Fluctuations & Correlations in Heavy Ion Collision

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1. Fluctuations : Some introductory remarks.
2. Results from Hadronic Model
3. Results from Quark Model
4. Discussion

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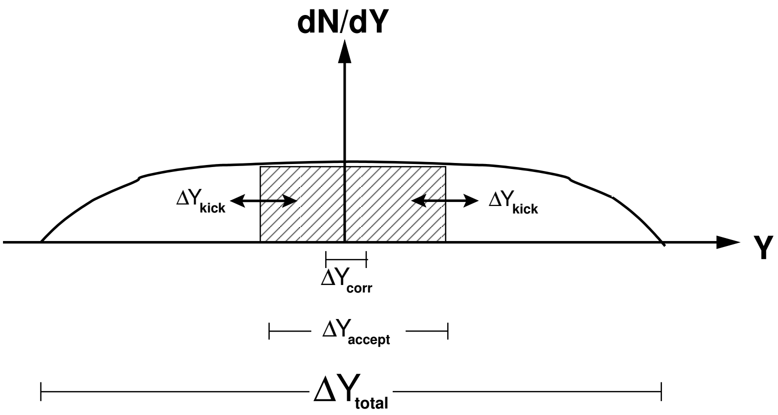
- The most efficient way to address fluctuations of a system created in a heavy-ion collision is via the study of event-by-event fluctuations.
- In addition, the study of fluctuations may reveal information beyond its thermodynamic properties.



Charge fluctuations will be able to tell us about the properties of the early stage of the system, the QGP, if the following criteria are met:

$$\Delta Y_{accept} \gg \Delta Y_{corr} \quad \text{and} \quad \Delta \mathrm{Y}_{\mathrm{total}} \gg \Delta \mathrm{Y}_{\mathrm{accept}} \gg \Delta \mathrm{Y}_{\mathrm{kick}}$$

V. Koch, arXiv : 0810.2520

Taylor Expansion of Pressure I

$$P(T,\mu_B,\mu_Q,\mu_S) = -\Omega(T,\mu_B,\mu_Q,\mu_S),$$

$$\frac{P(T,\mu_B,\mu_Q,\mu_S)}{T^4} = \sum_{n=i+j+k} c^{B,Q,S}_{i,j,k}(T)(\frac{\mu_B}{T})^i(\frac{\mu_Q}{T})^j(\frac{\mu_S}{T})^k$$

where,

$$c^{B,Q,S}_{i,j,k}(T) = \frac{1}{i!j!k!}\frac{\partial^i}{\partial(\frac{\mu_B}{T})^i}\frac{\partial^j}{\partial(\frac{\mu_Q}{T})^j}\frac{\partial^k(P/T^4)}{\partial(\frac{\mu_S}{T})^k}\Big|_{\mu_{B,Q,S}=0}$$

$$\mu_u = \mu_q + \frac{2}{3}\mu_Q, \quad \mu_d = \mu_q - \frac{1}{3}\mu_Q, \quad \mu_s = \mu_q - \frac{1}{3}\mu_Q - \mu_S$$

For diagonal Taylor coefficients we have used,

$$c_n^X = \frac{1}{n!}\frac{\partial^n\left(P/T^4\right)}{\partial\left(\frac{\mu_X}{T}\right)^n}; \qquad n = i + j$$

For off-diagonal Taylor coefficients we have used,

$$c_{i,j}^{X,Y} = \frac{1}{i!j!}\frac{\partial^{i+j}\left(P/T^4\right)}{\partial\left(\frac{\mu_X}{T}\right)^i\partial\left(\frac{\mu_Y}{T}\right)^j}$$

Diagonal and off-diagonal susceptibilities are respectively defined as,

$$\chi_{XY} = \frac{\partial^2(P/T^4)}{\partial(\mu_X/T)\partial(\mu_Y/T)} \qquad \chi_{XX} = \frac{\partial^2(P/T^4)}{\partial(\mu_X/T)^2}$$

- Pressure consists of two parts; one regular part and one non-analytic part.
$$P(T, \mu_u, \mu_d) = P_r(T, \mu_u, \mu_d) + P_s(\bar{t}, \bar{\mu}_u, \bar{\mu}_d)$$
with $\bar{t} = (T - T_C)/T_C$ and $\bar{\mu}_{u,d} = \mu_{u,d}/T$.
- $$t \equiv \bar{t} + A\mu_q^2 + B\mu_I^2$$
- From universal scaling behaviour;
$$P_s(\bar{t}, \bar{\mu}_u, \bar{\mu}_d) \sim t^{2-\alpha}$$
Then second and forth cumulant get contribution like;
$$(\partial^2 P_s/\partial\mu_X^2) \sim t^{1-\alpha} + \text{regular} \quad \text{and} \quad (\partial^4 P_s/\partial\mu_X^4) \sim t^{-\alpha} + \text{regular}$$

S. Ejiri *et. al.* Phys. Lett. B 633 (2006) 275.

Hadron Resonance Gas Model I

$$\ln Z^{id} = \sum_i \ln Z_i^{id}, \tag{1}$$

$$\ln Z_i^{id} = \frac{Vg_i}{2\pi^2} \int_0^\infty \pm p^2\, dp \ln[1 \pm \exp(-(E_i - \mu_i)/T)], \tag{2}$$

$$P_i^{id} = \frac{T}{V} \ln Z_i^{id} = \pm \frac{g_i T}{2\pi^2} \int_0^\infty p^2\, dp \ln[1 \pm \exp(-(E_i - \mu_i)/T)], \tag{3}$$

In excluded volume EVHRG model pressure can be written as

$$P(T,\mu_1,\mu_2,..) = \sum_i P_i^{id}(T,\hat{\mu}_1,\hat{\mu}_2,..), \tag{4}$$

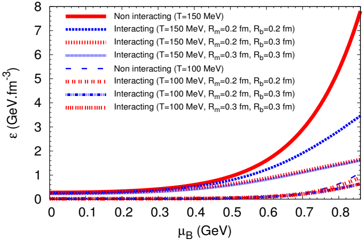
where for i th particle chemical potential is

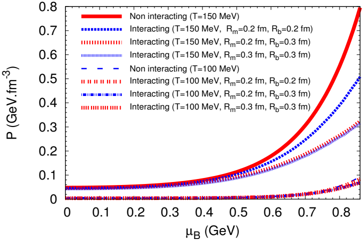
$$\hat{\mu}_i = \mu_i - V_{ev,i} P(T,\mu_1,\mu_2,..). \tag{5}$$

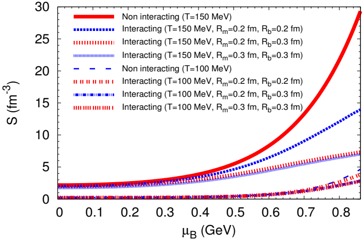
$P(T,\mu_1,\mu_2,..)$ is suppressed compared to the P^{id} because of suppression of effective chemical potential. Particles number density, entropy density and energy density are suppressed by a factor

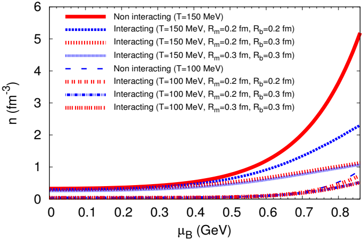
$$\frac{1}{1+\sum_k V_{ev,k} n_k^{id}(T,\hat{\mu}_k)}. \tag{6}$$

Thermodynamic Quantities

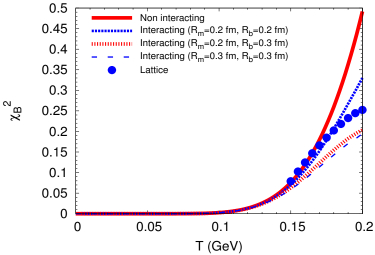


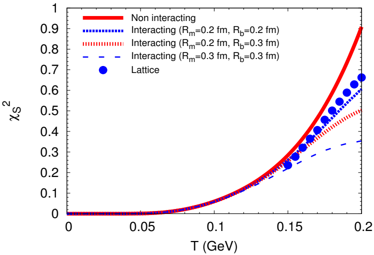


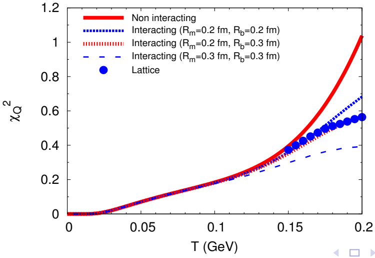




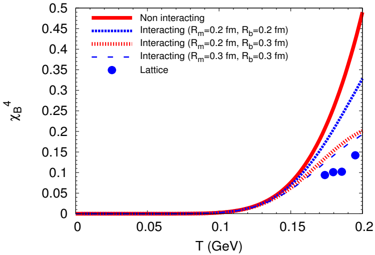
Second order susceptibilities

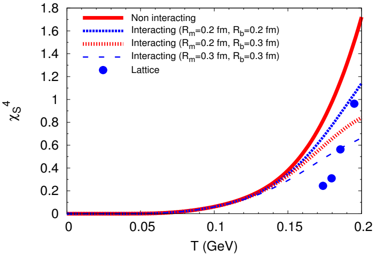


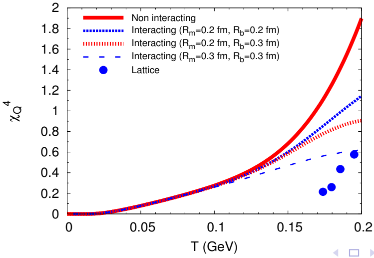




Fourth order susceptibilities

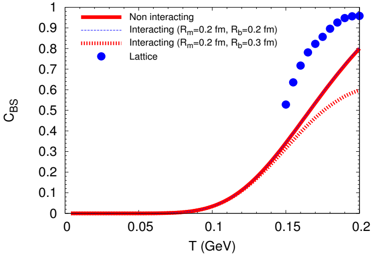


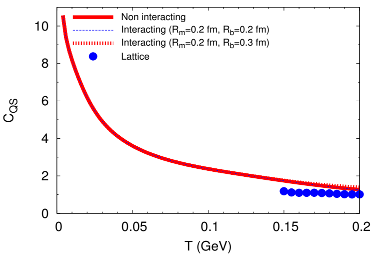


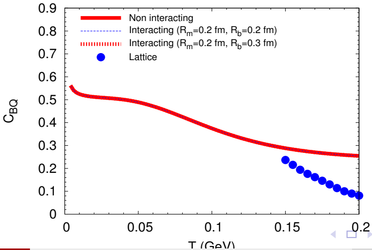


Offdiagonal susceptibilities

$$C_{BS} = -3\frac{\chi^{11}_{BS}}{\chi^2_S}, \qquad C_{QS} = 3\frac{\chi^{11}_{QS}}{\chi^2_S}, \qquad C_{BQ} = \frac{\chi^{11}_{BQ}}{\chi^2_B}. \tag{7}$$





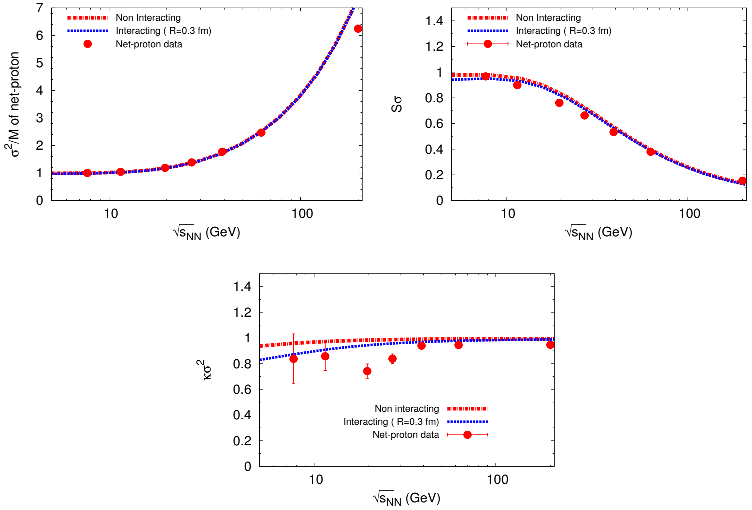


Experimental Observables

$$\sigma = \sqrt{\langle (N - \langle N \rangle)^2 \rangle}$$
$$s = \frac{\langle (N - \langle N \rangle)^3 \rangle}{\sigma^3}$$
$$\kappa = \frac{\langle (N - \langle N \rangle)^4 \rangle}{\sigma^4} \tag{8}$$

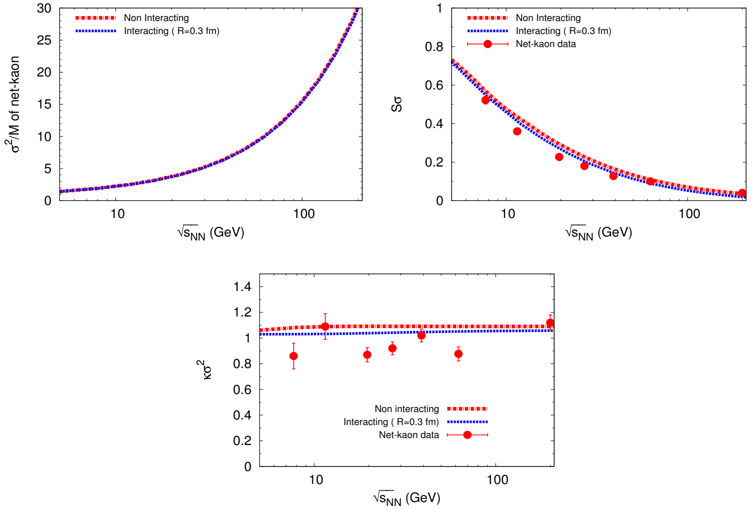
$$\frac{\chi^2_X}{\chi^1_X} = \frac{\sigma^2_X}{M_X}, \quad \frac{\chi^3_X}{\chi^2_X} = S_X \sigma_X, \quad \frac{\chi^4_X}{\chi^2_X} = \kappa_q \sigma^2_X. \tag{9}$$

Net proton

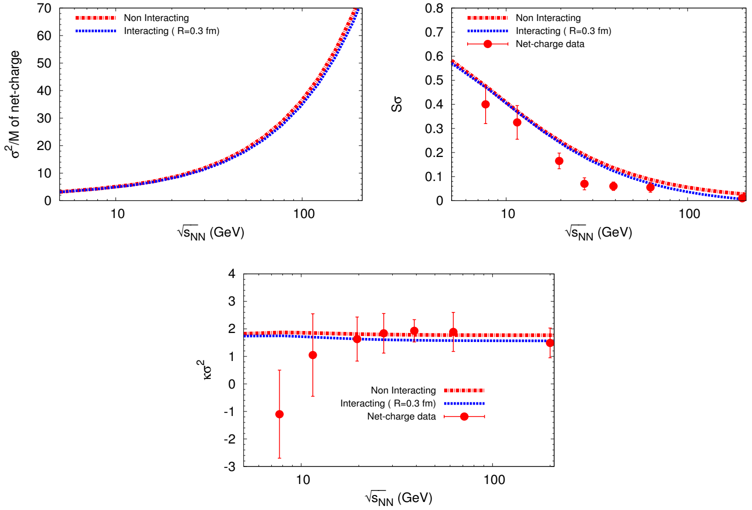


Non Interacting
Interacting (R=0.3 fm)
Net-proton data
σ²/M of net-proton
√sNN (GeV)
Sσ
√sNN (GeV)
Non interacting
Interacting (R=0.3 fm)
Net-proton data
κσ²
√sNN (GeV)

Net kaon


Non Interacting
Interacting (R=0.3 fm)
σ^2/M of net-kaon
$\sqrt{s_{NN}}$ (GeV)
Non Interacting
Interacting (R=0.3 fm)
Net-kaon data
$S\sigma$
$\sqrt{s_{NN}}$ (GeV)
Non interacting
Interacting (R=0.3 fm)
Net-kaon data
$\kappa\sigma^2$
$\sqrt{s_{NN}}$ (GeV)

Net charge



70
60
50
40
30
20
10
0
σ²/M of net-charge
Non Interacting
Interacting (R=0.3 fm)
10
100
√sNN (GeV)
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Sσ
Non Interacting
Interacting (R=0.3 fm)
Net-charge data
10
100
√sNN (GeV)
4
3
2
1
0
-1
-2
-3
κσ²
Non Interacting
Interacting (R=0.3 fm)
Net-charge data
10
100
√sNN (GeV)

A WORD OF CAUTION

$$T(\mu_B) = a - b\mu_B^2 - c\mu_B^4, \tag{10}$$

where $a = 0.166 \pm 0.002$ GeV, $b = 0.139 \pm 0.016$ GeV^{-1}, $c = 0.053 \pm 0.021$ GeV^{-3}.

$$\mu_X(\sqrt{s_{NN}}) = \frac{d_X}{1 + e_X\sqrt{s_{NN}}}, \tag{11}$$

X	d_X (GeV)	e_X (GeV^{-1})
B	1.308 ± 0.028	0.273 ± 0.008
S	0.214	0.161
Q	-0.0211	0.106

Motivation behind PNJL Model

- Nambu-Jona-Lasinio (NJL) model : Originally proposed for studying hadronic d.o.f. Later extended to quark d.o.f.
  Reproduces chiral symmetry breaking of QCD successfully through a non-vanishing chiral condensate.

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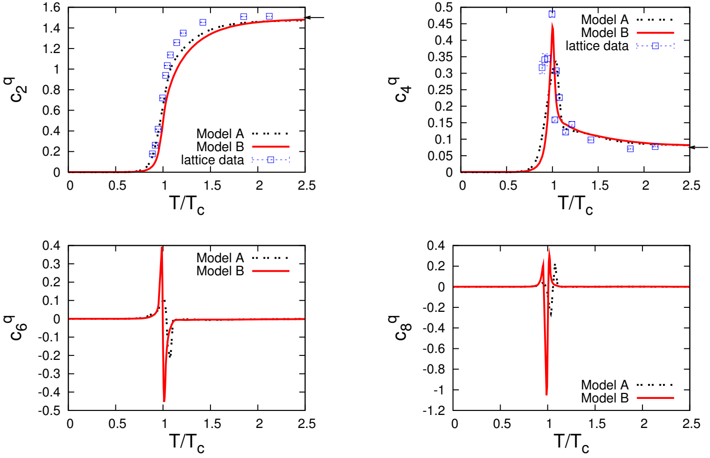
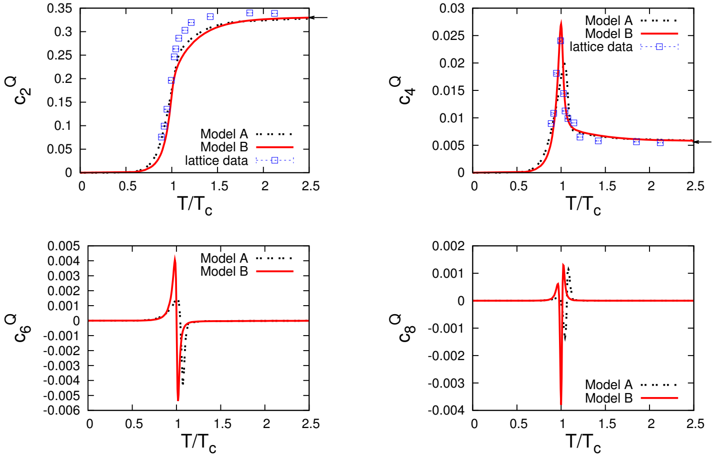
- Polyakov loop model : Originally proposed for pure gauge system. Reproduces confinement-deconfinement transition of QCD.
- Polyakov loop-Nambu-Jona-Lasinio (PNJL) model tied together these two aspects of QCD.

Taylor Coefficents for μ_q.....



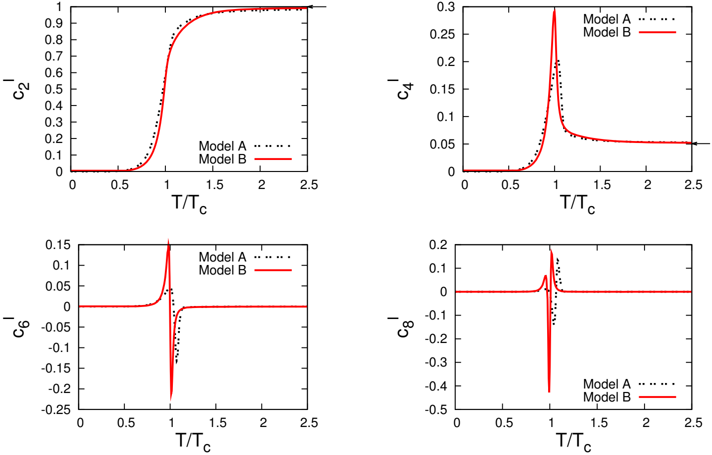
Lattice data taken from M. Cheng *et. al.* Phys. Rev. D 79, 074505 (2009).

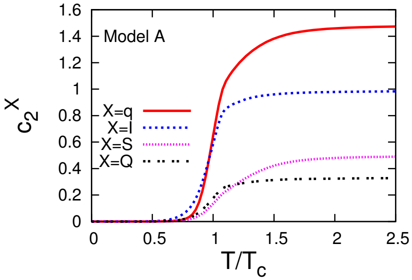
Taylor Coefficents for μ_Q.....

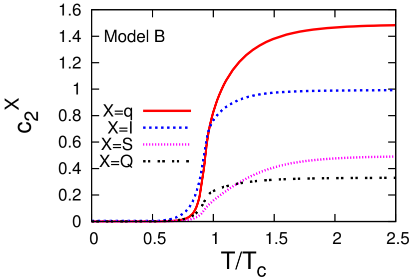


Lattice data taken from M. Cheng *et. al.* Phys. Rev. D 79, 074505 (2009).

Taylor Coefficents for μ_I.....



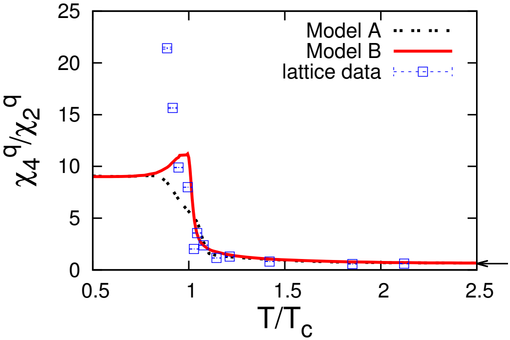




All diagonal Taylor coefficients show characteristic crossover $\Rightarrow$ QCD phase transition liberates quarks.

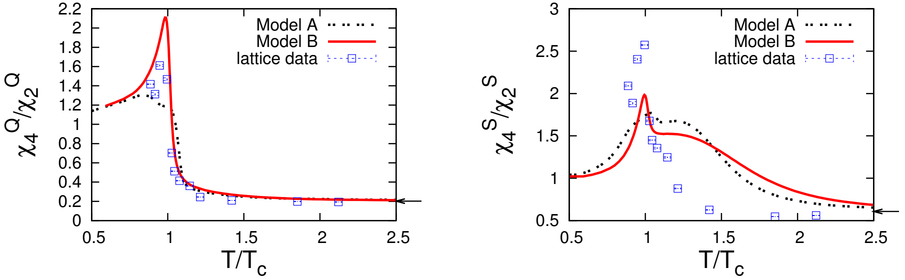
S. Gottlieb *et al.*, Phys. Rev. Lett. 59, 2247 (1987); R. V. Gavai *et al.*, Phys. Rev. D 40, 2743 (1989).

Kurtosis I



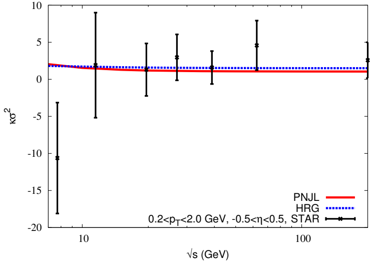
Kurtosis is a sensitive probe of deconfinement.
At low T kurtosis $R_q = (N_c B)^2 = 9$ and at high T it becomes unity in classical consideration and if corrected by quantum statistics $R_q = (6/\pi^2)$.

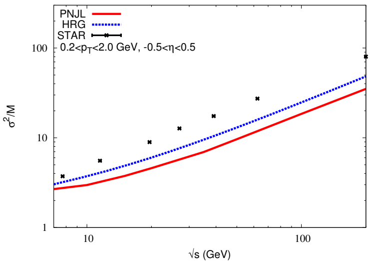
Kurtosis II

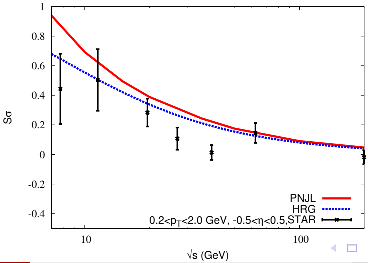


At low T, R_Q is dominated by charge fluctuations in pion sector resulting $R_Q = 1$. At high T, $R_Q = 2/\pi^2$ which is its SB limit.
Kurtosis for strange sector shows a peak at T_c. Model shows enhanced fluctuations after T_c and then converges to its SB limit.

Charge Fluctuations



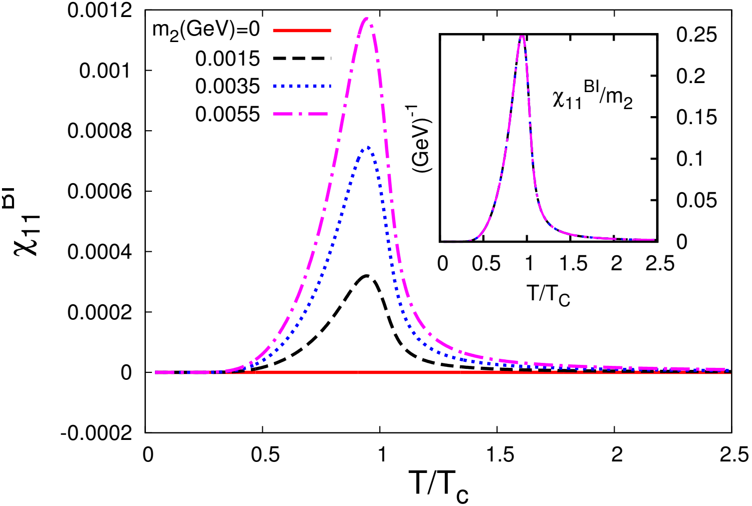




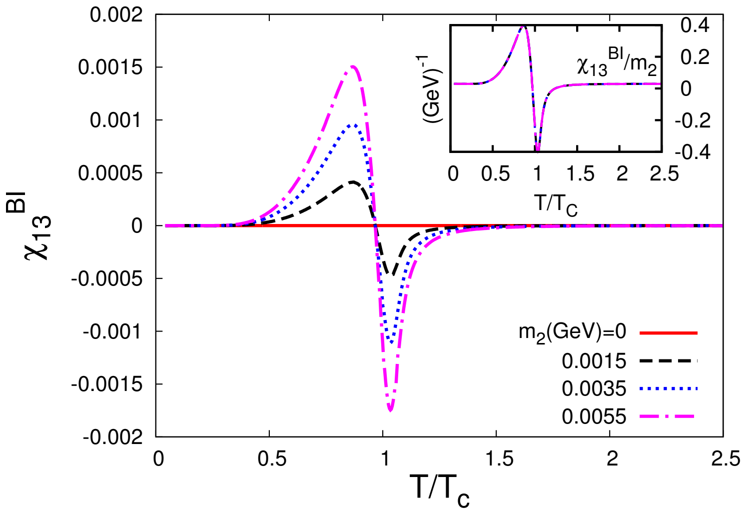
Isospin breaking and fluctuation

$$\hat{m} \equiv m_1 \mathbb{1}_{2\times 2} - m_2 \tau_3$$
$$= \begin{pmatrix} m_1 - m_2 & 0 \\ 0 & m_1 + m_2 \end{pmatrix} = \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix}.$$

2^{nd} order $B - I$ correlation along T



4^{th} order $B - I$ correlation along T



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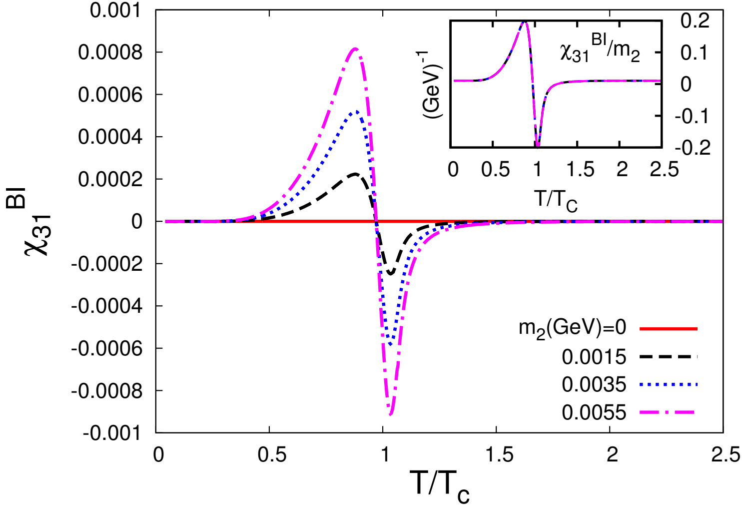
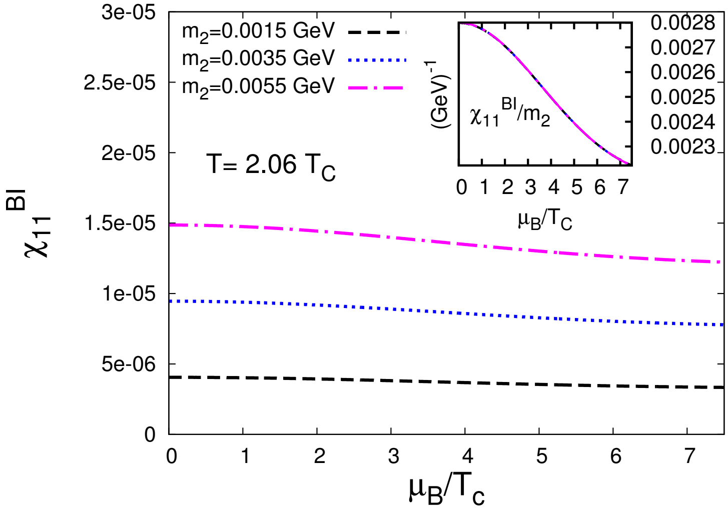
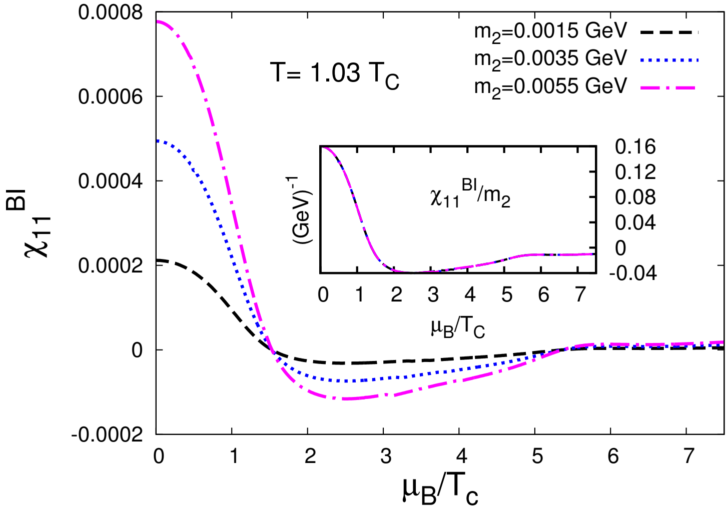


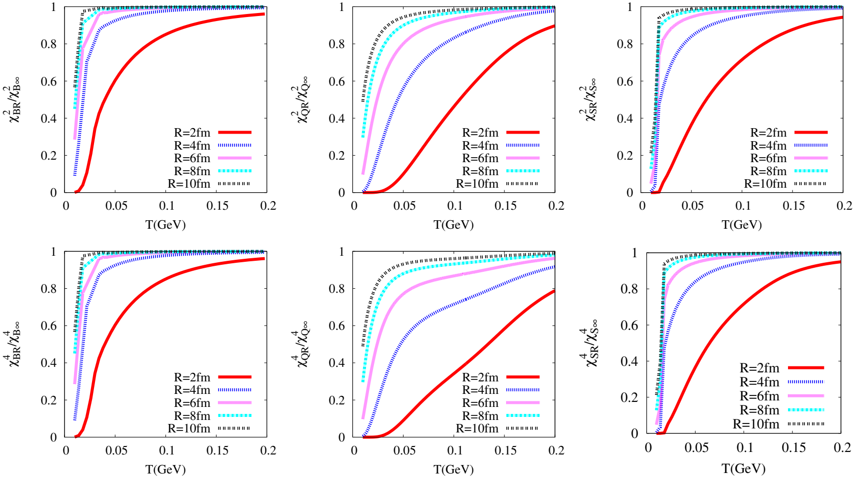
2^{nd} order $B - I$ correlation along μ_B



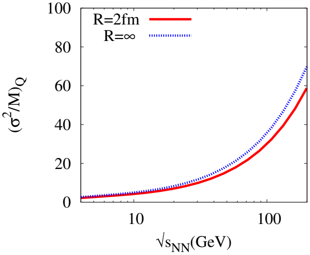
2^{nd} order $B - I$ correlation along μ_B

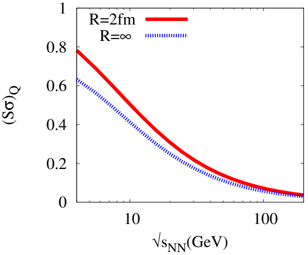


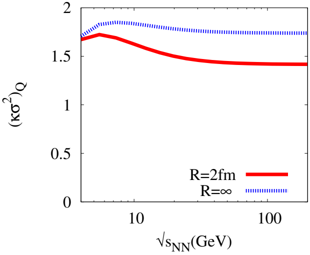
Susceptibilities at finite volume (Hadronic Phase)



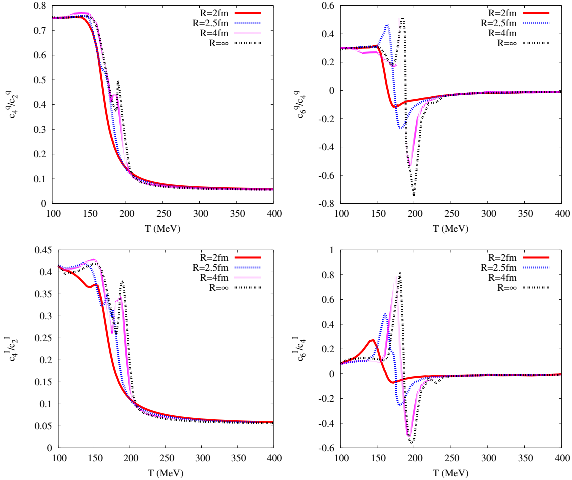
Kurtosis at finite volume







Ratios of susceptibilities



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- Explicit ISB through $m_u \neq m_d$ only. Off-diagonal susceptibilities exhibit non-monotonic behavior and almost linearly scales with $(m_d - m_u)$.
- Susceptibilities have a strong volume dependence.

List of collaborators

- Rajarshi Ray (Bose Institute)
- Sanjay K. Ghosh (Bose Institute)
- Sibaji Raha (Bose Institute)
- Anirban Lahiri (Bose Institute)
- Paramita Deb (Phys. Res. Lab.)
- Supriya Das (Bose Institute)
- Subhasis Samanta (Bose Institute)
- Kinkar Saha (Bose Institute)
- Sudipa Upadhyay (Bose Institute)

Thank You.