Atomic physics with twisted light

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Maxwell equations: Just a reminder

The classical electromagnetic field is described by electric and magnetic field vectors which satisfy Maxwell's equation (here written in SI units):


James Clerk Maxwell

$$\begin{cases} \boldsymbol{\nabla}\cdot\boldsymbol{E} = \dfrac{\rho}{\varepsilon_0} & \boldsymbol{\nabla}\times\boldsymbol{E} = -\dfrac{\partial\boldsymbol{B}}{\partial t} \\ \boldsymbol{\nabla}\cdot\boldsymbol{B} = 0 & \boldsymbol{\nabla}\times\boldsymbol{B} = \mu_0\boldsymbol{j} + \mu_0\varepsilon_0\dfrac{\partial\boldsymbol{E}}{\partial t} \end{cases}$$

It is convenient to generate electric and magnetic fields from scalar φ and vector $\boldsymbol{A}$ potentials:

$$\boldsymbol{E} = -\nabla\varphi - \frac{\partial\boldsymbol{A}}{\partial t} \qquad \boldsymbol{B} = \boldsymbol{\nabla}\times\boldsymbol{A}$$

(Potentials are not completely defined and we have a freedom to chose a gauge!)

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Wave equation and its solutions

For the electromagnetic field in vacuum (no currents, no charges) and within the Coulomb gauge, $\boldsymbol{\nabla} \cdot \boldsymbol{A} = 0$, the vector potential satisfies the wave equation:

$$\boldsymbol{\nabla}^2 \boldsymbol{A} - \frac{1}{c^2}\frac{\partial^2 \boldsymbol{A}}{\partial t^2} = 0$$

What are solutions of this equation?

Maxwell's Equations in Vacuum

Plane wave solution to wave equation

$\mathbf{E}(\mathbf{r}, t) = \mathbf{E}_o\, e^{i(\omega t - \mathbf{k}.\mathbf{r})}$ $\mathbf{E}_o$ constant vector

$\mu_0\varepsilon_0\partial^2\mathbf{E}/\partial t^2 = -\mu_0\varepsilon_0\omega^2\mathbf{E}$ $\mu_0\varepsilon_0\omega^2 = k^2$

$\omega = \pm k/(\mu_0\varepsilon_0)^{1/2} = \pm ck$ $\boxed{\omega/k = c = (\mu_0\varepsilon_0)^{-1/2} \text{ phase velocity}}$

Nature of Electromagnetic Waves

- A general plane wave with angular frequency ω travelling in the direction of the wave vector $\boldsymbol{k}$ has the form

$$\vec{E} = \vec{E}_0 \exp[i(\omega t - \vec{k}\cdot\vec{x})] \quad \vec{B} = \vec{B}_0 \exp[i(\omega t - \vec{k}\cdot\vec{x})]$$

(4.3)

When **A** satisfies this condition, we are said to be using the *Coulomb gauge*. This choice of gauge is convenient when no sources are present, which is the case considered here. One may then take $\phi = 0$, and **A** satisfies the wave equation

$$\nabla^2 \mathbf{A} - \frac{1}{c^2}\frac{\partial^2 \mathbf{A}}{\partial t^2} = 0 \qquad (4.4)$$

where c is the velocity of light *in vacuo*.

A monochromatic plane wave solution of (4.4) corresponding to the angular frequency ω (that is, to the frequency $\nu = \omega/(2\pi)$) is

$$\mathbf{A}(\mathbf{r}, t) = A_0(\omega)\hat{\boldsymbol{\varepsilon}} \cos(\mathbf{k}\cdot\mathbf{r} - \omega t + \delta_\omega) \qquad (4.5)$$

where **k** is the wave (or propagation) vector and δ_ω is a real constant phase. Substituting (4.5) in (4.4), it is found that the angular frequency ω and the wave …itude of the wave vector **k**) are related by

In many textbooks and lecture notes we find the plane wave solutions!

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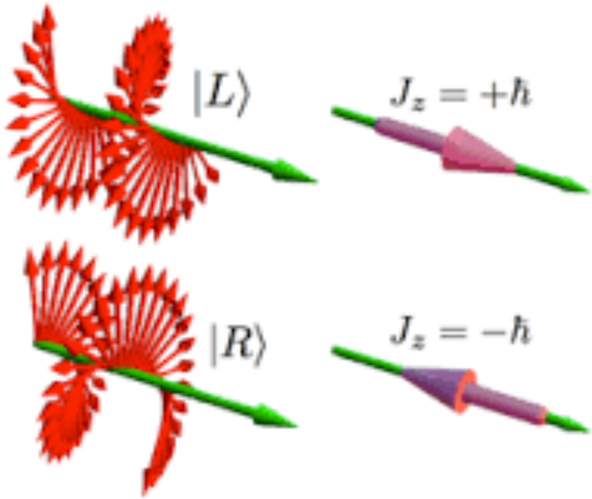
Plane-wave solutions of wave equation

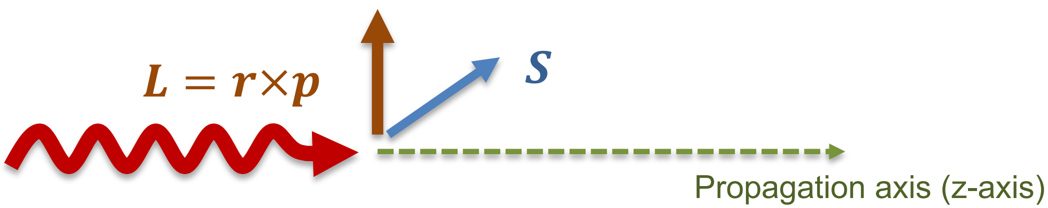


We usually employ the plane-wave solutions to describe propagation of the electromagnetic field:

$$\boldsymbol{A}(\boldsymbol{r}, t) = A_0 \boldsymbol{u}_\lambda \, e^{-i\omega t + i\boldsymbol{k}\boldsymbol{r}}$$

Quantum numbers: $\boldsymbol{k}, \omega, \lambda = \pm 1$

Just a reminder: $\lambda = \pm 1$ is the helicity of light. It projection of the spin of light onto its propagation axis. Since $\boldsymbol{L} = \boldsymbol{r} \times \boldsymbol{p}$ the projection of the orbital angular onto the propagation axis is *zero*.



Usually angular momentum is not even discussed in the analysis of plane waves.

Spherical-wave solutions of wave equation

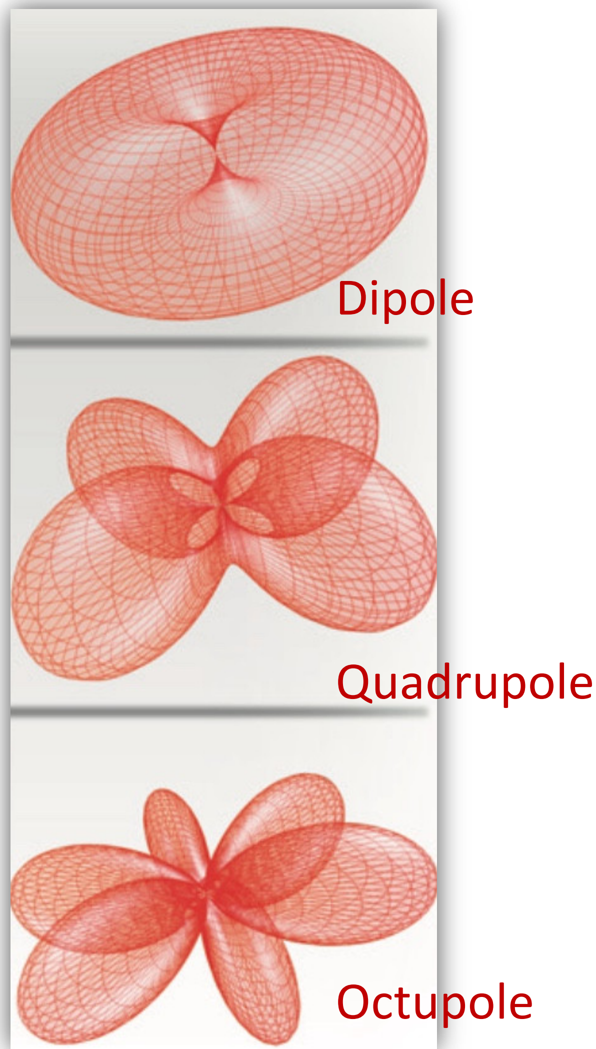
In atomic and nuclear physics we deal rather often with other class of solutions of the wave equation: spherical waves

$$a_{LM}^{(e,m)}(\boldsymbol{r},t) \propto j_L(kr) Y_{LM}^{(0,\pm 1)}(\theta,\varphi)$$

Vector spherical harmonics

They are characterized by quantum numbers: ω, L, M
No preferred direction of propagation anymore!

We want to find solution "in between" the plane- and spherical-waves!



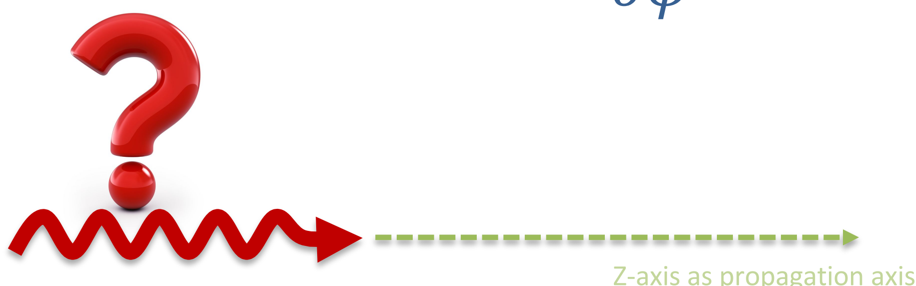
Twisted light solutions

We want to find solution of the wave-equation:

$$\boldsymbol{\nabla}^2 \boldsymbol{A} - \frac{1}{c^2}\frac{\partial^2 \boldsymbol{A}}{\partial t^2} = 0$$

That would be in the same time eigenfunction of the operator of projection of orbital angular momentum:

$$\hat{L}_z = -i\hbar \frac{\partial}{\partial \varphi}$$



How this solution looks like?

What is the set of commuting operators?

$$[\hat{L}_z, \hat{p}^2\,] = 0$$

$$[\hat{L}_z, \hat{p}_z\,] = 0$$

$$[\hat{L}_z, \hat{p}_{x,y}\,] \neq 0$$

$$[\hat{L}_z, \hat{p}_x^2 + \hat{p}_y^2] = 0$$

Quantum numbers:

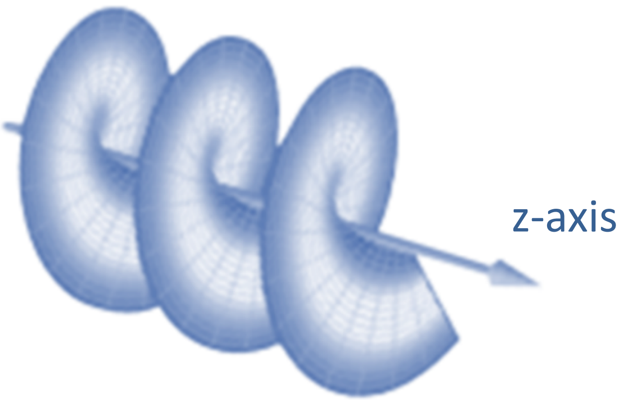
$$\omega, k_z, k_\perp = \sqrt{k_x^2 + k_y^2}, M$$

Twisted light solutions

The vector potential of the twisted wave reads as:

$$\boldsymbol{A}(\boldsymbol{r},t) \sim e^{-i\omega t + i k_z z}\, e^{iM\varphi}\, J_M(k_\perp r)$$

In contrast to the plane-wave we have an additional phase which depends on azimuthal angle φ!



This leads to the picture of rotating like a corkscrew phase-front. **That given the name: twisted light!**

What is the set of commuting operators?

$$[\hat{L}_z, \hat{p}^2] = 0$$

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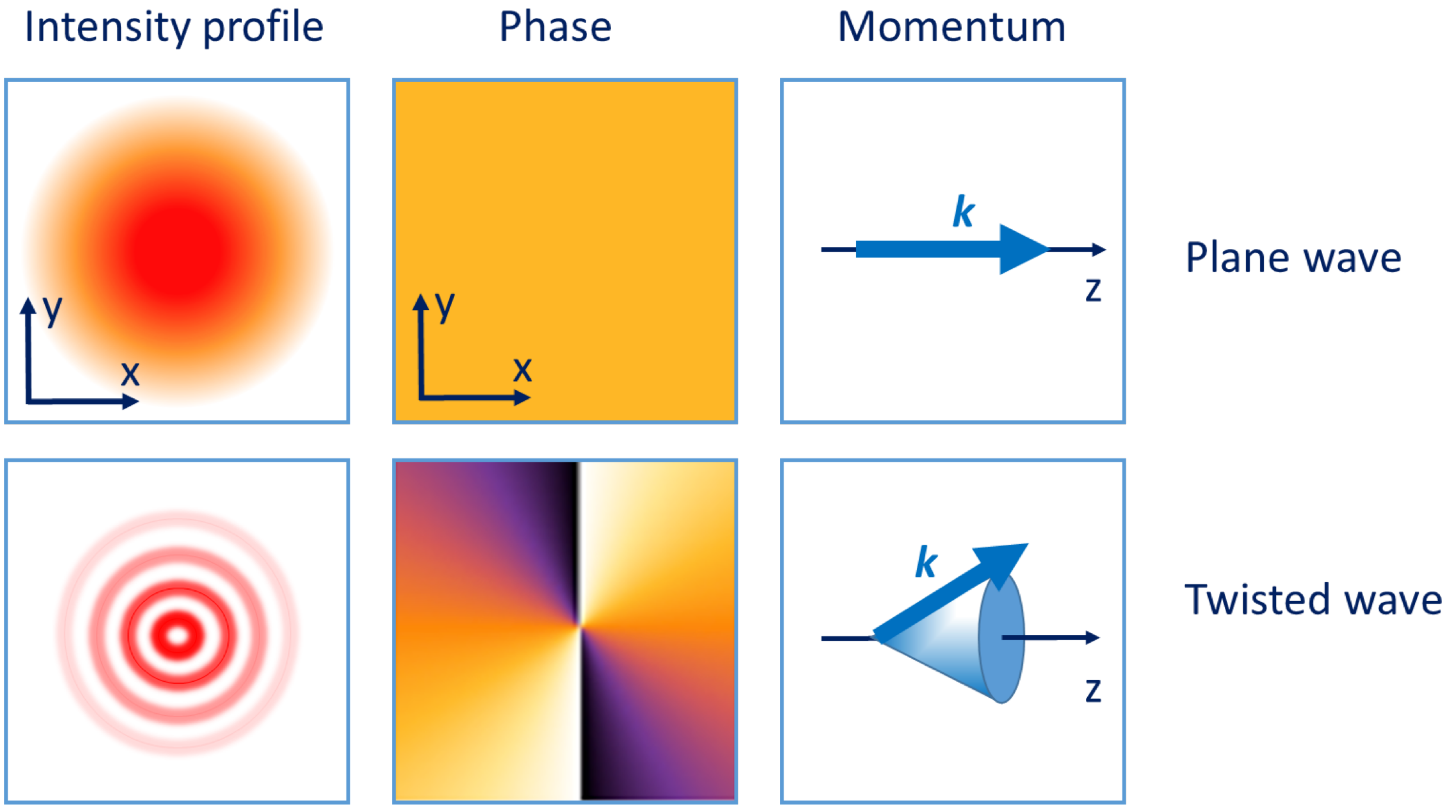
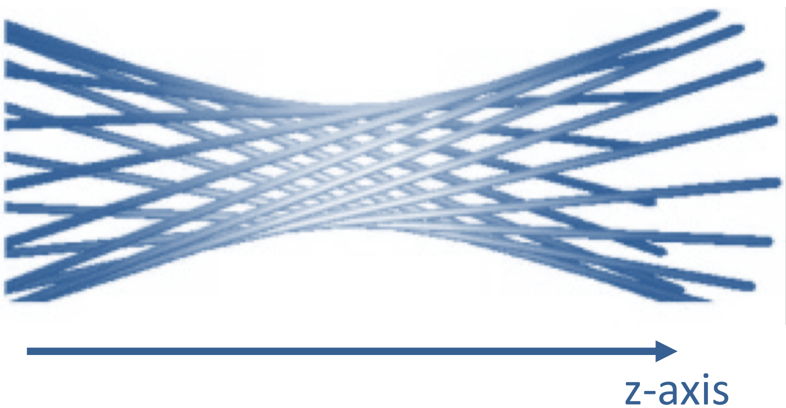
Quantum numbers:

$$\omega, k_z, k_\perp = \sqrt{k_x^2 + k_y^2}, M$$

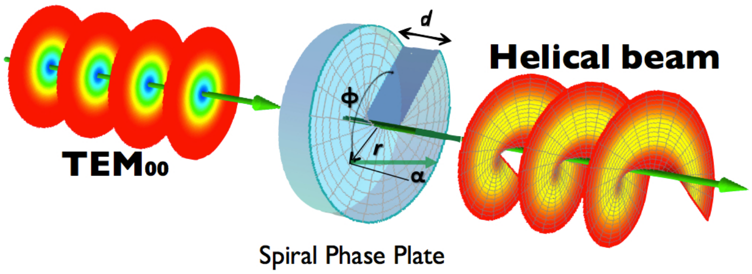
Twisted light: Basic properties

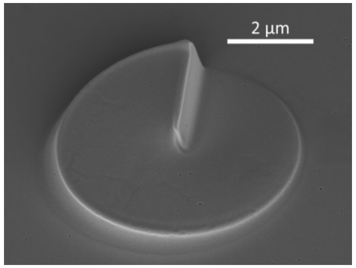




A vortex state is a <u>steady</u> interference pattern of plane waves which converges towards beam axis.

Production of twisted light



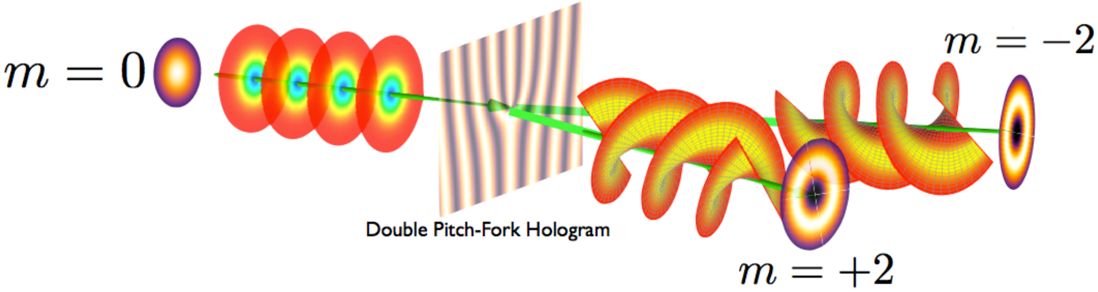


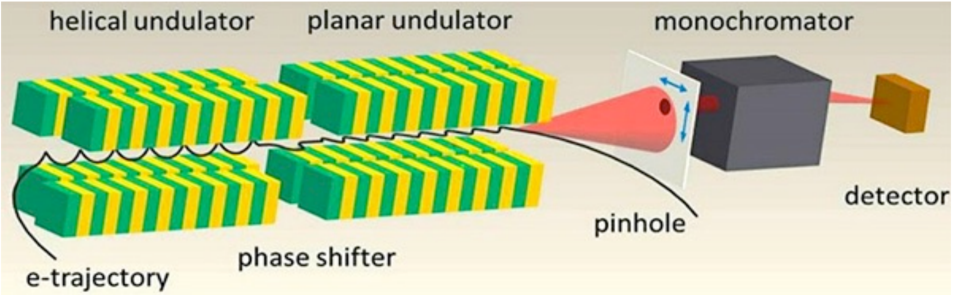
Spiral phase plates

Today can be fabricated directly on top of optical fibers

Computer-generated holograms

Allows one to produce twisted light with very large OAM projections



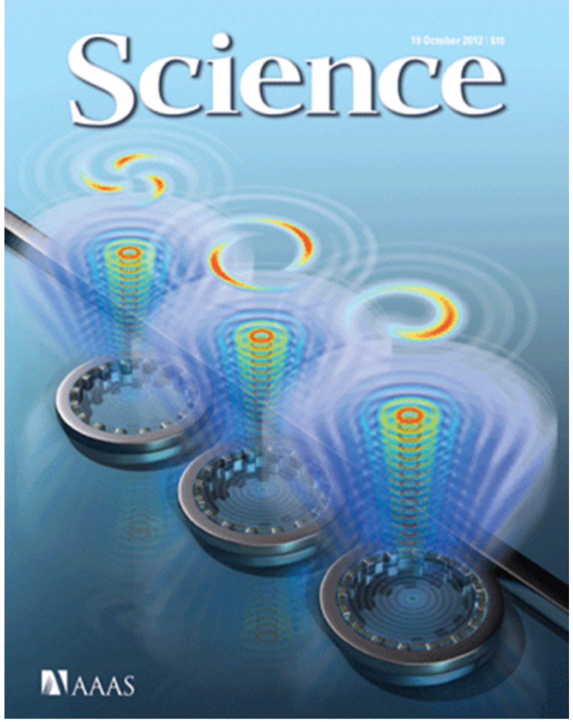


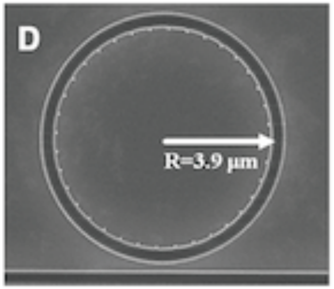
Helical undulators

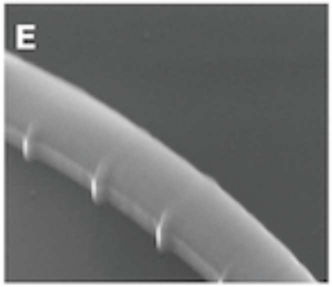
Generate twisted photons with energies up to 100 eV

Production of twisted light

While conventionally the twisted light is produced by optical elements such as plates and holograms, integrated arrays of emitters on a silicon chip have been recently demonstrated.





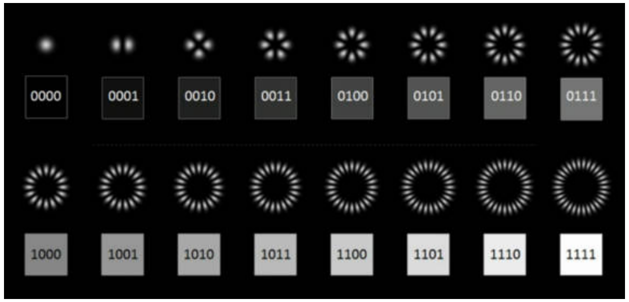


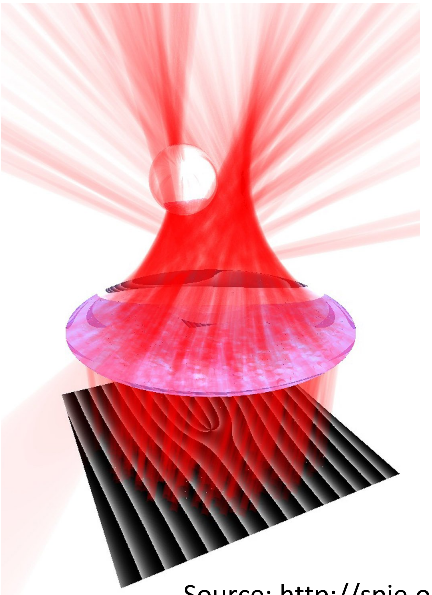
Much of the current interest is the ability to integrate optical vortex emitters into photonic integrated circuits!

Source: http://www.jwnc.gla.ac.uk/

Applications of twisted light

Classical and quantum information transfer:
multiplexing, free-space communications





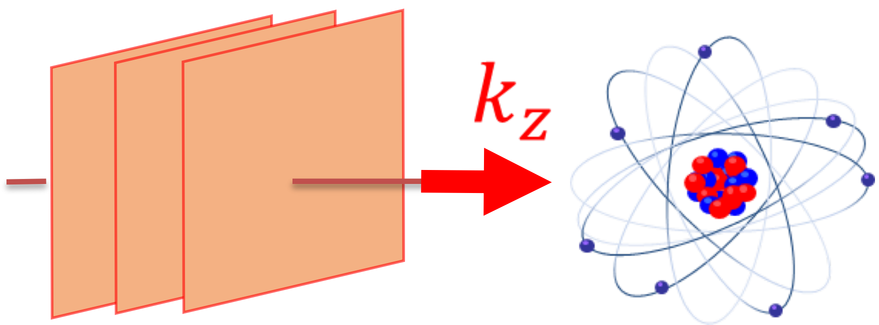
Source: http://spie.org/

Twisted light as an optical tweezer: manipulation of micro- and nano-particles

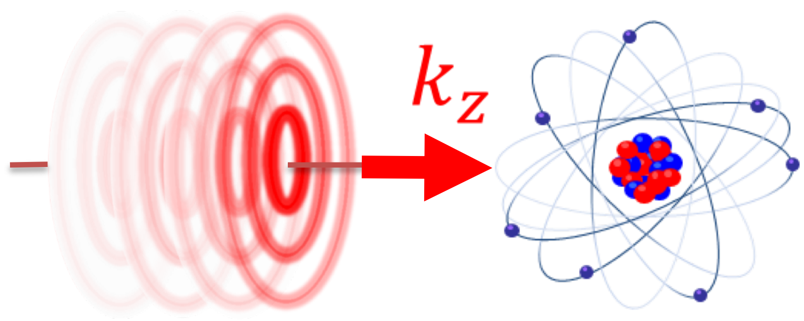
Source: www.ua-magazine.com

Cosmology and general relativity: rotating Kerr black holes, intergalactic gas emission

Interaction of twisted light with atoms



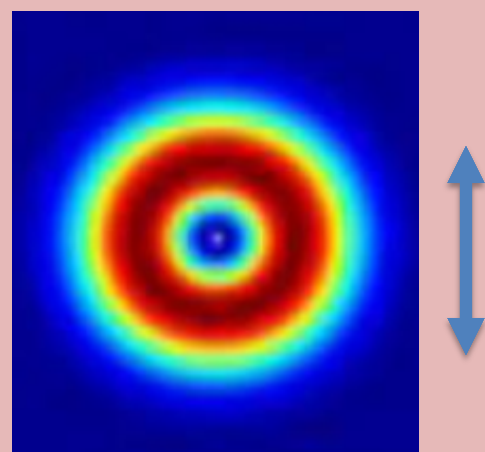
Incident plane-wave beam



Incident twisted beam

For almost a decade the opinion of community was: “no way!”

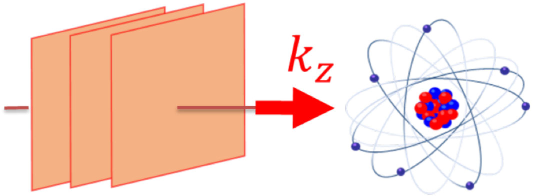
Argument: atom is too small!



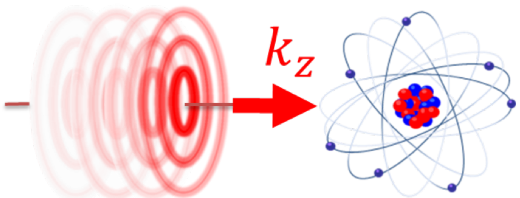
10-100 nm

Interaction of twisted light with matter

- Photoionization by twisted light
  - Ionization of a single atom and of mesoscopic target
- Photoexcitation by twisted light
  - Excitation of a single atom
- “Measurement” of the twistedness of emitted photons



Incident plane-wave beam



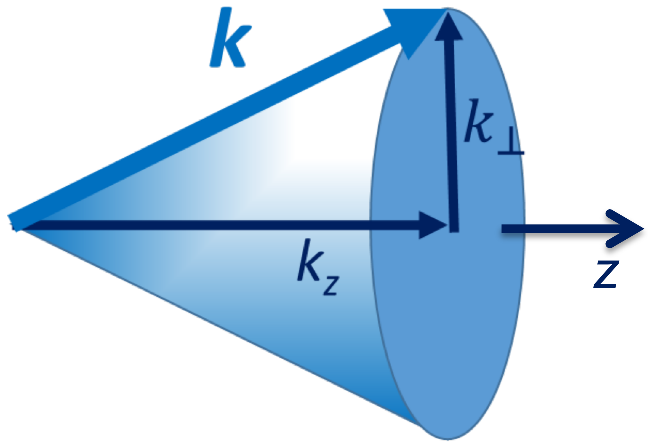
Incident twisted beam

Bessel light beams: Definition

In our study we will use the so-called Bessel states of light, which are solutions of the equation:

$$\hat{J}_z\, A(\boldsymbol{r}) = m\, A(\boldsymbol{r})$$

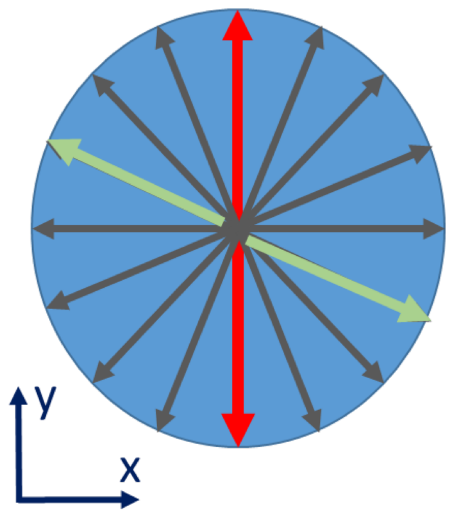
And, hence, are characterized by the projection of total angular momentum *m*.



- In the momentum space, the twisted Bessel light can be seen as a coherent superposition of plane waves with wave vectors ***k***, lying on a cone with a polar *opening* angle:

$$\theta_k = \arctan\left(\frac{|\boldsymbol{k}_\perp|}{k_z}\right)$$

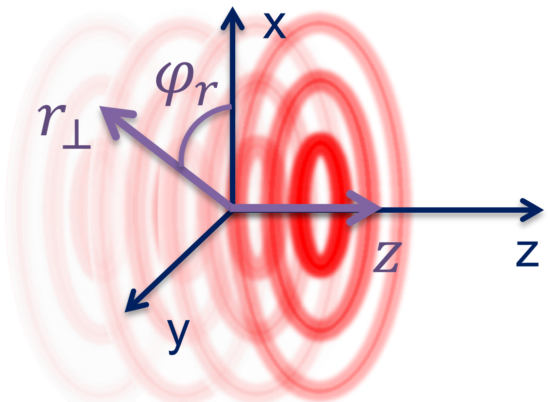
- If $\theta_k = 0$ the plane-wave is "recovered".



O. Matula, A. G. Hayrapetyan, V. G. Serbo, A. S., and S. Fritzsche, J. Phys. B **46** (2013) 205002

Bessel light beams: Poynting vector

One can obtain the Poynting vector of the Bessel light in cylindrical coordinates:



$$\boldsymbol{P}^{(tw)}(\boldsymbol{r}) = \boldsymbol{e}_{r_\perp} P_{r_\perp}(\boldsymbol{r}) + \boldsymbol{e}_{\varphi_r} P_{\varphi_r}(\boldsymbol{r}) + \boldsymbol{e}_z P_z(\boldsymbol{r})$$

With the components:

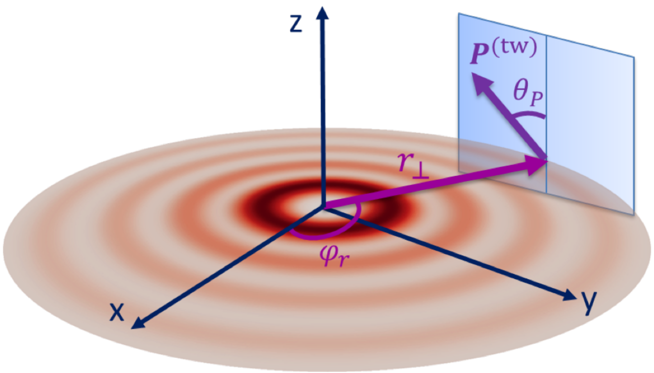
$$c_{\pm 1} = \frac{1}{2}(1 \pm \lambda \cos\theta_k)$$

$$\begin{cases} P_{r_\perp}(\boldsymbol{r}) = 0 \\ P_{\varphi_r}(\boldsymbol{r}) = \dfrac{k_\perp \omega^2}{4\pi} \sin\theta_k \, J_m(k_\perp r_\perp)\,[J_{m+1}(k_\perp r_\perp) c_{-1} + J_{m-1}(k_\perp r_\perp) c_{+1}] \\ P_z(\boldsymbol{r}) = \dfrac{k_\perp \omega^2 \lambda}{4\pi}\,[J^2{}_{m-1}(k_\perp r_\perp) c^2{}_1 - J^2{}_{m+1}(k_\perp r_\perp) c^2{}_{-1}] \end{cases}$$

We can see from these formulas that:

- $P_{r_\perp}(\boldsymbol{r})$ vanishes identically thus making explicit that the Bessel beams are non-diffractive
- $P_{\varphi_r}(\boldsymbol{r})$ and $P_z(\boldsymbol{r})$ depend only on the transverse coordinate $r_\perp$ but not on the angle φ_r

Bessel light beams: Poynting vector

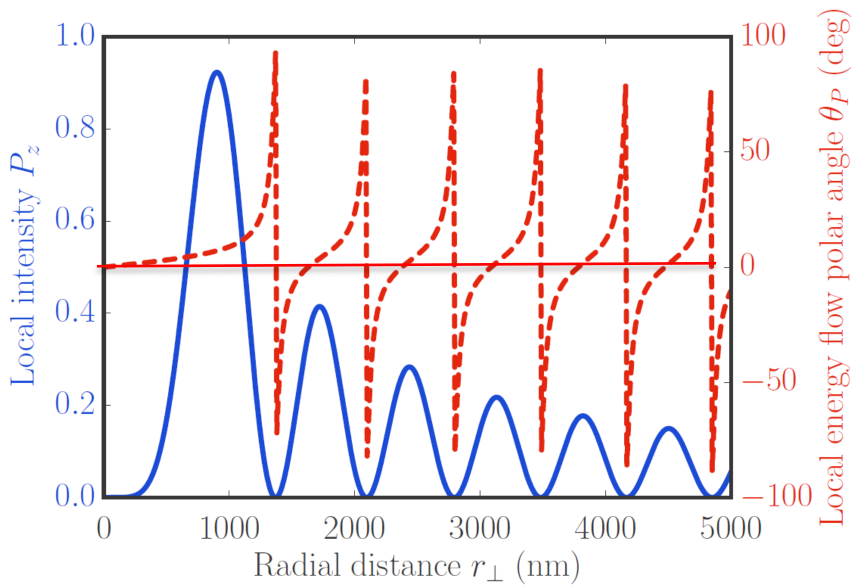


To better understand complex internal structure of Bessel beams, it is convenient to consider two *local* quantities:

- Intensity profile: $I(r_\perp) = |P_z(\boldsymbol{r})|$
- Direction of energy flow: $\theta_P = \arctan \frac{P_{\varphi r}}{P_z}$

Near the "rings" of high intensity energy flows predominantly in the forward (z-) direction.

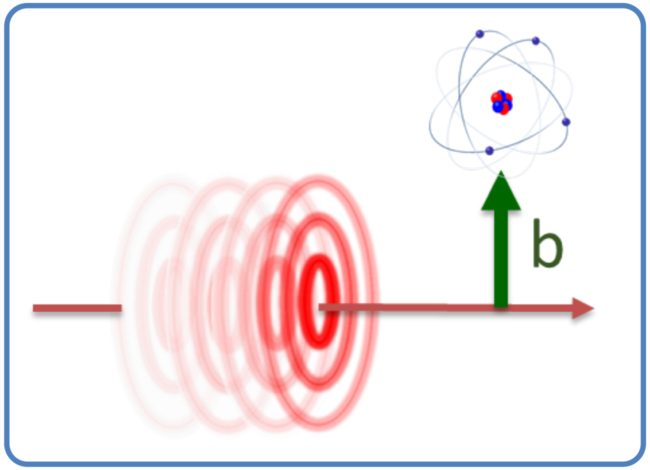
In the "dark" regions energy almost perpendicular or even backward flow may be observed!



A. S., D. Seipt, and S. Fritzsche, Phys. Rev. A **94** (2016) 033420

Photoionization: Theoretical background

We describe the atomic target quantum-mechanically and apply the first-order perturbation theory:



$$M_{if}^{(\mathrm{tw})} = \int \psi_f^\dagger(\boldsymbol{r})\,\boldsymbol{\alpha}\,\boldsymbol{A}_b(\boldsymbol{r})\psi_i(\boldsymbol{r})d\boldsymbol{r}$$

Vector-potential of the incident radiation including higher multipoles. (Not a dipole approximation!)

Initial- and final-state wave functions (relativistic and if necessary many-electron).

$$\boldsymbol{A}_b(\boldsymbol{r}) = \int a_{\kappa m}(\boldsymbol{k}_\perp)\,u_{\boldsymbol{k}\lambda}\,e^{i\boldsymbol{k}\boldsymbol{r}}e^{-i\boldsymbol{k}_\perp\boldsymbol{b}}\,\frac{d^2\boldsymbol{k}_\perp}{(2\pi)^2}$$

We can simplify these expressions within the non-relativistic dipole approximation.

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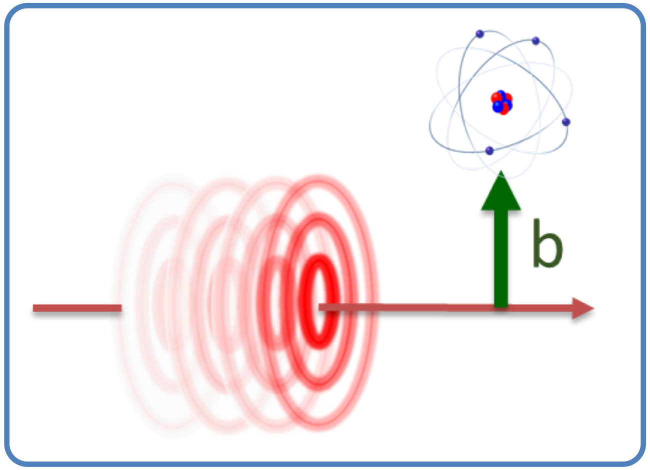
Photoionization: Electric dipole approximation

We describe the atomic target quantum-mechanically and apply the first-order perturbation theory:



$$M_{if}^{(\mathrm{tw})} = \int \psi_f^\dagger(\boldsymbol{r})\, \boldsymbol{\alpha}\, \boldsymbol{A}_b(\boldsymbol{r}) \psi_i(\boldsymbol{r}) d\boldsymbol{r}$$

Within the dipole approximation and for single-electron system we find:

$$M_{if} = \boldsymbol{A}_{E1}(\boldsymbol{b}) \cdot \boldsymbol{d}_{if}$$

The light vector potential at the *location* of a target atom

$$\boldsymbol{A}_{E1}(\boldsymbol{b}) = \int a_{\kappa m}(\boldsymbol{k}_\perp)\, \boldsymbol{u}_{k\lambda}\; e^{-i\boldsymbol{k}_\perp b}\, \frac{d^2\boldsymbol{k}_\perp}{(2\pi)^2}$$

The standard dipole matrix element

$$\boldsymbol{d}_{if} = \int \psi_f^\dagger(\boldsymbol{r})\, \widehat{\boldsymbol{p}}\, \psi_i(\boldsymbol{r}) d\boldsymbol{r}$$

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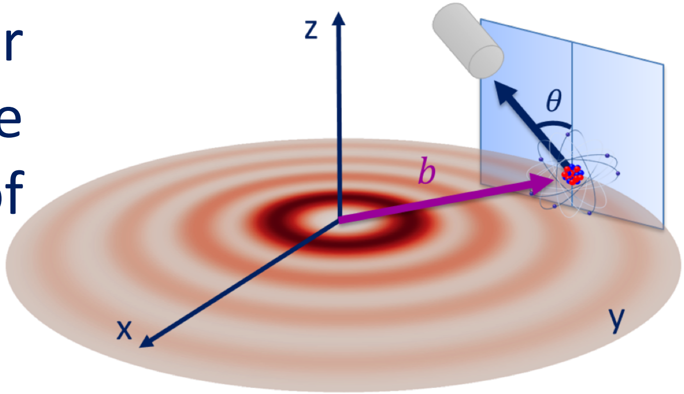
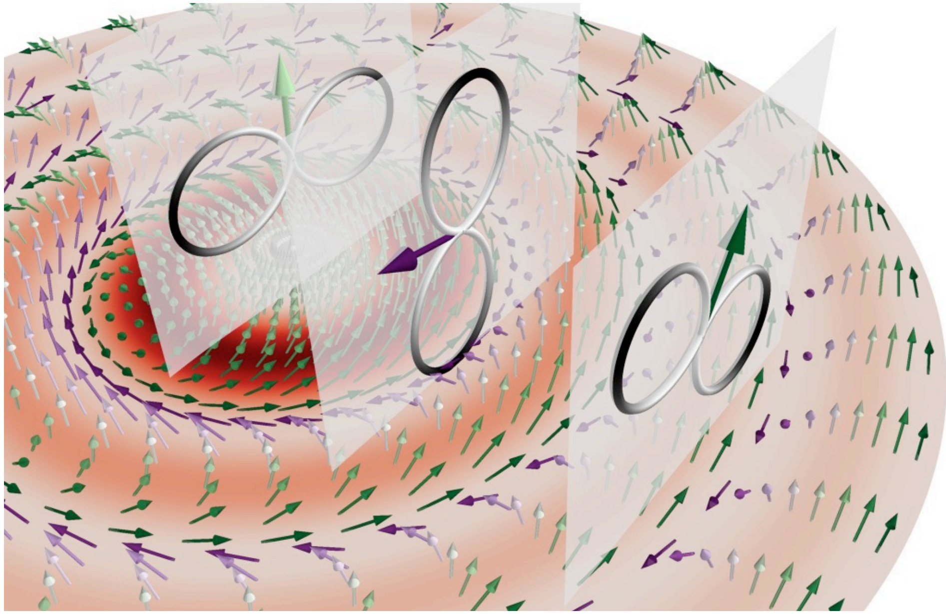
Photoionization by twisted light

In a plane, normal to the impact parameter vector $\boldsymbol{b}$, which is also a plane of the Poynting vector, the angular distribution of photo-electrons from alkalis:



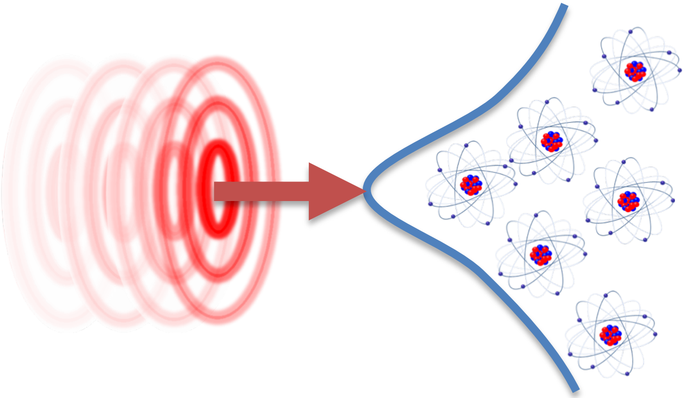
$$W(\theta) \propto \sin^2(\theta - \theta_P(\boldsymbol{b}))$$



Electron emission pattern is uniquely defined by the direction of the Poynting vector of the incident radiation at the position on a target atom!

A. S., D. Seipt, and S. Fritzsche, Phys. Rev. A **94** (2016) 033420

Ionization of mesoscopic target

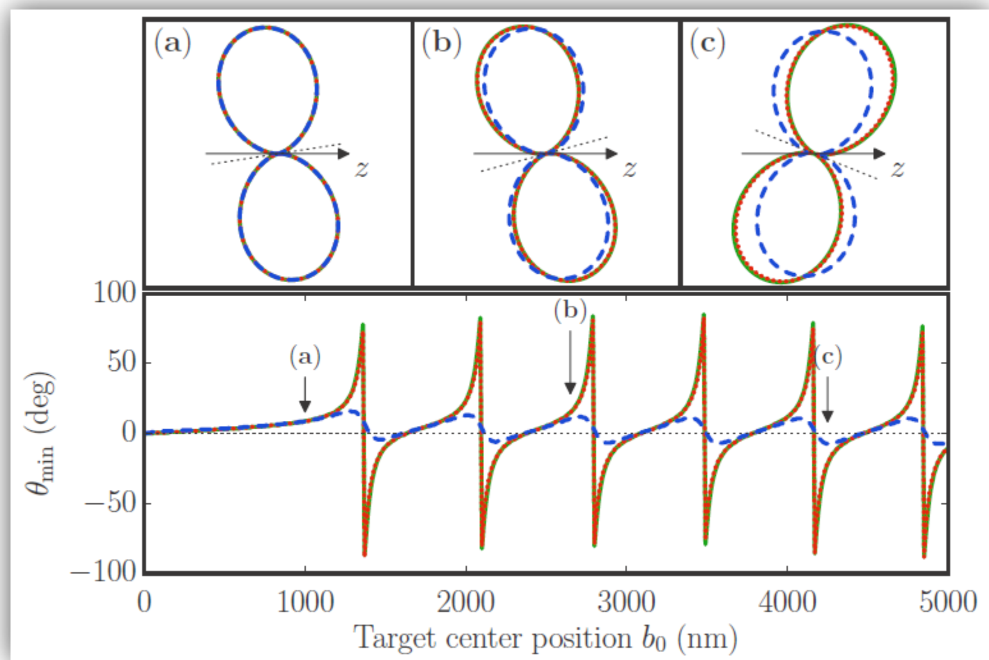


Of course, the proposed photoionization experiment with a single-atom-target is impossible by statistical issues.

Let us consider macroscopic target!

Photoionization of Na (3s) atoms by 5 eV twisted light (OAM=2) for different target sizes:

- Single atom
- 20 nm
- 100 nm



A. S., D. Seipt, and S. Fritzsche, Phys. Rev. A **94** (2016) 033420

Interaction of twisted light with matter

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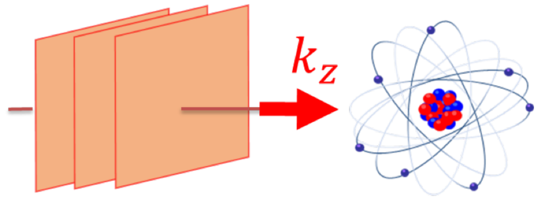
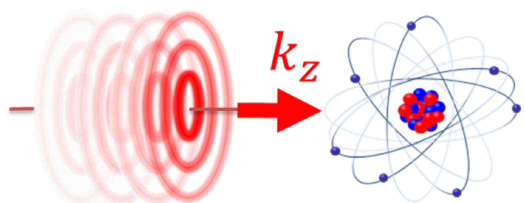


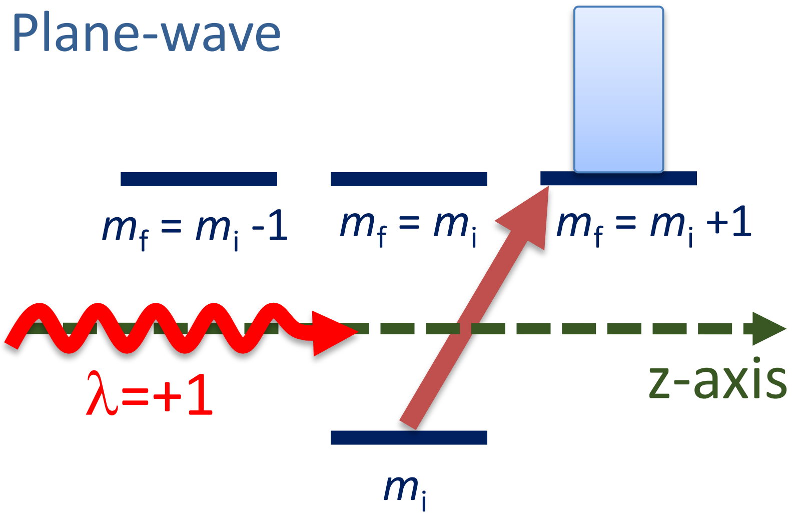
Incident plane-wave beam



Incident twisted beam

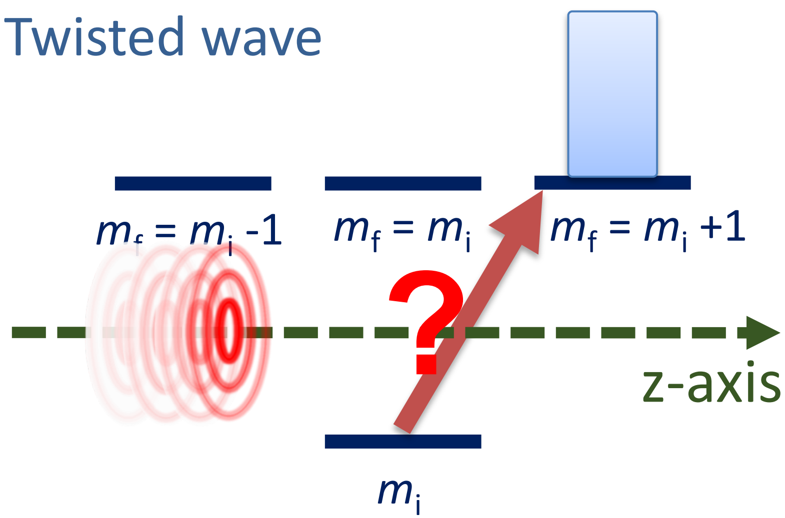
Photoexcitation: Selection rules

Plane-wave



Selection rule: $m_i + m = m_f$

Twisted wave



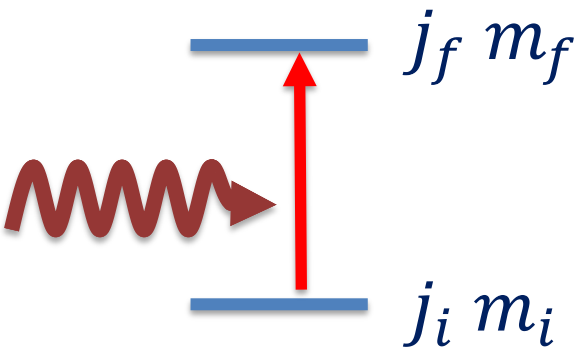
How twisted light "talks" to atom? And how we can describe this?

We study the fundamental process of photo-absorption of twisted light.

Does the absorption of twisted light affects the population of magnetic substates of excited atom?

Are there "selection rules" for the absorption of twisted light?

Origin of selection rules



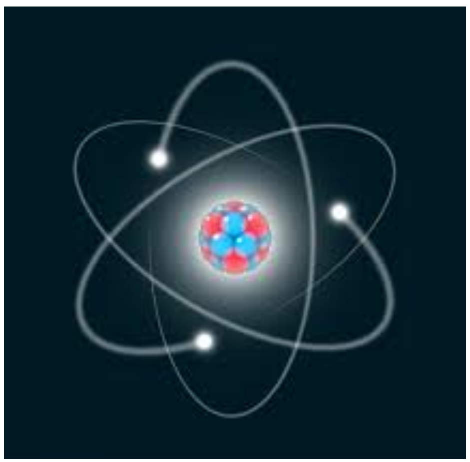
We derive the selections rules based on the analysis of transition matrix elements:

$$M_{if} = \int \psi_f^\dagger(\boldsymbol{r})\ \boldsymbol{\alpha}\ A(\boldsymbol{r})\psi_i(\boldsymbol{r})d\boldsymbol{r}$$

$$\propto \sum_{LM} \underbrace{\left\langle j_f\ m_f \middle| \alpha\ a_{LM}^{(e,m)} \middle| j_i\ m_i \right\rangle}\ \underbrace{W_{LM}}$$

What atom („buyer“) wants from a field: **structure and symmetry of an atom**

What field („seller“) can offer to an atom: **structure of light**



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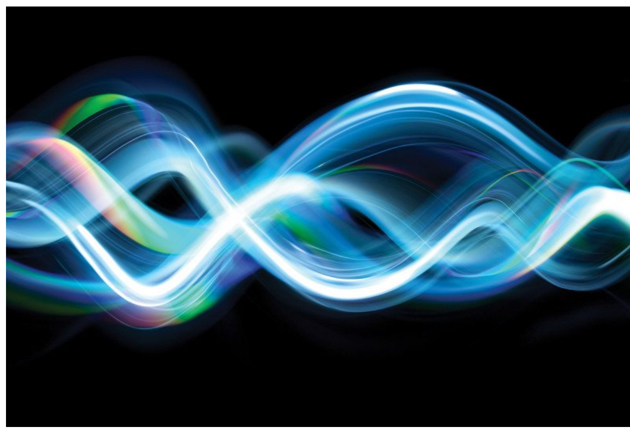
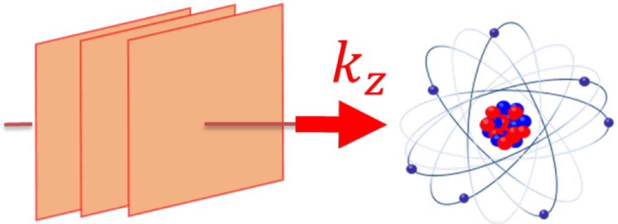


Plane-wave selection rules



Incident plane-wave beam

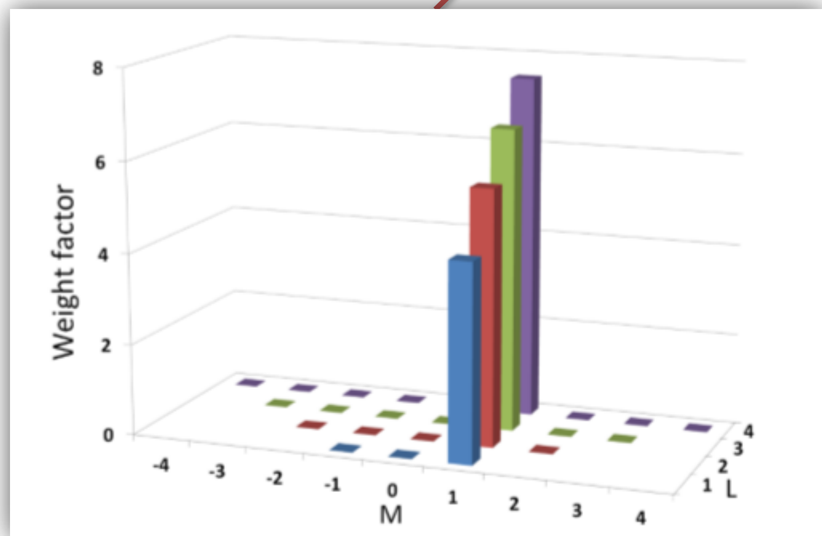
We derive the selections rules based on the analysis of transition matrix elements:

$$M_{if}^{\mathrm{pl}} = \int \psi_f^\dagger(\boldsymbol{r})\, \boldsymbol{\alpha}\, A(\boldsymbol{r}) \psi_i(\boldsymbol{r}) d\boldsymbol{r}$$

$$\propto \sum_{LM} \underbrace{\left\langle j_f\, m_f \middle| \alpha\, a_{LM}^{(e,m)} \middle| j_i\, m_i \right\rangle}\, \underbrace{W_{LM}}$$

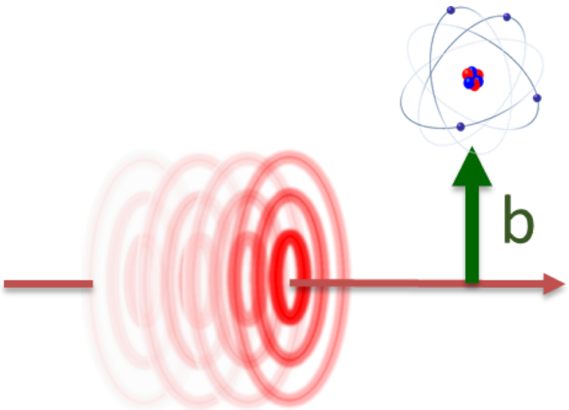
Momentum: $|j_i - j_f| \leq L \leq j_i + j_f$

Projection: $m_i + M = m_f$



Properties of plane-wave light don‘t depend on position!

Twisted “selection rules”



We derive the selections rules based on the analysis of transition matrix elements:

$$M_{if}^{\mathrm{tw}} = \int \psi_f^\dagger(\boldsymbol{r})\, \boldsymbol{\alpha}\, A(\boldsymbol{r}) \psi_i(\boldsymbol{r}) d\boldsymbol{r}$$

$$\propto \sum_{LM} \left\langle j_f\, m_f \middle| \alpha\, a_{LM}^{(e,m)} \middle| j_i\, m_i \right\rangle W_{LM}$$

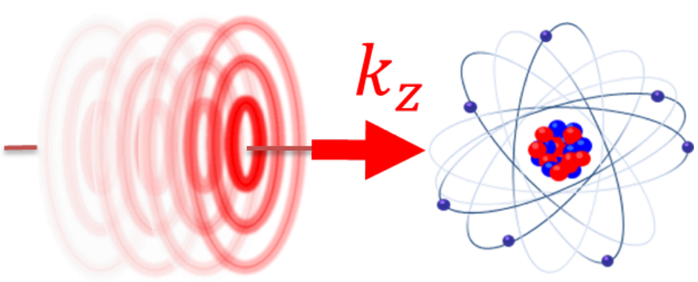
Momentum: $|j_i - j_f| \leq L \leq j_i + j_f$

Projection: $m_i + M = m_f$

Nothing new here!

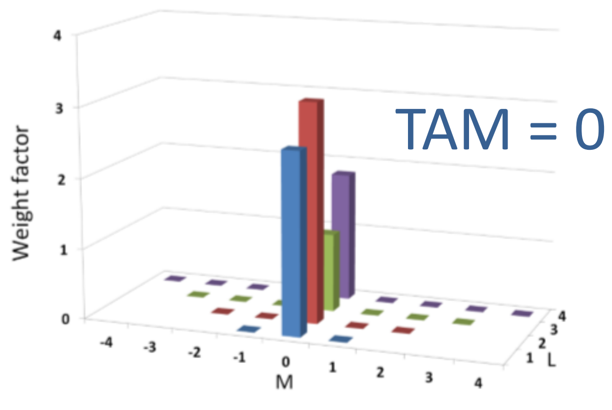
Strongly dependent on position within the wave-front!

Bessel "selection rules": *b = 0* case



$$M_{if}^{\mathrm{tw}} = \int \psi_f^\dagger(\boldsymbol{r})\, \boldsymbol{\alpha}\, A(\boldsymbol{r}) \psi_i(\boldsymbol{r}) d\boldsymbol{r}$$

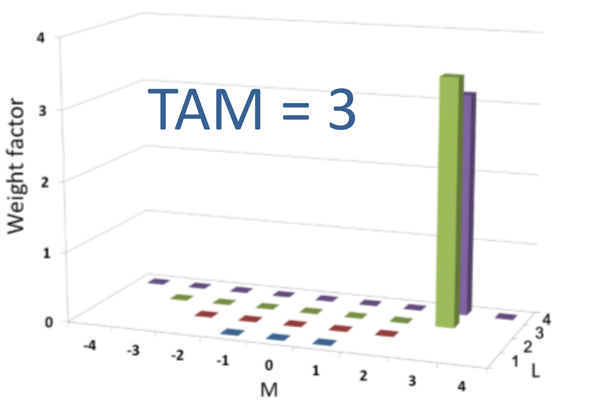
$$\propto \sum_{LM} \left\langle j_f\, m_f \middle| \alpha\, a_{LM}^{(e,m)} \middle| j_i\, m_i \right\rangle W_{LM}$$



For an atom at the beam axis (b=0) the structure of Bessel light suggests new selection rule:

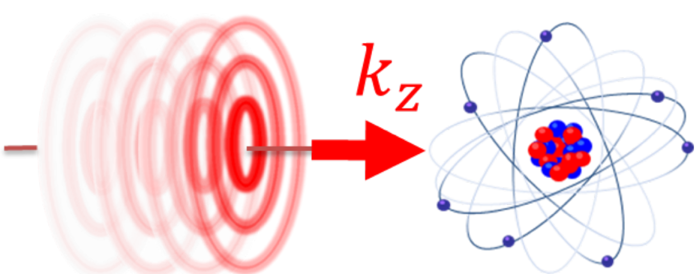
$$M = \text{TAM projection of beam}$$

$$L \geq |M|$$



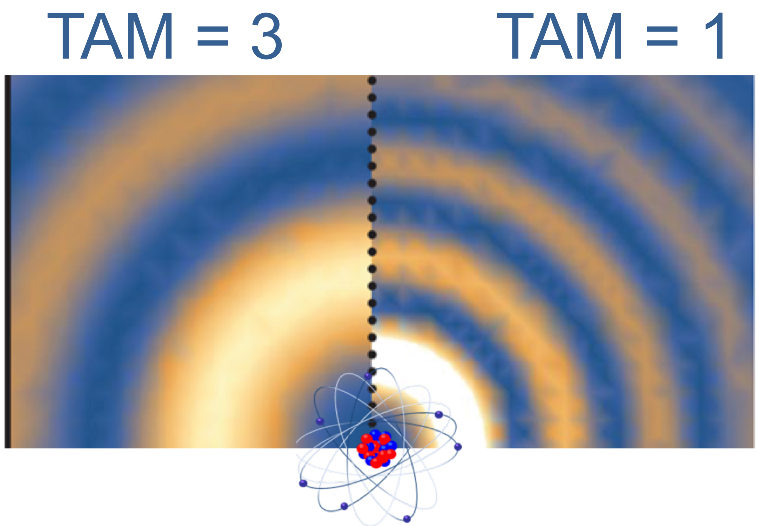
What allows us to operate with selection rules?

Bessel "selection rules": *b = 0* case



$$M_{if}^{\mathrm{tw}} = \int \psi_f^\dagger(\boldsymbol{r})\, \boldsymbol{\alpha}\, A(\boldsymbol{r}) \psi_i(\boldsymbol{r}) d\boldsymbol{r}$$

$$\propto \sum_{LM} \left\langle j_f\, m_f \middle| \alpha\, a_{LM}^{(e,m)} \middle| j_i\, m_i \right\rangle W_{LM}$$



For an atom at the beam axis (b=0) the structure of Bessel light suggests new selection rule:

$$M \;=\; \text{TAM projection of beam}$$

$$L \geq |M|$$

Atom, especially when located at the beam axis, experiences strongly inhomogenious field!

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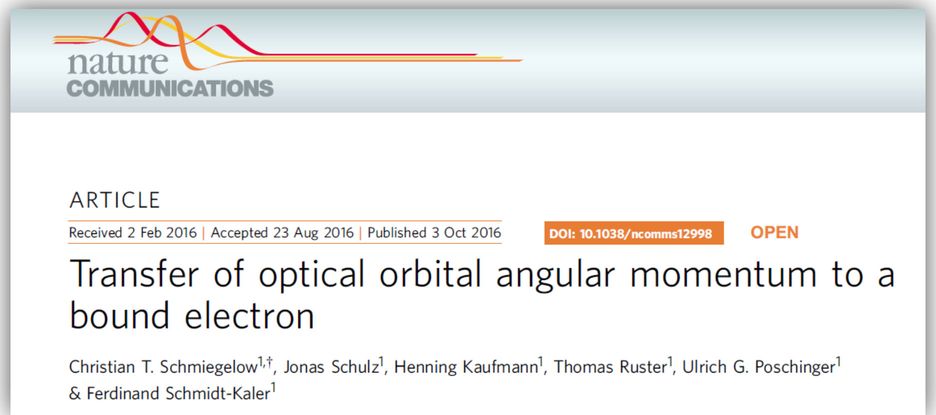
Experimental observation of the OAM transfer

Operation of transition selection rules by the orbital angular momentum has been demonstrated recently experimentally for a single trapped Ca^+ ion.

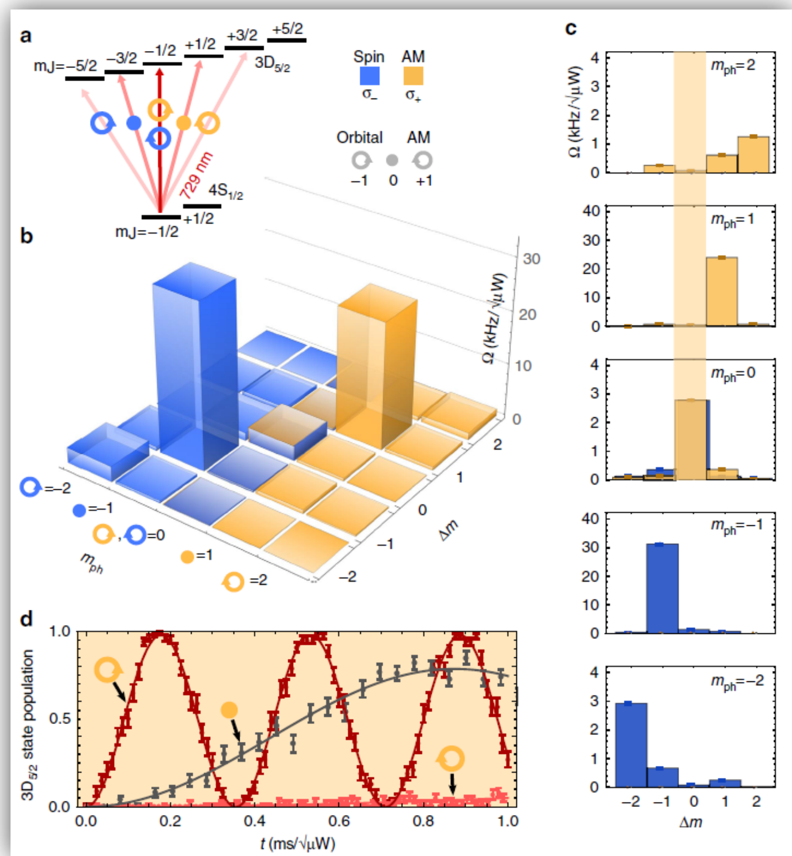

nature COMMUNICATIONS

ARTICLE

Received 2 Feb 2016 | Accepted 23 Aug 2016 | Published 3 Oct 2016 DOI: 10.1038/ncomms12998 OPEN

Transfer of optical orbital angular momentum to a bound electron

Christian T. Schmiegelow[1,†], Jonas Schulz[1], Henning Kaufmann[1], Thomas Ruster[1], Ulrich G. Poschinger[1] & Ferdinand Schmidt-Kaler[1]



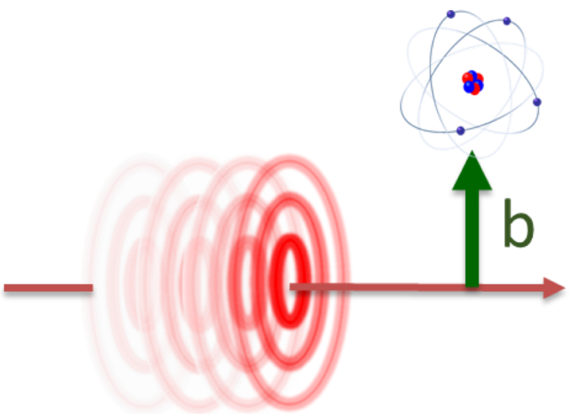
Experiment has been performed with Laguerre-Gaussian beam for which selection rule is written as:

$$m_i + m_l + \lambda = m_f$$

OAM projection

spin projection (helicity)

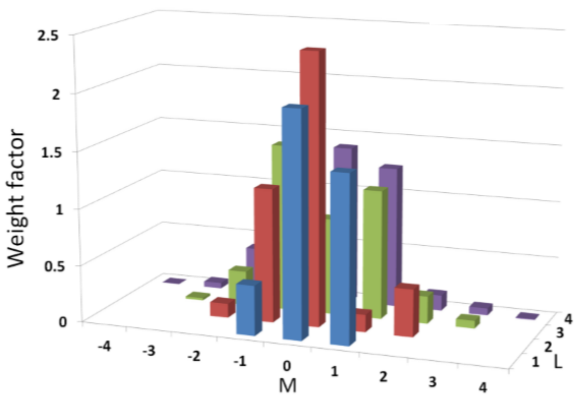
• Bessel "selection rules": *b ≠ 0* case

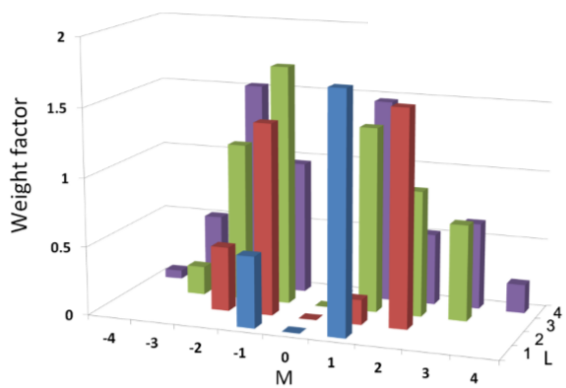


$$M_{if}^{\text{tw}} = \int \psi_f^{\dagger}(\boldsymbol{r})\, \boldsymbol{\alpha}\, A(\boldsymbol{r}) \psi_i(\boldsymbol{r}) d\boldsymbol{r}$$

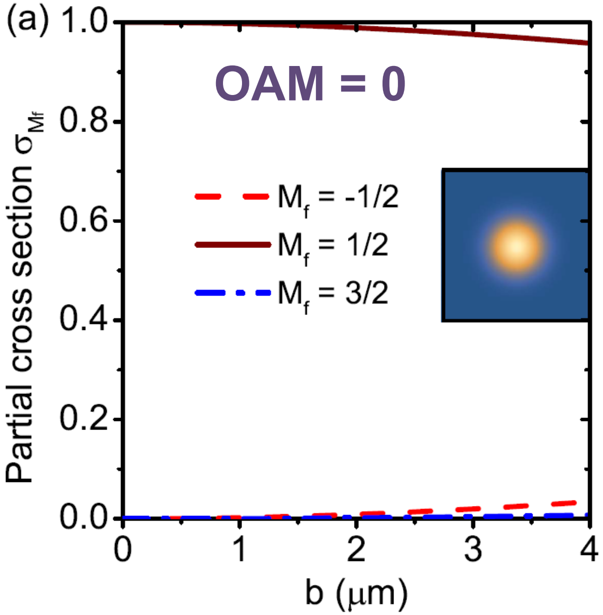
$$\propto \sum_{LM} \left\langle j_f\, m_f \middle| \alpha\, a_{LM}^{(e,m)} \middle| j_i\, m_i \right\rangle W_{LM}$$

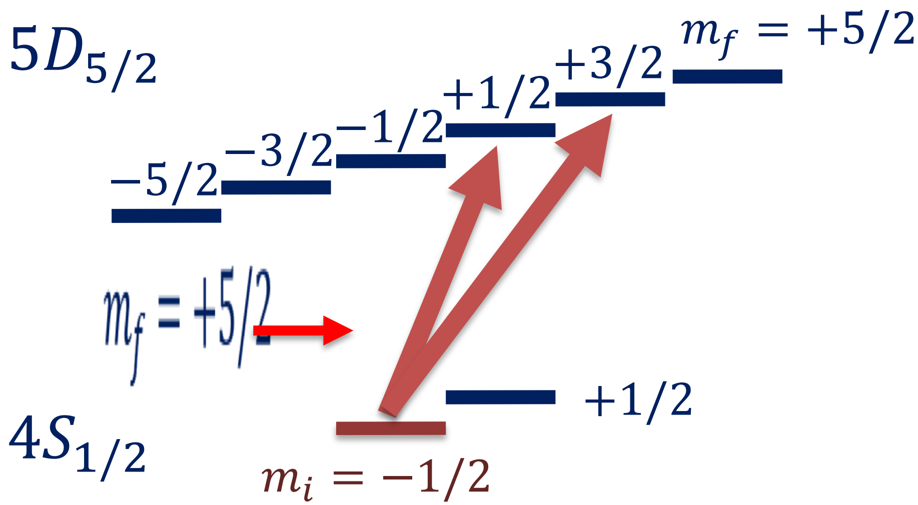
For an atom off the beam axis (b=0) the structure of Bessel light is even more complicated: various momenta and projections may arise depending on b.

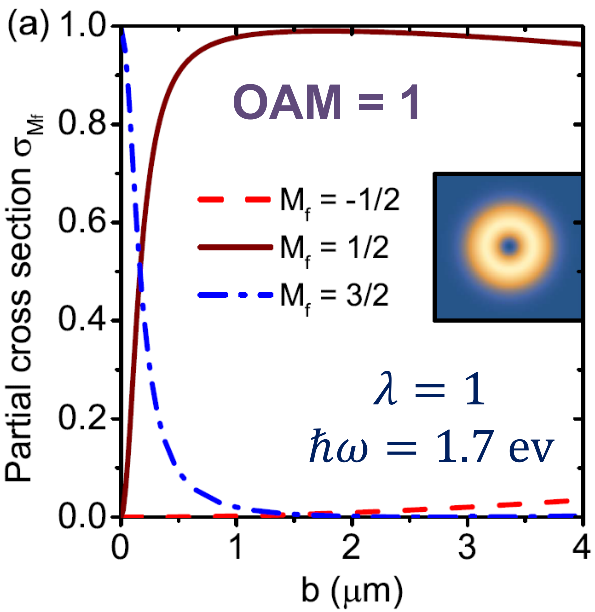




b-dependence of the "selection rule"





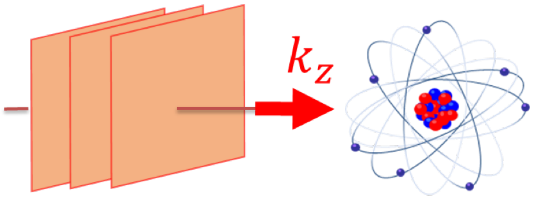


We have investigated dependence of the sublevel population and, hence, of the selection rule for the absorption of Laguerre- Gaussian beam.

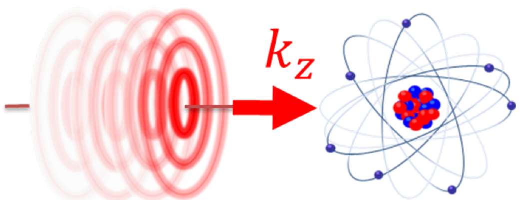
A. A. Peshkov, D. Seipt, A. S., and S. Fritzsche, Phys. Rev. A **96** (2017) 023407

Interaction of twisted light with matter

- Photoionization by twisted light
  - Ionization of a single atom and of mesoscopic target
- Photoexcitation by twisted light
  - Excitation of a single atom
- "Measurement" of the twistedness of emitted photons



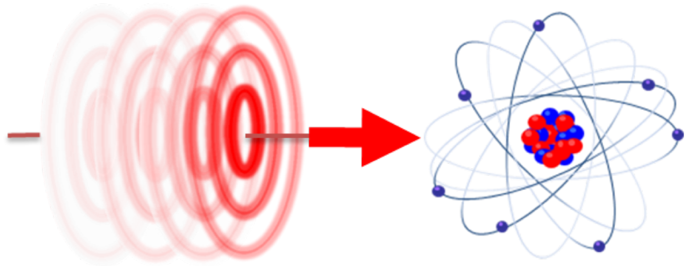
Incident plane-wave beam



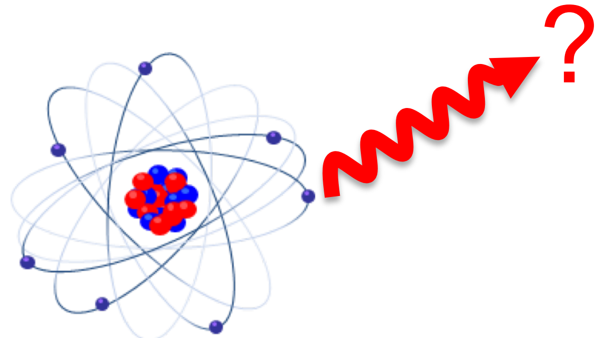
Incident twisted beam

OAM properties of emitted light?

So far most of the studies have dealt with incident twisted light



But what in known about twistedness of **emitted light**?



We need to develop theoretical approach to describe twistendess of photons, emitted in fundamental atomic processes!

But… how what is definition of twisted light?

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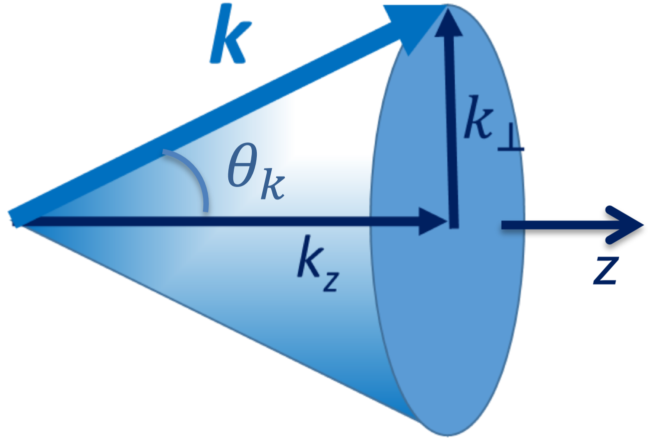
Bessel light beams: Definition

In our study we will use the so-called Bessel states of light, which are solutions of the equation:

$$\hat{J}_z\, A(\boldsymbol{r}) = m\, A(\boldsymbol{r})$$

And, hence, are characterized by the projection of total angular momentum *m*.



The vector potential of Bessel light state:

$$\boldsymbol{A}(\boldsymbol{r}) = \int a_{\kappa m}(\boldsymbol{k}_\perp)\, u_{\boldsymbol{k}\lambda}\, e^{i\boldsymbol{k}\boldsymbol{r}}\, \frac{d^2\boldsymbol{k}_\perp}{(2\pi)^2}$$

amplitude

plane-wave solution

$$a_{\kappa m}(k_\perp) = \sqrt{\frac{2\pi}{\kappa}}\, (-i)^m\, e^{im\varphi}\, \delta(k_\perp - \kappa)$$

Based on the analysis of Bessel photons we require:

- TAM > 1
- $\theta_k > 0$

O. Matula, A. G. Hayrapetyan, V. G. Serbo, A. S., and S. Fritzsche, J. Phys. B **46** (2013) 205002

Twistedness analysis: Theoretical background

To analyze the twistedness of the radiation emitted in atomic processes we propose to evaluate:

- **Mean value** and **dispersion** of the projection of the total angular momentum of light onto propagation direction $\hat{\boldsymbol{n}}_0$:

The mean value, written in matrix form, looks more complicated but allows analytical or numerical evaluation:

$$\langle \mathbf{J}\cdot\hat{\mathbf{n}}_0\rangle = \frac{\sum_{\lambda\lambda'\lambda''}\int d\mathbf{k}d\mathbf{k}'d\mathbf{k}''\,(8\omega\omega'\omega'')\,\langle \mathbf{k}\lambda\left|\rho^{(\mathrm{ph})}\right|\mathbf{k}'\lambda'\rangle\left\langle \mathbf{k}'\lambda'\left|\rho^{(\mathrm{det})}_{\hat{\mathbf{n}}_0}\right|\mathbf{k}''\lambda''\right\rangle\langle \mathbf{k}''\lambda''\left|(\mathbf{J}\cdot\hat{\mathbf{n}}_0)\right|\mathbf{k}\lambda\rangle}{\sum_{\lambda\lambda'}\int d\mathbf{k}d\mathbf{k}'\,(4\omega\omega')\,\langle \mathbf{k}\lambda\left|\rho^{(\mathrm{ph})}\right|\mathbf{k}'\lambda'\rangle\left\langle \mathbf{k}'\lambda'\left|\rho^{(\mathrm{det})}_{\hat{\mathbf{n}}_0}\right|\mathbf{k}\lambda\right\rangle}$$

Here the density matrix of emitted photons, $\left\langle k\lambda\middle|\hat{\rho}^{(\mathrm{ph})}\middle|k'\lambda'\right\rangle$, depends on particular process. But the matrix element of the operator $(\boldsymbol{J}\cdot\hat{\boldsymbol{n}}_0)$ can be evaluated in momentum space:

$$\langle \mathbf{k}'\lambda'\left|(\mathbf{J}\cdot\hat{\mathbf{n}}_0)\right|\mathbf{k}\lambda\rangle = \int d\mathbf{p}\,\mathbf{f}^{(\mathrm{pl})\dagger}_{\mathbf{k}'\lambda'}(\mathbf{p})\,(\mathbf{J}_p\cdot\hat{\mathbf{n}}_0)\,\mathbf{f}^{(\mathrm{pl})}_{\mathbf{k}\lambda}(\mathbf{p}) = \frac{1}{4\pi k^2}\delta(k-k')\frac{\delta_{\lambda\lambda'}}{2\omega}\sum_{\mu}(\hat{\mathbf{n}}_0)^{\mu}$$
$$\times\sum_{jm_jm'_j}(2j+1)\sqrt{j(j+1)}\,C^{jm'_j}_{jm_j\,1\mu}D^{j}_{m_j\lambda}(\varphi_k,\theta_k,0)D^{j*}_{m'_j\lambda}(\varphi'_k,\theta'_k,0)$$

Twistedness analysis: Theoretical background

To analyze the twistedness of the radiation emitted in atomic processes we propose to evaluate:

- **Mean value** and **dispersion** of the projection of the total angular momentum of light onto propagation direction $\hat{\boldsymbol{n}}_0 = \boldsymbol{k}/k$:

$$\langle \boldsymbol{J} \cdot \hat{\boldsymbol{n}}_0 \rangle = \frac{Tr\left(\hat{\rho}_\gamma \, \hat{\rho}_{\mathrm{det}} \, (\boldsymbol{J} \cdot \hat{\boldsymbol{n}}_0)\right)}{Tr\left(\hat{\rho}_\gamma \, \hat{\rho}_{\mathrm{det}}\right)} \qquad \Delta_J = \sqrt{\langle (\boldsymbol{J} \cdot \hat{\boldsymbol{n}}_0)^2 \rangle - \langle \boldsymbol{J} \cdot \hat{\boldsymbol{n}}_0 \rangle^2}$$

- **Mean value** and **dispersion** of the projection of the linear momentum of light onto propagation direction $\hat{\boldsymbol{n}}_0$:

$$\langle \boldsymbol{k} \cdot \hat{\boldsymbol{n}}_0 \rangle = \frac{Tr\left(\hat{\rho}_\gamma \, \hat{\rho}_{\mathrm{det}} \, (\boldsymbol{k} \cdot \hat{\boldsymbol{n}}_0)\right)}{Tr\left(\hat{\rho}_\gamma \, \hat{\rho}_{\mathrm{det}}\right)} \qquad \Delta_k = \sqrt{\langle (\boldsymbol{k} \cdot \hat{\boldsymbol{n}}_0)^2 \rangle - \langle \boldsymbol{k} \cdot \hat{\boldsymbol{n}}_0 \rangle^2}$$

V. Zaytsev, A.S, V. Shabaev, PRA *submitted*

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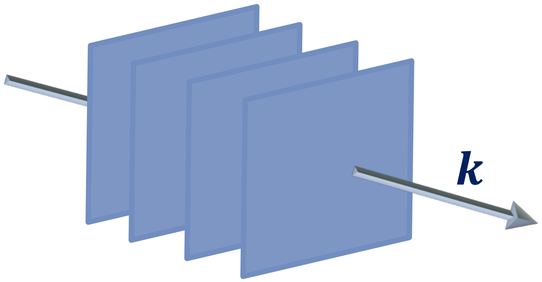
Twistedness analysis: Test cases

First we have applied our approach to the well-known test cases:

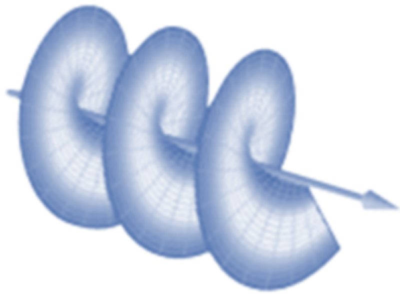
Plane wave



$$\langle \boldsymbol{J} \cdot \hat{\boldsymbol{n}}_0 \rangle = \lambda \qquad \langle (\boldsymbol{J} \cdot \hat{\boldsymbol{n}}_0)^2 \rangle = 1$$

$$\Delta_J = 0$$

Twisted wave



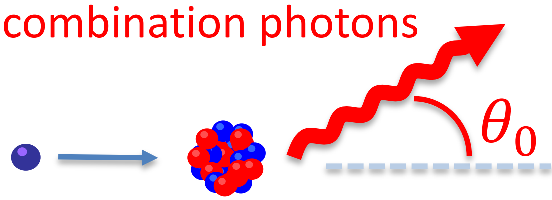
$$\langle \boldsymbol{J} \cdot \hat{\boldsymbol{n}}_0 \rangle = m \qquad \langle (\boldsymbol{J} \cdot \hat{\boldsymbol{n}}_0)^2 \rangle = m^2$$

$$\Delta_J = 0$$

Can we investigate now some basic atomic process?

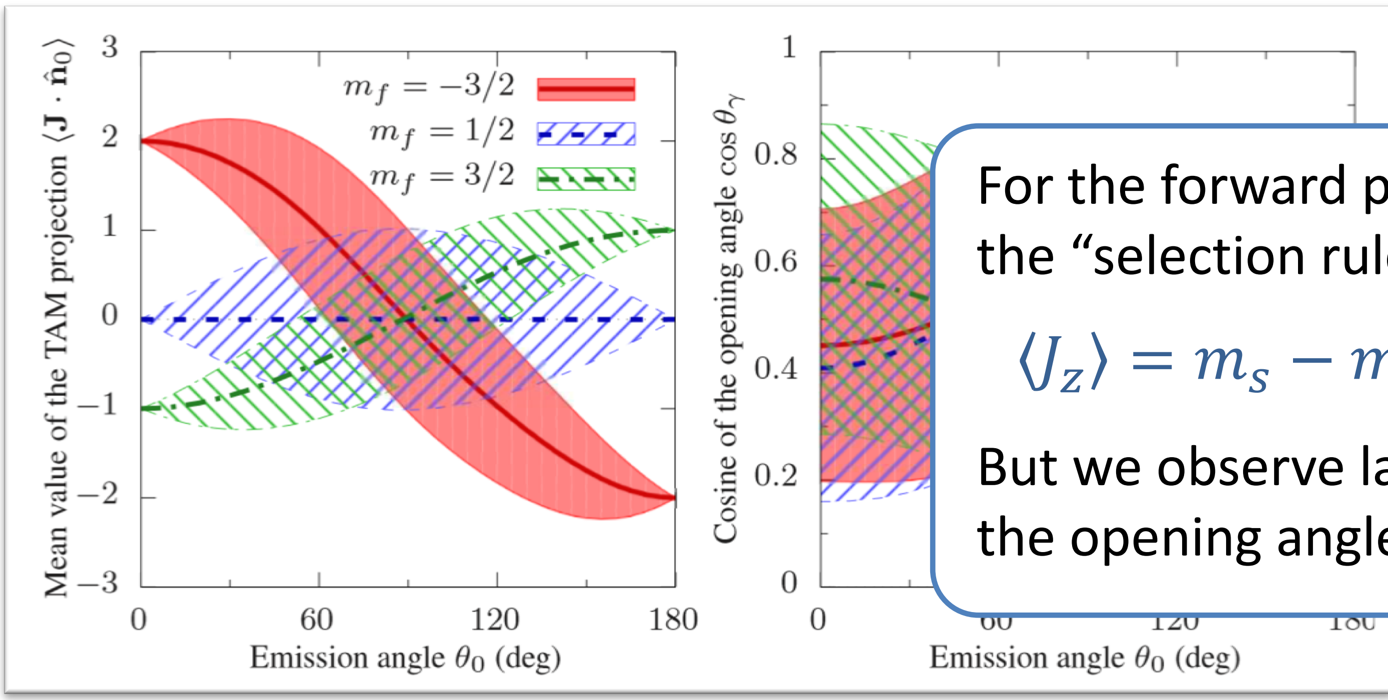
V. Zaytsev, A.S, V. Shabaev, PRA *submitted*

Radiative recombination of bare ions



Plane-wave, polarized ($m_s = 1/2$) electrons

We have studies RR of polarized electrons into various magnetic substates $2p_{3/2}(m_f)$ of finally hydrogen-like Ar ion. Incident electron energy is 2 keV.



For the forward photon emission the “selection rules” can be found:

$$\langle J_z \rangle = m_s - m_f \qquad \Delta_J = 0$$

But we observe large dispersion of the opening angle.

V. Zaytsev, A.S, V. Shabaev, PRA *submitted*

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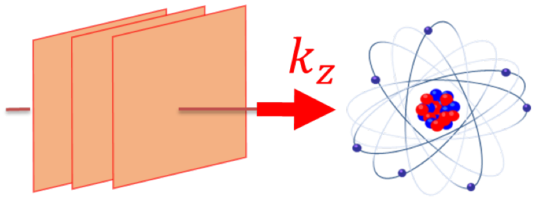
Summary

Twisted light beams provide a new tool for atomic physics studies and for many applications.

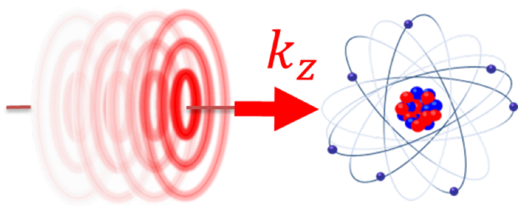
One need to better understand the interaction of twisted light with single atoms and atomic targets.

- Photoionization by twisted light
  - Ionization of a single atom and of mesoscopic target
    Probing (visualizing) the local energy flow



Incident plane-wave beam

- Photoexcitation
  - Excitation of a single atom
    Controlling of radiative selection rules



Incident twisted beam

- "Measurement" of the twistedness
  RR as a source of twisted γ-rays?

Many thanks to

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Anton Peshkov
Daniel Seipt
Andrey Volotka
HI-Jena

Vladimir Zaytsev
Vladimir Shabaev
St. Petersburg State University

Valery Serbo
Novoribisk State University

Thank you for your attention!

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