Exotic Nuclear Structures and Reactions from an Ab Initio Perspective

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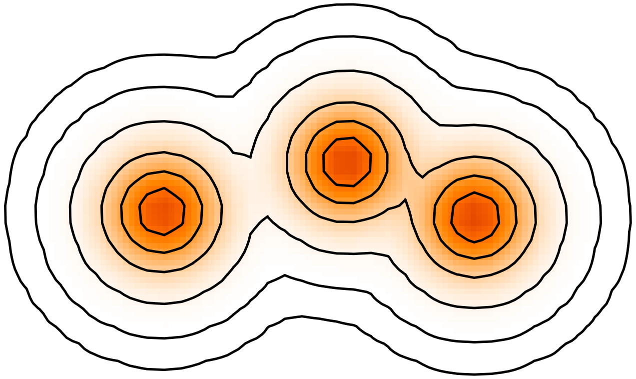
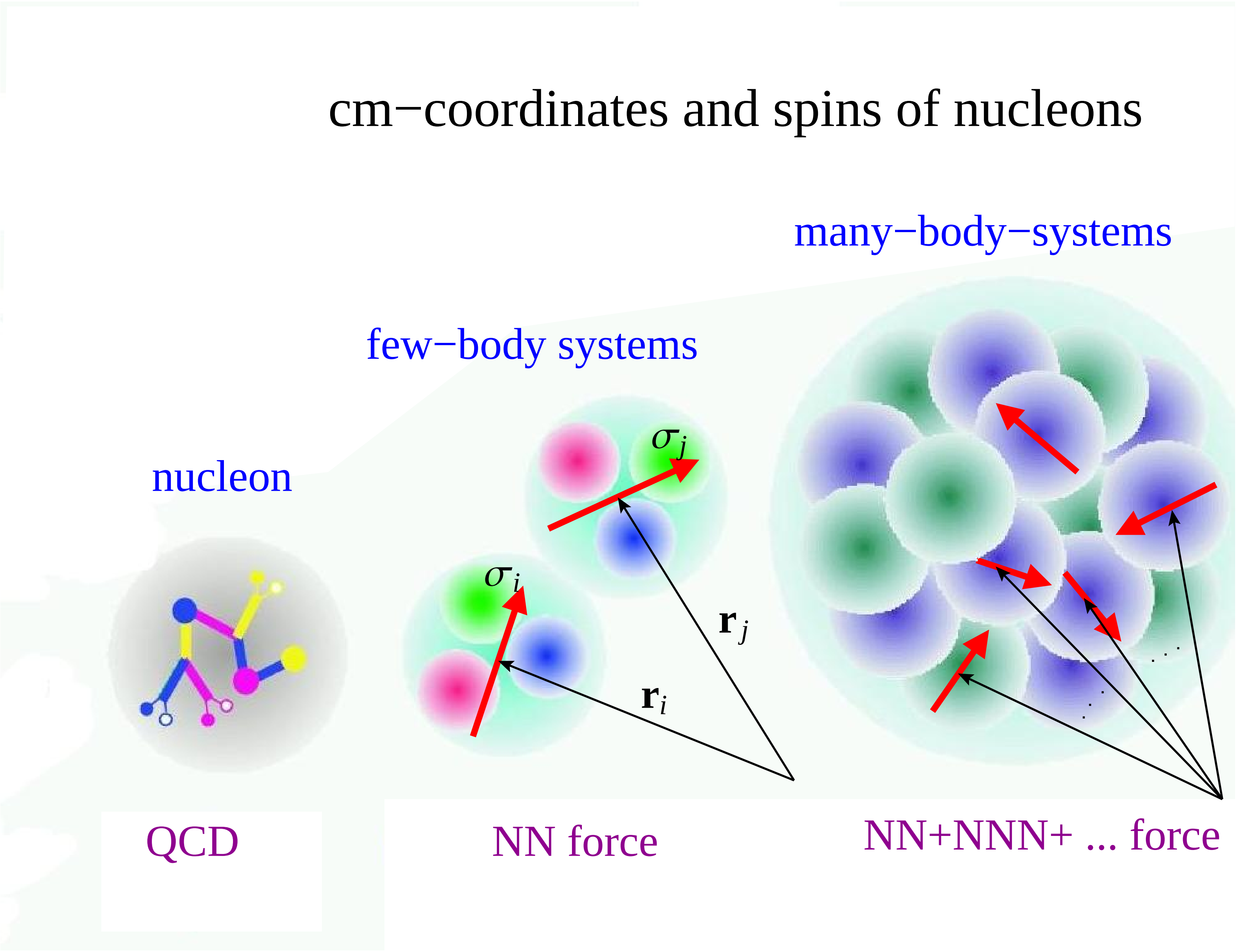


Nuclear Degrees of Freedom



Modern Nuclear Structure – Ab Initio

Ab Initio : from the beginning, without additional assumptions or special models

”beginning”

- c.m. positions and spins of nucleons $(\mathbf{r}_i, \sigma_i, \tau_i)$ as degrees of freedom
  $\Longrightarrow$ many-body state $\left|\widehat{\Psi}\right\rangle \;\epsilon\; \mathcal{H}$ Hilbert space
- interactions among nucleons approximated by potentials $\Longrightarrow V_{\mathrm{NN}} + V_{\mathrm{NNN}}$
  ”realistic” V_{NN} describes NN phase shifts and deuteron

Realistic NN-Potentials

QCD motivated

- symmetries, meson-exchange picture
- chiral effective field theory

short-range phenomenology

- short-range parametrisation or “contact” terms

experimental two-body data

- scattering phase-shifts & deuteron properties reproduced with high precision

supplementary three-nucleon force

- adjusted to spectra of light nuclei

Argonne V18

CD Bonn

Nijmegen I/II

Chiral N3LO

Argonne V18 + **Illinois 2**

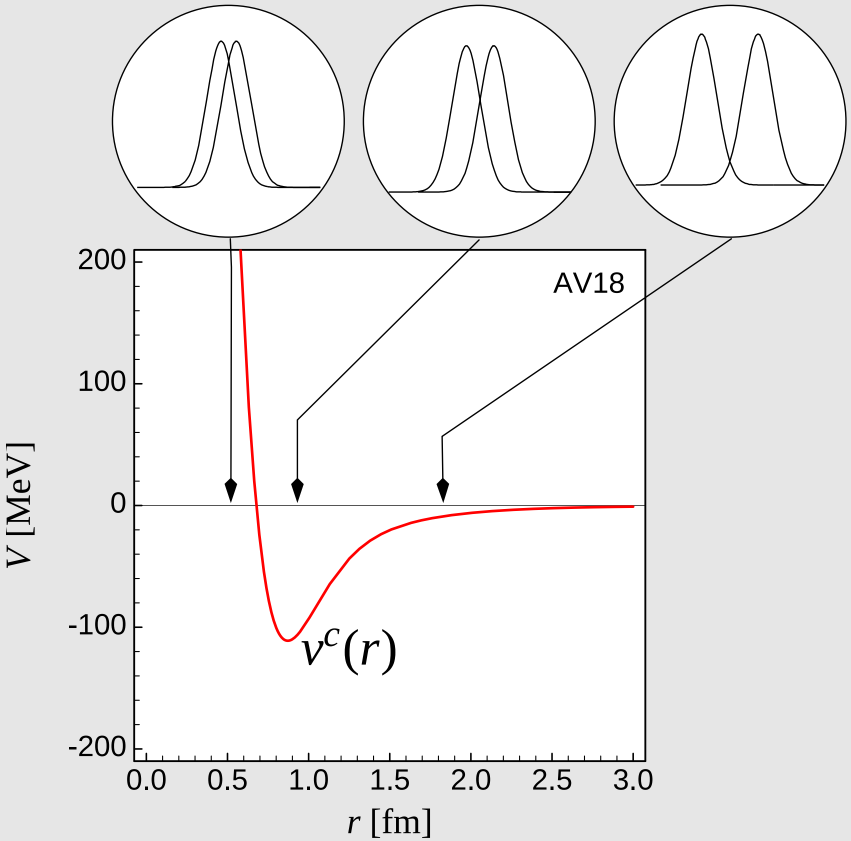
Chiral N3LO + **N2LO**

Potential and Nucleon Size

Nucleons are not pointlike !

Proton charge radius $\sqrt{\langle r^2 \rangle_e} = 0.86\ \mathrm{fm}$

Proton charge distribution and S=0, T=1 Potential



- proton size not small compared to interaction range
- half-density overlap at max attraction, average NN-distance

  $1.8\ \mathrm{fm} \approx 2\sqrt{\langle r^2 \rangle_e}$
- V_{NN} not elementary more like atom-atom potential
- expect three-body forces

Modern Nuclear Structure – Ab Initio

Ab initio treatment: solve many-body quantum problem

- $\underset{\sim}{H} \left| \widehat{\Psi}_n \right\rangle = E_n \left| \widehat{\Psi}_n \right\rangle$ with $\underset{\sim}{H} = \underset{\sim}{T} + \underset{\sim}{V}_{\mathrm{NN}} + \underset{\sim}{V}_{\mathrm{NNN}}$
- observables: energies E_n, moments $\left\langle \widehat{\Psi}_n \right| \underset{\sim}{A} \left| \widehat{\Psi}_n \right\rangle$, transitions $|\left\langle \widehat{\Psi}_k \right| \underset{\sim}{A} \left| \widehat{\Psi}_n \right\rangle|^2$ to be confronted with data

HOWEVER

Modern Nuclear Structure – Ab Initio

HOWEVER, there are conceptional problems

- realistic $\underset{\sim}{V}_{\mathrm{NN}}$ not unique !
  different phase-shift equivalent $\underset{\sim}{V}_{\mathrm{NN}}, \underset{\sim}{V}'_{\mathrm{NN}}, \underset{\sim}{V}''_{\mathrm{NN}}$ describe equally well the 2-body system
- $\underset{\sim}{V}_{\mathrm{NN}} + \underset{\sim}{V}_{\mathrm{NNN}} \iff \underset{\sim}{V}'_{\mathrm{NN}} + \underset{\sim}{V}'_{\mathrm{NNN}}$
  each NN-interaction needs its NNN-part to describe equally well the 3-body system

➨ in nuclear structure theory there is not the one genuine NN or NNN force

Modern Nuclear Structure – Ab Initio

and there are technical problems

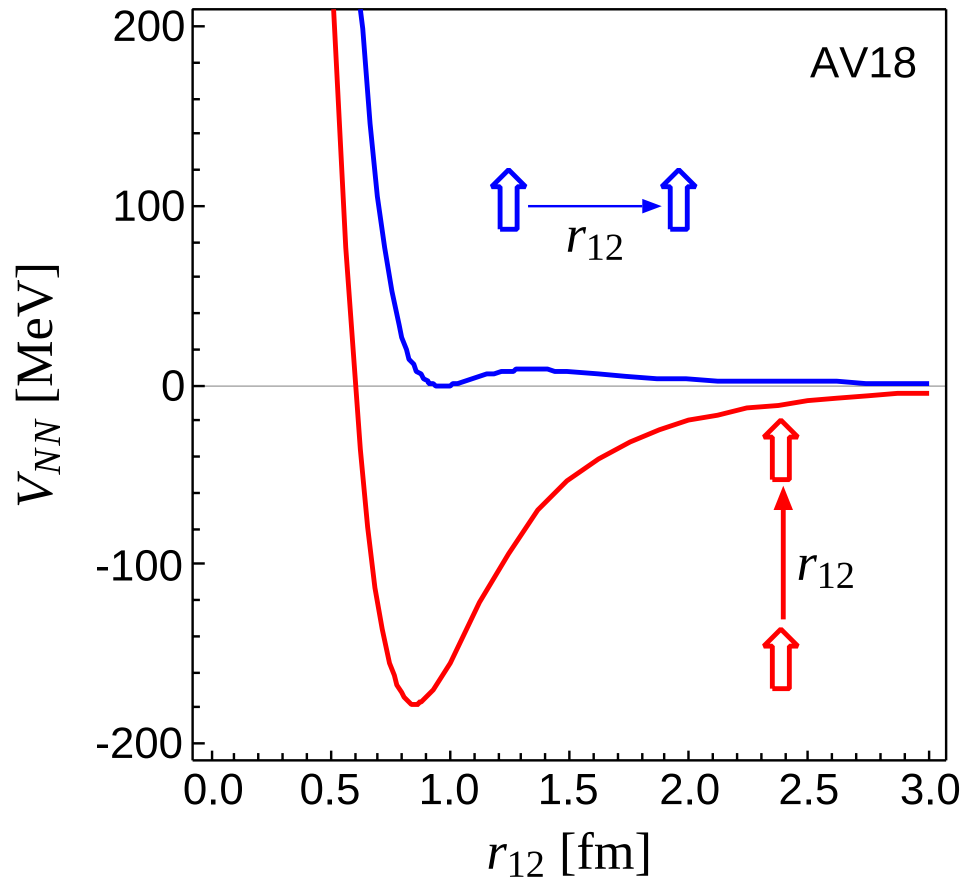
- $\underset{\sim}{H} \left| \widehat{\Psi}_n \right\rangle = E_n \left| \widehat{\Psi}_n \right\rangle$ cannot be solved numerically for larger mass numbers

Why ?

Realistic Nuclear Force

Argonne V18 ($S = 1,\ T = 0$)

spins parallel or perpendicular
to the relative distance vector



- strong repulsive core: nucleons can not get closer than ≈ 0.5 fm

➡ **central correlations**

- strong dependence on the orientation of the spins due to the tensor force

➡ **tensor correlations**

the nuclear force induces **strong short-range correlations** in the nuclear wave function

Modern Nuclear Structure – Ab Initio

and there are technical problems

- $\underset{\sim}{H} \left| \widehat{\Psi}_n \right\rangle = E_n \left| \widehat{\Psi}_n \right\rangle$ cannot be solved numerically for larger mass numbers

Solution: treat short-range correlations by effective interactions

- Approximation: Hilbert space $\mathcal{H} = \mathcal{H}_{\text{low-}k} \oplus \mathcal{H}_{\text{high-}k}$

  $\underset{\sim}{H}^{\text{eff}} \left| \Psi_n \right\rangle = E_n \left| \Psi_n \right\rangle$ with $\left| \Psi_n \right\rangle \epsilon\, \mathcal{H}_{\text{low-}k}$

- Unitary transformation $\left| \widehat{\Psi}_n \right\rangle = \underset{\sim}{U} \left| \Psi_n \right\rangle$ such that

  $\underset{\sim}{H}^{\text{eff}} = \underset{\sim}{U}^\dagger \underset{\sim}{H} \underset{\sim}{U}$ does not connect $\mathcal{H}_{\text{low-}k}$ with $\mathcal{H}_{\text{high-}k}$

  sounds great, but many-body forces appear

  $\underset{\sim}{H}^{\text{eff}} = \underset{\sim}{T} + \underset{\sim}{V}^{\text{eff}}_{\text{NN}} + \underset{\sim}{V}^{\text{eff}}_{\text{NNN}} + \underset{\sim}{V}^{\text{eff}}_{\text{NNNN}} + \underset{\sim}{V}^{\text{eff}}_{\text{NNNNN}} + \cdots$

- Other observables $\underset{\sim}{A}^{\text{eff}} = \underset{\sim}{U}^\dagger \underset{\sim}{A}\, \underset{\sim}{U}$

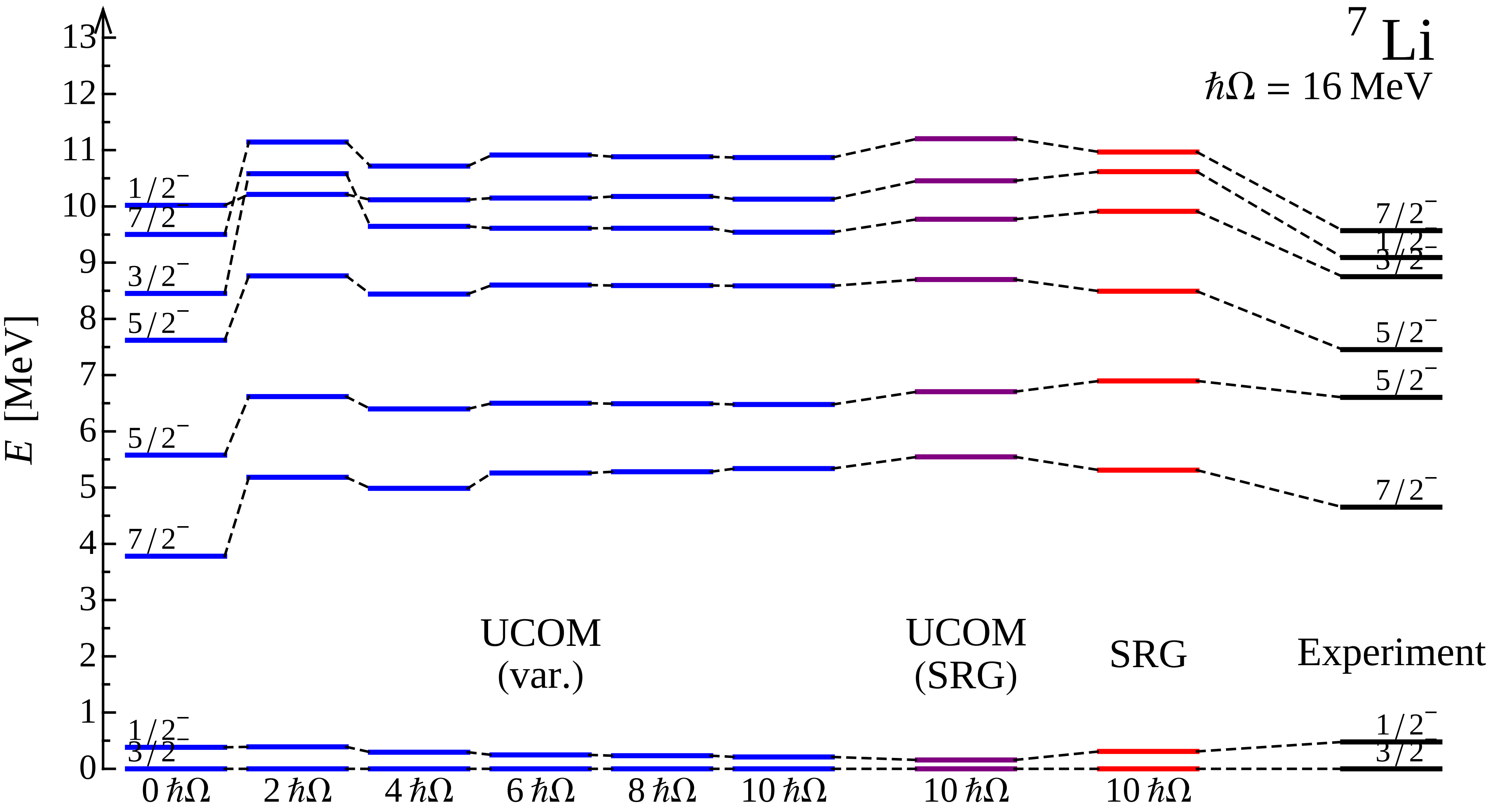
$\mathcal{H}_{\textbf{low-}k}$ Hilbert space: Harmonic Oscillator Basis

• •

No-Core Shell Model

Hartree Fock

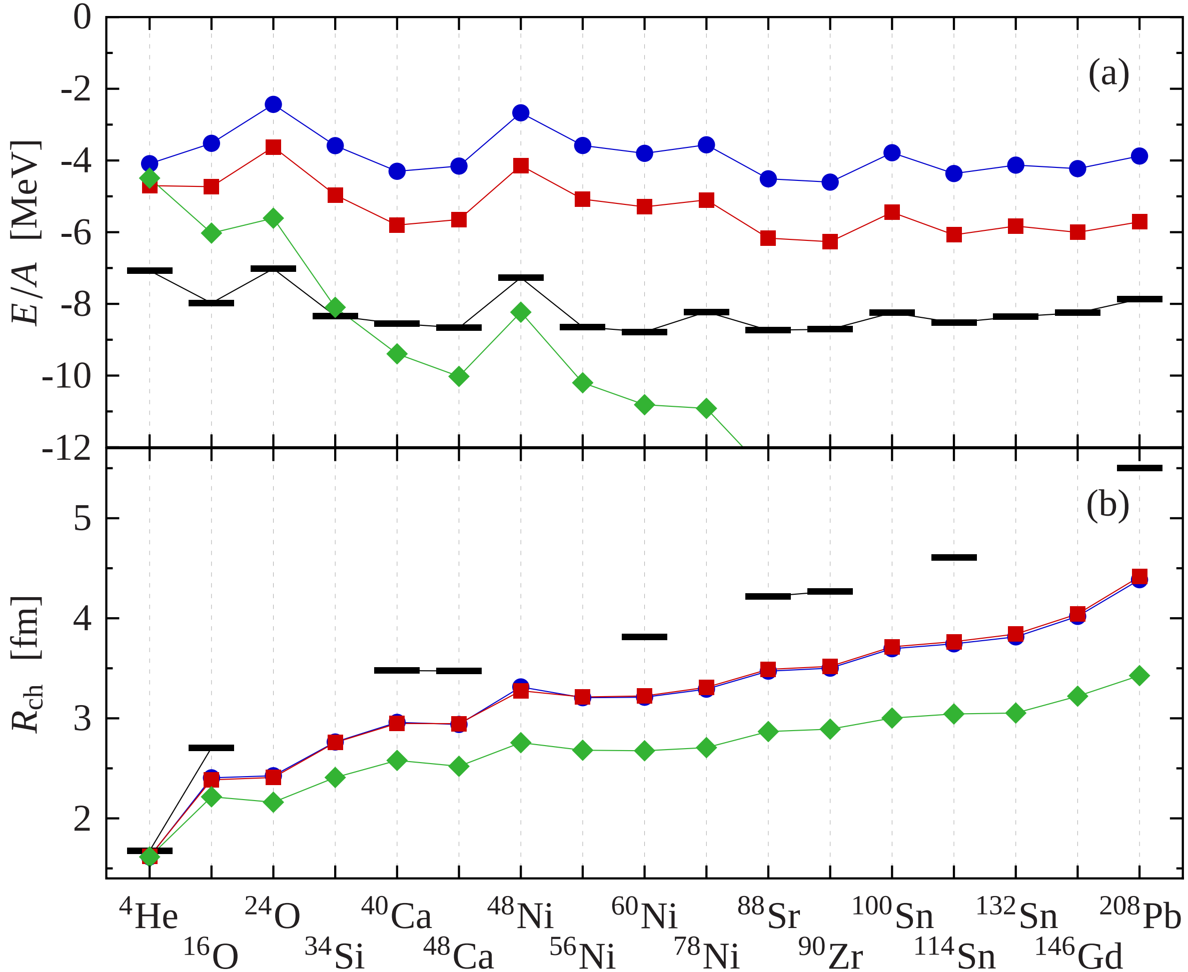
NCSM for ^{7}Li $\qquad H_{\text{eff}} = T + V_{\text{NN}}^{\text{eff}}$



- fast convergence of spectra, no scattering to high-k

Hartree-Fock $H_{\text{eff}} = T + V_{\text{NN}}^{\text{eff}}$

➨ Ab-Initio HF for A = 4 to 208



long-range correlations are missing

UCOM(var.)
UCOM(SRG)
SRG

Only two-body part $V_{\text{NN}}^{\text{eff}}$

but three- and higher-body eff. interactions cannot be completely neglected

especially for SRG (repulsive)

$\mathcal{H}_{\textbf{low-}k}$ Hilbert space:
Fermionic Molecular Dynamics

FMD many-body wave functions

Restore symmetries by projections

Variation After Projection (VAP)

Configuration mixing

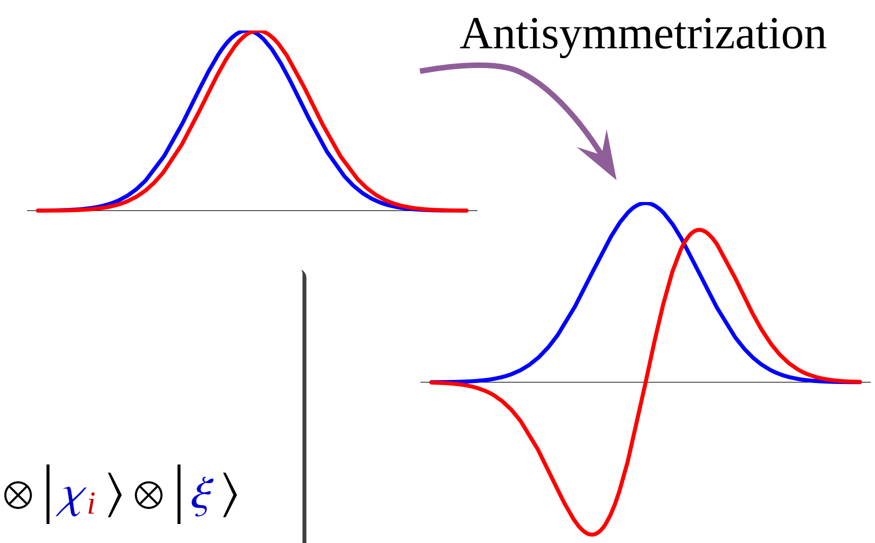
FMD Many-Body Hilbert Space

Fermionic

Slater determinant

$$\left| Q \right\rangle = \underset{\sim}{\mathcal{A}}\Big(\left| q_1 \right\rangle \otimes \cdots \otimes \left| q_A \right\rangle\Big)$$

➨ antisymmetrized A-body state



Molecular

single-particle states

$$\left\langle \mathbf{x} \middle| q \right\rangle = \sum_i c_i \exp\left\{-\frac{(\mathbf{x}-\mathbf{b}_i)^2}{2a_i}\right\} \otimes \left| \chi_i \right\rangle \otimes \left| \xi \right\rangle$$

➨ Gaussian wave-packets in phase-space, spin is free, isospin is fixed

➨ Hilbert space contains shell-model, clusters, halos, scattering states

Dynamics in Hilbert space

spanned by one or several non-orthogonal $\left| Q^{(a)} \right\rangle$

$$\left| \Psi; J^\pi M \right\rangle = \sum_{a,K'} \psi_{aK'} \, \underset{\sim}{P}^{J^\pi}_{MK'} \underset{\sim}{P}^{\mathbf{P}=0} \left| Q^{(a)} \right\rangle$$

variational principle $\rightarrow$ $Q^{(a)} = \{ q_\nu^{(a)}, \nu = 1 \cdots A \}, \; \psi_{aK'}$

Multi-Configuration Mixing

➨ most general projected state for multi-configuration calculations

$$\left| \Psi; J^\pi M \right\rangle = \sum_{aK} \psi_{aK} \, \underset{\sim}{P}^\pi \underset{\sim}{P}^J_{MK} \underset{\sim}{P}^{\mathbf{P}=0} \left| Q^{(a)} \right\rangle$$

➨ task: find a set of intrinsic states $\left\{ \left| Q^{(a)} \right\rangle, a = 1, \ldots, N \right\}$ that describe the physical situation well

Multi-configuration calculations

$$\underset{\sim}{H} \left| J^\pi M, n \right\rangle = E_n^{J^\pi} \left| J^\pi M, n \right\rangle$$

➨ **diagonalize** Hamiltonian in this set of non-orthogonal projected intrinsic states

$$\sum_{bK'} \left\langle Q^{(a)} \right| \underset{\sim}{H} \underset{\sim}{P}^{J^\pi}_{KK'} \underset{\sim}{P}^{\mathbf{P}=0} \left| Q^{(b)} \right\rangle \cdot c^{(n)}_{bK'} = E_n^{J^\pi} \sum_{bK'} \left\langle Q^{(a)} \right| \underset{\sim}{P}^{J^\pi}_{KK'} \underset{\sim}{P}^{\mathbf{P}=0} \left| Q^{(b)} \right\rangle \cdot c^{(n)}_{bK'}$$

➨ energy levels $E_n^{J^\pi}$ and eigenstates $\left| J^\pi M, n \right\rangle$ describing nuclear many-body system

$$\left| J^\pi M, n \right\rangle = \sum_{bK'} c^{(n)}_{bK'} \, \underset{\sim}{P}^\pi \underset{\sim}{P}^J_{MK'} \underset{\sim}{P}^{\mathbf{P}=0} \left| Q^{(b)} \right\rangle$$

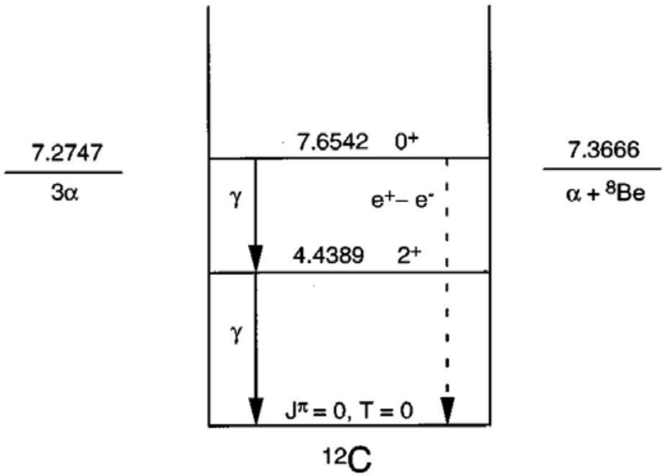
Cluster States in ^{12}C

Astrophysical Motivation

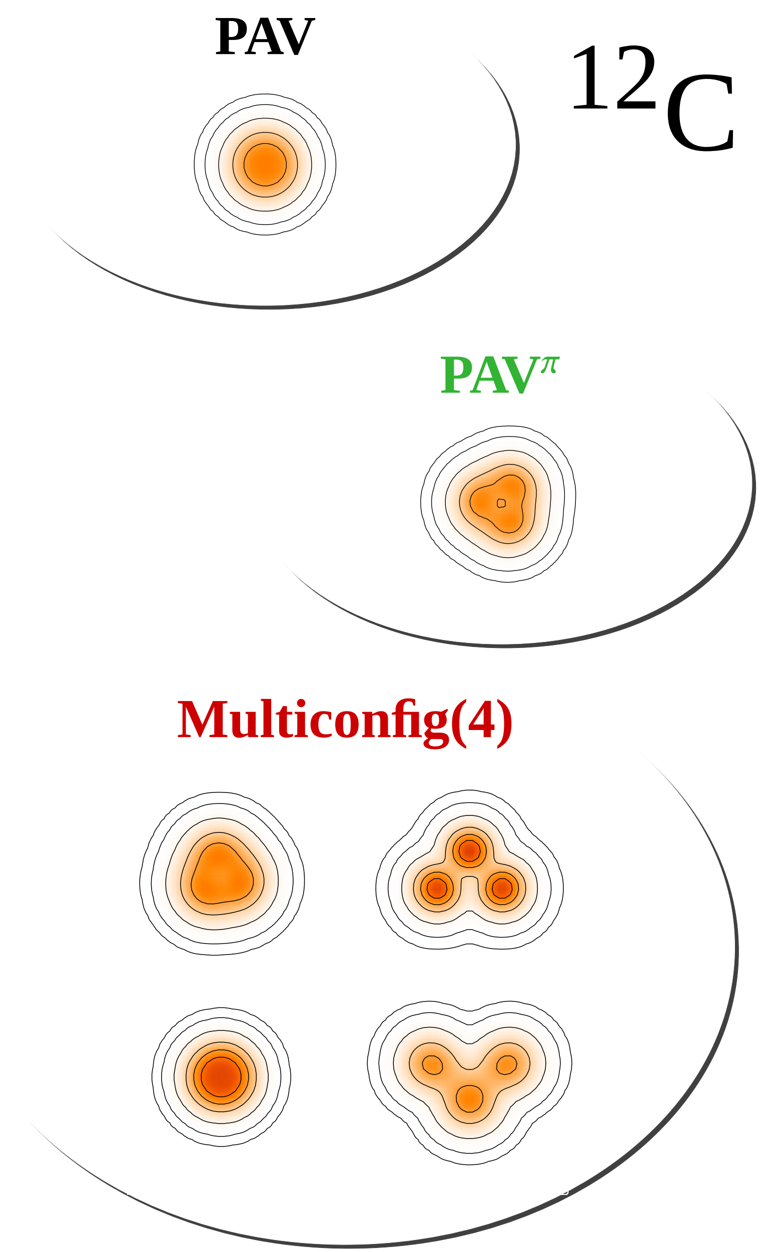
Structure

- Is the Hoyle state a α-cluster state ?
- Other excited 0^+ and 2^+ states

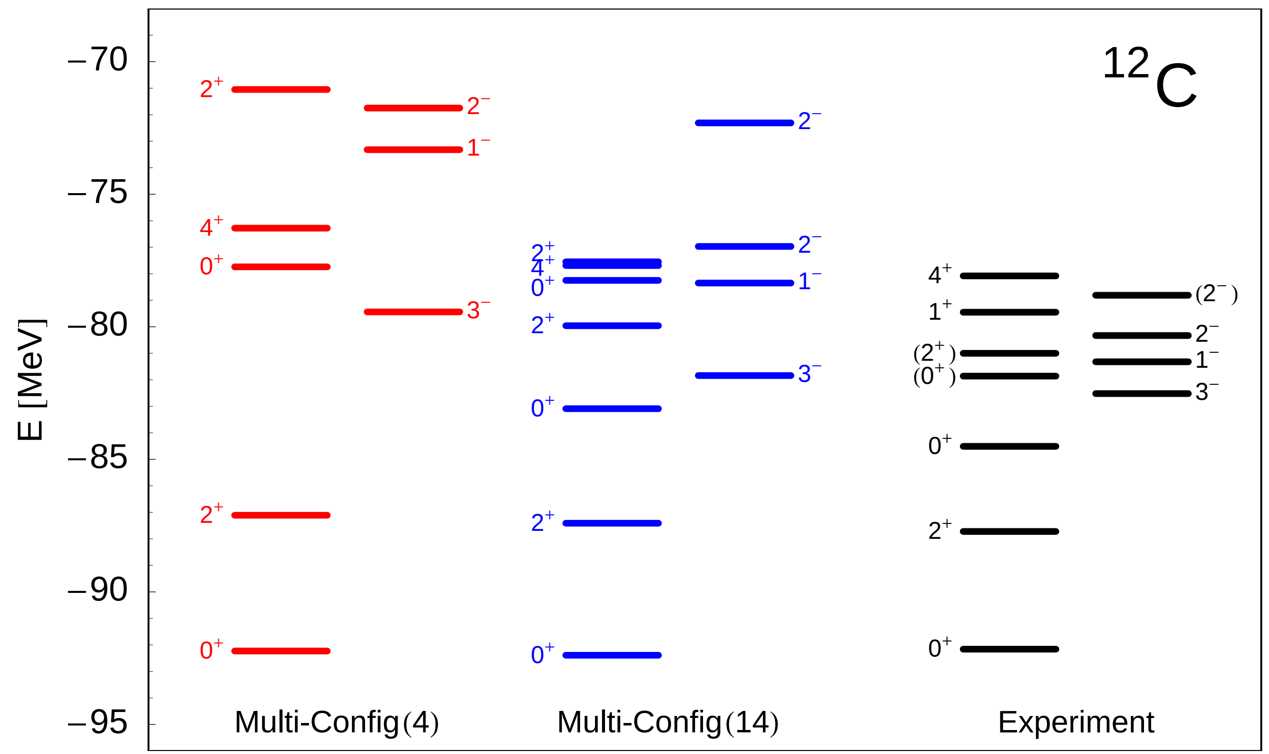
➨ Analyze wave functions in harmonic oscillator basis



FMD - Variation, PAV$^\pi$, Multiconfig.

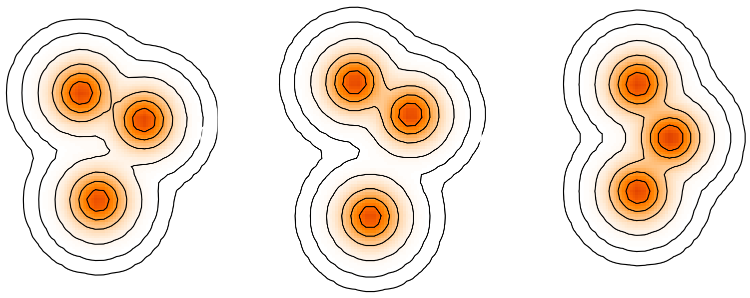


	E [MeV]	r_{charge} [fm]	$B(E2)$ [e^2fm^4]
PAV	-81.4	2.36	-
PAV$^\pi$	-88.5	2.51	36.3
Multiconfig(4)	-92.2	2.52	42.8
Multiconfig(14)	-92.4	2.52	42.9
Exp	-92.2	2.47	39.7 ± 3.3



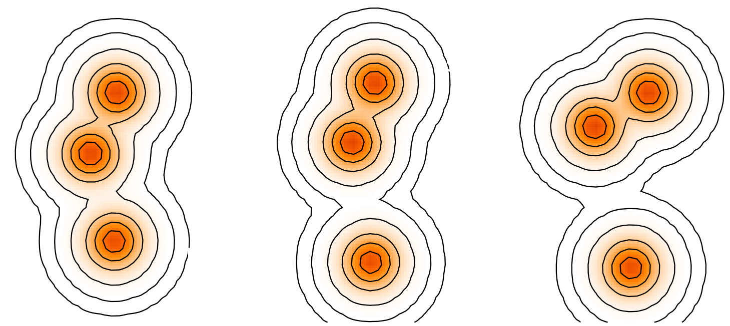
^{12}C 0^+ states

0_2^+ Hoyle state



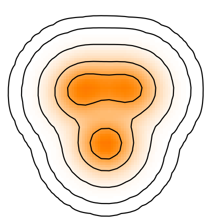
$|\langle \cdot | 0_2^+ \rangle| = 0.76$ $|\langle \cdot | 0_2^+ \rangle| = 0.71$ $|\langle \cdot | 0_2^+ \rangle| = 0.50$

0_3^+ state

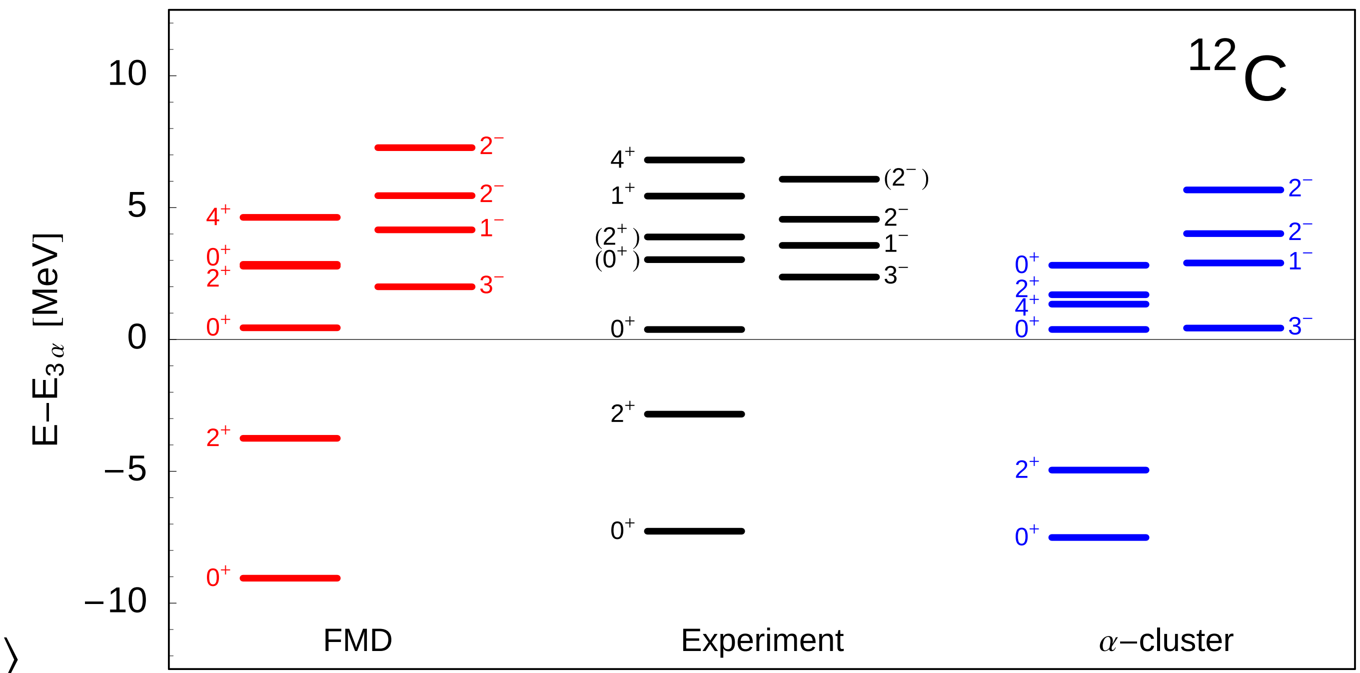


$|\langle \cdot | 0_3^+ \rangle| = 0.69$ $|\langle \cdot | 0_3^+ \rangle| = 0.65$ $|\langle \cdot | 0_3^+ \rangle| = 0.44$

0_1^+ ground state

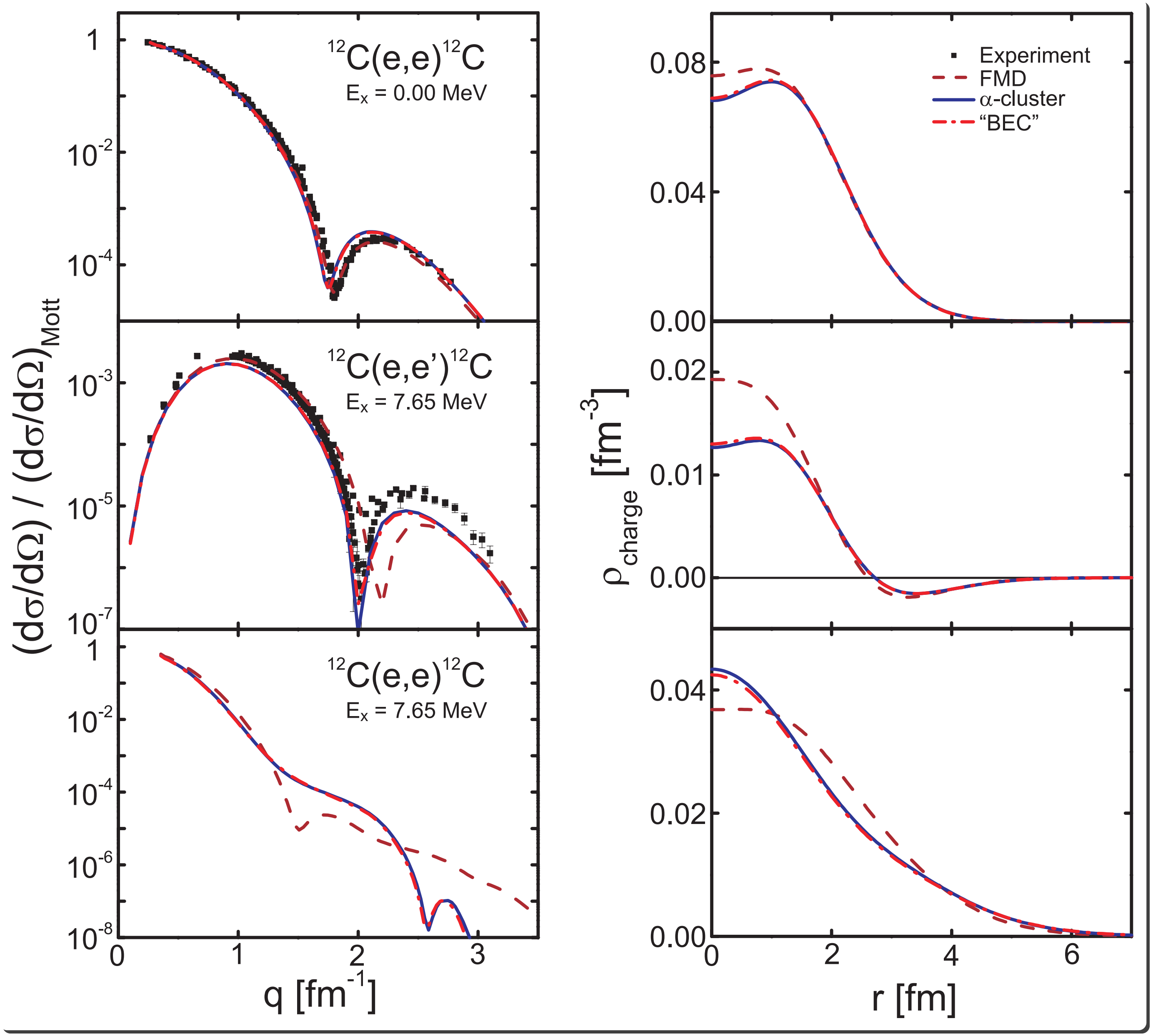


$|\langle \cdot | 0_1^+ \rangle| = 0.94$



$$|J^\pi M, n\rangle = \sum_{a,K'} c^{(n)}_{aK'} \underset{\sim}{P}^{J^\pi}_{MK'} \underset{\sim}{P}^{\mathbf{P}=0} |Q^{(a)}\rangle$$

^{12}C Hoyle State in Electron Scattering



- calculate formfactors, center-of-mass treated properly, formfactor is a A-body operator

$$F(\mathbf{q}) = \sum_i \langle \Psi_a \,|\, e^{i\mathbf{q}\cdot(\underset{\sim}{\mathbf{x}}_i - \underset{\sim}{\mathbf{X}})} \,|\, \Psi_b \rangle$$

- compare to experiment in Distorted Wave Born Approximation
- α-cluster and "BEC" calculated with mod. Volkov interation

M. Chernykh, H. Feldmeier, P. von Neumann-Cosel, T. Neff, A. Richter, PRL **98**, 032501 (2007)

"BEC" formfactors: Y. Funaki et al. EPJA 28(2006)259 and private communication

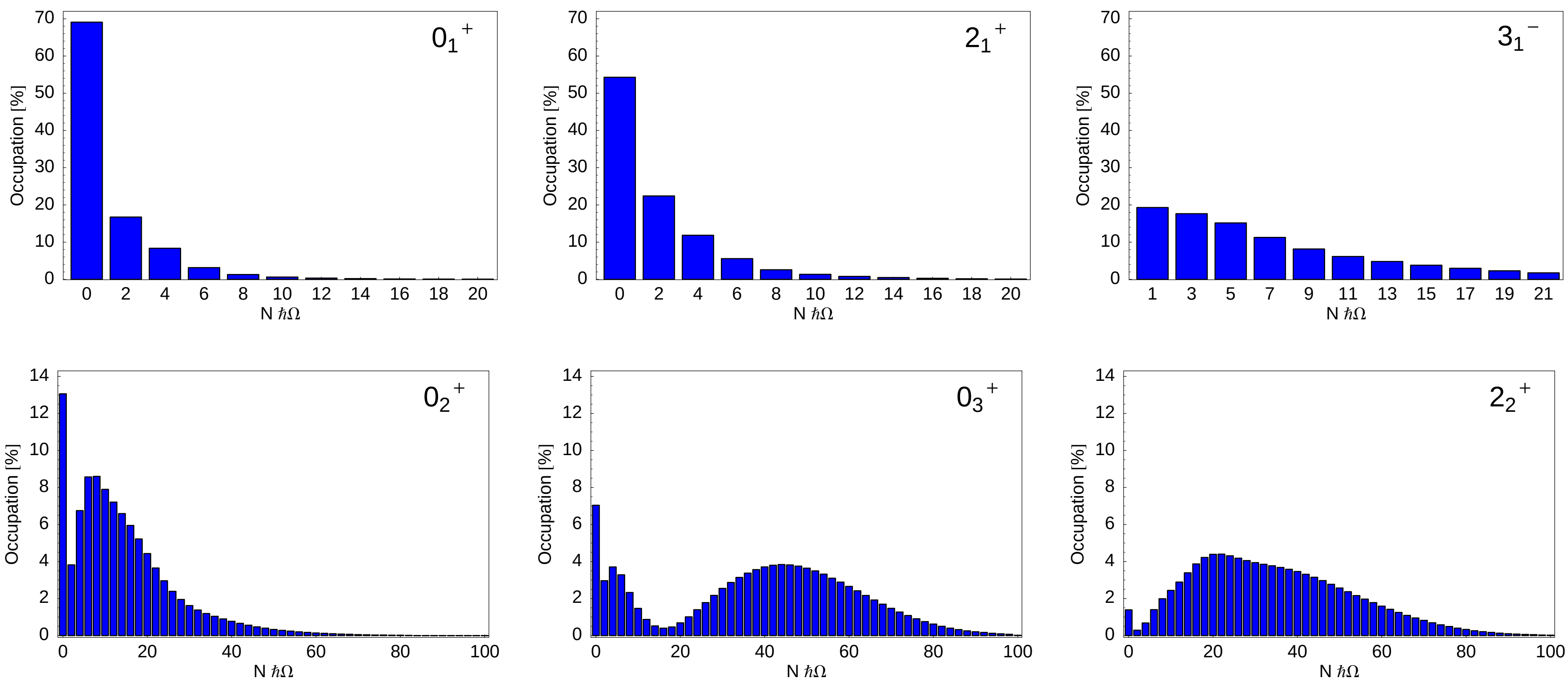
Cluster States in ^{12}C

Harmonic Oscillator $N\,\hbar\Omega$ Excitations

Occupation probabilities of spaces with N harmonic oscillator quanta

$$\mathrm{Occ}(N) = \langle \Psi \,|\, \delta\Big(\textstyle\sum_{i=1}^{A}\big(\underset{\sim}{H}^{HO}(i)/\hbar\Omega - 3/2\big) - N\Big) \,|\, \Psi \rangle$$

FMD



$\mathcal{H}_{\text{low-}k}$ Hilbert space:
Fermionic Molecular Dynamics

Beryllium Isotopes

- α-clustering, halos in ^{11}Be and ^{14}Be, $N = 8$ shell closure ?

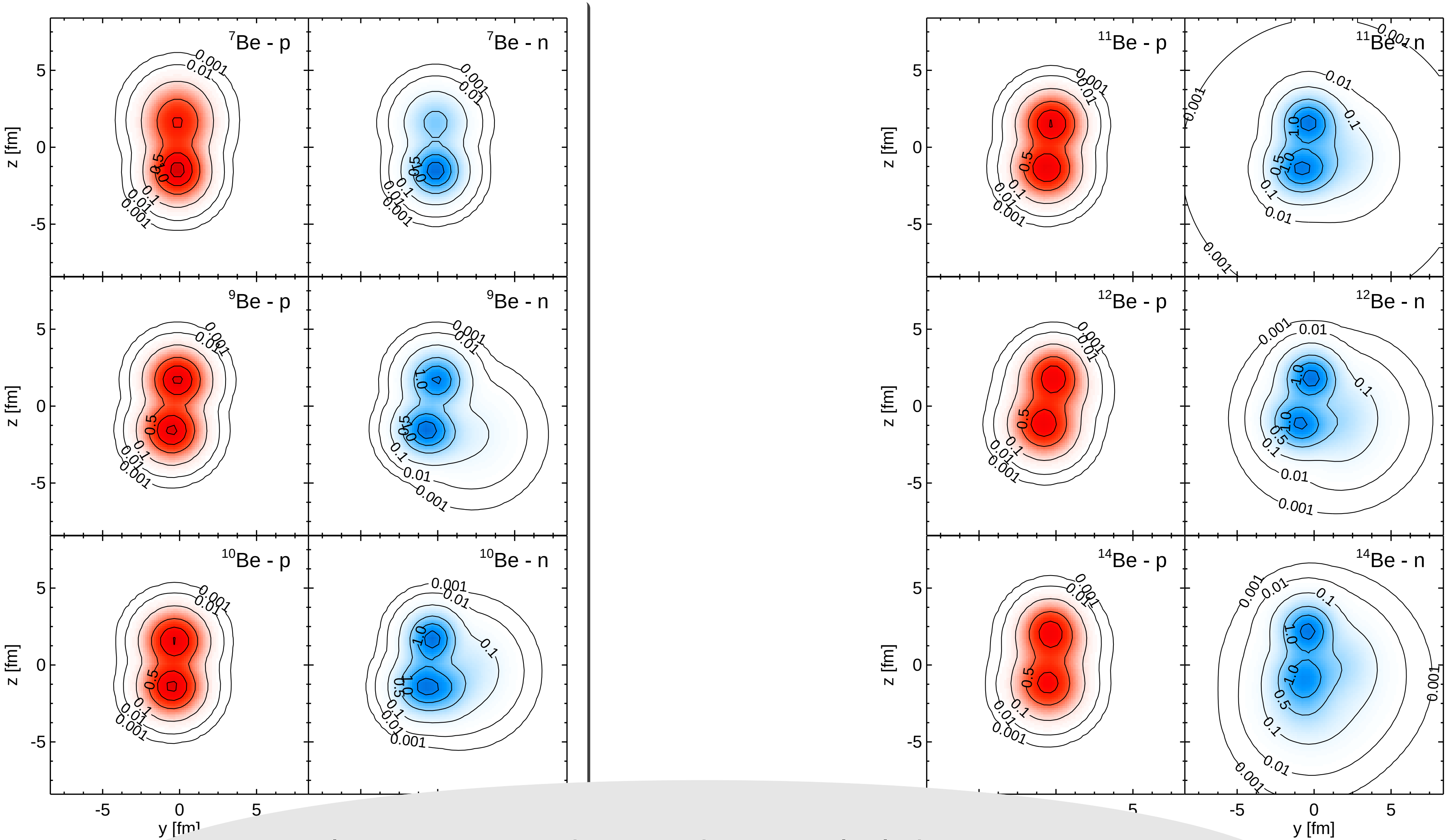
Observables

- energies
- charge and matter radii, electromagnetic transitions

Thomas Neff
Results still preliminary !

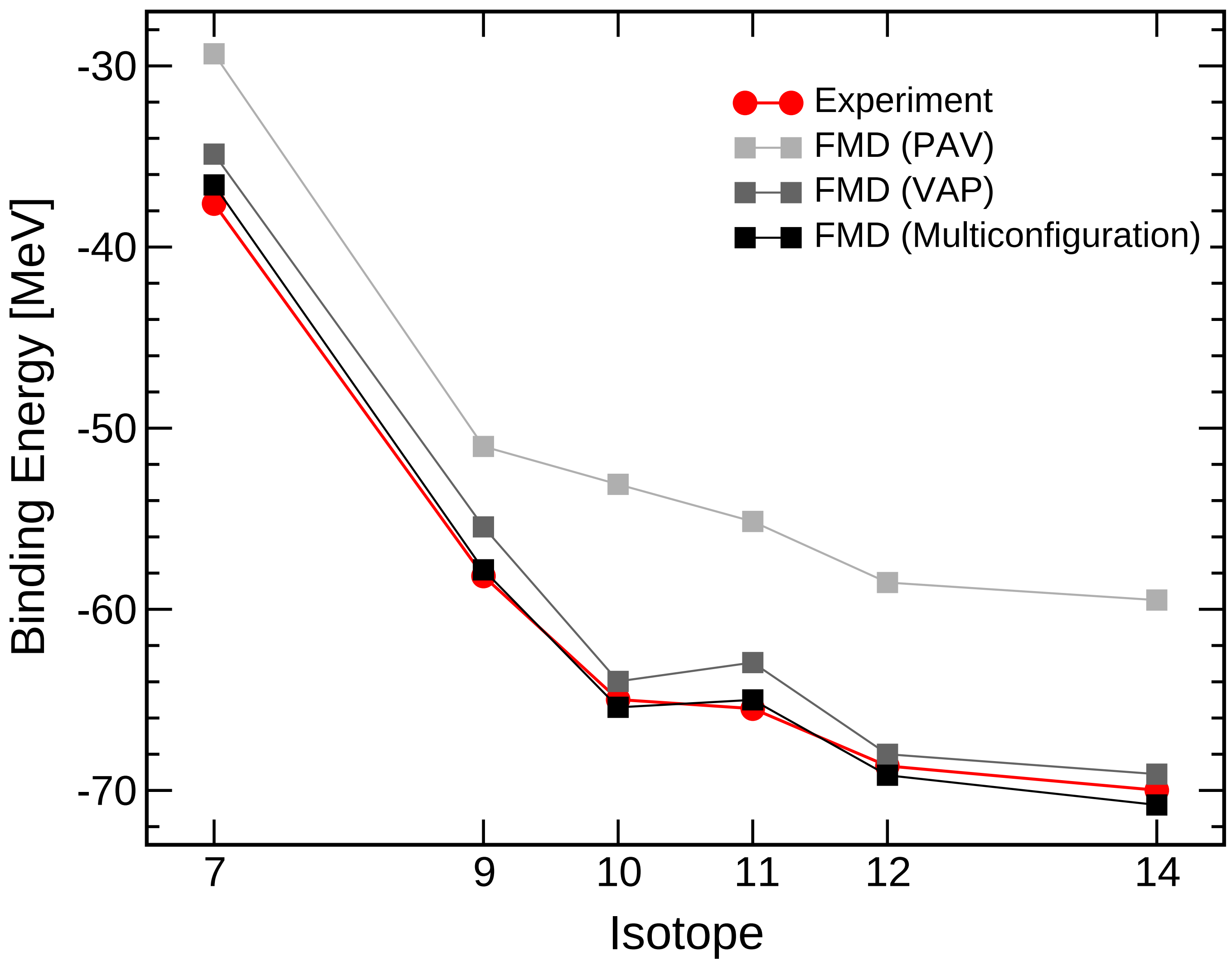
Beryllium Isotopes

Variation after Projection



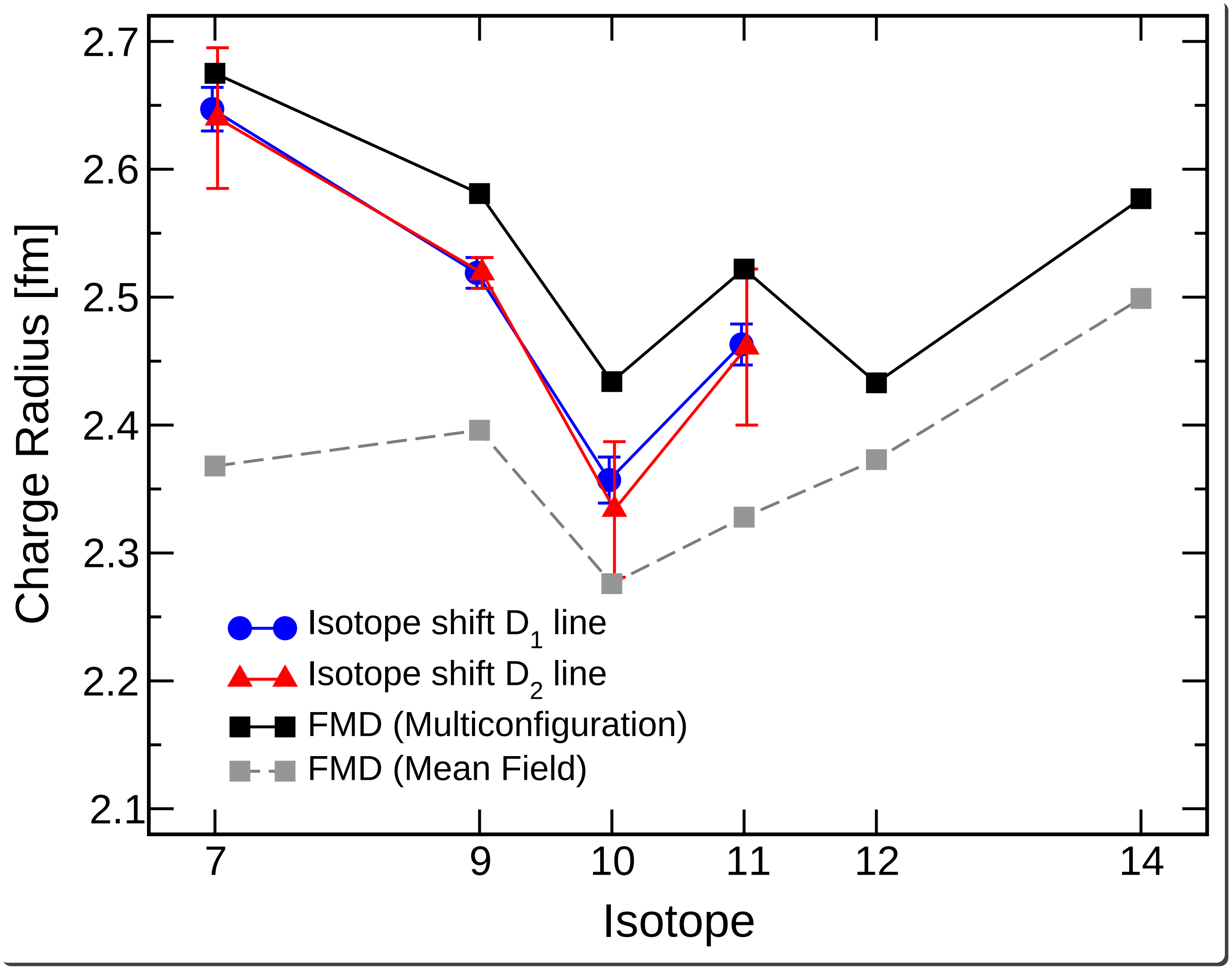
- cluster structure shows up in VAP calculations
- *s*-wave configuration dominant in ^{11}Be

Beryllium Isotopes
Binding energies



- large correlation energies due to cluster structure
- loosely bound systems gain most by configuration mixing

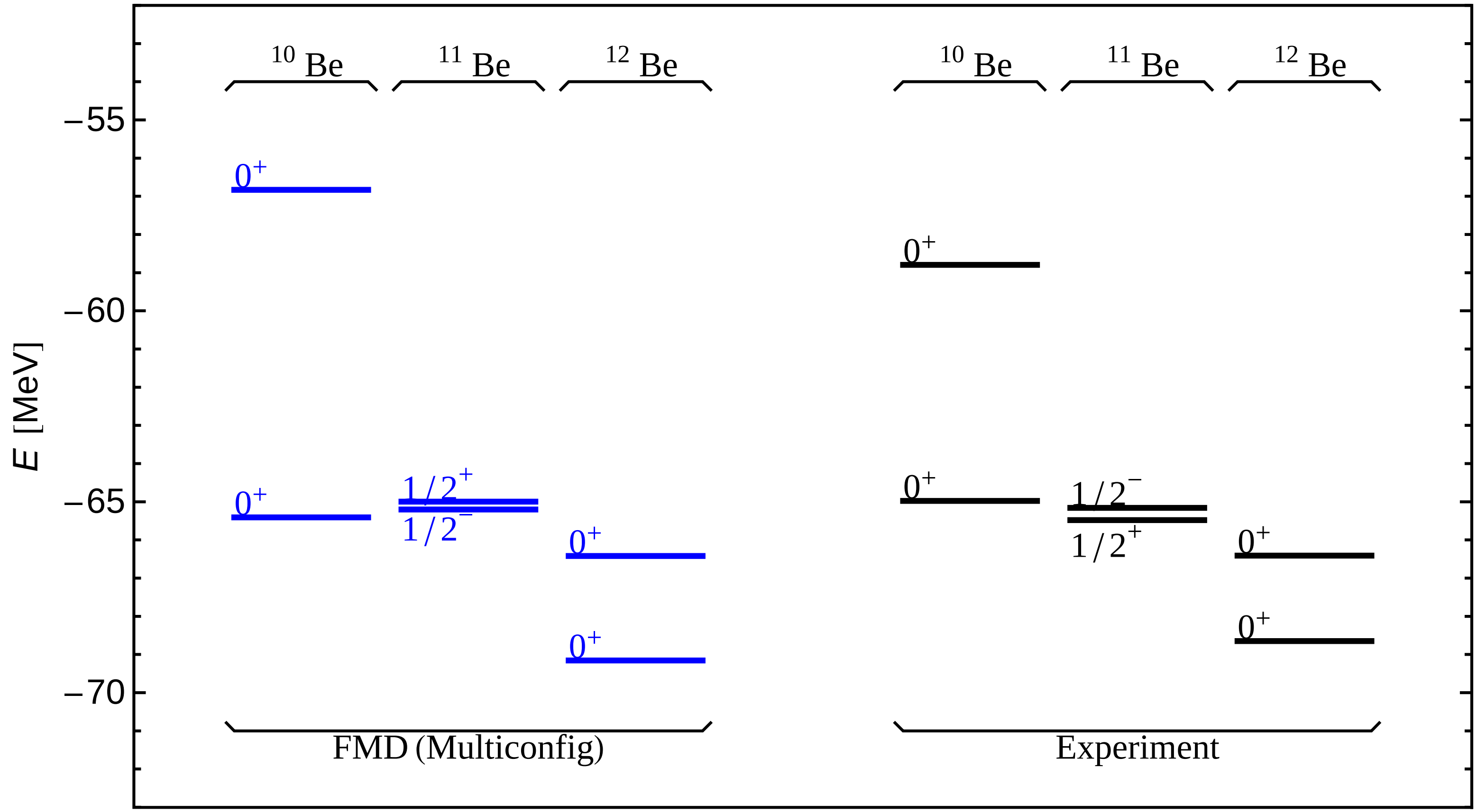
Charge Radii



Nörtershäuser *et al.*, Phys. Rev. Lett. **102**, 243002 (2009)

Zakova, Neff, *et al.*, J. Phys. G, accepted for publication

$N = 8$ Shell Closure ?



- ”almost correct” level ordering in ^{11}Be
- ^{12}Be ground state dominated by p^2 configuration, sizeable admixture of s^2 and d^2 configurations which strongly mix

Electromagnetic transitions

^{10}Be

	FMD(Multiconfig)	Experiment
$B(E2; 2_1^+ \rightarrow 0_1^+)$	11.27 $e^2\text{fm}^4$	10.2 ± 1.0 $e^2\text{fm}^4$
$B(E2; 0_2^+ \rightarrow 2_1^+)$	4.99 $e^2\text{fm}^4$	3.2 ± 1.9 $e^2\text{fm}^4$
$B(E1; 0_2^+ \rightarrow 1_1^-)$	0.013 $e^2\text{fm}^2$	0.013 ± 0.004 $e^2\text{fm}^2$

^{11}Be

	FMD(Multiconfig)	Experiment
$B(E1; 1/2_1^+ \rightarrow 1/2_1^-)$	0.020 $e^2\text{fm}^2$	0.099 ± 0.010 $e^2\text{fm}^2$

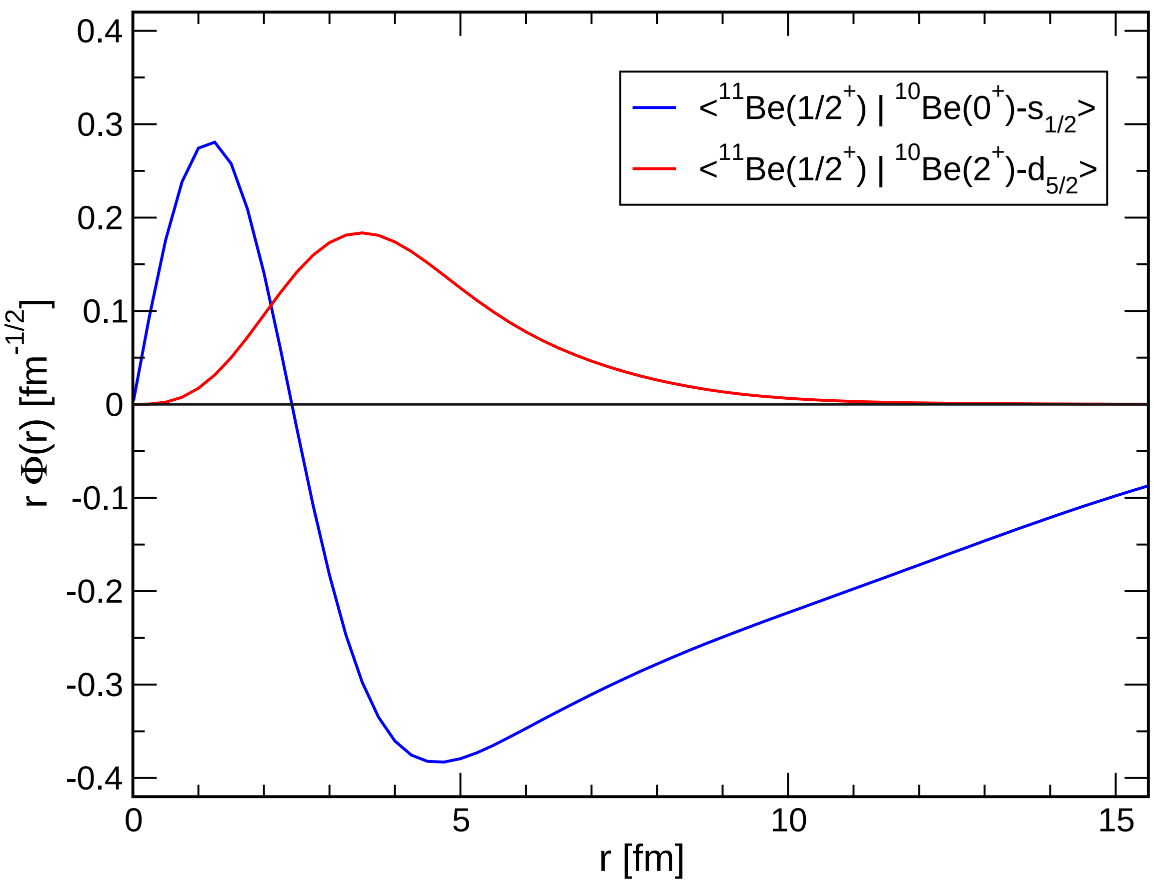
^{12}Be

	FMD(Multiconfig)	Experiment
$B(E2; 2_1^+ \rightarrow 0_1^+)$	8.27 $e^2\text{fm}^4$	8.0 ± 3.0 $e^2\text{fm}^4$
$B(E2; 0_2^+ \rightarrow 2_1^+)$	6.50 $e^2\text{fm}^4$	7.0 ± 0.6 $e^2\text{fm}^4$
$M(E0; 0_1^+ \rightarrow 0_2^+)$	1.05 $e\text{fm}^2$	0.87 ± 0.03 $e\text{fm}^2$
$B(E1; 0_1^+ \rightarrow 1_1^-)$	0.08 $e^2\text{fm}^2$	0.051 ± 0.003 $e^2\text{fm}^2$

Nakamura *et al.*, Phys. Lett. **B394**, 11 (1997).
Shimoura *et al.*, Phys. Lett. **B654**, 87 (2007).

Iwasaki *et al.*, Phys. Lett. **B491**, 8 (2000).

^{11}Be-^{10}Be Overlaps



- extended s-wave halo
- $s_{1/2}$ spectroscopic factor overestimated compared to results obtained from knockout and transfer reactions

Spectroscopic Factors

^{11}Be	^{10}Be	l_j	S
$1/2^+$	0^+	$s_{1/2}$	0.937
	2^+	$d_{5/2}$	0.094
	2^+	$d_{3/2}$	0.007
$5/2^+$	0^+	$d_{5/2}$	0.543
	2^+	$s_{1/2}$	0.329
	2^+	$d_{5/2}$	0.243
$1/2^-$	0^+	$p_{1/2}$	0.805
	2^+	$p_{3/2}$	0.779

Reactions

Program

- FMD Hilbert space should contain besides bound states, also resonances and scattering states
- Implement boundary conditions
- Phase shifts, capture cross section

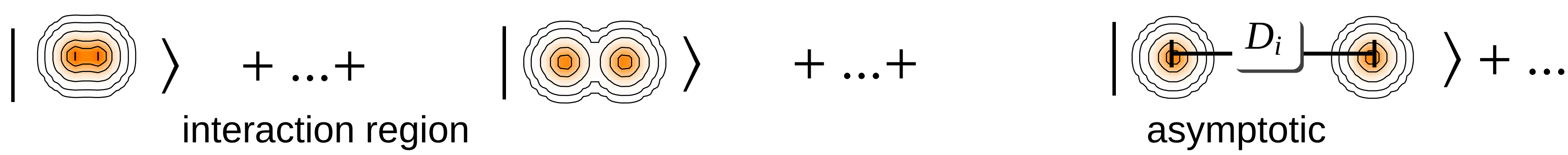
^{3}He$(\alpha,\gamma)^7$Be reaction

Many-Body Hilbert Space for Scattering

Localized FMD states can represent many-body scattering states

- ➨ asymptotic states product of “frozen” FMD states $\left(\mathcal{A} \left| {}^3\mathrm{He}, -D_i/2 \right\rangle \otimes \left| {}^4\mathrm{He}, +D_i/2 \right\rangle \right)$
- ➨ FMD states for compound system in the interaction region $\left(\left| {}^7\mathrm{Be} \right\rangle , \left| {}^7\mathrm{Be}^* \right\rangle \; \ldots \right)$

scattering state:



Boundary conditions

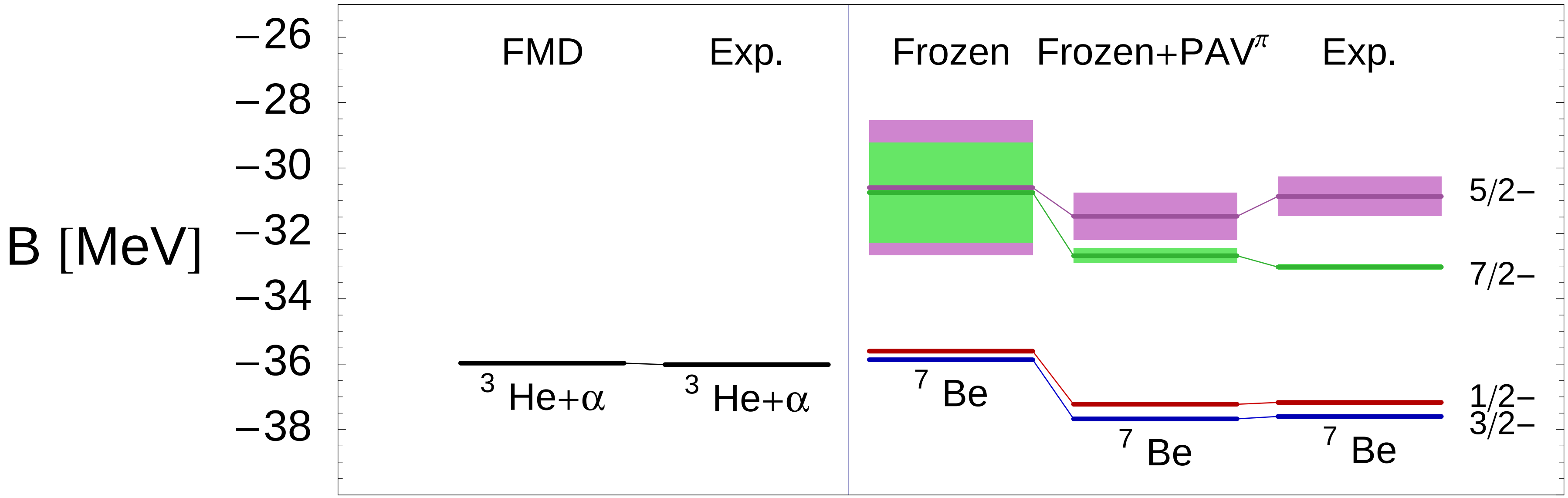
- matching to the Coulomb solution of two point-like nuclei
- ➨ phase shifts for scattering or widths of resonances

^{7}Be Levels Bound and in Continuum

- boundary condition outgoing wave only, **Gamov** state

$$\langle r \,|\, \Psi, [\ell \tfrac{1}{2}] J^\pi \rangle \xrightarrow{r\to\infty} iF_\ell(kr) + G_\ell(kr), \quad k = +\sqrt{2\mu Z}$$

➨ complex eigenvalue $Z = E - i\Gamma/2$



interaction slightly adjusted
to give correct threshold

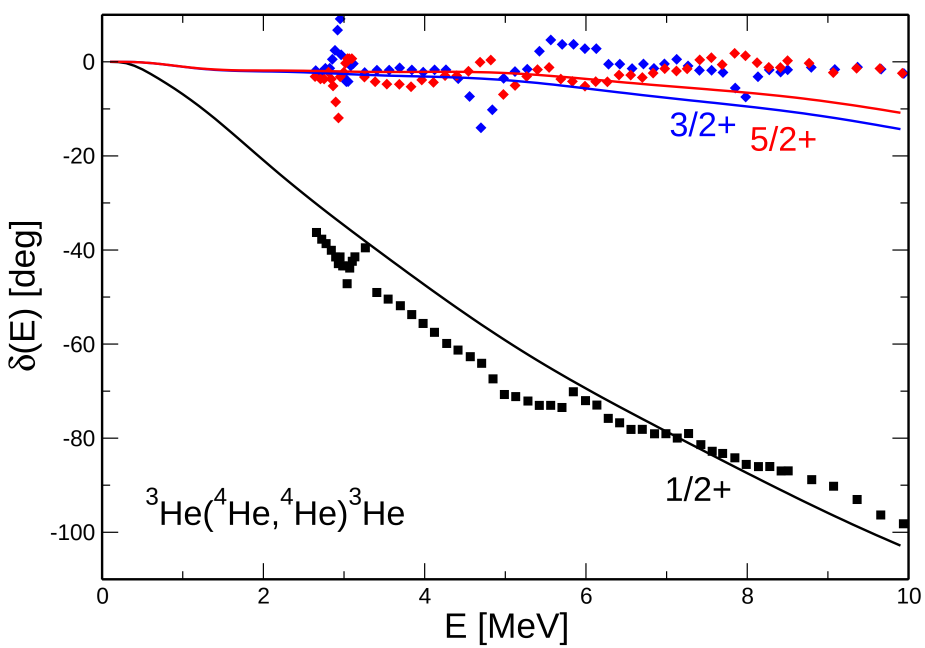
^{3}He – ^{4}He phase shifts

- boundary condition Coulomb scattering solutions

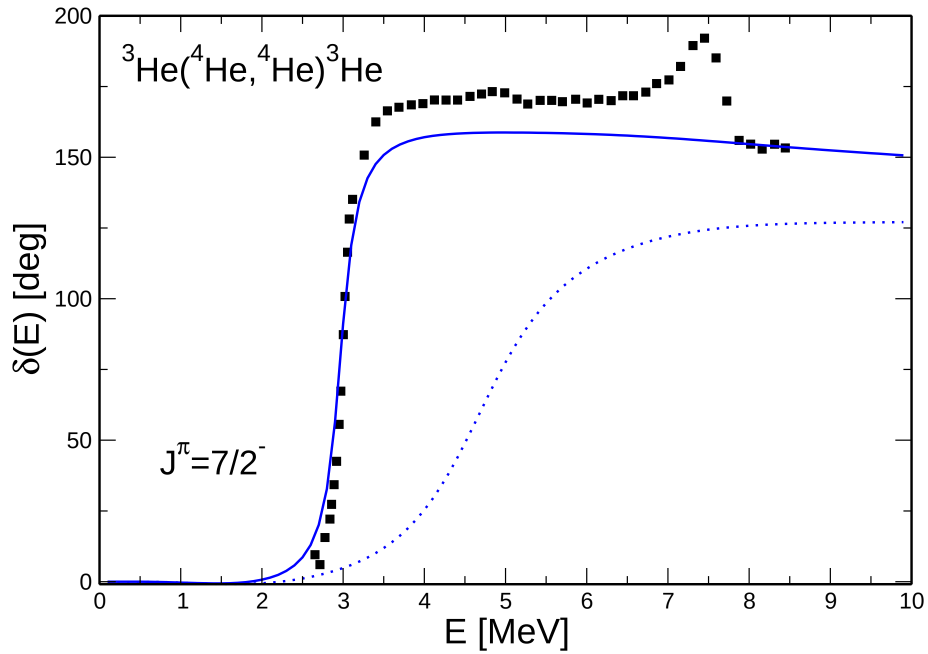
$$\left\langle r \middle| \Psi, [\ell \tfrac{1}{2}] J^\pi \right\rangle \xrightarrow{r\to\infty} F_\ell(kr) + \tan(\delta_\ell(k))\, G_\ell(kr), \quad k = +\sqrt{2\mu E}$$

➨ phase shift $\delta(E)$

non-resonant



resonant

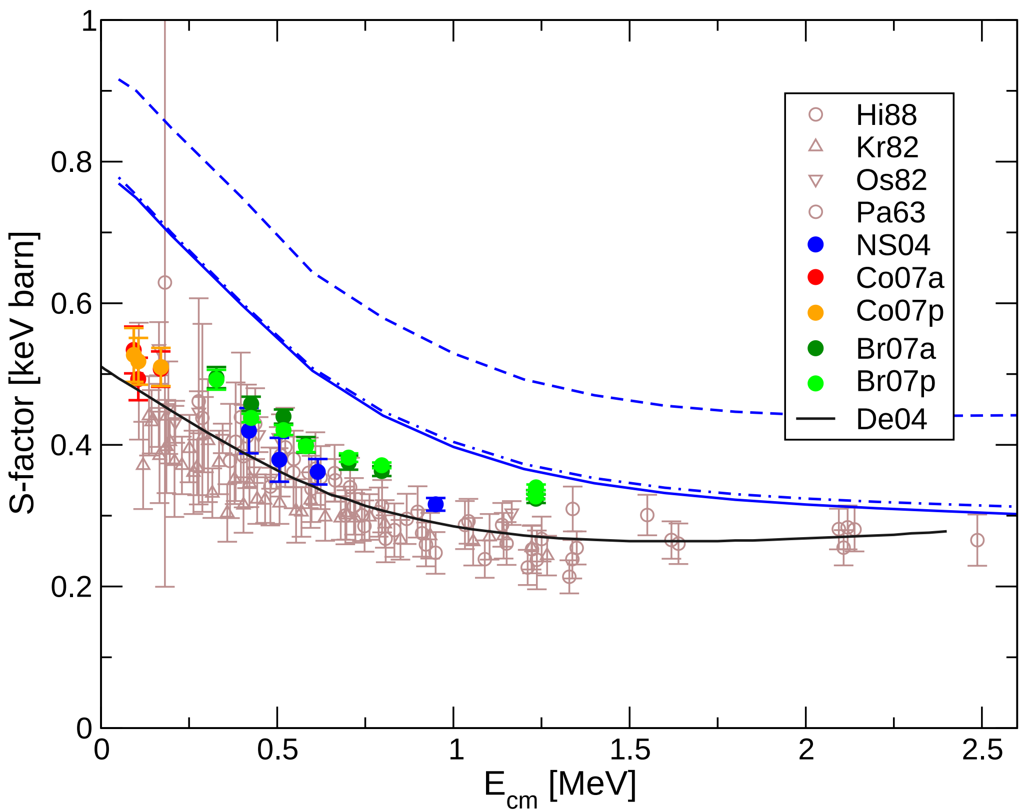


······ frozen states only

S-Factor of Radiative Capture

preliminary

- Capture from $1/2^+$, $3/2^+$ and $5/2^+$ scattering states into $3/2^-$ and $1/2^-$ bound states
- ^{7}Be described by single PAV$^\pi$ configuration (dashed line) or VAP configurations for $3/2^-$ and $1/2^-$ (dash dotted line) and additional $5/2^-$ and $7/2^-$ VAP configurations (solid line)



interaction slightly adjusted to give correct threshold

New data

(LUNA, Seattle and Weizmann)

R-matrix fit to old data (—)

Descouvement et al. (2004)

Concluding Remarks

How to build a nuclear theory

1) Choose proper relevant degrees of freedom
   e.g. c.m. positions & spins of nucleons,
   one-body & pairing densities for larger nuclei
2) Find corresponding Hamiltonian (energy expressed in terms of deg. of freedom)
3) Calculate observables: energies, transitions, moments, cross sections, ...
   compare with data, make predictions

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Nucleons are complex many-body systems, not point like

- NN-interaction is not a priori given, not fundamental
  is an effective interaction, depends on many-body Hilbert space
- NNN-interactions are needed
  $\left(\underset{\sim}{V}_{\mathrm{NN}} + \underset{\sim}{V}_{\mathrm{NNN}}\right)$ has to be treated consistently
- 2- and 3-body systems do not uniquely fix $\left(\underset{\sim}{V}_{\mathrm{NN}} + \underset{\sim}{V}_{\mathrm{NNN}}\right)$

Concluding Remarks

Novel concepts & methods progressed nuclear structure substantially in the last decade

- Chiral-PT, NN + NNN consistent
- Phase-shift equivalent low-momentum effective interactions
  UCOM, SRG, $V_{\text{low-}k}$, tame short-range correlations
- Exact few-body methods, Fadeev Yakubowski, hypersherical harmonics, ...
- No-core shell model, Coupled Cluster, Importance sampling, ...
  medium and long-range correlations by configuration mixing
- Exotic states, clusters, halos, require non-standard Hilbert spaces, FMD, ...
- First ab initio microscopic nucleus-nucleon scattering, nucleus-nucleus scattering
- Ab initio energy density functionals ?
- ...

FAIR will be a key promoter driving nuclear structure theory to new frontiers

Thanks to my Collaborators

- A. Cribeiro, K. Langanke, T. Neff, D. Weber
  GSI Darmstadt

- H. Hergert, R. Roth
  Institut für Kernphysik, TU Darmstadt

Helium Isotopes ^{4}He – ^{8}He

Structure

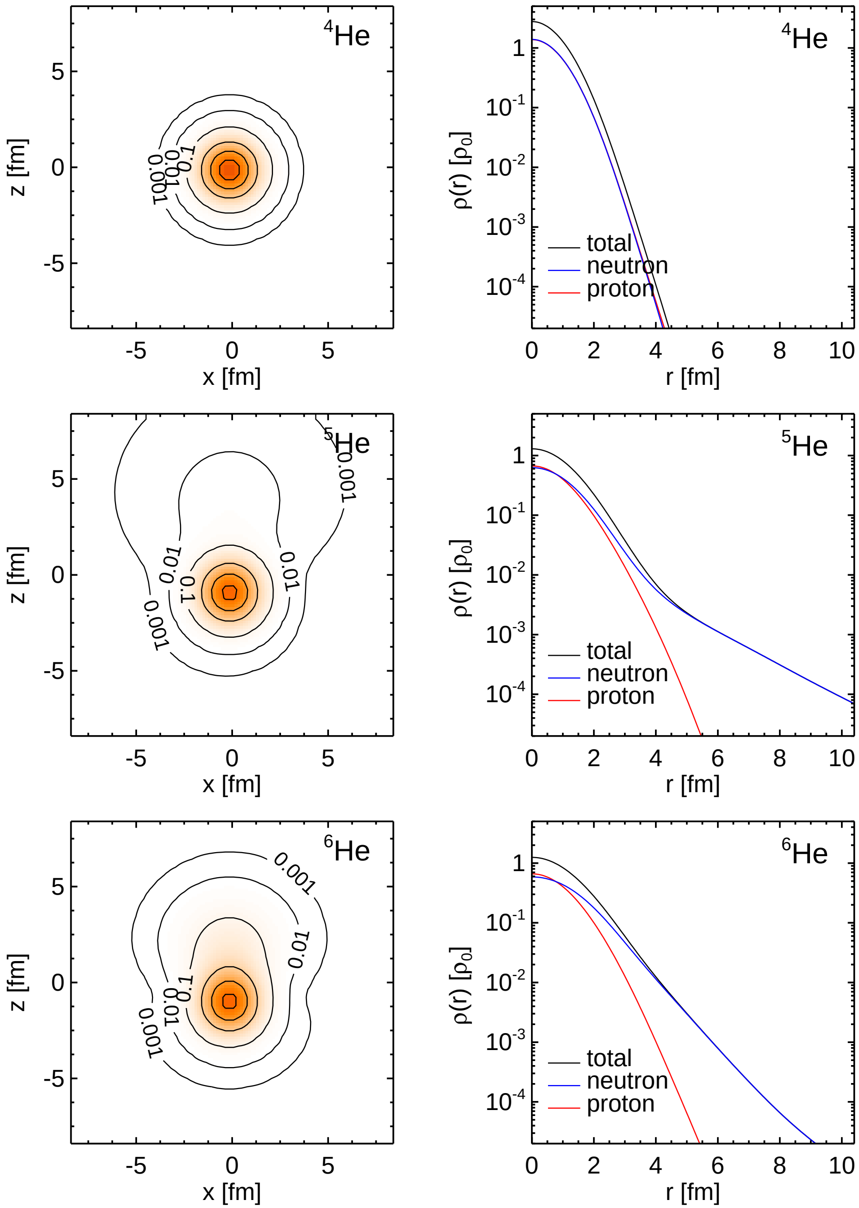
- Borromean behaviour
- Zero-point oscillation of soft dipole mode

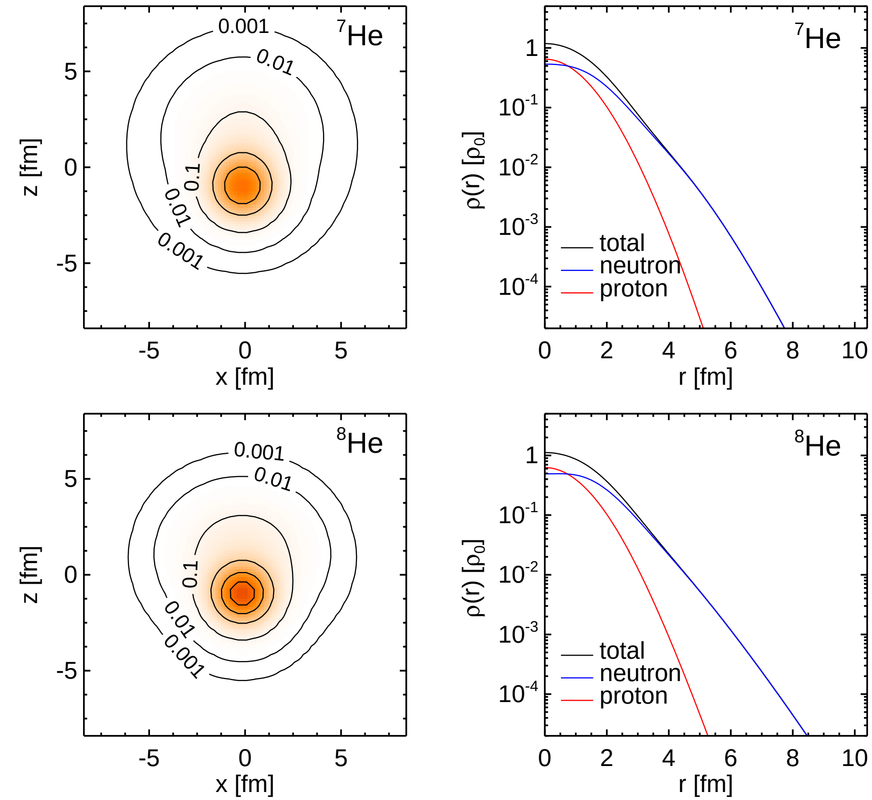
Observables

- Charge radii
- Matter radii
- Proton, neutron densities

Helium Isotopes

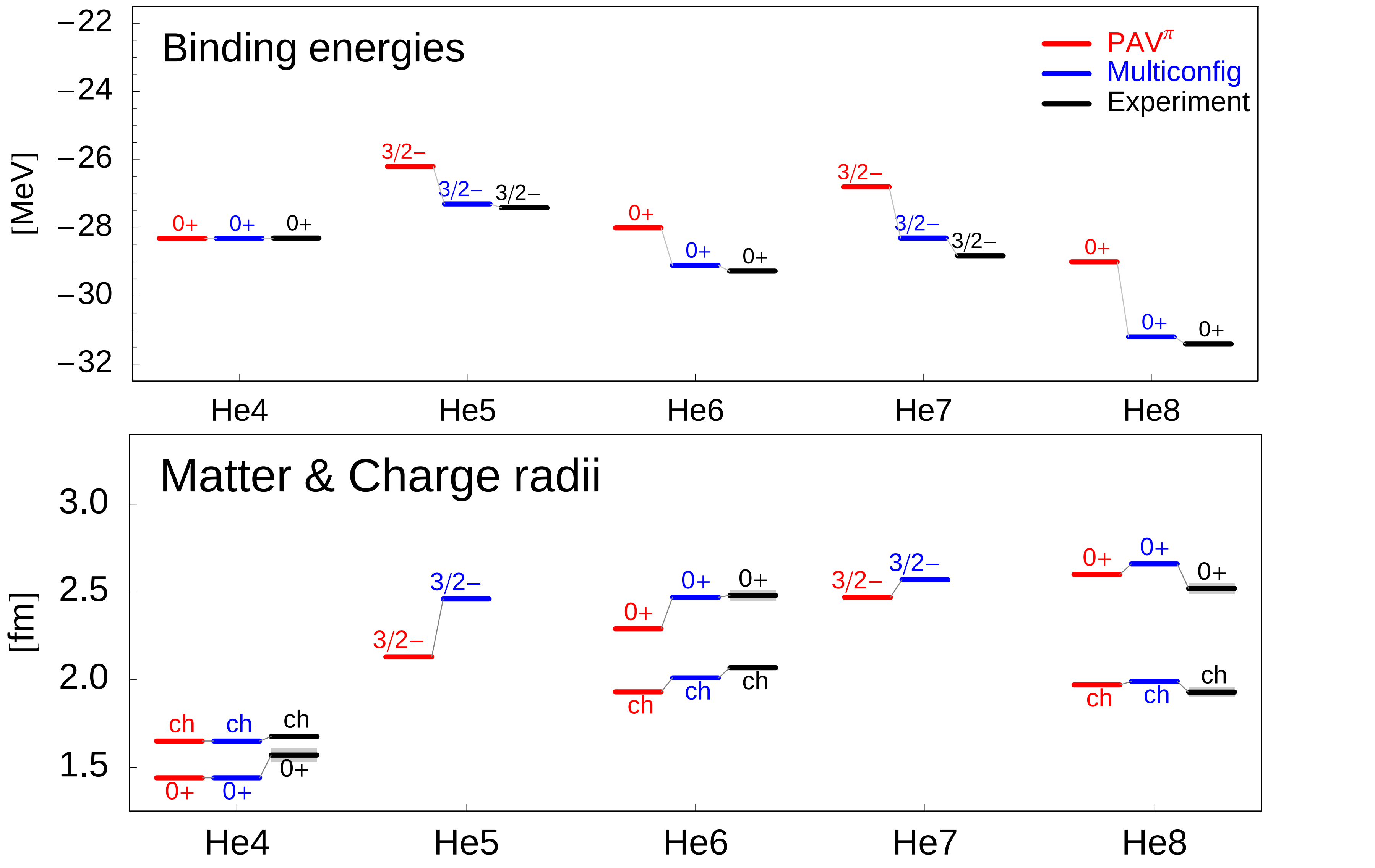
dipole and quadrupole constraints





- intrinsic densities of VAP$^\pi$ states
  $$\left| Q^{\pm} \right\rangle = \tfrac{1}{2}\left(1 \pm \underset{\sim}{\Pi}\right)\left| Q \right\rangle$$
- radial densities from multiconfiguration calculations

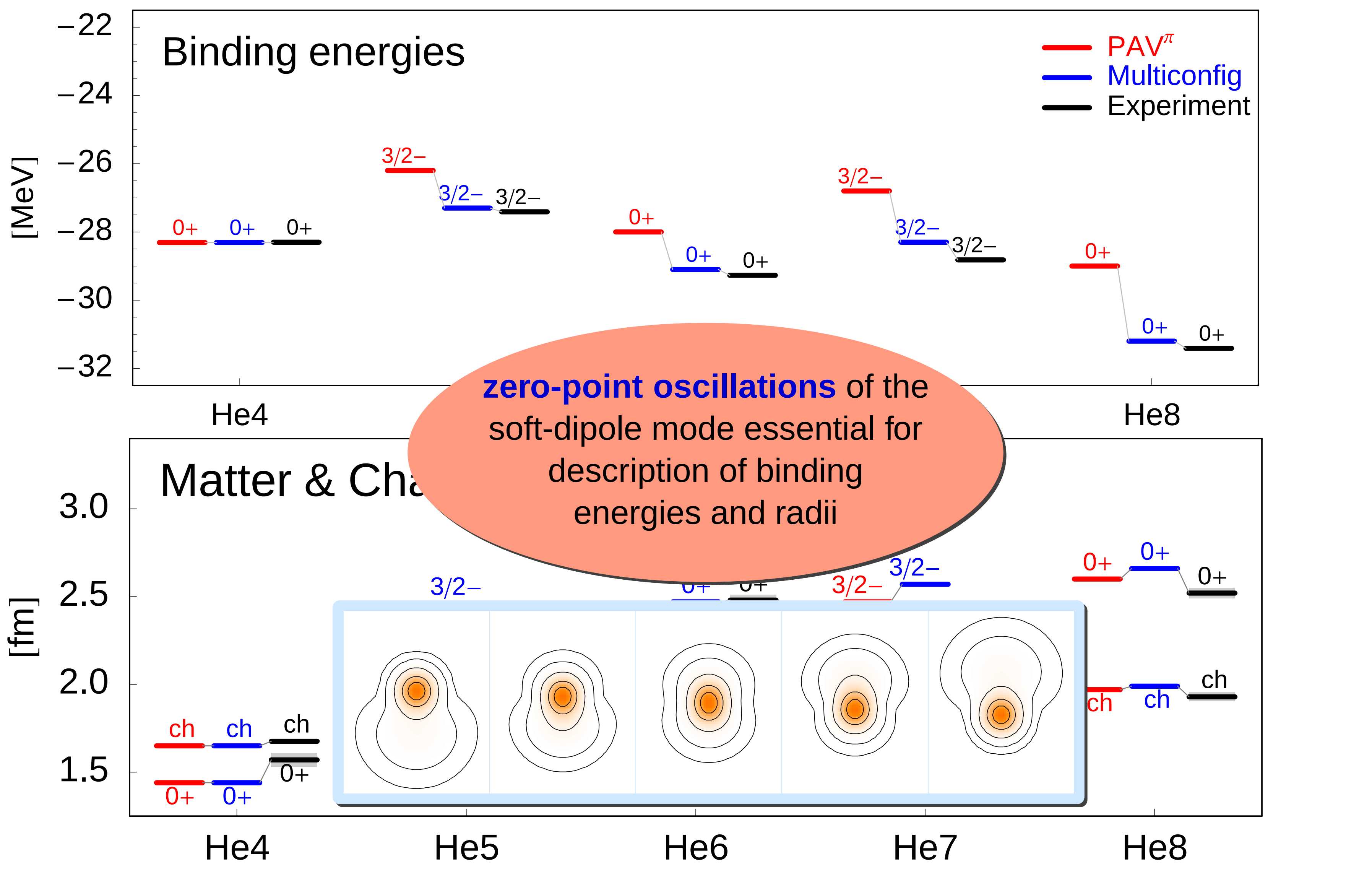
Helium Isotopes



Exp: Ozawa,Suzuki,Tanihata, NP**A693**(2001)32; Raman,Nestor,Tikkanen, Atomic Data and Nucl. Data Tables **78**(2001)1

^{6}He and ^{8}He charge radius: P. Mueller et al, Phys. Rev. Lett. **99** (2007) 252501

Helium Isotopes



zero-point oscillations of the soft-dipole mode essential for description of binding energies and radii

Exp: Ozawa,Suzuki,Tanihata, NP**A693**(2001)32; Raman,Nestor,Tikkanen, Atomic Data and Nucl. Data Tables **78**(2001)1

^{6}He and ^{8}He charge radius: P. Mueller et al, Phys. Rev. Lett. **99** (2007) 252501

Neon Isotopes ^{17}Ne – ^{22}Ne

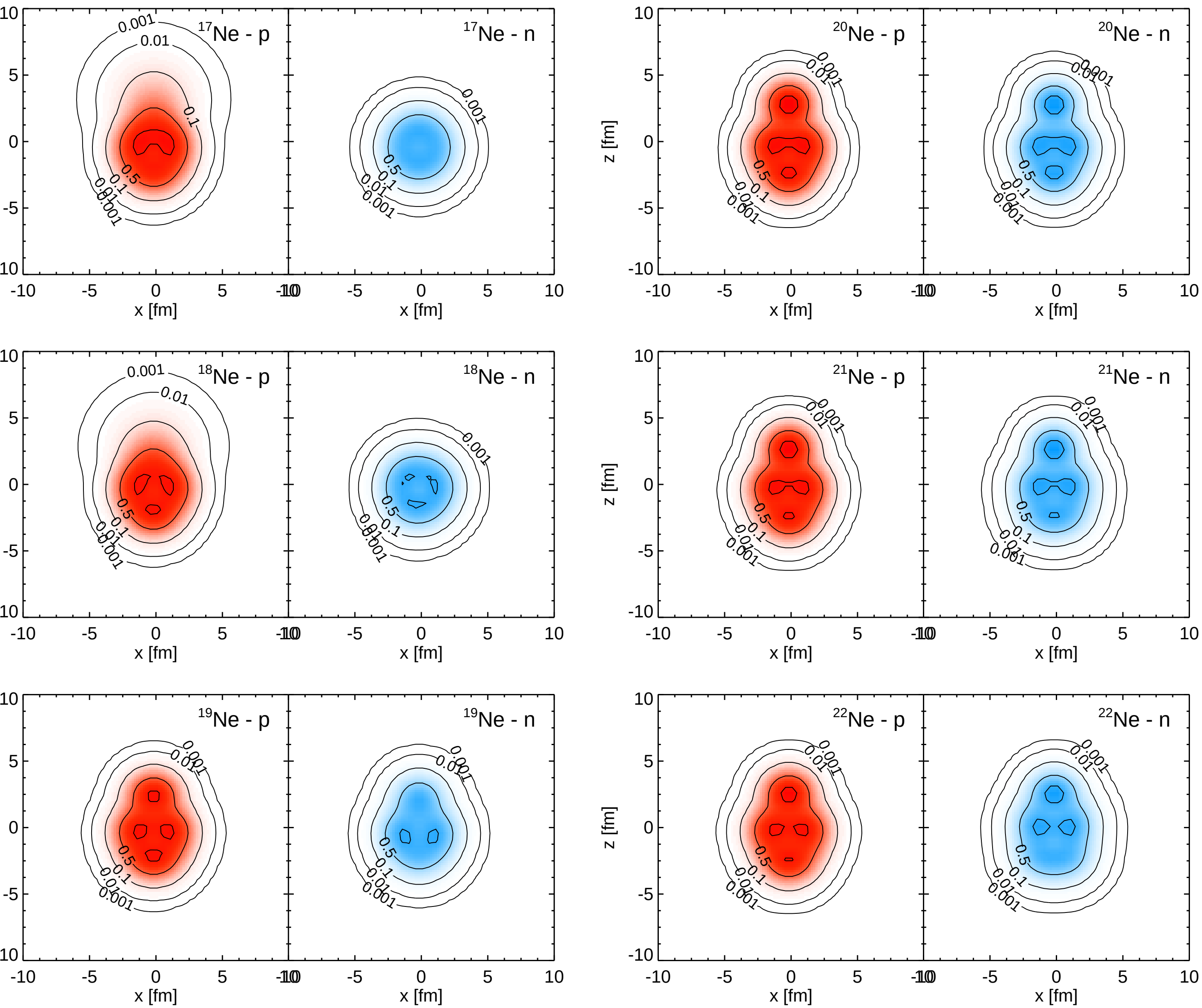
Structure

- s^2/d^2 **occupation in** ^{17}Ne **and** ^{18}Ne
- ^{3}He **and** ^{4}He **cluster admixtures**

Observables

- **Charge Radii**
- **Matter Radii**
- **Is** ^{17}Ne **a Halo nucleus ?**

Variation after Parity Projection VAP$^{\pi}$



Intrinsic proton/neutron densities of dominant FMD state

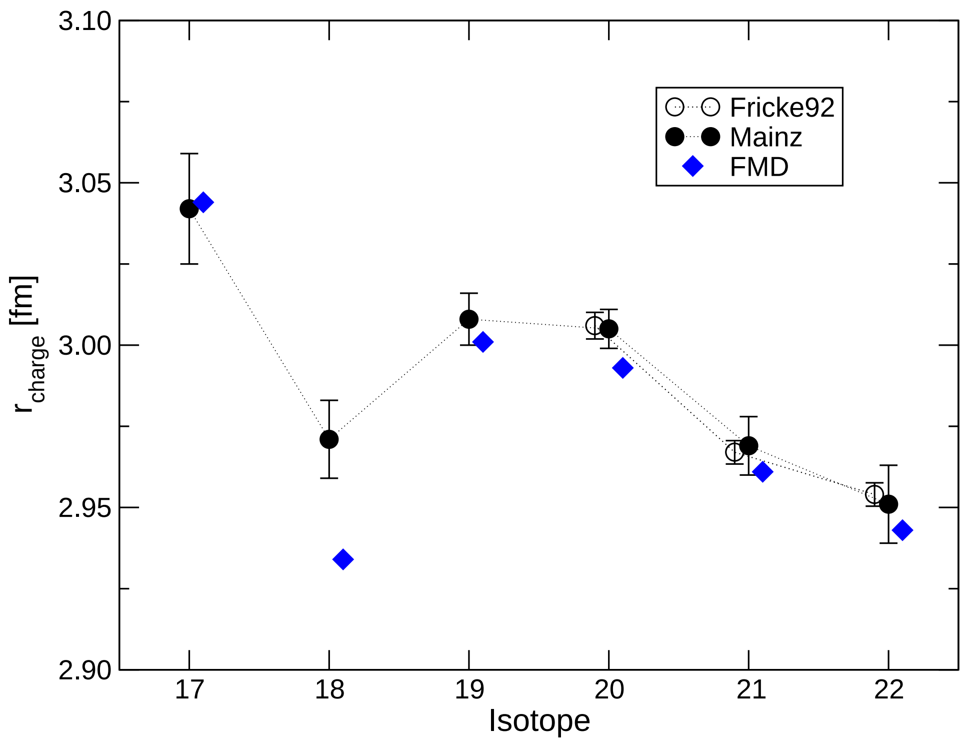
- Variation after parity projection on positive and negative parity
- Crank strength of spin-orbit force, changes properties of single-particle orbits and their occupations
- "s^2" and "d^2" minima in ^{17,18}Ne
- explicit cluster configurations:

^{17}Ne: ^{14}O-^{3}He
^{18}Ne: ^{14}O-^{4}He
^{19}Ne: ^{16}O-^{3}He, ^{15}O-^{4}He

^{20}Ne: ^{16}O-^{4}He
^{21}Ne: "^{17}O"-^{4}He
^{22}Ne: "^{18}O"-^{4}He

$\left| Q^{\pm} \right\rangle$ minima

Charge Radii



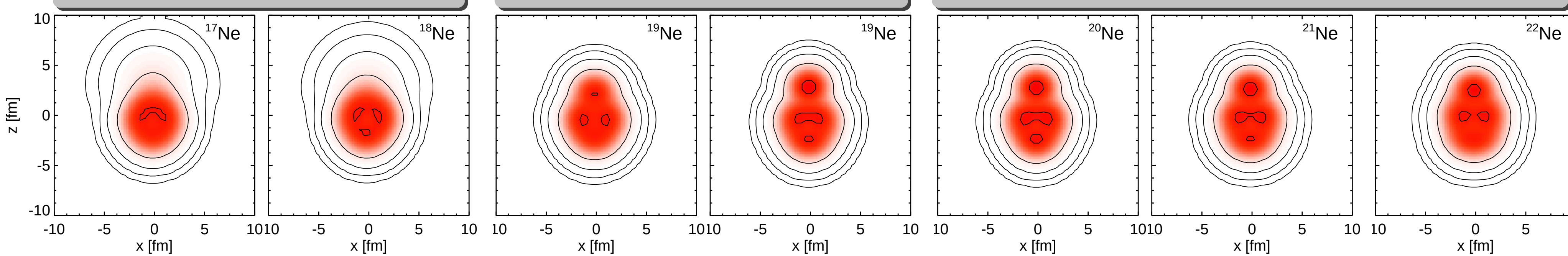
- charge radii of ^{17,18}Ne depend strongly on s^2/d^2 occupations
- cluster admixtures responsible for large charge radii in $^{19-22}$Ne
- measurements of charge radii by COLLAPS@ISOLDE

W. Geithner, T. Neff, *et al.*, submitted to PRL

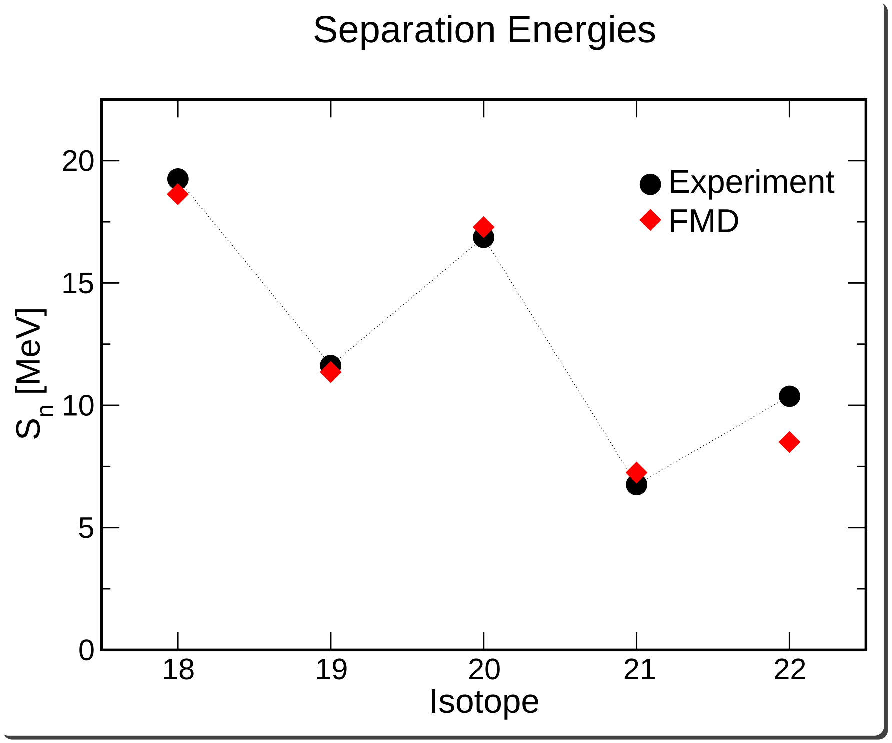
^{17,18}Ne: s^2/d^2 admixture

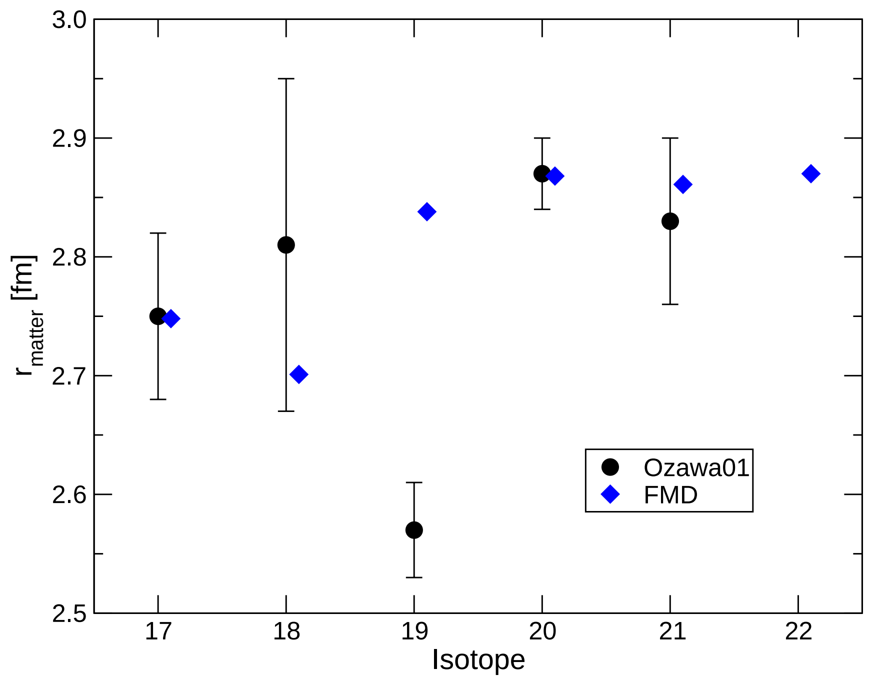
^{19}Ne: ^{3}He, α clustering

$^{20-22}$Ne: α clustering



Separation Energies and Matter Radii



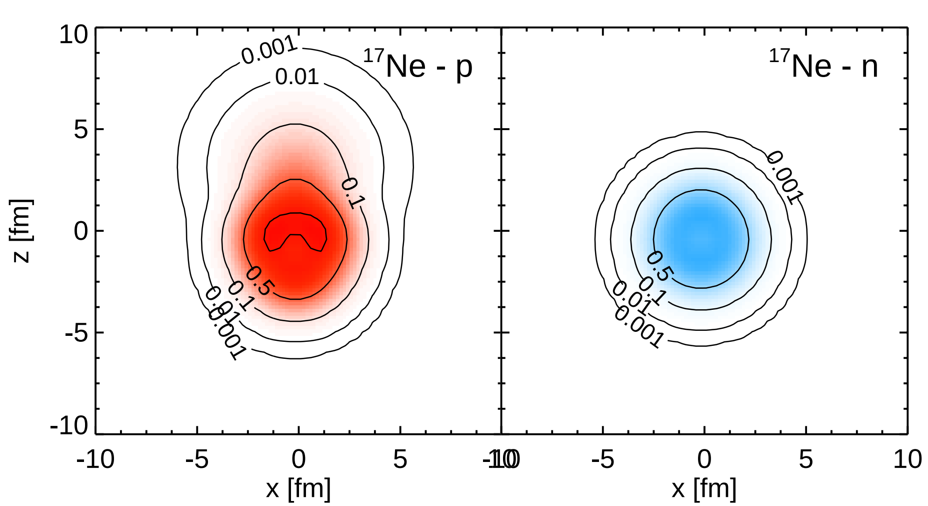


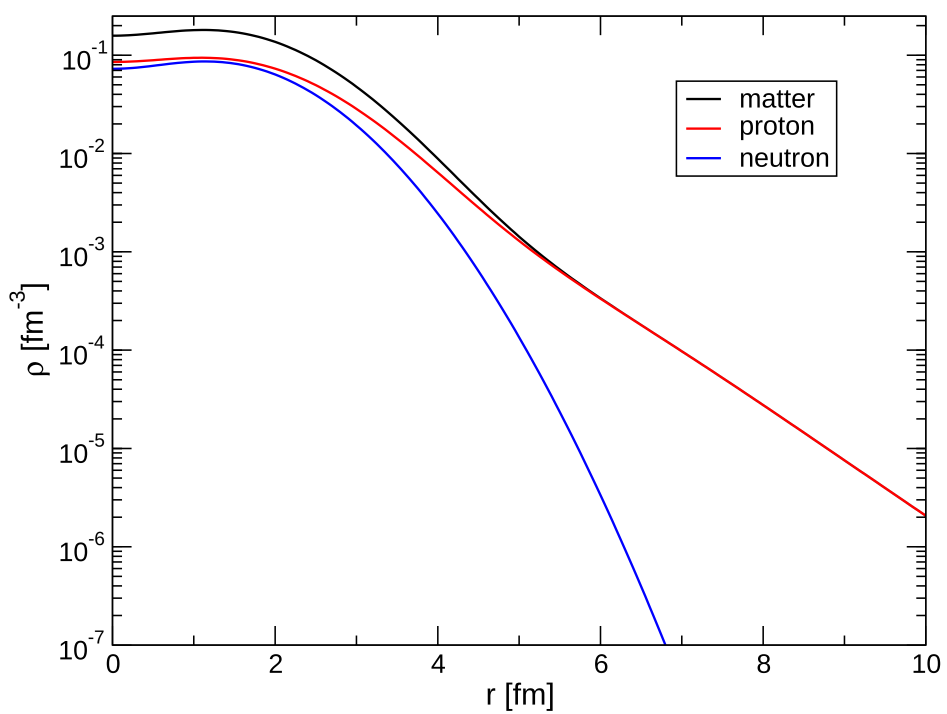
- matter radii from interaction cross sections

  A. Ozawa et al., Nuc. Phys. **A693** (2001) 32

- good agreement with exception of ^{19}Ne

^{17}Ne Halo ?



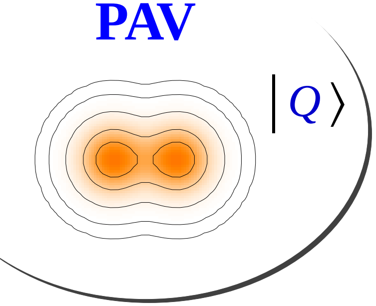


	FMD	Experiment
r_{ch}[fm]	3.03	3.042(17)
r_{mat}[fm]	2.75	2.75(7)
$B(E2; \frac{1}{2}^- \rightarrow \frac{3}{2}^-)[e^2\text{fm}^4]$	76.7	66^{+18}_{-25}
$B(E2; \frac{1}{2}^- \rightarrow \frac{5}{2}^-)[e^2\text{fm}^4]$	119.8	124(18)
occupancy s^2	40%	
occupancy d^2	55%	

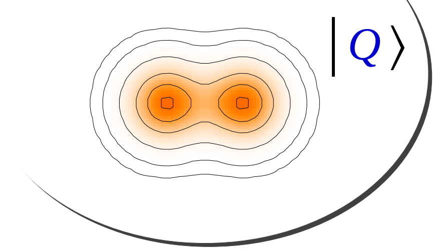
- proton skin $r_p - r_n$ = 0.45 fm
- 40% probability to find a proton at $r > 5$ fm

FMD - Projection, Variation after Proj., Multiconfiguration

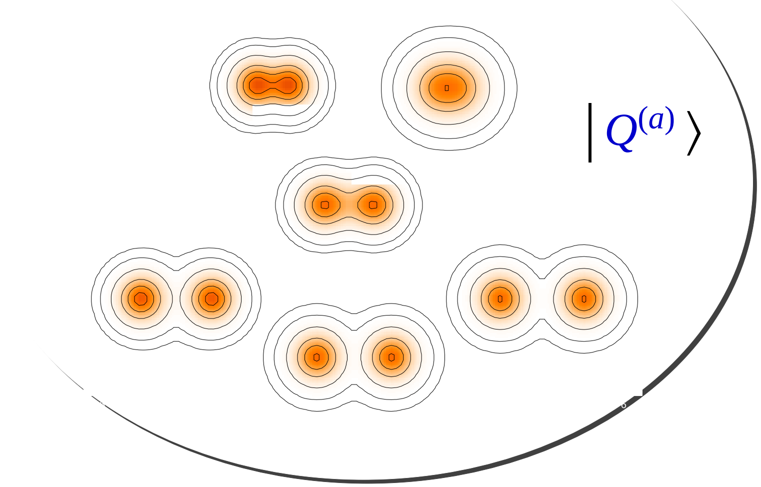
PAV



VAP



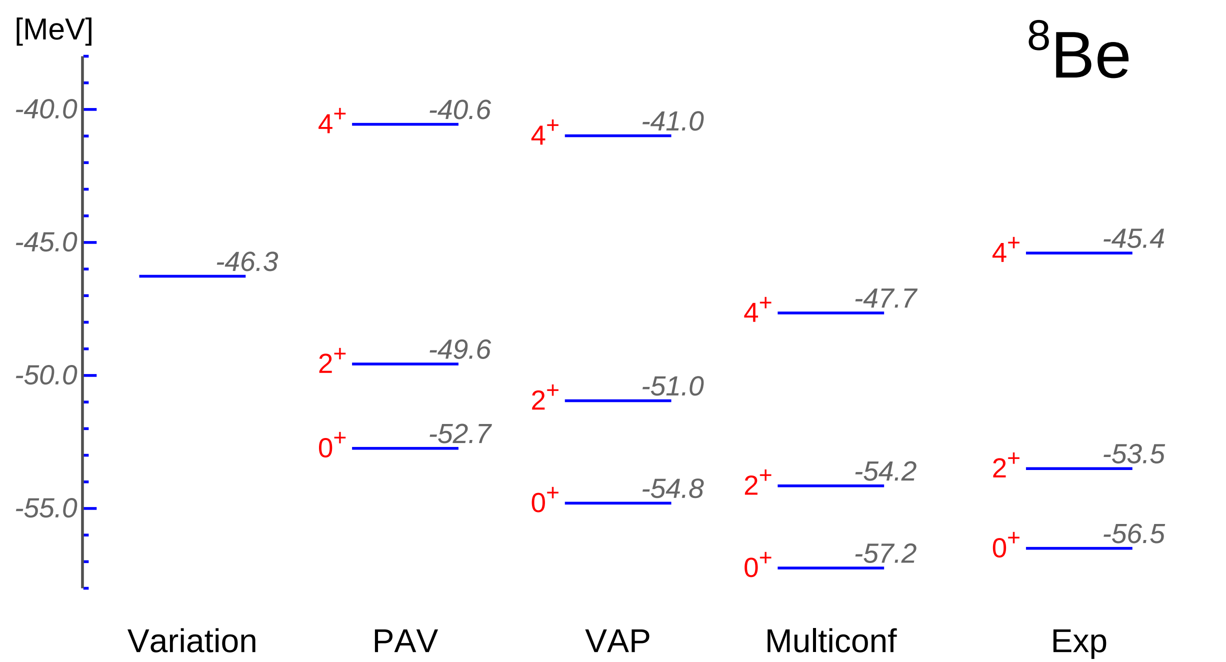
Multiconfig



$$\left| J^\pi M, n \right\rangle = \sum_{a,K'} c^{(n)}_{aK'} \, \underset{\sim}{P}^{J^\pi}_{MK'} \underset{\sim}{P}^{\mathbf{P}=0} \left| Q^{(a)} \right\rangle$$

Radius and Quadrupole Moment as Generator Coordinates

	r_{charge} [fm]	Q [fm^2]	$B(E2)$ [e^2fm^4]
PAV	2.39	-6.25	9.31
VAP	2.49	-8.02	15.36
Multiconfig	2.74	-11.88	30.39



Collective Coordinate Representation

Size Measure

- Operator $\underset{\sim}{B}$ measures extension of the system

$$\underset{\sim}{B} = \frac{1}{A^2} \sum_{i<j=1}^{A} \left(\underset{\sim}{x}(i) - \underset{\sim}{x}(j)\right)^2$$

Asymptotic Interpretation $\quad r \gg R_{C1} + R_{C2}$

- Eigenvalues relate to relative distance r (for each $J^\pi M$)

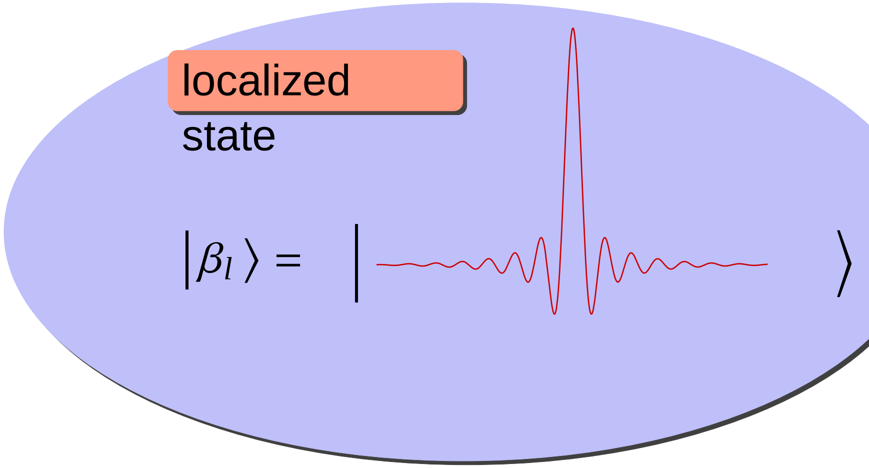
$$\underset{\sim}{B} \left| \beta_l \right\rangle = \beta_l \left| \beta_l \right\rangle$$

$$\Rightarrow \beta(r) = \frac{1}{A}\left(\frac{A_1 A_2}{A} r^2 + A_1 R_{C1}^2 + A_2 R_{C2}^2\right) \quad \Rightarrow \quad r_l \leftrightarrow \beta_l$$

- Eigenvectors are localized in β and r

$$\left\langle \beta_l \right| \underset{\sim}{B}^2 \left| \beta_l \right\rangle - \left\langle \beta_l \right| \underset{\sim}{B} \left| \beta_l \right\rangle^2 = 0$$

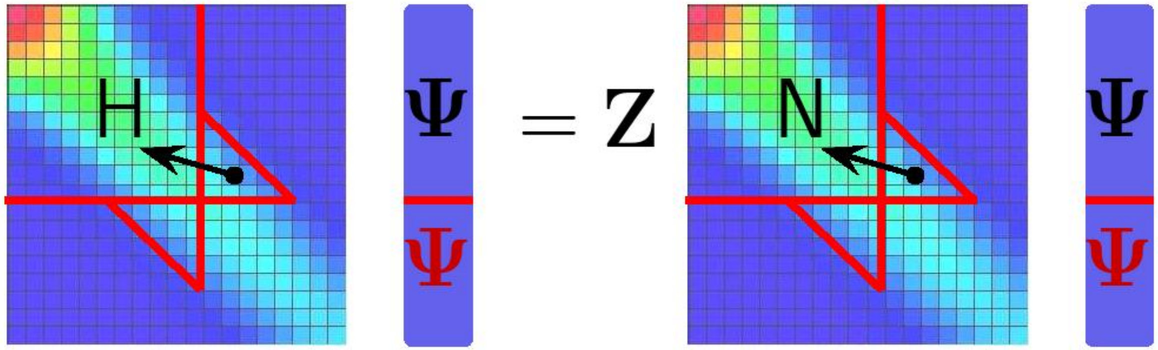
$$\Rightarrow \Psi(r_l) := \left\langle \beta_l \middle| J^\pi M; \Psi \right\rangle$$ **relative wave function**



Boundary Conditions 1

Implement boundary conditions using the Collective Coordinate Representation

- Eigenvalue problem for scattering state $\left| J^\pi M; \Psi \right\rangle$



$$\left| J^\pi M; \Psi \right\rangle = \sum_{aK}^{N} \psi_{aK} \, \underset{\sim}{P}^{J^\pi}_{MK} \underset{\sim}{P}^{\mathbf{P}=0} \left| Q^{(a)} \right\rangle + \sum_{aK=N+1}^{N+n} \psi_{aK} \, \underset{\sim}{P}^{J^\pi}_{MK} \underset{\sim}{P}^{\mathbf{P}=0} \left| Q^{(a)} \right\rangle$$

- Express unknown ψ_{aK} by known asymptotic solution $\left\langle r \middle| w \right\rangle = w(r)$ like

$$\frac{\left\langle \beta_N \middle| \left[\underset{\sim}{H} , \underset{\sim}{B} \right]^s \middle| J^\pi M; \Psi \right\rangle}{\left\langle \beta_N \middle| J^\pi M; \Psi \right\rangle} \overset{!}{=} \frac{\left\langle r_N \middle| \left[\frac{1}{2\mu} \left(-\frac{d^2}{dr^2} + \frac{\ell(\ell+1)}{r^2} \right) + \frac{Z_1 Z_2 e^2}{r} , \beta(r) \right]^s \middle| w \right\rangle}{\left\langle r_N \middle| w \right\rangle} \qquad s = 1, \cdots , n$$

FMD many-body world = asymptotic point charge world

➼ Hamiltonian and Overlap matrix get modified
both depend on complex eigenvalue Z

Boundary Conditions 2

Different boundary conditions — Different physical situations

- **Whittaker** function

$$\langle r | w \rangle = W_\ell(kr) \quad , \quad k = +\sqrt{-2\mu E}$$

➨ **bound state** with tail tunneling into Coulomb barrier, $E < 0$

- **outgoing** Coulomb scattering solution

$$\langle r | w \rangle = iF_\ell(kr) + G_\ell(kr) \quad , \quad k = +\sqrt{2\mu Z}$$

➨ **Gamov state** with resonance energy and width $Z = E - i\Gamma/2$

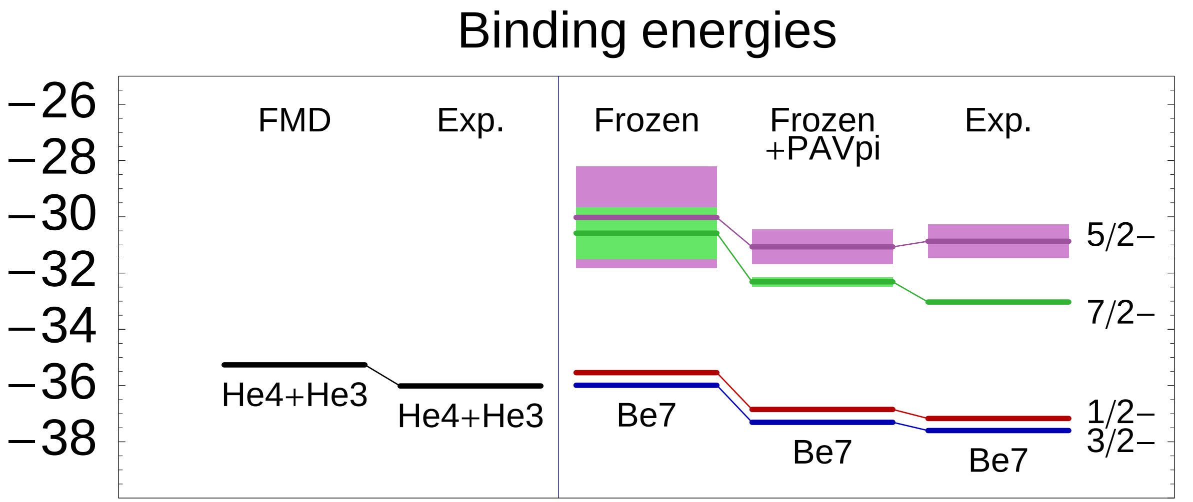
- Coulomb **scattering** solution with phase shift

$$\langle r | w \rangle = F_\ell(kr) + \tan\left(\delta_\ell(E)\right) G_\ell(kr) \quad , \quad k = +\sqrt{2\mu E}$$

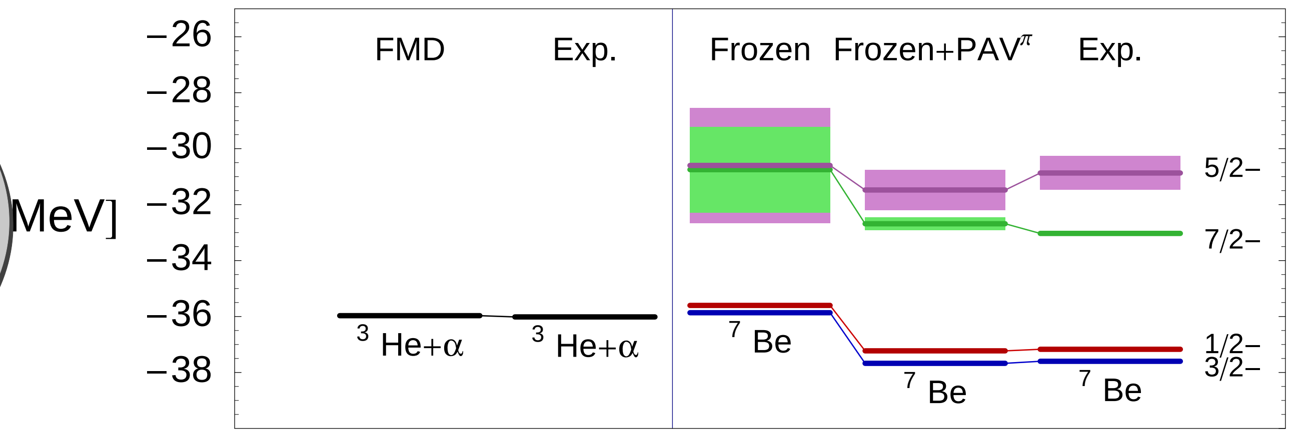
➨ **continuum solution** with phase shift $\delta_\ell(E)$, $E > 0$

^{7}Be Levels Bound and in Continuum

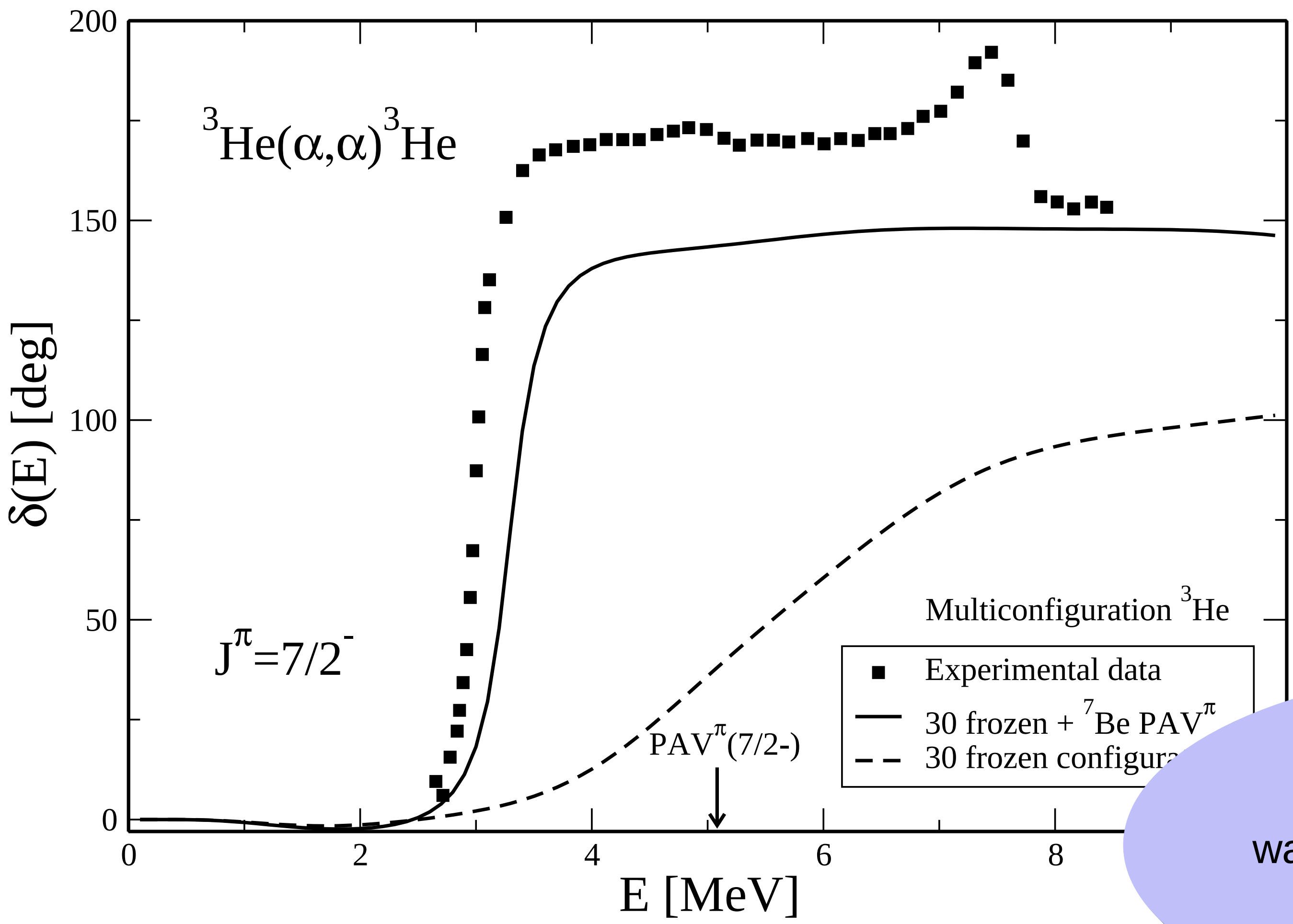
- implement boundary conditions using the **Gamov** state, outgoing only
- ➨ Hamiltonian and Overlap matrix get modified, complex eigenvalue



- single Slater determinant gives poor description for ^{3}He
- ➨ use multiconfiguration state for ^{3}He



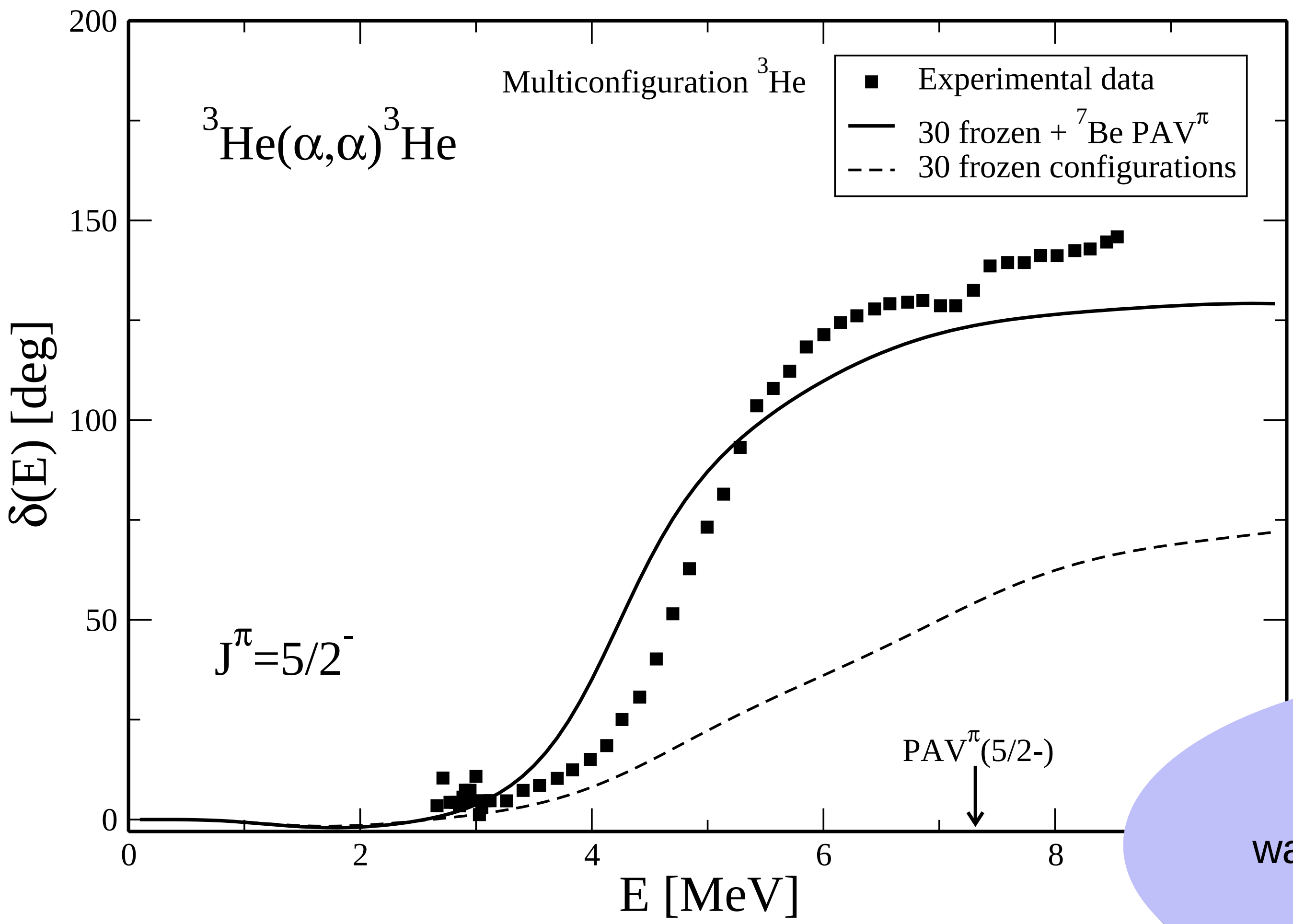
^{7}Be Phase Shift $7/2^-$ Resonance



★ **Resonance** ★
wave function large in interior
PAV$^\pi$ state essential

Data: R. J. Spiger, T. A. Tombrello, Phys. Rev. 163(1967)162

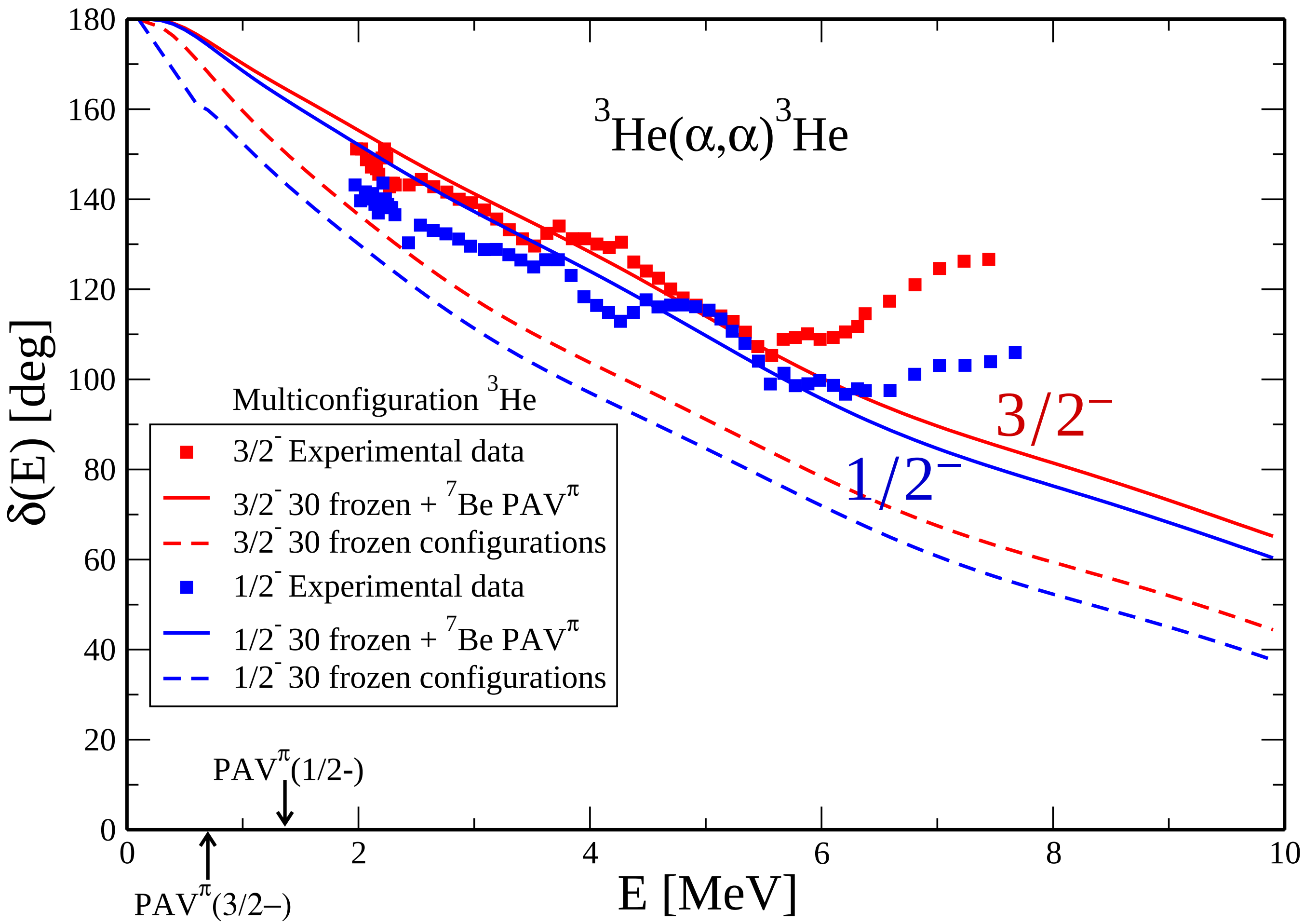
^{7}Be Phase Shift $5/2^-$ Resonance



★ **Resonance** ★
wave function large in interior
PAV$^\pi$ state essential

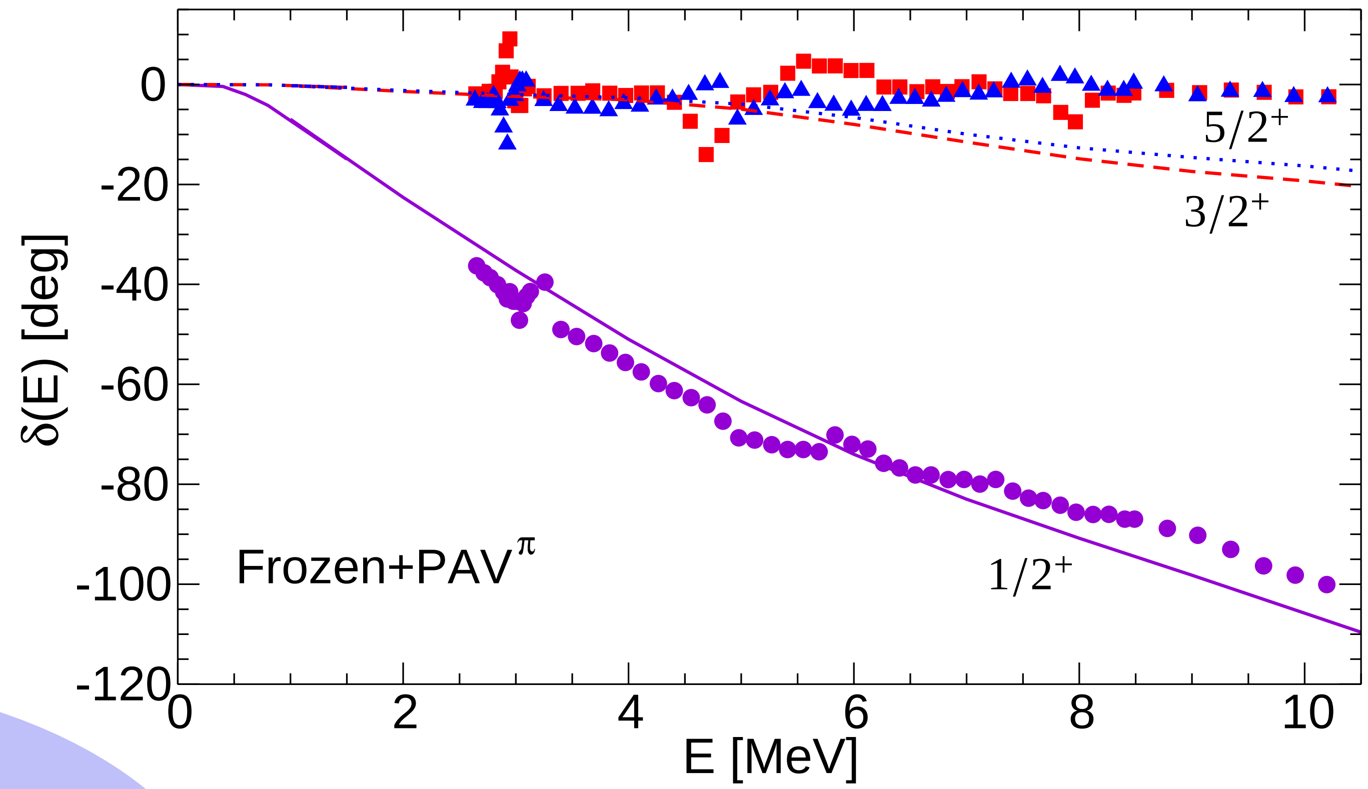
Data: R. J. Spiger, T. A. Tombrello, Phys. Rev. 163(1967)162

^{7}Be Phase Shifts, nonresonant



Data: R. J. Spiger, T. A. Tombrello, Phys. Rev. 163(1967)162

^{7}Be Phase Shifts, nonresonant



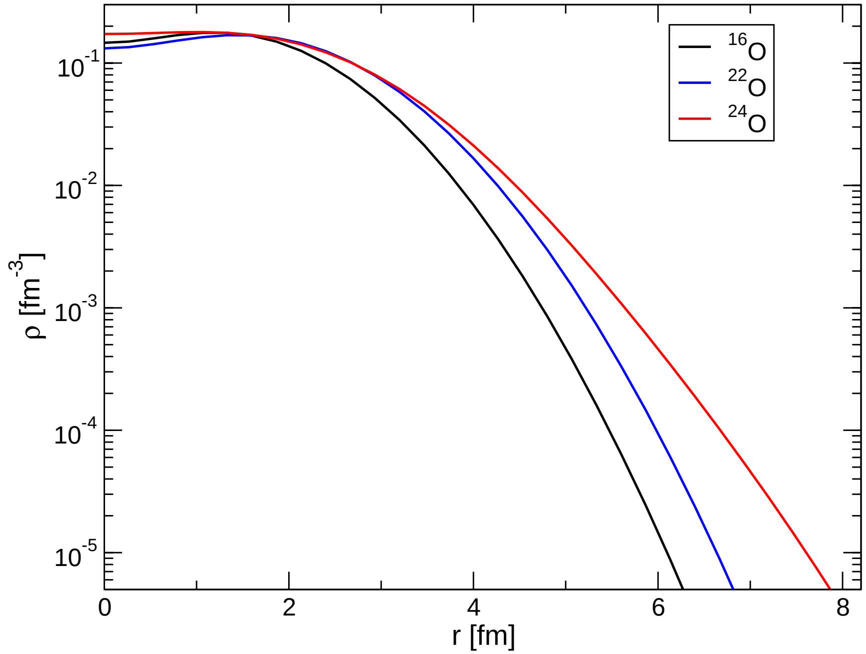
➨ remaining task:

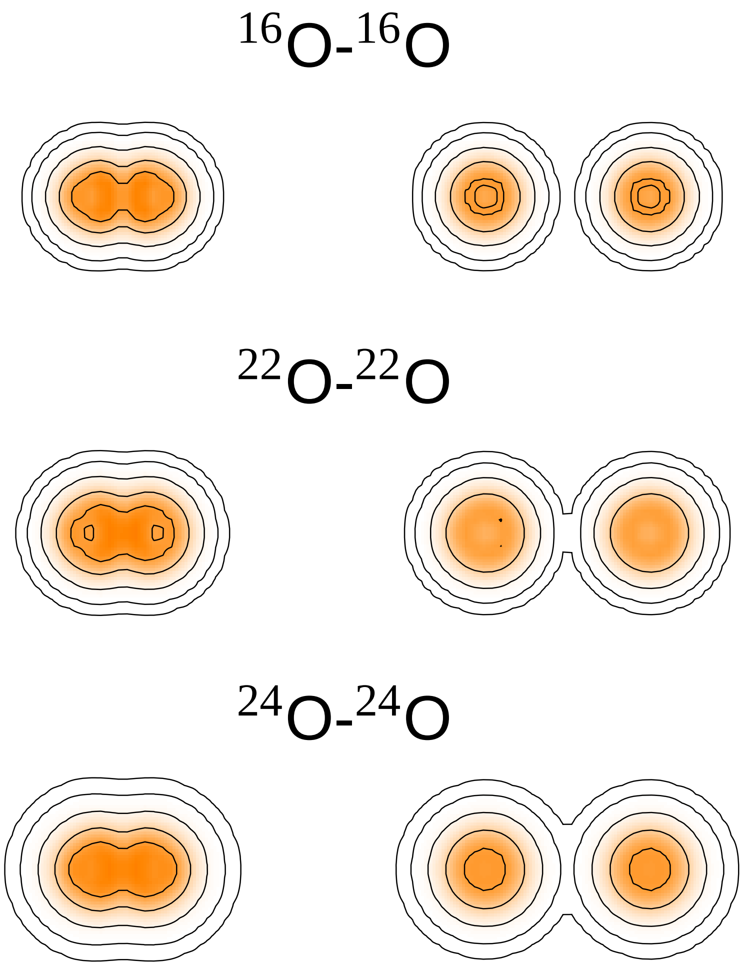
calculate **dipole transitions** from the scattering states to the bound states to obtain the cross-sections and the **S-factor**

Data: R. J. Spiger, T. A. Tombrello, Phys. Rev. 163(1967)162

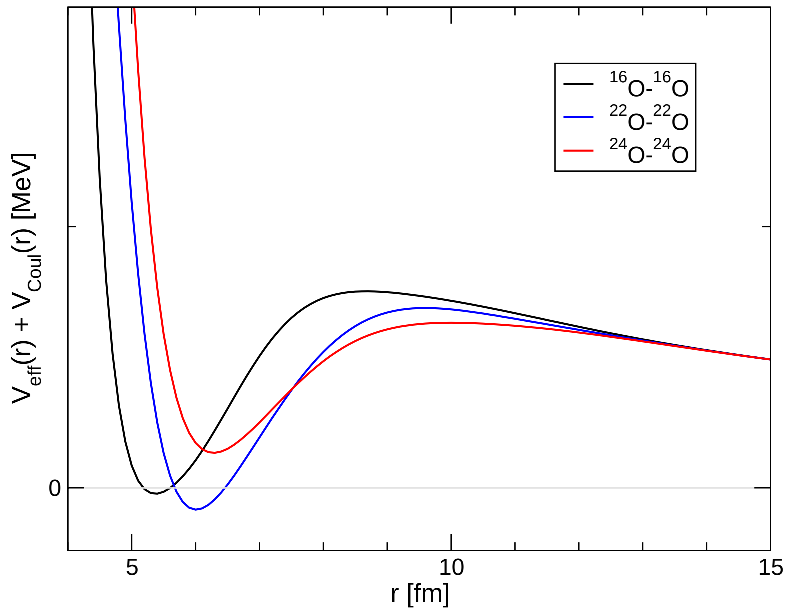
Microscopic Nucleus-Nucleus Interactions

- Fermionic Molecular Dynamics (FMD) many-body states
- Effective nucleon-nucleon interaction derived from realistic Argonne V18 interaction





S-Factors



Microscopically derived
Nucleus-Nucleus potentials

Astrophysical S-factor

$$S(E) = \sigma(E)\, E\, e^{2\pi\eta}$$