

# Transverse Dynamics

G. Franchetti, GSI

12/4/2017

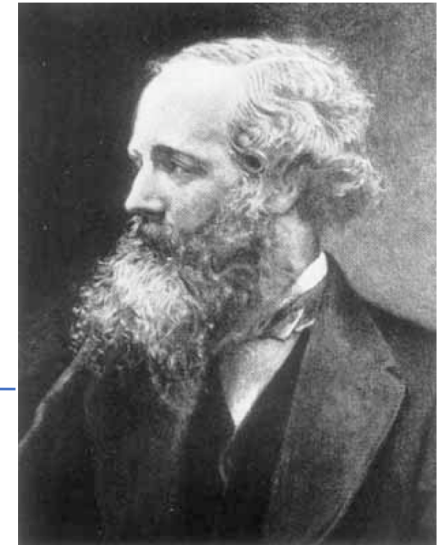
# Motion of particles

*I. Newton*



$$F = \frac{dp}{dt}$$

*J.C. Maxwell*

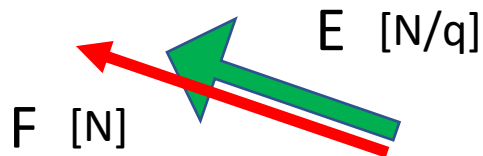


$$F = qE + qv \times B$$

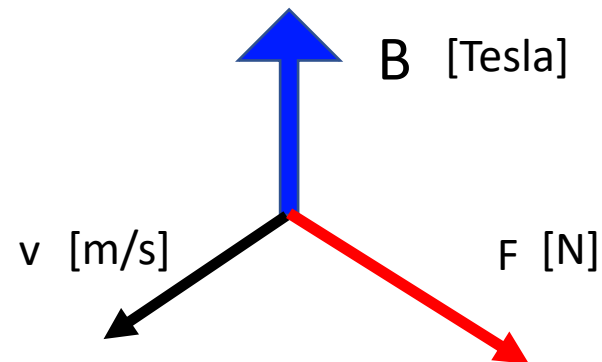
# The Lorentz force

$$F = qE + qv \times B$$

Electric field



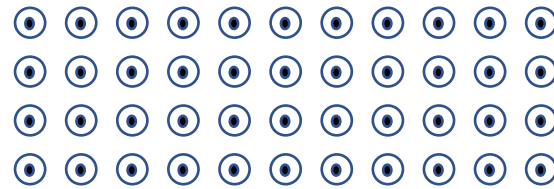
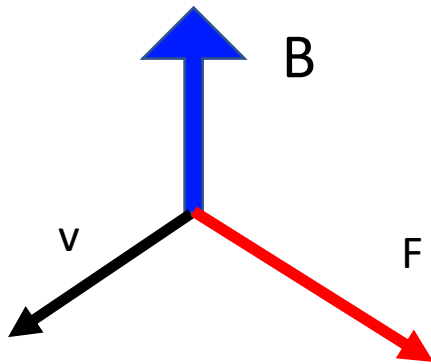
Magnetic field



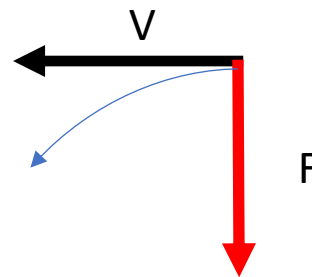
Magnetic field does not  
make work

# The Lorentz force

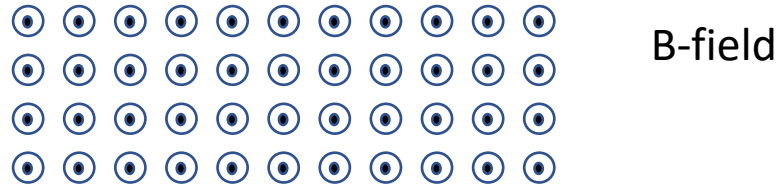
$$F = qE + qv \times B$$



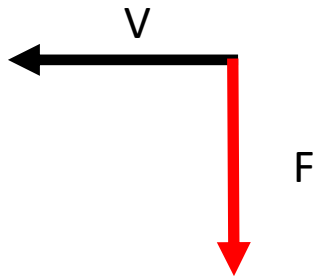
B-field



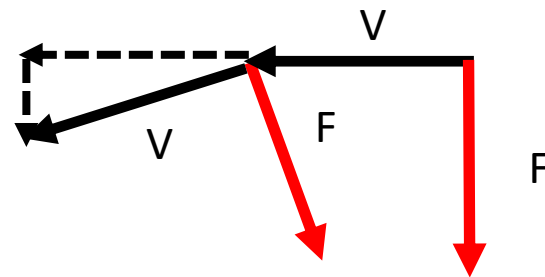
# The Lorentz force

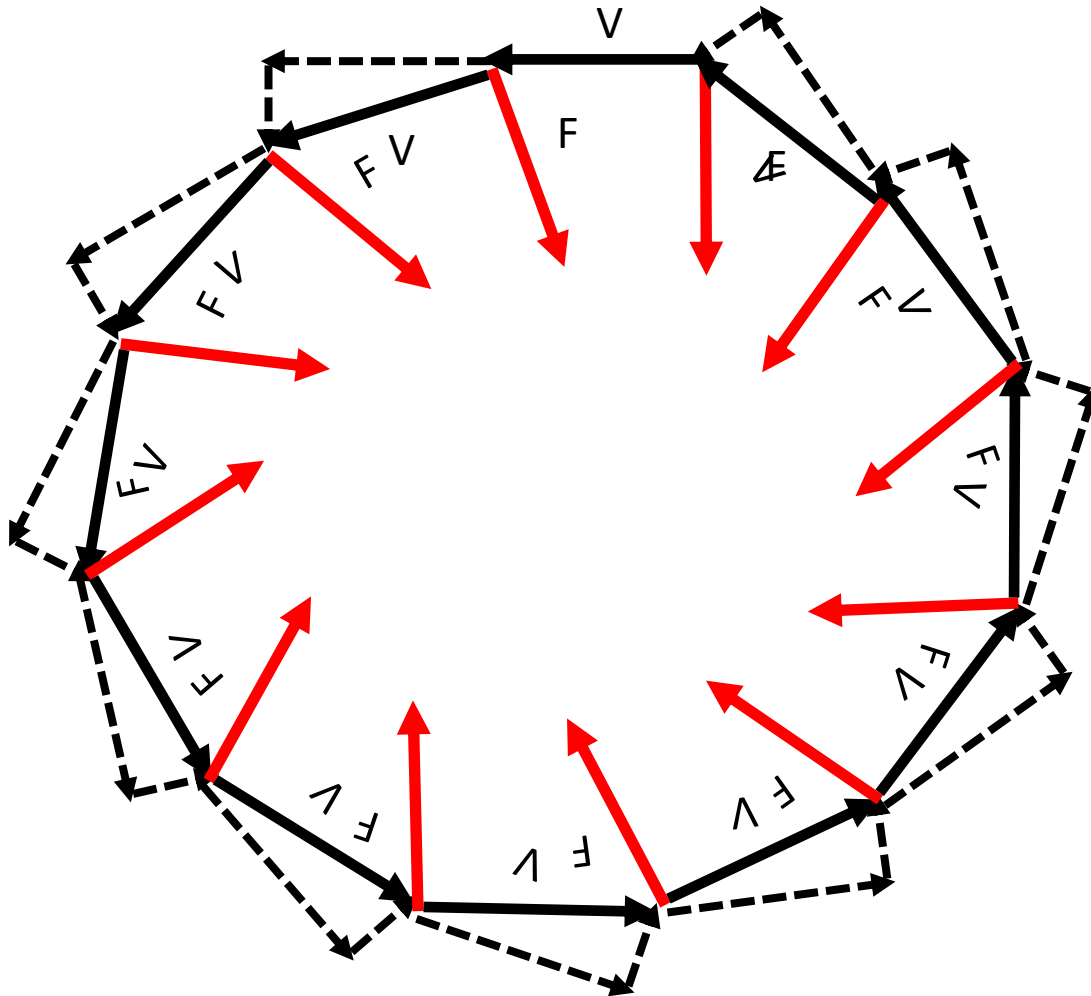


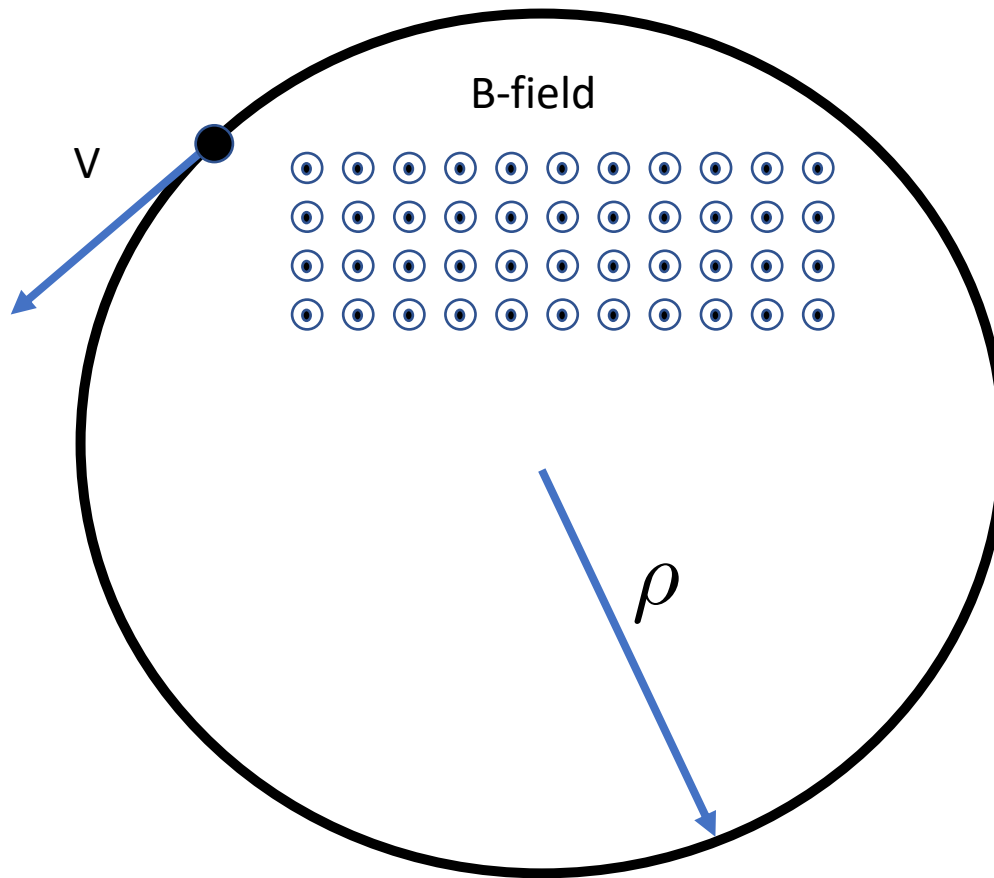
At  $t = 0$



At  $t = \Delta t$





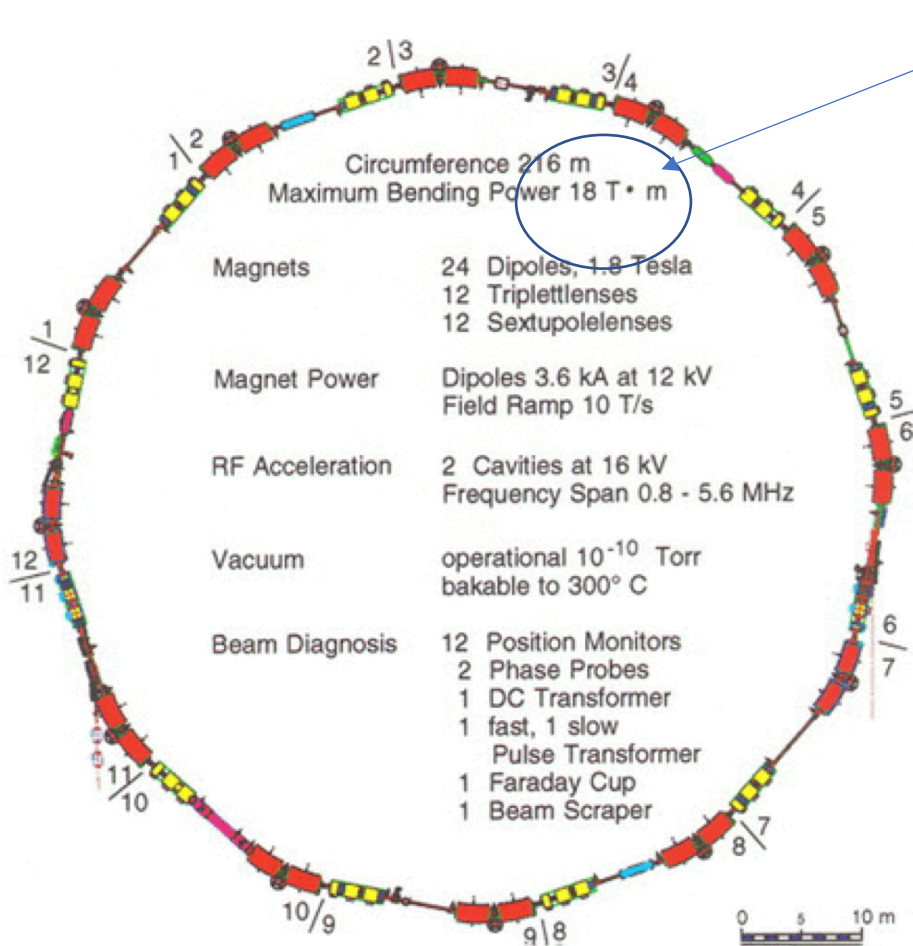


$$m\gamma v = q\rho B$$

$$\frac{p}{q} = \rho B$$

# Example: SIS18

## SIS18 Sektionen



maximum

$$B\rho = 18 \text{ Tm}$$

$$B = 1.8 \text{ Tesla}$$



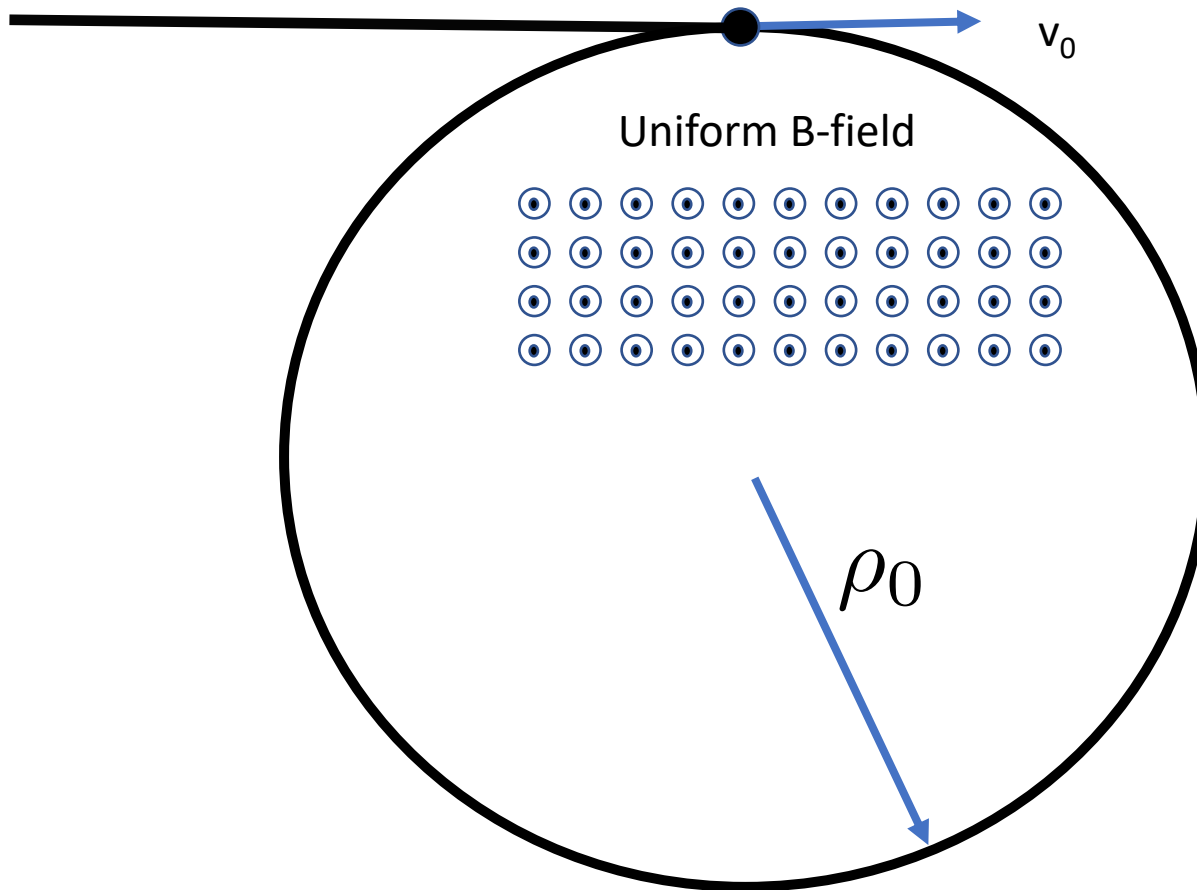
Bending radius = 10 m



But this for the correct particle energy !

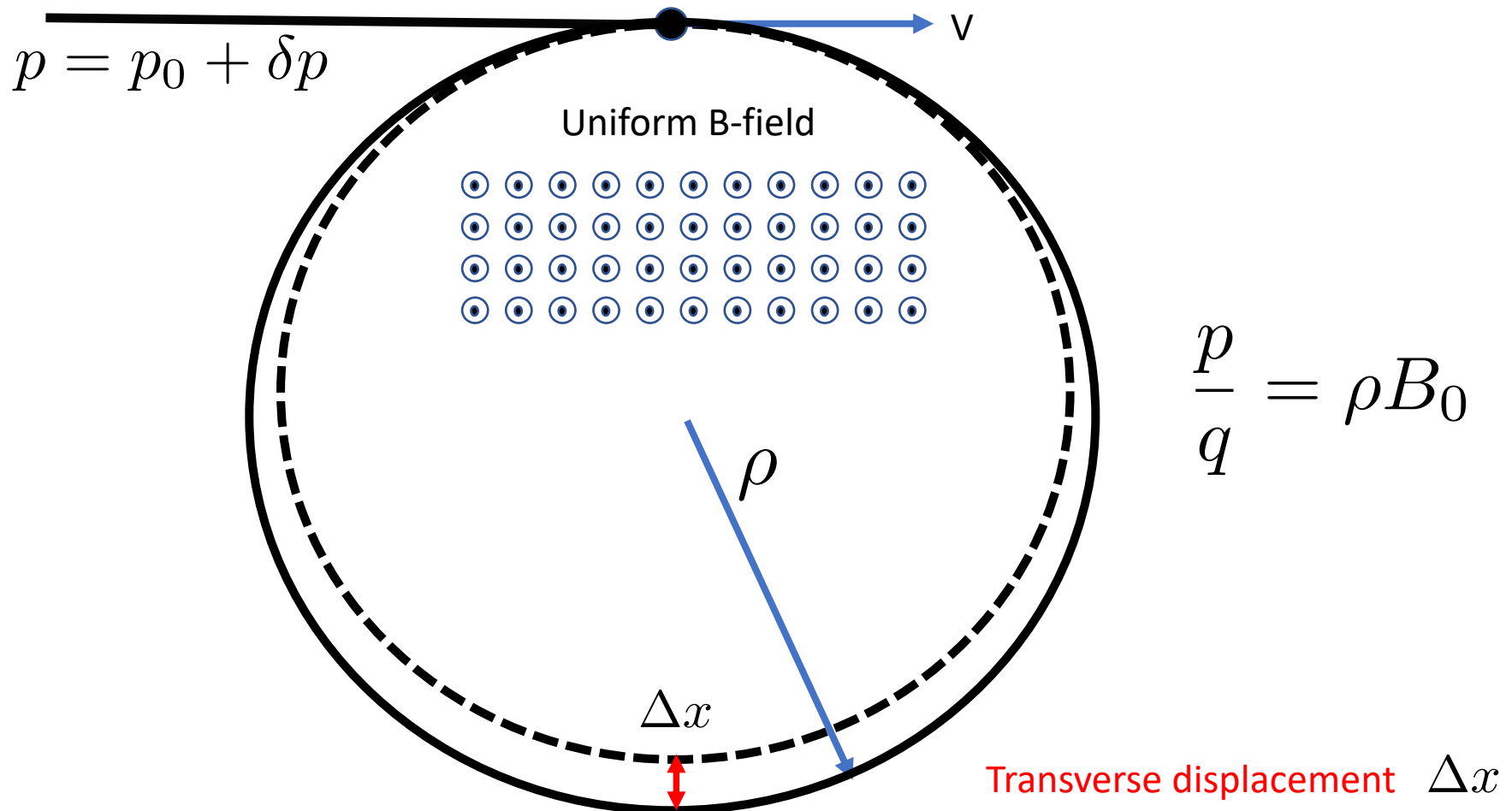


# Energy match: simple example



$$\frac{p_0}{q} = \rho_0 B_0$$

# Energy mismatch: simple example



# Simple estimate

$$\frac{p}{q} = \rho B_0 \quad \Rightarrow \quad \frac{\delta \rho}{\rho} = \frac{\delta p}{p} \quad \Rightarrow \quad \Delta x = 2\rho \frac{\delta p}{p}$$

For SIS18 type of machine

Mismatch  $\frac{\delta p}{p} = 10^{-3} \quad \Rightarrow \quad \Delta x = 2 \text{ cm} \quad (\text{big!})$

# What if B is wrong?

$$\frac{p}{q} = \rho B_0 \quad \Rightarrow \quad \frac{\delta \rho}{\rho} = -\frac{\delta B}{B_0} \quad \Rightarrow \quad \Delta x = 2\rho \frac{\delta p}{p}$$

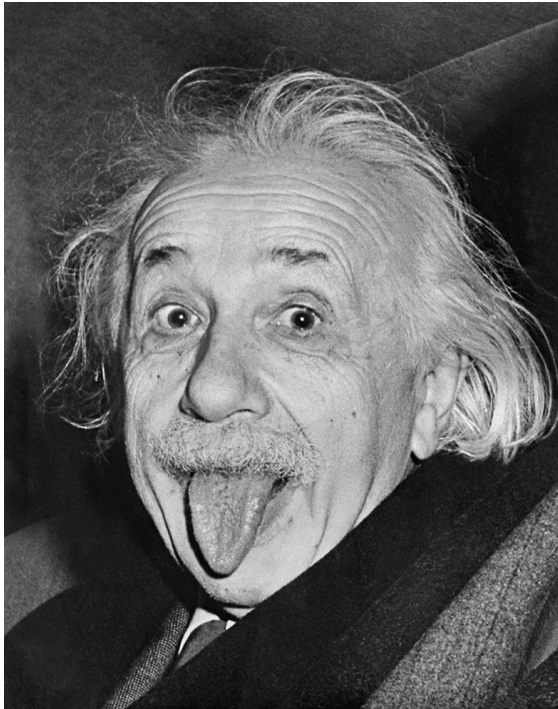
For SIS18 type of machine

$$\text{"wrong" } B \quad \frac{\delta B}{B_0} = 10^{-3} \quad \Rightarrow \quad \Delta x = 2 \text{ cm} \quad (\text{big!})$$

dB/B have to be very small  $\rightarrow$  smaller than  $10^{-4}$

# Energy $\leftrightarrow$ Momentum

$$E = mc^2$$



$$E = mc^2 + E_k$$



**Kinetic energy**  
of the atom

$$E^2 = (mc^2)^2 + (pc)^2$$



energy



momentum

S10 U09 TH/HTP 40AR18+ 300.10 schnell\_1H4 S02BE1 Expert

|                             |                             |                         |
|-----------------------------|-----------------------------|-------------------------|
| InjEnergie: 11.321          | Ext-Energie: 300.1          | Profilgit.Trig Inj.     |
| B-Rho [Tm]: 1.07849         | B-Rho [Tm]: 5.96516         | U-RpA [kV]: 18.0        |
| Frequenz: 854.902           | Frequenz: 3620.049          | U-RpE [kV]: 20.0        |
| Ist-Frequ.: 854.902         | sigma(-1..1): 0.0           | U-Flat [kV]: 10.0       |
| Taupunkt: 0.0               | QH: 4.34                    | rad.Pos.Ext.: -2.5      |
| QH: 4.34                    | QU: 3.27                    | Zyklus-Zeit: 2.555      |
| QU: 3.27                    | dTK7BC1Lmrad: 0.0           | Harmonische: 4.0        |
| ChopFenst.: 60.0            | dTK7MU5 mrad: 0.0           | dS12MU3Inrad: -1.0      |
| Bump.Flank: 150.0           | rad.Pos.Inj.: 0.0           | dS12ME1Inrad: 5.5       |
| Chop.Uerz.: 35.0            |                             |                         |
| dU-Ready: 10000             | t-Ramp[ms]: 64.0            | B-Punkt[T/s]: 1.30831   |
| BumpAmplmm: 90.0            |                             | Teilchen /µA: 0.162E+07 |
|                             |                             | U-Inj [kV]: 2.0         |
| AnzInjekt.: 1               | MMIKühlZ.ms: 1000.0         |                         |
|                             |                             | e-Kühler 0/1: 0         |
| IKüBumpXmm: 0.0             |                             | Kü-dp/p-Inj.: 0.0       |
| IKüBumpX': 0.0              |                             | Kühlerf.Inj.: 600.0     |
| IKüBumpYmm: 0.0             | InjektPlateau: 2.0          | Kühlerst.[A]: 0.3       |
| IKüBumpY': 0.0              |                             | Wartezeit[ms]: 0.0      |
| Sollb. Ampl.[mm]: 5.5       | SollStg.Zeit[ms]: 32.0      |                         |
| Kickwinkel[mrad]: 6.2       | SollFallZeit[ms]: 32.0      |                         |
| Kickstart(0-360): 76.5      | Extr.Teilchen/µA: 0.383E+06 |                         |
| Bypass [mrad]: 5.0          |                             |                         |
| HF-Trigger: 1-Gap, 2-Master | 2                           |                         |
| mit/ohne ESR-Synchron. 1/0: | 0                           |                         |

INIT  
PROTOKOLL  
TFS  
RESET  
SAVE  
RESTORE  
SOLL\_SOLL  
COPY  
EXIT

Eingaben an Geraete InitWertAnzeige alter Zustand

Injection energy  
Rigidity  
 $E_k = 11.321 \text{ MeV/u}$   
 $B r = 1.07849 \text{ Tm}$

# Argon: Mass 40, Charge + 18

SISMODI

Injection energy

$$\begin{aligned} E_k &= 11.321 \text{ MeV/u} \\ &= 11.321 \times 10^6 \text{ e/u} \\ &= 1.81382 \times 10^{-12} \text{ J/u} \end{aligned}$$

Rigidity

$$B r = 1.07849 \text{ Tm}$$

This is 0.1% off →  
There is a radial  
Displacement of beam  
orbit

$$m_p = 1.66056 \times 10^{-27} \text{ Kg}$$

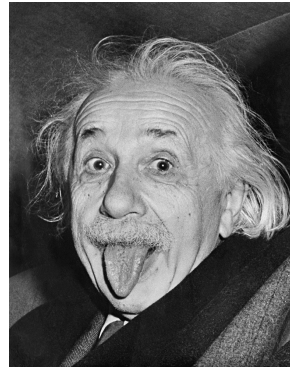
$$\begin{aligned} E &= 40 m_p c^2 + E_k \cdot 40 \\ &= 5.96976 \times 10^{-9} + 7.25529 \times 10^{-11} \text{ J} \\ &= 6.04232 \times 10^{-9} \text{ J} \end{aligned}$$



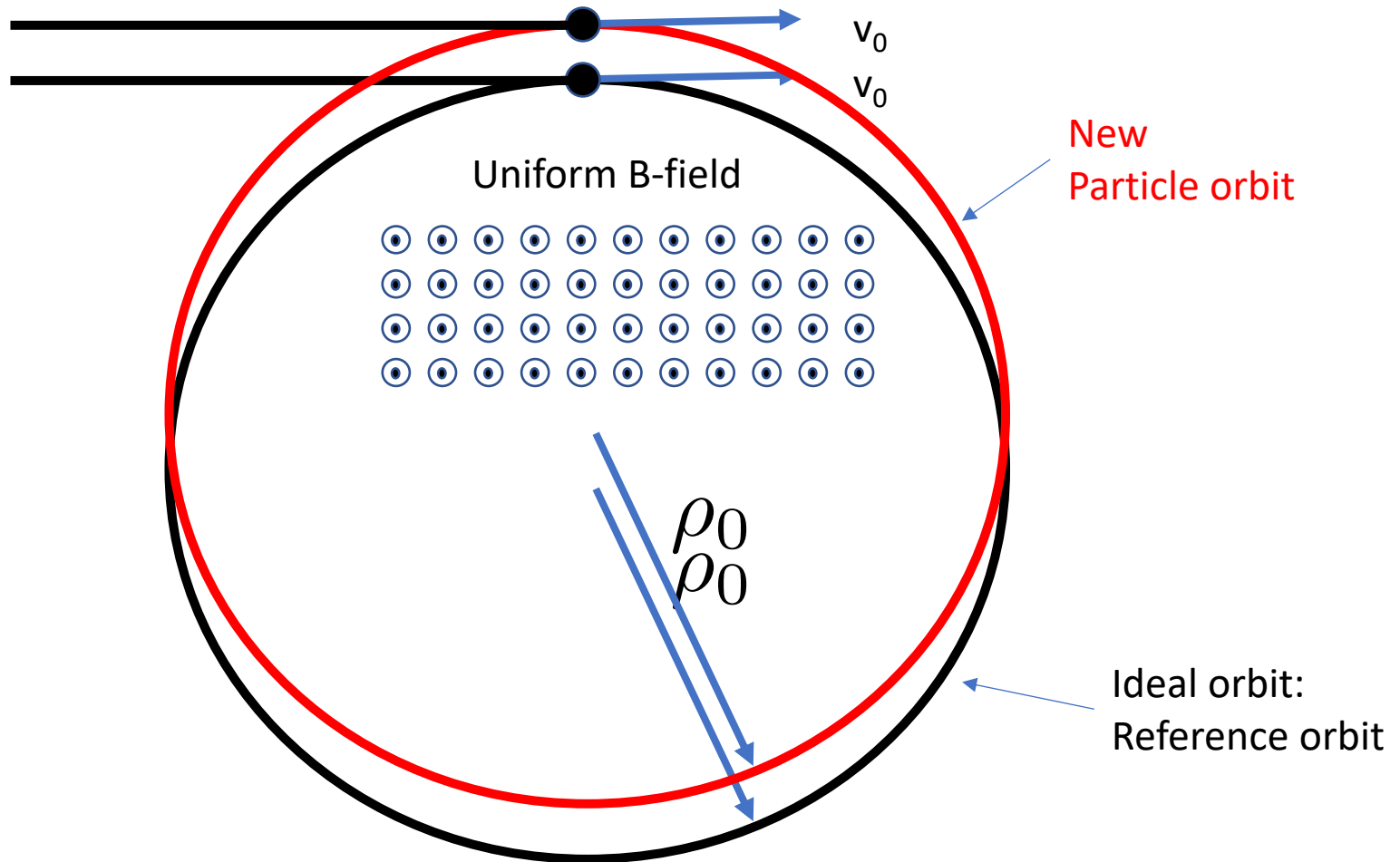
$$E^2 = (mc^2)^2 + (pc)^2$$

$$p = 3.11398 \times 10^{-18} \text{ Kg m / s}$$

$$\frac{p_0}{q} = \rho_0 B_0 \quad B r = 1.07977 \text{ Tm}$$

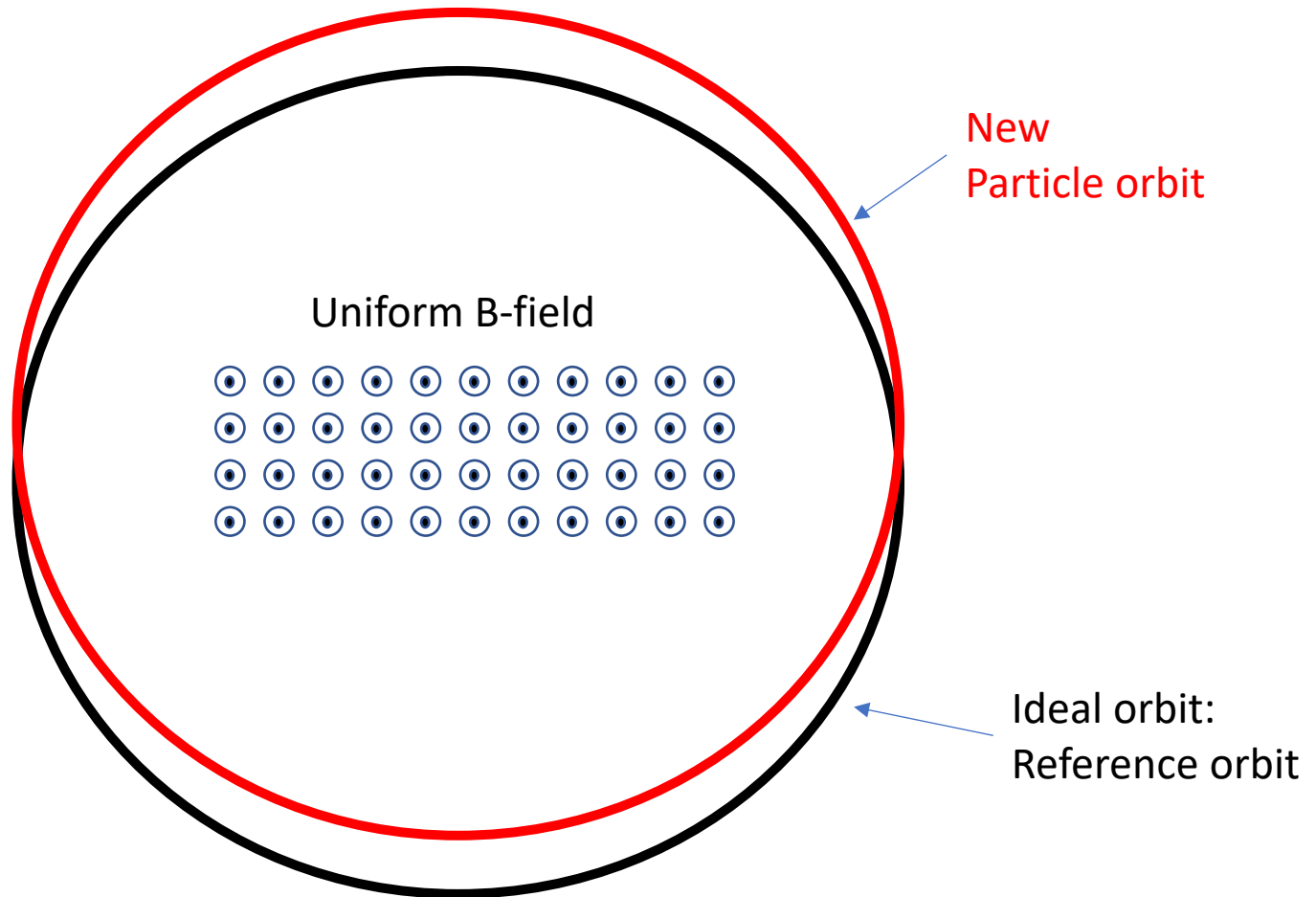


# Problem: a particle not in the correct position

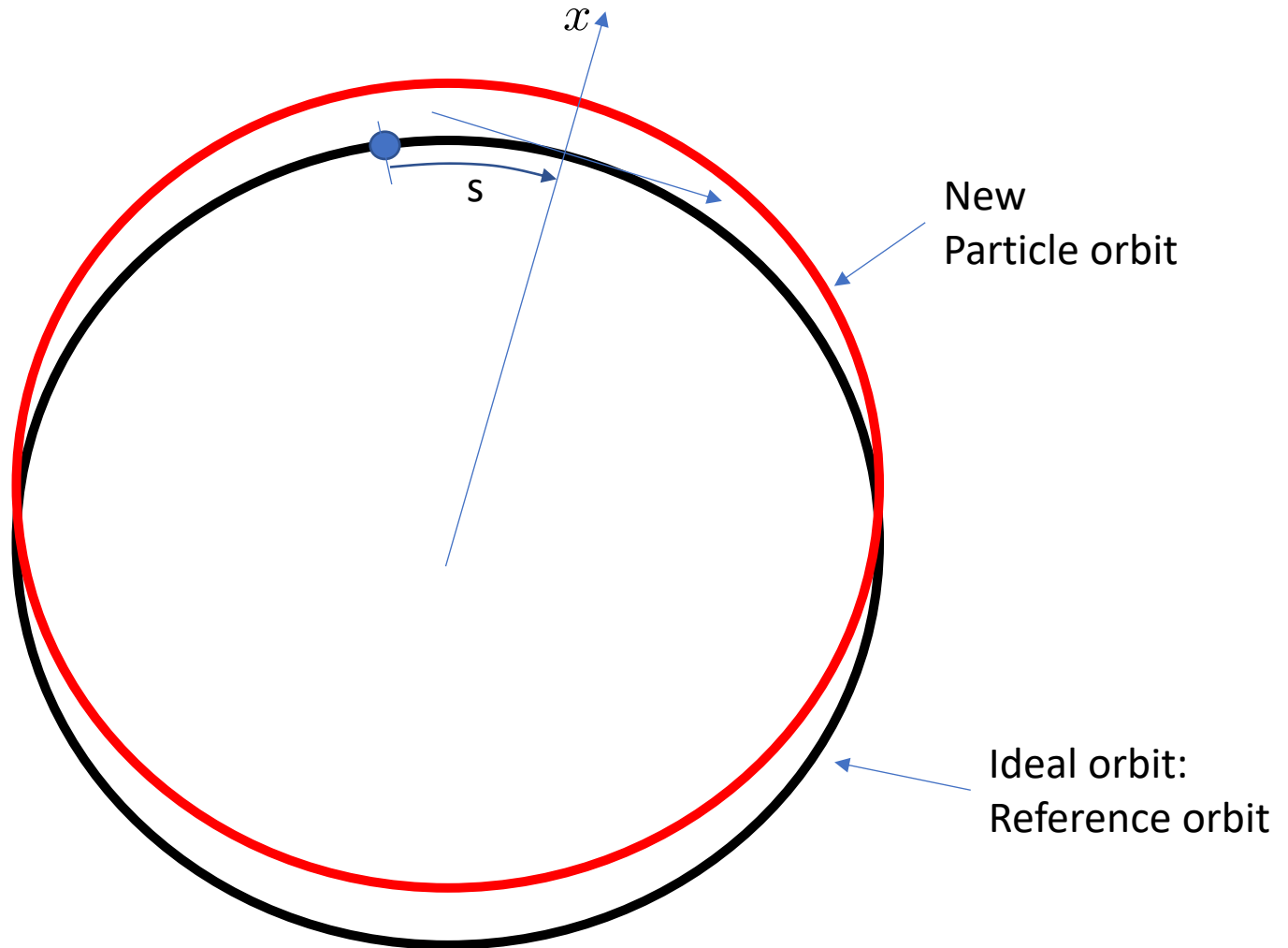




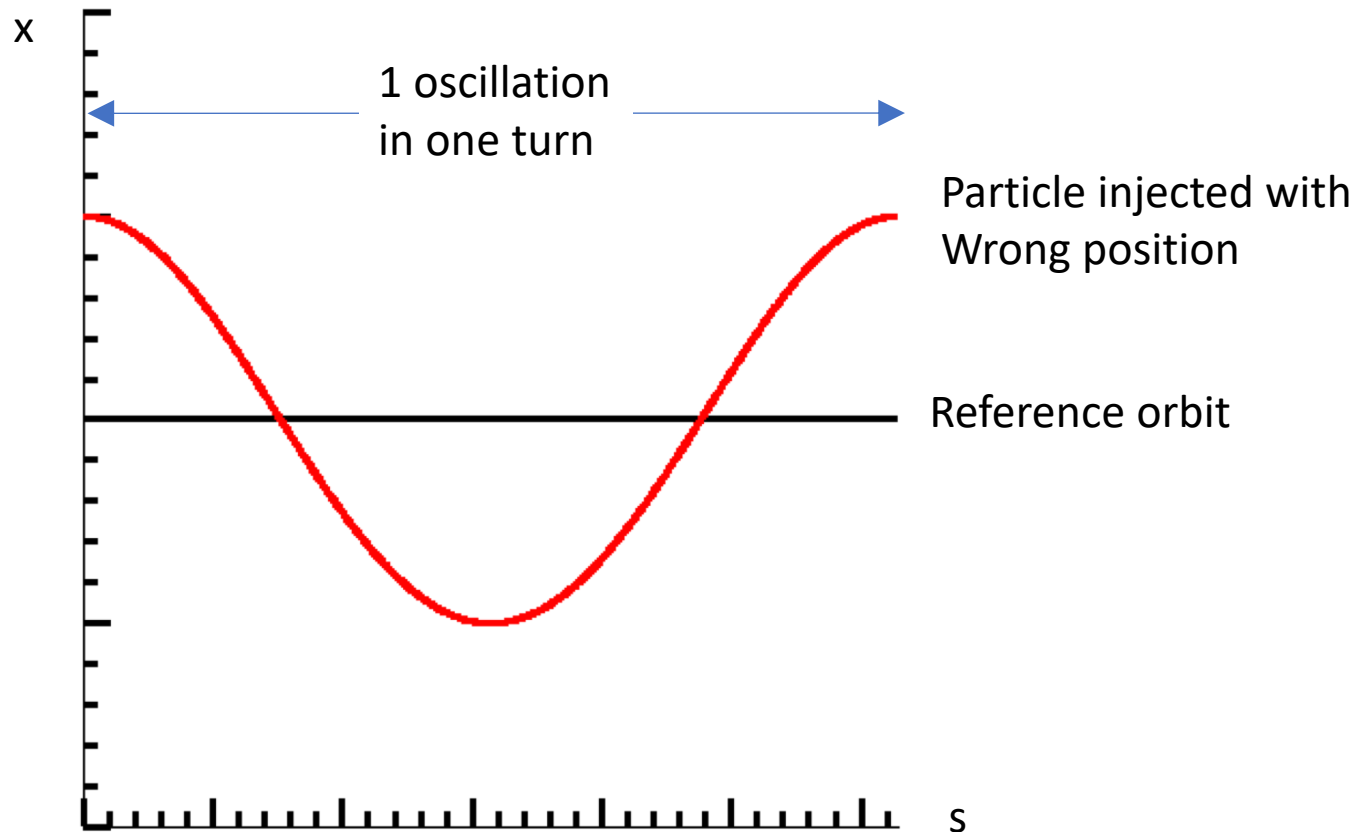
# Problem: a particle not in the correct position



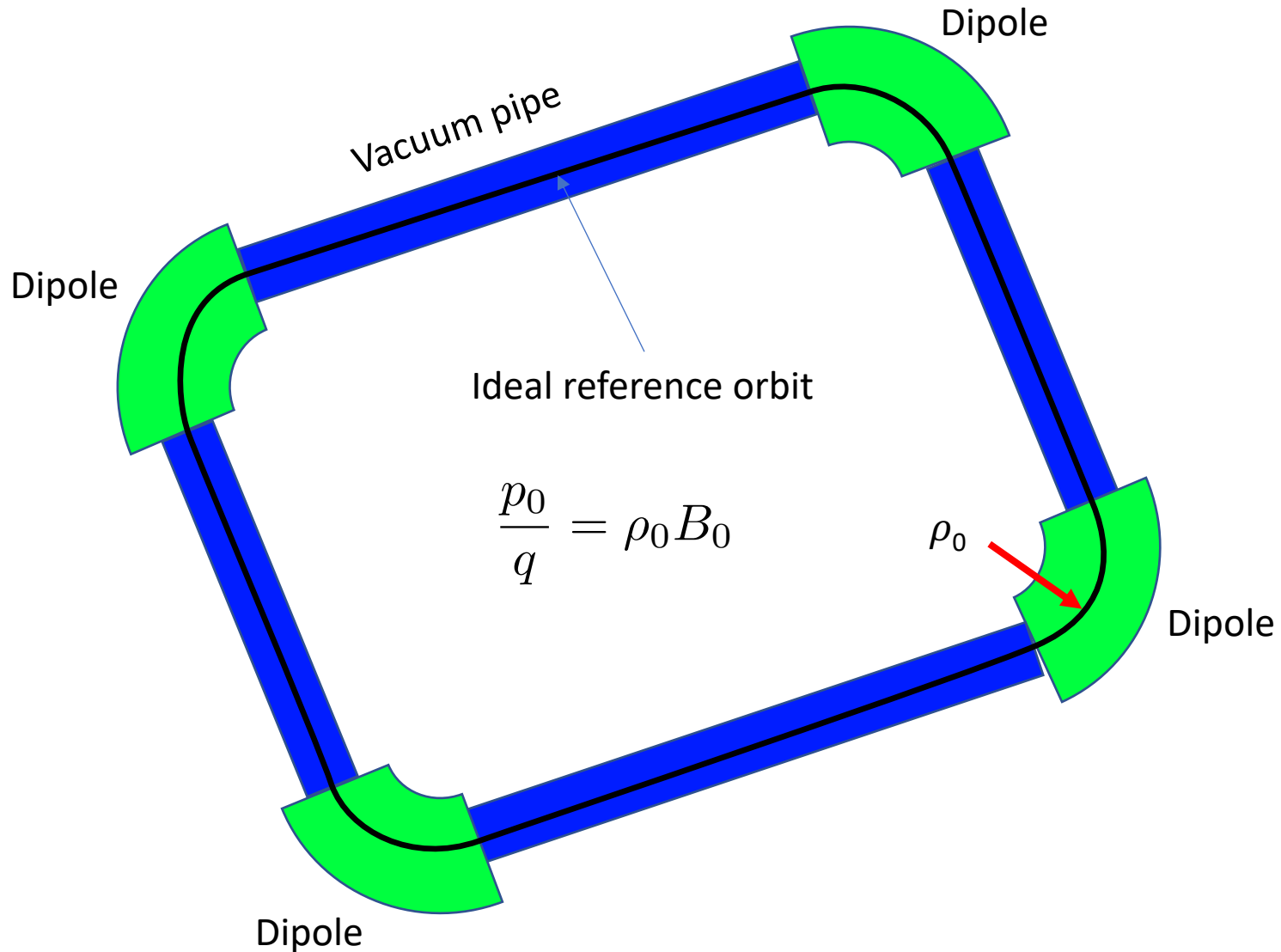
# From the reference orbit



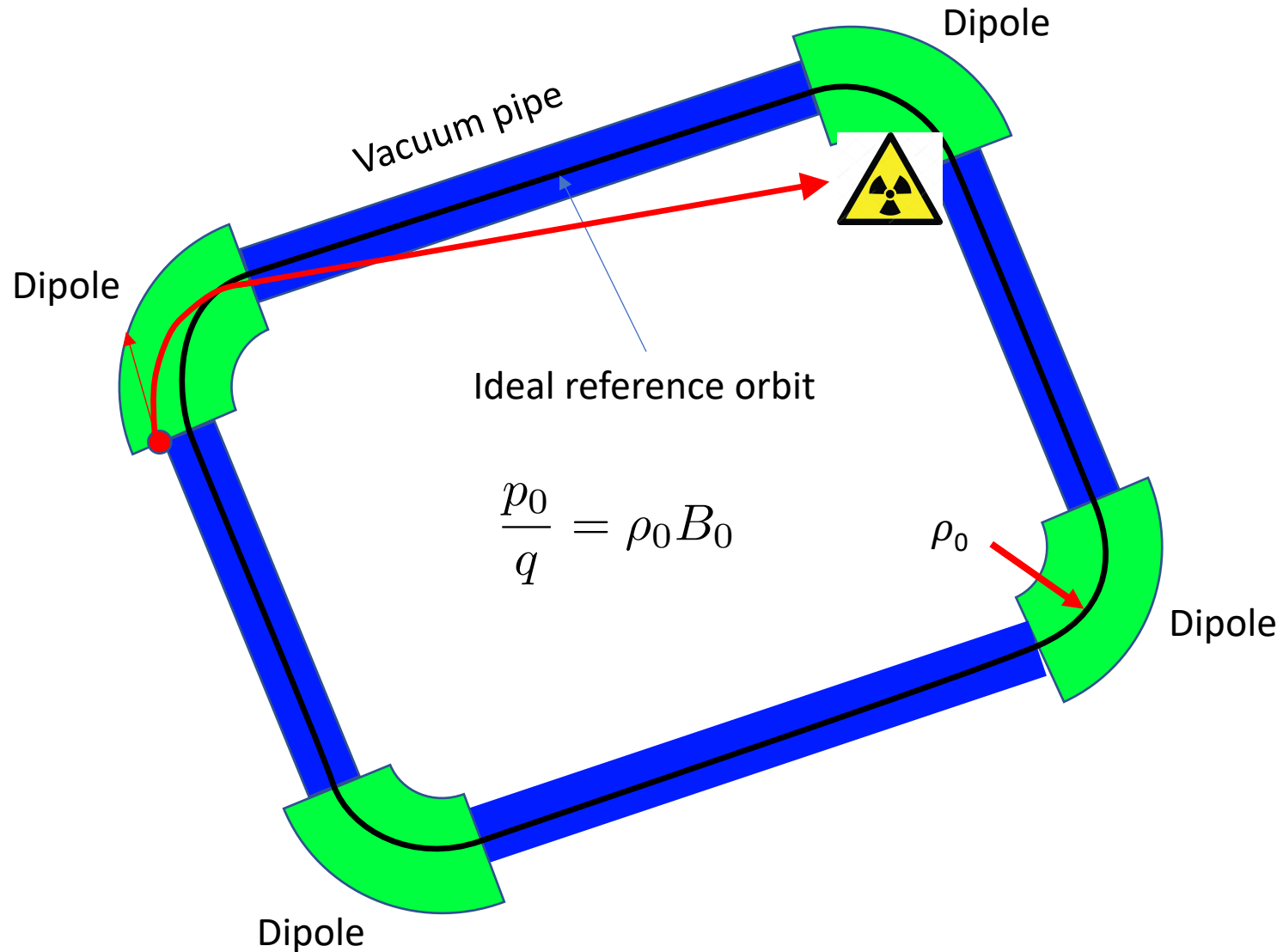
# From the reference orbit



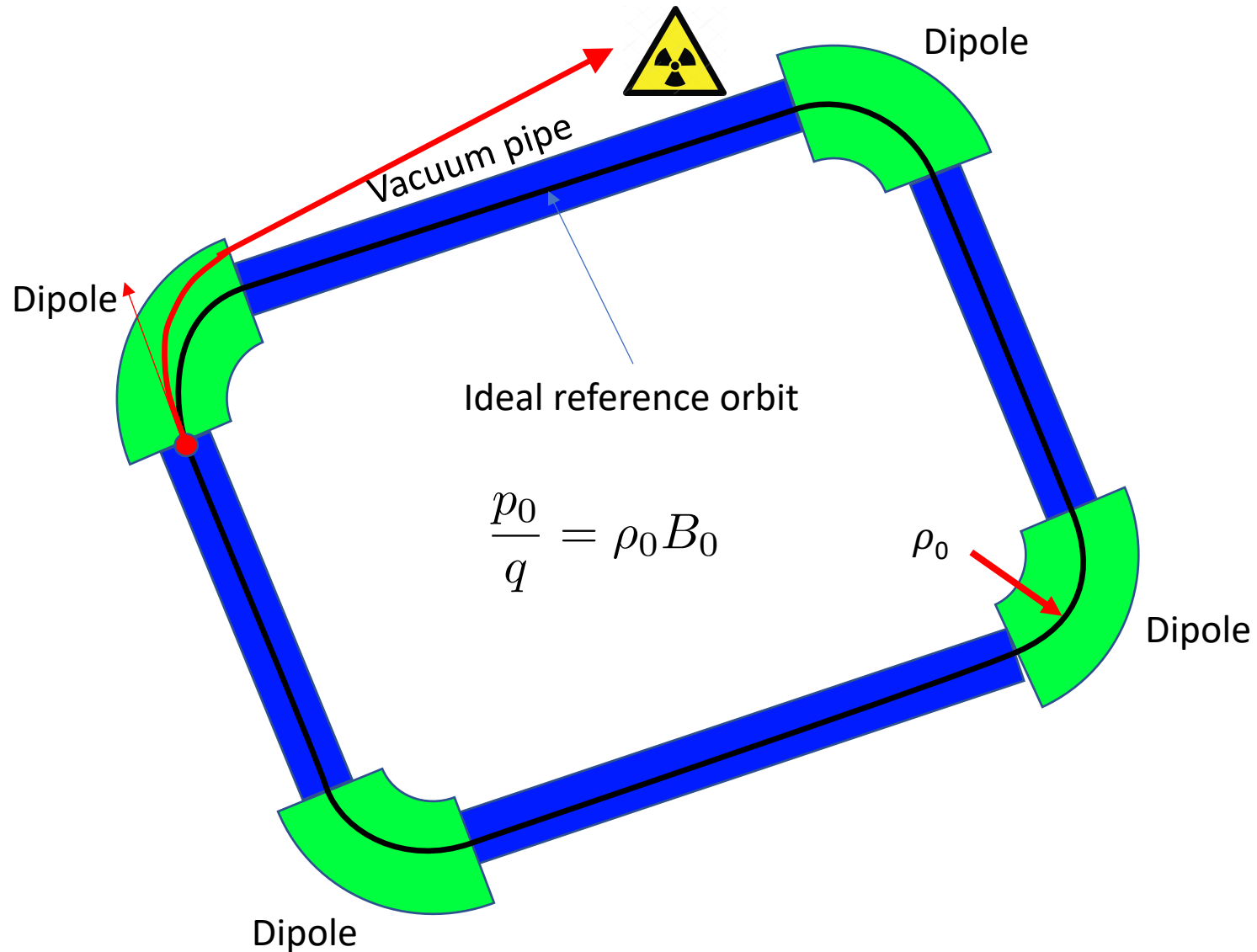
# A Ring with drifts



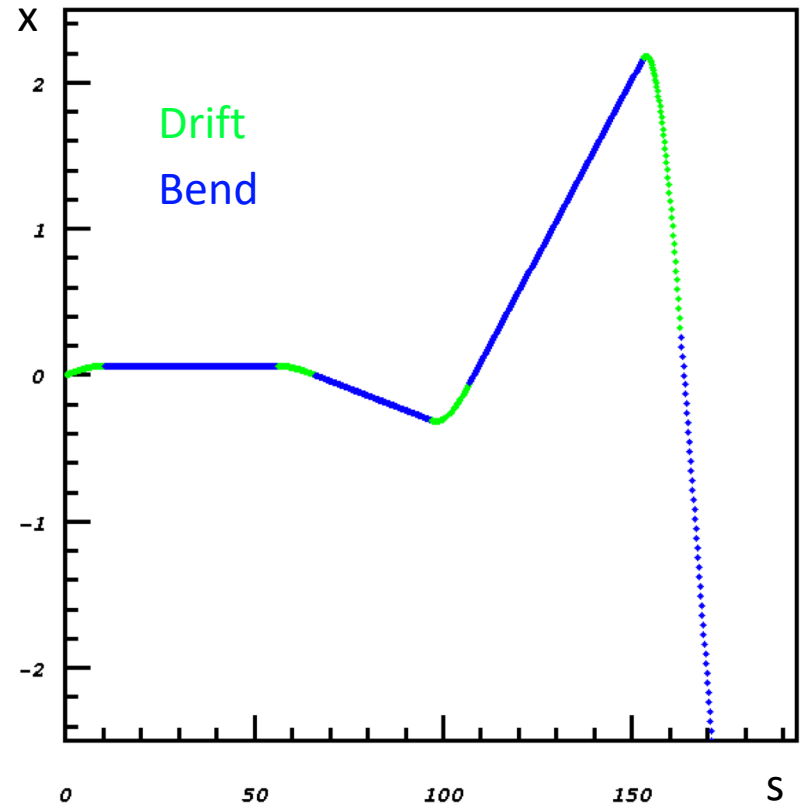
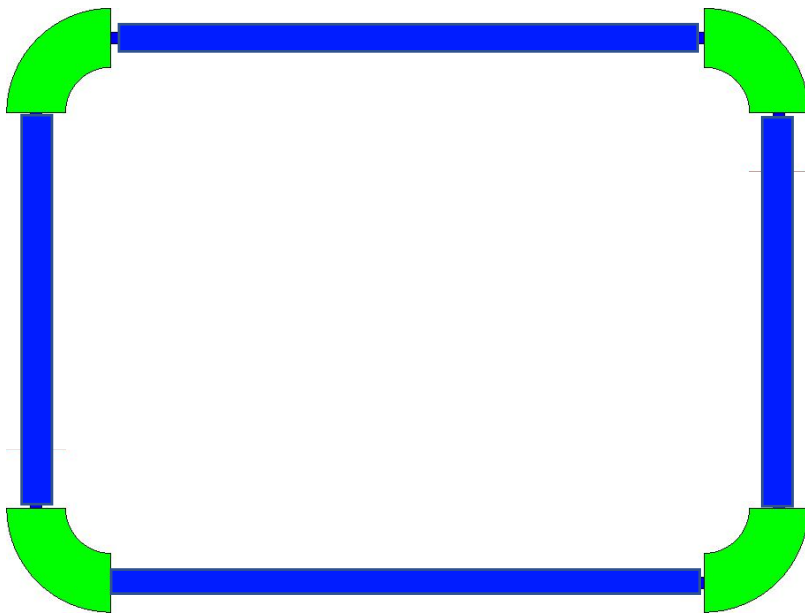
# Problems: particle in wrong position



# Problems: particle with wrong energy



# From the reference frame of the reference orbit

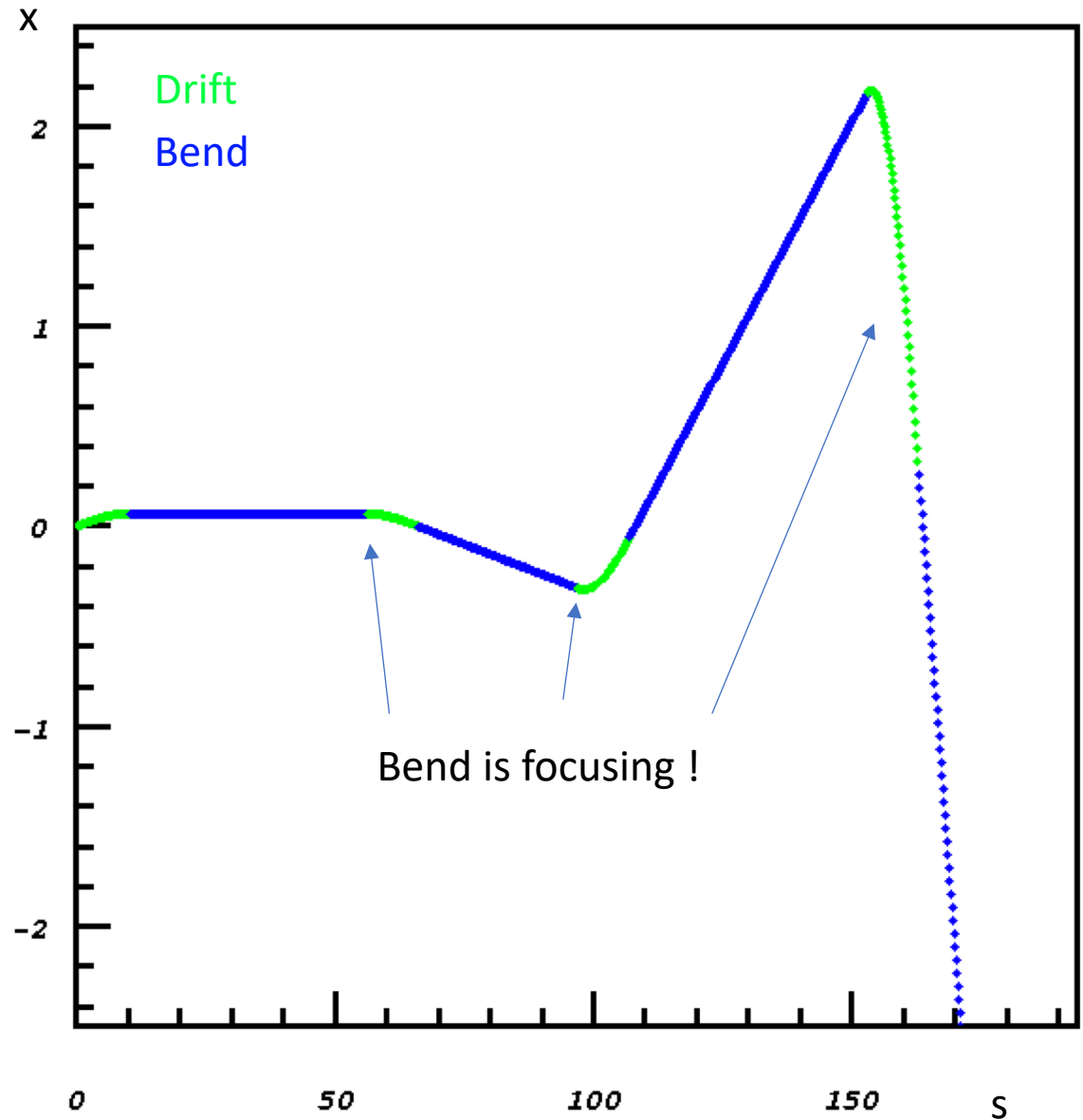


Motion unstable !!

# Observation

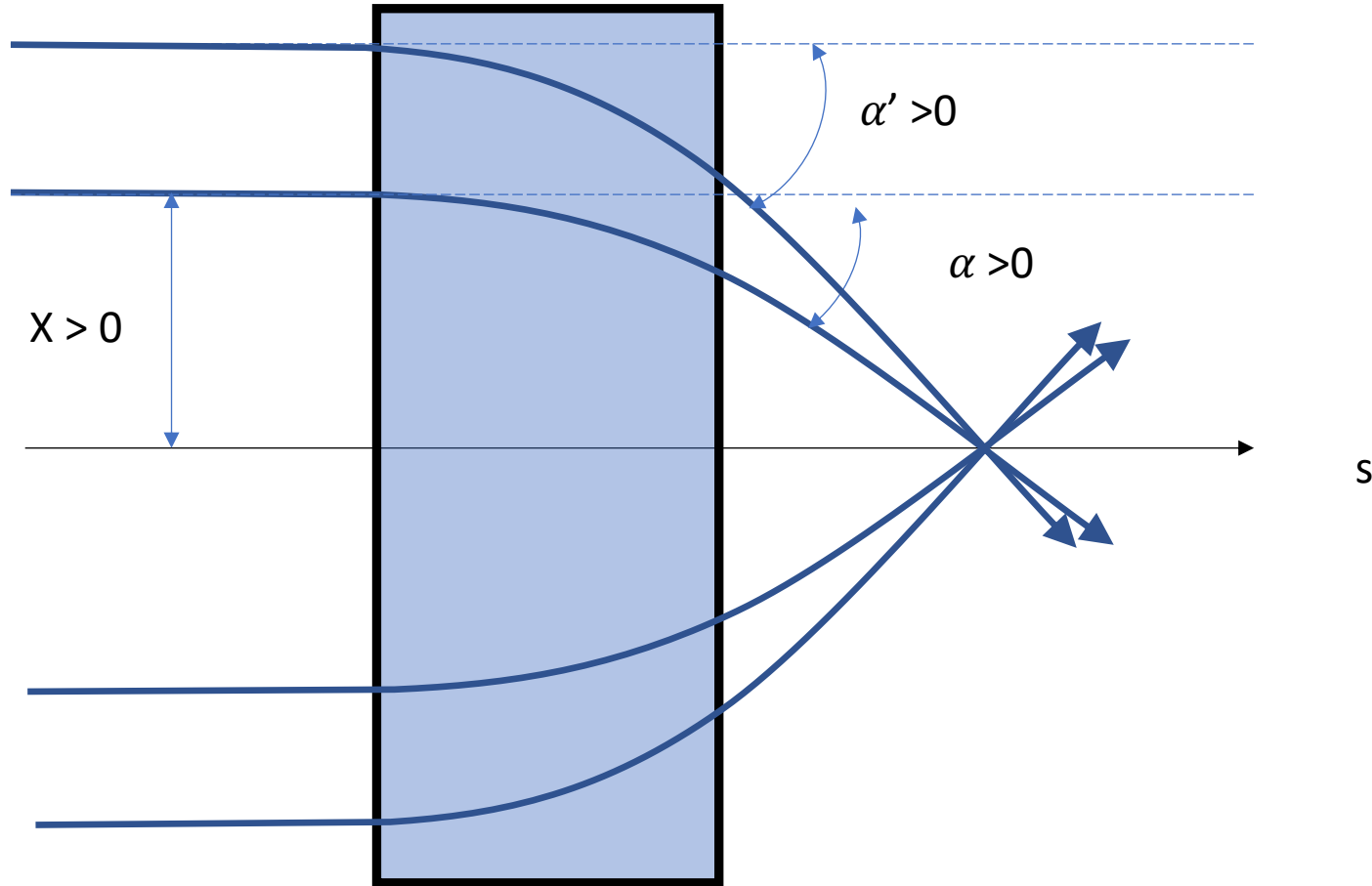
There is an  
“over-focusing”

Therefore transverse  
motion is unstable  
only the design particle  
is “accepted”

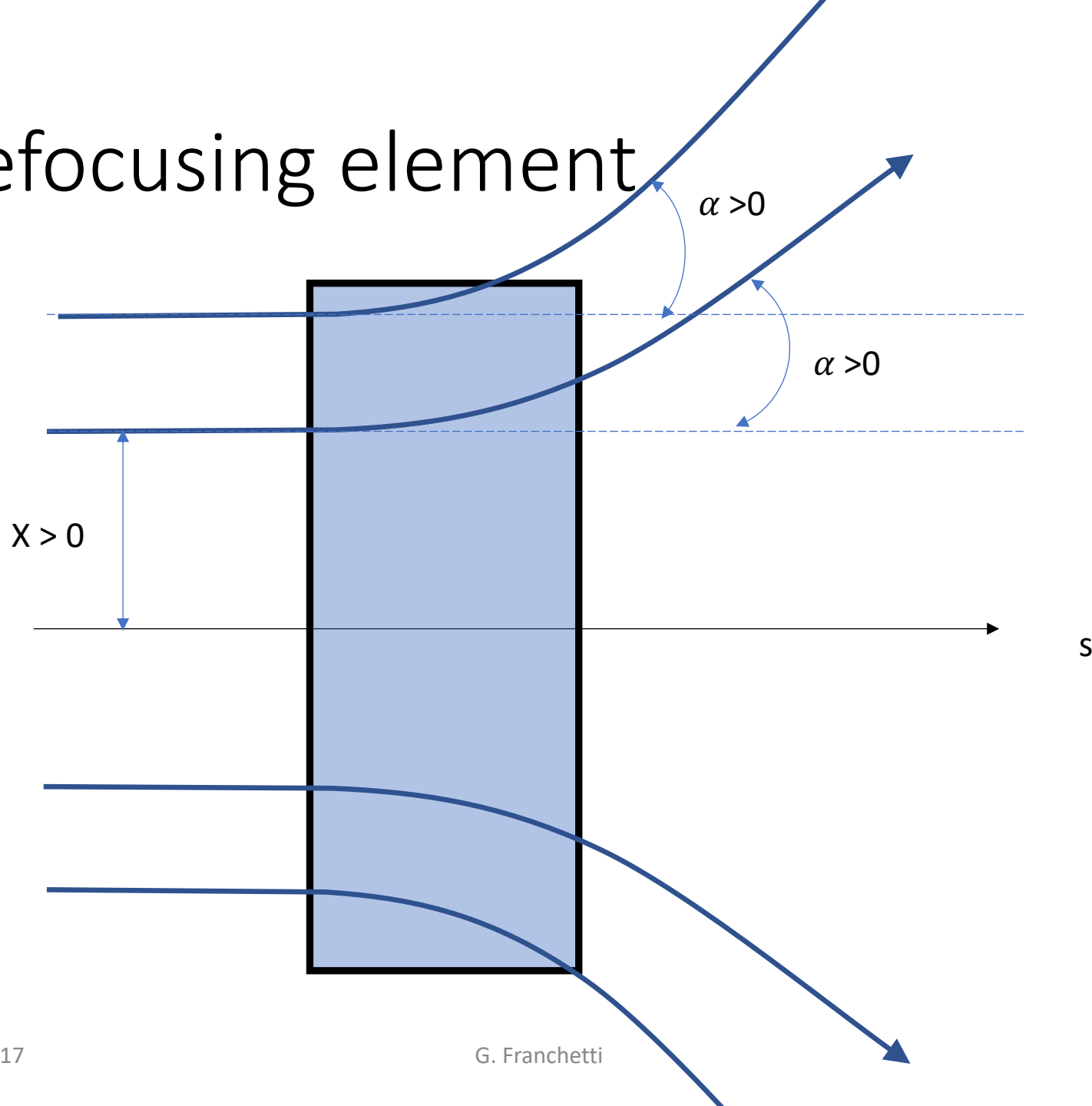




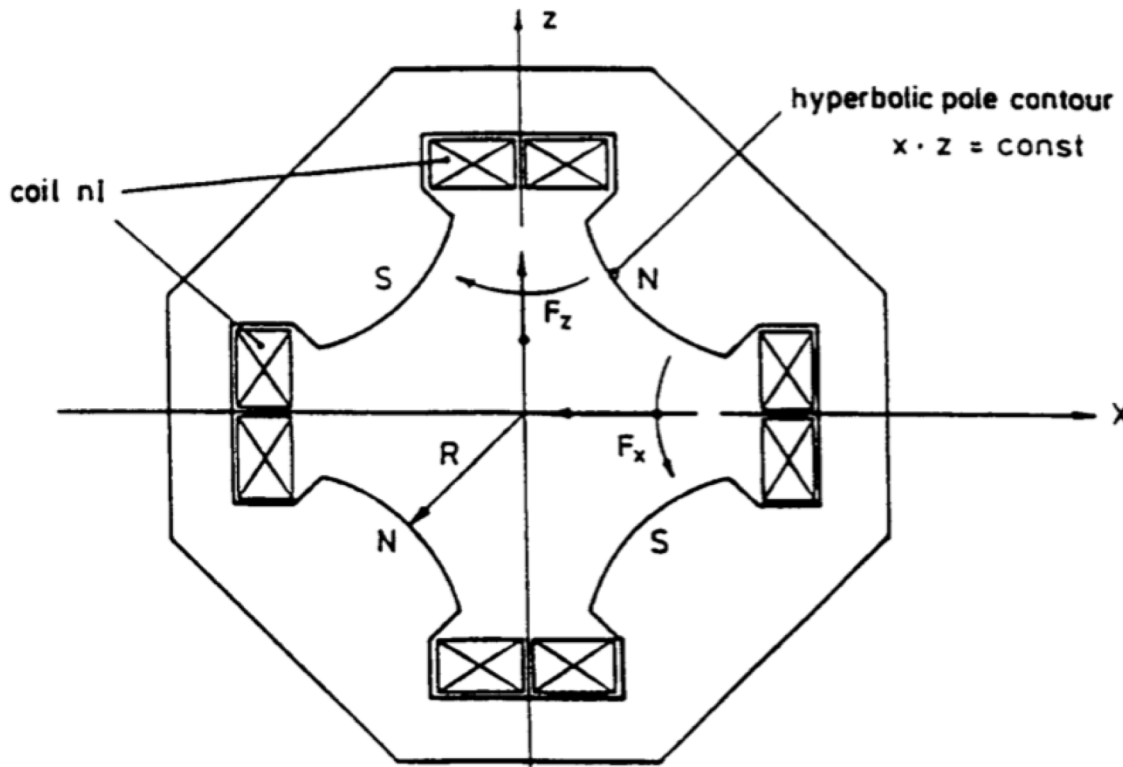
# Focusing element



# Defocusing element



# Quadrupoles



$$B_x = \alpha y$$

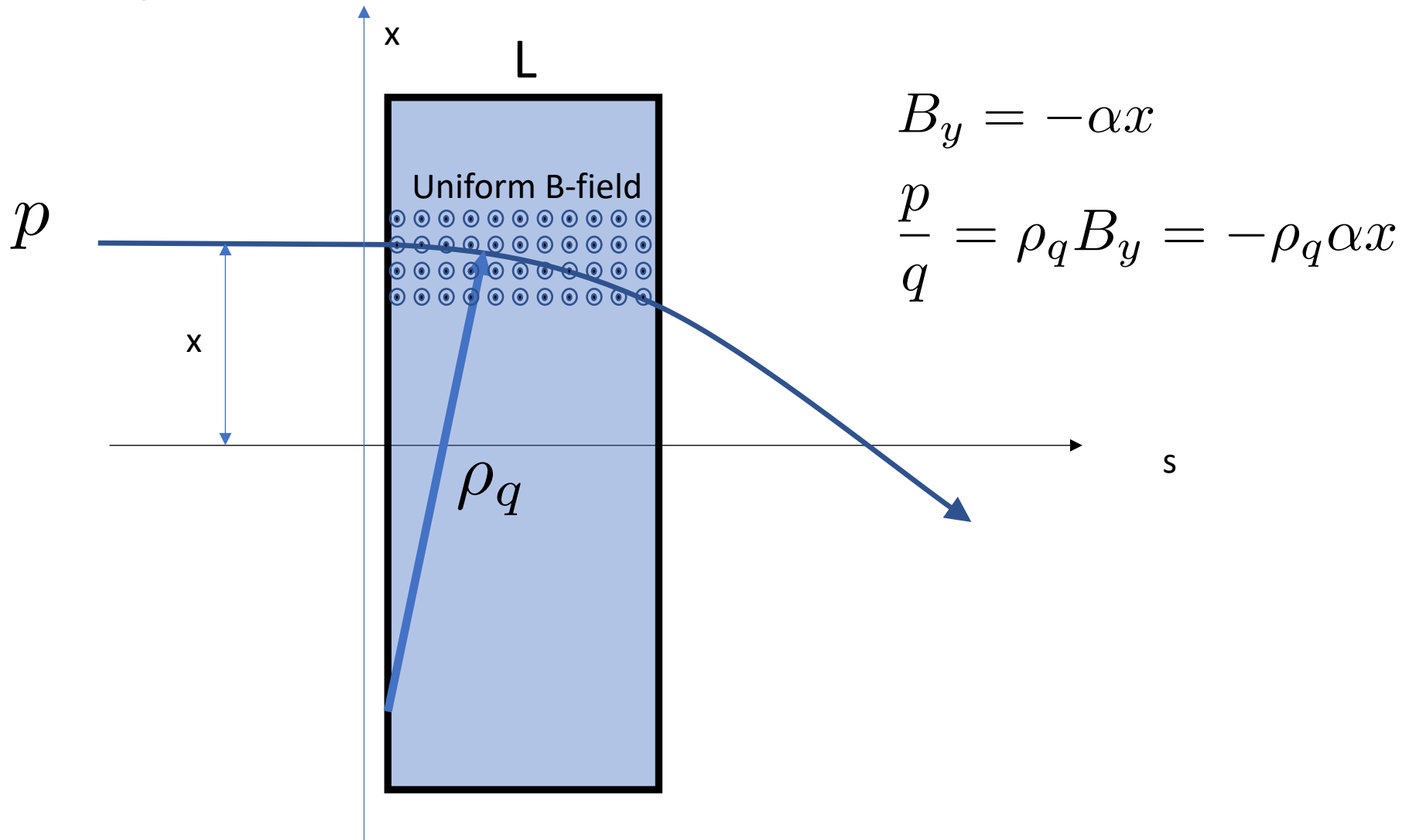
$$B_y = -\alpha x$$



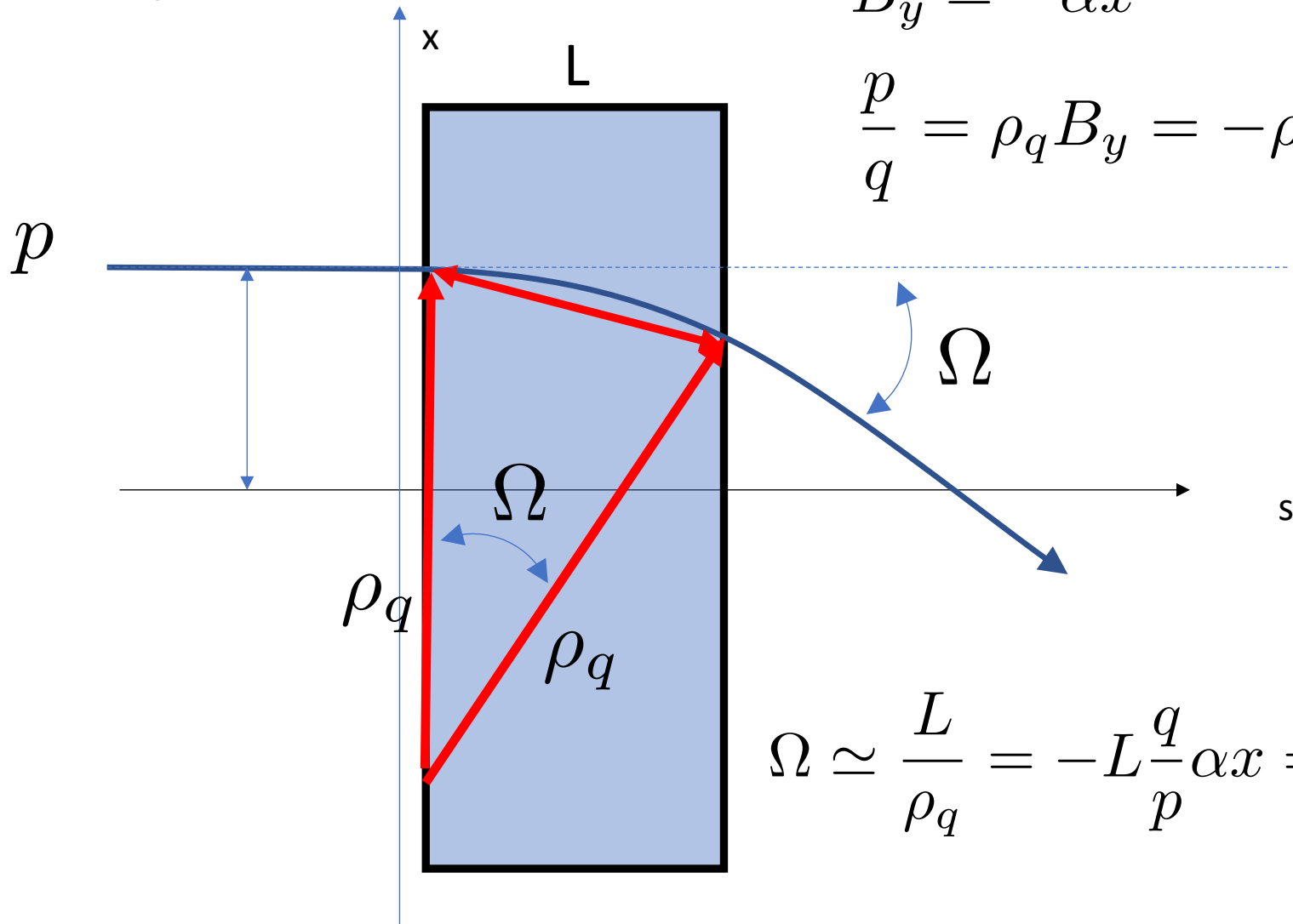
It is set by the  
Current on the coils

$$B_z \simeq 0$$

# Dynamics



# Dynamics

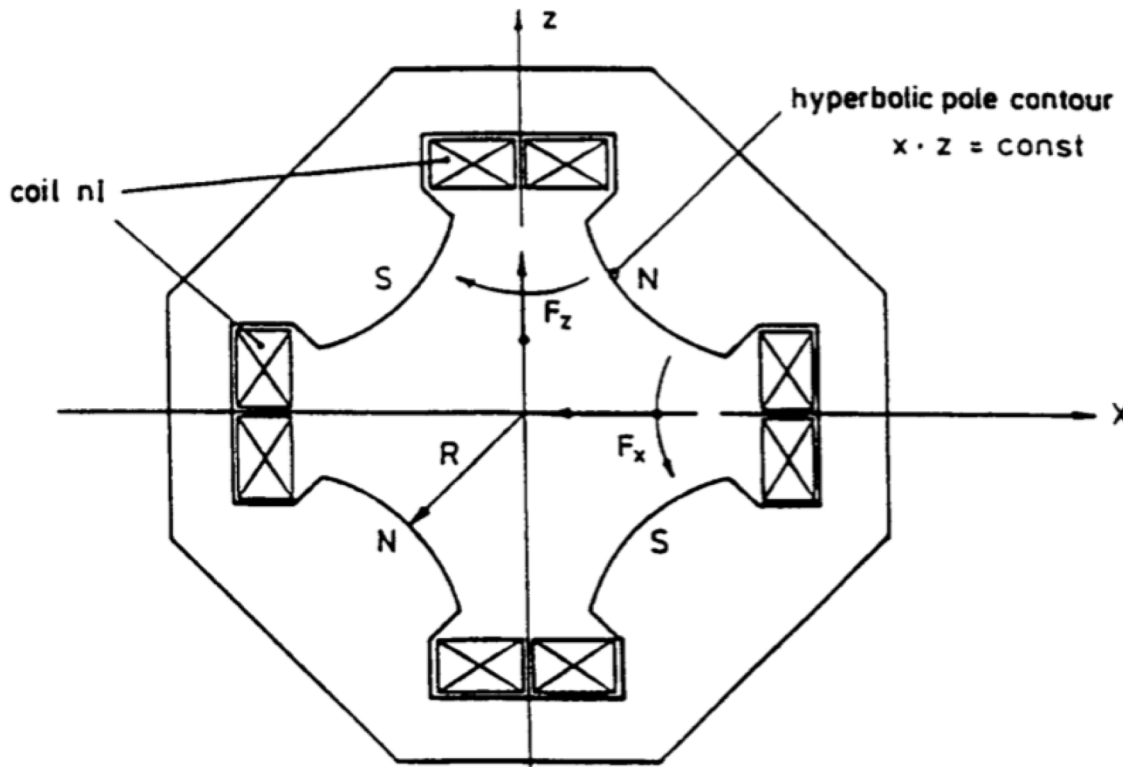


$$B_y = -\alpha x$$

$$\frac{p}{q} = \rho_q B_y = -\rho_q \alpha x$$

$$\Omega \simeq \frac{L}{\rho_q} = -L \frac{q}{p} \alpha x = -L \frac{\alpha}{B \rho} x$$

# Quadrupoles



$$B_x = \alpha y$$

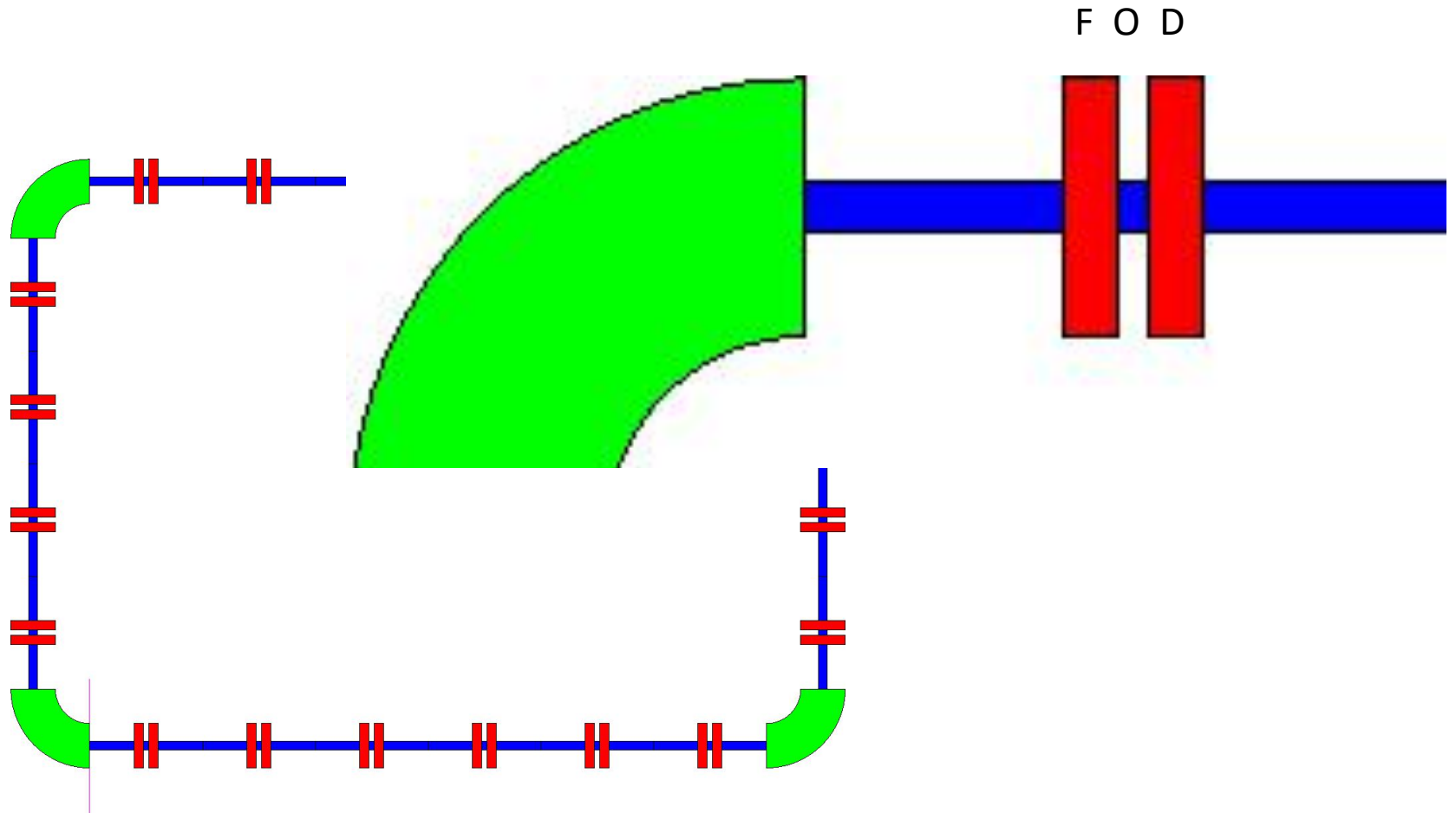
$$B_y = -\alpha x$$

The bending angle depends on the quantity

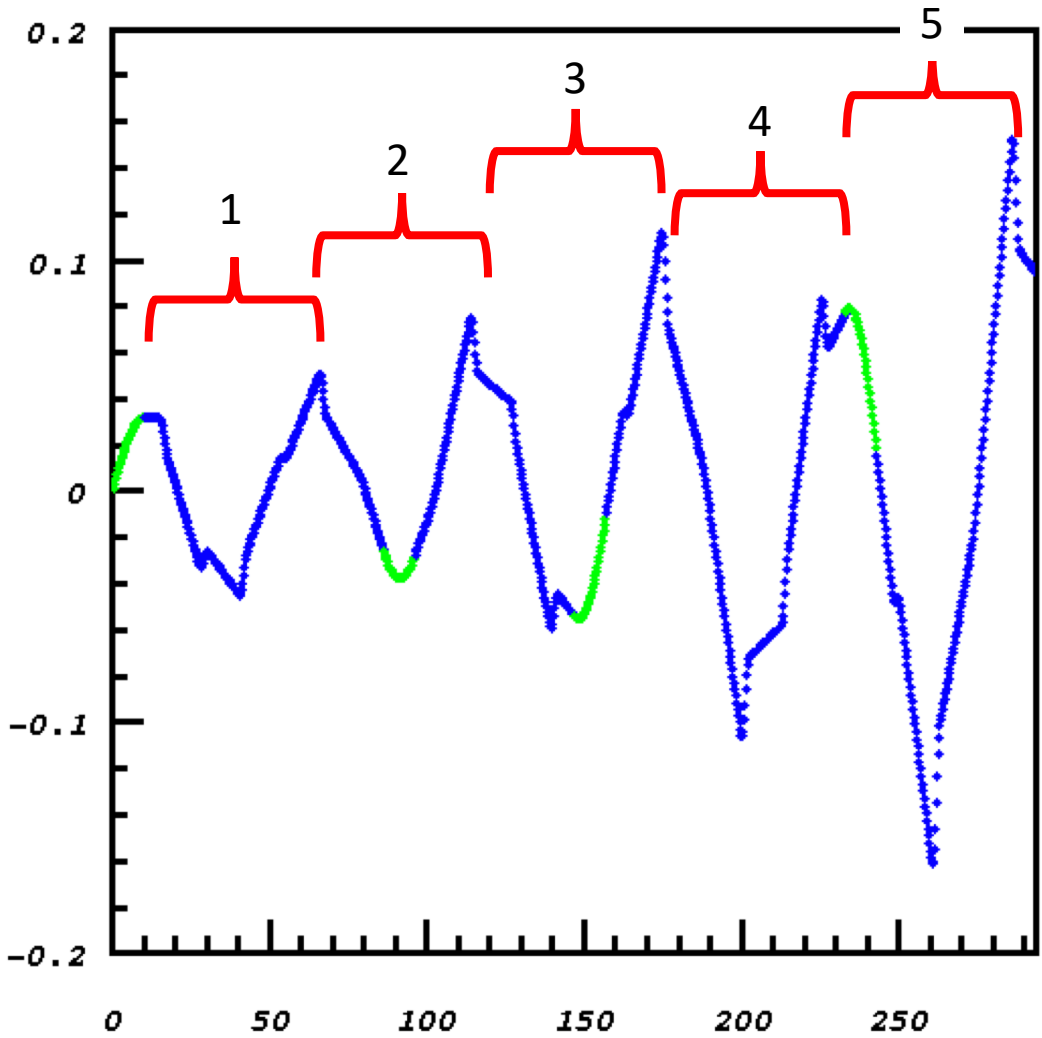
$$k = \frac{\alpha}{B\rho}$$

If "k" is constant  $\rightarrow$   
particles are focused  
In the same way

# FODO structure



# Stable dynamics



5 oscillations + something

TUNE= number of oscillations

$$Q_x = 5.15$$



S10 U09 TH/HTP 40AR18+ 300.10 schnell\_1H4 S02BE1 Expert

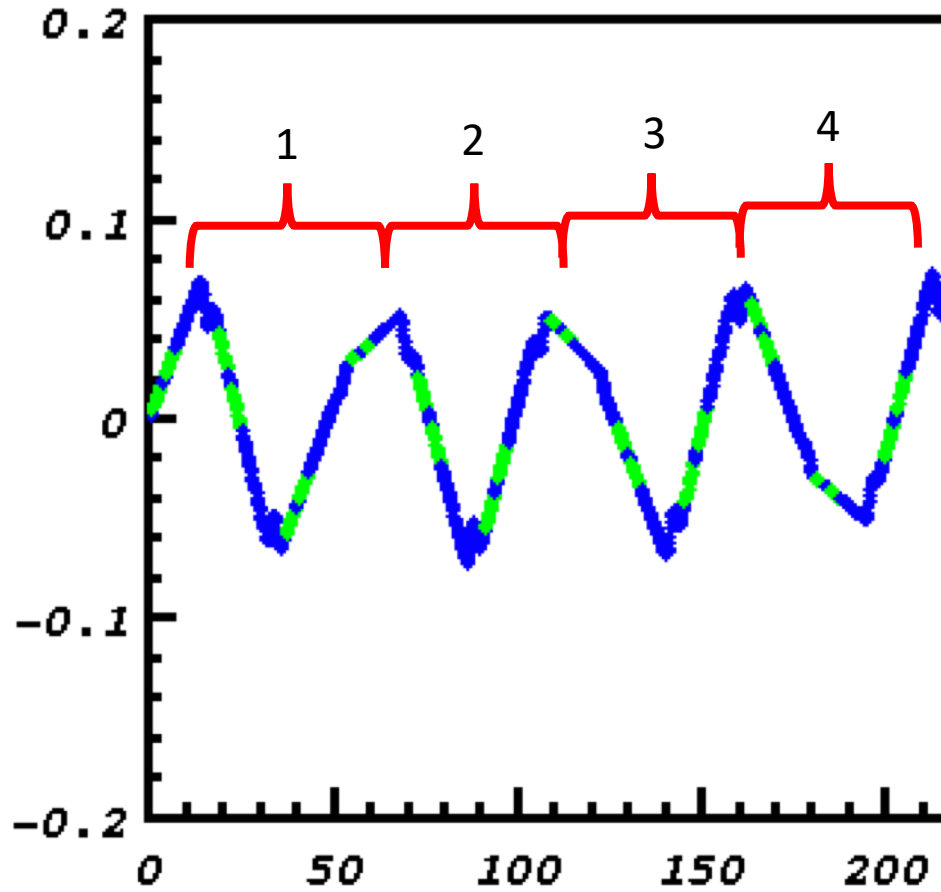
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|-----------------------------|-----------------------------|-------------------------|
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| Frequenz : 854.902          | Frequenz : 3620.049         | U-RpE [kU]: 20.0        |
| Ist-Frequ.: 854.902         |                             | U-Flat [kU]: 10.0       |
| Taupunkt : 0.0              | sigma(-1..1): 0.0           | rad.Pos.Ext.: -2.5      |
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| QU : 3.27                   | QU : 3.27                   | Harmonische : 4.0       |
| ChopFenst.: 0.0             | dTK7BC1Lmrad: 0.0           | dS12MU3Inrad: -1.0      |
| Bump.Flank: 150.0           | dTK7MU5 mrad: 0.0           | dS12ME1Inrad: 5.5       |
| Chop.Verz.: 35.0            | rad.Pos.Inj.: 0.0           |                         |
| dU-Ready : 10000            |                             | B-Punkt[T/s]: 1.30831   |
| BumpAmplmm: 90.0            | t-Ramp[ms]: 64.0            | Teilchen /μA: 0.162E+07 |
|                             |                             | U-Inj [kU]: 2.0         |
| AnzInjekt.: 1               | MMIKühlZ.ms : 1000.0        |                         |
| IKüBumpXmm: 0.0             |                             | e-Kühler 0/1: 0         |
| IKüBumpX': 0.0              |                             | Kü-dp/p-Inj.: 0.0       |
| IKüBumpYmm: 0.0             | InjektPlateau: 2.0          | Kühlerf.Inj.: 600.0     |
| IKüBumpY': 0.0              |                             | Kühlerst.[A]: 0.3       |
|                             |                             | Wartezeit[ms]: 0.0      |
| Sollb. Ampl.[mm]: 5.5       | SollStg.Zeit[ms]: 32.0      |                         |
| Kickwinkel[mrad]: 6.2       | SollFallZeit[ms]: 32.0      |                         |
| Kickstart(0-360): 76.5      | Extr.Teilchen/μA: 0.383E+06 |                         |
| Bypass [mrad]: 5.0          |                             |                         |
| HF-Trigger: 1-Gap, 2-Master | 2                           |                         |
| mit/ohne ESR-Synchron. 1/0: | 0                           |                         |

INIT  
PROTOKOLL  
TFS  
RESET  
SAVE  
RESTORE  
SOLL\_SOLL  
COPY  
EXIT

Eingaben an Geraete InitWertAnzeige alter Zustand

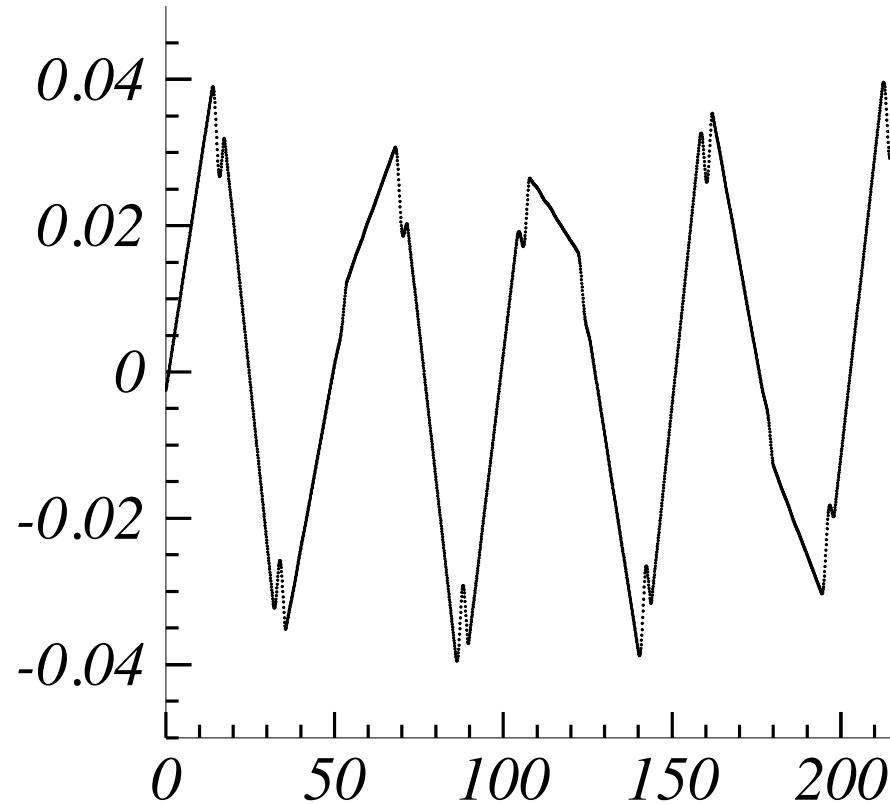
4.34 oscillations  
4.27 oscillations

$$Q_x = 4.34$$



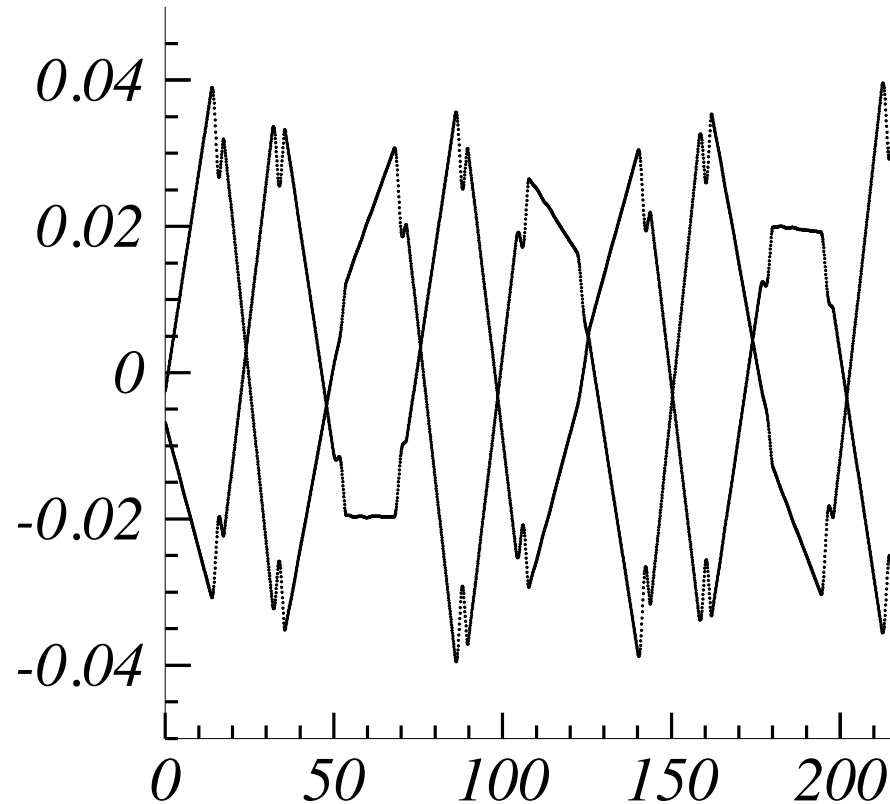
# Envelope

One particle  
With a certain  
Transverse energy  
E



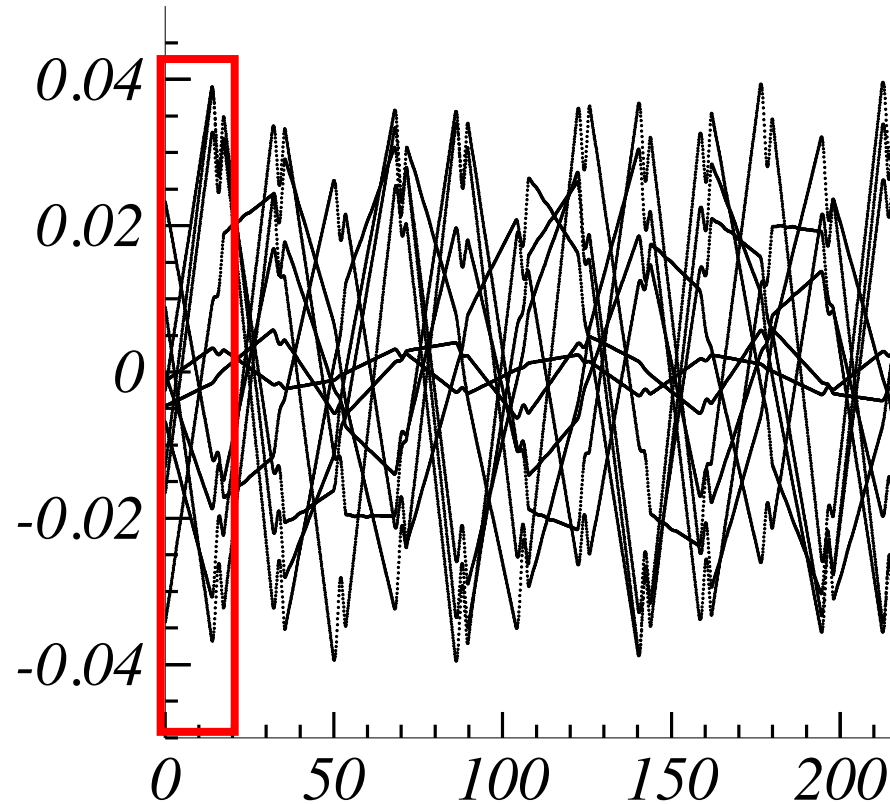
# Envelope

2 particles  
With a certain  
Transverse energy  
E



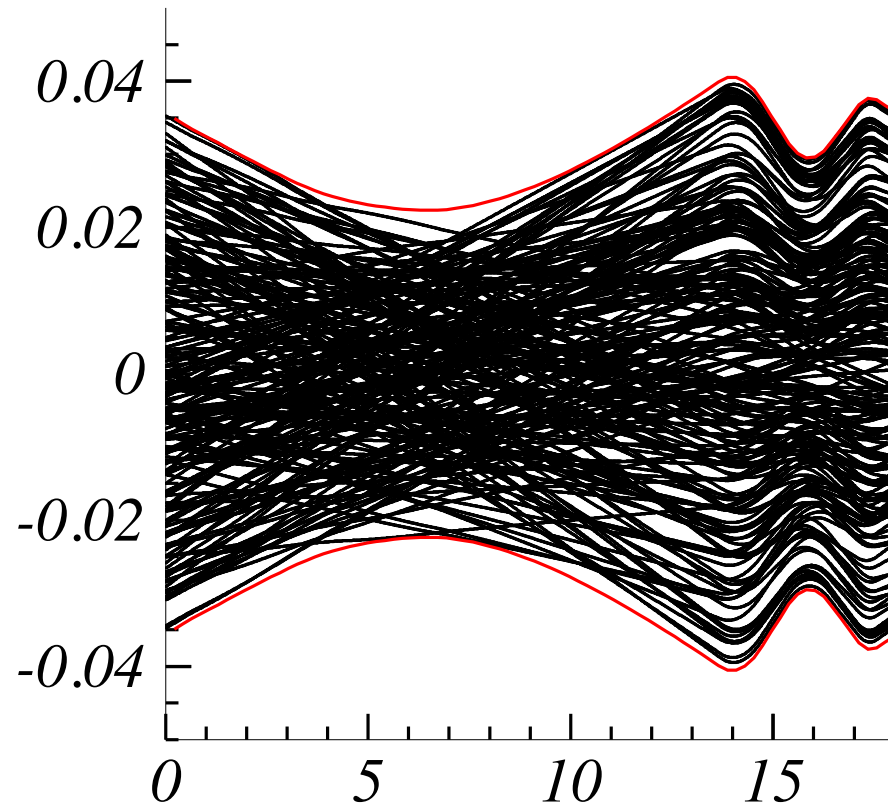
# Envelope

10 particles  
With a certain  
Transverse energy  
E



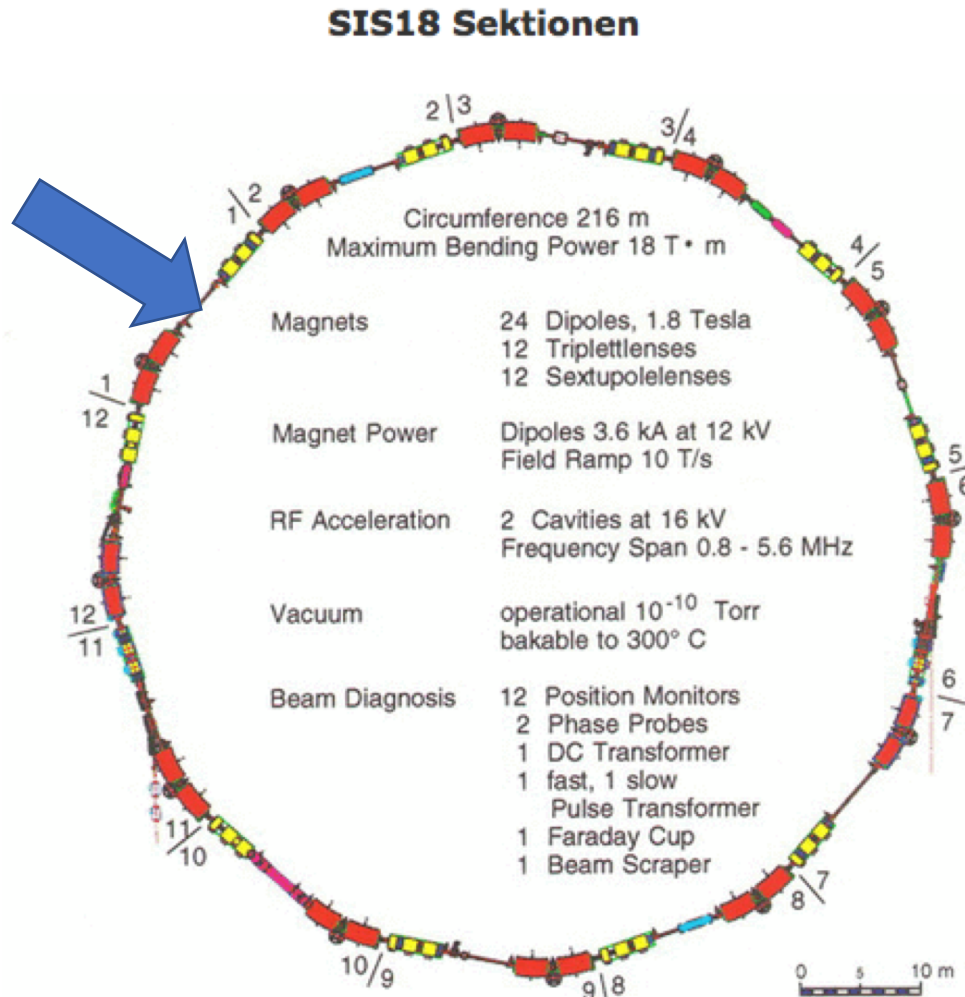
# Envelope

200 particles  
With a certain  
Transverse energy  
 $E$

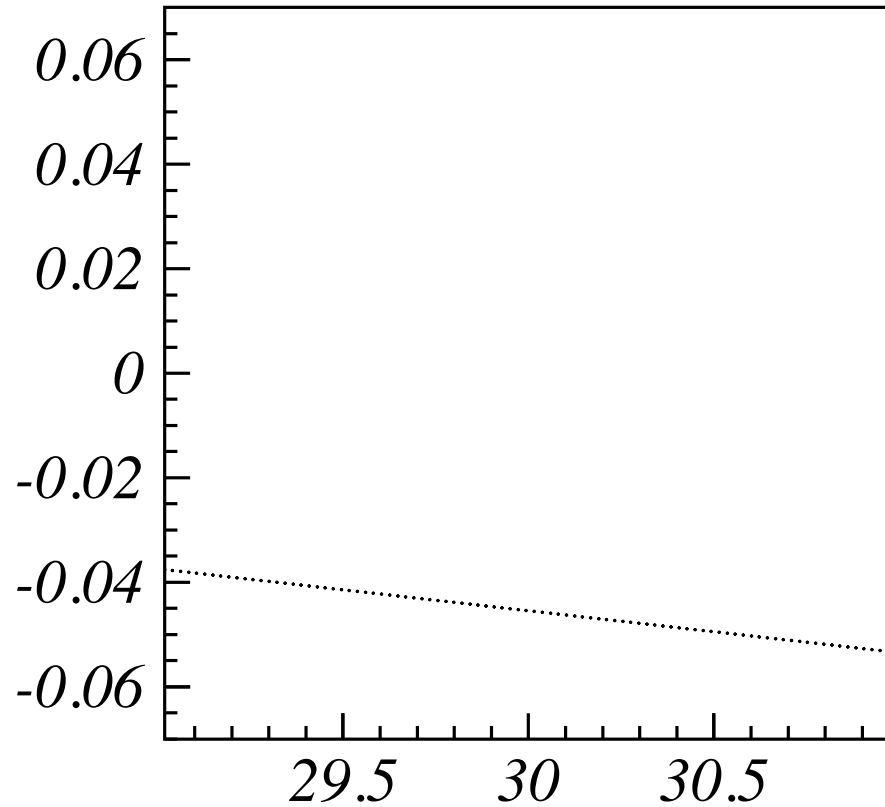


# Transverse particle energy ?

Look at trajectories  
turn after turn  
At length  
30 meters

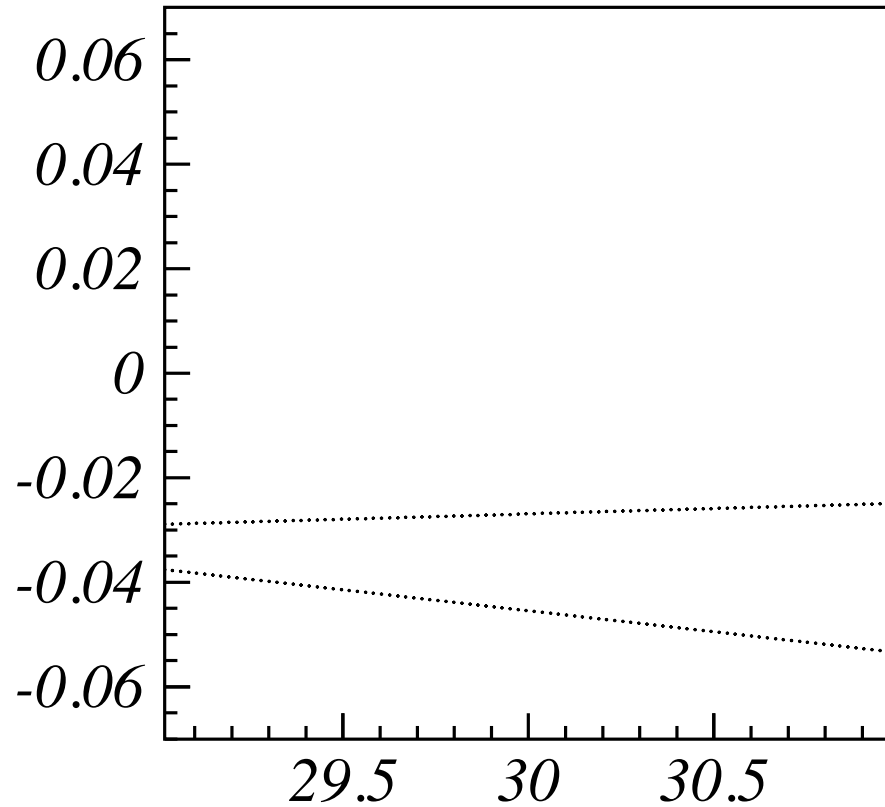


# Particle trace first turn

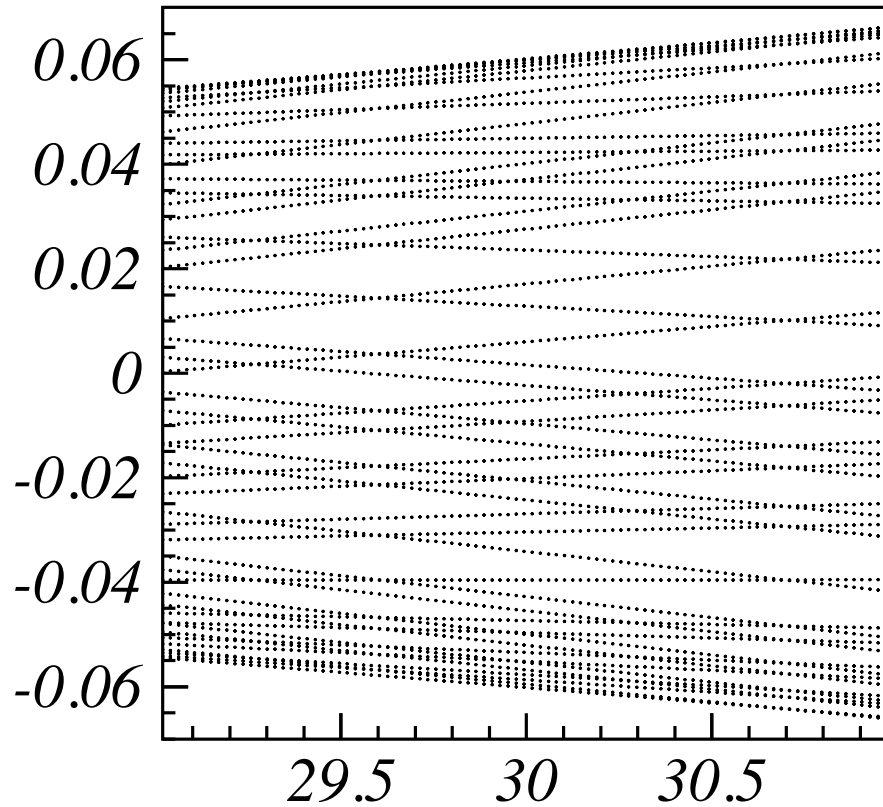




# Particle trace second turn



# Particle trace 50 turns

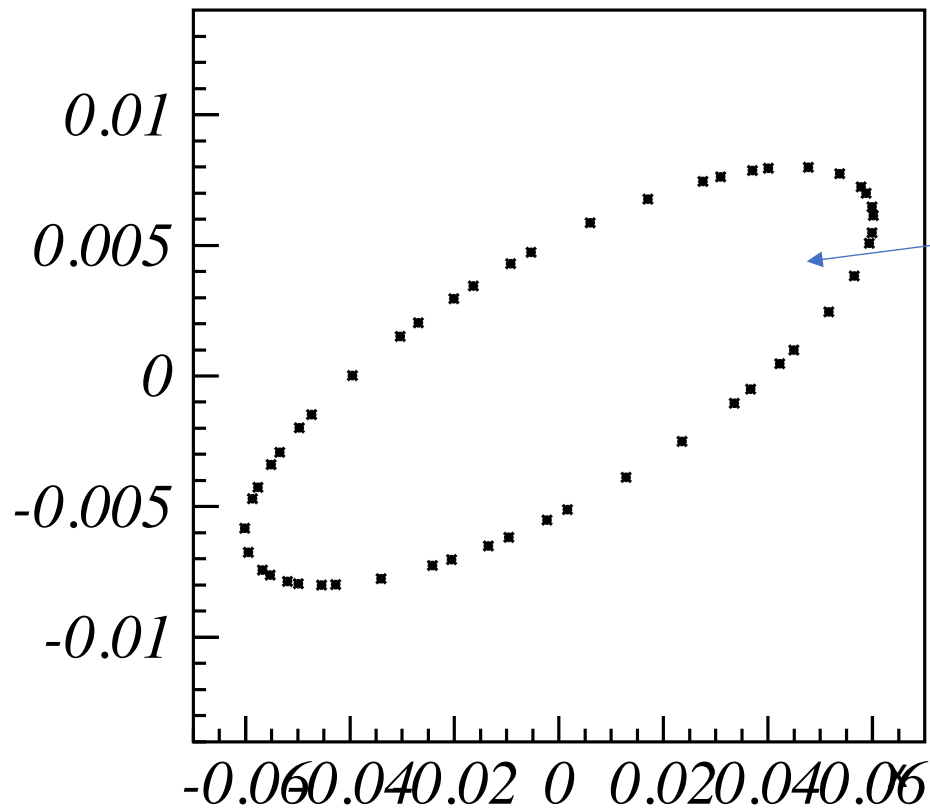


It is the same  
particle !

The energy is  
The same

# Plot $x$ and $x'$

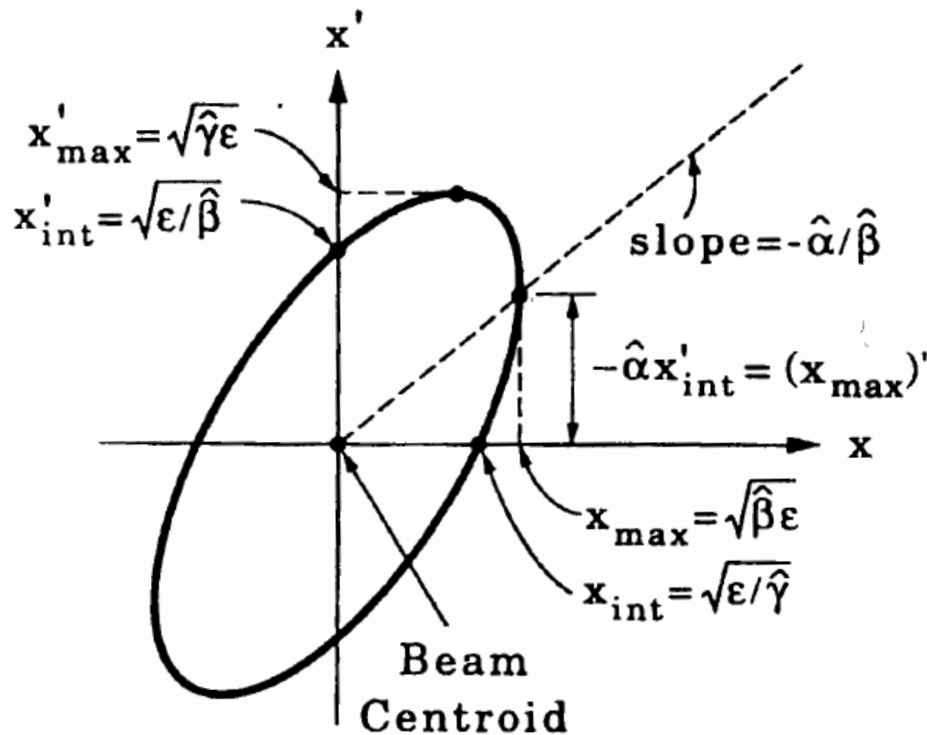
$x'$



This is the  
Same particle  
Turn after turn

The ellipse  
means that the  
energy is  
preserved

# Feature of the ellipse



$$\epsilon_x = \gamma_x x^2 + 2\alpha_x x x' + \beta_x x'^2$$

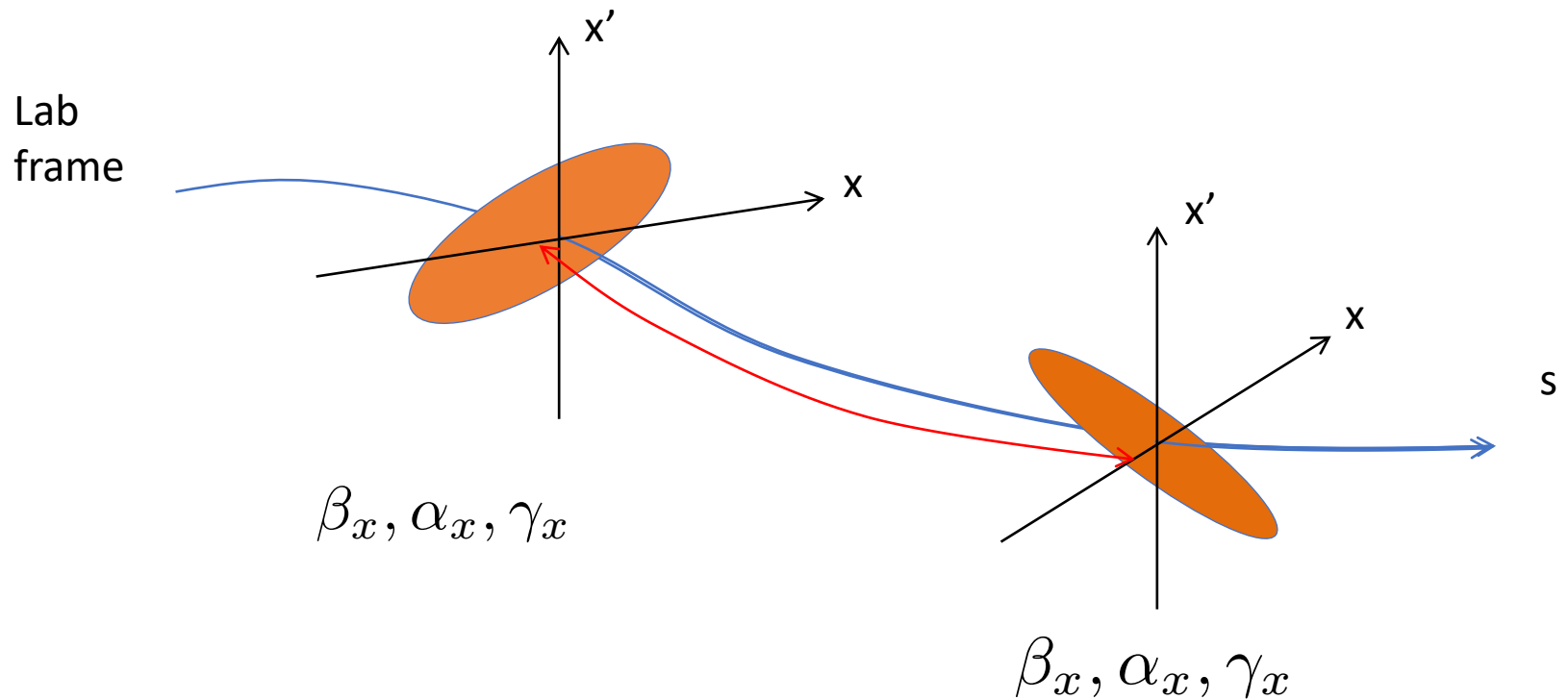
The orientation of the ellipse  
Depends on the parameters

$$\beta_x, \alpha_x, \gamma_x$$

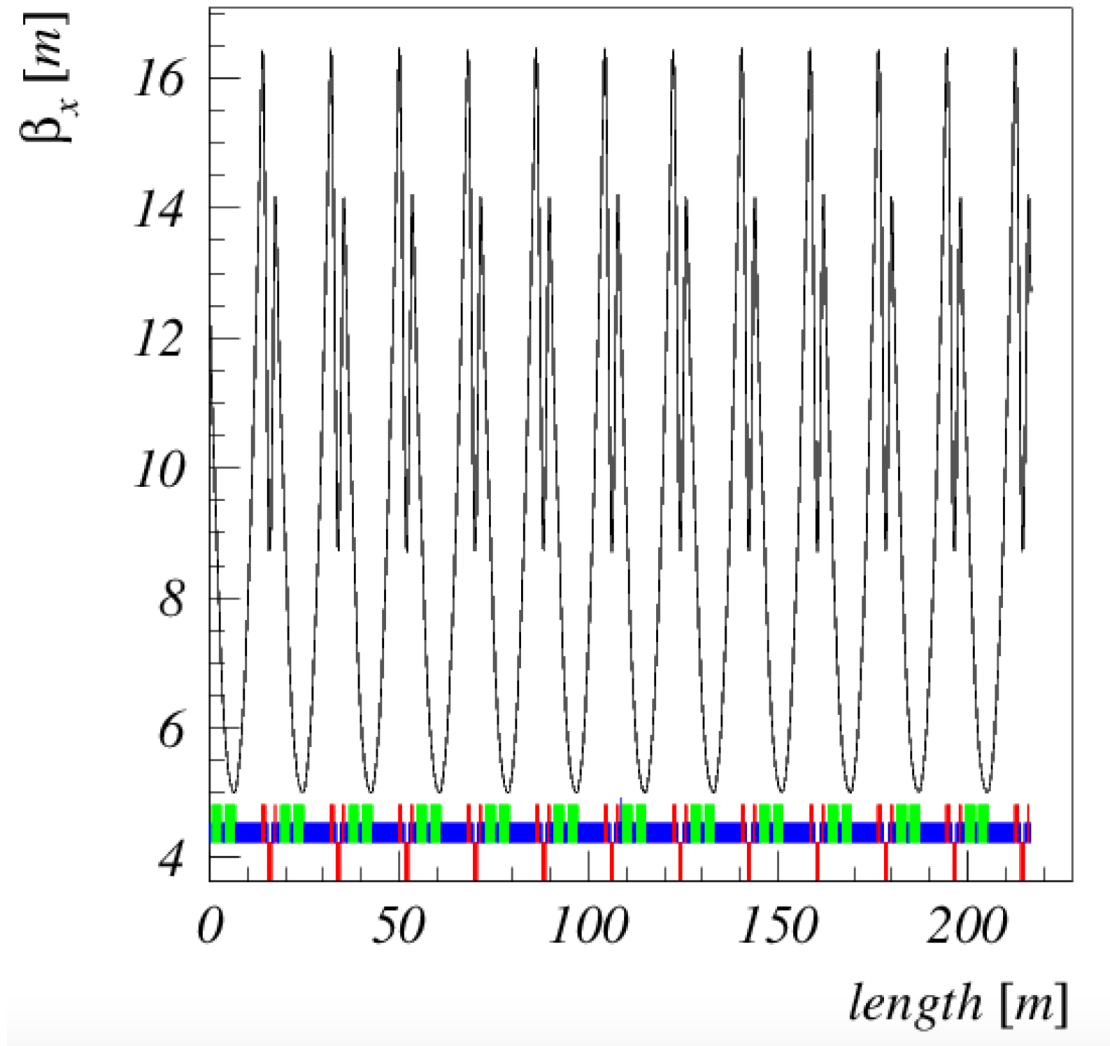
$$\gamma_x \beta_x - \alpha_x^2 = 1$$

$$\epsilon_x = \frac{\text{Area of the ellipse}}{\pi}$$

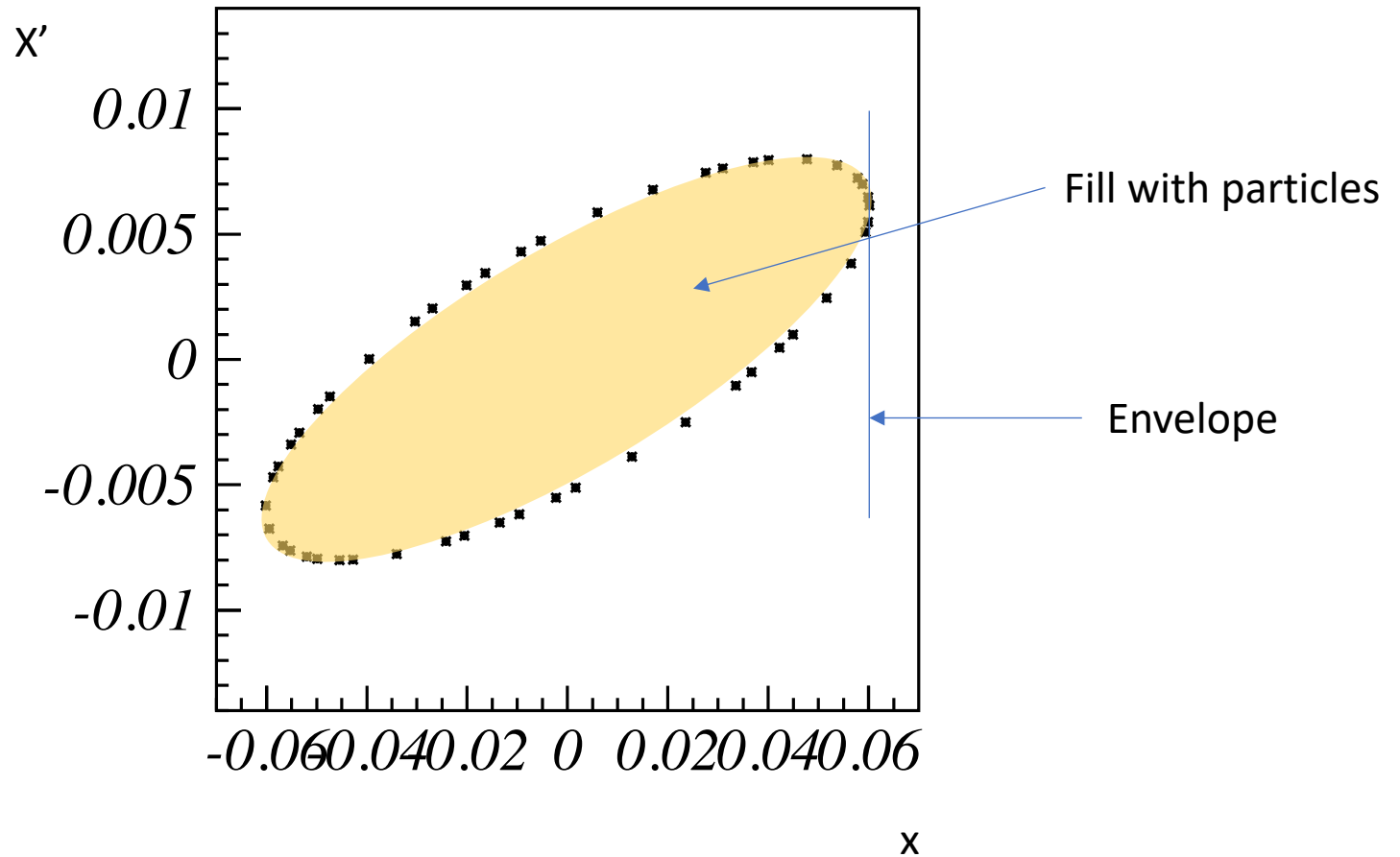
# The ellipse rotate and stretch



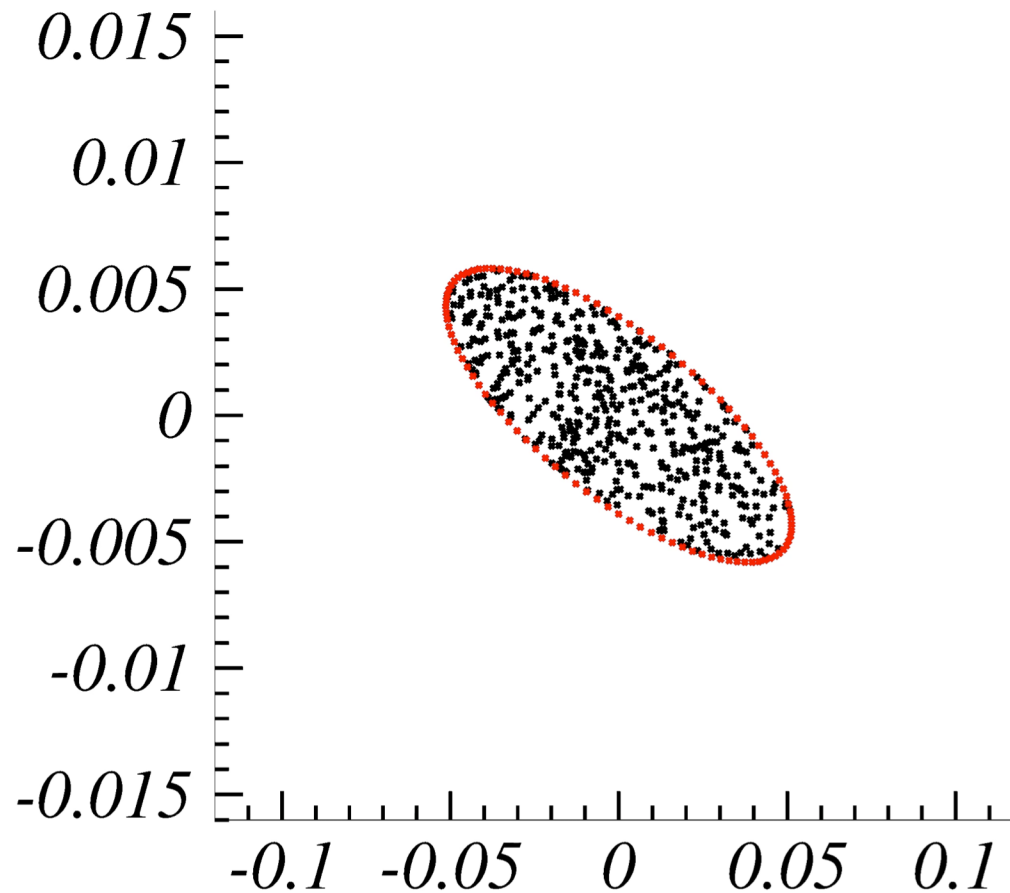
# Optical functions



# Injection of a beam: matched

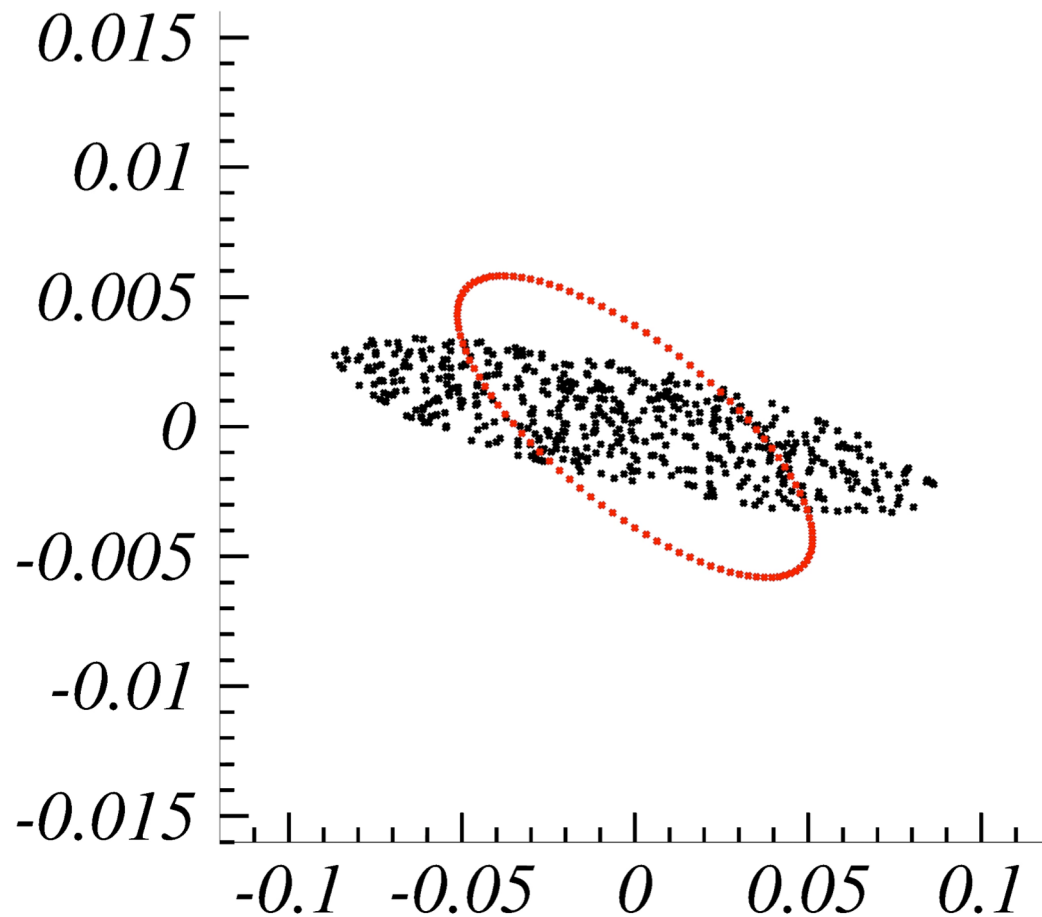


# Injection of a beam: matched

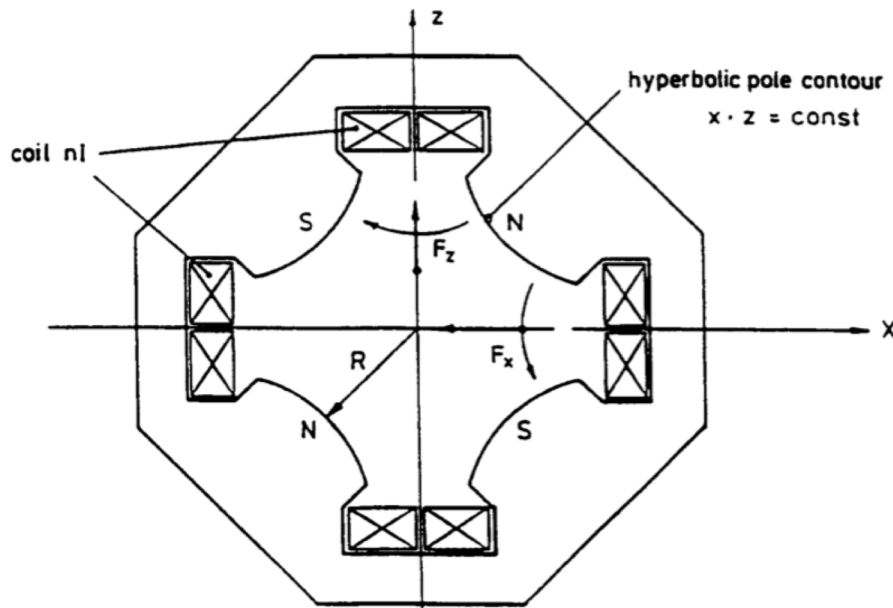




# Mismatched beam: consequences



# Second order effects



$$k = \frac{\alpha}{B\rho}$$

$$\frac{p}{q} = \rho B$$

$$\frac{p}{p_0} = \frac{B\rho}{B\rho_0}$$

$$k = k_0 \frac{p_0}{p}$$

The tune is  
Changed !!

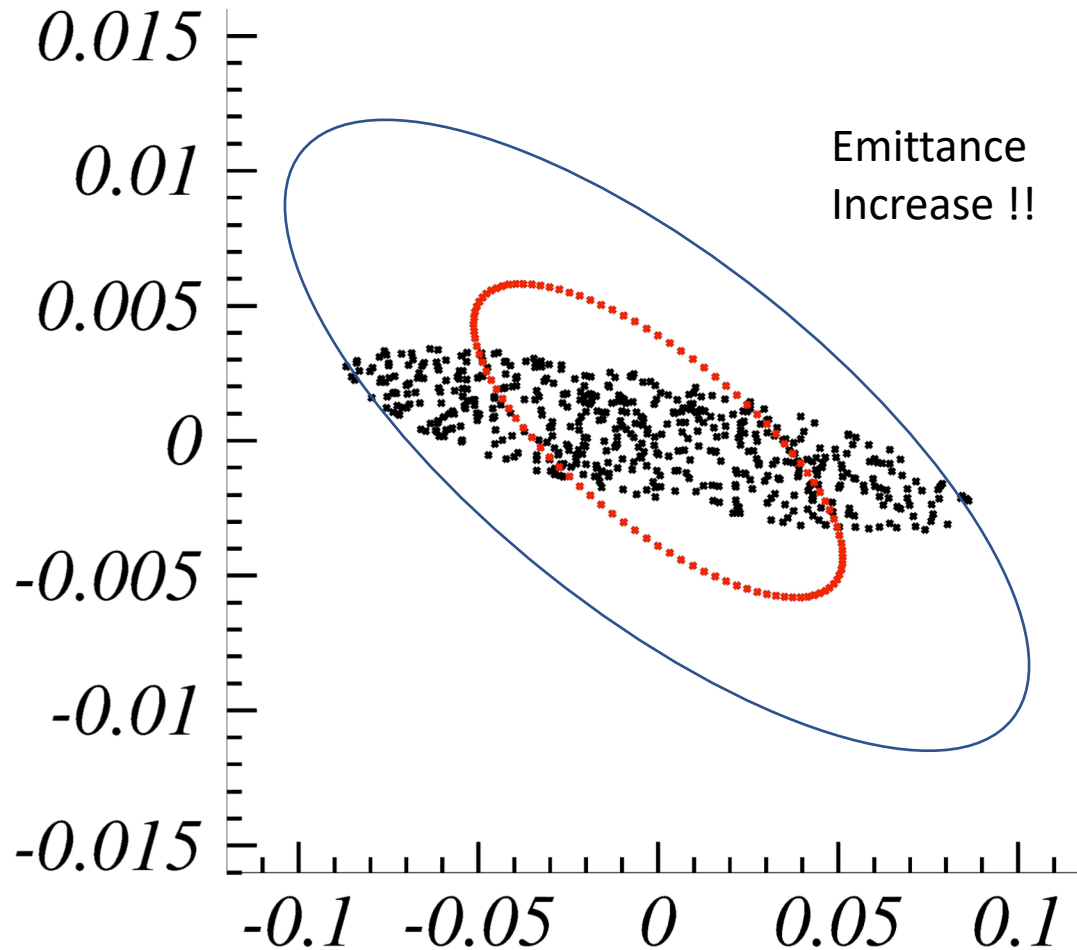
# Second order effects

$$k = k_0 \frac{p_0}{p} \quad \longrightarrow \quad k = \frac{k_0}{1 + \delta p/p_0} \quad \text{Chromaticity}$$

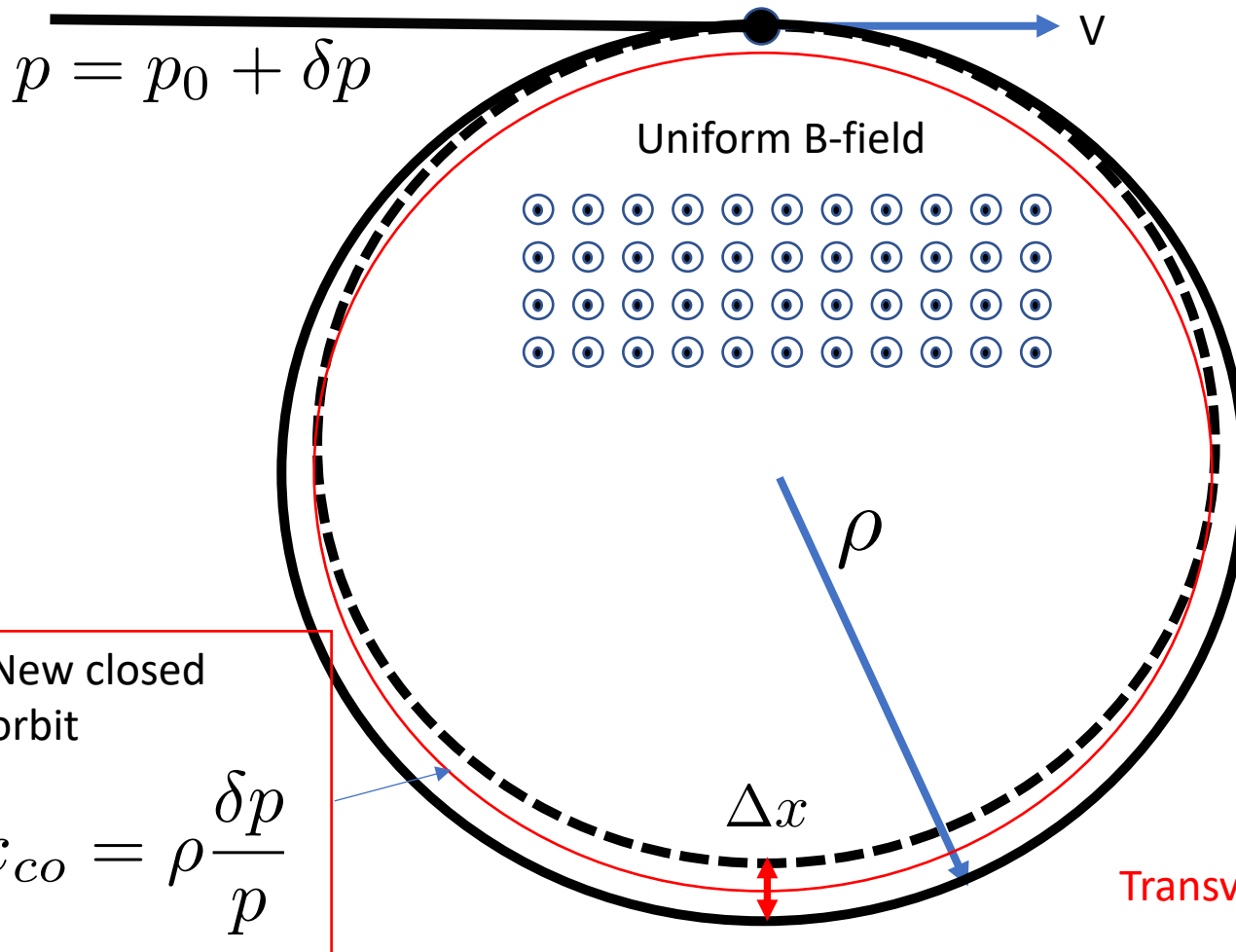
Tune is dependent on the off-momentum: each particle have a slightly different tune

$$Q_x(p) = Q_{x0} + \xi \frac{\delta p}{p}$$

# High order effects: $dp/p = 10^{-3}$



# Energy mismatch: now closed orbit

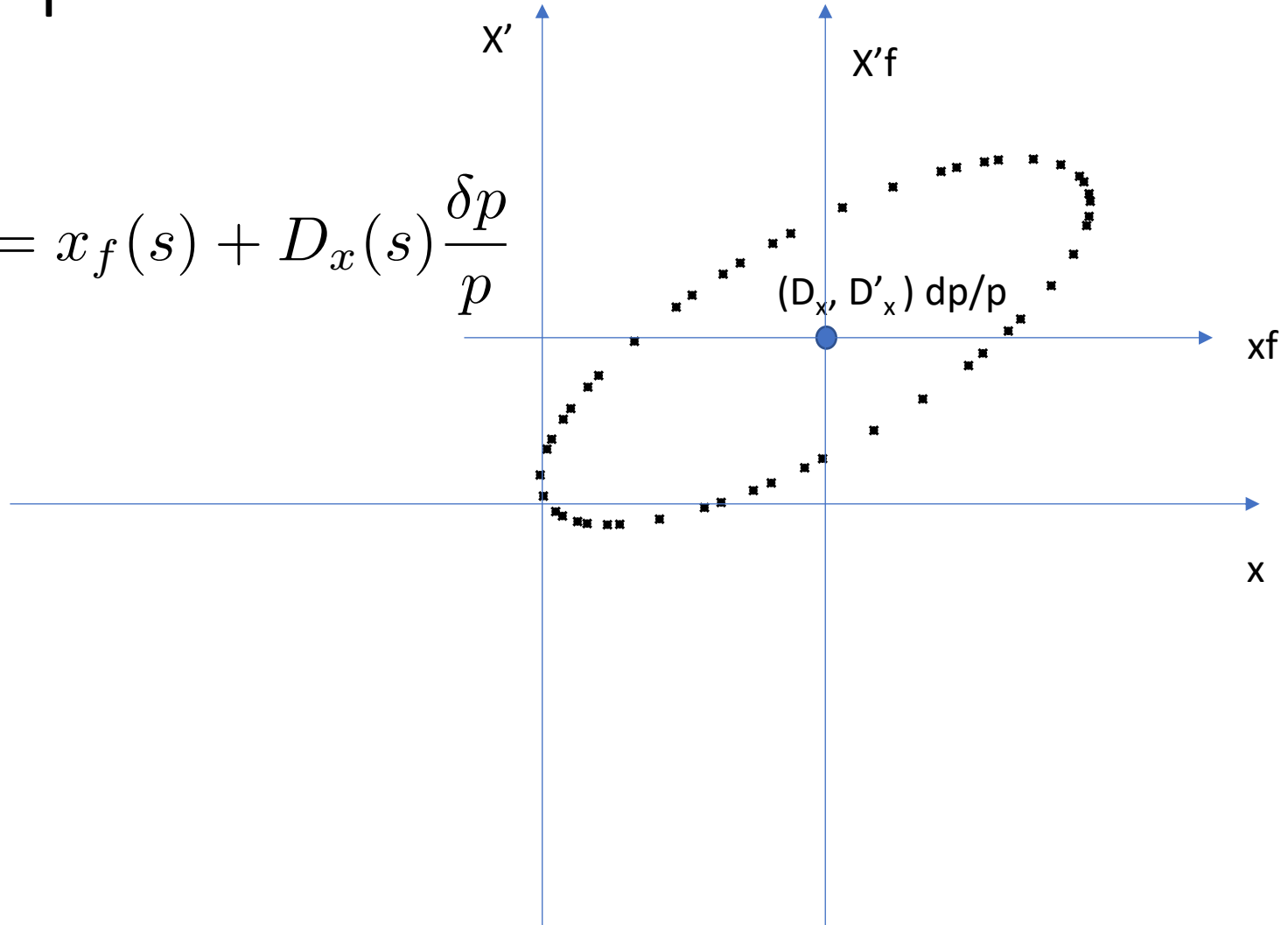


$$\frac{p}{q} = \rho B_0$$

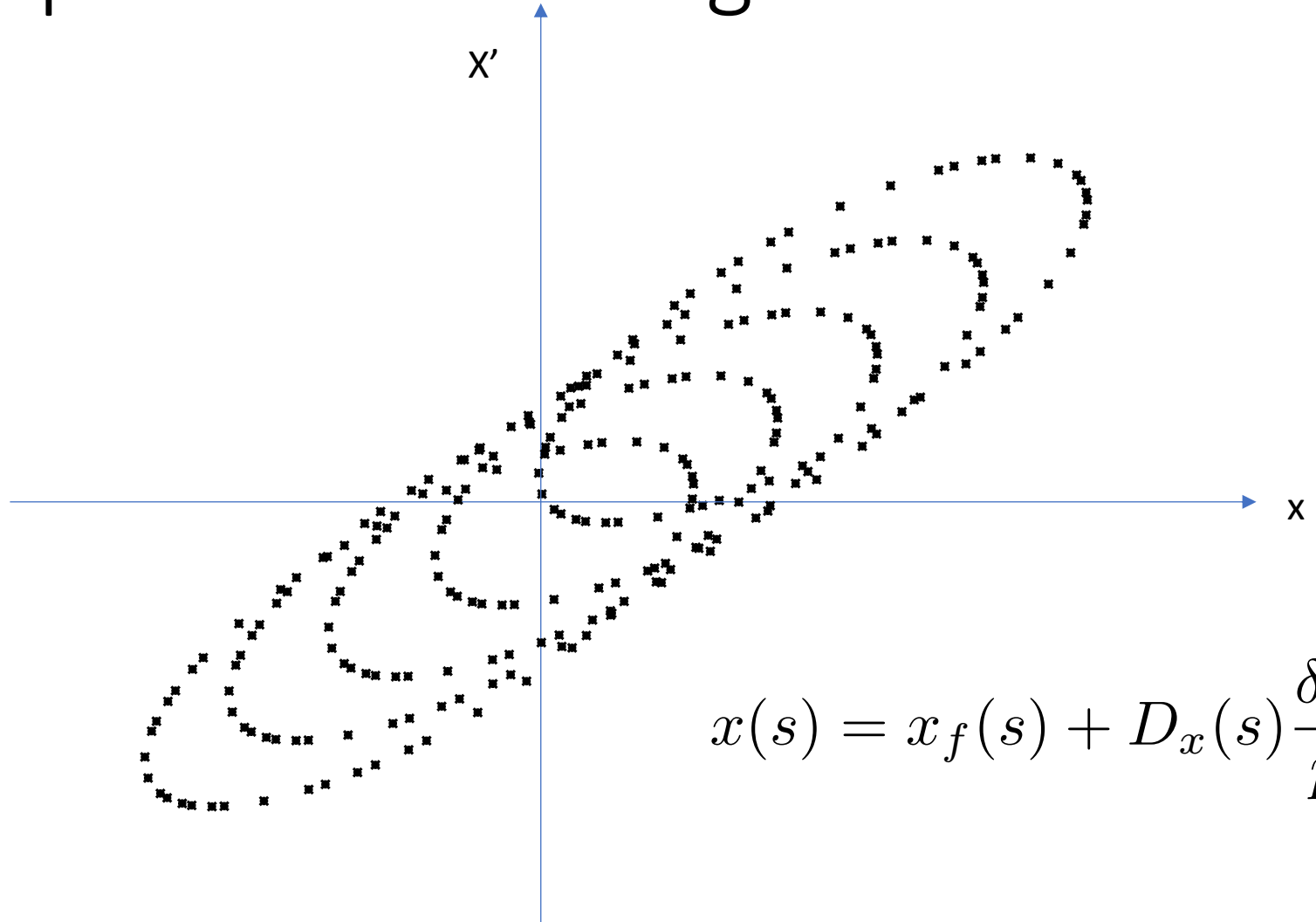
$$\Delta x = 2\rho \frac{\delta p}{p}$$

# Dispersion

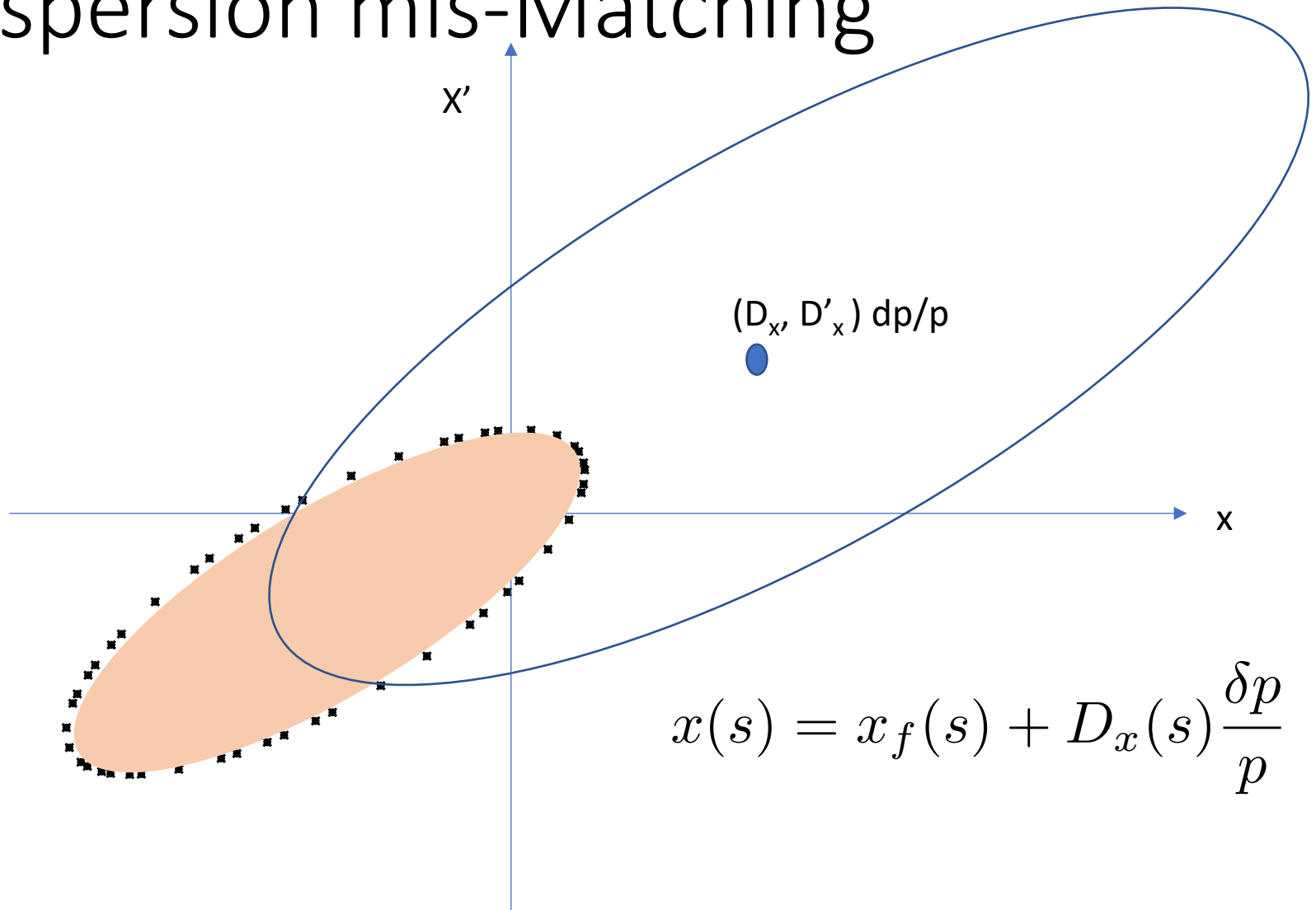
$$x(s) = x_f(s) + D_x(s) \frac{\delta p}{p}$$



# Dispersion Matching



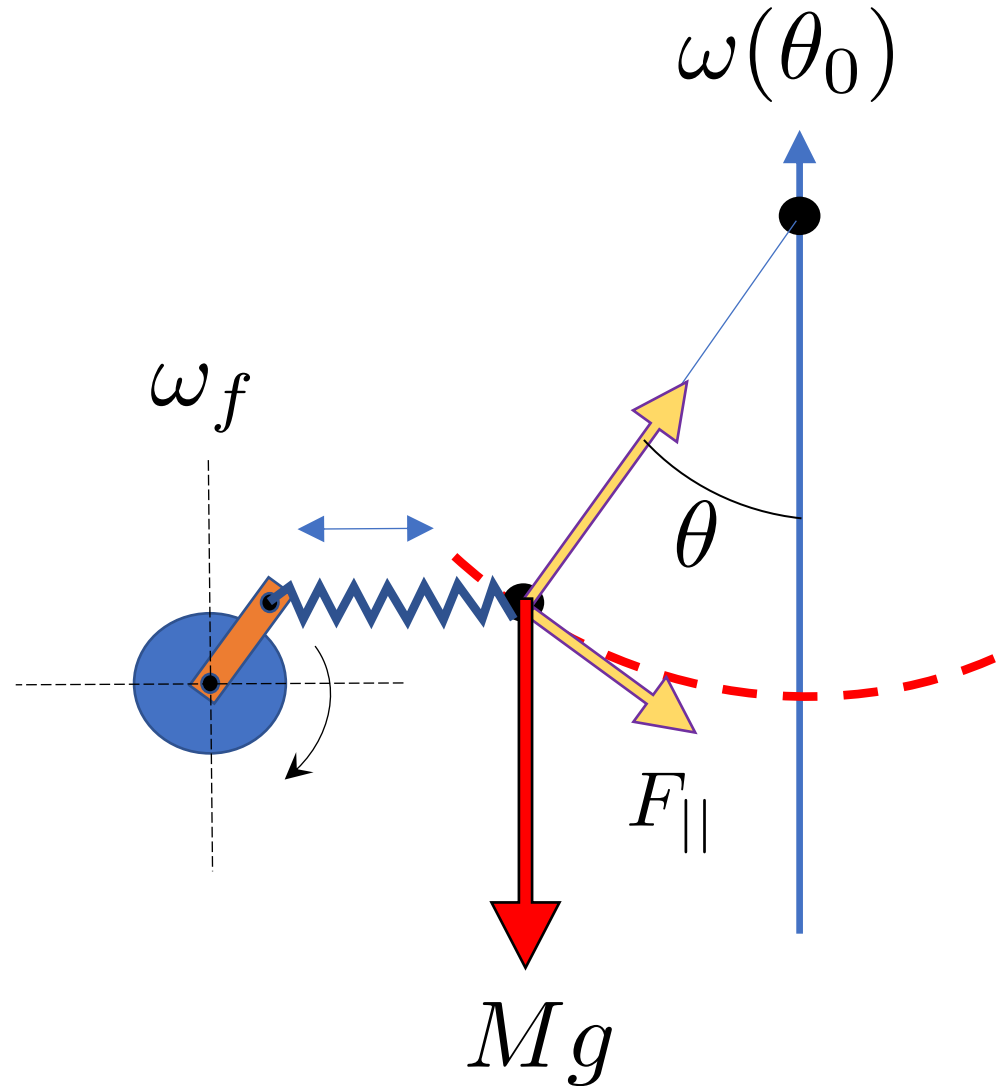
# Dispersion mis-Matching



$$x(s) = x_f(s) + D_x(s) \frac{\delta p}{p}$$



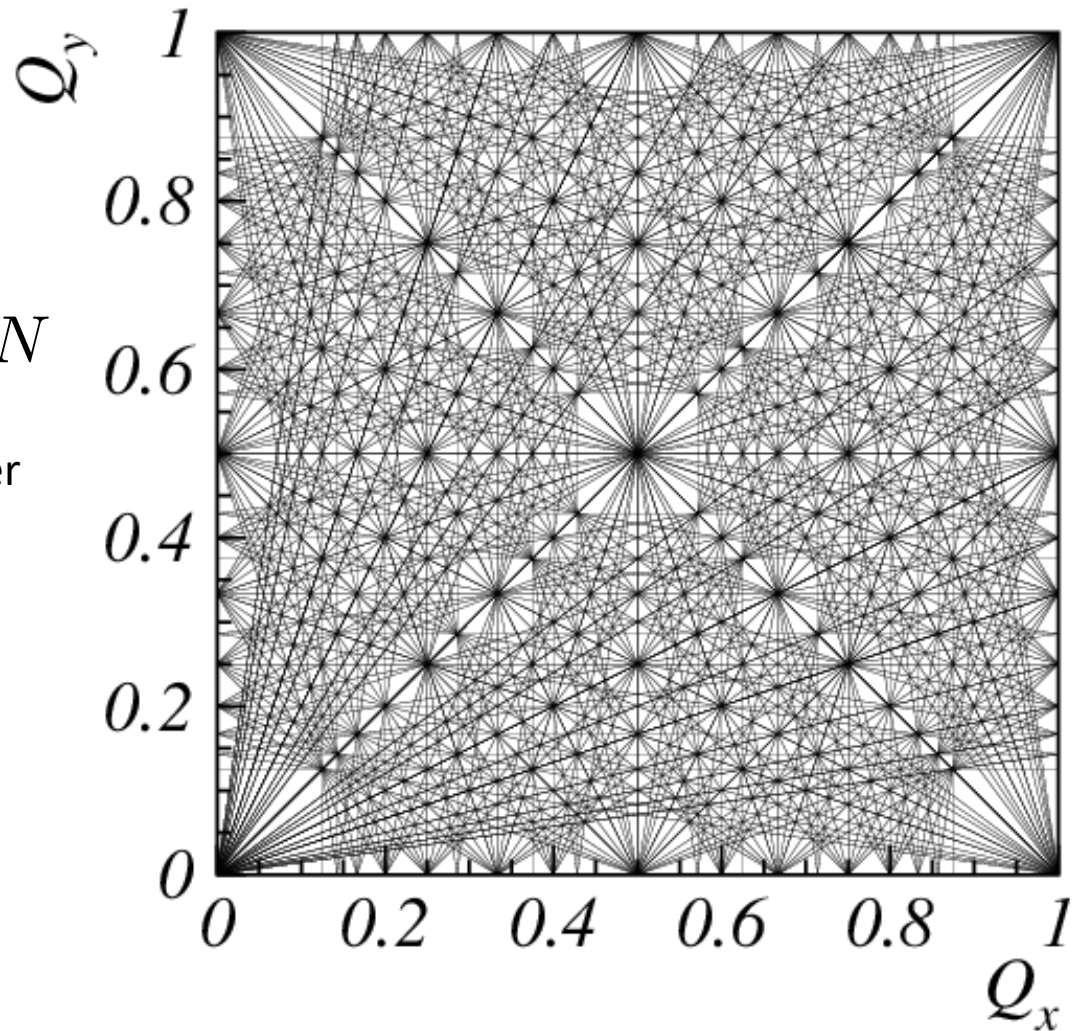
# Driven systems $\rightarrow$ Resonances



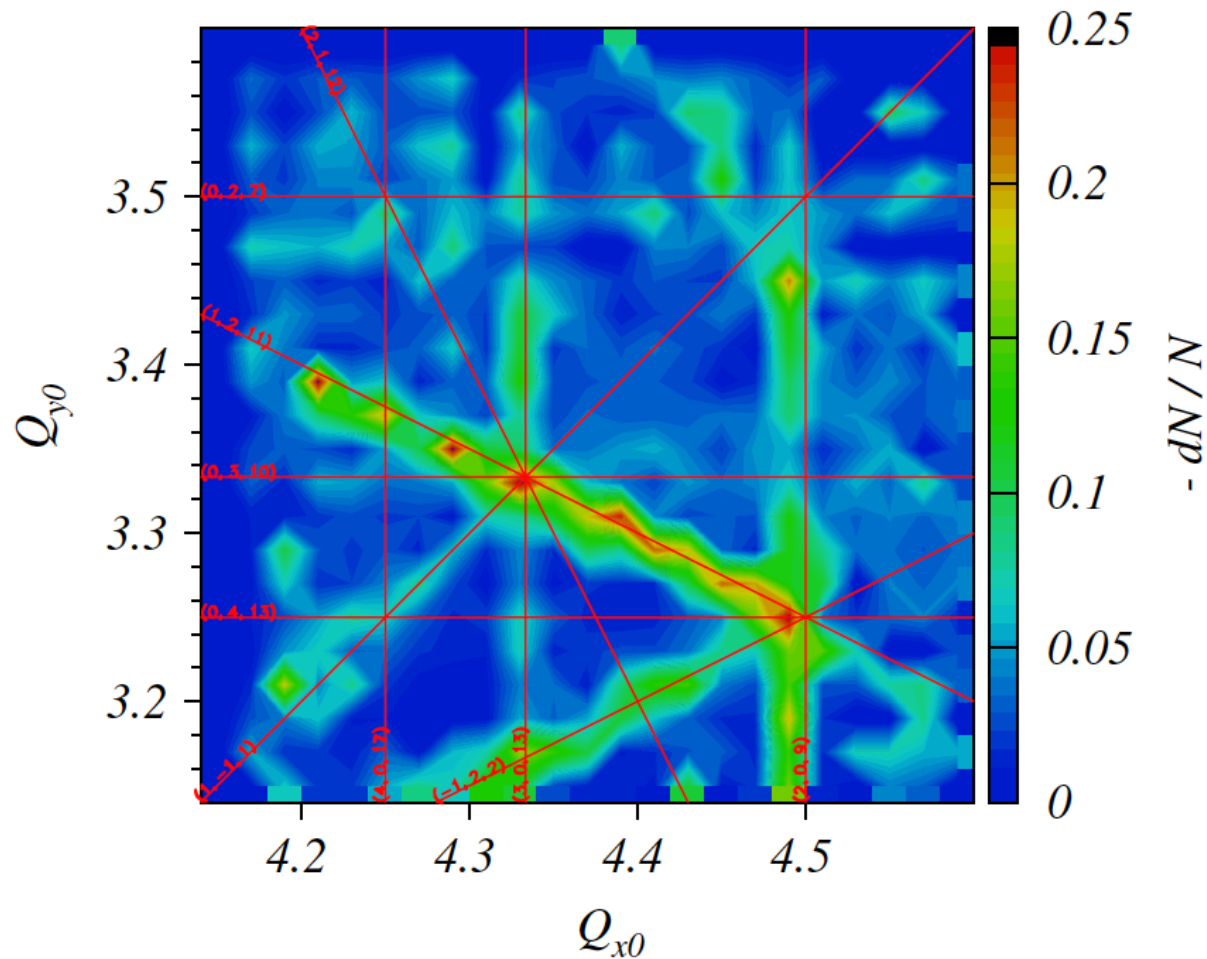
# Resonances on working diagram

$$nQ_x + mQ_y = N$$

$$|n| + |m| = \text{order}$$



# Resonances $\rightarrow$ SIS18



# Not covered now

Courant-Snyder

COD

Theory of resonances

High intensity

Ripples

**Mentoring Program → COMING SOON !**