Beam-beam effects in circular colliders

with strong emphasis on measurements, tools and methods

(derived from a CAS lecture)

1. **Linear colliders (single pass, no damping ...)**
2. **Lepton colliders (multipass, damping)**
3. **Hadron colliders (multipass, no damping)**

Reading material

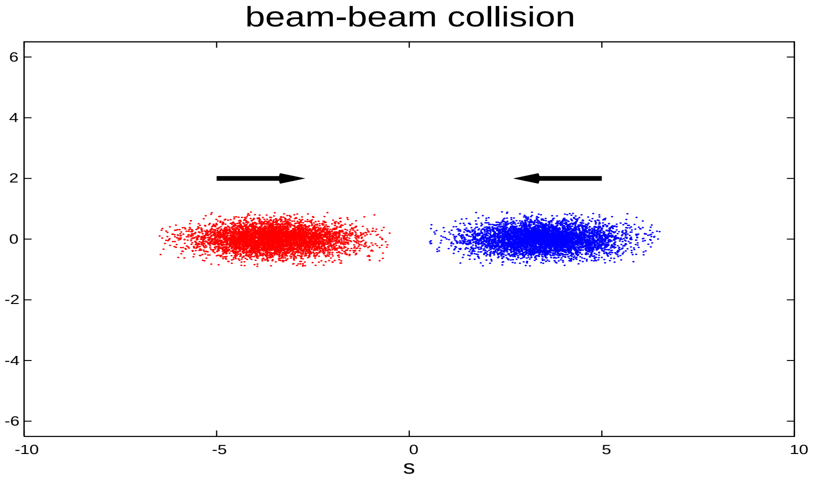
[1]L. Evans, "Beam-beam interactions", in Proceedings CERN Accelerator school: Antiprotons for Colliding-beam Facilities, CERN, 1983.

[2]W. Herr, "Beam-beam Interactions", in Proceedings CERN Accelerator School: Advanced Accelarator Physics Course, Trondheim, 2013, CERN-2014-009.

[3]W. Herr, "Mathematical and Numerical Methods for Nonlinear Dynamis", in CERN Accelerator School, Trondheim, 2013, CERN-2014-009.

[4]Proceedings "ICFA beam-beam workshop", 18. - 22. March 2013, ed. W. Herr, CERN-2014-004.

Beams in collision



Typically:

 **0.001% (or less) of particles have useful interactions**

 **99.999% (or more) of particles are just perturbed**

Note: typical numbers for hadron collisions, leptons a lot worse

Some challenges (beam-beam related, incomplete):

Circular colliders:

1. **Beams are re-used - LHC:** $\geq 5 \cdot 10^{10}$ **beam-beam interactions per production run (fill)** → **challenge for the beam dynamics (many different types of beam-beam effects to be understood and controlled)**
2. **Must guarantee stability and beam quality for a long time**
3. **Particle distributions change as result of interaction (results in time dependent forces ..)**
4. **One critical perfomance parameter: high luminosity !**
5. **...**

Unfortunately, despite all progress not all aspects are well understood and a general theory does not exist [1, 2].

Luminosity: $$L = \frac{N_1 N_2 f n_B}{4\pi\sigma_x\sigma_y} = \frac{N_1 N_2 f n_B}{4\pi \cdot \sigma_x\sigma_y}$$

High luminosity is not good for beam-beam effects ...
Beam-beam effects are not good for high luminosity ...

Menu:

- **Overview: which effects are important for present and future machines (LEP, PEP, Tevatron, RHIC, LHC, FCC, linear colliders, ...)**
- **Qualitative and physical picture of the effects**

Derivations in:
Proceedings, Advanced CAS, Trondheim (2013)
http://cern.ch/Werner.Herr/CAS2011_Chios/bb/bb1.pdf

Studying beam-beam effects - how to proceed

- Need to know the forces
- Apply concepts of non-linear dynamics
- Apply concepts of multi-particle dynamics
- Analytical models and simulation techniques well developed in the last 20 years (but still a very active field of research)

LHC is a wonderful epitome as it exhibits many of the features revealing beam-beam problems

First step: Fields and Forces

Need fields $\vec{E}$ and $\vec{B}$ of opposing beam with a particle distribution $\rho(x, y, z)$

In rest frame (denoted $'$) only electrostatic field: $\vec{E}'$, and $\vec{B}' \equiv 0$

Derive potential $U(x, y, z)$ from ρ and Poisson equation:

$$\Delta U(x, y, z) = -\frac{1}{\epsilon_0}\rho(x, y, z)$$

The electrostatic fields become:

$$\vec{E}' = -\nabla U(x, y, z)$$

Transform into moving frame to get $\vec{B}$ and calculate Lorentz force

Example Gaussian distribution (a simplification !):

$$\rho(x, y, z) = \frac{NZ_1 e}{\sigma_x \sigma_y \sigma_z \sqrt{2\pi}^3} \exp\left(-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2} - \frac{z^2}{2\sigma_z^2}\right)$$

For 2D case the potential becomes:

$$U(x, y, \sigma_x, \sigma_y) = \frac{NZ_1 e}{4\pi\epsilon_0} \int_0^\infty \frac{\exp(-\frac{x^2}{2\sigma_x^2+q} - \frac{y^2}{2\sigma_y^2+q})}{\sqrt{(2\sigma_x^2 + q)(2\sigma_y^2 + q)}} dq$$

Once known, can derive $\vec{E}$ and $\vec{B}$ fields and therefore forces

For arbitrary distribution (non-Gaussian): difficult (or impossible), numerical solution required

Force for round Gaussian beams

Simplification 1: round beams → $\sigma_x = \sigma_y = \sigma$

Simplification 2: very relativistic → $\beta \approx$ **1**

- **One finds: Only components E_r and B_Φ are non-zero**
- **Force has only radial component, i.e. depends only on distance r from bunch centre where:** $r^2 = x^2 + y^2$

$$F_r(r) = -\frac{Ne^2(1+\beta^2)}{2\pi\epsilon_0 \cdot r}\left[1 - \exp(-\frac{r^2}{2\sigma^2})\right]$$

For $\sigma_x \neq \sigma_y$ the forces are more complicated:

$$E_x = \frac{ne}{2\epsilon_0 \sqrt{2\pi(\sigma_x^2 - \sigma_y^2)}} \mathrm{Im} \left[\mathrm{erf} \left(\frac{x + iy}{\sqrt{2(\sigma_x^2 - \sigma_y^2)}} \right) - e^{\left(-\frac{x^2}{2\sigma_x^2} + \frac{y^2}{2\sigma_y^2} \right)} \mathrm{erf} \left(\frac{x\frac{\sigma_y}{\sigma_x} + iy\frac{\sigma_x}{\sigma_y}}{\sqrt{2(\sigma_x^2 - \sigma_y^2)}} \right) \right]$$

$$E_y = \frac{ne}{2\epsilon_0 \sqrt{2\pi(\sigma_x^2 - \sigma_y^2)}} \mathrm{Re} \left[\mathrm{erf} \left(\frac{x + iy}{\sqrt{2(\sigma_x^2 - \sigma_y^2)}} \right) - e^{\left(-\frac{x^2}{2\sigma_x^2} + \frac{y^2}{2\sigma_y^2} \right)} \mathrm{erf} \left(\frac{x\frac{\sigma_y}{\sigma_x} + iy\frac{\sigma_x}{\sigma_y}}{\sqrt{2(\sigma_x^2 - \sigma_y^2)}} \right) \right]$$

The function erf(t) is the complex error function

$$\mathrm{erf(t)} \; = \; e^{-t^2} \left[1 + \frac{2i}{\sqrt{\pi}} \int_0^t e^{z^2} \; dz \right]$$

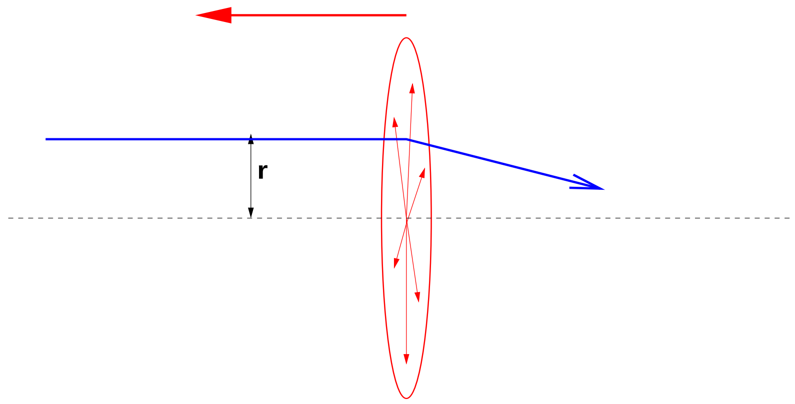
The magnetic field components follow from:

$$B_y \; = \; -\beta_r E_x / c \quad \text{and} \quad B_x \; = \; \beta_r E_y / c$$

Assumption/simplification: we shall continue with round beams ...

The forces will result in a deflection: "beam-beam kick"

- We are careless[a] and use (r, r', x, x', y, y') as coordinates
- We need the deflections (kicks $\Delta x'$, $\Delta y'$) of the particles:



Incoming particle (from left) deflected by force from opposite beam (from right)

Deflection depends on the distance r to the centre of the fields

[a]One should always use canonical variables, but here x, x' more convenient

After a short calculation (integration along bunch):

Using the classical particle radius:

$$r_0 = e^2/4\pi\epsilon_0 mc^2$$

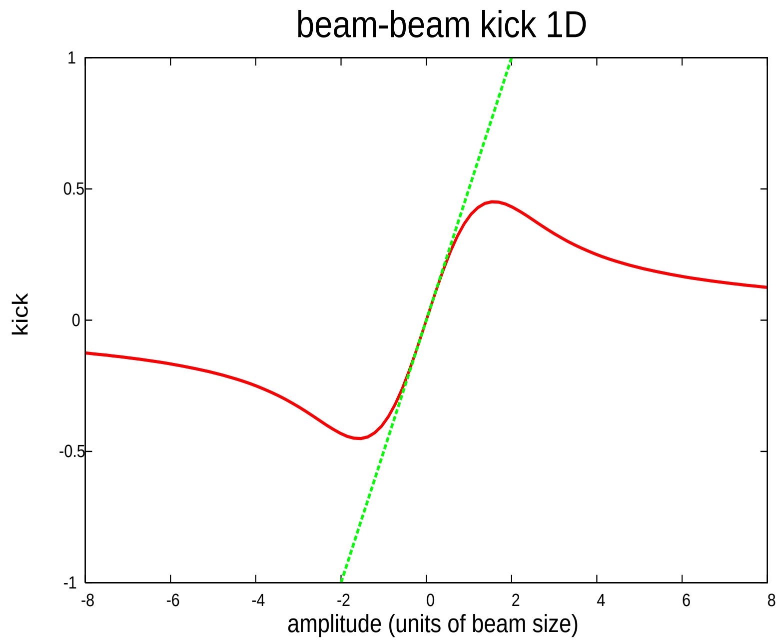
we have (radial kick and in Cartesian coordinates):

$$\Delta r' = -\frac{2Nr_0}{\gamma} \cdot \frac{r}{r^2} \cdot \left[1 - \exp(-\frac{r^2}{2\sigma^2})\right]$$

$$\Delta x' = -\frac{2Nr_0}{\gamma} \cdot \frac{x}{r^2} \cdot \left[1 - \exp(-\frac{r^2}{2\sigma^2})\right]$$

$$\Delta y' = -\frac{2Nr_0}{\gamma} \cdot \frac{y}{r^2} \cdot \left[1 - \exp(-\frac{r^2}{2\sigma^2})\right]$$

Form of the kick (as function of amplitude)



- **For small amplitudes: linear force (like "beam-beam quadrupole")**
- **For large amplitudes: very non-linear force**

Can one quantify the beam-beam strength ?

Tune shift by the "beam-beam quadrupole" may be a good indicator

- **Use the slope of quadrupole force (kick $\Delta r'$) at zero amplitude**
- **This defines: beam-beam parameter ξ**
- **For head-on interactions (general case, non round beams):**

$$\xi_{x,y} = \frac{N \cdot r_o \cdot \beta^*_{x,y}}{2\pi\gamma\sigma_{x,y}(\sigma_x + \sigma_y)}$$

Note: it is independent of β^*, important for (circular) colliders

some examples: LEP - LHC (most recent)

	LEP (e^+e^-)	LHC (pp) (2017)
Beam sizes	$\approx$ 200μm · 4μm	$\approx$ 11μm · 11μm
Intensity N	4.0 · 10^{11}/bunch	1.40 · 10^{11}/bunch
Energy	100 GeV	6500 GeV
$\epsilon_x \cdot \epsilon_y$	($\approx$) 20 nm · 0.2 nm	0.4 nm · 0.4 nm
$\beta^*_x \cdot \beta^*_y$ (nominal)	($\approx$) 1.25 m · 0.05 m	0.30 m · 0.30 m
Crossing angle	0.0	340 μrad
Beam-beam parameter(ξ)	0.0700	0.0070 (0.0037)

Unlike often assumed: Linear tune shift ΔQ_{bb} from beam-beam interaction proportional, but not equal to ξ

Still, how big is the tune shift for a given ξ

Take only the linear part ("beam-beam quadrupole") and add to the lattice

Transformation matrix of a thin quadrupole (beam-beam is really thin):

$$\begin{pmatrix} 1 & 0 \\ \frac{1}{-f} & 1 \end{pmatrix}$$

For small amplitudes linear force like a quadrupole with focal length f

$$\frac{1}{f} = \frac{\Delta x'}{x} = \frac{N r_0}{\gamma \sigma^2} = \left[\frac{\xi \cdot 4\pi}{\beta^*} \right]$$

Use the "Full Turn Matrix" without beam-beam:

$$\begin{pmatrix} \cos(2\pi(Q)) & \beta^* \sin(2\pi(Q)) \\ -\frac{1}{\beta^*} \sin(2\pi(Q)) & \cos(2\pi(Q)) \end{pmatrix}$$

Add "beam-beam thin lens", i.e. the (linear) beam-beam focusing:

$$\begin{pmatrix} \cos(2\pi Q) & \beta_0^* \sin(2\pi Q) \\ -\frac{1}{\beta_0^*} \sin(2\pi Q) & \cos(2\pi Q) \end{pmatrix} \circ \begin{pmatrix} 1 & 0 \\ \frac{1}{-f} & 1 \end{pmatrix}$$

allow for a change of the tune Q and β in the resulting matrix:

should become $$\begin{pmatrix} \cos(2\pi(Q+\Delta Q)) & \beta^* \sin(2\pi(Q+\Delta Q)) \\ -\frac{1}{\beta^*} \sin(2\pi(Q+\Delta Q)) & \cos(2\pi(Q+\Delta Q)) \end{pmatrix}$$

Solving this equation gives us (like a "tuning quadrupole"):

$$\cos(2\pi(Q + \Delta Q)) = \cos(2\pi Q) - \frac{\beta_0^*}{2f}\sin(2\pi Q) \quad \text{and} \quad \frac{\beta^*}{\beta_0^*} = \sin(2\pi Q)/\sin(2\pi(Q + \Delta Q))$$

$$\rightarrow \quad \boxed{\left[\frac{\beta^*}{\beta_0^*} = \frac{\sin(2\pi Q)}{\sin(2\pi(Q+\Delta Q))}\right] = \frac{1}{\sqrt{1+4\pi\xi\ \cot(2\pi Q)-4\pi^2\xi^2}}}$$

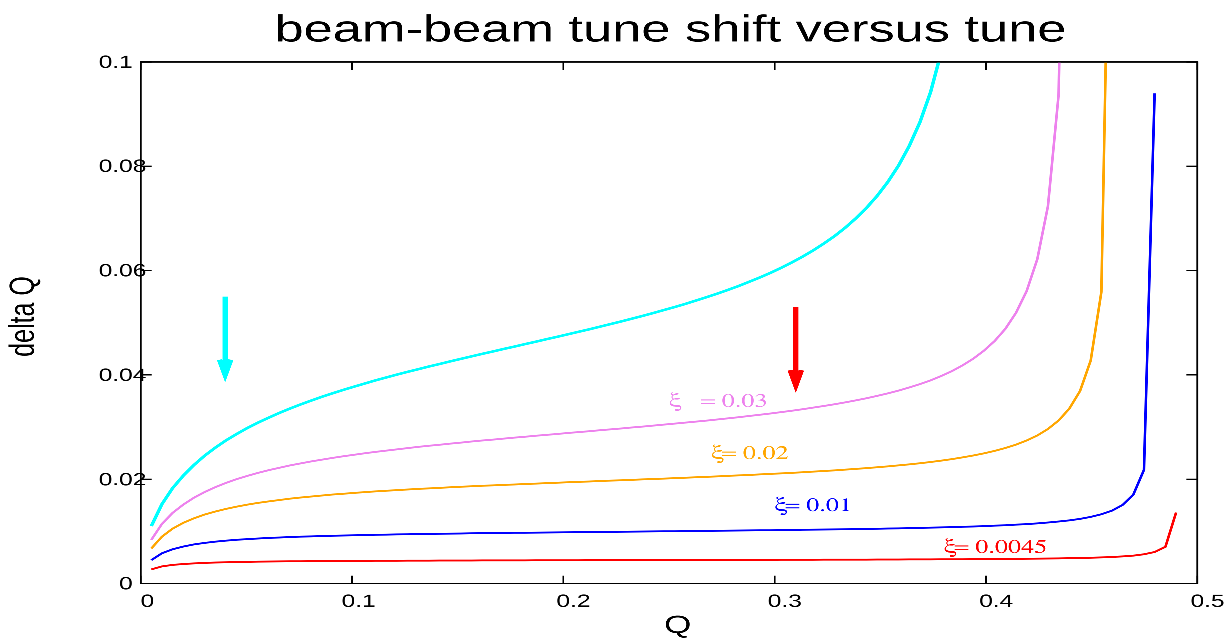
At a "real" quadrupole: β at quadrupole changes slightly, (usually assumed constant)

At beam-beam interaction:

Both ΔQ and β depend also on ξ and tune Q (must not be ignored)

β can become significantly <u>smaller</u> or <u>larger</u> at interaction point

This is called "Dynamic β"



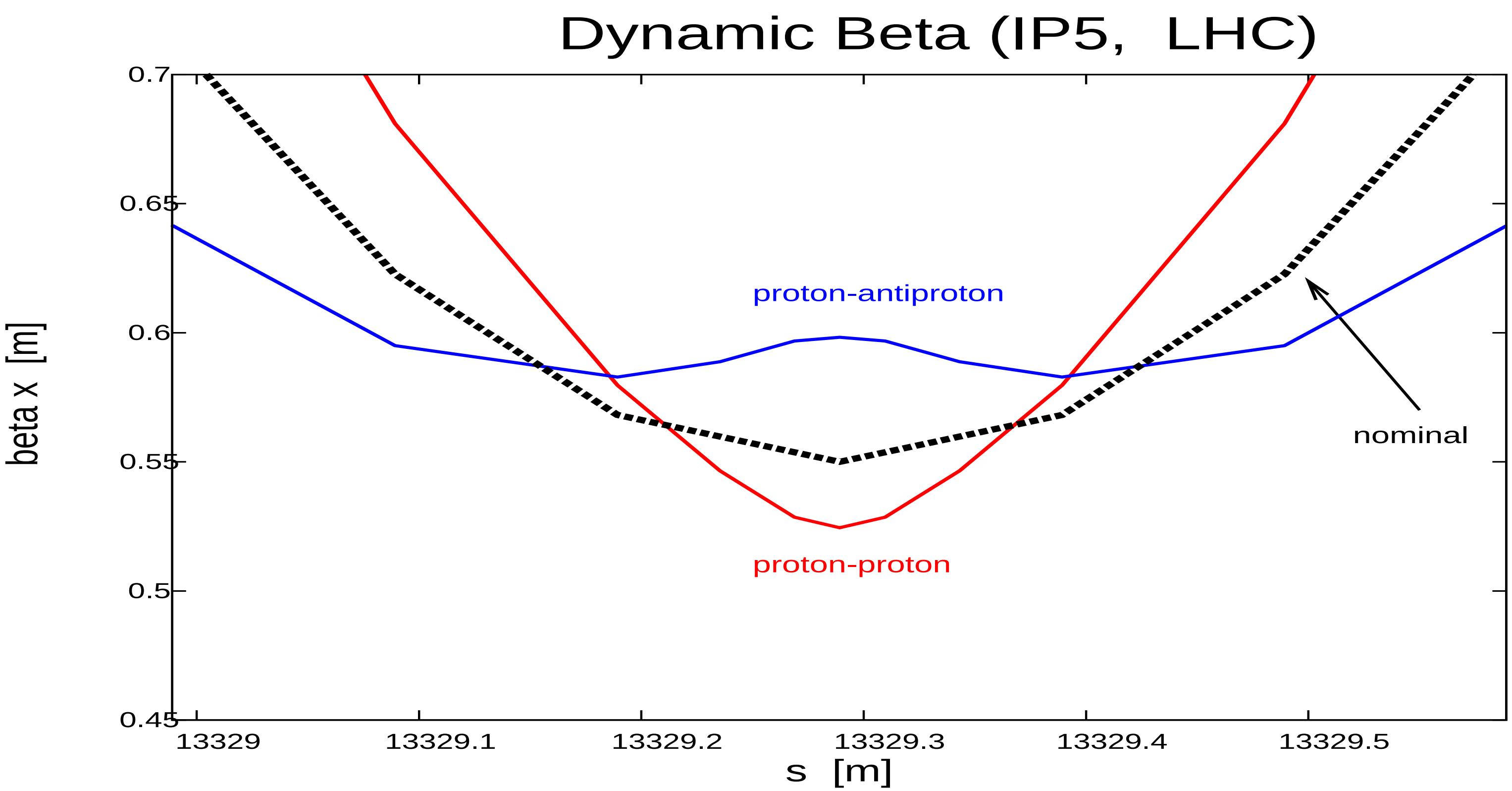
LEP working point, close to integer (vertical plane):

ΔQ_y **decreased: 0.07** $\Longrightarrow$ $\approx$ **0.04 !!!**

β^* **decreased: 5 cm** $\Longrightarrow$ $\approx$ **2.5 - 2.8 cm (Luminosity !)**

LHC working point, far from integer:

$\Delta Q \approx \xi$, **Weaker effects on** β^* ➡



Dynamic β in LHC, computed for pp and $p\bar{p}$

(with standard LHC parameters)

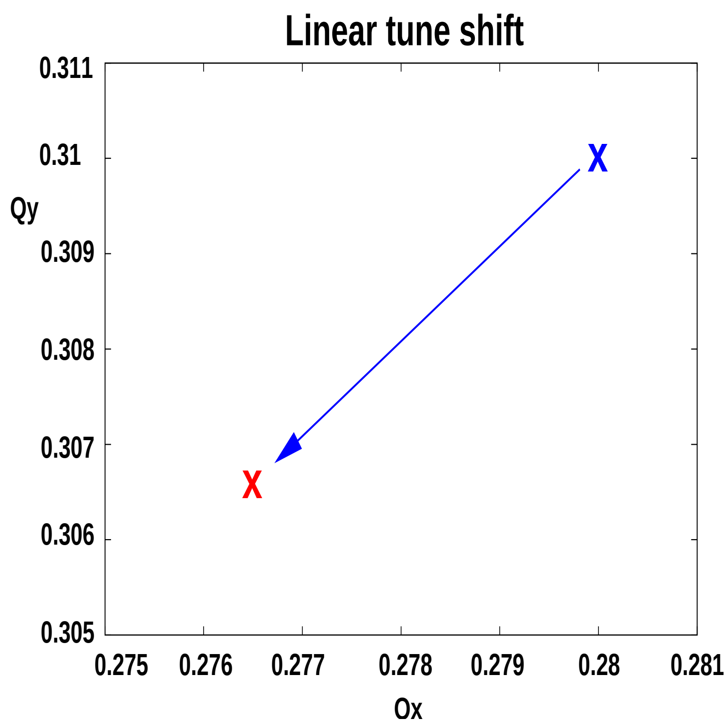
β^* : **0.55 m** $\Longrightarrow$ **0.52 m**

beam-beam linear tune shift in working diagramm

- Start with standard working point, no beam-beam
- With beam-beam: Tune shift in both planes
- LHC (equally charged beams)

  Tune shift is negative (pp)

  Whole beam moves to new tune

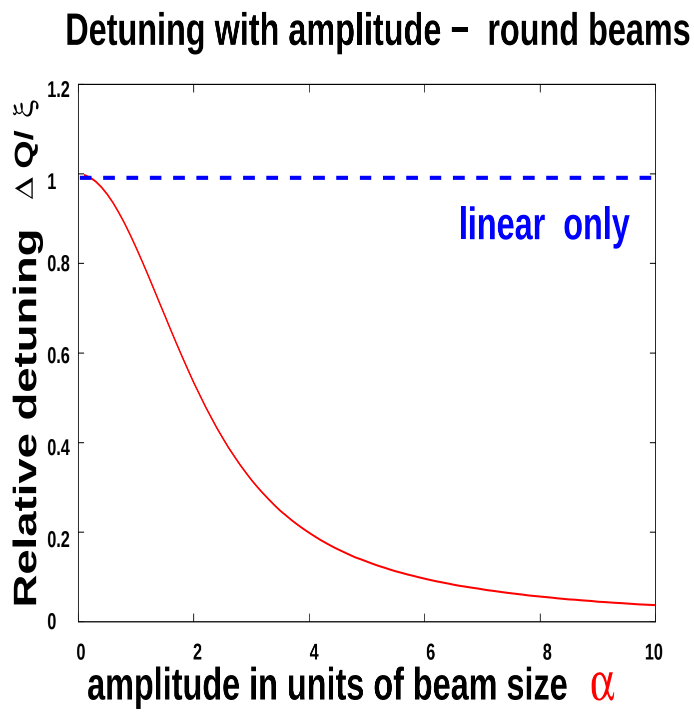


LHC is/was round (in most hadron colliders) → equal tune change in both planes

Usually not the case for leptons

Non-linear force: Amplitude detuning

- $\Delta\ Q$ depends on normalized amplitude α in units of beam size
- Different particles have different tunes
- Largest effect for **small** amplitudes ($\Delta Q\ \approx\ \xi$)



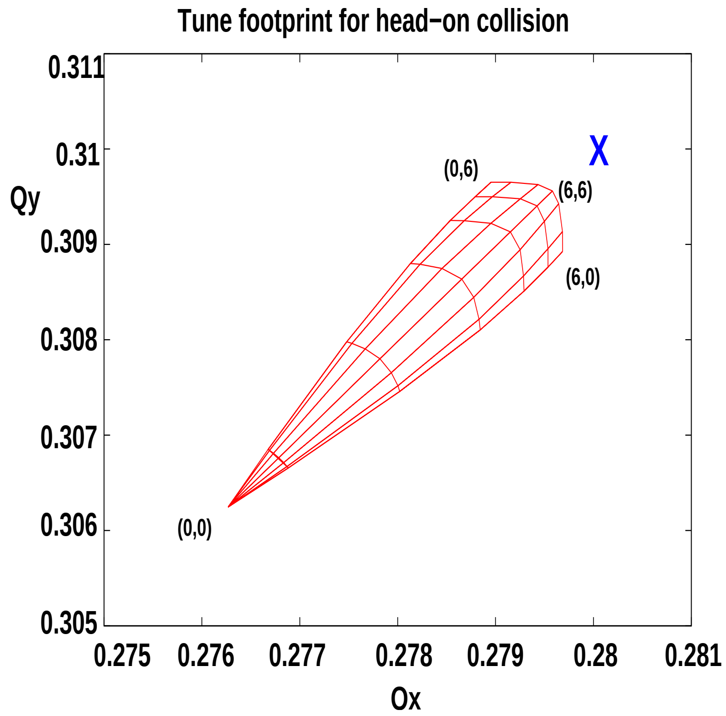
with $\alpha = \frac{a}{\sigma}$ we get: $\Delta Q/\xi = \frac{4}{\alpha^2}\left[1 - I_0(\frac{\alpha^2}{4}) \cdot e^{\frac{-\alpha^2}{4}}\right]$

Non-Linear tune shift (two dimensions)

- Start with standard working point
- Tunes depend on x and y amplitudes
- No single tune in the beam:

  Tunes are "spread out"

  Point becomes a footprint



More complicated in case of unequal beams

Total tune spread is $\approx$ 0.004 (one IP) !   Are we worried ??

First some slang: weak-strong and strong-strong

Both beams are very strong (strong-strong):

- **Both beam are affected and change during a beam-beam interaction:**

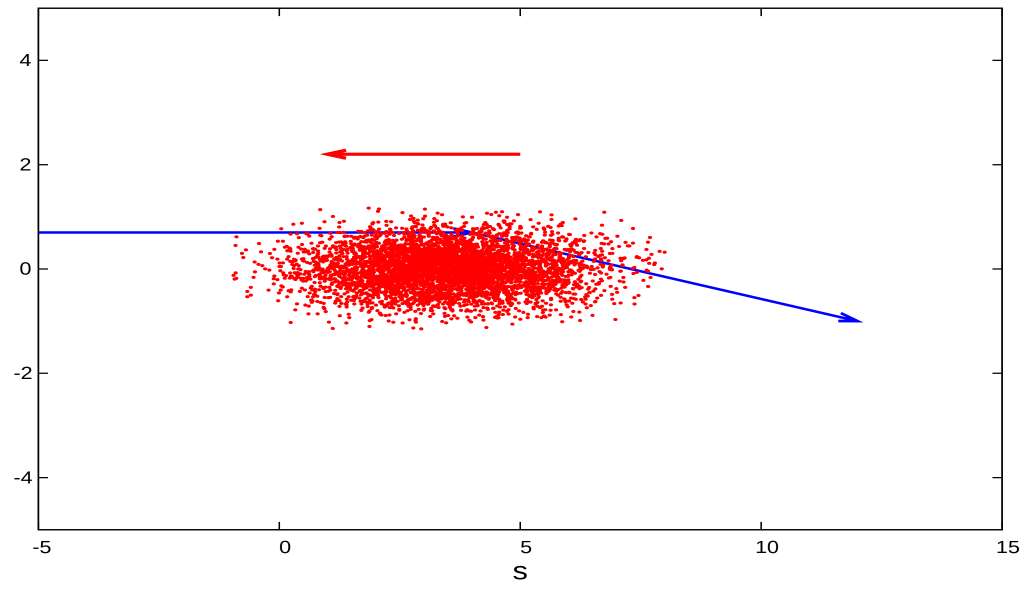
  Beam 1 changes beam 2, beam 2 changes beam 1 →
  beam-beam effects change every time the beams "meet"

- **Examples: LHC, LEP, RHIC, ... (FCC ?)**
- **Evaluation of effects challenging (need to be self-consistent)**

One beam much stronger (weak-strong):

- **Only the weak beam is affected and changed due to beam-beam interaction**
- **Examples: SPS collider, Tevatron, ...**

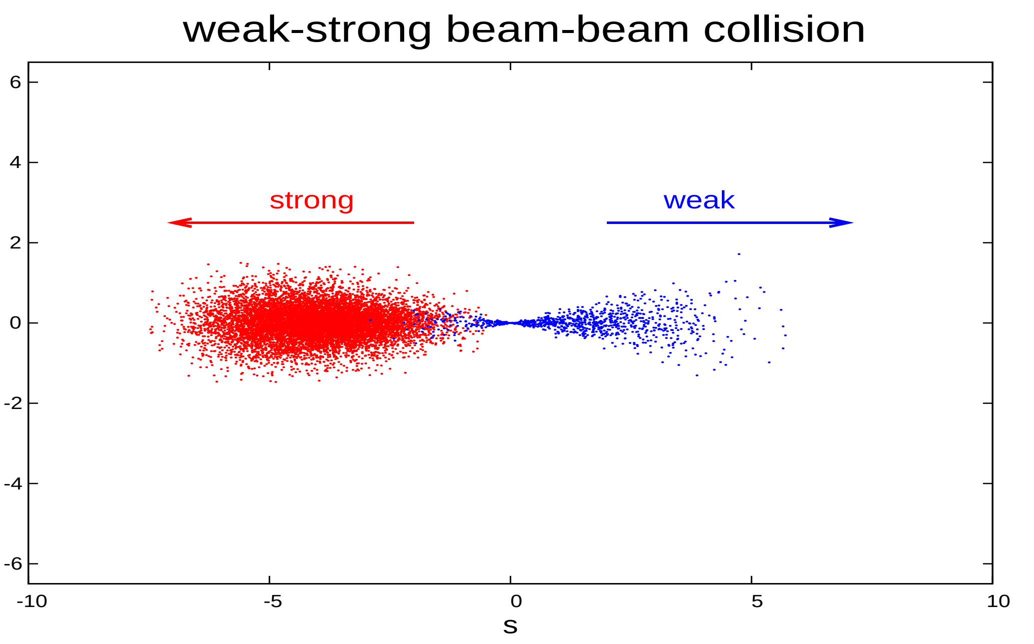
weak-strong beam-beam collision



Counter-rotating beam unaffected and treated as a static field

Equivalent to treat single particles, tracking etc.

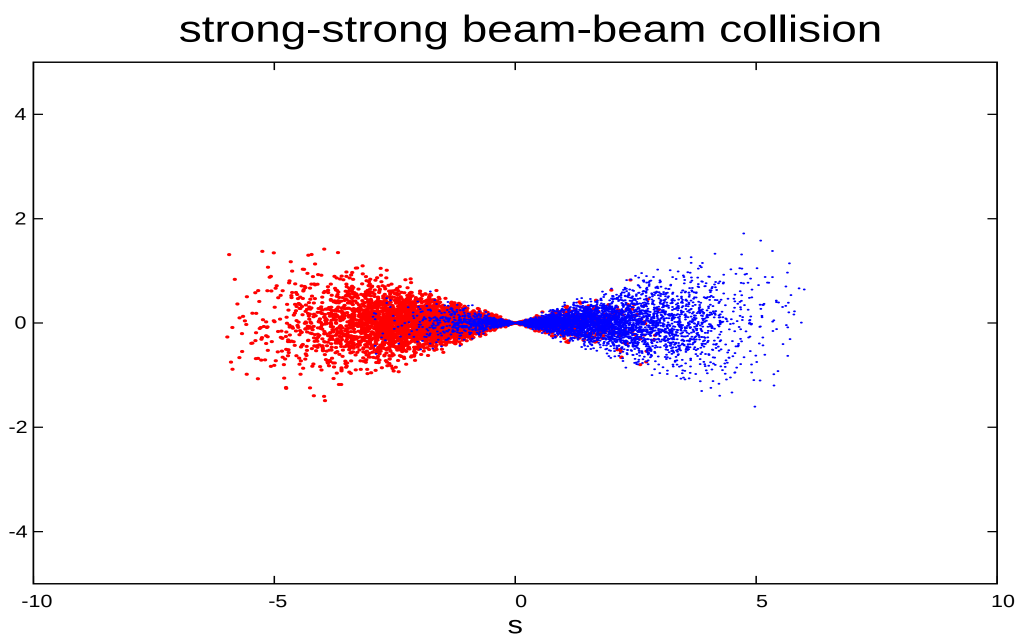
this is usually done to study single particle stability



Counter-rotating beam unaffected and treated as a static field

Weak beam can be strongly perturbed or destroyed

Note: unequal beam sizes also dangerous (e.g. SPS collider)



Both beams are (maybe heavily) distorted or destroyed

Size, shape, density, (losses ?) ...

Always treat both <u>beams</u> - not particles (self-consistently)

In tracking studies usually ignored (but check)

Important for coherent effects and LHC beams

How to get it self-consistent, one has to consider the change of beam size and particle distribution:

1. **Simulation**
   - **Multi-particle tracking**
   - **Gives desired results, but requires computing resources and very careful analysis (numerical problems, intelligent choice of field solver (!), ...)**
2. **As complement: coupled Vlasov equations**
   - **Usually difficult to solve analytically, need perturbative treatment, but still ...**
   - **Just a sketch used for LHC coherent effects**

First we consider head-on collision of one bunch per beam a and b

Particle distributions ψ^a and ψ^b mutually changed by interaction (by the "other parts")

Interaction depends on particle distributions

- Beam ψ^a solution depends on beam ψ^b
- Beam ψ^b solution depends on beam ψ^a

Can one find a self-consistent solution ?

What is the equation of motion ?

→ For distribution function: Vlasov equation

$$\frac{\partial \psi^a}{\partial t} = -q_x p_x \frac{\partial \psi^a}{\partial x} + force \frac{\partial \psi^a}{\partial p_x}$$

for beam a:

$$\frac{\partial \psi^a}{\partial t} = -q_x p_x \frac{\partial \psi^a}{\partial x} + \left(\frac{\partial p_x}{\partial t}\right)\frac{\partial \psi^a}{\partial p_x}$$

$$\frac{\partial \psi^a}{\partial t} = -q_x p_x \frac{\partial \psi^a}{\partial x} + \underbrace{\left(q_x x + \delta_p(t)\cdot 4\cdot \pi \xi_x \ \textbf{p.v.} \int_{-\infty}^{+\infty} \frac{\rho^b(x';t)}{x - x'} dx'\right)}_{\text{force from beam b on beam a}} \frac{\partial \psi^a}{\partial p_x}$$

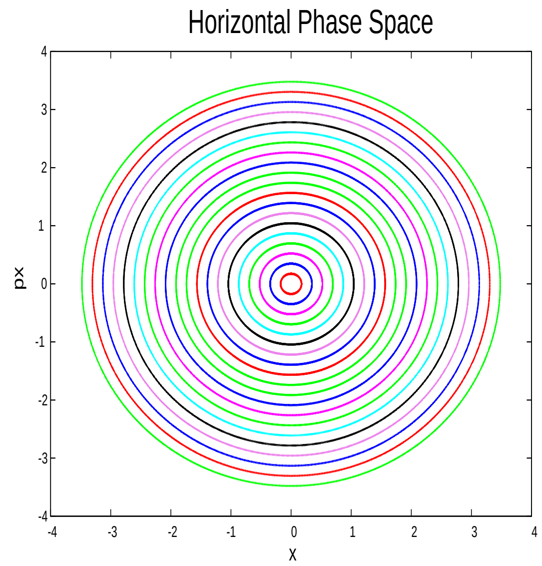
$$\rho^b(x;t) = \int_{-\infty}^{\infty} \psi^b(x, p_x; t)\, dp_x$$

The same thing for beam b: two coupled differential equations for beam distributions $\psi^a(x, p_x)$ and $\psi^b(x, p_x)$

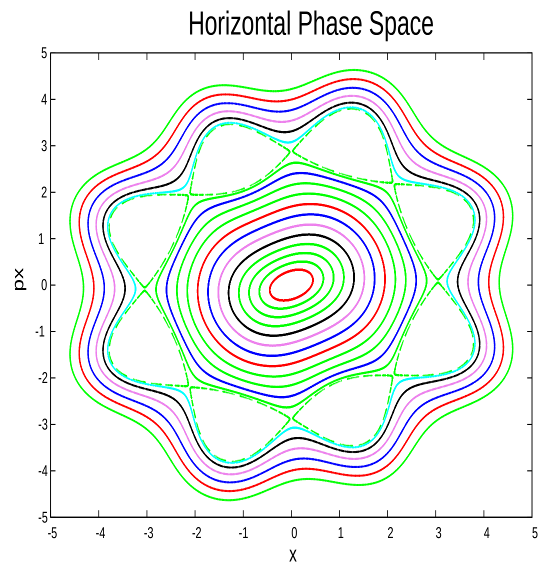
Normally cannot find <u>exact</u> solution, numerical solutions required, powerful methods exist (examples in backup slides and details in [3])

Fortunately: most important only for <u>coherent</u> beam-beam effects, can often be ignored otherwise (Yokoya, 1990)

- **Without beam-beam**
- **All particle have the same tune (all on circles)**



- **With (head-on) beam-beam**
- **Tune depends on amplitude**
- **For some amplitudes they are on resonances**



Can one reconstruct the phase space ?

A powerful technique, heavily used to analyse LHC beam-beam problems: Lie transformation based on a Hamiltonian treatment followed by a normal form analysis. Without derivation (short computation in back-up slides, all details see e.g. [3]) one gets for the "effective Hamiltonian" h:

$$h = \overbrace{-\mu J}^{\text{no beam-beam}} + \left(\sum_n c_n(J) \cdot in\mu \cdot \frac{1}{1 - e^{-in\mu}} \cdot e^{in\Psi} \right)$$

or (c_n are Fourier components of the force, see backup slides):

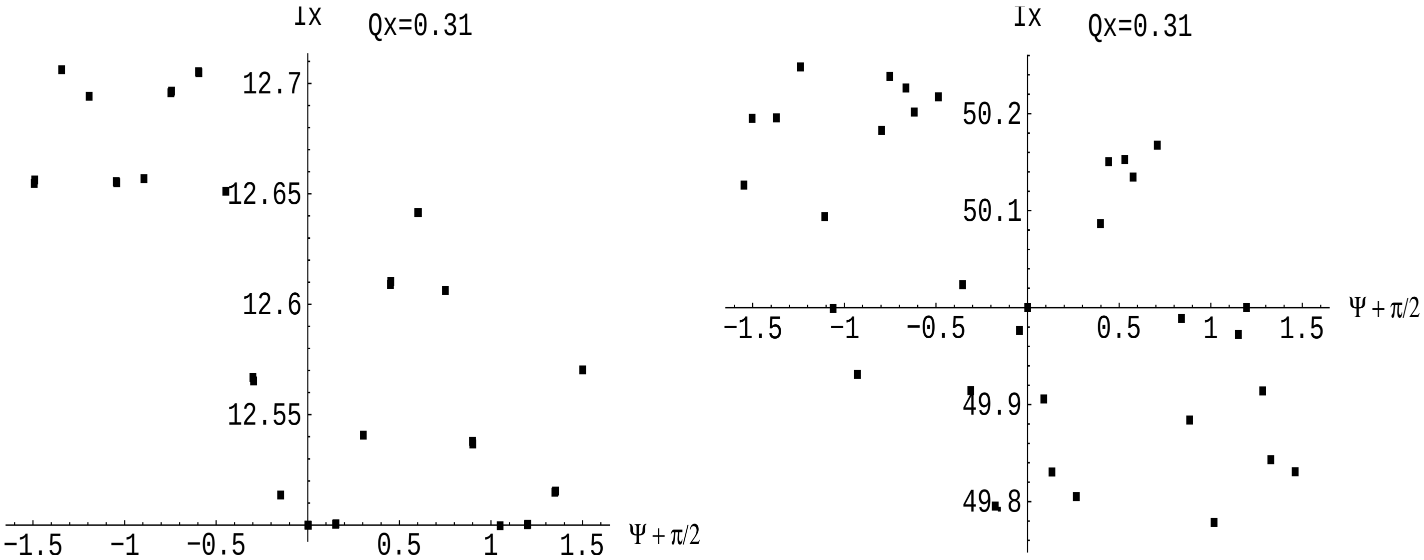
$$h = -\mu J + \left(\sum_n c_n(J) \frac{n\mu}{2\sin(\frac{n\mu}{2})} \; e^{(in\Psi + i\frac{n\mu}{2})} \right)$$

the tune shift with amplitude follows immediately with:

$$\Delta\mu(J) = -\frac{1}{2\pi} \frac{dc_0(J)}{dJ} \qquad \text{(}J\text{ is now action variable)}$$

Note: once you have h you have (almost) <u>everything</u> (see [3])

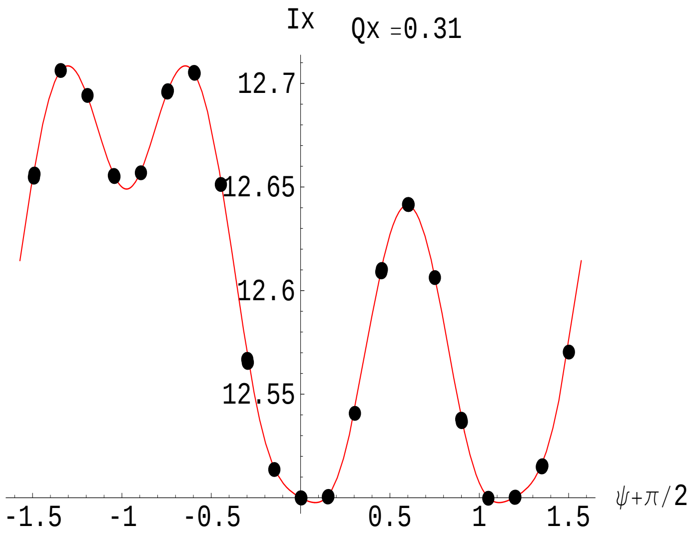
Invariant from tracking: Poincaré section of one IP

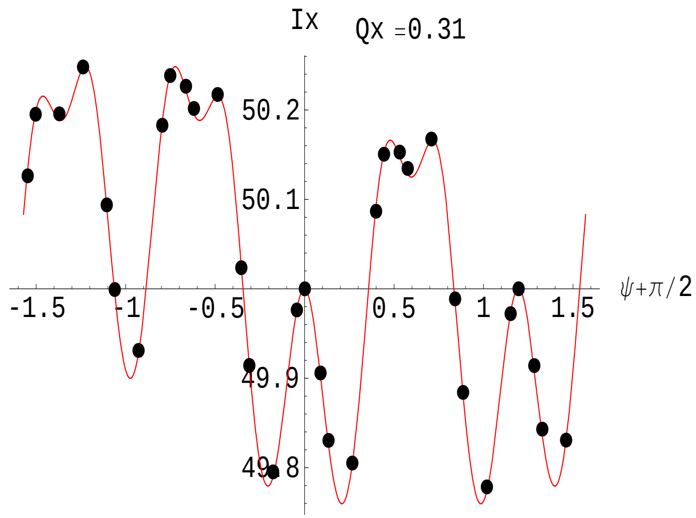


- **Phase space (action-angle) coordinates plotted each turn**
- **Shown for particle amplitudes of 5σ_x and 10σ_x**

Without beam-beam: a straight line

Invariant versus tracking: one IP





➡ **Shown for particle amplitudes of 5σ_x and 10σ_x**

one can reproduce and analyse the motion ...

works also for more than one interaction point (see backup slides), for LHC we treat up to 124 interactions per turn

Problems with hadron machines

- **Hadron (e.g. protons) machines have no or very little damping**

  No equilibrium emittance - no hard beam-beam limit (unlike lepton colliders and common believe) just gets worse and worse ...

- **Very hard to exceed 0.01**

- **Losses or lifetime extremely hard (zzz) to predict, a prediction within a factor 2 is pretty good ..**

The next problem

$$\Longrightarrow \quad \mathcal{L} = \frac{N_1 N_2 f \cdot n_B}{4\pi\sigma_x\sigma_y}$$

How to collide many bunches (for high $\mathcal{L}$) ??

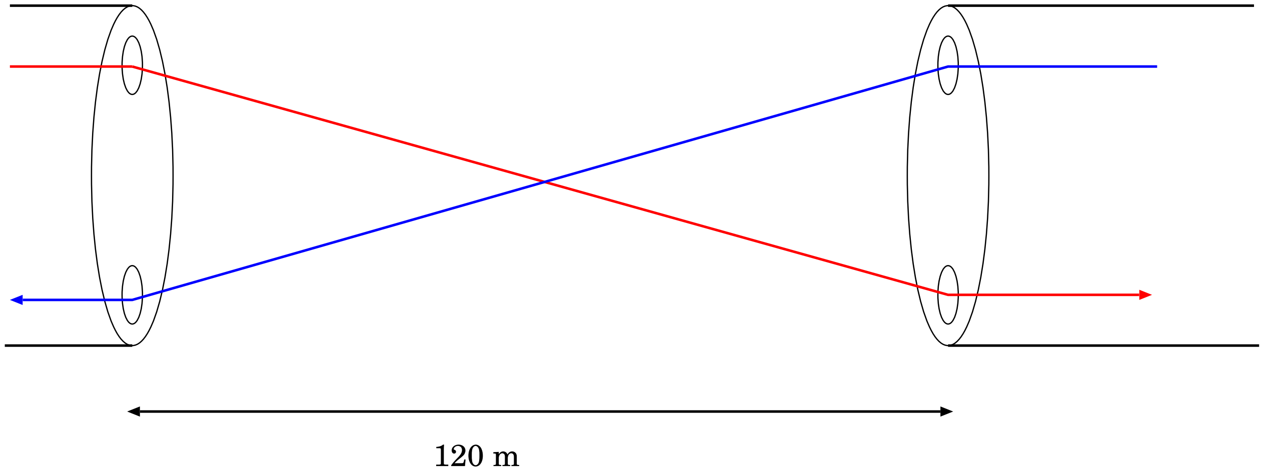
Must avoid unwanted collisions !!

Separation of the beams:

- **"Pretzel" scheme (SPS ,LEP, Tevatron, Cornell)**
- **Bunch trains (LEP,PEP, ...)**

- **Choose Crossing angle for LHC**

Two beams, 2808 bunches each, every 25 ns In common part of the chamber around the 4 experiments



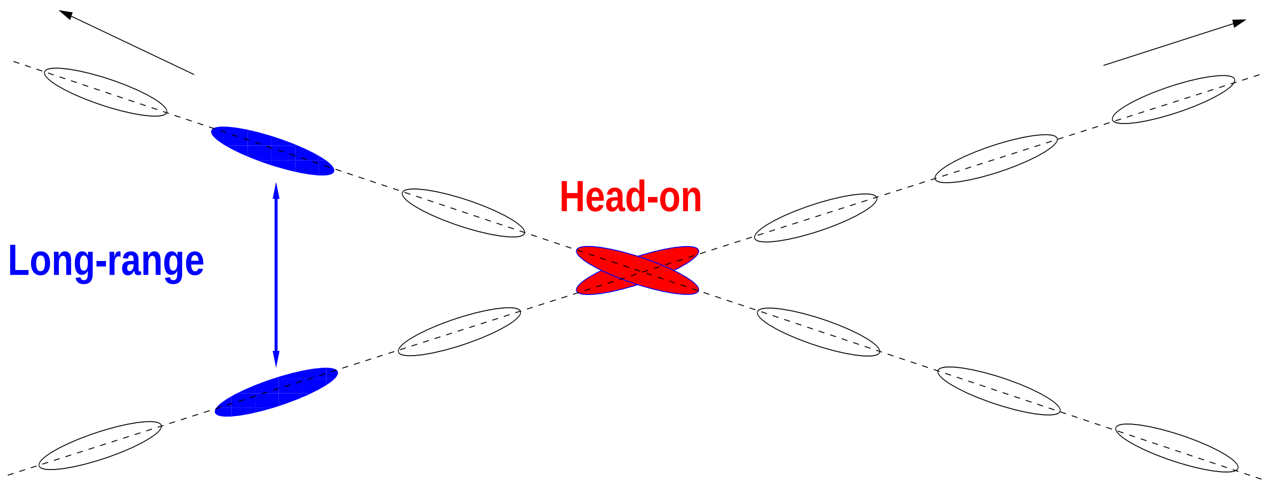
Beams have to exchange between inner and outer beam pipes

Over 120 m: about 30 parasitic interactions

Four IPs: a total of 124 beam-beam interactions !!

Need local separation: Established with 2 horizontal and 2 vertical crossings

Crossing angles (example LHC)



Beams separated, but still same vacuum chamber

Particles experience distant (weak) forces

Separation typically 6 - 12 σ (note: beam size growths linearly with distance to collision point, so does the separation with the crossing angle)

→ **We get so-called long range interactions**

What is special about them ?

In addition to the head-on effects:

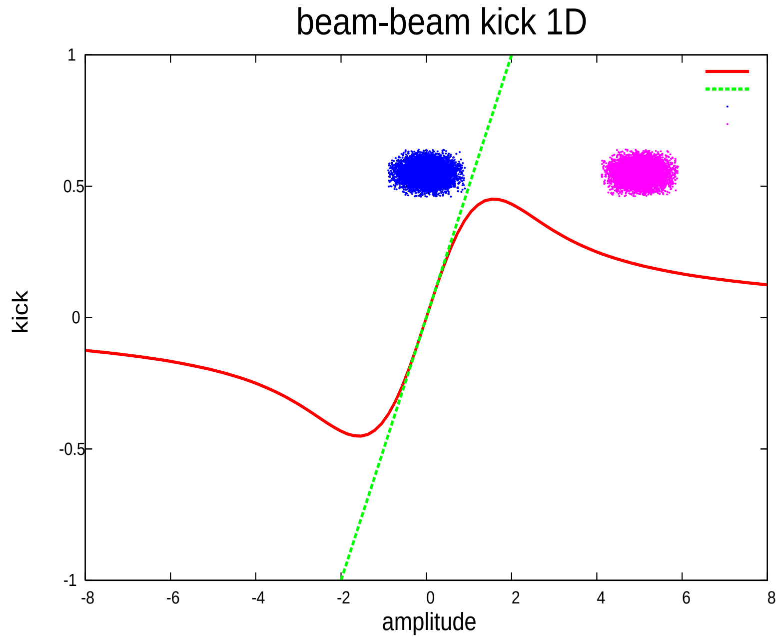
- Break symmetry between planes, stronger resonance excitation
- Mostly affect particles at large amplitudes, i.e. the ones we expect to loose
- Cause effects on closed orbit, tune, chromaticity, .. (to come)
- Special case: PACMAN effects
- Tune shift has opposite sign in plane of separation

  e.g. case with a horizontal crossing angle (LHC):

  Horizontal tune shift positive, vertical tune shift negative

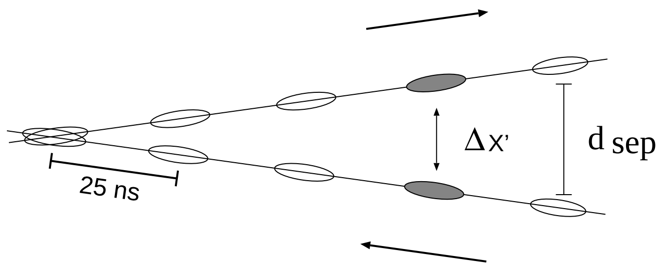
Why opposite tune shift ???

What do the particles "see":



- **What counts: local slope has opposite sign for large separation oscillation now not around the centre, but around the separated orbit**
- **Opposite sign for focusing in plane of separation !**

Quantitatively: Long range kick



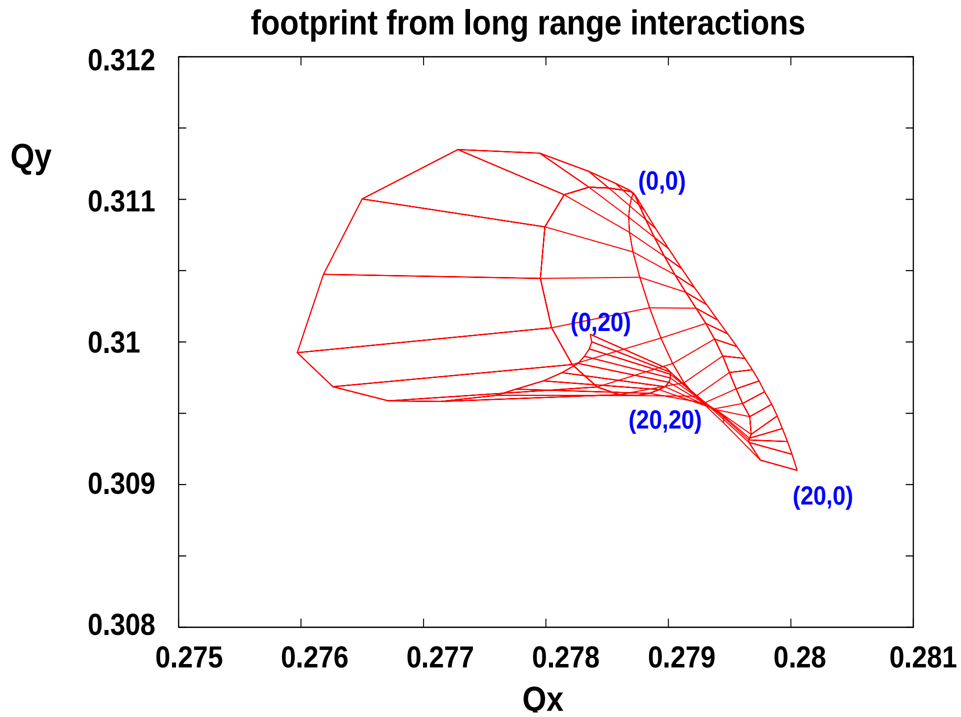
→ **Modified "kick" with horizontal separation d:**

$$\Delta x'(x+d, y, r) = -\frac{2Nr_0}{\gamma} \cdot \frac{(x+d)}{r^2} \left[1 - \exp(-\frac{r^2}{2\sigma^2})\right]$$

(with: $r^2 = (x+d)^2 + y^2$**)**

Red flag: to use this expression, e.g. in a simulation, there is a small complication, was used incorrectly in the past (before 1990 and in Chao Handbook), if interested ask offline

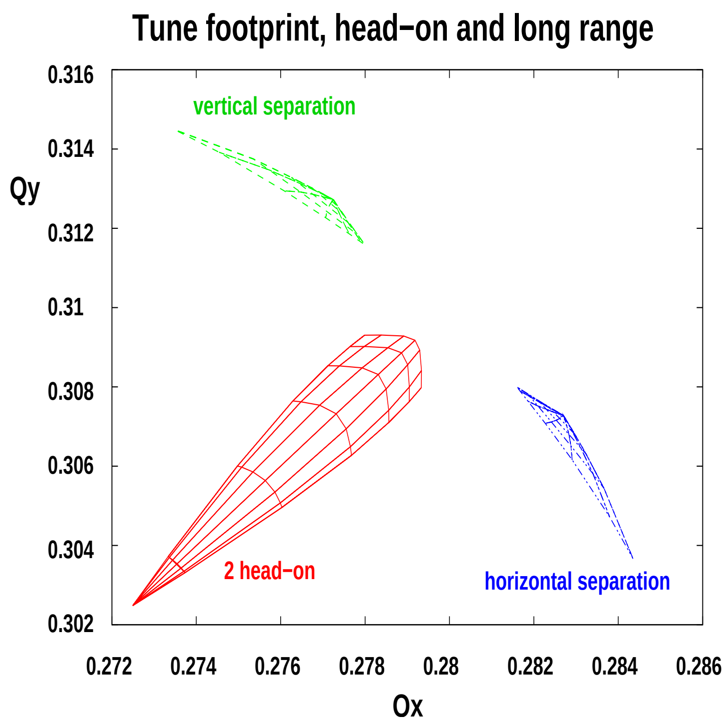
- Tune shift large for largest amplitudes (where non-linearities are strong)
- Size proportional to $\frac{1}{d^2}$
- We should expect problems at small separation
- Footprint is very asymmetric



One observes a "folding" (can easily be understood from the picture)

For small separation, the size of the footprint can be large → particle losses

- Compare foot print for different contributions
- Seem to be totally separated
- For horizontal and vertical separation go opposite directions



Can one take advantage of that ??

(Note: a second head-on collision just doubles the size of the footprint)

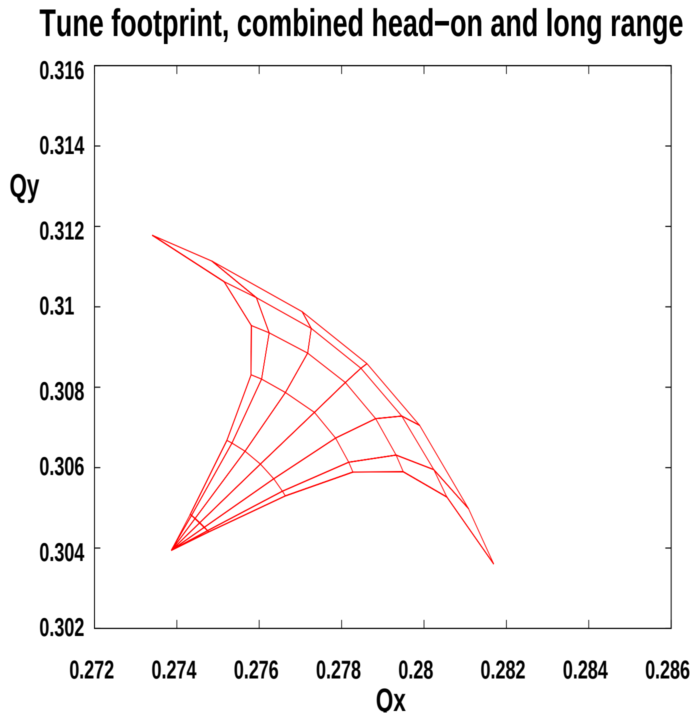
Two interaction points:

Two head-on footprints

\+ Horizontal long range

\+ Vertical long range

= Symmetric



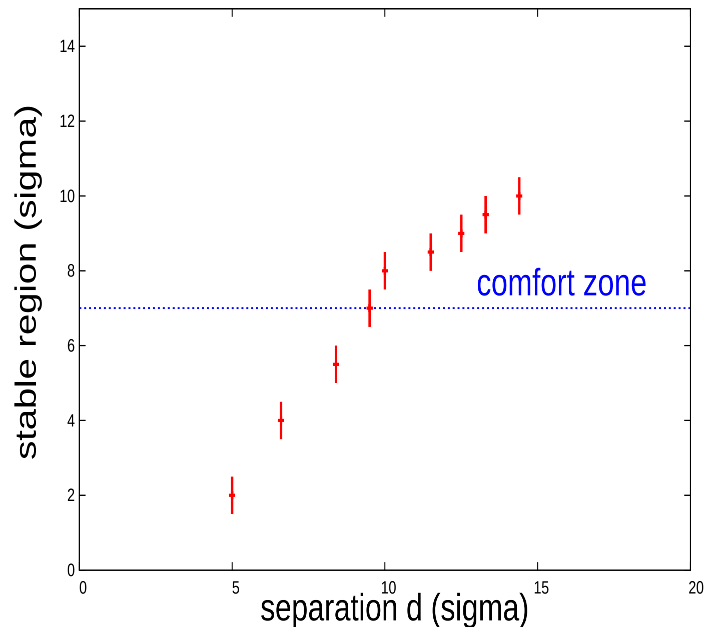
- Alternating (i.e. one vertical and horizontal each), implemented in the LHC, tune spread around 0.01 ! and it works ...
- Seems to get some compensation, i.e. overall footprint slightly decreased and is symmetric
- Looks like a very minor improvement, but see later

Small crossing angle $\Longleftrightarrow$ small separation $\Longleftrightarrow$ big problem ?

Stable region (a.k.a Dynamic Aperture) versus separation in units of beam size σ

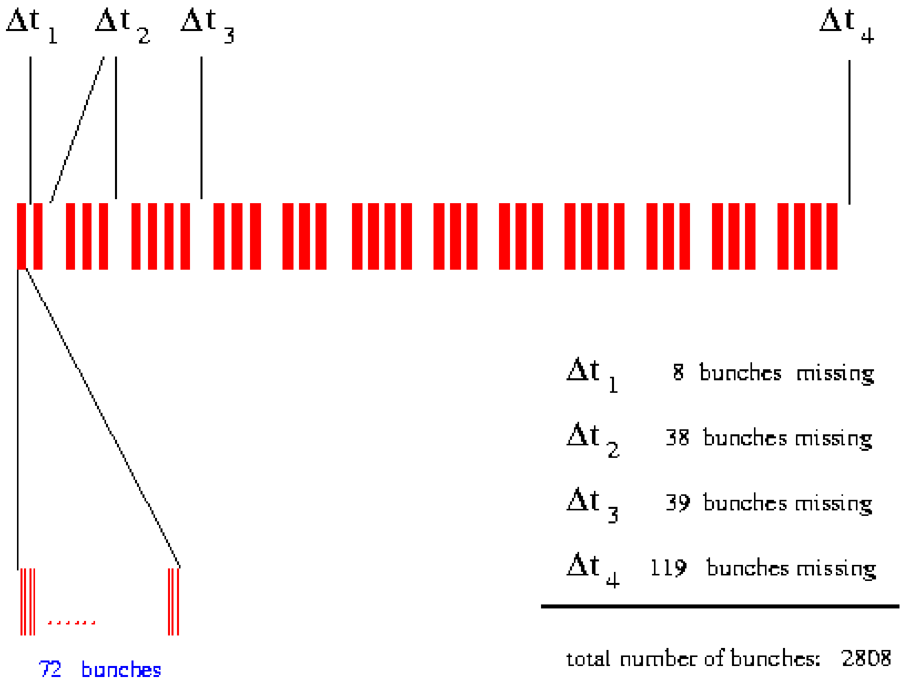
(from simulations)

Minimum separation for LHC:
$\approx$ 10 σ (design value)



For too small separation: particles may be lost and/or bad liftime

More jargon: PACMAN bunches



- **Trains not continuous: gaps for injection, extraction, dump ..**
- **Nominal 2808 of 3564 possible bunches, optimized now**

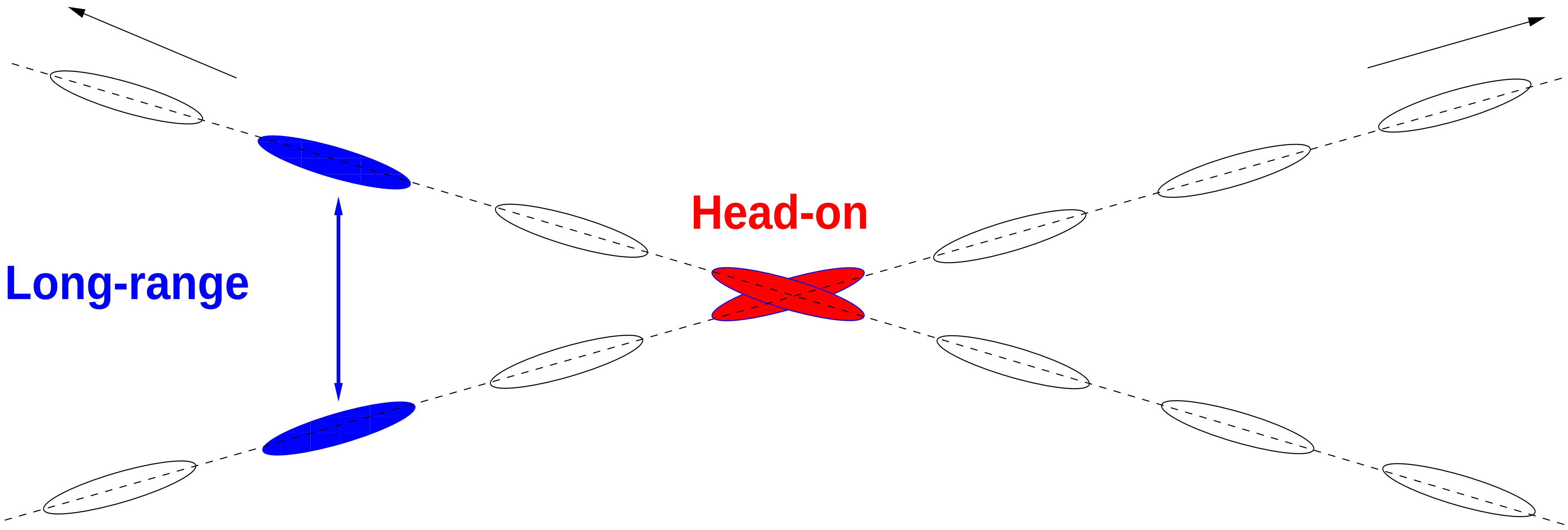
Trains for beam 1 and beam 2 are symmetric:

At interaction point:

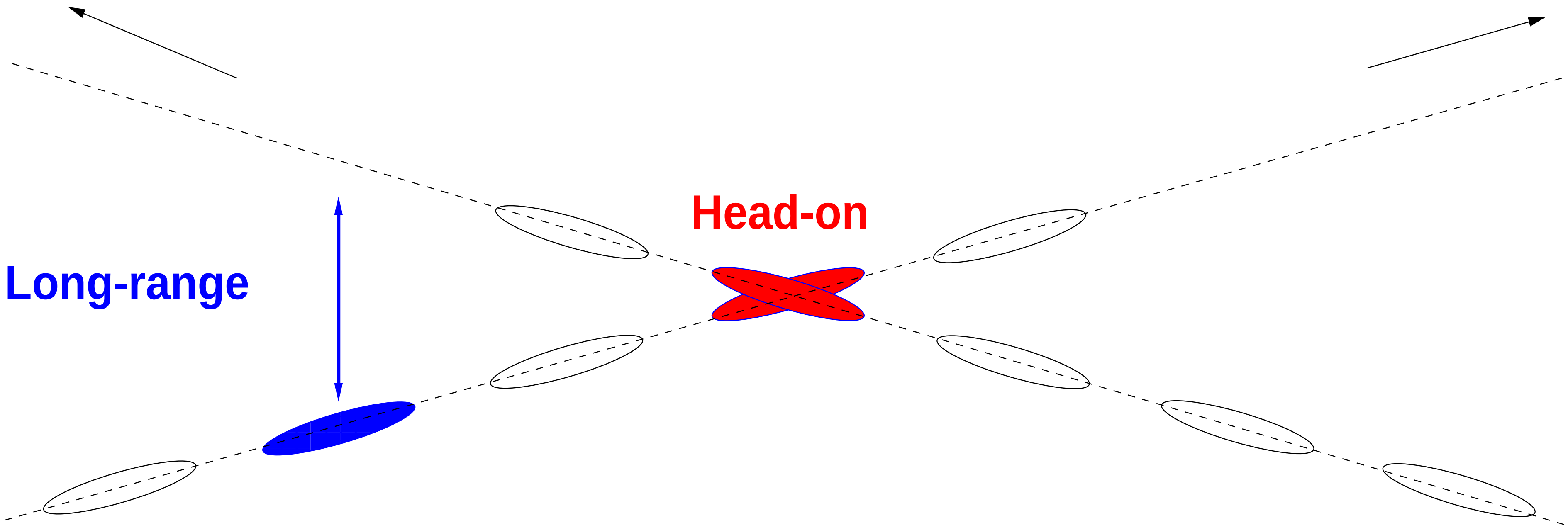
bunch 1 meets bunch 1,   bunch 2 meets bunch 2, etc.

hole 1 meets hole 1,   hole 2 meets hole 2, etc.

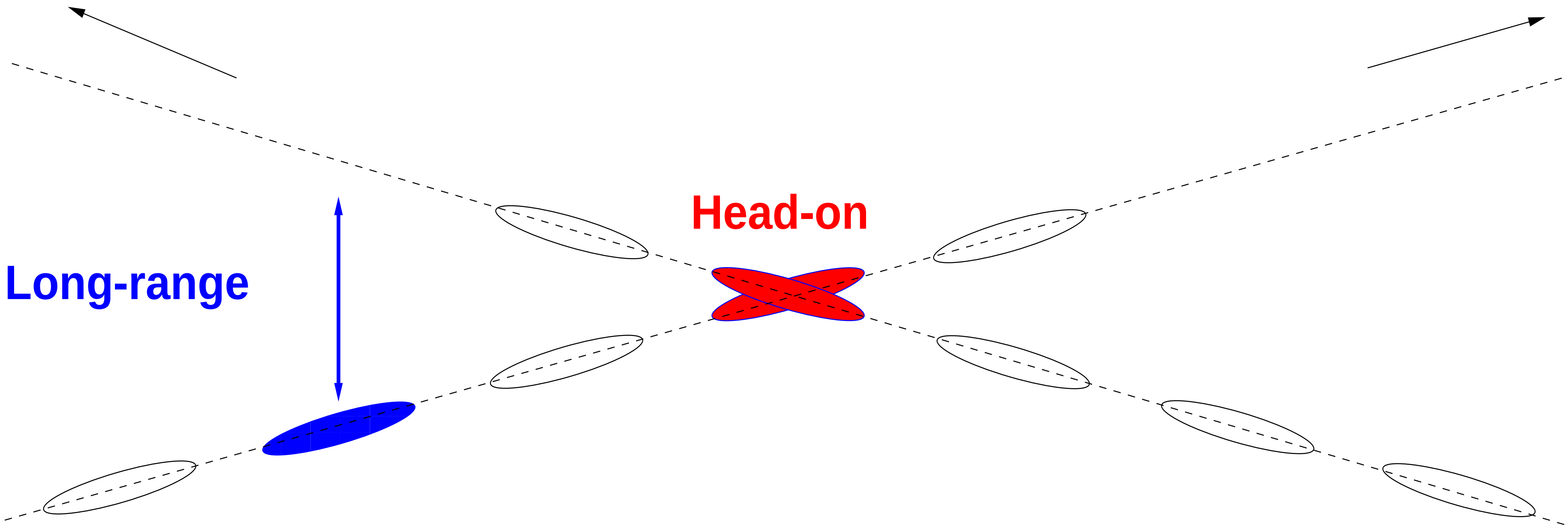
But not always ...



What we want ...



What we get ... (because of the gaps)



- Some <u>Bunches</u> meet <u>holes</u> (at beginning and end of batch)
- Cannot be avoided
- Worst case: less than half of long range collisions (depends on collision scheme and gaps)

When a bunch meets a "hole":

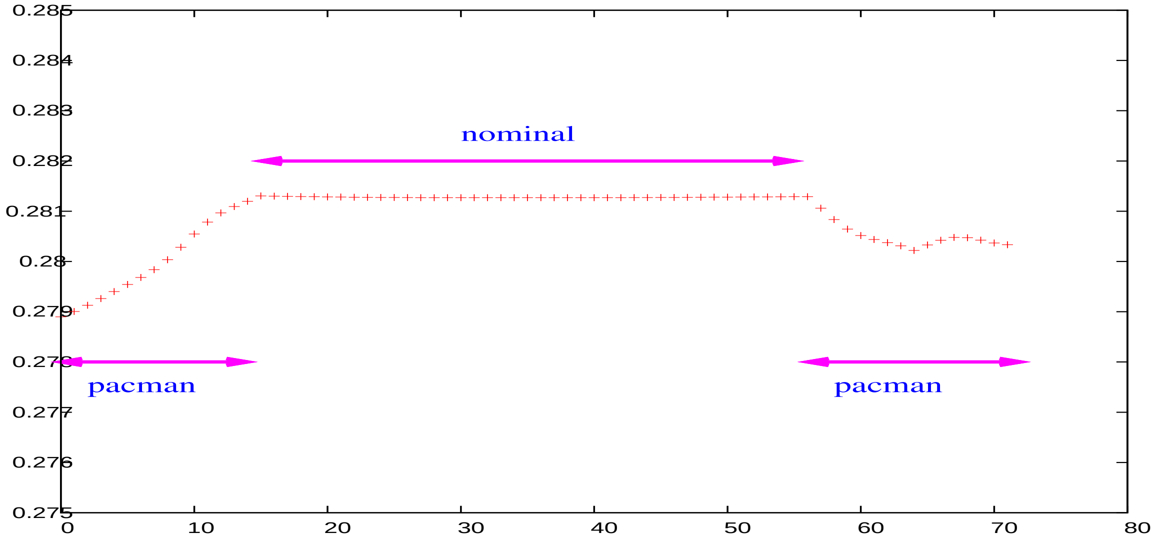
- Miss some long range interactions → PACMAN bunches
- They see fewer unwanted interactions in total
- Different integrated beam-beam effect
- Long range effects for different bunches will be different:

→ Different tune, chromaticity, orbit ...

- May be more difficult to optimize

Note: this case is specific LHC, but something similar happens in other machines in some form ..

Example: tune along the train, two horizontal (H + H) crossings

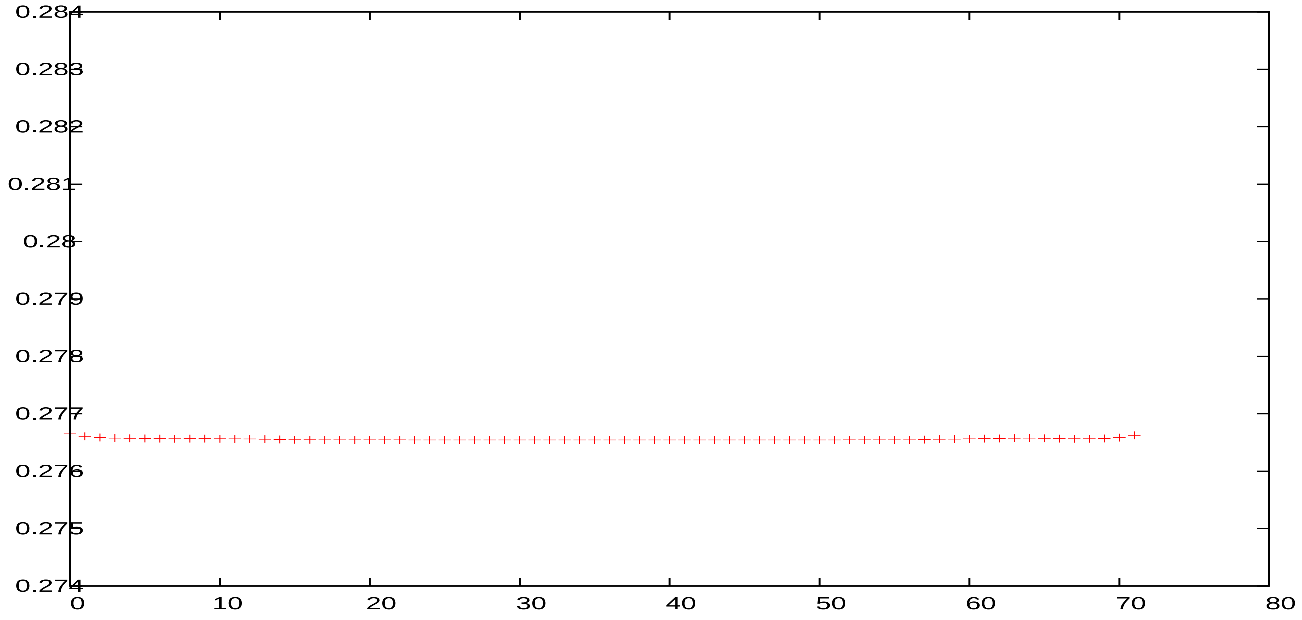


Horizontal tune along bunch train (computed, all bunches assumed equal)

Tune spread between bunches rather large ($\approx$ 0.0025), effects add up for several crossings

Too large ($\Delta Q \approx 0.01$) for 4 IPs

Example: tune along the train, alternating (H + V) crossings

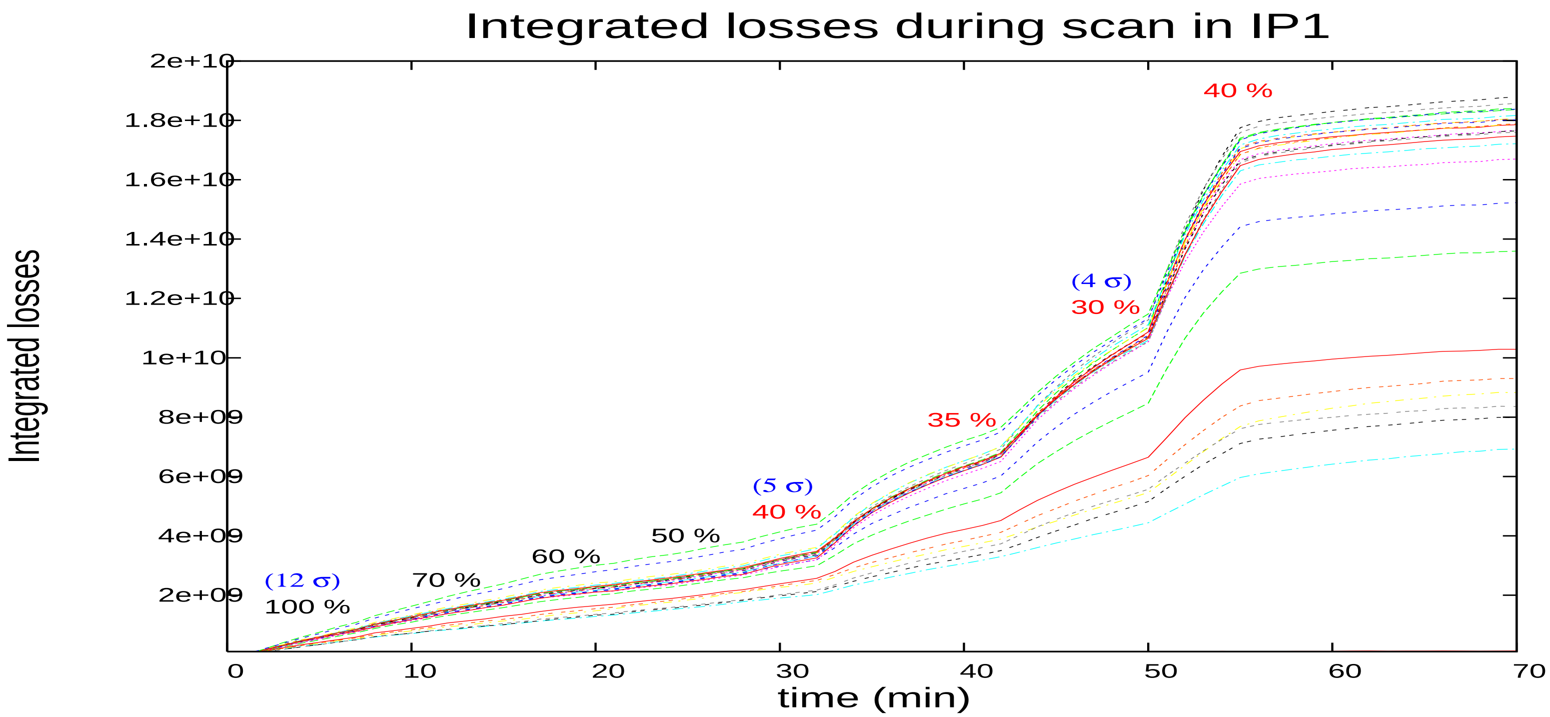


Horizontal tune along bunch train (computed)

Tune spread has disappeared due to compensation by alternating crossings (1 horizontal and 1 vertical)

This is the <u>real</u> reason for alternating crossings in the LHC !

PACMANs can be measured (without compensation):



Recent measurement: Separation (crossing angle) slowly decreased from 12 σ to 4 σ

Bunches with <u>more</u> long range collisions suffer first

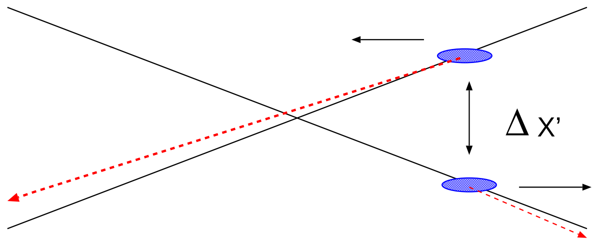
Closed orbit effects

Starting from the kick $\Delta x'$ for long range interactions:

$$\Delta x'(x+d, y, r) = -\frac{2Nr_0}{\gamma} \cdot \frac{(x+d)}{r^2}\left[1 - \exp(-\frac{r^2}{2\sigma^2})\right]$$

For well separated beams ($d \gg \sigma$) the force (kick) has an amplitude independent contribution:[a]

$$\Delta x' = \frac{const.}{d} \cdot \left[1 - \frac{x}{d} + O\left(\frac{x^2}{d^2}\right) + ...\right]$$



[a]This is one of the the complications mentioned before ...

This constant and amplitude independent kick changes the orbit !

Has been observed in LEP with bunch trains (and was bad)

So should be evaluated by computation, however:

Change of orbit → change of separation → change of orbit ...
Change of tune → change of separation → change of tune ...

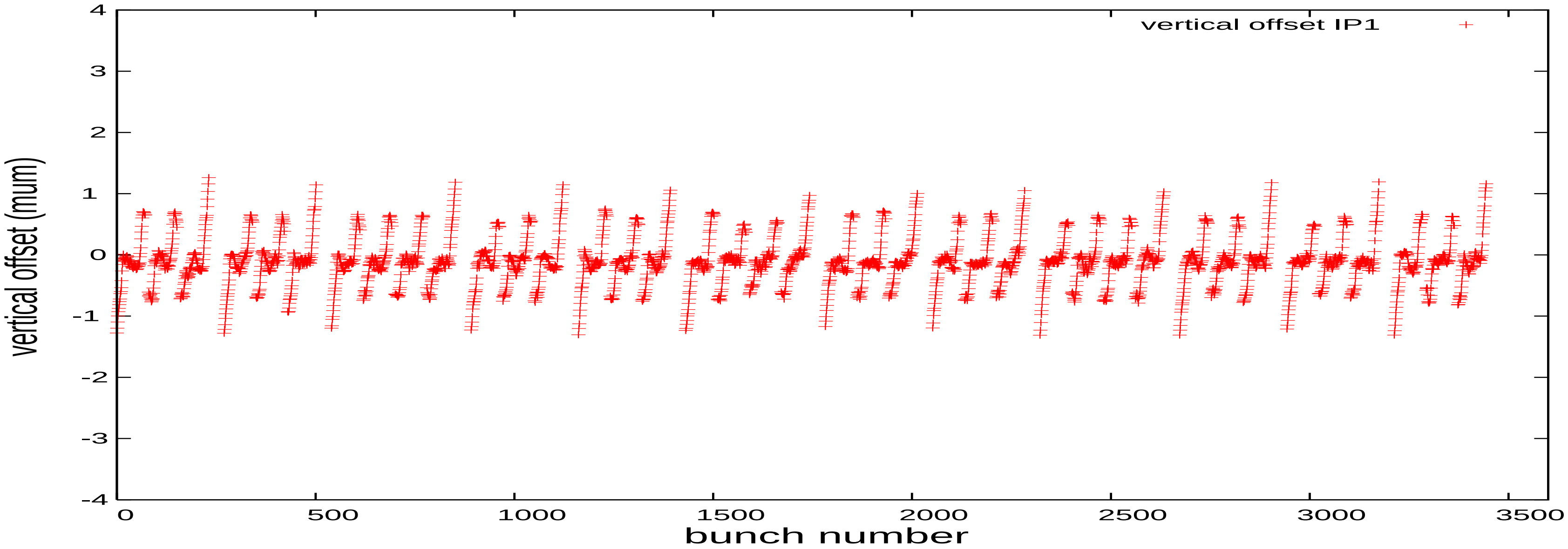
All PACMAN bunches will be on different orbits

In LEP: 8 bunches,   in LHC: 2808 bunches

Requires the self-consistent computation of 5616 orbit !

(first time done in 2001, calculation took 45 minutes for equal bunches, can handle unequal bunches as well)

PACMAN Orbit effects: calculation

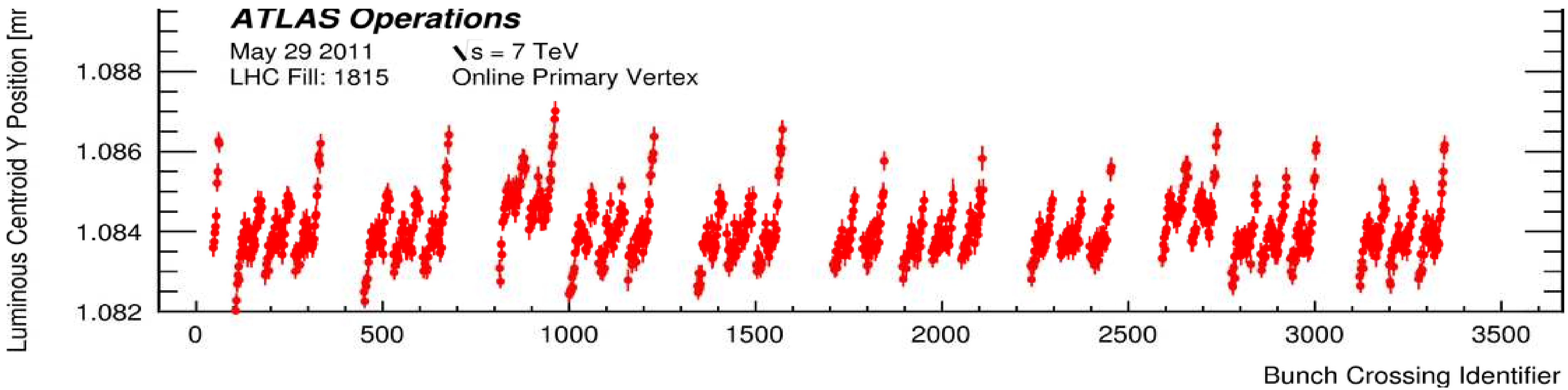


- Predicted orbits from self-consistent computation
- Vertical offset expected at collision points, sizeable with respect to beam size, loss of luminosity: for smaller β^* it gets worse and worse

  Does it have anything to do with reality ?
- Cannot be resolved with beam position measurement, but ..

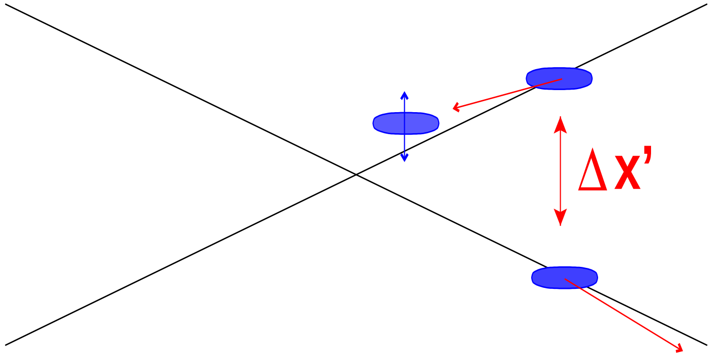
PACMAN Orbit effects: measurement

2011-07-05 file:///afs/cern.ch/user/z/zwe/Desktop/PNG/bcid_vs_posY_pm_posYErr.png #1



- **Measured vertex centroid in ATLAS detector**
- **Very good agreement with computation**

Coherent beam-beam effect (very short)



<u>When bunches are well separated:</u>

All particles in a bunch "see" the same kick

<u>Whole</u> bunch sees a kick as an entity (coherent kick)

The coherent kick of separated beams can excite coherent dipole oscillations

All bunches couple because each bunch "sees" many opposing bunches: many coherent modes possible !

When bunches are separated much less than one σ (quasi head-on interactions):

Remember orbit kick ➡ all particles "see" the same kick

$$\Delta x' = \frac{const.}{d} \cdot \left[1 - \overbrace{\frac{x}{d} + O\left(\frac{x^2}{d^2}\right) + \ldots}^{other\ parts} \right]$$

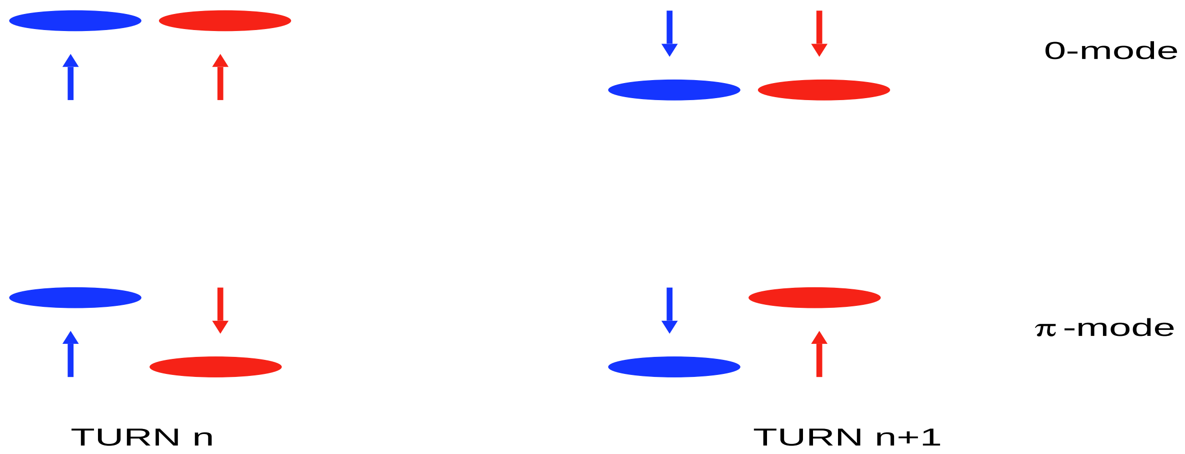
There is "one part" of the kick that is the same for all particles

This part also excites dipolar oscillations

There are "other parts" which are different for the particles

These parts can change the particle <u>distribution</u>

Simplest case - one bunch per beam:



Coherent modes: two bunches are "locked" in a coherent oscillation, turn by turn, can be either:

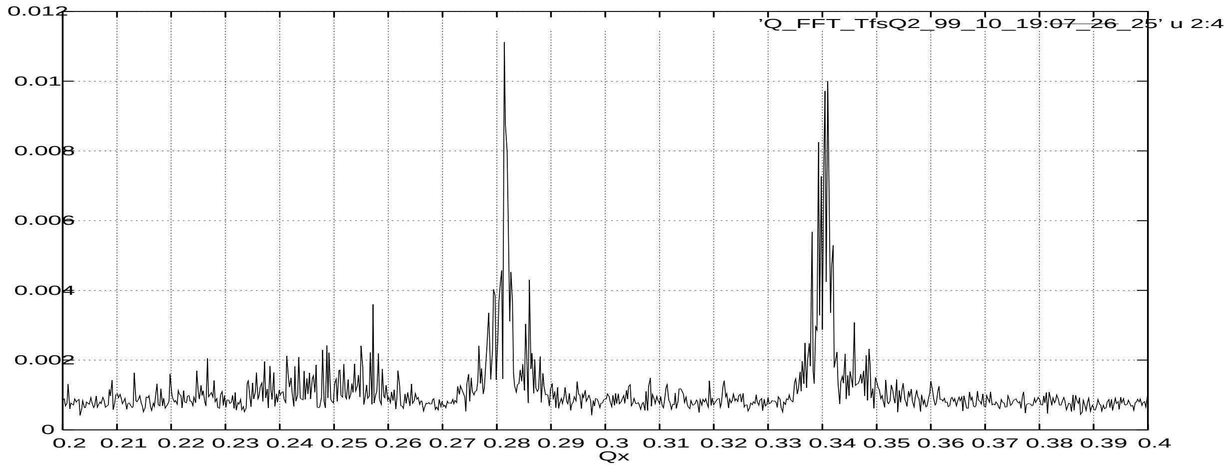
Two bunches oscillate "in phase": 0-mode

Two bunches oscillate "out of phase": π-mode

0-mode has no tune shift and is stable

π-mode has large tune shift and can be unstable

What was measured: LEP



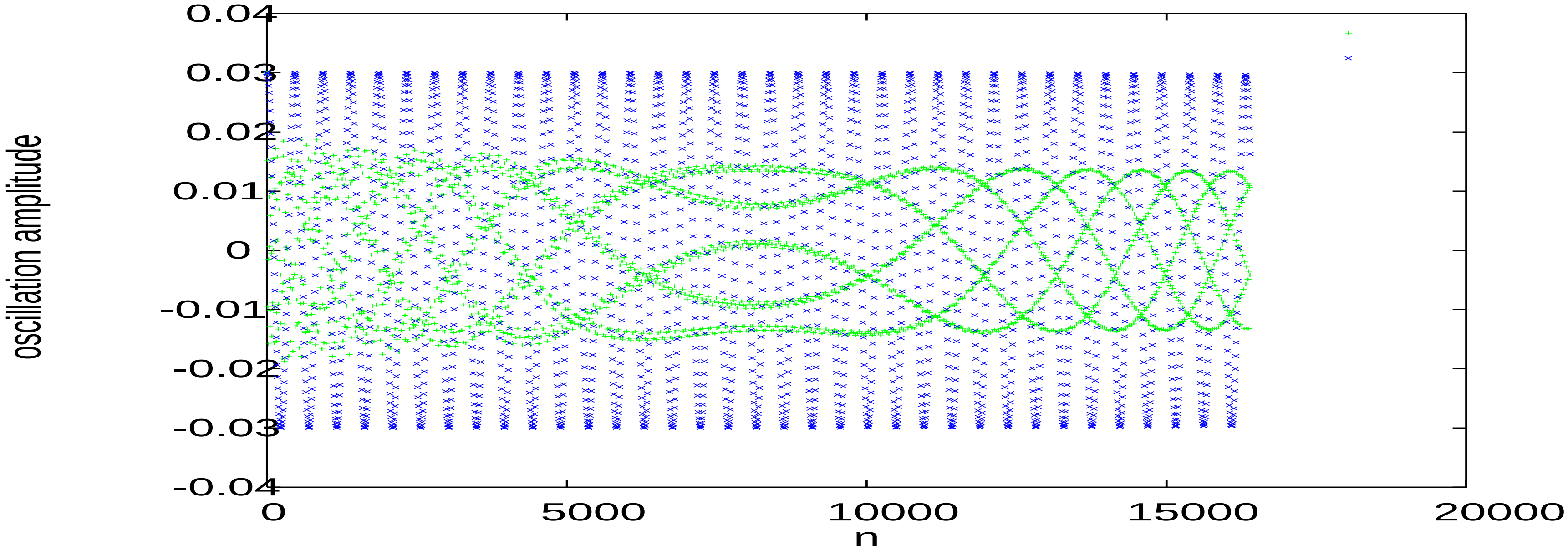
$$\Uparrow \qquad\qquad \Uparrow$$

$$0:\ 0.28 \qquad\qquad \pi:\ 0.34$$

Both modes clearly visible - they are real !

History: first seen at DESY (A. Piwinski), detailed calculation by Yokoya (1990)

Just the result of the self-consistent calculation, see earlier:



What is shown: difference and sum of bunch centres

$< \psi^a > \; - \; < \psi^b >$ π **- mode**

$< \psi^a > \; + \; < \psi^b >$ 0 **- mode**

How to deal with the problems ?

Every "Coherent Motion" requires 'organized' motion of many/all particles

Requires a "high degree of symmetry" (between the beams)

Possible countermeasure break the symmetry e.q. by:

- Different bunch intensity

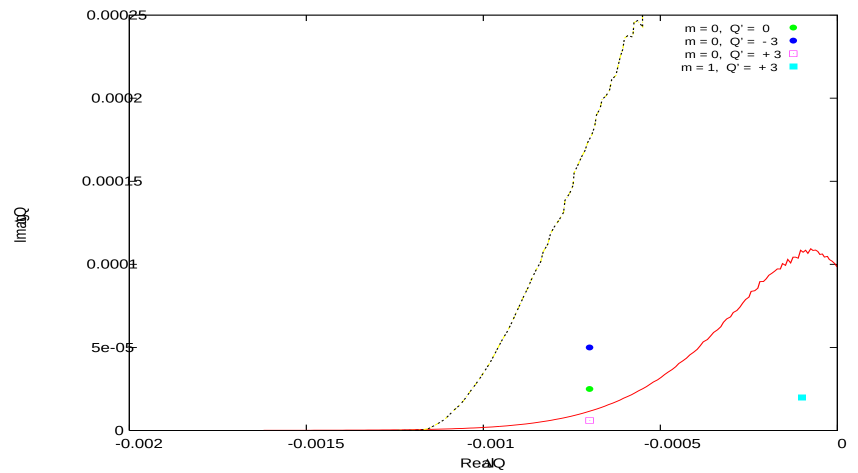
  (results in different tune shifts for the two beams)

- Different nominal tunes of the two beams

➤ Increasing differences: Y becomes smaller and smaller and π-mode is Landau damped

➤ LHC: not seen, beams and collisions scheme not symmetric enough and π-mode is Landau damped

More on Landau damping - here head tail modes



- Stabilization of collective instabilities with colliding beams

- (Head-on) collisions for identical tune spread much more efficient than octupoles for Landau damping, in particular for high energies[a]

  LHC beams rather unstable during adjustent, new procedures proposed and evaluated

➡ **Beam-beam collisions save the day for the LHC !**

[a] see e.g. W.Herr, "Landau Damping", in Proceedings CAS Trondheim (2013)

Wrap up I

- Beam-beam interaction the largest complication for beam dynamics in colliders
- Many different effects have to be considered and evaluated
- A full theory does not (yet) exist, but enormous progress in last 25 years, mostly due to :
  - Experience from previous hadron colliders (SPS, Tevatron, LEP)
  - LHC: Experimental evidence and operational experience
  - Needs to design the LHC and to study the various complications, not present in previous colliders, requiring to enter uncharted territories
  - Resulting in the development of novel tools and methods

Wrap up II

A proper evaluation depends on powerful tools, many of them developped for nonlinear beam dynamis, beam-beam and other applications:

- **Lie methods and normal form analysis**
- **Simulation tools, not mentioned, e.g. PIC codes, FMM (recent, vital for Long Range calculations), for field calculations)**
- **Numerical solvers for Vlasov equation**
- **Not treated (see e.g. [3]), symplectic integrators**
- **Not treated (see e.g. [3]), but very powerful:**
  Truncated Power Series Algebra (TPSA) based on Differential Algebra for exact calculation of derivatives

* **<u>Foreseen:</u> Integer Algebra for <u>exact</u> long term tracking**

If interested: attend the topical CAS course on "Numerical Methods for Accelerators", 11. - 23. November 2018 in Thessaloniki

BACKUP SLIDES 2

- **Kick ($\Delta r'$): angle by which the particle is deflected radially during the passage through the bunch**

- **Forces are different <u>along</u> the bunch: integration of force over the collision, i.e. duration of passage Δt (assuming: m_1=m_2 and Z_1=$-Z_2$= 1):**

$$\text{Newton's law :} \quad \Delta r' = \frac{1}{mc\beta\gamma}\int_{-\frac{\Delta t}{2}}^{+\frac{\Delta t}{2}} F_r(r,s,t)dt$$

with:

$$F_r(r,s,t) = -\frac{Ne^2(1+\beta^2)}{\sqrt{(2\pi)^3}\epsilon_0 r\sigma_s}\left[1-\exp(-\frac{r^2}{2\sigma^2})\right]\cdot\left[\exp(-\frac{(s+vt)^2}{2\sigma_s^2})\right]$$

Assumption: longitudinal density distribution is Gaussian with σ_s

Hamiltonian for beam-beam interaction

The Hamiltonian for a linear one turn map is:

$$f_2 = -\frac{\mu}{2}\left(\frac{x^2}{\beta} + \beta p_x\right)$$

If $F(x)$ is the potential for the beam-beam force, the non-linear map with beam-beam is written (as a Lie transformation:

$$e^{:h:} = e^{:f_2:}\, e^{F:}$$

where h is the wanted overall effective Hamiltonian and

$$f(x) = \frac{2Nr_0}{\gamma x}\left(1 - e^{\frac{-x^2}{\sigma^2}}\right) \quad \Rightarrow \quad F(x) = \int_0^x f(u)du$$

Transforming to action/angle variables J and Φ as:

$$x = \sqrt{2J\beta}\sin\Phi, \quad p_x = \sqrt{\frac{2J}{\beta}}\cos\Phi$$

The potential $F(x)$ can be written as a Fourier series:

$$F(x) \quad = \quad \sum_{n=-\infty} \infty c_n(J) e^{in\Phi}$$

where

$$c_n(J) \quad = \quad \frac{1}{2\pi} \int_0^{2\pi} e^{-in\Phi} \; F(x) \; d\Phi$$

For the standard formalism to compute the effective Hamiltonian h see [3].

Tools and methods:

Analysis and purpose:

- **Evaluate stability of solution**
- **Calculate frequency spectra of oscillations**
- **Identify discrete spectral lines of oscillations**

Tools:

- **Numerical integration of Vlasov equation**
- **Multi-particle and multi-bunch tracking**
- **Perturbation theory**

Numerical integration

- Vlasov equation is Partial Differential Equation
- Aim: find distribution and its time evolution
- Use: numerical integration with Finite Difference Methods
- Basic concept:
  - Replace derivative by finite differences
  - Represent continuous function $\psi(x, t)$ by two-dimensional grid u_j^n (t $\rightarrow$ n, x $\rightarrow$ j)

Example 1:

$$\frac{\delta u}{\delta t} = \lambda \frac{\delta^2 u}{\delta x^2}, \qquad u(x, t = 0) = u_j^0 = f(x) \qquad \textbf{becomes :}$$

$$\frac{u_j^{n+1} - u_j^n}{\Delta t} = \lambda \frac{u_{j+1}^n - 2u_j^n + u_{j-1}^n}{\Delta x^2}$$

$$\Longrightarrow \qquad u_j^{n+1} = u_j^n + \frac{\lambda \Delta t}{\Delta x^2}(u_{j+1}^n - 2u_j^n + u_{j-1}^n)$$

spacial step Δx: $x_j = j \cdot \Delta x$

time step Δt for t: $t_n = n \cdot \Delta t$

initial distribution: $u_j^0 = f(x_j)$

Example 2:

$$\frac{\delta^2 u}{\delta t^2} = \lambda^2 \frac{\delta^2 u}{\delta x^2}, \qquad u(x, t = 0) = u_j^0 = f(x) \qquad \textbf{becomes :}$$

$$\frac{u_j^{n+1} - 2u_j^n + u_{j-1}^n}{\Delta t^2} = \lambda^2 \frac{u_{j+1}^n - 2u_j^n + u_{j-1}^n}{\Delta x^2}$$

$$\Longrightarrow \qquad u_j^{n+1} = 2(1 - \left(\frac{\lambda \Delta t}{\Delta x}\right)^2)u_j^n + \left(\frac{\lambda \Delta t}{\Delta x}\right)^2 (u_{j+1}^n + u_{j-1}^n) - u_j^{n-1}$$

Exercise → try to solve:

$$\frac{\delta^2 u}{\delta t^2} = 4\frac{\delta^2 u}{\delta x^2}, \qquad 0 < x < 1, \qquad u(x, 0) = \sin(\pi x)$$

Strategy for numerical integration

- **Use finite difference scheme**
- **Back to our problem which looks like (per beam):**

$$\frac{\partial\psi(x,p_x;t)}{\partial t} = -A(x;t)\frac{\partial\psi(x,p_x;t)}{\partial p_x} - B(p_x)\frac{\partial\psi(x,p_x;t)}{\partial x},$$

- **In each substep integrate in one direction (operator splitting):**

$$\begin{aligned}\frac{\partial\psi(x,p_x;t)}{\partial t} &= -A(x;t)\frac{\partial\psi(x,p_x;t)}{\partial p_x}\\ \frac{\partial\psi(x,p_x;t)}{\partial t} &= -B(p_x)\frac{\partial\psi(x,p_x;t)}{\partial x}\end{aligned}$$

- **Discretise $\psi(x,p_x;t)$ on the grid (i,j,n): U_{ij}^n (n = t/Δt)**

Grid calculations:

Each substep equation is of the type: $\frac{\delta u}{\delta t} = \lambda \frac{\delta f(u)}{\delta p_x}$

with: $f(u) = A(x) \cdot u$

- u_{ij}^n **is the discretisation at time** $t = n\Delta t$ **of the density** $\psi(x, p_x; t)$ **for** $x = i\Delta x$ **and** $y = j\Delta p_x$
- **Use the Lax-Wendroff scheme which looks like (e.g. first half-step in j-direction):**

$$u_{ij}^{n+1/2} = u_{ij}^n - \frac{A(x;t)}{2} \frac{\Delta t}{\Delta p_x} (u_{ij+1}^n - u_{ij-1}^n) + \frac{1}{2} \left(A(x;t) \frac{\Delta t}{\Delta p_x} \right)^2 (u_{ij+1}^n - 2u_{ij}^n + u_{ij-1}^n)$$

Exercise → try to derive (hint: Taylor expansion of u in t)

Grid calculations:

- **Putting it all together with $f = A(x)u$ and a similar half-step for $g = B(p_x)u$ (now going the i-direction) one gets**

  $u_{ij}^{n} \longrightarrow u_{ij}^{n+\frac{1}{2}} \longrightarrow u_{ij}^{n+1}$

- **Small complication: in presence of discontinuities this method may generate oscillations**

- **Remedy: introduce artificial 'viscosity'**

Strategy for simulations

- **Represent bunches by macro particles (10^4)**
- **Track each particle individually around machine**
- **At interaction points evaluate force from other beam on each particle (that is where space charge effects are treated similar)**
- **In principle: for each particle in beam a calculate the integral over $\rho^b(x', t)$**
- **In practice not possible, unless one makes assumptions (e.g. Gaussian beams etc.), need other techniques**

FIELD COMPUTATION

Solve Poisson equation for potential $\Phi(x, y)$ with charge distribution $\rho(x, y)$:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\Phi(x, y) = -2\pi\rho(x, y)$$

formally possible with Green's function:

$$\Phi(x, y) = \int G(x - x', y - y')\rho(x, y)dx'dy'$$

and (for open boundary):

$$G(x - x', y - y') = -\frac{1}{2}\ln[(x - x')^2 + (y - y')^2]$$

Techniques for field (force) calculation

- **Soft Gaussian approximation: Assume Gaussian distribution with varying centre and width (fast but not precise, but o.k. for incoherent studies)**
- **Particle-particle methods: (precise but slow, typical: N = 10^4)**
- **Particle-mesh methods: evaluate field on a finite mesh (precise, but slow for separated beams)**
- **Hybrid Fast Multipole Methods (HFMM): recent method, precise and much better for separated beams and beam halos**

PARTICLE-PARTICLE METHODS (PP)

- **Simple: accumulate forces by finding the force F(i,j) between particle i and particle j**
- **Problem: computational cost is O**(N_p^2)
- **For our problems typically** $N_p \geq 10^4$
- **Used sometimes in astrophysics**
- **For** $N_p < 10^3$ **and for close range dynamics good**

PARTICLE-MESH METHODS (PM)

- Approximate force as field quantity on a mesh.
- Differential operators are replaced by finite difference approximations.
- Particles (i.e. charges) are assigned to nearby mesh points (various methods).
- Problems:
  - Computational cost is O$(N_g \ln(N_g))$
  - Bad to study close encounters
  - Not ideal when mesh is largely empty (e.g. long range interactions)

PARTICLE-MESH METHODS

- **Main steps:**
  - **Assign charges to mesh points (NG, TSC, CIC)**
  - **Solve field equation on the mesh (many variants)**
  - **Calculate force from mesh defined potential**
  - **Interpolate force on grid ($N_g \cdot N_g$) to find force on particle**

PARTICLE ASSIGNMENT METHODS

- **NGP: (Nearest Grid Point), densities at mesh points are assigned by the total amount of charge surrounding the grid point, divided by the cell volume. Drawback are discontinuous forces.**
- **CIC: (Cloud in cell), involve 2^K nearest neighbours, (K = dimension of the problem), give continuous forces.**
- **TSC: (Triangular Shaped Cloud), use assignment interpolation function that is piecewise quadratic.**

IN PRACTICE ...

- **Must look at:**
  - **Stability**
  - **Noise reduction (short scale fluctuations due to granularity)**
  - **Number of particles**
  - **Size of grid cells**
  - **. . .**

A MORE RECENT APPROACH

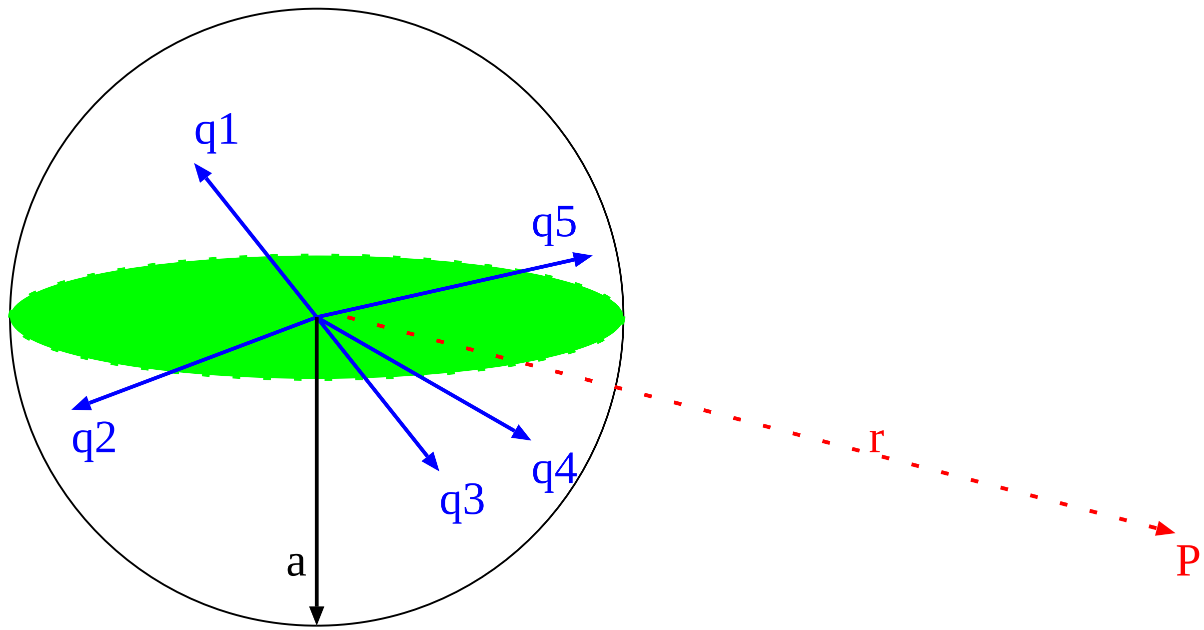
- **Fast Multipole Method (FMM)**
- **Derived from particle-particle methods, i.e. particles are not on a grid**
- **Tree code: treat far-field and short-field effects separately**
- **Relies on composing multipole expansions**
- **Computing cost: between O**(N_p) **and O**$(N_p \ln(N_p))$

Fast Multipole Method

- **Well-known multipole expansion at point P for k point charges q_i:**

$$\Phi(\vec{r}) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \frac{M_l^m}{r^{l+1}} Y_l^m(\theta, \phi)$$

$$\textbf{with}: \quad M_l^m = \sum_{i=1}^{k} q_i a_i^l Y_l^{*m}(\alpha_i, \beta_i)$$

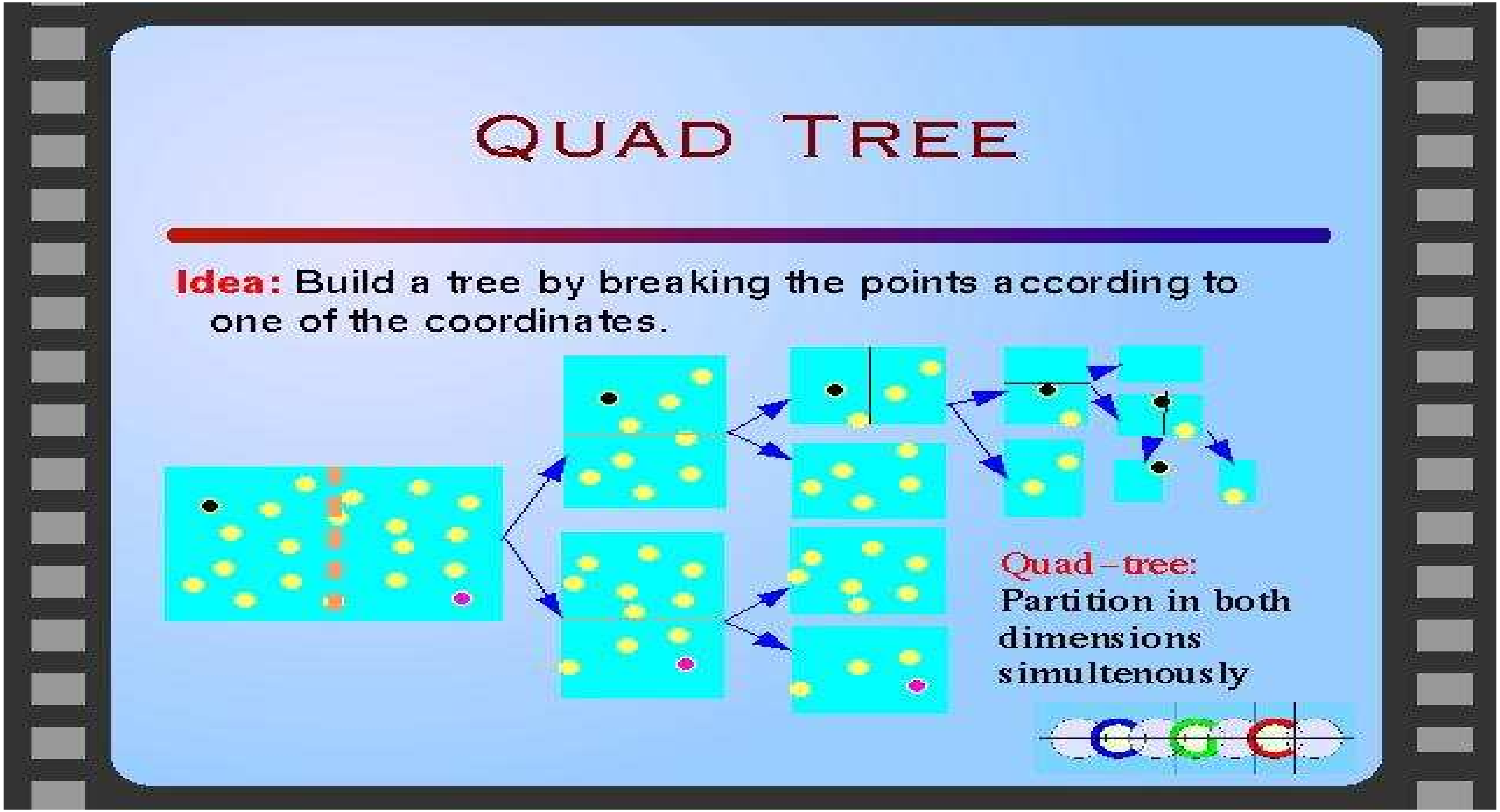


- **Order determined by desired accuracy**

FMM PROCEDURE

- **Hierarchical spatial decomposition into small cells and sub-cells (e.g. quad-tree)**
- **Multipole expansion for each sub-cell**
- **Expansions in cells are combined to represent effect of larger and larger groups of particles**
- **A ’calculus’ is defined to relocate and combine multipole expansion**
- **Far-field effects are combined with near-field effects to give potential (and field) at every particle**

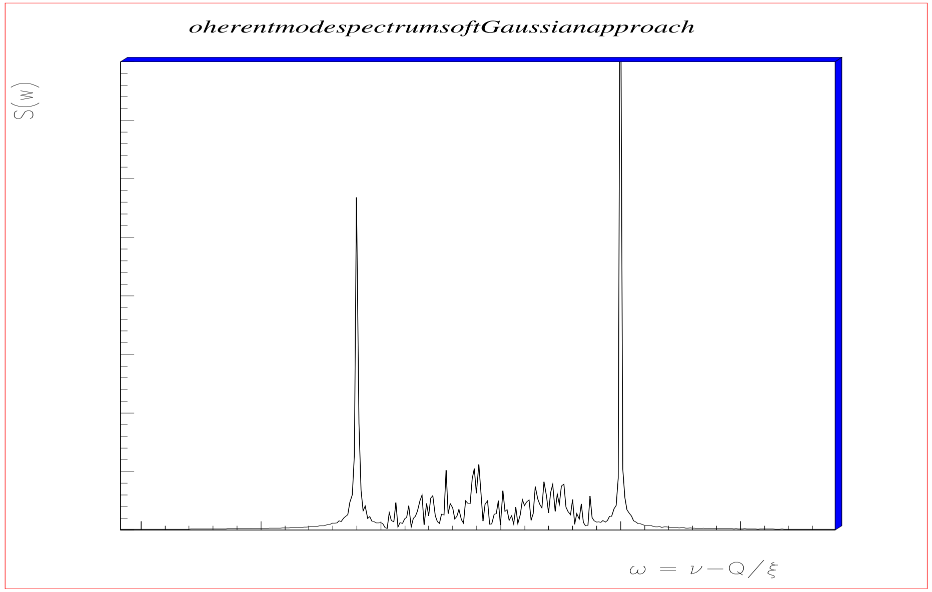
QUAD-TREE DIVISION



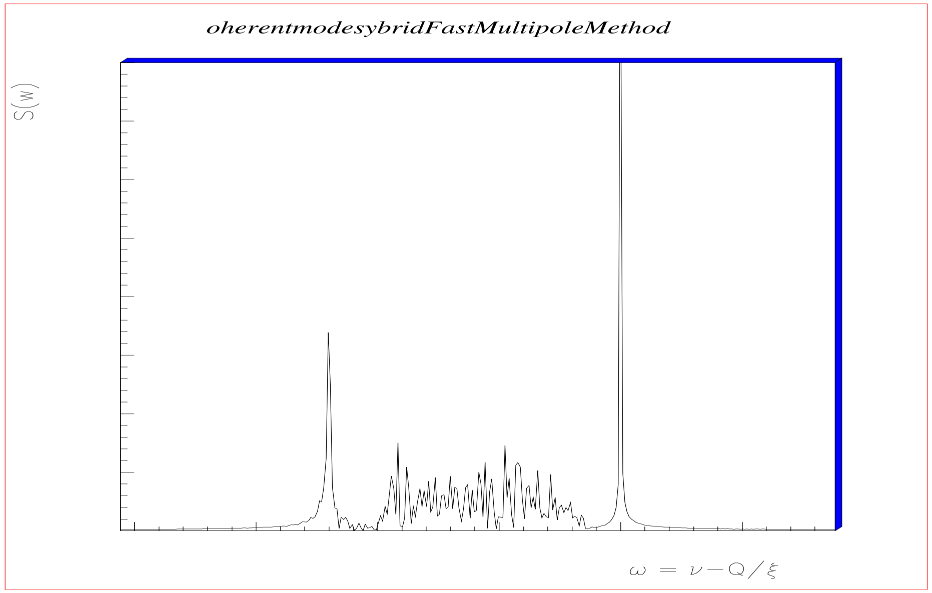
A VARIATION: HFMM

- **Hybrid Fast Multipole Method: FMM with a grid**
- **Assign particles on a grid, use FMM to calculate fields at grid points**
- **Particles may or may not be assigned to a grid**
- **Particle outside the grid are treated with standard FMM**
- **Precision is excellent and O(N_g) when all particles are on the grid**
- **Ideal for separated beams**

Is a Gaussian good enough ??
(or: why all this effort ?)



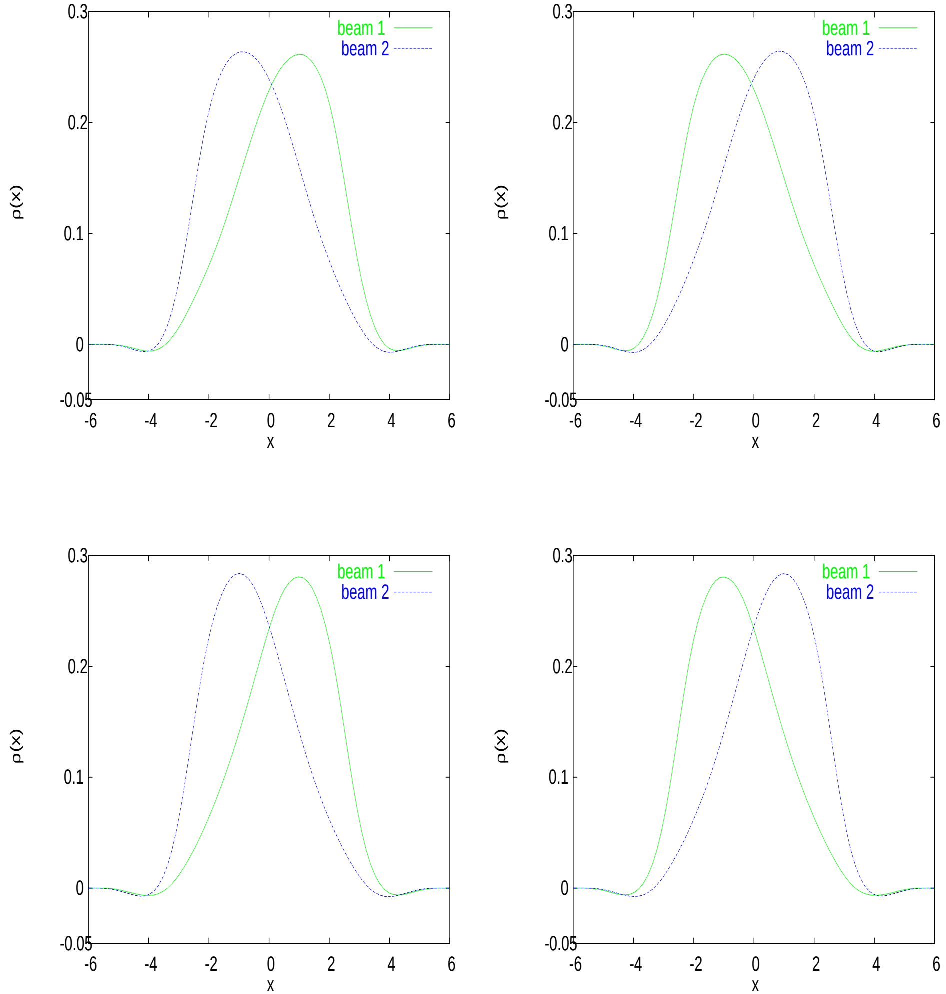
oherentmodespectrumsoftGaussianapproach
S(w)
$\omega = \nu - Q/\xi$



oherentmodesybridFastMultipoleMethod
S(w)
$\omega = \nu - Q/\xi$

- Factor is 1.1 (and not 1.214)

Why not ??



- "Skewness" important !
- Mostly core participates in oscillation
- Exercise: why does that change the frequency ?

Strategy of perturbation theory

- $\psi^a(x, p_x; t) \;=\; \psi^a_s \;+\; \psi^a_o(x, p_x; t)$
- **Go to I_x and ϕ_x (action and angle)**
- **Fourier expansion:**
  $\psi^a_o(I_x, \phi_x; t) \;=\; \sum_m \; exp(im\,\phi_x \;-\; \nu\,t) \;\cdot\; e^{(-I_x/2)} \;\cdot\; f^a_m(I_x)$
- **In Vlasov equation:** $i\frac{\partial}{\partial t}\vec{f} \;=\; \xi \cdot A \cdot \vec{f}$
- **With** $\vec{f} \;=\; (f^a, f^b)$ **and** $f \;\sim\; exp(-\xi\,\lambda\,t)$
- **Eigenvalues of** $\lambda\,\vec{f} \;=\; A\,\vec{f}$ **related to mode frequencies**
- **Can obtain eigenmodes** $\psi^a_{o,\lambda}(I_x, \phi_x; t)$