[[kk]_{0}^{2}\rho]_{0}$
5	•	$C_{00,2000}^{2000}$	$[[kk]_0 \rho]_0$	1	$[[kk]_0 \rho]_0$
6	٠	$C_{00,2202}^{2202}$	$[[kk]_2 \rho]_2$	1	$[[kk]_2 \rho]_2$
7		$C_{20,1110}^{1110}$	[ks]0	$[\nabla \nabla]_0$	[ks]0
8		$C_{22,1112}^{1110}$	[ks]0	$[\nabla \nabla]_2$	[ks]2
9		$C_{00,3110}^{1110}$	[ks]0	1	$[[kk]_0 ks]_0$
10	•	$c_{20,1111}^{1111}$	[ks]1	$[\nabla \nabla]_0$	[ks]1
11	•	$C_{22,1111}^{1111}$	[ks]1	$[\nabla \nabla]_2$	$[ks]_1$
12		$C_{22,1112}^{1111}$	[ks]1	$[\nabla \nabla]_2$	[ks]2
13	•	$C_{00,3111}^{1111}$	[ks]1	1	$[[kk]_0 ks]_1$
14		$C_{20,1112}^{1112}$	[ks]2	$[\nabla \nabla]_0$	[ks]2
15		$C_{22,1112}^{1112}$	[ks]2	$[\nabla \nabla]_2$	[ks]2
16		$C_{00,3112}^{1112}$	[ks]2	1	$[[kk]_0 ks]_2$
17		$c_{00,3312}^{1112}$	[ks]2	1	$[[k[kk]_2]_3s]_2$
18	٠	C0000 31,1111	[<i>ρ</i>]0	$[\nabla \nabla]_0 \nabla$	[ks]1
19	•	C ⁰⁰⁰⁰ C ^{11,3111}	[<i>ρ</i>]0	∇	$[[kk]_0 ks]_1$
20	•	C ²⁰⁰⁰ C ¹¹ ,1111	$[[kk]_0 \rho]_0$	∇	$[ks]_1$
21	•	C ²²⁰² C ¹¹ ,1111	$[[kk]_2 \rho]_2$	∇	$[ks]_1$
22		C ²²⁰² 11,1112	$[[kk]_2 \rho]_2$	∇	[ks]2

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DME expansions

Number of terms increases exponentially



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More constraints, Galilean and gauge invariance

Galilean invariance

Relativistic effects are neglected

Local gauge invariance

For local forces

$$v(\mathbf{r}_1',\mathbf{r}_2',\mathbf{r}_1,\mathbf{r}_2) = \delta(\mathbf{r}_1'-\mathbf{r}_1)\delta(\mathbf{r}_2'-\mathbf{r}_2)v(\mathbf{r}_1,\mathbf{r}_2)$$

such as Gogny, Coulomb, and Yukawa.

Symmetries introduce constraints between the parameters



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New code HOSPHE

- Spherical HO basis
- All terms up to 6th order in derivatives
- Mean field + linear response
- Density expanded in multipoles (zero multipole for MF and higher for LR)
- To appear in Comp. Phys. Comm., B.G. Carlsson *et al* arXiv:0912.3230.



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Summary empirical functional

- General sixth order EDF
- Restricted only by symmetries
- Elegant notation introduced
- Fourth order and (Galilean,Gauge)=(48,30) free parameters. Skyrme (2nd order) has 18
- Code publicly available
- B. G. Carlsson et. al. Phys. Rev. C78, 044326 (2008)
- B. G. Carlsson et al, To appear in Comp. Phys. Comm., arXiv:0912.3230. (2010)

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DME methods

- Introduced by Negele and Vautherin [1] to relate microscopic theory to Skyrme type models
- G-matrix to Skyrme functional
- Transforms direct and exchange energy to expansions in derivatives of the DM
- Corrective contributions to the Slater approximation for exchange
- Leads to much simpler and faster calculations for 2 and 3 body exchange parts

New stuff

- Test convergence of DME methods (first time beyond 2nd order)
- new expansion method
- [1] Negele and Vautherin PRC 5, 1472 (1972)

Gogny effective interaction as test case

$$V_{12} = Contact part + Coulomb part + \sum_{k=1}^{2} e^{-(\mathbf{r}_1 - \mathbf{r}_2)^2/\mu_k^2} (W_k + B_k P_\sigma - H_k P_\tau - M_k P_\sigma P_\tau)$$

• Accuracy for masses 0.798
MeV
• Range < 3 fm
• Good test for DME methods
$$V_{next} = \frac{1}{2} \int_{-10}^{10} \int_{-1$$

Gradient expansion of the direct term

Direct (Hartree) term (neglecting spin and isospin)

$$\mathcal{E}_{dir}^{int} = \frac{1}{2} \int \int \rho(\mathbf{r}_1) \,\rho(\mathbf{r}_2) \,v(\mathbf{r}_1,\mathbf{r}_2) d\mathbf{r}_1 d\mathbf{r}_2. \tag{1}$$

Taylor expansion in powers of $\textbf{r}=\textbf{r}_1-\textbf{r}_2$ around $\textbf{R}=\frac{1}{2}\left(\textbf{r}_1+\textbf{r}_2\right)$

$$\rho(\mathbf{r}_{1}) = \rho\left(\mathbf{R} + \frac{\mathbf{r}}{2}\right) = e^{\frac{1}{2}\mathbf{r}\cdot\nabla}\rho(\mathbf{R}) = \sum_{n} \frac{1}{n!} \left(\frac{\mathbf{r}}{2}\cdot\nabla\right)^{n}\rho(\mathbf{R})$$
$$= \sum_{nL} a_{nL} r^{n} \left[Y_{L}(\hat{r}), \hat{D}_{nL}\right]_{0} \rho(\mathbf{R}).$$

expanding both densities and inserting in Eq. 1 gives

$$\mathcal{E}_{dir}^{int} = \sum_{nn'L} \int \underbrace{\left(\int \frac{a_{nL}a_{n'L}}{\sqrt{1+2L}} r^{n+n'+2} v(r) dr\right)}_{C_L^{nn'}} \left[D_{nL}\rho\left(\mathbf{R}\right), D_{n'L}\rho\left(\mathbf{R}\right)\right]_0 d\mathbf{R}$$



[1] R.K. Bhaduri and D.W.L. Sprung NPA 297, 365 (1978)

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Direct term accuracy for the Gogny D1S force

- Test with densities from Skyrme SLy4 functional
- nine nuclei: 4 He, 16 O, 40,48 Ca, 56,78 Ni, 100,132 Sn and 208 Pb
- Exact direct energy versus gradient expansion

- First test going beyond second order
- Accuracy surprisingly good
- Total RMS at sixth order is 0.479 MeV



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Exchange term

The direct term can be calculated exactly but for the exchange term the DME makes calculations much faster.

Some proposals for the exchange part:

- NV: corrective factors to Slater using Bessel fkn expansion of $\rho(\mathbf{r}_1, \mathbf{r}_2)$
- PSA: Recent proposal improving the NV expansion for the spin part [1]
- DT: Our method, improved convergence, works well for all channels without rotational averaging.
- [1] B. Gebremarian et. al. arxiv 0910.4979 (2009)

DT expansion for the exchange term

The density in the exchange (Fock) term

$$\mathcal{E}_{\text{exc}}^{\text{int}} = \frac{1}{2} \int \int \rho(\mathbf{r}_1, \mathbf{r}_2) \rho(\mathbf{r}_2, \mathbf{r}_1) v(|\mathbf{r}_1 - \mathbf{r}_2|) d\mathbf{r}_1 d\mathbf{r}_2$$

can be expanded in powers of $\textbf{r}=\textbf{r}_1-\textbf{r}_2$ around $\textbf{R}=\frac{1}{2}\left(\textbf{r}_1+\textbf{r}_2\right)$ as:

$$\begin{split} \rho(\mathbf{R},\mathbf{r}) &= \bar{\rho}(\mathbf{R},\mathbf{r}) + e^{-\frac{r^2}{\sigma^2}} \left[e^{i\mathbf{r}\cdot\hat{\mathbf{k}}} e^{\frac{(\mathbf{r}_1-\mathbf{r}_2)^2}{\sigma^2}} (\rho(\mathbf{r}_1,\mathbf{r}_2) - \bar{\rho}(\mathbf{r}_1,\mathbf{r}_2)) \right] \bigg|_{\mathbf{r}_1 = \mathbf{r}_2 = \mathbf{R}} \\ &= \sum_{nLm} \pi_{nL}^m \left(r \right) \left[Y_L \left(\hat{r} \right), \, \hat{K}_{nL} \rho(\mathbf{r}_1,\mathbf{r}_2) \bigg|_{\mathbf{r}_1 = \mathbf{r}_2 = \mathbf{R}} \right]_0. \end{split}$$

around a model density (Slater mixed density)

$$\bar{\rho}(\mathbf{r}_1, \mathbf{r}_2) = \rho(\mathbf{R}) \, \frac{3j_1\left(k_F r\right)}{k_F r}$$

Gives the exchange energy as quasi-local expansion

$$\mathcal{E}_{exc}^{int} \simeq \sum_{t,nn'L} \int d\mathbf{R} \, C_{nL}^{n'} \left[\rho_{n'L}(\mathbf{R}), \rho_{nL}(\mathbf{R}) \right]_{0},$$

Idea behind the DT expansion

$$ho(\mathbf{r}_1,\mathbf{r}_2)$$
 as fkn of $\mathbf{r}=\mathbf{r}_1-\mathbf{r}_2$ for $\mathbf{R}=0$



- Exact density from analytical model of HO with smeared occupancies
- The model density gives a good average approximation
- Shaded regions indicate relevant range

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Idea behind the DT expansion

$$\rho(\mathbf{r}_1, \mathbf{r}_2)$$
 as fkn of $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$ for $\mathbf{R} = 0$



 Quadratic Taylor expansion improves for r < 1.5 fm

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Idea behind the DT expansion

$$\rho(\mathbf{r}_1, \mathbf{r}_2)$$
 as fkn of $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$ for $\mathbf{R} = \mathbf{0}$



 NV Bessel-function expansion is somewhat better

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DME expansions

Idea for the DT expansion

$$\rho(\mathbf{r}_1, \mathbf{r}_2)$$
 as fkn of $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$ for $\mathbf{R} = 0$



 DT improves short range while preserving the long range part

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Comparison of NV and DT expansions

- We fix the damping width *a* in the DT by fitting the sixt order expansion to symmetric nuclear matter which gives $a \approx 4/k_F$
- Better convergence than NV
- Avoids rotational averaging in the non-local direction, hence has the potential to be more accurate than NV



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Exchange term accuracy for the Gogny D1S force

Test case: SLy4 for nine nuclei: ⁴He, ¹⁶O, ^{40,48}Ca, ^{56,78}Ni, ^{100,132}Sn and ²⁰⁸Pb

- First test of convergence properties (beyond 2nd order)
- 2nd order: PSA best, RMS = 14.09 MeV
- 4th order: DT best, RMS = 1.54 MeV
- 6th order: DT best, RMS = 0.36 MeV



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Conclusions

- First test of DME methods beyond 2nd order
- new method with excellent convergence properties
- Can be used to approximate both direct and exchange
- at sixth order the number of active terms are doubled (gauge symmetry) compared to Skyrme calculations, hence the computer time only doubles.

- B. G. Carlsson and J. Dobaczewski, submitted to PRL, arXiv:1003.2543v1 (2010)
- J. Dobaczewski, B.G. Carlsson and M. Kortelainen, submitted to JPG, arXiv:1002.3646v1 (2010)

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Plans for the future

- Derive simple and fast Hartree-Fock functional from chiral effective field theory (in collaboration with T. Duguet)
- Add RPA correlations using iterative HOSPHE-LR (Monday presentation by J. Toivanen)
- Add remaining correlations using LDA (second Born, GW)
- sp states from the response function (Hedin style)

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Thank You