Exotic Nuclei and Beyond - An Ab Initio Theory Perspective -

Robert Roth











 $H |\Psi_n\rangle = E_n |\Psi_n\rangle$

solve nuclear many-body problem based on realistic interactions using controlled and improvable truncations with quantified theoretical uncertainties



$\mathsf{H} | \Psi_n \rangle = E_n | \Psi_n \rangle$

What is the nuclear Hamiltonian?

nuclear forces, chiral effective field theory, three-body interactions, consistency and convergence,...

What about these many-body states?

degrees of freedom, Hilbert space and truncations, single-particle basis, bound states vs. continuum,...

How to solve this equation?

large-scale diagonalization, decoupling approaches, truncations and convergence, uncertainty quantification...

Hamiltonian

$H = T - T_{cm} + V_{NN} + V_{3N} + \cdots$

Nuclear Interactions from Chiral EFT

Weinberg, van Kolck, Machleidt, Entem, Meißner, Epelbaum, Krebs, Bernard,...

- low-energy effective field theory for relevant degrees of freedom (π,N) based on symmetries of QCD
- explicit long-range pion dynamics
- unresolved short-range physics absorbed in contact terms, low-energy constants fit to experiment
- hierarchy of consistent NN, 3N, 4N,... interactions and electroweak operators
- many recent developments
 - improved NN up to N4LO+
 - 3N interaction up to N3LO
 - 4N interaction at N3LO
 - improved fits and error analysis
 - order-by-order uncertainty quantification





How to solve this equation?

large-scale diagonalization, decoupling approaches, truncations and convergence, uncertainty quantification...

Unitary Transformations



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NUCLEAR PHYSICS A

A unitary correlation operator method

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Abstract

The short range repulsion between nucleons is treated by a unitary correlation operator which shifts the nucleons away from each other whenever their uncorrelated positions are within the repulsive core. By formulating the correlation as a transformation of the relative distance between particle pairs, general analytic expressions for the correlated wave functions and correlated op-

Unitary Transformations

eigenvalue problem of H and expectation value of observable O
 H | Ψ_n > = E_n | Ψ_n >
 O_n = (Ψ_n | O | Ψ_n)

- invent a **unitary operator U** with $U U^{\dagger} = 1$ and use it
 - $U^{\dagger}HUU^{\dagger}|\Psi_{n}\rangle = E_{n}U^{\dagger}|\Psi_{n}\rangle \qquad O_{n} = \langle \Psi_{n}|UU^{\dagger}OUU^{\dagger}|\Psi_{n}\rangle$

identify transformed operators and states and rewrite equations

$$|\tilde{\Psi}_n\rangle = U^{\dagger} |\Psi_n\rangle, \qquad \tilde{H} = U^{\dagger} H U, \qquad \tilde{O} = U^{\dagger} O U$$

 $\tilde{H}|\tilde{\Psi}_n\rangle = E_n|\tilde{\Psi}_n\rangle$ $O_n = \langle \tilde{\Psi}_n|\tilde{O}|\tilde{\Psi}_n\rangle$

Unitary Transformations

eigenvalue problem of H and expectation value of observable O

 $\mathsf{H} | \Psi_n \rangle = E_n | \Psi_n \rangle$

 $O_n = \langle \Psi_n | O | \Psi_n \rangle$



Bohrloch - 14



Korrelieste Wellenfunktion (Index em weggelassen)

$$x X(x) = \left(\frac{dR}{dx}\right)^{1/2} R_{-}(x) \mathcal{G}(R_{-}(x))$$

/

$$R_{-}(x) := R(Y(x) - 1) \int R_{-}(R_{+}(x)) = r$$

$$R_{+}(x) := R(Y(x) + 1) \int R_{+}(R_{-}(x)) = x$$

$$\frac{p'(x)!}{dx} = \frac{dR_{-}(x)}{dx} = \frac{S(R_{-}(x))}{S(x)}$$

$$R'_{+}(x)! = \frac{dR_{+}(x)}{dx} = \frac{S(R_{+}(x))}{S(x)}$$

$$R'_{-}(x) = \frac{1}{R'_{+}(x)}$$
Wobii $r = R_{-}(x)$

Koordinatantransformation:

$$\tau = R_{-}(x)$$

 $x = R_{+}(r)$ $dx = R'_{+}(r) dr$; $\frac{d}{dx} = \frac{1}{R'_{+}(r)} \frac{d}{dr}$

Unitary Correlation Operator Method

Feldmeier, Neff, Roth, Schnack,...

unitary transformation designed to imprint central and tensor correlations into nuclear many-body states

explicit ansatz for unitary operator with central and tensor correlations

$$\mathbf{U} = \mathbf{C}_{\Omega}\mathbf{C}_{r} = \exp\left(-i\sum_{j< k} \mathbf{g}_{\Omega, jk}\right) \exp\left(-i\sum_{j< k} \mathbf{g}_{r, jk}\right)$$

central correlations: generated by distance-dependent shift in the relative coordinate of a nucleon pair

$$g_r = \frac{1}{2} [s(r) q_r + q_r s(r)] \qquad q_r = \frac{1}{2} [\frac{\vec{r}}{r} \cdot \vec{q} + \vec{q} \cdot \frac{\vec{r}}{r}]$$

tensor correlations: alignment of spatial orientation of nucleon pair depending on spin direction and orientation

$$g_{\Omega} = \frac{3}{2} \vartheta(r) \left[(\vec{\sigma}_1 \cdot \vec{q}_{\Omega}) (\vec{\sigma}_2 \cdot \vec{r}) + (\vec{q}_{\Omega} \leftrightarrow \vec{r}) \right] \qquad \vec{q}_{\Omega} = \vec{q} - \frac{\vec{r}}{r} q_r$$

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Similarity Renormalization Group

Glazek, Wilson, Wegner, Perry, Bogner, Furnstahl, Hergert, Roth,...

continuous unitary transformation of Hamiltonian towards diagonal form with respect to uncorrelated basis

consistent unitary transformation of Hamiltonian and observables

$$H_{\alpha} = U_{\alpha}^{\dagger} H U_{\alpha} \qquad O_{\alpha} = U_{\alpha}^{\dagger} O U_{\alpha}$$

• evolution equations for H_{α} and U_{α} with continuous flow parameter α

$$\frac{d}{d\alpha}H_{\alpha} = [\eta_{\alpha}, H_{\alpha}] \qquad \qquad \frac{d}{d\alpha}O_{\alpha} = [\eta_{\alpha}, O_{\alpha}] \qquad \qquad \frac{d}{d\alpha}U_{\alpha} = -U_{\alpha}\eta_{\alpha}$$

physics-guided choice dynamic generator η_α, e.g. for momentum prediagonalization

$$\eta_{\alpha} = (2\mu)^2 [T_{int}, H_{\alpha}]$$





















CE - 1



CLUSTER - ENTWICKLUNG

Versuchemstand: 1Q> = ens 1Q0> In Slates determinante

S = St feilchen Fahlerbaltend

Effection Operator: Deff := e is o e-is

Erwashings weit ; <QIQIQ> = <Qole²⁵ Qe⁻¹⁵ |Qo> = <Qol Qeg |Qo> Cluster Entwichlung von Daff $Q_{eff} = \sum_{n=1}^{\infty} Q_{eff}^{(n)}$, Product zustände $\mathcal{D}_{eff}^{(n)} := \frac{1}{n!} \sum_{k \neq w} \langle k_{k} k_{2} \cdots k_{n} | \mathcal{Q}_{eff} - \sum_{m=0}^{m} \mathcal{Q}_{eff}^{(m)} | \ell_{i} \ell_{i} \cdots \ell_{n} \rangle a_{k_{i}}^{\dagger} a_{k_{i}}^{\dagger} a_{e_{n}}^{\dagger} d_{e_{2}}^{\dagger} d_{e_{1}}^{\dagger} d_{e_{2}}^{\dagger} d_{e_{1}}^{\dagger} d_{e_{2}}^{\dagger} d_{e_{1}}^{\dagger} d_{e_{2}}^{\dagger} d_{e_{2}}^{\dagger} d_{e_{1}}^{\dagger} d_{e_{2}}^{\dagger} d_{e_{1}}^{\dagger} d_{e_{2}}^{\dagger} d_{e_{1}}^{\dagger} d_{e_{2}}^{\dagger} d_{e_{2}}^{$

Hamiltonian in A-Body Space

evolution induces *n*-body contributions H_α^[n] to Hamiltonian

$$H_{\alpha} = H_{\alpha}^{[1]} + H_{\alpha}^{[2]} + H_{\alpha}^{[3]} + H_{\alpha}^{[4]} + \cdots$$

- truncation of cluster series formally destroys unitarity and invariance of observables (independence of α)
- flow-parameter variation provides diagnostic tool to assess neglected contributions of higher particle ranks

SRG-Evolved Hamiltonians

NNonly: use initial NN, keep evolved NN

NN+3N_{ind} : use initial NN, keep evolved NN+3N

NN+3N_{full} : use initial NN+3N, keep evolved NN+3N

NN+3N_{full}+4N_{ind} : use initial NN+3N, keep evolved NN+3N+4N

Many-Body Problem

No-Core Shell Model

Barrett, Vary, Navrátil, Maris, Nogga, Roth,...

no-core shell model is the most universal and powerful ab initio approach for light nuclei (up to A≈25)

• idea: solve eigenvalue problem of Hamiltonian represented in model space of HO Slater determinants truncated w.r.t. HO excitation energy $N_{max}\hbar\Omega$

$$\begin{pmatrix} \vdots \\ C_{l'}^{(n)} \\ \vdots \end{pmatrix} = E_n \begin{pmatrix} \vdots \\ C_{l'}^{(n)} \\ \vdots \end{pmatrix}$$

No-Core Shell Model

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- idea: solve eigenvalue problem of Hamiltonian represented in model space of HO Slater determinants truncated w.r.t. HO excitation energy $N_{\text{max}}\hbar\Omega$
 - convergence of observables w.r.t. N_{max} is the only limitation and source of uncertainty
- Importance truncation: reduce NCSM model space to physically relevant basis states and extrapolate to full space a posteriori
 - increases the range of applicability of NCSM significantly

alternative basis: optimize basis to enhance model-space convergence or to include continuum physics

- single-particle basis: natural orbitals, Gamow states
- many-body basis: resonating group method with binary clusters

NCSM: Convergence - Natural Orbitals

J. Müller, A. Tichai, K. Vobig, R. Roth, in prep.



NAT basis, NN+3N(500), α =0.08 fm⁴, e_{max} =12

NCSM: Oxygen Isotopes

J. Müller, A. Tichai, K. Vobig, R. Roth, in prep.





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Decoupling Methods

advent of novel ab initio approaches targeting the ground state of medium-mass nuclei very efficiently

idea: decouple reference state from particle-hole excitations by a unitary or similarity transformation of Hamiltonian



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Tsukiyama, Bogner, Schwenk, Hergert,...

- In-Medium Similarity Renormalisation Group: decouple many-body reference state from particle-hole excitations by SRG transformation
 - normal-ordered A-body Hamiltonian truncated at the two-body level
 - open and closed-shell nuclei can be targeted directly

Hagen, Papenbrock, Dean, Piecuch, Binder,...

- Coupled-Cluster Theory: ground-state is parametrised by exponential wave operator acting on single-determinant reference state
 - truncation at doubles level (CCSD) with corrections for triples contributions
 - directly applicable for closed-shell nuclei, equations-of-motion methods for open-shell

In-Medium SRG

Tsukiyama, Bogner, Schwenk, Hergert,...



Hamiltonian and generator in normal order with respect to single or multideterminant reference state, omit residual three-body piece

$$H(s) = E(s) + \sum_{ij} f_j^i(s) \tilde{A}_j^i + \frac{1}{4} \sum_{ijkl} \Gamma_{kl}^{ij}(s) \tilde{A}_{kl}^{ij} + \frac{1}{36} \sum_{ijklmn} W_{lmn}^{ijk}(s) \tilde{A}_{lmn}^{ijk}$$

define generator to suppress off-diagonal contributions that couple reference state to ph excitations

$$\gamma(s) = [H(s), H^{d}(s)] = [H^{od}(s), H^{d}(s)]$$

Gebrerufael, Vobig, Hergert, Roth,...



Gebrerufael, et al.; PRL 118, 152503 (2017)



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IM-NCSM: Oxygen Isotopes



IM-NCSM: Oxygen Isotopes

Vobig, Gebrerufael, Roth; in prep.



IM-NCSM: Excitation Spectra



- IM-NCSM and direct NCSM in excellent agreement for converged states
- first excited 0⁺ states in ¹²C and ¹⁶C differ

Hypernuclei

Ab Initio Hypernuclear Structure



- precise data on ground states & spectroscopy of hypernuclei
- ab initio few-body and phenomen. shell-model, mean-field or cluster-model calculations done so far
- chiral YN & YY interactions at (N)LO are available

time to transfer ab initio toolbox to hypernuclei

Ab Initio Toolbox for Hypernuclei

Wirth et al.; PRL 113, 192502 (2014); PRL 117, 182501 (2016)

Hamiltonian from chiral EFT

- NN+3N: standard chiral Hamiltonian (Entem&Machleidt, Navrátil)
- YN: LO chiral interaction (Haidenbauer et al.), NLO in progress

Similarity Renormalization Group

- consistent SRG-evolution of NN, 3N, YN interactions
- using particle basis and including $\Lambda\Sigma$ -coupling (larger matrices)
- Λ - Σ mass difference and $p\Sigma^{\pm}$ Coulomb included consistently

Importance Truncated No-Core Shell Model

- include explicit $(p, n, \Lambda, \Sigma^+, \Sigma^0, \Sigma^-)$ with physical masses
- larger model spaces easily tractable with importance truncation
- all p-shell single-Λ hypernuclei are accessible

Application: $^{7}_{\Lambda}$ Li

Wirth et al.; PRL 113, 192502 (2014); PRL 117, 182501 (2016)



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Application: $^{7}_{\Lambda}$ Li

Wirth et al.; PRL 113, 192502 (2014); PRL 117, 182501 (2016)



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Application: $^{9}_{\Lambda}Be$

Wirth et al.; PRL 113, 192502 (2014); PRL 117, 182501 (2016)



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Application: $^{13}_{\Lambda}C$

Wirth et al.; PRL 113, 192502 (2014); PRL 117, 182501 (2016)



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Light Neutron-Rich Hypernuclei

Wirth et al.; PLB 779, 336 (2018)



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Suppression of Λ - Σ Conversion

Wirth et al.; PRL 117, 182501 (2016)



• design SRG-generator that suppresses the Λ - Σ conversion exclusively

- Σ admixture in the wave functions eliminated or "integrated out"
- same large induced YNN interactions as in standard SRG

Suppression of Λ - Σ Conversion

Implications for the Hyperon Puzzle

- neutron stars reach densities, where hyperon production should be energetically favorable
- \blacksquare including explicit As with AN interaction softens EOS does not support 2M \odot neutron star
- phenomenological fix: introduce strongly repulsive ANN interaction

fine**art** america

Conclusions

A Look Back...

past 20 years have seen dramatic progress in ab initio many-body methods for nuclear structure and reactions

...extensions of NCSM, coupled-cluster theory, in-medium SRG, self-consistent Green's function, many-body perturbation theory

a number of important developments are in progress

...spectroscopy of open-shell nuclei, merging NCSM and IM-SRG, derivation of valence-space interactions, broad range of observables

the reach of ab initio methods has grown tremendously

...medium-mass and heavy nuclei, low-lying and collective excitations, continuum effects and reaction observables, resonances, hypernuclei

for the next few years the focus will move towards improvements of the chiral interactions

...consistent higher orders, systematic study of order-by-order convergence, inclusion of consistent currents, improved fitting strategies

rigorous quantification of theoretical uncertainties will play an important role

...propagation of uncertainties from chiral EFT inputs to nuclear structure observables, full quantification of many-body uncertainties

Iots of relevant physics predictions...

All My Best Wishes to...

... and special thanks for so many things

GSI Theory Excursion 2002

Epilogue

thanks to my group and my collaborators

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