Radiative and weak decays of decuplet baryons

Måns Holmberg

Uppsala University

Stockholm, June 2018

Outline

Chiral symmetry

- The three lightest quarks are approximate massless on a hadronic scale

$$\begin{pmatrix} m_u \\ m_d \\ m_s \end{pmatrix} \ll 1\,\mathrm{GeV} \leq \begin{pmatrix} m_c \\ m_b \\ m_t \end{pmatrix}$$

- The Lagrangian of massless quark splits up in a purely right-handed and a purely left-handed part

$$\bar{q}Mq = \bar{q}_R M q_L + \bar{q}_L M q_R$$

- $SU(3)_L \times SU(3)_R \times U(1)_V$ symmetry in chiral limit
- Symmetry is spontaneously broken to $SU(3)_V \times U(1)_V$

$\rightsquigarrow$ Expect 8 Goldstone bosons $\Rightarrow$ Pseudoscalar meson octet

- Good starting point for low-energy QCD

Chiral perturbation theory (χPT)

- An effective field theory

$$\mathcal{L} = \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \mathcal{L}^{(3)} + \cdots$$

- Neglect irrelevant degrees of freedom
- Expand in powers of momenta and quark masses
- Rely on symmetries to construct the effective Lagrangian

$\rightsquigarrow$ Chiral symmetry heavily reduces number of terms!

- A power counting scheme determines the relevance of each diagram
- Predictive power once low energy constants (LECs) are known
- External sources are naturally introduced: v^μ, a^μ, s, p

χPT for baryons

- The octet baryons are collected in

$$B = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix}$$

- The decuplet is expressed by a totally symmetric flavor tensor

$$T^{111} = \Delta^{++}, \quad T^{112} = \frac{1}{\sqrt{3}}\Delta^+,$$

$$T^{122} = \frac{1}{\sqrt{3}}\Delta^0, \quad T^{222} = \Delta^-,$$

$$T^{113} = \frac{1}{\sqrt{3}}\Sigma^{*+}, \quad T^{123} = \frac{1}{\sqrt{6}}\Sigma^{*0}, \quad T^{223} = \frac{1}{\sqrt{3}}\Sigma^{*-},$$

$$T^{133} = \frac{1}{\sqrt{3}}\Xi^{*0}, \quad T^{233} = \frac{1}{\sqrt{3}}\Xi^{*-}, \quad T^{333} = \Omega\,.$$

Leading order (LO) Lagrangian for baryons

$$
\begin{aligned}
\mathcal{L}^{(1)}_{\text{baryon}} &= \mathrm{tr}\left(\bar{B}\left(i\not{D} - m_{(8)}\right) B\right) \\
&+ \bar{T}^{\mu}_{abc}\left(i\gamma_{\mu\nu\alpha}(D^{\alpha} T^{\nu})^{abc} - \gamma_{\mu\nu}\, m_{(10)}\,(T^{\nu})^{abc}\right) \\
&+ \frac{D}{2}\,\mathrm{tr}(\bar{B}\,\gamma^{\mu}\,\gamma_5\,\{u_{\mu}, B\}) + \frac{F}{2}\,\mathrm{tr}(\bar{B}\,\gamma^{\mu}\,\gamma_5\,[u_{\mu}, B]) \\
&+ \frac{h_A}{2\sqrt{2}}\,\left(\epsilon^{ade}\;\bar{T}^{\mu}_{abc}\,(u_{\mu})^{b}_{d}\,B^{c}_{e} + \epsilon_{ade}\,\bar{B}^{e}_{c}\,(u^{\mu})^{d}_{b}\;T^{abc}_{\mu}\right) \\
&- \frac{H_A}{2}\;\bar{T}^{\mu}_{abc}\gamma_{\nu}\gamma_5\,(u^{\nu})^{c}_{d}\;T^{abd}_{\mu}
\end{aligned}
$$

- The Goldstone bosons (related to u_{μ}) are encoded in

$$
\Phi = \Phi^{\dagger} = \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}\,\eta & \sqrt{2}\,\pi^{+} & \sqrt{2}\,K^{+} \\ \sqrt{2}\,\pi^{-} & -\pi^0 + \frac{1}{\sqrt{3}}\,\eta & \sqrt{2}\,K^{0} \\ \sqrt{2}\,K^{-} & \sqrt{2}\,\bar{K}^{0} & -\frac{2}{\sqrt{3}}\,\eta \end{pmatrix}.
$$

Radiative decays $B^*(J = 3/2) \to B\gamma$

- Interactions with photons can be studied by

$$v^\mu \to eA^\mu \begin{pmatrix} \frac{2}{3} & 0 & 0 \\ 0 & -\frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{3} \end{pmatrix} .$$

- At NLO, only one term contributes ($f_-^{\mu\nu} = 0$)

$$\mathcal{L}_{\text{int}}^{(2)} = ic_M \epsilon_{ade} \bar{B}_c^e \gamma_\mu \gamma_5 (f_+^{\mu\nu})_b^d T_\nu^{abc} + ic_E \epsilon_{ade} \bar{B}_c^e \gamma_\mu (f_-^{\mu\nu})_b^d T_\nu^{abc} + \text{h.c.}$$

⇝ Magnetic vector transition form factor related to c_M.

⇝ Electric axial-vector transition form factor related to c_E.

- The partial decay width at tree level is

$$\Gamma_{B^* \to \gamma B} = \frac{c^2}{6\pi} p_{\text{cm}}^3 \frac{E_B + m_{B^*}}{m_{B^*}} .$$

Radiative decays $B^*(J = 3/2) \to B\gamma$

We can fit c_M to data and make predictions

Decay	$c/(c_M e)$	BR [%]	$\lvert c_M \rvert$ [GeV^{-1}]
$\Delta \to N\gamma$	$2/\sqrt{3}$	0.60 ± 0.05	2.00 ± 0.03
$\Sigma^{*+} \to \Sigma^+\gamma$	$-2/\sqrt{3}$	0.70 ± 0.17	1.89 ± 0.08
$\Sigma^{*-} \to \Sigma^-\gamma$	0	< 0.024	—
$\Sigma^{*0} \to \Sigma^0\gamma$	$1/\sqrt{3}$	$\mathbf{0.18\pm0.01}$	—
$\Sigma^{*0} \to \Lambda\gamma$	-1	1.25 ± 0.13	1.89 ± 0.05
$\Xi^{*0} \to \Xi^0\gamma$	$-2/\sqrt{3}$	$\mathbf{4.0\pm0.3}$	—
$\Xi^{*-} \to \Xi^-\gamma$	0	< 4	—

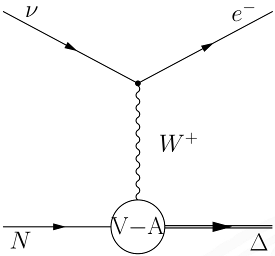
$\Sigma^{*-} \to \Sigma^-\gamma$, $\Xi^{*0} \to \Xi^0\gamma$ vanishes due to U-spin symmetry

(predictions in boldface)

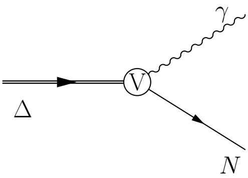
M. Holmberg, SL, arXiv:1802.05168 [hep-ph], to appear in EPJ A

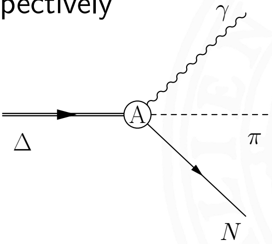
Axial-vector transition form factors

- Interesting for scattering neutrino-nucleon to electron-Delta
- Low energies: want to know deviation from LO result $\rightsquigarrow$ LEC c_E



- Vector and axial-vector transition form factors contribute also to $\Delta \to N\gamma$ and $\Delta \to N\pi\gamma$, respectively

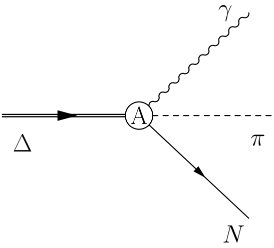




Axial-vector TFFs and three-body decays

Problems:

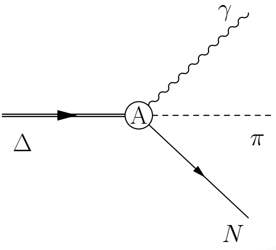
- Needs to be disentangled from bremsstrahlung
- Hard to measure for broad Delta



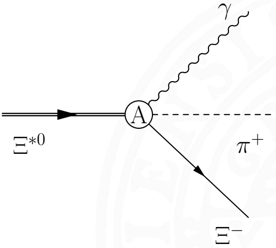
Axial-vector TFFs and three-body decays

Problems:

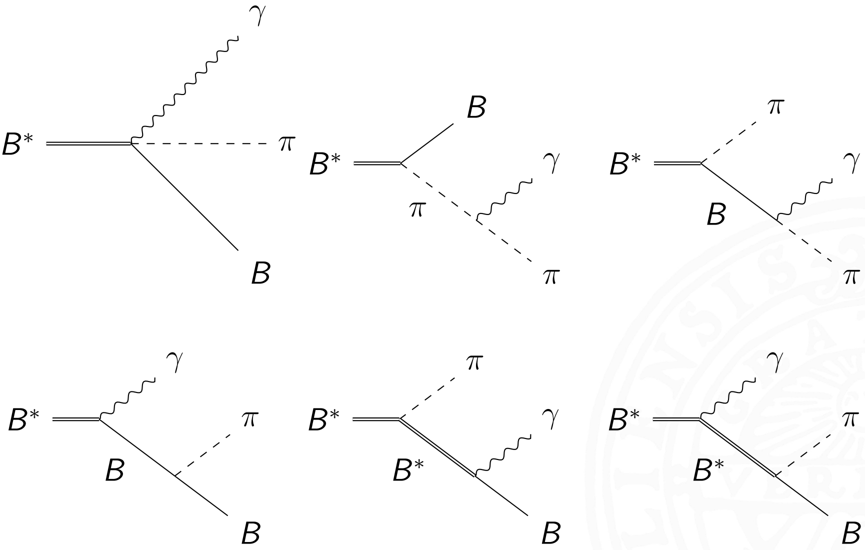
- Needs to be disentangled from bremsstrahlung
- Hard to measure for broad Delta



$\hookrightarrow$ Get some clue from radiate three-body decays of hyperons, e.g. cascades



Three body decays $B^*(J = 3/2) \to B\gamma\pi$



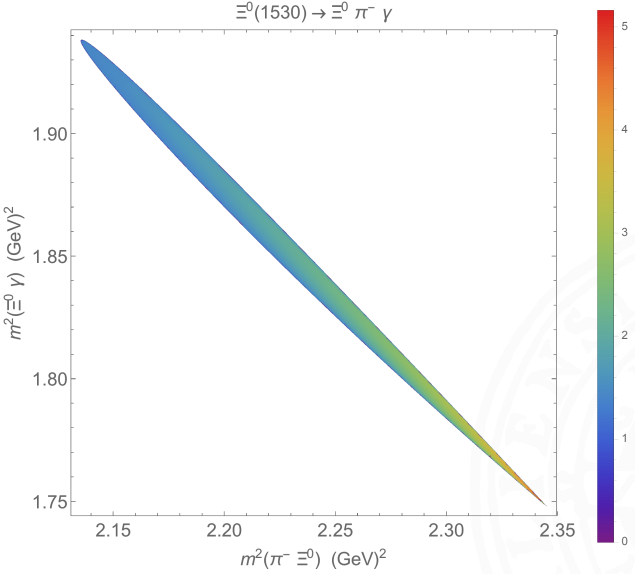
Three body decays $B^*(J = 3/2) \to B\gamma\pi$

Preliminary predictions (none of these are measured!)

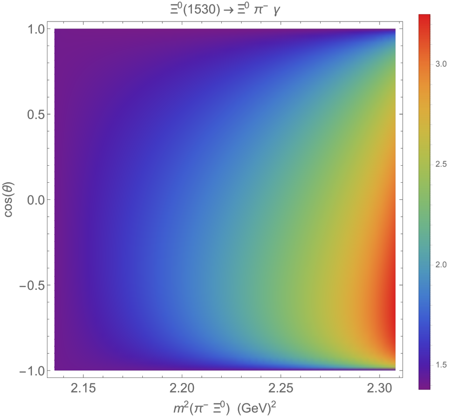
Decay	BR	Decay	BR
$\Sigma^{*+} \to \Sigma^+ \pi^0 \gamma$	1.1×10^{-6}	$\Xi^{*-} \to \Xi^- \pi^0 \gamma$	7.9×10^{-6}
$\Sigma^{*+} \to \Sigma^0 \pi^+ \gamma$	3.6×10^{-5}	$\Xi^{*-} \to \Xi^0 \pi^- \gamma$	1.3×10^{-3}
$\Sigma^{*+} \to \Lambda \pi^+ \gamma$	—	$\Xi^{*0} \to \Xi^- \pi^+ \gamma$	1.1×10^{-3}
$\Sigma^{*-} \to \Sigma^- \pi^0 \gamma$	6.0×10^{-7}	$\Xi^{*0} \to \Xi^0 \pi^0 \gamma$	1.8×10^{-6}
$\Sigma^{*-} \to \Sigma^0 \pi^- \gamma$	4.3×10^{-5}	$\Delta^{++} \to p \pi^+ \gamma$	1.7×10^{-3}
$\Sigma^{*-} \to \Lambda \pi^- \gamma$	—	$\Delta^+ \to p \pi^0 \gamma$	6.6×10^{-5}
$\Sigma^{*0} \to \Sigma^+ \pi^- \gamma$	5.7×10^{-5}	$\Delta^+ \to n \pi^+ \gamma$	7.4×10^{-4}
$\Sigma^{*0} \to \Sigma^- \pi^+ \gamma$	3.2×10^{-5}	$\Delta^0 \to p \pi^- \gamma$	1.0×10^{-3}
$\Sigma^{*0} \to \Sigma^0 \pi^0 \gamma$	2.5×10^{-8}	$\Delta^0 \to n \pi^0 \gamma$	7.2×10^{-6}
$\Sigma^{*0} \to \Lambda \pi^0 \gamma$	3.5×10^{-6}	$\Delta^- \to n \pi^- \gamma$	2.3×10^{-3}

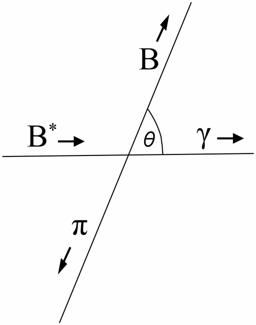
(Photon energy cut at 25 MeV)

Three body decays $B^*(J = 3/2) \to B\gamma\pi$



Three body decays $B^*(J = 3/2) \to B\gamma\pi$



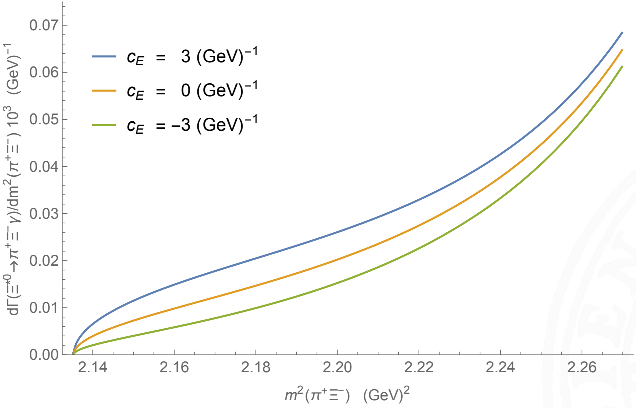


- Blows up when photon energy approaches zero

$$m^2(\pi B) = M_{B^*}^2 - 2M_{B^*}E_\gamma$$

Three body decays $B^*(J = 3/2) \to B\gamma\pi$

- Consider $\Xi^{*0}(1530) \to \Xi^- \pi^+ \gamma$



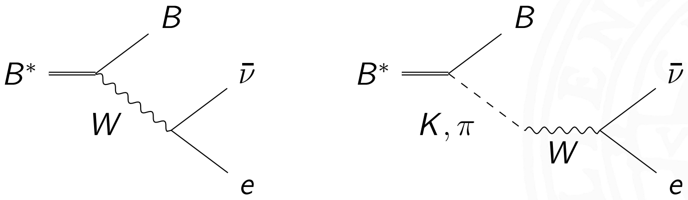
- Branching ratio $1.3 \cdot 10^{-3}$ (cut off photon energy at 25 MeV)

Weak decays $B^*(J = 3/2) \to Be\bar{\nu}$

- Interactions with W-bosons obtained by

$$l_\mu = v_\mu - a_\mu \to -\frac{g_w}{\sqrt{2}} W_\mu^+ \begin{pmatrix} 0 & V_{ud} & V_{us} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \text{h.c.}$$

- At NLO, decay depends on 3 LECs (c_M, c_E, h_A)

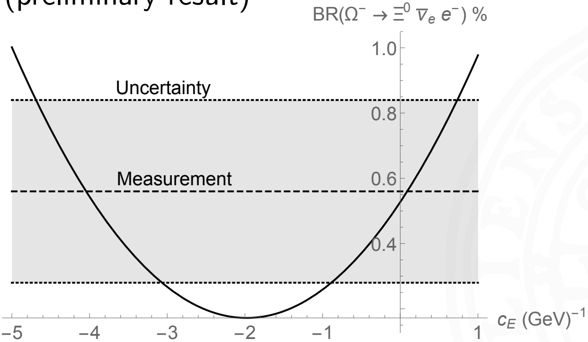


Decay $\Omega^- \to \Xi^0 \bar{\nu}_e e^-$

- The BR of $\Omega^- \to \Xi^0 \bar{\nu}_e e^-$ is known: $(0.56 \pm 0.28)\%$
- Two solutions of the LEC c_E

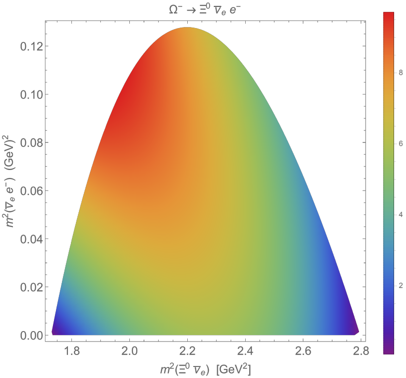
$$\frac{\Gamma_{\Omega^- \to \Xi^0 \bar{\nu}_e e^-}}{\Gamma_{\text{tot}}} = \alpha h_A^2 + \beta c_M^2 + \gamma c_E^2 + \delta h_A c_E$$

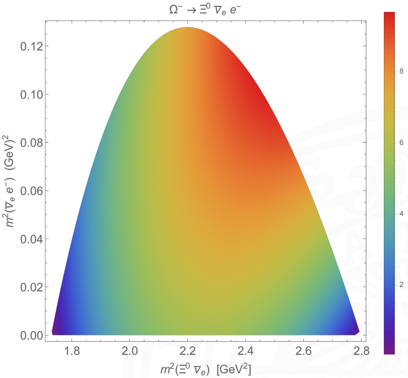
(preliminary result)



Decay $\Omega^- \to \Xi^0 \bar{\nu}_e e^-$

The Dalitz plot distinguishes between the two solutions of c_E





Summary

- χPT is a versatile tool for describing low-energy QCD
- Predicts a 4% BR of $\Xi^{*0} \to \Xi^0\gamma$
- Radiative three-body decays can be used to learn more about several LECs

$\hookrightarrow$ However, BR are generally small, especially for neutral final states

- The BR of $\Xi^{*-} \to \Xi^0 \pi^- \gamma$, $\Xi^{*0} \to \Xi^- \pi^+ \gamma$ are of order 10^{-3}

$\hookrightarrow$ Can be used to pin down c_E

- The weak decay $\Omega^- \to \Xi^0 e^- \bar{\nu}_e$ can also be used to pin down c_E

$\hookrightarrow$ Need higher quality measurements

Backup slide: U-spin

No explicit flavor breaking at NLO, as $\mathcal{M} \sim m_G^2 \sim \mathcal{O}(p^2)$

U-spin, which interchanges d and s quarks, is conserved

Need $\mathcal{O}(p^4)$ terms $\rightarrow$ N^{3}LO at tree level or N^{2}LO loops